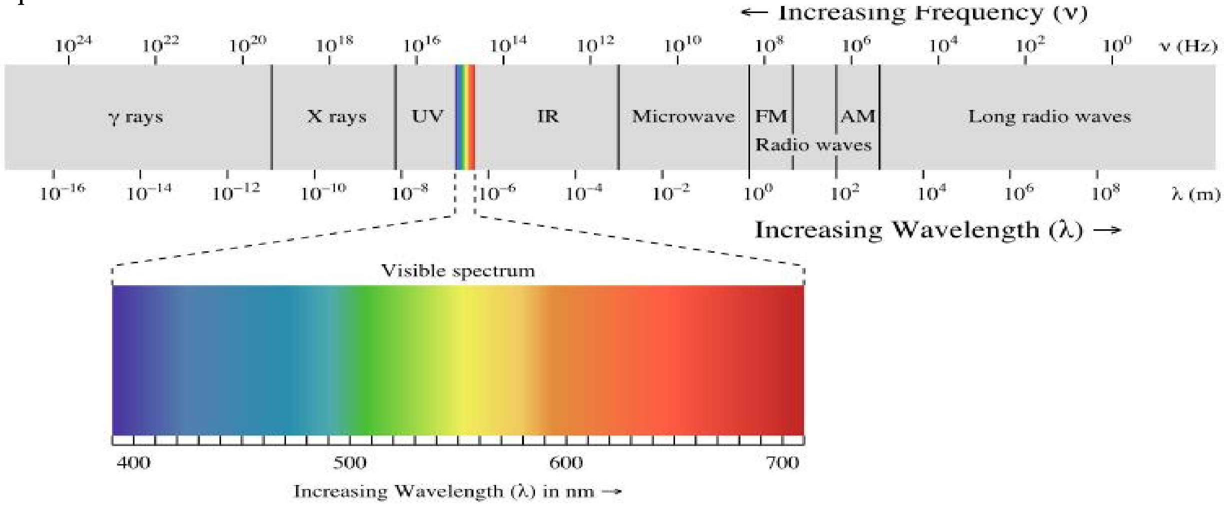
UNIT I
WAVEGUIDES & RESONATORS

INTRODUCTION

Microwaves are electromagnetic waves with wavelengths ranging from 1 mm to 1 m, or frequencies between 300 MHz and 300 GHz.



Apparatus and techniques may be described qualitatively as "microwave" when the wavelengths of signals are roughly the same as the dimensions of the equipment, so that lumped-element circuit theory is inaccurate. As a consequence, practical microwave technique tends to move away from the discrete resistors, capacitors, and inductors used with lower frequency radio waves. Instead, distributed circuit elements and transmission-line theory are more useful methods for design, analysis. Open-wire and coaxial transmission lines give way to waveguides, and lumped-element tuned circuits are replaced by cavity resonators or resonant lines. Effects of reflection, polarization, scattering, diffraction, and atmospheric absorption usually associated with visible light are of practical significance in the study of microwave propagation. The same equations of electromagnetic theory apply at all frequencies.

While the name may suggest a micrometer wavelength, it is better understood as indicating wavelengths very much smaller than those used in radio broadcasting. The boundaries between far infrared light, terahertz radiation, microwaves, and ultra-high-frequency radio waves are fairly arbitrary and are used variously between different fields of study. The term microwave generally refers to "alternating current signals with frequencies between 300 MHz (3×108 Hz) and 300 GHz (3×1011 Hz)."[1] Both IEC standard 60050 and IEEE standard 100 define "microwave" frequencies starting at 1 GHz (30 cm wavelength).

Electromagnetic waves longer (lower frequency) than microwaves are called "radio waves". Electromagnetic radiation with shorter wavelengths may be called "millimeter waves", terahertz radiation or even *T-rays*. Definitions differ for millimeter wave band, which the IEEE defines as 110 GHz to 300 GHz.

MICROWAVE FREQUENCY BANDS

The microwave spectrum is usually defined as electromagnetic energy ranging from approximately 1 GHz to 1000 GHz in frequency, but older usage includes lower frequencies. Most common applications are within the 1 to 40 GHz range. Microwave frequency bands, as defined by the Radio Society of Great Britain (RSGB), are shown in the table below:

Microwave frequency bands

Designation	Frequency range
L band	1 to 2 GHz
S band	2 to 4 GHz
C band	4 to 8 GHz
X band	8 to 12 GHz
Ku band	12 to 18 GHz
K band	18to 26.5GHz
Ka band	26.5to 40GHz

Discovery

The existence of electromagnetic waves, of which microwaves are part of the frequency spectrum, was predicted by James Clerk Maxwell in 1864 from his equations. In 1888, Heinrich Hertz was the first to demonstrate the existence of electromagnetic waves by building an apparatus that produced and detected microwaves in the UHF region. The design necessarily used horse-and-buggy materials, including a horse trough, a wrought iron point spark, Leyden jars, and a length of zinc gutter whose parabolic cross-section worked as a reflection antenna. In 1894 J. C. Bose publicly demonstrated radio control of a bell using millimetre wavelengths, and conducted research into the propagation of microwaves.

Plot of the zenith atmospheric transmission on the summit of Mauna Kea throughout the entire gigahertz range of the electromagnetic spectrum at a precipitable water vapor level of 0.001 mm. (simulated)

Frequency range

The microwave range includes ultra-high frequency (UHF) (0.3–3 GHz), super high frequency (SHF) (3–30 GHz), and extremely high frequency (EHF) (30–300 GHz) signals.

Above 300 GHz, the absorption of electromagnetic radiation by Earth's atmosphere is so great that it is effectively opaque, until the atmosphere becomes transparent again in the so-called infrared and optical window frequency ranges.

Microwave Sources

Vacuum tube based devices operate on the ballistic motion of electrons in a vacuum under the influence of controlling electric or magnetic fields, and include the magnetron, klystron, travelling wave tube (TWT), and gyrotron. These devices work in the density modulated mode, rather than the current modulated mode. This means that they work on the basis of clumps of electrons flying ballistically through them, rather than using a continuous stream.

A maser is a device similar to a laser, except that it works at microwave frequencies.

Solid-state sources include the field-effect transistor, at least at lower frequencies, tunnel diodes and Gunn diodes

ADVANTAGES OF MICROWAVES

Communication

- Before the advent of fiber optic transmission, most long distance telephone calls were carried via microwave point-to-point links through sites like the AT&T Long Lines. Starting in the early 1950's, frequency division multiplex was used to send up to 5,400 telephone channels on each microwave radio channel, with as many as ten radio channels combined into one antenna for the *hop* to the next site, up to 70 km away.
- Wireless LAN protocols, such as Bluetooth and the IEEE 802.11 specifications, also use microwaves in the 2.4 GHz ISM band, although 802.11a uses ISM band and U-NII frequencies in the 5 GHz range. Licensed long-range (up to about 25 km) Wireless Internet Access services can be found in many countries (but not the USA) in the 3.5–4.0 GHz range.
- Metropolitan Area Networks: MAN protocols, such as WiMAX (Worldwide Interoperability for Microwave Access) based in the IEEE 802.16 specification. The IEEE 802.16 specification was designed to operate between 2 to 11 GHz. The commercial implementations are in the 2.3GHz, 2.5 GHz, 3.5 GHz and 5.8 GHz ranges.
- Wide Area Mobile Broadband Wireless Access: MBWA protocols based on standards specifications such as IEEE 802.20 or ATIS/ANSI HC-SDMA (e.g. iBurst) are designed to operate between 1.6 and 2.3 GHz to give mobility and in-building penetration characteristics similar to mobile phones but with vastly greater spectral efficiency.
- Cable TV and Internet access on coaxial cable as well as broadcast television use some of the lower microwave frequencies. Some mobile phone networks, like GSM, also use the lower microwave frequencies.
- Microwave radio is used in broadcasting and telecommunication transmissions because, due to their short wavelength, highly directive antennas are smaller and therefore more practical than they would be at longer wavelengths (lower frequencies). There is also

more bandwidth in the microwave spectrum than in the rest of the radio spectrum; the usable bandwidth below 300 MHz is less than 300 MHz while many GHz can be used above 300 MHz. Typically, microwaves are used in television news to transmit a signal from a remote location to a television station from a specially equipped van.

Remote Sensing

- Radar uses microwave radiation to detect the range, speed, and other characteristics of remote objects. Development of radar was accelerated during World War II due to its great military utility. Now radar is widely used for applications such as air traffic control, navigation of ships, and speed limit enforcement.
- A Gunn diode oscillator and waveguide are used as a motion detector for automatic door openers (although these are being replaced by ultrasonic devices).
- Most radio astronomy uses microwaves.
- Microwave imaging; *see Photoacoustic imaging in biomedicine*

Navigation

Global Navigation Satellite Systems (GNSS) including the American Global Positioning System (GPS) and the Russian (GLONASS) broadcast navigational signals in various bands between about 1.2 GHz and 1.6 GHz.

Power

- A microwave oven passes (non-ionizing) microwave radiation (at a frequency near 2.45 GHz) through food, causing dielectric heating by absorption of energy in the water, fats and sugar contained in the food. Microwave ovens became common kitchen appliances in Western countries in the late 1970s, following development of inexpensive cavity magnetrons.
- Microwave heating is used in industrial processes for drying and curing products.
- Many semiconductor processing techniques use microwaves to generate plasma for such purposes as reactive ion etching and plasma-enhanced chemical vapor deposition (PECVD).
- Microwaves can be used to transmit power over long distances, and post-World War II research was done to examine possibilities. NASA worked in the 1970s and early 1980s to research the possibilities of using Solar power satellite (SPS) systems with large solar arrays that would beam power down to the Earth's surface via microwaves.
- Less-than-lethal weaponry exists that uses millimeter waves to heat a thin layer of human skin to an intolerable temperature so as to make the targeted person move away. A two-second burst of the 95 GHz focused beam heats the skin to a temperature of 130 F (54 C)

at a depth of 1/64th of an inch (0.4 mm). The United States Air Force and Marines are currently using this type of Active Denial System.[2]

APPLICATIONS OF MICROWAVE ENGINEERING

- Antenna gain is proportional to the electrical size of the antenna. At higher frequencies, more antenna gain is therefore possible for a given physical antenna size, which has important consequences for implementing miniaturized microwave systems.
- More bandwidth can be realized at higher frequencies. Bandwidth is critically important because available frequency bands in the electromagnetic spectrum are being rapidly depleted.
- Microwave signals travel by line of sight are not bent by the ionosphere as are lower frequency signals and thus satellite and terrestrial communication links with very high capacities are possible.
- Effective reflection area (radar cross section) of a radar target is proportional to the target’s electrical size. Thus generally microwave frequencies are preferred for radar systems.
- Various molecular, atomic, and nuclear resonances occur at microwave frequencies, creating a variety of unique applications in the areas of basic science, remote sensing, medical diagnostics and treatment, and heating methods.
- Today, the majority of applications of microwaves are related to radar and communication systems. Radar systems are used for detecting and locating targets and for air traffic control systems, missile tracking radars, automobile collision avoidance systems, weather prediction, motion detectors, and a wide variety of remote sensing systems.
- Microwave communication systems handle a large fraction of the world’s international and other long haul telephone, data and television transmissions.
- Most of the currently developing wireless telecommunications systems, such as direct broadcast satellite (DBS) television, personal communication systems (PCSs), wireless local area networks (WLANS), cellular video (CV) systems, and global positioning satellite (GPS) systems rely heavily on microwave technology.

WAVEGUIDE :

The transmission line can’t propagate high range of frequencies in GHz due to skin effect. Waveguides are generally used to propagate microwave signal and they always operate beyond certain frequency that is called “*cut off frequency”*. so they behaves as high pass filter.

Types of waveguides: -

(1)rectangular waveguide
(2)cylindrical waveguide
(3)elliptical waveguide
(4)parallel waveguide

RECTANGULAR WAVEGUIDE :

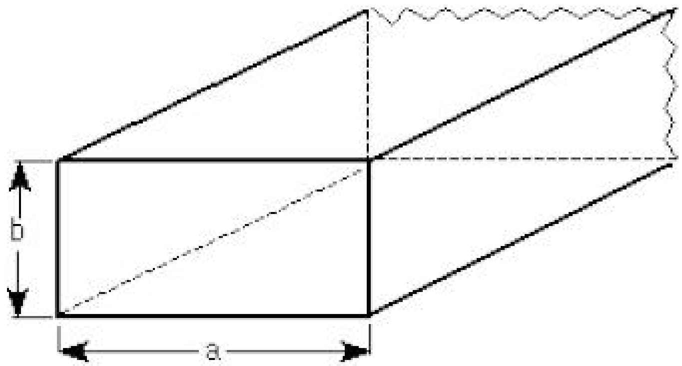
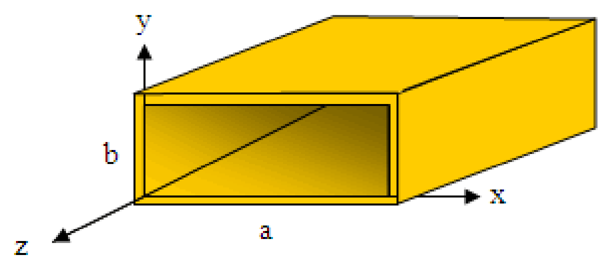


Figure 1. The Rectangular Waveguide



Let us assume that the wave is travelling along z-axis and field variation along z-direction is equal to e-Υz,where z=direction of propagation and Υ= propagation constant.

Assume the waveguide is lossless (α=0) and walls are perfect conductor (σ=∞). According to maxwell's equation:

$$\nabla \times H = J + \partial D/\partial t \text{ and } \nabla \times E = -\partial B/ .$$

$$\text{So } \nabla \times H = J\omega \in E --- (1.a) ,$$

$$\nabla \times E = -J\omega\mu H. \text{ ---------(1.b)}$$

Expanding equation (1),

$$\begin{vmatrix} Ax & Ay & Az \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ Hx & Hy & Hz \end{vmatrix} = J\omega \in [ExAx + EyAy + EzAz]$$

By equating coefficients of both sides we get,

$$\frac{\partial}{\partial y}Hz - \frac{\partial}{\partial z}Hy = J\omega \in Ex \quad \text{-----------------2(a)}$$

$$-\frac{\partial}{\partial x}Hz + \frac{\partial}{\partial z}Hx = J\omega \in Ey \quad \text{-----------2(b)}$$

$$\frac{\partial}{\partial x}Hy - \frac{\partial}{\partial y}Hx = J\omega \in Ez \quad \text{-----------------2(c)}$$

As the wave is travelling along z-direction and variation is along $-\Upsilon z$ direction

$$\Rightarrow \frac{\partial}{\partial z}(e^{-\Upsilon z}) = -\gamma e^{-\Upsilon z} \; .$$

Comparing above equations, $\partial/\partial z = -\Upsilon$

So by putting this value of $\partial/\partial z$ in equations 2(a,b,c),we will get

$$\frac{\partial}{\partial y}Hz + \Upsilon\, Hy = j\omega \in Ex \quad \text{-----------------3(a)}$$

$$\frac{\partial}{\partial x}Hz + \Upsilon\, Hx = -j\omega \in Ey \quad \text{-----------3(b)}$$

$$\frac{\partial}{\partial x}Hy - \frac{\partial}{\partial y}Hx = j\omega \in Ez \quad \text{-----------------3(c)}$$

Similarly from relation $\nabla \times E = -j\omega\mu H$ and $\partial/\partial z = -\Upsilon$,we will get

$$\frac{\partial}{\partial y}Ez + \Upsilon\, Ey = -j\omega\mu Hx \quad \text{-----------------4(a)}$$

$$\frac{\partial}{\partial x}Ez + \Upsilon\, Ex = j\omega\mu Hy \quad \text{-----------4(b)}$$

$$\frac{\partial}{\partial x}Ey - \frac{\partial}{\partial y}Ex = -j\omega\mu Hz \quad \text{-----------------4(c)}$$

From equation sets of (3) ,

we will get : $\partial/\partial y Hz + \Upsilon\, Hy = j\omega \in Ex$

$$Ex = \frac{1}{j\,\omega \in}\left[\frac{\partial}{\partial y}Hz + \Upsilon\, Hy\right] \quad \text{---------(5)}$$

From equation sets of (4) ,we will get : $\partial/\partial x Ez + \Upsilon\, Ex = j\omega\mu Hy$

$$Ex = \frac{1}{\Upsilon}\left[j\omega\mu Hy - \frac{\partial}{\partial x}Ez\right] \quad \text{---------(6)}$$

Equating equations (5) and (6), we will get

$$=> \frac{1}{j\,\omega \in}[\frac{\partial}{\partial y}Hz + \Upsilon\, Hy] = \frac{1}{\Upsilon}[j\omega\mu Hy - \frac{\partial}{\partial x}Ez]$$

$$=> \frac{\Upsilon}{j\,\omega \in}\frac{\partial}{\partial y}Hz + \frac{\Upsilon^2}{j\,\omega \in}Hy = j\omega\mu Hy - \frac{\partial}{\partial x}Ez$$

$$=> (\frac{\Upsilon^2}{j\,\omega \in} - j\omega\mu)Hy = -\frac{\partial}{\partial x}Ez - \frac{\Upsilon}{j\,\omega \in}\frac{\partial}{\partial y}Hz$$

$$=> (\frac{\Upsilon^2+\omega^2\mu \in}{j\omega \in})\ Hy = -\frac{\partial}{\partial x}Ez - \frac{\Upsilon}{j\,\omega \in}\frac{\partial}{\partial y}Hz$$

$$\text{Let } (\Upsilon^2 + \omega^2\mu \in) = h^2$$

$$=> (\frac{h^2}{j\omega \in})Hy = -\frac{\partial}{\partial x}Ez - \frac{\Upsilon}{j\omega \in}\frac{\partial}{\partial y}Hz$$

$$Hy = -\frac{j\omega \in}{h^2}\frac{\partial}{\partial x}Ez - \frac{\Upsilon}{h^2}\frac{\partial}{\partial y}Hz \text{ ------------ } (7)$$

Similarly we will get by simplifying other equations

$$Hx = -\frac{j\omega \in}{h^2}\frac{\partial}{\partial y}Ez - \frac{\Upsilon}{h^2}\frac{\partial}{\partial x}Hz \text{ ------------ } (8)$$

$$Ex = -\frac{j\omega\mu}{h^2}\frac{\partial}{\partial y}Hz - \frac{\Upsilon}{h^2}\frac{\partial}{\partial x}Ez \text{ ------------ } (9)$$

$$Ey = \frac{j\omega\mu}{h^2}\frac{\partial}{\partial x}Hz - \frac{\Upsilon}{h^2}\frac{\partial}{\partial y}Ez \text{ ----------- } (10)$$

FIELD SOLUTIONS FOR TRANSVERSE MAGNETIC FIELD IN RECTANGULAR WAVEGUIDE :

Hz=0 and Ez≠0

EXYZ = (C1 coskxx+C2sin$k$xx)(C3coskyy+C4sin$k$yy) $e{-}\gamma z$

The values of C1,C2,C3,C4,Kx,Ky are found out from boundary equations.as we know that the tangential component of E are constants across the boundary, then

$$E= \begin{cases} 0, x = 0 \text{ and } x = a \\ 0, y = 0 \text{ and } y = b \end{cases}$$

AT x=0 AND y=0 ;

E=C1C3$e{-}\gamma z$=0 but we know that $e{-}\gamma z$≠0 wave is travelling along z-direction.

So either C1=0 or C3=0 otherwise C1C3=0

AT x=0 AND y=b ;

$$E= C_1(C_3\cos kyb + C_4\sin k_y b) e^{-\gamma z}=0$$

So $C_1C_{3=}0$

So equation (g)becomes

$$E_{XYZ}= (C_2\sin k_x x \times C_4\sin k_y y)e^{-\gamma z}\text{------(i)}$$

Hence for x=0,E=0

$$\text{So } (C_2\sin k_x a \times C_4\sin k_y y)e^{-\gamma z} = 0$$

$$=>\sin kxa = 0 => k_x=\frac{m*\pi}{a}$$

In equation (i) for y=b=>E=0;

$$\text{So } (C_2\sin k_x x \times C_4\sin k_y b)e^{-\gamma z} = 0$$

$$=>\sin kyb = 0 => k_y=\frac{n*\pi}{b}$$

So finally solutions for TRANSVERSE MAGNETIC MODE is given by

$$E_{z=}C\ (\sin(\frac{m*\pi}{a})x \times \sin(\frac{n*\pi}{b})y \times e^{-\gamma z}$$

Where C2 $\times C$4=C

CUT-OFF FREQUENCY :

It is the minimum frequency after which propagation occurs inside the waveguide.

As we know that => Kx^2 +ky^2+kz^2=k^2

=> Kx^2 +ky^2= k^2-kz^2

=> Kx^2 +ky^2=k^2+γ2

As we know that β=$-j\omega\ \mu\varepsilon$ *and* $k2$=$\beta2$

So we will get that : => Kx^2 +ky^2=k^2+γ2=$\omega2\mu\varepsilon$+γ2

$$\text{So } \gamma = \sqrt{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon\right]}$$

At f=fc or w=wc ,at cut off frequency propagation is about to start. So γ=0

$$=>0=\sqrt{\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2-\omega c^2\mu\varepsilon\right]}$$

$$=>\omega c^2\mu\varepsilon=\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2$$

$$W_c=1/\sqrt{\mu\varepsilon}\left(\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right)^{1/2}$$

So $f_c=1/2\pi\sqrt{\mu\varepsilon}\left(\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right)^{1/2}$ ---------***cut off frequency equation***

where m=n=0,1,2,3……

$$\text{At free space } f_c=1/2\pi\sqrt{\mu o\varepsilon o}\left(\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right)^{1/2}$$

$$=>f_c=c/2\left(\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right)^{1/2}\text{---}$$

CUT – OFF WAVELENGTH:

This is given by

$$y\!\!/c=\frac{c}{f}=2\times\left(\frac{1}{\left(\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right)^{1/2}}\right)$$

DOMINANT MODE :

The mode having lowest cut-off frequency or highest cut-off wavelength is called DOMINANT MODE.

The mode can be TM01,TM10,TM11,But for TM10 and TM01,wave can't exist.

Hence TM11has lowest cut-off frequency and is the DOMINANT MODE in case of all TM modes only.

PHASE CONSTANT :

As we know that

$$\gamma=\left(\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2-\omega^2\mu\varepsilon\right)^{1/2}$$

So

$$j\beta=\sqrt{\omega^2\mu\varepsilon-\omega c^2\mu\varepsilon}$$

This condition satisfies that only $\omega c2\mu\varepsilon>\omega 2\mu\varepsilon$

$$\beta = \sqrt{\omega^2\mu\varepsilon - \omega c^2\mu\varepsilon}$$

PHASE VELOCITY :

It is given by Vp=ω/ β

$$V_p = \frac{\omega}{(\omega^2\mu\varepsilon - \omega c^2\mu\varepsilon)}$$

$$V_p = 1/\left[\sqrt{\omega\varepsilon}(1 - fc^2/f^2)\right]$$

GUIDE WAVELENGTH :

It is given by

$$\lambda g = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{\omega^2\mu\varepsilon - \omega c^2\mu\varepsilon}}$$

$$\lambda g = 1/\sqrt{(f^2\mu\varepsilon - fc^2\mu\varepsilon)}$$

$$\lambda g = \frac{c}{f}/(1 - fc^2/f^2)$$

$$\lambda g = \frac{\lambda o}{[1 - \left(\frac{\lambda o}{\lambda c}\right)^2]^{\frac{1}{2}}}$$

$$\frac{1}{\lambda g^2} = 1/\lambda o^2 - \frac{1}{\lambda c^2}$$

SOLUTIONS OF TRANSVERSE ELECTRIC MODE :

Here Ez=0 and Hz≠0

$$Hz = (B1\cos kxx + B2\sin kxx)(B3\cos kyy + B4\sin kyy)\, e^{-\gamma z}$$

B1,B2,B3,B4,KX,KY are found from boundary conditions.

$$Ex = 0 \; for \; y = 0 \; and \; y = b$$
$$Ey = 0 \; for \; x = 0 \; and \; x = a$$

At x=0 and y=0 ;

$$Ey = \frac{J\omega\mu}{h^2}\frac{\partial}{\partial x}Hz - \frac{\Upsilon}{h^2}\frac{\partial}{\partial y}Ez \quad ,\text{as} \frac{\Upsilon}{h^2}\frac{\partial}{\partial y}Ez = 0$$

$$\text{So Ey} = \frac{J\omega\mu}{h^2}\frac{\partial}{\partial x}Hz$$

Here

$$\frac{\partial}{\partial x}Hz = [B1 * kx * (-\sin kxx) + B2 * kx * \cos k\,xx)(\,B3\cos kyy + B4\sin k\,yy\,)\,e^{-\gamma z}]$$

So

$$Ey = \frac{J\omega\mu}{h^2}[B1 * kx * (-\sin kxx) + B2 * kx * \cos k\,xx)(\,B3\cos kyy + B4\sin k\,yy\,)\,e^{-\gamma z}]$$

At x=0, $\partial/\partial x Hz$=0

0=[B2*Kx][(B3cosKyy+B4sinKyy)$e{-}\gamma z$]

From this B2=0

so

$$Ex = -\frac{j\omega\mu}{h^2}\frac{\partial}{\partial v}Hz - \frac{Y}{h^2}\frac{\partial}{\partial x}Ez \qquad [\text{as}\ \frac{Y}{h^2}\frac{\partial}{\partial x}Ez = 0];$$

$$Ex = -\frac{j\omega\mu}{h^2}\frac{\partial}{\partial y}Hz$$

Here

$$\frac{\partial}{\partial y}Hz = [B1\,(\cos kxx) + B2\sin k\,xx)(-\,B3 * ky * \sin kyy + B4 * ky * \cos k\,yy\,)\,e^{-\gamma z}]$$

$$Ex = -\frac{J\omega\mu}{h^2}[B1\,(\cos kxx) + B2\sin k\,xx)(-\,B3 * ky * \sin kyy + B4 * ky * \cos k\,yy\,)\,e^{-\gamma z}]$$

$$B1\,(\cos kxx) + B2\sin k\,xx)(-\,B3 * ky * \sin kyy + B4 * ky * \cos k\,yy\,)\,e^{-\gamma z} = 0$$

At y=0 , $\partial/\partial y Hz$=0

At y=0 , $\frac{\partial}{\partial y} Hz = 0$

$$[\text{B1}\,(\cos kxx) + \text{B2}\sin k\,\text{xx})(\text{B4} * \text{ky})e^{-\gamma z} = 0$$

From this B_4=0

So H_z= $\text{B1}\,(\cos kxx) * \ \text{B3}\cos kyy\ \ * e^{-\gamma z}$

$$\frac{\partial}{\partial x} Hz = [\text{B1} * \text{kx} * (-\sin kxx)\,(\,\text{B3}\cos kyy\,)\,e^{-\gamma z}]$$

Here we know that at x=a,E_y=0

So E_y= $\frac{J\omega\mu}{h^2}[-\text{B1} * \text{kx} * (-\sin kxa) * \ \ \text{B3}\cos kyy * \ \ e^{-\gamma z}]$=0

$$\sin kxa = 0 => kx = \frac{m\pi}{a}$$

At y=b,E_x=0

So E_x=$-\frac{J\omega\mu}{h^2}[\text{B1}\,(\cos kxx)\,(\text{B3} * \text{ky} * \sin kyb)\,e^{-\gamma z}]$=0

so sinkyb=0=>ky=$n\pi b$

So the general TRANSVERSE ELECTRIC MODE solution is given by

$$H_z=\text{B}\,(\cos\frac{m\pi}{a}x)\,(\cos\frac{n\pi}{b}y)\,e^{-\gamma z}$$

Where B=B1B3

CUT-OFF FREQUENCY :

The cut-off frequency is given as

$$f_c=c/2\left(\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right)^{1/2}$$ -

DOMINANT MODE :

The mode having lowest cut-off frequency or highest cut-off wavelength is called DOMINANT MODE.here TE00 where wave can’t exist.

So fc(TE01)=c/2b

fc(TE10)=c/2a

for rectangular waveguide we know that a>b

so TE10 is the dominant mode in all rectangular waveguide.

DEGENERATE MODE :

The modes having same cut-off frequency but different field equations are called degenerate modes.

WAVE IMPEDANCE :

Impedance offered by waveguide either in TE mode or TM mode when wave travels through ,it is called wave impedance.
For TE mode

$$\eta TE = \frac{\eta i}{\sqrt{(1 - \frac{fc^2}{f^2})}}$$

And

$$\eta TM = \eta i * \sqrt{(1 - \frac{fc^2}{f^2})}$$

Where ηi=intrinsic impedance=377ohm=120π

CYLINDRICAL WAVEGUIDES

A circular waveguide is a tubular, circular conductor. A plane wave propagating through a circular waveguide results in transverse electric (TE) or transverse magnetic field(TM) mode. Assume the medium is lossless(α=0) and the walls of the waveguide is perfect conductor(σ=∞). The field equations from MAXWELL'S EQUATIONS are:-

$$\nabla xE=-j\omega\mu H \text{ ------(1.a)}$$
$$\nabla xH=j\omega\epsilon E \text{ -----(1.b)}$$

Taking the first equation,

$$\nabla xE=-j\omega\mu H$$

Expanding both sides of the above equation in terms of cylindrical coordinates, we get

$$\frac{1}{\rho} * \begin{vmatrix} A\rho & \rho A\varphi & Az \\ \partial/\partial\rho & \partial/\partial\varphi & \partial/\partial z \\ E\rho & \rho E\varphi & Ez \end{vmatrix} = -j\omega\mu[H\rho A\rho + H\varphi A\varphi + HzAz]$$

Equating :-

$$\frac{1}{\rho}\left\{\frac{\partial(Ez)}{\partial\varphi}-\frac{\partial(\rho E\varphi)}{\partial z}\right\}=-j\omega\mu H\rho \quad \text{------ (2.a)}$$

$$\left\{\frac{\partial(E\rho)}{\partial z}-\frac{\partial(Ez)}{\partial\rho}\right\}=-j\omega\mu H\varphi \quad \text{------ (2.b)}$$

$$\frac{1}{\rho}\left\{\frac{\partial(\rho E\varphi)}{\partial\rho}-\frac{\partial(E\rho)}{\partial\varphi}\right\}=-j\omega\mu Hz \quad \text{------- (2.c)}$$

Similarly expanding ∇xH=jωϵE,

$$\frac{1}{\rho}*\begin{vmatrix} A\rho & \rho A\varphi & Az \\ \partial/\partial\rho & \partial/\partial\varphi & \partial/\partial z \\ H\rho & \rho H\varphi & Hz \end{vmatrix}=j\omega\epsilon[E\rho A\rho+E\varphi A\varphi+EzAz]$$

$$\frac{1}{\rho}\left\{\frac{\partial(Hz)}{\partial\varphi}-\frac{\partial(\rho H\varphi)}{\partial z}\right\}= j\omega\epsilon E\rho \quad \text{------ (3.a)}$$

$$\left\{\frac{\partial(H\rho)}{\partial z}-\frac{\partial(Hz)}{\partial\rho}\right\}=-j\omega\epsilon E\varphi \quad \text{------- (3.b)}$$

$$\frac{1}{\rho}\left\{\frac{\partial(\rho H\varphi)}{\partial\rho}-\frac{\partial(H\rho)}{\partial\varphi}\right\}=j\omega\epsilon Ez \quad \text{------- (3.c)}$$

Let us assume that the wave is propagating along z direction. So,

$$Hz=e^{-} ;$$

$$=> \frac{\partial Hz}{\partial z}=-\gamma e^{-\gamma z}$$

$$\boxed{\frac{\partial}{\partial z}=-\gamma}$$

Putting in equation 2 and 3:

$$\left\{\frac{\partial(Ez)}{\partial\varphi}+\gamma\rho E\varphi\right\}=-j\omega\mu\rho H\rho \quad \text{------ (4.a)}$$

$$\left\{\gamma E\rho+\frac{\partial(Ez)}{\partial\rho}\right\}=j\omega\mu\rho H\varphi \quad \text{------- (4.b)}$$

$$\left\{\frac{\partial(\rho E\varphi)}{\partial\rho}-\frac{\partial(E\rho)}{\partial\varphi}\right\}=-j\omega\mu\rho Hz \quad \text{------- (4.c)}$$

And

$$\frac{1}{\rho}\left\{\frac{\partial(Hz)}{\partial\varphi}+\gamma\rho H\varphi\right\}=\mathrm{j}\omega\epsilon\mathrm{E}\rho \quad \text{------ (5.a)}$$

$$\left\{\gamma H\rho+\frac{\partial(Hz)}{\partial\rho}\right\}=\mathrm{j}\omega\epsilon\mathrm{E}\varphi \quad \text{------- (5.b)}$$

$$\frac{1}{\rho}\left\{\frac{\partial(\rho H\varphi)}{\partial\rho}-\frac{\partial(H\rho)}{\partial\varphi}\right\}=\mathrm{j}\omega\epsilon\mathrm{E}z \quad \text{------- (5.c)}$$

Now from eq(4.a) and eq(5.b),we get

$$\mathrm{H}\rho=\frac{1}{-\mathrm{j}\omega\mu\rho}\left\{\frac{\partial(Ez)}{\partial\varphi}+\gamma\rho E\varphi\right\};\ \mathrm{H}\rho=\frac{1}{\gamma}[-\mathrm{j}\omega\epsilon\mathrm{E}\varphi-\frac{\partial(Hz)}{\partial\rho}]$$

$$\therefore \frac{1}{-\mathrm{j}\omega\mu\rho}\left\{\frac{\partial(Ez)}{\partial\varphi}+\gamma\rho E\varphi\right\}=\frac{1}{\gamma}[-\mathrm{j}\omega\epsilon\mathrm{E}\varphi-\frac{\partial(Hz)}{\partial\rho}]$$

$$\Rightarrow \frac{1}{\rho}\frac{\partial(Ez)}{\partial\varphi}+\gamma E\varphi=-\frac{\omega^2\mu\varepsilon}{\gamma}\mathrm{E}\varphi+\frac{\mathrm{j}\omega\mu}{\gamma}\frac{\partial\mathrm{Hz}}{\partial\rho}$$

$$\Rightarrow \left(\frac{\gamma^2+\omega^2\mu\varepsilon}{\gamma}\right)\mathrm{E}\varphi=\frac{\mathrm{j}\omega\mu}{\gamma}\frac{\partial\mathrm{Hz}}{\partial\rho}-\frac{1}{\rho}\frac{\partial\mathrm{Hz}}{\partial\rho}$$

Let $(\gamma 2+\omega 2\mu\varepsilon)$ =h2=Kc2 ;

For lossless medium $\alpha=0;\gamma=j\beta$;

Now the final equation for Eφ is

$$\mathrm{E}\varphi=\frac{-\mathrm{j}}{\mathrm{Kc}^2}\left(\frac{\beta}{\rho}\frac{\partial\mathrm{Ez}}{\partial\varphi}-\omega\mu\frac{\partial\mathrm{Hz}}{\partial\rho}\right) \quad \text{----- (6.a)}$$

$$\mathrm{H}\varphi=\frac{-\mathrm{j}}{\mathrm{Kc}^2}\left(\omega\varepsilon\frac{\partial\mathrm{Ez}}{\partial\rho}+\frac{\beta}{\rho}\frac{\partial\mathrm{Hz}}{\partial\varphi}\right) \quad \text{----- (6.b)}$$

$$\mathrm{E}\rho=\frac{-\mathrm{j}}{\mathrm{Kc}^2}\left(\frac{\omega\mu}{\rho}\frac{\partial\mathrm{Hz}}{\partial\varphi}+\beta\frac{\partial\mathrm{Ez}}{\partial\rho}\right) \quad \text{----- (6.c)}$$

$$\mathrm{H}\rho=\frac{\mathrm{j}}{\mathrm{Kc}^2}\left(\frac{\omega\varepsilon}{\rho}\frac{\partial\mathrm{Ez}}{\partial\varphi}-\beta\frac{\partial\mathrm{Hz}}{\partial\rho}\right) \quad \text{------ (6.d)}$$

Equations (6.a),(6.b),(6.c),(6.d) are the field equations for cylindrical waveguides.

TE MODE IN CYLINDRICAL WAVEGUIDE :-

For TE mode, Ez=0, Hz≠0.
As the wave travels along z-direction,$e^{-\gamma z}$ is the solution along z-direction.

As, $\gamma^2 + \omega^2\mu\varepsilon = h^2$; ;

$$-\beta^2+\omega^2\mu\epsilon = Kc^2;$$

$$-\beta^2+K^2 = Kc^2 \qquad (\text{as } K^2= \omega^2\mu\varepsilon)$$

According to maxwell's equation, the laplacian of Hz :

$$\nabla^2 H_z = -\omega^2\mu\varepsilon H_z;$$

$$\nabla^2 H_z + \omega^2\mu\varepsilon H_z = 0;$$

$$\nabla^2 H_z + K^2 H_z = 0$$

Expanding the above equation, we get:

$$\frac{\partial^2 Hz}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial Hz}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 HZ}{\partial \varphi^2} + \frac{\partial^2 Hz}{\partial z^2} + K^2 H_z = 0;$$

Now $H_Z = e^{-\gamma z}$; $\frac{\partial^2 Hz}{\partial z^2} = (-\gamma)^2\ e^{-\gamma z}$; $\frac{\partial^2 Hz}{\partial z^2} = -\beta^2\ e^{-\gamma z}$;

$$\frac{\partial^2 Hz}{\partial z^2} = -\beta^2 Hz \qquad => \frac{\partial^2}{\partial z^2} = -\beta^2;$$

Putting this value in the above equations, we get:-

$$\frac{\partial^2 Hz}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial Hz}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 HZ}{\partial \varphi^2} - \beta^2 Hz + K^2 H_z = 0;$$

$$\frac{\partial^2 Hz}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial Hz}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 HZ}{\partial \varphi^2} + Kc^2 H_z = 0;\ \ [\text{as } -\beta^2+K^2 = Kc^2\ ]$$

$$\frac{\partial^2 Hz}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial Hz}{\partial \rho} + Kc^2 H_z = -\frac{1}{\rho^2}\frac{\partial^2 HZ}{\partial \varphi^2};$$

The partial differential with respect to ρ and φ in the above equation are equal only when the individuals are constant (Let it be Ko2).

$$-\frac{1}{\rho^2}\frac{\partial^2 HZ}{\partial\varphi^2} = K_o^{\,2}$$

$$\frac{\partial^2 HZ}{\partial\varphi^2} + \rho^2 K_o^{\,2} = 0$$

Solutions to the above differential equation is:-

Hz = B1sin (Koφ)+B2cos(Koφ)-----{solution along φ direction}.

Now,

$$\frac{\partial^2 Hz}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial Hz}{\partial\rho} + (Kc^2 H_z - K_o^{\,2}) = 0$$

This equation is similar to BESSEL'S EQUATION,so the solution of this equation is
Hz=CnJn(Kc ρ) --------{solution along ρ-direction}
Hence,
$Hz = Hz(\rho)Hz(\varphi)e^{-\gamma z}$;
So, the final solution is,

$$Hz = CnJn(K_c\,\rho)[\,B_1\sin(K_o\varphi) + B_2\cos(K_o\varphi)]\,e^{-\gamma z}$$

Applying boundary conditions:-

At ρ=a, Eφ=0 => ∂Hz/∂ρ=0,

=> $J_n(K_c\,\rho) = 0$

=> $J_n(K_c\,a) = 0$

If the roots of above equation are defined as P_{mn}',then

$K_c = P_{mn}/a$;

$$Hz = CnJn\left(\frac{P_{mn}'}{a}\rho\right)[\,B_1\sin(K_o\varphi) + B_2\cos(K_o\varphi)]\,e^{-\gamma z}$$

CUT-OFF FREQUENCY :- It is the minimum frequency after which the propagation occurs inside the cavity.

∴ γ=0;

But we know that

$$\gamma^2 + \omega^2\mu\varepsilon = Kc^2;$$

$$\omega^2 \mu\varepsilon = Kc^2$$

$$\omega^2 = \frac{Kc^2}{\mu\varepsilon}$$

$$2\pi f_c = \sqrt{\frac{Kc^2}{\mu\varepsilon}}$$

$$f_c = \frac{1}{2\pi}\sqrt{\frac{Kc^2}{\mu\varepsilon}}$$

$$\therefore \quad f_c = \frac{Pmn'}{2\pi a\sqrt{\mu\varepsilon}}$$

CUTOFF WAVELENGTH :-

$$\frac{c}{fc} = \frac{\frac{1}{\sqrt{\mu\varepsilon}}}{\frac{Pmn'}{2\pi a\sqrt{\mu\varepsilon}}} = \frac{2\pi a}{Pmn'}$$

The experimental values of Pmn' are:-

n \ m	1	2	3
0	3.832	7.016	10.173
1	1.841	5.331	8.536
2	3.054	6.706	9.969
3	3.054	6.706	9.970

As seen from this table,TE11 mode has the lowest cut off frequency, hence **TE11** is the dominating mode.

MICROWAVE COMPONENTS

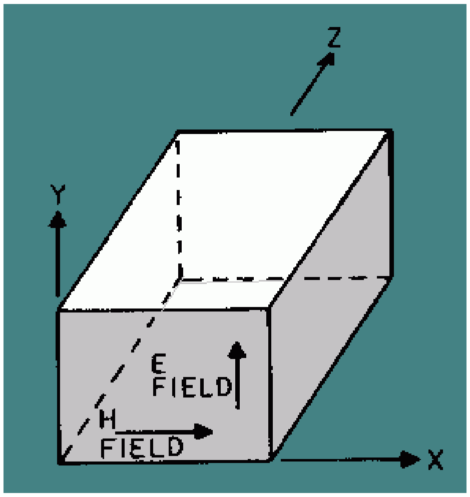
MICROWAVE RESONATOR:

- They are used in many applications such as oscillators, filters, frequency meters, tuned amplifiers and the like.
- A microwave resonator is a metallic enclosure that confines electromagnetic energy and stores it inside a cavity that determines its equivalent capacitance and inductance and from the energy dissipated due to finite conductive walls we can determine the equivalent resistance.
- The resonator has finite number of resonating modes and each mode corresponds to a particular resonant frequency.
- When the frequency of input signal equals to the resonant frequency, maximum amplitude of standing wave occurs and the peak energy stored in the electric and magnetic field are calculated.

RECTANGULAR WAVEGUIDE CAVITY RESONATOR:

Resonator can be constructed from closed section of waveguide by shorting both ends thus forming a closed box or cavity which store the electromagnetic energy and the power can be dissipated in the metallic walls as well as the dielectric medium

DIAGRAM:



The geometry of rectangular cavity resonator spreads as

$$0 \le x \le a;$$
$$0 \le y \le b;$$
$$0 \le z \le d$$

Hence the expression for cut-off frequency will be

$$\omega_0^2 \mu \epsilon = (m\pi/a)^2 + (n\pi/b)^2 + (l\pi/d)^2$$

$$\omega_0 = \frac{1}{\sqrt{\mu\epsilon}} [(m\pi/a)^2 + (n\pi/b)^2 + (l\pi/d)^2]^{1/2}$$

$$f_0 = \frac{1}{2\pi\sqrt{\mu\epsilon}} [(m\pi/a)^2 + (n\pi/b)^2 + (l\pi/d)^2]^{1/2}$$

$$f_0 = \frac{c'}{2} [(m\pi/a)^2 + (n\pi/b)^2 + (l\pi/d)^2]^{1/2}$$

This is the expression for resonant frequency of cavity resonator.

The mode having lowest resonant frequency is called DOMINANT MODE and for TE AND TM the dominant modes are TE-101 and TM-110 respectively.

QUALITY FACTOR OF CAVITY RESONATOR:

- $Q=2\pi \times \frac{\text{maximum energy stored per cycle}}{\text{energy dissipated per cycle}}$

FACTORS AFFECTING THE QUALITY FACTOR:

Quality factor depends upon 2 factors:
Lossy conducting walls

Lossy dielectric medium of a waveguide

1) LOSSY CONDUCTING WALL:

The Q-factor of a cavity with lossy conducting walls but lossless dielectric medium i.e. $\sigma c \neq \infty$ and $\sigma = 0$

Then $Qc = (2\omega 0\ We/Pc)$
Where w0-resonant frequency
We-stored electrical energy
Pc-power loss in conducting walls

2) LOSSY DIELECTRIC MEDIUM:

The Q-factor of a cavity with lossy dielectric medium but lossless conducting walls i.e. $\sigma c = \infty$ and $\sigma \neq 0$

$$Q_d = 2\omega_0 W_e / P_d = (1/\tan\delta)$$

Where $\tan\delta = \varsigma\omega\epsilon$

Pd=power loss in dielectric medium
When both the conducting walls and the dielectric medium are lossy in nature then

Total power loss = Pc+Pd

$$1/Q_{total} = 1/Q_c + 1/Q_d$$

$$\text{or } Q_{total} = 1/(1/Q_c + 1/Q_d)$$

MICROSTRIP

Microstrip transmission line is a kind of "high grade" printed circuit construction, consisting of a track of copper or other conductor on an insulating substrate. There is a "backplane" on the other side of the insulating substrate, formed from similar conductor. A picture (37kB). Looked at end on, there is a "hot" conductor which is the track on the top, and a "return" conductor which is the backplane on the bottom. Microstrip is therefore a variant of 2-wire transmission line. If one solves the electromagnetic equations to find the field distributions, one finds very nearly a completely TEM (transverse electromagnetic) pattern. This means that there are only a few regions in which there is a component of electric or magnetic field in the direction of wave propagation. There is a picture of these field patterns (incomplete) in T C Edwards "Foundations for Microstrip Circuit Design" edition 2 page 45. See the booklist for further bibliographic details. The field pattern is commonly referred to as a Quasi TEM pattern.

Under some conditions one has to take account of the effects due to longitudinal fields. An example is geometrical dispersion, where different wave frequencies travel at different phase velocities, and the group and phase velocities are different. The quasi TEM pattern arises because of the interface between the dielectric substrate and the surrounding air. The electric field lines have a discontinuity in direction at the interface.

The boundary conditions for electric field are that the normal component (ie the component at right angles to the surface) of the electric field times the dielectric constant is continuous across the boundary; thus in the dielectric which may have dielectric constant 10, the electric field suddenly drops to 1/10 of its value in air. On the other hand, the tangential component (parallel to the interface) of the electric field is continuous across the boundary.

In general then we observe a sudden change of direction of electric field lines at the interface, which gives rise to a longitudinal magnetic field component from the second Maxwell's equation, curl E = - dB/dt. Since some of the electric energy is stored in the air and some in the dielectric, the effective dielectric constant for the waves on the transmission line will lie somewhere between that of the air and that of the dielectric. Typically the effective dielectric constant will be 50-85% of the substrate dielectric constant. As an example, in (notionally) air spaced microstrip the velocity of waves would be c = 3 * 10^8 metres per second. We have to divide this figure by the square root of the effective dielectric constant to find the actual wave velocity for the real microstrip line.

At 10 GHz the wavelength on notionally air spaced microstrip is therefore 3 cms; however on a substrate with effective dielectric constant of 7 the wavelength is 3/(sqrt{7}) = 1.13cms.

WAVEGUIDE CUTOFF FREQUENCY:

-waveguide cutoff frequency is an essential parameter for any waveguide - it does not propagate signals below this frequency. It is easy to understand an calculate with our equations.

The cutoff frequency is the frequency below which the waveguide will not operate.

Accordingly it is essential that any signals required to pass through the waveguide do not extend close to or below the cutoff frequency.

The waveguide cutoff frequency is therefore one of the major specifications associated with any waveguide product.

Waveguide cutoff frequency basics

Waveguides will only carry or propagate signals above a certain frequency, known as the cut-off frequency. Below this the waveguide is not able to carry the signals. The cut-off frequency of the waveguide depends upon its dimensions. In view of the mechanical constraints this means that waveguides are only used for microwave frequencies. Although it is theoretically possible to build waveguides for lower frequencies the size would not make them viable to contain within normal dimensions and their cost would be prohibitive.

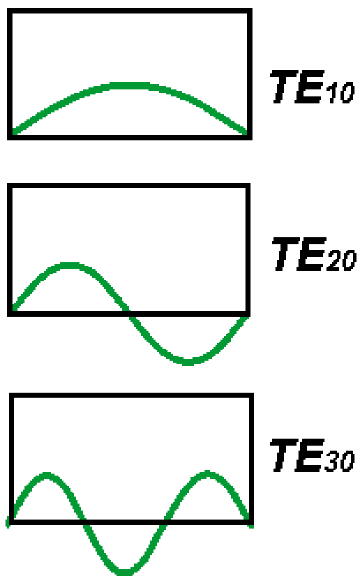
As a very rough guide to the dimensions required for a waveguide, the width of a waveguide needs to be of the same order of magnitude as the wavelength of the signal being carried. As a result, there is a number of standard sizes used for waveguides as detailed in another page of this tutorial. Also other forms of waveguide may be specifically designed to operate on a given band of frequencies

What is waveguide cutoff frequency? - the concept

Although the exact mechanics for the cutoff frequency of a waveguide vary according to whether it is rectangular, circular, etc, a good visualisation can be gained from the example of a rectangular waveguide. This is also the most widely used form.

Signals can progress along a waveguide using a number of modes. However the dominant mode is the one that has the lowest cutoff frequency. For a rectangular waveguide, this is the TE10 mode.

The TE means transverse electric and indicates that the electric field is transverse to the direction of propagation.



TE modes for a rectangular waveguide

The diagram shows the electric field across the cross section of the waveguide. The lowest frequency that can be propagated by a mode equates to that were the wave can "fit into" the waveguide.

As seen by the diagram, it is possible for a number of modes to be active and this can cause significant problems and issues. All the modes propagate in slightly different ways and therefore if a number of modes are active, signal issues occur.

It is therefore best to select the waveguide dimensions so that, for a given input signal, only the energy of the dominant mode can be transmitted by the waveguide. For example: for a given frequency, the width of a rectangular guide may be too large: this would cause the TE20 mode to propagate.

As a result, for low aspect ratio rectangular waveguides the TE20 mode is the next higher order mode and it is harmonically related to the cutoff frequency of the TE10 mode. This relationship and attenuation and propagation characteristics that determine the normal operating frequency range of rectangular waveguide.

Rectangular waveguide cutoff frequency

Although waveguides can support many modes of transmission, the one that is used, virtually exclusively is the TE10 mode. If this assumption is made, then the calculation for the lower cutoff point becomes very simple:

$$f_c = \frac{c}{2a}$$

Where

fc = rectangular waveguide cutoff frequency in Hz
c = speed of light within the waveguide in metres per second
a = the large internal dimension of the waveguide in metres

It is worth noting that the cutoff frequency is independent of the other dimension of the waveguide. This is because the major dimension governs the lowest frequency at which the waveguide can propagate a signal.

Circular waveguide cutoff frequency

the equation for a circular waveguide is a little more complicated (but not a lot).

$$f_c = \frac{1.8412\ c}{2\pi a}$$

where:
fc = circular waveguide cutoff frequency in Hz
c = speed of light within the waveguide in metres per second
a = the internal radius for the circular waveguide in metres

Although it is possible to provide more generic waveguide cutoff frequency formulae, these ones are simple, easy to use and accommodate, by far the majority of calculations needed.