Unit -3
Structures of FIR and IIR systems

STRUCTURES FOR THE REALIZATION OF DISCRETE-TIME SYSTEMS

The major factors that influence our choice o f a specific realization are computational complexity, memory requirements, and finite-word-length effects in the computations.

STRUCTURES FOR FIR SYSTEMS

In general, an FIR system is described by the difference equation

$$y(n) = \sum_{k=0}^{M-1} b_k x(n-k) \tag{7.2.1}$$

or, equivalently, by the system function

$$H(z) = \sum_{k=0}^{M-1} b_k z^{-k} \tag{7.2.2}$$

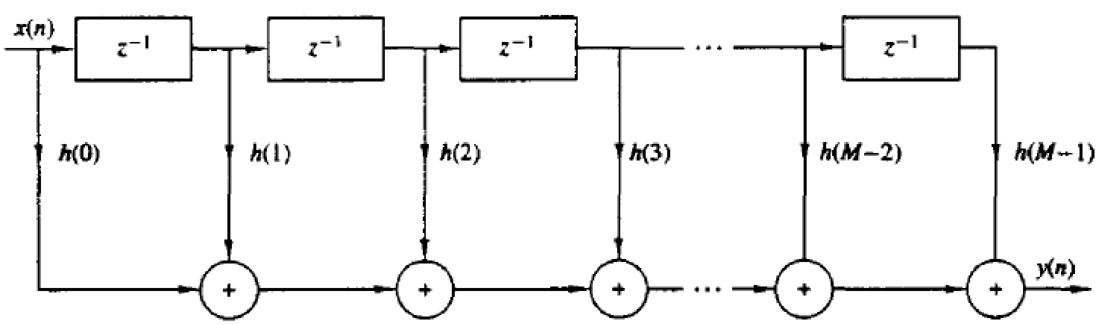
Furthermore, the unit sample response of the FIR system is identical to the coefficients $\{b_k\}$, that is,

$$h(n) = \begin{cases} b_n, & 0 \le n \le M-1 \\ 0, & \text{otherwise} \end{cases}$$

Direct-Form Structure

The direct form realization follows immediately from the non recursive difference equation given below

$$y(n) = \sum_{k=0}^{M-1} h(k)x(n-k)$$



Cascade-Form Structures

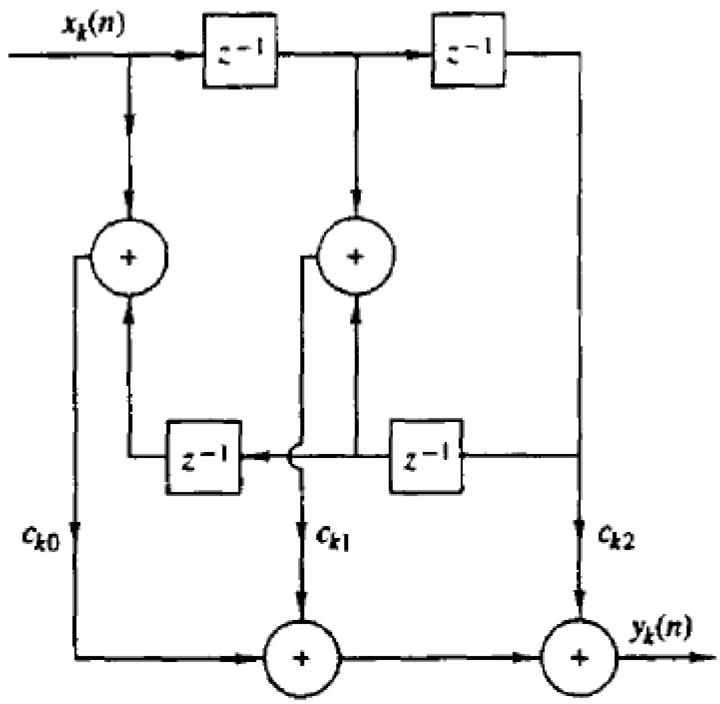
The cascaded realization follows naturally system function given by equation. It is simple matter to factor H(z) into second order FIR system so that

$$H(z) = \prod_{k=1}^{K} H_k(z)$$

where

$$H_k(z) = b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2} \qquad k = 1, 2, \ldots, K$$

$$\begin{aligned} H_k(z) &= c_{k0}(1 - z_k z^{-1})(1 - z_k^* z^{-1})(1 - z^{-1}/z_k)(1 - z^{-1}/z_k^*) \\ &= c_{k0} + c_{k1}z^{-1} + c_{k2}z^{-2} + c_{k1}z^{-3} + z^{-4} \end{aligned}$$



Frequency-Sampling Structures

The frequency-sampling realization is an alternative structure for an FIR filter in which the parameters that characterize the filter are the values o f the desired frequency response instead of the impulse response *h(n).* To derive the frequency sampling structure, we specify the desired frequency response at a set o f equally spaced frequencies, namely

$$\omega_k = \frac{2\pi}{M}(k+\alpha) \qquad k = 0, 1, \ldots, \frac{M-1}{2} \quad M \text{ odd}$$

$$k = 0, 1, \ldots, \frac{M}{2} - 1 \quad M \text{ even}$$

$$\alpha = 0 \text{ or } \tfrac{1}{2}$$

The frequency response of the system is given by

$$H(\omega) = \sum_{n=0}^{M-1} h(n) e^{-j\omega n}$$

$$H(k+\alpha) = H\left(\frac{2\pi}{M}(k+\alpha)\right)$$

$$= \sum_{n=0}^{M-1} h(n) e^{-j2\pi(k+\alpha)n/M} \qquad k = 0, 1, \ldots, M-1$$

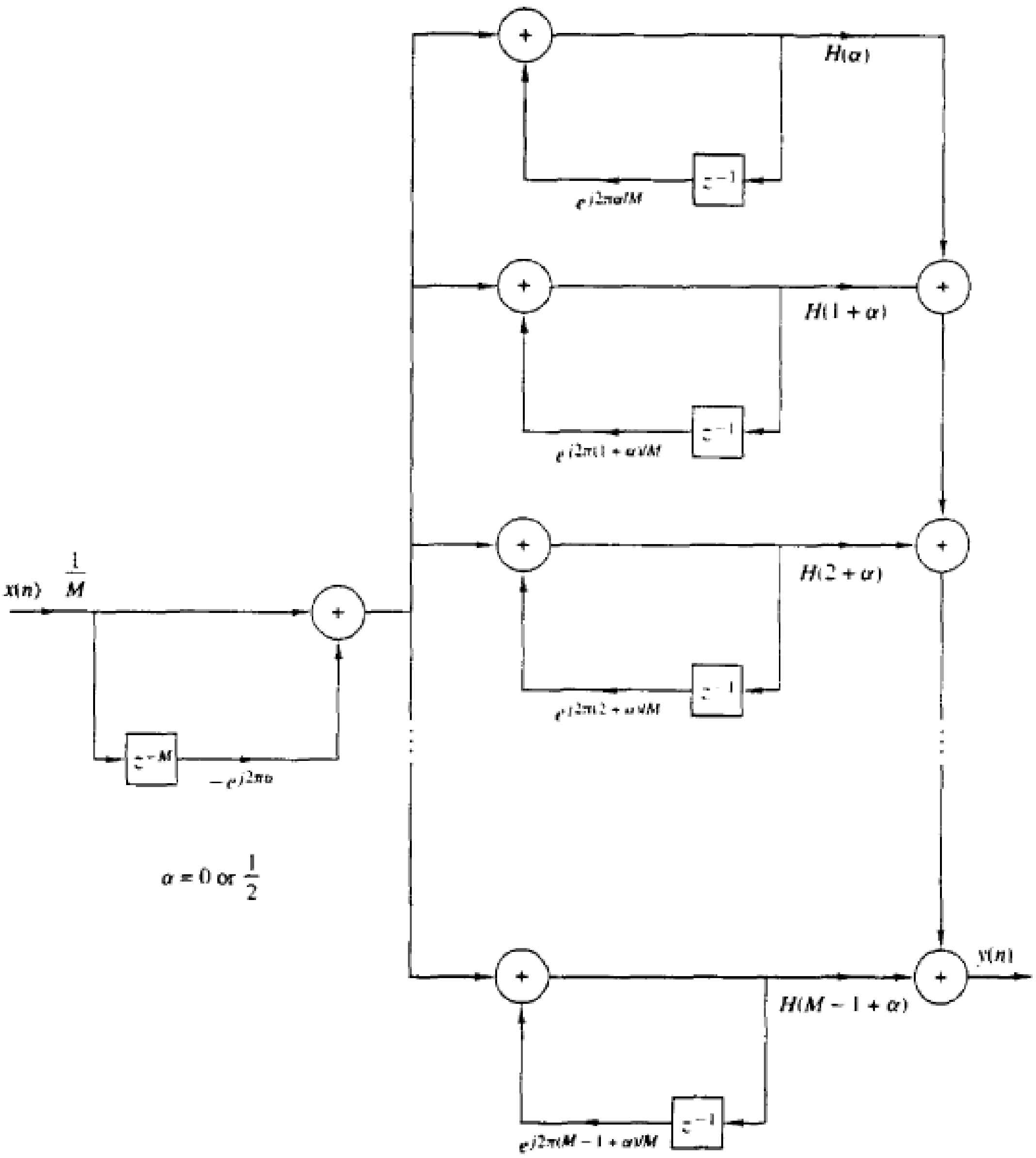
$$h(n) = \frac{1}{M} \sum_{k=0}^{M-1} H(k+\alpha) e^{j2\pi(k+\alpha)n/M} \qquad n = 0, 1, \ldots, M-1$$

$$H(z) = \sum_{n=0}^{M-1} h(n) z^{-n}$$

$$= \sum_{n=0}^{M-1} \left[\frac{1}{M} \sum_{k=0}^{M-1} H(k+\alpha) e^{j2\pi(k+\alpha)n/M}\right] z^{-n}$$

$$H(z) = \sum_{k=0}^{M-1} H(k+\alpha) \left[\frac{1}{M} \sum_{n=0}^{M-1} (e^{j2\pi(k+\alpha)/M} z^{-1})^n\right]$$

$$= \frac{1 - z^{-M} e^{j2\pi\alpha}}{M} \sum_{k=0}^{M-1} \frac{H(k+\alpha)}{1 - e^{j2\pi(k+\alpha)/M} z^{-1}}$$



Lattice Structure

In this section w e introduce another F IR filter structure, called the lattice filter or Lattice realization. Lattice filters are used extensively in digital speech processing And in the implementation of adaptive filters. Let us begin the development by considering a sequence of FIR filters with system functions

$$H_m(z) = A_m(z) \qquad m = 0, 1, 2, \ldots, M-1 \tag{7.2.17}$$

where, by definition, $A_m(z)$ is the polynomial

$$A_m(z) = 1 + \sum_{k=1}^{m} \alpha_m(k) z^{-k} \qquad m \geq 1 \tag{7.2.18}$$

and $A_0(z) = 1$. The unit sample response of the mth filter is $h_m(0) = 1$ and $h_m(k) = \alpha_m(k)$, $k = 1, 2, \ldots, m$. The subscript m on the polynomial $A_m(z)$ denotes the degree of the polynomial. For mathematical convenience, we define $\alpha_m(0) = 1$.

If $\{x(n)\}$ is the input sequence to the filter $A_m(z)$ and $\{y(n)\}$ is the output sequence, we have

$$y(n) = x(n) + \sum_{k=1}^{m} \alpha_m(k) x(n-k) \tag{7.2.19}$$

Next, let us consider an FIR filter for which $m = 2$. In this case the output from a direct-form structure is

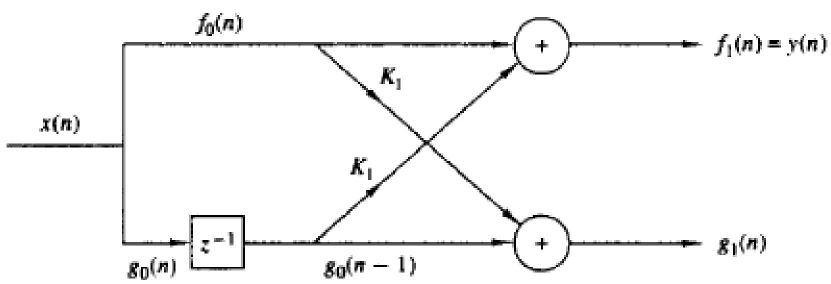
$$y(n) = x(n) + \alpha_2(1) x(n-1) + \alpha_2(2) x(n-2) \tag{7.2.22}$$

By cascading two lattice stages as shown in Fig. 7.10, it is possible to obtain the same output as (7.2.22). Indeed, the output from the first stage is

$$\begin{aligned} f_1(n) &= x(n) + K_1 x(n-1) \\ g_1(n) &= K_1 x(n) + x(n-1) \end{aligned} \tag{7.2.23}$$

The output from the second stage is

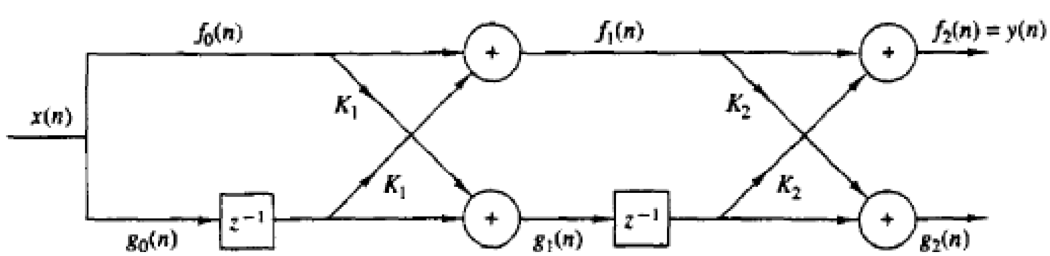
$$\begin{aligned} f_2(n) &= f_1(n) + K_2 g_1(n-1) \\ g_2(n) &= K_2 f_1(n) + g_1(n-1) \end{aligned} \tag{7.2.24}$$



$f_0(n) = g_0(n) = x(n)$

$f_1(n) = f_0(n) + K_1 g_0(n-1) = x(n) + K_1 x(n-1)$

$g_1(n) = K_1 f_0(n) + g_0(n-1) = K_1 x(n) + x(n-1)$



$$f_2(n) = x(n) + K_1 x(n-1) + K_2[K_1 x(n-1) + x(n-2)]$$
$$= x(n) + K_1(1+K_2)x(n-1) + K_2 x(n-2)$$

The general form of lattice structure for m stage is given by'

$$f_0(n) = g_0(n) = x(n)$$
$$f_m(n) = f_{m-1}(n) + K_m g_{m-1}(n-1) \qquad m = 1, 2, \ldots, M-1$$
$$g_m(n) = K_m f_{m-1}(n) + g_{m-1}(n-1) \qquad m = 1, 2, \ldots, M-1$$

Conversion of lattice coefficients to direct-form filter coefficients. The direct-form FIR filter coefficients $\{\alpha_m(k)\}$ can be obtained from the lattice coefficients $\{K_i\}$ by using the following relations:

$$A_0(z) = B_0(z) = 1 \tag{7.2.47}$$
$$A_m(z) = A_{m-1}(z) + K_m z^{-1} B_{m-1}(z) \qquad m = 1, 2, \ldots, M-1 \tag{7.2.48}$$
$$B_m(z) = z^{-m} A_m(z^{-1}) \qquad m = 1, 2, \ldots, M-1 \tag{7.2.49}$$

Conversion of direct-form FIR filter coefficients to lattice coefficients. Suppose that we are given the FIR coefficients for the direct-form realization or, equivalently, the polynomial $A_m(z)$, and we wish to determine the corresponding lattice filter parameters $\{K_i\}$. For the m-stage lattice we immediately obtain the parameter $K_m = \alpha_m(m)$. To obtain K_{m-1} we need the polynomials $A_{m-1}(z)$ since, in general, K_m is obtained from the polynomial $A_m(z)$ for $m = M-1, M-2, \ldots, 1$. Consequently, we need to compute the polynomials $A_m(z)$ starting from $m = M-1$ and "stepping down" successively to $m = 1$.

$$K_m = \alpha_m(m) \quad \alpha_{m-1}(0) = 1$$
$$\alpha_{m-1}(k) = \frac{\alpha_m(k) - K_m \beta_m(k)}{1 - K_m^2}$$
$$= \frac{\alpha_m(k) - \alpha_m(m)\alpha_m(m-k)}{1 - \alpha_m^2(m)} \qquad 1 \le k \le m-1$$

STRUCTURES FOR IIR SYSTEMS

In this section we consider different IIR system s structures described by the difference equation given by the system function. Just as in the case o f FIR system s, there are several types o f structures or realizations, including direct-form structures, cascade-form structures, lattice structures, and lattice-ladder structures. In addition, IIR systems lend them selves to a parallel form realization. We begin by describing two direct-form realizations.

DIRECT FORM STRUCTURES:

The rational system function as given by (7.1.2) that characterizes an IIR system can be viewed as two systems in cascade, that is,

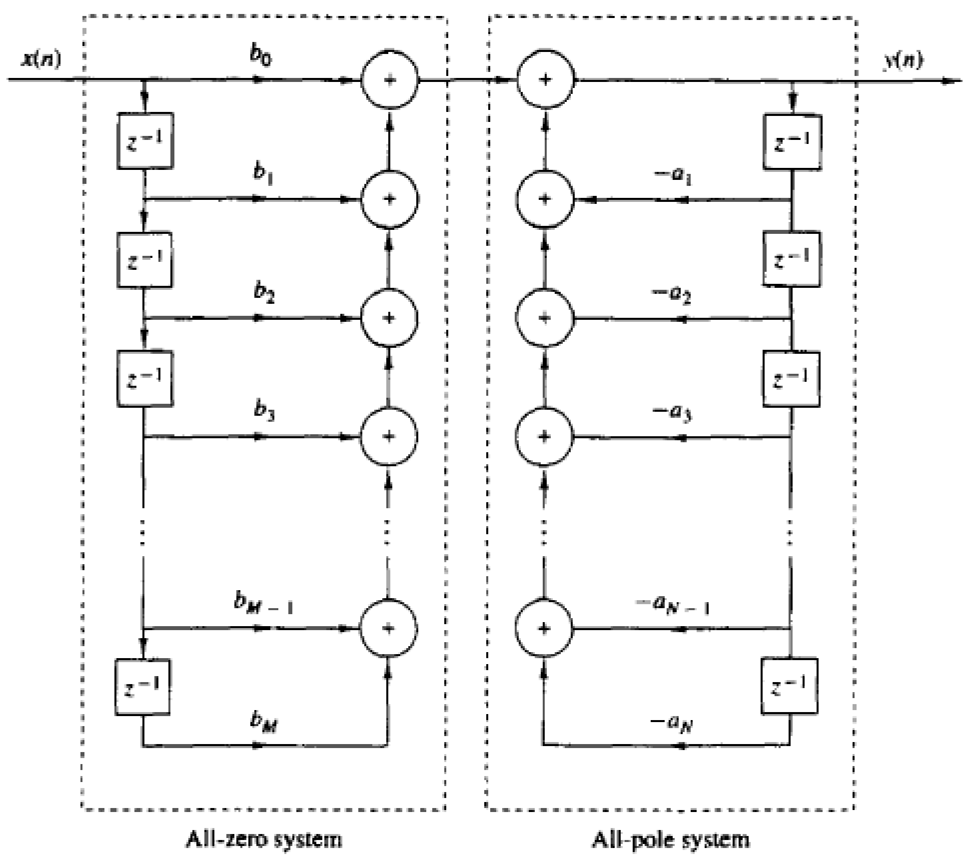
$$H(z) = H_1(z)H_2(z) \tag{7.3.1}$$

where $H_1(z)$ consists of the zeros of $H(z)$, and $H_2(z)$ consists of the poles of $H(z)$,

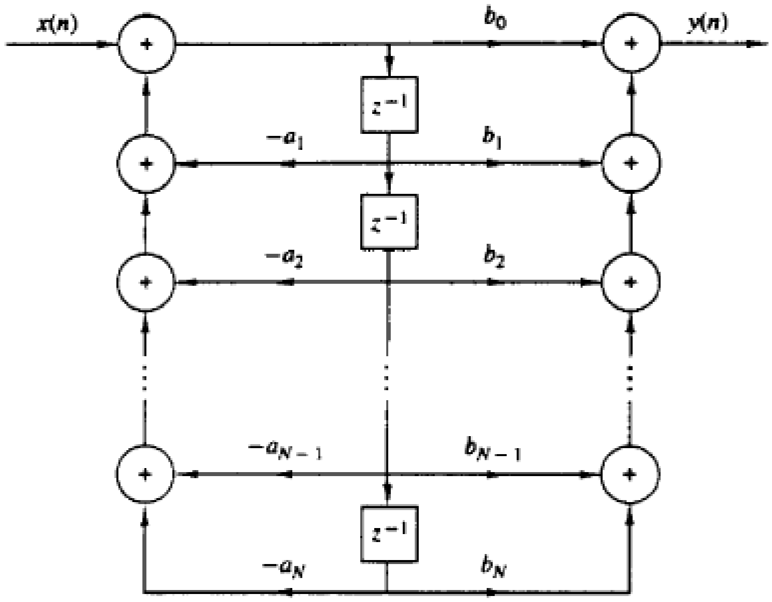
$$H_1(z) = \sum_{k=0}^{M} b_k z^{-k} \tag{7.3.2}$$

and

$$H_2(z) = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}} \tag{7.3.3}$$

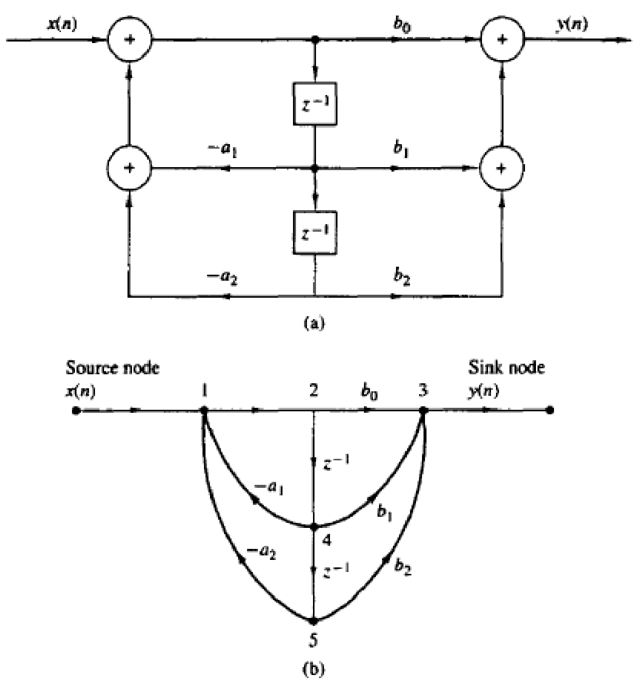


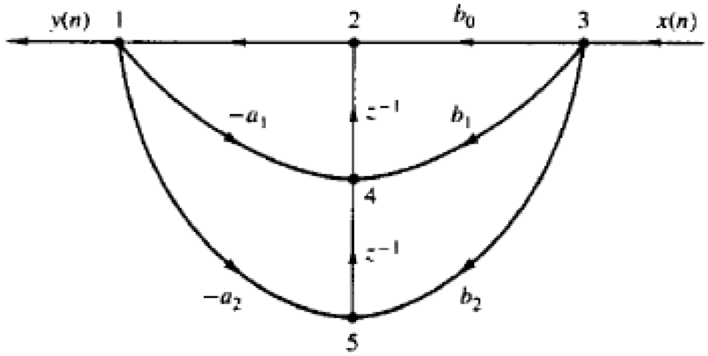
DIRECT FORM II



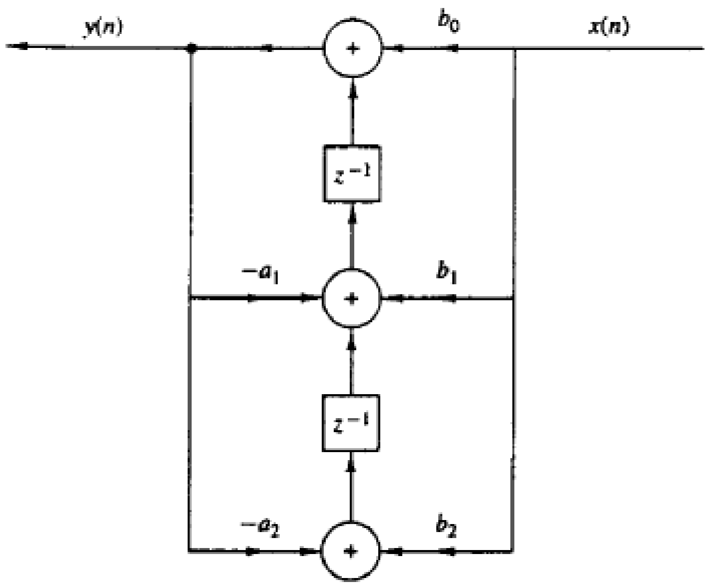
Signal Flow Graphs and Transposed Structures

A signal flow graph provides an alternative, N but equivalent, graphical representation to a block diagram structure that we have been using to illustrate various system realizations. T he basic elements o f a flow graph are branches and nodes. A signal flow graph is basically a set o f directed branches that connect at nodes. By definition, the signal out of a branch is equal to the branch gain (system function) times the signal into the branch. Furthermore, the signal at anode o f a flow graph is equal to the sum o f the signals from all branches connecting to the node.





(a)



(b)

Cascade-Form Structures

Let us consider a high-order IIR system with system function given by equation. Without loss o f generality we assume that $N > M$. T h e system can be factored into a cascade o f second-order subsystem s, such that $H(z)$ can b e expressed as

$$H(z) = \prod_{k=1}^{K} H_k(z)$$

where K is the integer part of $(N+1)/2$. $H_k(z)$ has the general form

$$H_k(z) = \frac{b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2}}{1 + a_{k1}z^{-1} + a_{k2}z^{-2}}$$

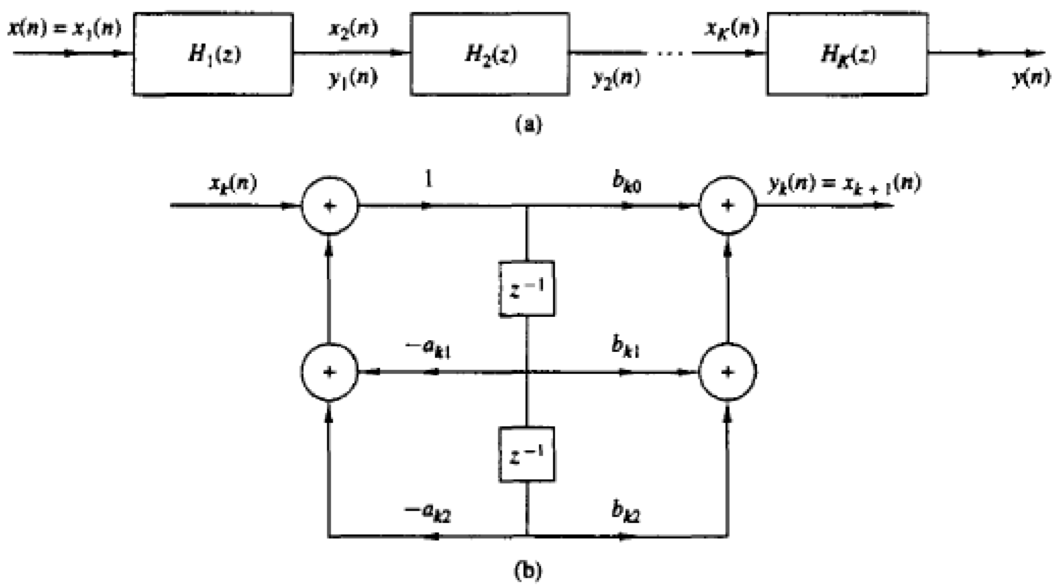
The general form of the cascade structure is illustrated in Fig. 7.19. If we use the direct form II structure for each of the subsystems, the computational algorithm for realizing the IIR system with system function $H(z)$ is described by the following set of equations.

$$y_0(n) = x(n) \tag{7.3.16}$$

$$w_k(n) = -a_{k1}w_k(n-1) - a_{k2}w_k(n-2) + y_{k-1}(n) \qquad k = 1, 2, \ldots, K \tag{7.3.17}$$

$$y_k(n) = b_{k0}w_k(n) + b_{k1}w_k(n-1) + b_{k2}w_k(n-2) \qquad k = 1, 2, \ldots, K \tag{7.3.18}$$

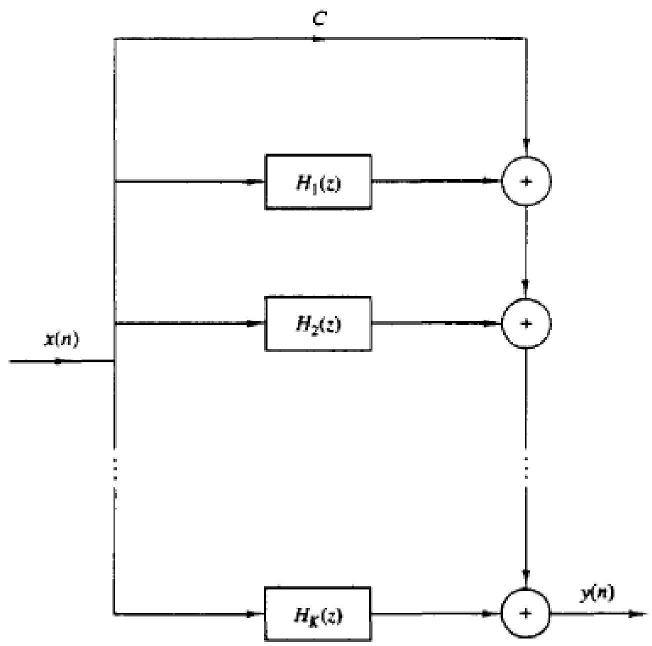
$$y(n) = y_K(n) \tag{7.3.19}$$



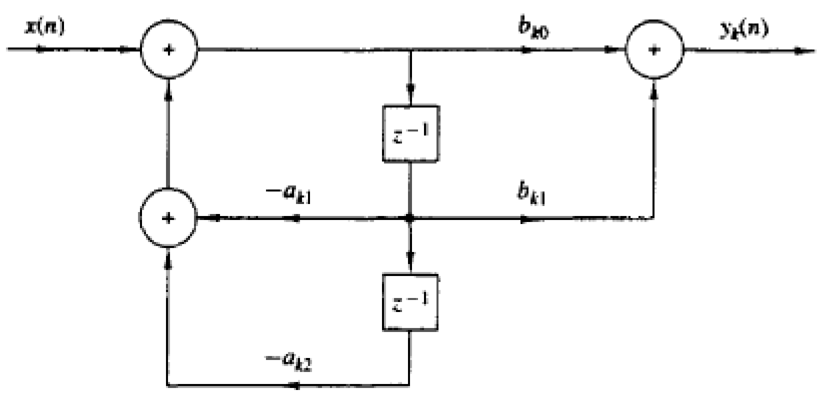
Parallel-Form Structures

A parallel-form realization o f an IIR system can be obtained by performing a partial-fraction expansion o f *H(z)* . Without loss o f generality, w e again assume that $N > M$ and that the poles are distinct. Then, by performing a partial-fraction expansion o f *H(z)*, we obtain the result

$$H(z) = C + \sum_{k=1}^{N} \frac{A_k}{1 - p_k z^{-1}}$$



The realization of second order form is given by



The general form of parallel form of structure is f\given by

$$w_k(n) = -a_{k1}w_k(n-1) - a_{k2}w_k(n-2) + x(n) \qquad k = 1, 2, \ldots, K$$

$$y_k(n) = b_{k0}w_k(n) + b_{k1}w_k(n-1) \qquad k = 1, 2, \ldots, K$$

$$y(n) = Cx(n) + \sum_{k=1}^{K} y_k(n)$$

Lattice and Lattice-Ladder Structures for IIR Systems

In Section 7.2.4 we developed a lattice filter structure that is equivalent to an FIR system. In this section we extend the development to IIR systems.

Let us begin with an all-pole system with system function

$$H(z) = \frac{1}{1 + \sum_{k=1}^{N} a_N(k) z^{-k}} = \frac{1}{A_N(z)} \tag{7.3.26}$$

The direct form realization of this system is illustrated in Fig. 7.23. The difference equation for this IIR system is

$$y(n) = -\sum_{k=1}^{N} a_N(k) y(n-k) + x(n) \tag{7.3.27}$$

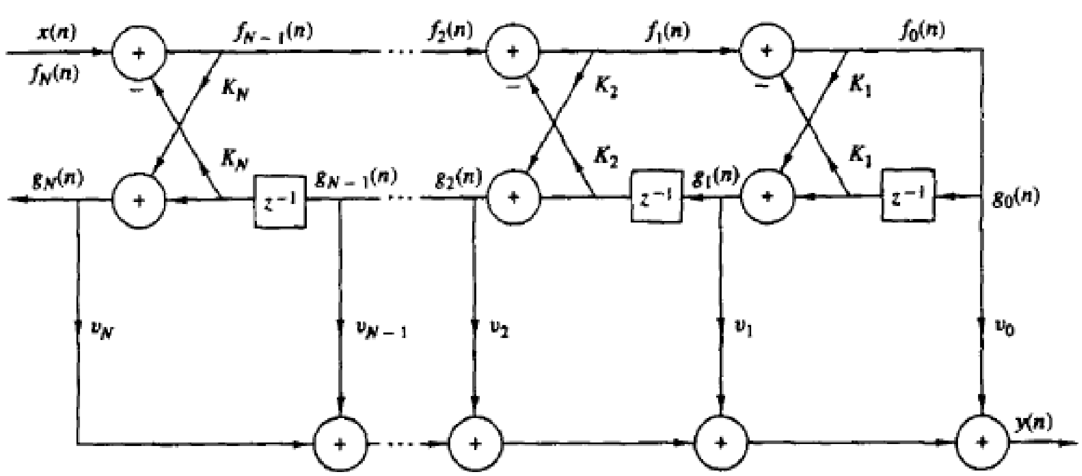
It is interesting to note that if we interchange the roles of input and output [i.e., interchange $x(n)$ with $y(n)$ in (7.3.27)], we obtain

$$x(n) = -\sum_{k=1}^{N} a_N(k) x(n-k) + y(n)$$

or, equivalently,

$$y(n) = x(n) + \sum_{k=1}^{N} a_N(k) x(n-k) \tag{7.3.28}$$

We note that the equation in (7.3.28) describes an FIR system having the system function $H(z) = A_N(z)$, while the system described by the difference equation in (7.3.27) represents an IIR system with system function $H(z) = 1/A_N(z)$.



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