Pattern Recognition

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Reference Books

- R.O.Duda, P.E.Hart and D.G.Stork, ‘Pattern Classification’, Johy Wiley, 2002
- C.M.Bishop, ‘Pattern Recognition and Machine Learning’, Springer, 2006.
- C. M. Bishop, ‘Neural Networks for Pattern Recognition’, Oxford University Press, Indian Edition, 2003.
- R.O.Duda and P.E. Hart, ‘Pattern Classification and Scene Analysis’, Wiley, 1973

Pattern Recognition

A basic attribute of people – categorisation of sensory input

Examples of Pattern Recognition tasks

- ‘Reading’ facial expressions
- Recognising Speech
- Reading a Document
- Identifying a person by fingerprints
- Diagnosis from medical images
- Wine tasting

Machine Recognition of Patterns

- Feature extractor makes some measurements on the input pattern.
- X is called *Feature Vector*. Often, $X \in \Re^n$.
- Classifier maps each feature vector to a class label.
- Features to be used are problem-specific.

Some Examples of PR Tasks

- Character Recognition
  - Pattern – Image.
  - Class – identity of character
  - Features: Binary image, projections (e.g., row and column sums), Moments etc.

Some Examples of PR Tasks

- Speech Recognition
  - Pattern – 1-D signal (or its sampled version)
  - Class – identity of speech units
  - Features – LPC model of chunks of speech, spectral info, cepstrum etc.
  - Pattern can become a *sequence* of feature vectors.

Examples contd...

- Fingerprint based identity verification
  - Pattern – image plus a identity claim
  - Class – Yes / No
  - Features: Position of Minutiae, Orientation field of ridge lines etc.

Examples contd...

- Video-based Surveillance
  - Pattern – video sequence
  - Class – e.g., level of alertness
  - Features – Motion trajectories, Parameters of a prefixed model etc.

Examples contd...

- Credit Screening
  - Pattern – Details of an applicant (for, e.g., credit card)
  - Class – Yes / No
  - Features: income, job history, level of credit, credit history etc.

Examples contd...

- Imposter detection (of, e.g., credit card)
  - Pattern – A sequence of transactions
  - Class – Yes / No
  - Features: Amount of money, locations of transactions, times between transactions etc.

Examples contd...

- Document Classification
  - Pattern – A document and a query
  - Class – Relevant or not (in general, rank)
  - Features – word occurrence counts, word context etc.
- Spam filtering, diagnostics of machinery etc.

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- Some techniques available to extract ‘more relevant’ quantities from the initial measurements. (e.g., PCA)
- After feature extraction each pattern is a vector
- Classifier is a function to map such vectors into class labels.
- Many general techniques of classifier design are available.
- Need to test and validate the final system.

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- Often, $\mathcal{X} = \Re^n$. Convenient to take $M = 2$. Then we take the labels as $\{0, 1\}$ or $\{-1, 1\}$.
- Then, any binary valued function on $\mathcal{X}$ is a classifier.
  - What h to choose? We want **correct** or **optimal** classifier.
  - How to design classifiers?
  - How to judge performance?
  - How to provide performance guarantees?

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- The 2-class problem is the basic problem.
- We will also look at M-class classifiesrs.

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- Design of classifier:
  We have to choose a specific form for the classifier.
  What values to use for parameters such as a, b, c?

Designing Classifiers

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- Need to decide how feature vector values determine the class.
  (How different marks reflect goodness of candidate)
- In most applications, not possible to design classifier from ’physics of the problem’.
- The difficulties are
  - Lot of variability in patterns of a single class
  - Variability in feature vector values
  - Feature vectors of patterns from different classes can be arbitrarily close.
  - Noise in measurements

Designing Classifiers contd...

- Often the only information available for the design is – A training set of example patterns.
- Training set: $\{(X_i, y_i), \quad i = 1, \ldots, \ell\}$.
  Here X_i is an example feature vector of class y_i.

Designing Classifiers contd...

- Often the only information available for the design is – A training set of example patterns.
- Training set: $\{(X_i, y_i), \quad i = 1, \ldots, \ell\}$.
  Here X_i is an example feature vector of class y_i.
- Generation of training set – Take representative patterns of known category (data collection) and obtain the feature vectors. (choice of feature measurements).
- Now *learn* an *appropriate* function h as classifier. (Model choice)
- Test and validate the classifier on more data.

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- : Features:
  - x_1: Marks based on academic record
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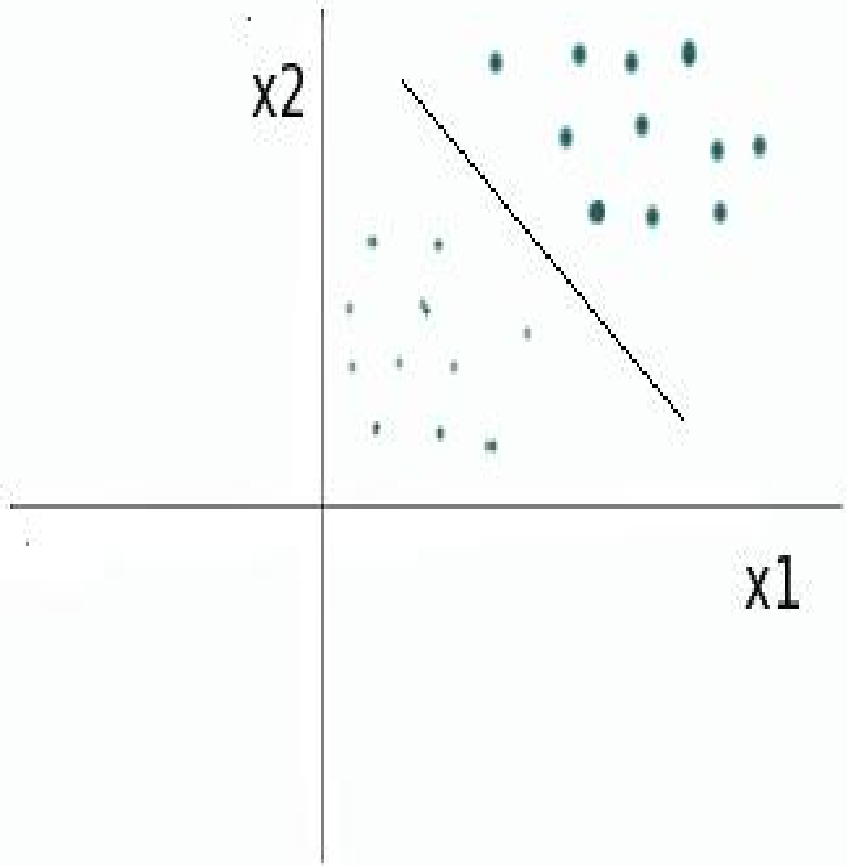
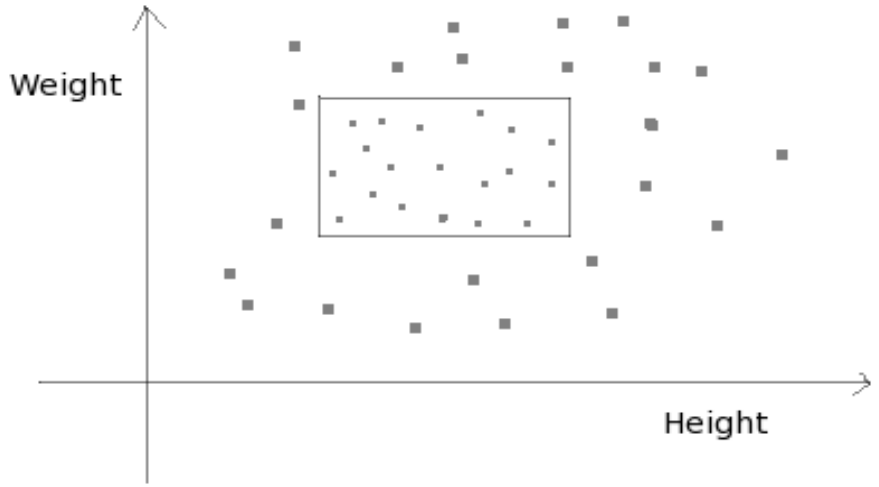
- A Classifier: $ax_1 + bx_2 > c \Rightarrow$ ‘Good’
  We have chosen a specific form for the classifier.
- Design of classifier: What values to use for a, b, c?
- **Information available: ‘experience’ – history of past candidates**

Training Set



Another example problem

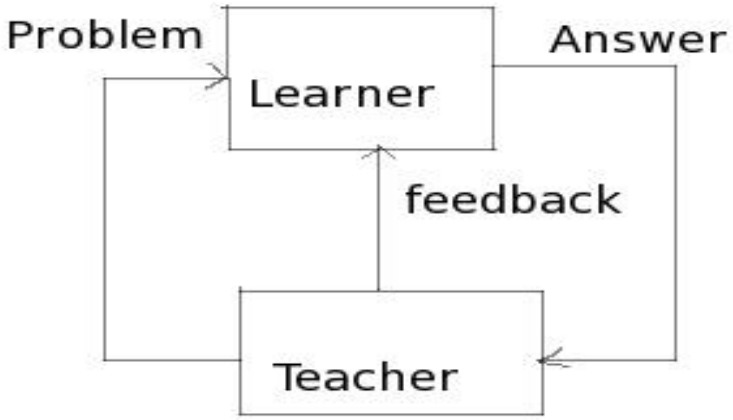
- Problem: recognize persons of ‘medium build’
- Features: Height and Weight



The classifier is ‘nonlinear’ here

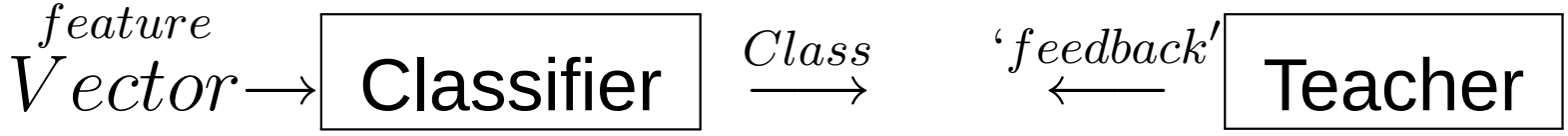
Learning from Examples

- Designing a classifier is a typical problem of learning from examples. (Also called learning with a teacher).



- Nature of feedback from teacher determines difficulty of the learning problem.

In the context of Pattern Recognition



- *Supervised learning* – Teacher gives the true class label for each feature vector
- *Reinforcement Learning* – noisy assessment of performance, e.g., correct/incorrect
- *Unsupervised learning* – no teacher input (Clustering problems)

- When the class labels of training patterns as given by ‘teacher’ are noisy, we consider it as supervised learning with label noise or classification noise.
- Many classifier design algorithms do supervised learning.

Function Learning

- Closely related problem. Output is continuous-valued rather than discrete as in classifiers.
- Here training set examples could be $\{(X_i, y_i), \;\; i = 1, \ldots, \ell\}$, $X_i \in \mathcal{X}$, $y_i \in \Re$.

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- Here training set examples could be $\{(X_i, y_i),\ \ i = 1, \ldots, \ell\}$, $X_i \in \mathcal{X}, y_i \in \Re$.
- The prediction variable, y, is continuous; rather than taking finitely many values. (There can be noise in examples).
- Similar learning techniques needed to infer the underlying functional relationship between X and y. (Regression function of y on X).

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- Time series prediction: Given a series $x_1, x_2, \cdots$, find a function to predict x_n.

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  - Predict stock prices, exchange rates etc.
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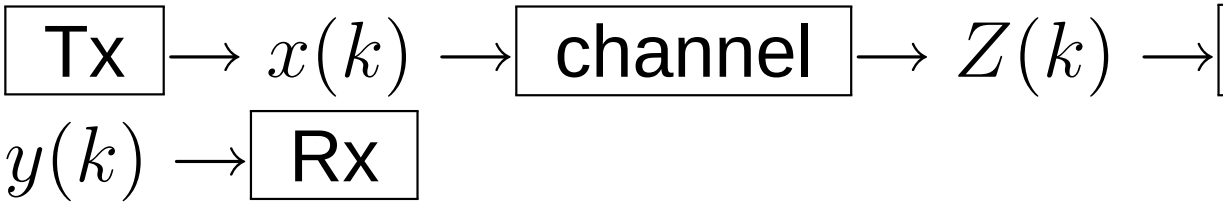
  - Predict stock prices, exchange rates etc.
  - Linear prediction model used in speech analysis
- More general predictors can use other variables also.
  - Predict rainfall based on measurements and (possibly) previous years' data.
  - In general, System Identification. (An application: smart sensors)

Examples contd... : Equaliser



We want $y(k) = x(k)$. Design (or adapt) the filter to achieve this

We can choose a filter as

$$y(k) = \sum_{i=1}^{T} a_i Z(k-i)$$

Find 'best' a_i – a function learning problem.

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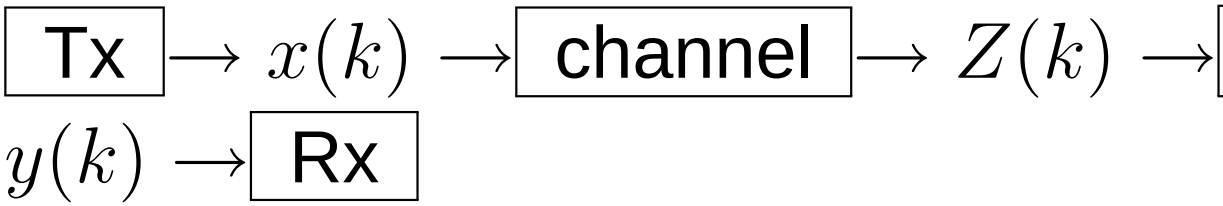

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Training set: $\{(x(k), Z(k)), k = 1, 2, \cdots, N\}$

How do we know $x(k)$ at the receiver end?

Prior agreements (Protocols)

Learning from examples

- Learning from examples is basic to both classification and regression.
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- We essentially want to fit a 'best' function: $y = f(X)$.
- Suppose $X \in \Re$. Then this is a familiar 'curve-fitting' problem.
- Model choice (choice of form for f) is very important here.
- If we choose f to be a polynomial, higher the degree 'better' would be the performance on training data.
- But that is not the real performance – A polynomial of degree ℓ would give zero error!!

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Learning from examples – Generalization

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- We ‘know’ the class label of patterns (or the values for prediction variable) in the training set.
- Errors on the training set do not necessarily tell how good is the classifier.
- Any classifier that amounts to only storing the training set is useless.
- Interested in the ‘generalization abilities’ – how does our classifier perform on unseen or new patterns.

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Design of Classifiers

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- Statistical Pattern Recognition – An approach where the variabilities are captured through probabilistic models.
- There are other approaches, e.g., syntactic pattern recognition, fuzzy-set based methods etc.
- In this course we consider classification and regression (function learning) problems in the statistical framework.

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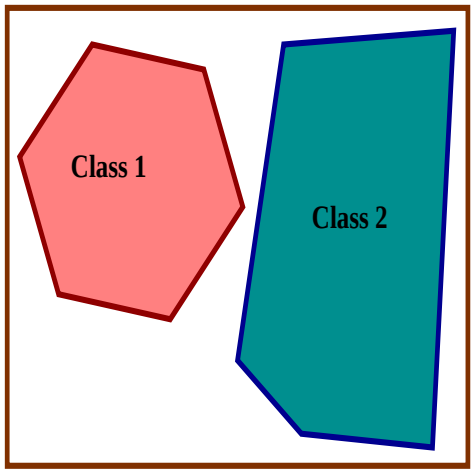
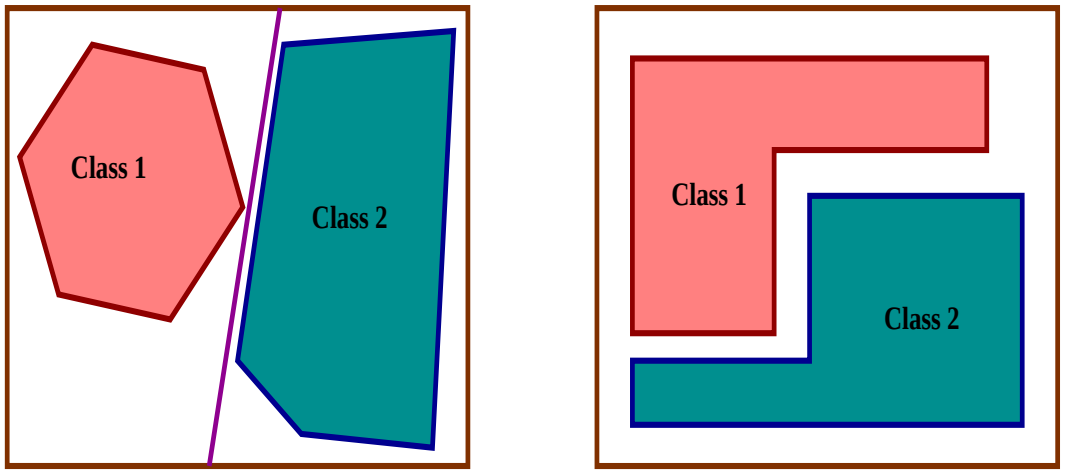
- f_i are called *class conditional densities*.
- Let $\mathbf{X} = (X_1, \cdots, X_n) \in \Re^n$ represent the feature vector.
- Then f_i is the (conditional) joint density of the random variables $X_1, \cdots, X_n$ given that $\mathbf{X}$ is from class-i.

- Class conditional densities model the variability in the feature values.
- For example, the two classes can be uniformly distributed in the two regions as shown. (The two classes are *separable* here).

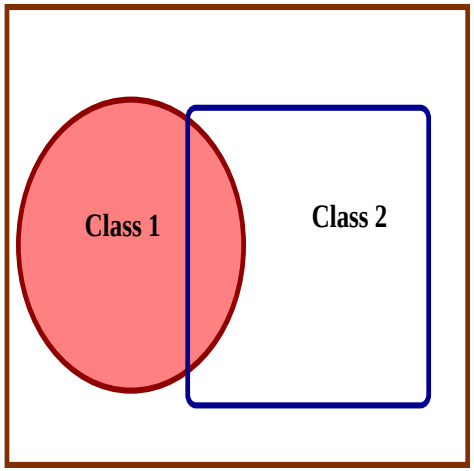


- When class regions are separable, an important special case is linear separability.



- The classes in the left panel above are linearly separable (can be separated by a line) while those in the right panel are not linearly separable (though separable).

- In general, the two class conditional densities can overlap. (The same value of feature vector can be from different classes with different probabilities)



- The statistical viewpoint gives us one way of looking for ‘optimal’ classifier.
- We can say we want a classifier that has least probability of misclassifying a random pattern (drawn from the underlying distributions).

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- $q_0(X) > q_1(X)$ would imply that the feature vector X is more likely to come from class-0 rather than class-1.
- Hence, intuitively, such a classifier should minimize probability of error in classification.

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- For a feature vector X, let $y(X)$ denote the class label of X. In general, $y(X)$ would be random.
- Now we want to assign a figure of merit to each possible classifier in $\mathcal{H}$.

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- Here we are treating all errors as same. We will

Statistical PR contd.

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Statistical PR contd.

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- Let $p_i = \text{Prob}[y(X) = i]$. Called *prior probabilities*.
- Recall, *posterior probabilities*, $q_i(X) = \text{Prob}[y(X) = i \mid X]$. Now, by Bayes theorem

$$q_i(X) = f_i(X)p_i \ / \ Z$$

where $Z = f_0(X)p_0 + f_1(X)p_1$ is the normalising constant

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- We will prove optimality of Bayes classifier later.

story so far

- We consider PR as a two step process – Feature measurement/extraction and Classification
- A classifier is to map feature vectors to Class labels.
- The main difficulty in designing classifiers is due to large variability in feature vector values.
- The main information we have for the design is a training set of examples.
- Function learning is a closely related problem.
- In both cases we need to learn from (training) examples.

story so far

- The statistical view is: model the variation in feature vectors from a given class through probability models.
- One objective can be: Find a classifier that has least probability of error.
- For example, Bayes Classifier is the optimal one if we know class conditional densities.

Organization of the course

- Overview of classifier learning strategies
- Nearest Neighbour classification rule
- Bayes Classifier for minimizing risk.
- Some variations on this theme (e.g. Neymann-Pearson Classifier)
- Techniques for estimating class conditional densities to implement Bayes classifier.

Organization of the course

- Linear discriminant functions
- Learning linear discriminant functions (Perceptron, LMS, logistic regression)
- Linear least-squares regression
- Simple overview of statistical learning theory
- Empirical risk minimization and VC theory

Organization of the course

- Learning nonlinear classifiers
- Neural Networks (Backpropagation, RBF networks)
- Support Vector Machines
- Kernel-based methods
- Feature extraction and dimensionality reduction (PCA)
- Boosting and ensemble classifiers (AdaBoost)

Thank You!