

ANALOG COMMUNICATIONS SYSTEMS

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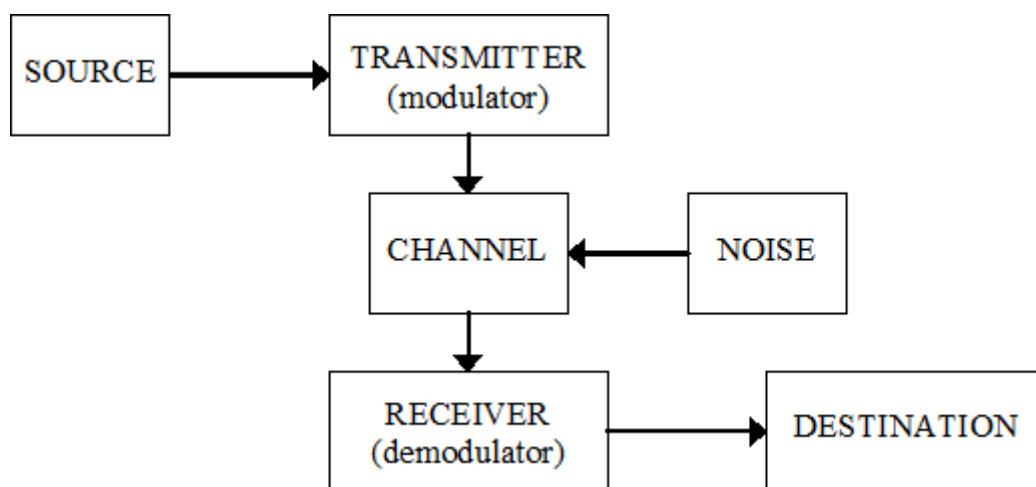
# UNIT-I

## Introduction to Communication System

Communication is the process by which information is exchanged between individuals through a medium.

Communication can also be defined as the transfer of information from one point in space and time to another point.

The basic block diagram of a communication system is as follows.



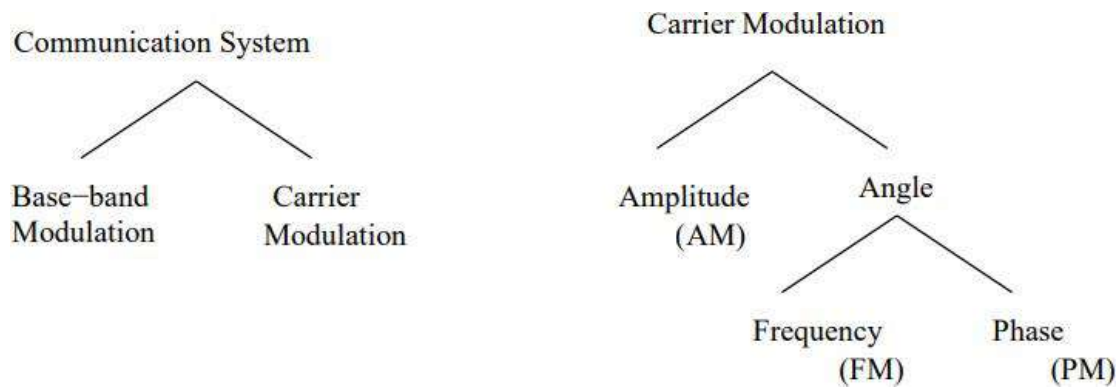
- **Transmitter:** Couples the message into the channel using high frequency signals.
- **Channel:** The medium used for transmission of signals
- **Modulation:** It is the process of shifting the frequency spectrum of a signal to a frequency range in which more efficient transmission can be achieved.
- **Receiver:** Restores the signal to its original form.
- **Demodulation:** It is the process of shifting the frequency spectrum back to the original baseband frequency range and reconstructing the original form.

### Modulation:

Modulation is a process that causes a shift in the range of frequencies in a signal.

- Signals that occupy the same range of frequencies can be separated.
- Modulation helps in noise immunity, attenuation - depends on the physical medium.

The below figure shows the different kinds of analog modulation schemes that are available



Modulation is operation performed at the transmitter to achieve efficient and reliable information transmission.

For analog modulation, it is frequency translation method caused by changing the appropriate quantity in a carrier signal.

It involves two waveforms:

- A modulating signal/baseband signal – represents the message.
- A carrier signal – depends on type of modulation.

•Once this information is received, the low frequency information must be removed from the high frequency carrier. •This process is known as “Demodulation”.

#### **Need for Modulation:**

- Baseband signals are incompatible for direct transmission over the medium so, modulation is used to convey (baseband) signals from one place to another.
- Allows frequency translation:
  - Frequency Multiplexing
  - Reduce the antenna height
  - Avoids mixing of signals
  - Narrowbanding
- Efficient transmission
- Reduced noise and interference

#### **Types of Modulation:**

Three main types of modulations:

##### **Analog Modulation**

- **Amplitude modulation**

Example: Double sideband with carrier (DSB-WC), Double- sideband suppressed carrier (DSB-SC), Single sideband suppressed carrier (SSB-SC), vestigial sideband (VSB)

- **Angle modulation (frequency modulation & phase modulation)**

Example: Narrow band frequency modulation (NBFM), Wideband frequency modulation (WBFM), Narrowband phase modulation (NBPM), Wideband phase modulation (WBPM)

### **Pulse Modulation**

- Carrier is a train of pulses
- Example: Pulse Amplitude Modulation (PAM), Pulse width modulation (PWM) , Pulse Position Modulation (PPM)

### **Digital Modulation**

- Modulating signal is analog
  - Example: Pulse Code Modulation (PCM), Delta Modulation (DM), Adaptive Delta Modulation (ADM), Differential Pulse Code Modulation (DPCM), Adaptive Differential Pulse Code Modulation (ADPCM) etc.
- Modulating signal is digital (binary modulation)
  - Example: Amplitude shift keying (ASK), frequency Shift Keying (FSK), Phase Shift Keying (PSK) etc

### **Frequency Division Multiplexing**

Multiplexing is the name given to techniques, which allow more than one message to be transferred via the same communication channel. The channel in this context could be a transmission line, *e.g.* a twisted pair or co-axial cable, a radio system or a fibre optic system *etc.*

FDM is derived from AM techniques in which the signals occupy the same physical 'line' but in different frequency bands. Each signal occupies its own specific band of frequencies all the time, *i.e.* the messages share the channel **bandwidth**.

- FDM – messages occupy **narrow** bandwidth – all the time.

Multiplexing requires that the signals be kept apart so that they do not interfere with each other, and thus they can be separated at the receiving end. This is accomplished by separating the signal either in frequency or time. The technique of separating the signals in frequency is referred to as frequency-division multiplexing (FDM), whereas the technique of separating the signals in time is called time-division multiplexing. In this section, we discuss frequency division multiplexing systems, referred hereafter as FDM.

Fig. 1 shows the block diagram of FDM system. As shown in the Fig. 1, input message signals, assumed to be of the low-pass type are passed through input low-pass filters. This filtering action removes high-frequency components that do not contribute significantly to signal representation but may disturb other message signals that share the common channel. The filtered message signals are then modulated with necessary carrier frequencies with the help of modulators. The most commonly method of modulation in FDM is single sideband modulation, which requires a bandwidth that is approximately equal to that of original message signal. The band pass filters following the modulators are used to restrict the band of each modulated wave to its prescribed range. The outputs of band-pass filters are combined in parallel which form the input to the common channel.

At the receiving end, bandpass filters connected to the common channel separate the message signals on the frequency occupancy basis. Finally, the original message signals are recovered by individual demodulators.

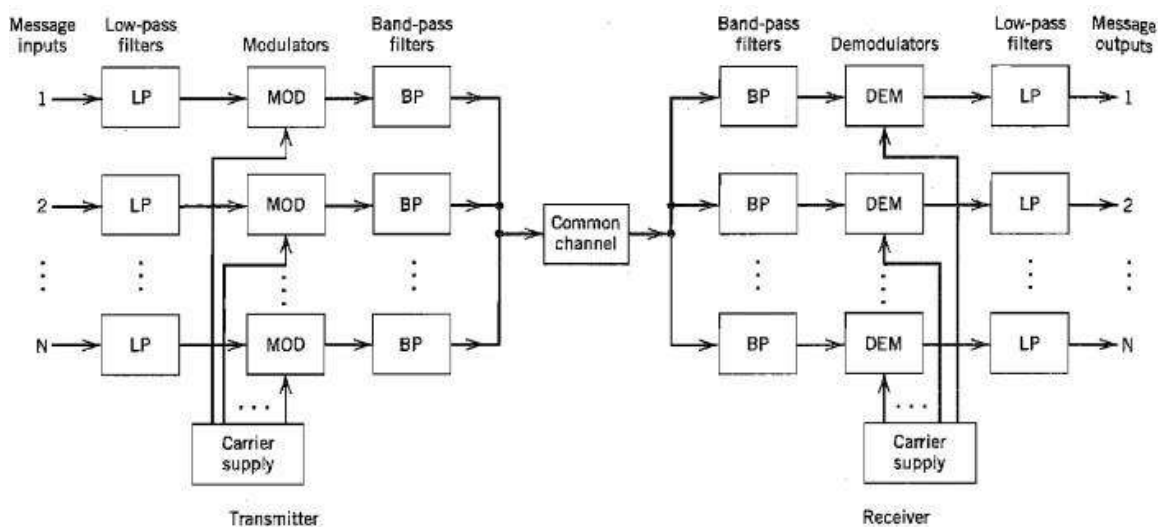


Fig.1. Frequency Division Multiplexing

### Amplitude Modulation (AM)

Amplitude Modulation is the process of changing the amplitude of a relatively high frequency carrier signal in accordance with the amplitude of the modulating signal (Information).

The carrier amplitude varied linearly by the modulating signal which usually consists of a range of audio frequencies. The frequency of the carrier is not affected.

- Application of AM - Radio broadcasting, TV pictures (video), facsimile transmission
- Frequency range for AM - 535 kHz – 1600 kHz
- Bandwidth - 10 kHz

### Various forms of Amplitude Modulation

- Conventional Amplitude Modulation (Alternatively known as Full AM or Double Sideband Large carrier modulation (DSBLC) /Double Sideband Full Carrier (DSBFC)
- Double Sideband Suppressed carrier (DSBSC) modulation
- Single Sideband (SSB) modulation
- Vestigial Sideband (VSB) modulation

### Time Domain and Frequency Domain Description

It is the process where, the amplitude of the carrier is varied proportional to that of the message signal.

Let  $m(t)$  be the base-band signal,  $m(t) \longleftrightarrow M(\omega)$  and  $c(t)$  be the carrier,  $c(t) = A_c \cos(\omega_c t)$ .  $f_c$  is chosen such that  $f_c \gg W$ , where  $W$  is the maximum frequency component of  $m(t)$ . The amplitude modulated signal is given by

$$s(t) = A_c [1 + k_a m(t)] \cos(\omega_c t)$$

Fourier Transform on both sides of the above equation

$$S(\omega) = \pi A_c / 2 (\delta(\omega - \omega_c) + \delta(\omega + \omega_c)) + k_a A_c / 2 (M(\omega - \omega_c) + M(\omega + \omega_c))$$

$k_a$  is a constant called amplitude sensitivity.

$k_a m(t) < 1$  and it indicates percentage modulation.

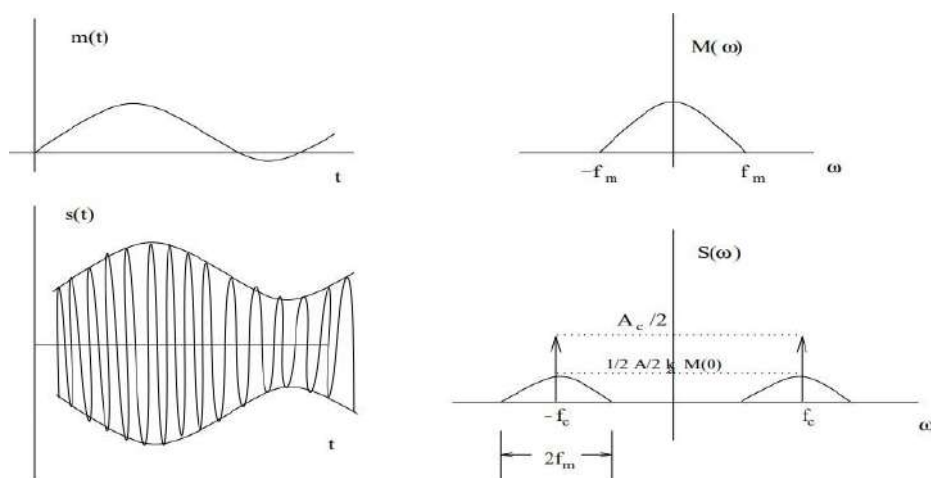


Fig.2. Amplitude modulation in time and frequency domain

### Single Tone Modulation:

Consider a modulating wave  $m(t)$  that consists of a single tone or single frequency component given by

$$m(t) = A_m \cos(2\pi f_m t) \quad \dots\dots\dots(1)$$

where  $A_m$  is peak amplitude of the sinusoidal modulating wave

$f_m$  is the frequency of the sinusoidal modulating wave

Let  $A_c$  be the peak amplitude and  $f_c$  be the frequency of the high frequency carrier signal. Then the corresponding single-tone AM wave is given by

$$s(t) = A_c [1 + m \cos(2\pi f_m t)] \cos(2\pi f_c t) \quad \dots\dots\dots(2)$$

Let  $A_{\max}$  and  $A_{\min}$  denote the maximum and minimum values of the envelope of the modulated wave. Then from the above equation (2.12), we get

$$\frac{A_{\max}}{A_{\min}} = \frac{A_c(1+m)}{A_c(1-m)}$$

$$m = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$$

Expanding the equation (2), we get

$$s(t) = A_c \cos(2\pi f_c t) + \frac{1}{2} mA_c \cos[2\pi(f_c + f_m)t] + \frac{1}{2} mA_c \cos[2\pi(f_c - f_m)t]$$

The Fourier transform of  $s(t)$  is obtained as follows.

$$s(f) = \frac{1}{2} A_c [\delta(f - f_c) + \delta(f + f_c)] + \frac{1}{4} mA_c [\delta(f - f_c - f_m) + \delta(f + f_c + f_m)] \\ + \frac{1}{4} mA_c [\delta(f - f_c + f_m) + \delta(f + f_c - f_m)]$$

Thus the spectrum of an AM wave, for the special case of sinusoidal modulation consists of delta functions at  $\pm f_c$ ,  $f_c \pm f_m$ , and  $-f_c \pm f_m$ . The spectrum for positive frequencies is as shown in figure

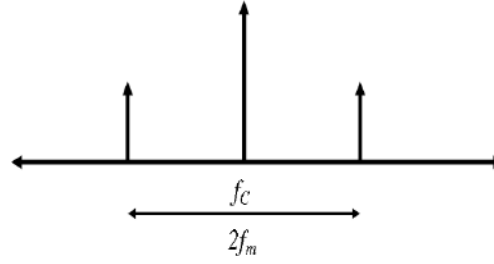


Fig.3. Frequency Domain characteristics of single tone AM

### Power relations in AM waves:

Consider the expression for single tone/sinusoidal AM wave

$$s(t) = A_c \cos(2\pi f_c t) + \frac{1}{2} m A_c \cos[2\pi(f_c + f_m)t] + \frac{1}{2} m A_c \cos[2\pi(f_c - f_m)t] \quad \text{.....(1)}$$

This expression contains three components. They are carrier component, upper side band and lower side band. Therefore Average power of the AM wave is sum of these three components.

Therefore the total power in the amplitude modulated wave is given by

$$P_t = \frac{V_{car}^2}{R} + \frac{V_{LSB}^2}{R} + \frac{V_{USB}^2}{R} \quad \text{.....(2)}$$

Where all the voltages are rms values and R is the resistance, in which the power is dissipated.

$$P_C = \frac{V_{car}^2}{R} = \frac{\left(\frac{A_c}{\sqrt{2}}\right)^2}{R} = \frac{A_c^2}{2R}$$

$$P_{LSB} = \frac{V_{LSB}^2}{R} = \left(\frac{mA_c}{2\sqrt{2}}\right)^2 \frac{1}{R} = \frac{m^2 A_c^2}{8R} = \frac{m^2}{4} P_c$$

$$P_{USB} = \frac{V_{USB}^2}{R} = \left(\frac{mA_c}{2\sqrt{2}}\right)^2 \frac{1}{R} = \frac{m^2 A_c^2}{8R} = \frac{m^2}{4} P_c$$

Therefore total average power is given by

$$\begin{aligned}
 P_t &= P_c + P_{LSB} + P_{USB} \\
 P_t &= P_c + \frac{m^2}{4} P_c + \frac{m^2}{4} P_c \\
 P_t &= P_c \left( 1 + \frac{m^2}{4} + \frac{m^2}{4} \right) \\
 P_t &= P_c \left( 1 + \frac{m^2}{2} \right) \quad \dots\dots\dots(3)
 \end{aligned}$$

**The ratio of total side band power to the total power in the modulated wave is given by**

$$\begin{aligned}
 \frac{P_{SB}}{P_t} &= \frac{P_c (m^2 / 2)}{P_c (1 + m^2 / 2)} \\
 \frac{P_{SB}}{P_t} &= \frac{m^2}{2 + m^2} \quad \dots\dots\dots(4)
 \end{aligned}$$

**This ratio is called the efficiency of AM system**

### **Generation of AM waves:**

Two basic amplitude modulation principles are discussed. They are square law modulation and switching modulator.

### **Square Law Modulator**

When the output of a device is not directly proportional to input throughout the operation, the device is said to be non-linear. The Input-Output relation of a non-linear device can be expressed as

$$V_o = a_0 + a_1 V_{in} + a_2 V_{in}^2 + a_3 V_{in}^3 + a_4 V_{in}^4 + \dots\dots\dots$$

When the input is very small, the higher power terms can be neglected. Hence the output is approximately given by  $V_o = a_0 + a_1 V_{in} + a_2 V_{in}^2$

When the output is considered up to square of the input, the device is called a square law device and the square law modulator is as shown in the figure 4

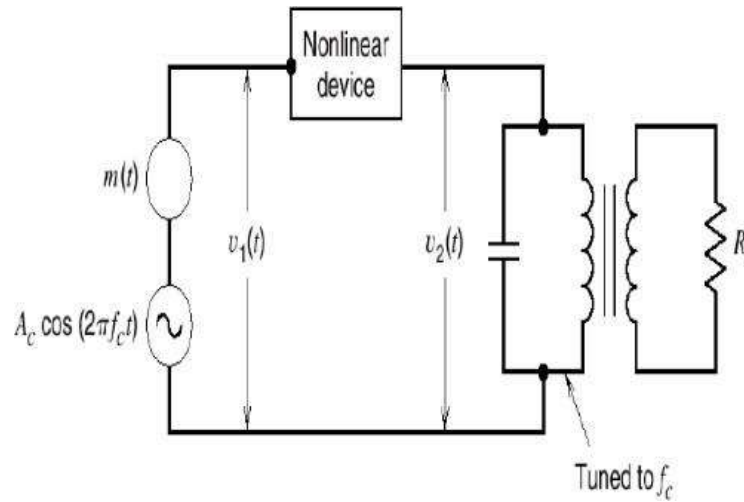


Fig.4. Square Law Modulator

Consider a non-linear device to which a carrier  $c(t)=A_c \cos(2\pi f_c t)$  and an information signal  $m(t)$  are fed simultaneously as shown in figure 4. The total input to the device at any instant is

$$V_{in} = c(t) + m(t)$$

$$V_{in} = A_c \cos 2\pi f_c t + m(t)$$

As the level of the input is very small, the output can be considered up to square of the input, i.e.,  $V_o = a_0 + a_1 V_{in} + a_2 V_{in}^2$

$$V_o = a_0 + a_1 [A_c \cos 2\pi f_c t + m(t)] + a_2 [A_c \cos 2\pi f_c t + m(t)]^2$$

$$V_o = a_0 + a_1 A_c \cos 2\pi f_c t + a_1 m(t) + \frac{a_2 A_c^2}{2} (1 + \cos 4\pi f_c t) + a_2 [m(t)]^2 + 2a_2 m(t) A_c \cos 2\pi f_c t$$

$$V_o = a_0 + a_1 A_c \cos 2\pi f_c t + a_1 m(t) + \frac{a_2 A_c^2}{2} \cos 4\pi f_c t + a_2 m^2(t) + 2a_2 m(t) A_c \cos 2\pi f_c t$$

Taking Fourier transform on both sides, we get

$$V_o(f) = (a_0 + \frac{a_2 A_c^2}{2}) \delta(f) + \frac{a_1 A_c}{2} [\delta(f - f_c) + \delta(f + f_c)] + a_1 M(f) + \frac{a_2 A_c^2}{4} [\delta(f - 2f_c) + \delta(f + 2f_c)] + a_2 M(f) + a_2 A_c [M(f - f_c) + M(f + f_c)]$$

Therefore the square law device output 0 V consists of the dc component at  $f = 0$ . The information signal ranging from 0 to W Hz and its second harmonics are signal at  $f_c$  and  $2f_c$ .

Frequency band centered at  $f_c$  with a deviation of  $\pm W$ , Hz.

The required AM signal with a carrier frequency  $f_c$  can be separated using a band pass filter at the out put of the square law device. The filter should have a lower cut-off frequency ranging between  $2W$  and  $(f_c - W)$  and upper cut-off frequency between  $(f_c + W)$  and  $2f_c$

Therefore the filter out put is

$$s(t) = a_1 A_c \cos 2\pi f_c t + 2a_2 A_c m(t) \cos 2\pi f_c t$$

$$s(t) = a_1 A_c \left[ 1 + 2 \frac{a_2}{a_1} m(t) \right] \cos 2\pi f_c t$$

If  $m(t) = A_m \cos 2\pi f_m t$ , we get

$$s(t) = a_1 A_c \left[ 1 + 2 \frac{a_2}{a_1} A_m \cos 2\pi f_m t \right] \cos 2\pi f_c t$$

Comparing this with the standard representation of AM signal,

$$s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$

Therefore modulation index of the output signal is given by

$$m = 2 \frac{a_2}{a_1} A_m$$

The output AM signal is free from distortion and attenuation only when  $(f_c - W) > 2W$  or  $f_c > 3W$ .

Spectrum is as shown below

### Switching Modulator

Consider a semiconductor diode used as an ideal switch to which the carrier signal  $c(t) = A_c \cos(2\pi f_c t)$  and information signal  $m(t)$  are applied simultaneously as shown figure

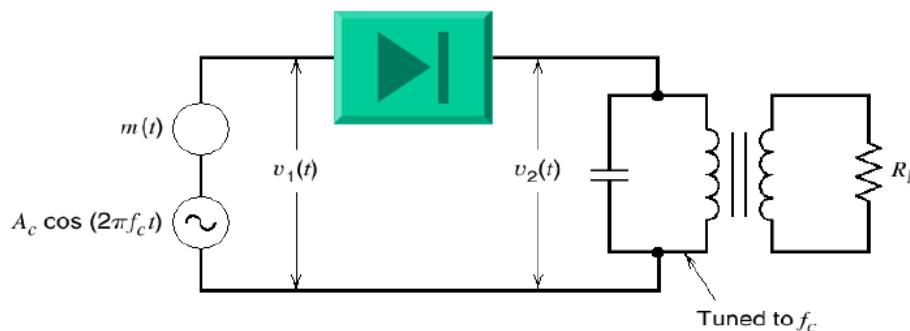


Fig.5. Switching Modulator

The total input for the diode at any instant is given by

$$v_1 = c(t) + m(t)$$

$$v_1 = A_c \cos 2\pi f_c t + m(t)$$

When the peak amplitude of  $c(t)$  is maintained more than that of information signal, the operation is assumed to be dependent on only  $c(t)$  irrespective of  $m(t)$ .

When  $c(t)$  is positive,  $v_2=v_1$  since the diode is forward biased. Similarly, when  $c(t)$  is negative,  $v_2=0$  since diode is reverse biased. Based upon above operation, switching response of the diode is periodic rectangular wave with an amplitude unity and is given by

$$p(t) = \frac{1}{2} + \frac{1}{\pi} \sum_{n=-\infty}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos(2\pi f_c t (2n-1))$$

$$p(t) = \frac{1}{2} + \frac{2}{\pi} \cos(2\pi f_c t) - \frac{2}{3\pi} \cos(6\pi f_c t) + -$$

$n=0,1$   $n=-1,+2$

Therefore the diode response  $V_o$  is a product of switching response  $p(t)$  and input  $v_1$ .

$$v_2 = v_1 * p(t)$$

$$V_2 = [A_c \cos 2\pi f_c t + m(t)] \left[ \frac{1}{2} + \frac{2}{\pi} \cos 2\pi f_c t - \frac{2}{3\pi} \cos 6\pi f_c t + - - \right]$$

Applying the Fourier Transform, we get

$$\begin{aligned} V_2(f) = & \frac{A_c}{4} [\delta(f - f_c) + \delta(f + f_c)] + \frac{M(f)}{2} + \frac{A_c}{\pi} \delta(f) \\ & + \frac{A_c}{2\pi} [\delta(f - 2f_c) + \delta(f + 2f_c)] + \frac{1}{\pi} [M(f - f_c) + M(f + f_c)] \\ & - \frac{A_c}{6\pi} [\delta(f - 4f_c) + \delta(f + 4f_c)] - \frac{A_c}{3\pi} [\delta(f - 2f_c) + \delta(f + 2f_c)] \\ & - \frac{1}{3\pi} [M(f - 3f_c) + M(f + f_c)] \end{aligned}$$

The diode output  $v_2$  consists of

a dc component at  $f=0$ .

Information signal ranging from 0 to  $w$  Hz and infinite number of frequency bands centered at  $f, 2f_c, 3f_c, 4f_c, \dots$

The required AM signal centred at  $f_c$  can be separated using band pass filter. The lower cut off-frequency for the band pass filter should be between  $w$  and  $f_c - w$  and the upper cut-off frequency between  $f_c + w$  and  $2f_c$ . The filter output is given by the equation

$$S(t) = \frac{A_c}{2} \left[ 1 + \frac{4}{\pi} \frac{m(t)}{A_c} \right] \cos 2\pi f_c t$$

For a single tone information, let  $m(t) = A_m \cos(2\pi f_m t)$

$$S(t) = \frac{A_c}{2} \left[ 1 + \frac{4}{\pi} \frac{A_m}{A_c} \cos 2\pi f_m t \right] \cos 2\pi f_c t$$

Therefore modulation index,  $m = \frac{4}{\pi} \frac{A_m}{A_c}$

The output AM signal is free from distortions and attenuations only when  $f_c - w > w$  or  $f_c > 2w$ .

### Detection of AM waves

Demodulation is the process of recovering the information signal (base band) from the incoming modulated signal at the receiver. There are two methods, they are Square law Detector and Envelope Detector

#### Square Law Detector

Consider a non-linear device to which the AM signal  $s(t)$  is applied. When the level of  $s(t)$  is very small, output can be considered upto square of the input.

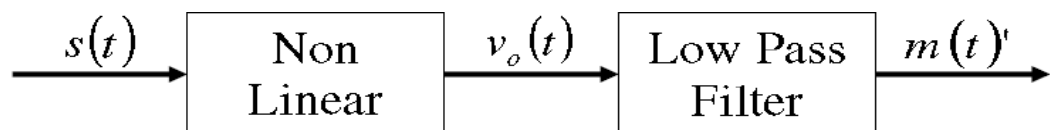


Figure: Demodulation of AM using square law device

$$\text{Therefore } V_o = a_o + a_1 V_{in} + a_2 V_{in}^2$$

If  $m(t)$  is the information signal (0-wHz) and  $c(t) = A_c \cos(2\pi f_c t)$  is the carrier, input AM signal to the non-linear device is given by

$$s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$

$$V_o = a_o + a_1 s(t) + a_2 [s(t)]^2$$

$$V_o = a_o + a_1 A_c \cos 2\pi f_c t + a_1 A_c K_a m(t) \cos 2\pi f_c t + a_2 [A_c \cos 2\pi f_c t + A_c K_a m(t) \cos 2\pi f_c t]^2$$

Applying Fourier transform on both sides, we get

$$\begin{aligned} V_o(f) = & \left[ a_o + \frac{a_2 A_c^2}{2} \right] \delta(f) + \frac{a_1 A_c}{2} [\delta(f - f_c) + \delta(f + f_c)] \\ & + \frac{a_1 A_c K_a}{2} [M(f - f_c) + M(f + f_c)] + \frac{a_2 A_c^2 K_a^2}{4} [M(f - 2f_c) + M(f + 2f_c)] \\ & + \frac{a_2 A_c^2 K_a^2}{2} \left[ M(f) \right]_{\pm 2W} + \frac{a_2 A_c^2 K_a^2}{2} [M(f - 2f_c) + M(f + 2f_c)] \\ & + \frac{a_2 A_c^2}{4} [\delta(f - 2f_c) + \delta(f + 2f_c)] + a_2 A_c^2 K_a [M(f)] \end{aligned}$$

The device output consists of a dc component at  $f = 0$ , information signal ranging from 0-W Hz and its second harmonics and frequency bands centered at  $f_c$  and  $2f_c$ . The required information can be separated using low pass filter with cut off frequency ranging between W and  $f_c - w$ . The filter output is given by

$$m'(t) = \left( a_o + \frac{a_2 A_c^2}{2} \right) + a_2 A_c^2 K_a m(t) + \frac{a_2 A_c^2 K_a^2 m^2(t)}{2}$$

DC component + message signal + second harmonic

The dc component (first term) can be eliminated using a coupling capacitor or a transformer. The effect of second harmonics of information signal can be reduced by maintaining its level very low. When  $m(t)$  is very low, the filter output is given by

$$m^1(t) = a_2 A_c^2 K_a m(t)$$

When the information level is very low, the noise effect increases at the receiver, hence the system clarity is very low using square law demodulator.

### Envelope Detector

It is a simple and highly effective system. This method is used in most of the commercial AM radio receivers. An envelope detector is as shown below.

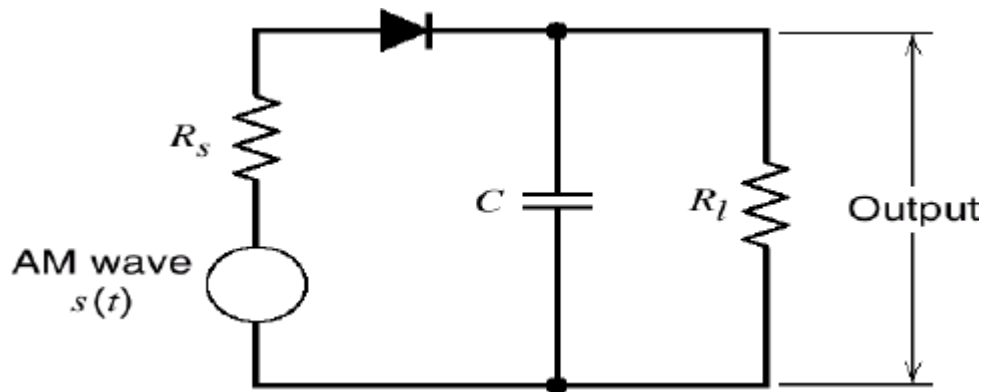


Fig.7. Envelope Detector

During the positive half cycles of the input signals, the diode D is forward biased and the capacitor C charges up rapidly to the peak of the input signal. When the input signal falls below this value, the diode becomes reverse biased and the capacitor C discharges through the load resistor  $R_L$ .

The discharge process continues until the next positive half cycle. When the input signal becomes greater than the voltage across the capacitor, the diode conducts again and the process is repeated.

The charge time constant  $(r_f + R_s)C$  must be short compared with the carrier period, the capacitor charges rapidly and thereby follows the applied voltage up to the positive peak when the diode is conducting. That is the charging time constant shall satisfy the condition,

$$(r_f + R_s)C \ll \frac{1}{f_c}$$

On the other hand, the discharging time-constant  $R_L C$  must be long enough to ensure that the capacitor discharges slowly through the load resistor  $R_L$  between the positive peaks of the carrier wave, but not so long that the capacitor voltage will not discharge at the maximum rate of change of the modulating wave.

That is the discharge time constant shall satisfy the condition,

$$\frac{1}{f_c} \ll R_L C \ll \frac{1}{W}$$

Where 'W' is band width of the message signal. The result is that the capacitor voltage or detector output is nearly the same as the envelope of AM wave.

#### **Advantages and Disadvantages of AM:**

##### **Advantages of AM:**

- Generation and demodulation of AM wave are easy.
- AM systems are cost effective and easy to build.

**Disadvantages:**

- AM contains unwanted carrier component, hence it requires more transmission power.
- The transmission bandwidth is equal to twice the message bandwidth.

To overcome these limitations, the conventional AM system is modified at the cost of increased system complexity. Therefore, three types of modified AM systems are discussed.

**DSBSC (Double Side Band Suppressed Carrier) modulation:**

In DSBSC modulation, the modulated wave consists of only the upper and lower side bands. Transmitted power is saved through the suppression of the carrier wave, but the channel bandwidth requirement is the same as before.

**SSBSC (Single Side Band Suppressed Carrier) modulation:** The SSBSC modulated wave consists of only the upper side band or lower side band. SSBSC is suited for transmission of voice signals. It is an optimum form of modulation in that it requires the minimum transmission power and minimum channel bandwidth. Disadvantage is increased cost and complexity.

**VSB (Vestigial Side Band) modulation:** In VSB, one side band is completely passed and just a trace or vestige of the other side band is retained. The required channel bandwidth is therefore in excess of the message bandwidth by an amount equal to the width of the vestigial side band. This method is suitable for the transmission of wide band signals.

## DSB-SC MODULATION

### DSB-SC Time domain and Frequency domain Description:

DSBSC modulators make use of the multiplying action in which the modulating signal multiplies the carrier wave. In this system, the carrier component is eliminated and both upper and lower side bands are transmitted. As the carrier component is suppressed, the power required for transmission is less than that of AM.

If  $m(t)$  is the message signal and  $c(t) = A_c \cos(2\pi f_c t)$  is the carrier signal, then DSBSC modulated wave  $s(t)$  is given by

$$s(t) = c(t) m(t)$$
$$s(t) = A_c \cos(2\pi f_c t) m(t)$$

Consequently, the modulated signal  $s(t)$  undergoes a phase reversal, whenever the message signal  $m(t)$  crosses zero as shown below.

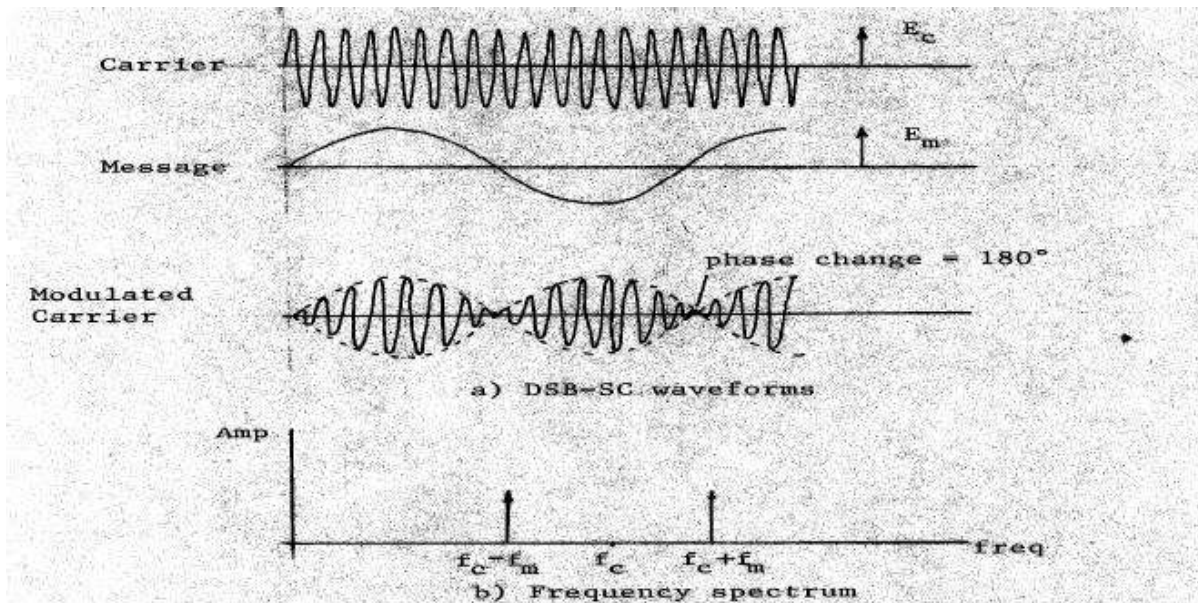


Fig.1. (a) DSB-SC waveform (b) DSB-SC Frequency Spectrum

The envelope of a DSBSC modulated signal is therefore different from the message signal and the Fourier transform of  $s(t)$  is given by

$$S(f) = \frac{A_c}{2} [M(f - f_c) + M(f + f_c)]$$

For the case when base band signal  $m(t)$  is limited to the interval  $-W < f < W$  as shown in figure below, we find that the spectrum  $S(f)$  of the DSBSC wave  $s(t)$  is as illustrated below. Except for a change in scaling factor, the modulation process simply translates the spectrum of the base band signal by  $f_c$ . The transmission bandwidth required by DSBSC modulation is the same as that for AM.

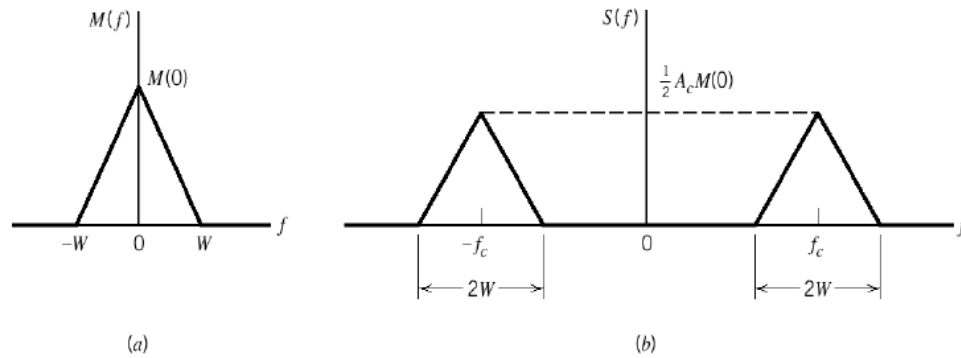


Figure: Message and the corresponding DSBSC spectrum

### Generation of DSBSC Waves:

#### Balanced Modulator (Product Modulator)

A balanced modulator consists of two standard amplitude modulators arranged in a balanced configuration so as to suppress the carrier wave as shown in the following block diagram. It is assumed that the AM modulators are identical, except for the sign reversal of the modulating wave applied to the input of one of them. Thus, the output of the two modulators may be expressed as,

$$s_1(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$

$$s_2(t) = A_c [1 - k_a m(t)] \cos(2\pi f_c t)$$

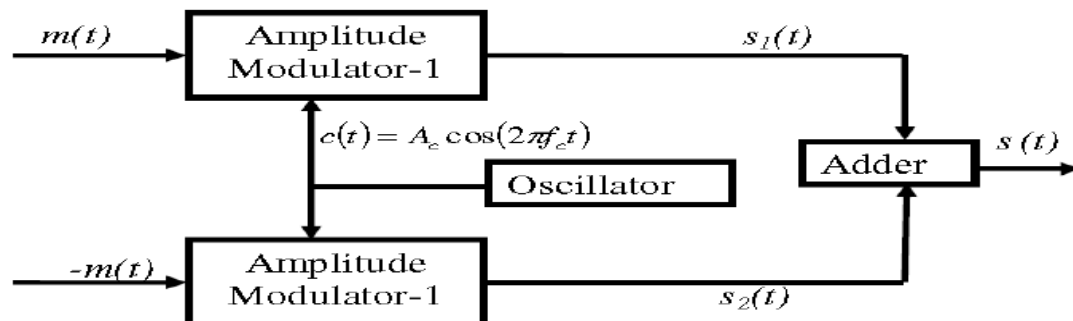


Fig .3 : Balanced modulator

Subtracting  $s_2(t)$  from  $s_1(t)$ ,

$$s(t) = s_1(t) - s_2(t)$$

$$s(t) = 2k_a m(t) A_c \cos(2\pi f_c t)$$

Hence, except for the scaling factor  $2ka$ , the balanced modulator output is equal to the product of the modulating wave and the carrier.

### Ring Modulator

Ring modulator is the most widely used product modulator for generating DSBSC wave and is shown below.

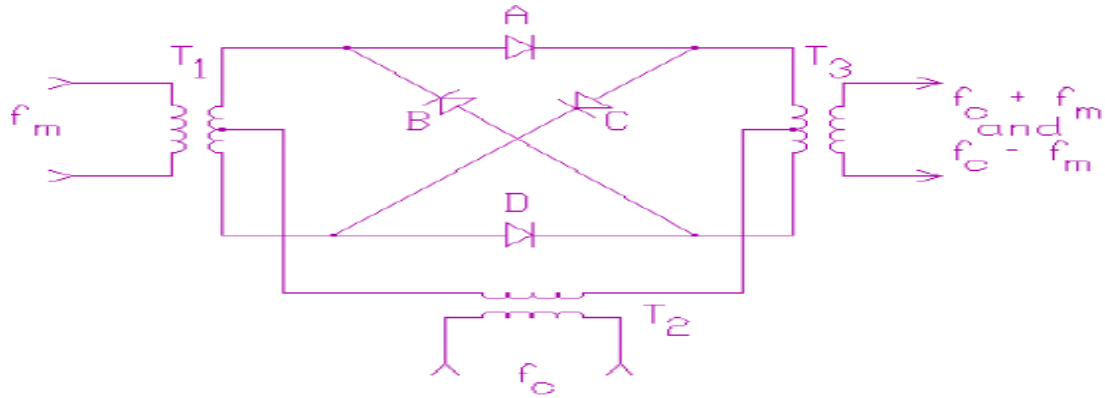


Fig.4 : Ring modulator

The four diodes form a ring in which they all point in the same direction. The diodes are controlled by square wave carrier  $c(t)$  of frequency  $f_c$ , which is applied longitudinally by means of two center-tapped transformers. Assuming the diodes are ideal, when the carrier is positive, the outer diodes D1 and D2 are forward biased where as the inner diodes D3 and D4 are reverse biased, so that the modulator multiplies the base band signal  $m(t)$  by  $c(t)$ . When the carrier is negative, the diodes D1 and D2 are reverse biased and D3 and D4 are forward, and the modulator multiplies the base band signal  $-m(t)$  by  $c(t)$ .

Thus the ring modulator in its ideal form is a product modulator for square wave carrier and the base band signal  $m(t)$ . The square wave carrier can be expanded using Fourier series as

$$c(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos(2\pi f_c t (2n-1))$$

Therefore the ring modulator out put is given by

$$s(t) = m(t)c(t)$$

$$s(t) = m(t) \left[ \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos(2\pi f_c t (2n-1)) \right]$$

From the above equation it is clear that output from the modulator consists entirely of modulation products. If the message signal  $m(t)$  is band limited to the frequency band  $-w < f < w$ , the output spectrum consists of side bands centred at  $f_c$ .

## Detection of DSB-SC waves:

### Coherent Detection:

The message signal  $m(t)$  can be uniquely recovered from a DSBSC wave  $s(t)$  by first multiplying  $s(t)$  with a locally generated sinusoidal wave and then low pass filtering the product as shown.

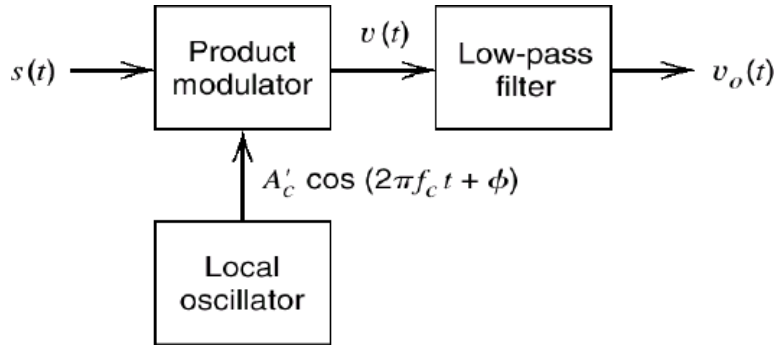


Fig.5 : Coherent detector

It is assumed that the local oscillator signal is exactly coherent or synchronized, in both frequency and phase, with the carrier wave  $c(t)$  used in the product modulator to generate  $s(t)$ . This method of demodulation is known as coherent detection or synchronous detection.

Let  $A_c^{-1} \cos(2\pi f_c t + \phi)$  be the local oscillator signal, and  $s(t) = A_c \cos(2\pi f_c t) m(t)$  be the DSBSC wave. Then the product modulator output  $v(t)$  is given by

$$v(t) = A_c A_c^{-1} \cos(2\pi f_c t) \cos(2\pi f_c t + \phi) m(t)$$

$$v(t) = \frac{A_c A_c^{-1}}{4} \cos(4\pi f_c t + \phi) m(t) + \frac{A_c A_c^{-1}}{2} \cos(\phi) m(t)$$

The first term in the above expression represents a DSBSC modulated signal with a carrier frequency  $2f_c$ , and the second term represents the scaled version of message signal. Assuming that the message signal is band limited to the interval  $-w < f < w$ , the spectrum of  $v(t)$  is plotted as shown below.

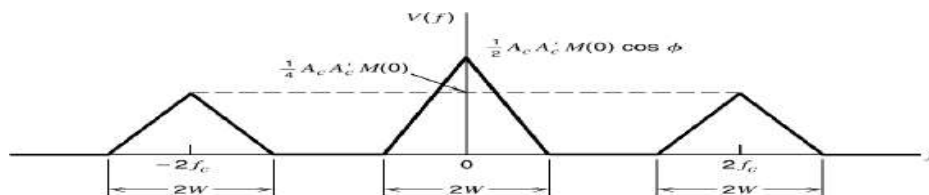


Fig.6. Spectrum of output of the product modulator

From the spectrum, it is clear that the unwanted component (first term in the expression) can be removed by the low-pass filter, provided that the cut-off frequency of the filter is greater than  $W$  but less than  $2f_c - W$ . The filter output is given by

$$v_o(t) = \frac{A_c A_c^1}{2} \cos(\phi) m(t)$$

The demodulated signal  $v_o(t)$  is therefore proportional to  $m(t)$  when the phase error  $\phi$  is constant.

### Costas Receiver (Costas Loop):

Costas receiver is a synchronous receiver system, suitable for demodulating DSBSC waves. It consists of two coherent detectors supplied with the same input signal,

that is the incoming DSBSC wave  $s(t) = A_c \cos(2\pi f_c t) m(t)$  but with individual local oscillator signals that are in phase quadrature with respect to each other as shown below.

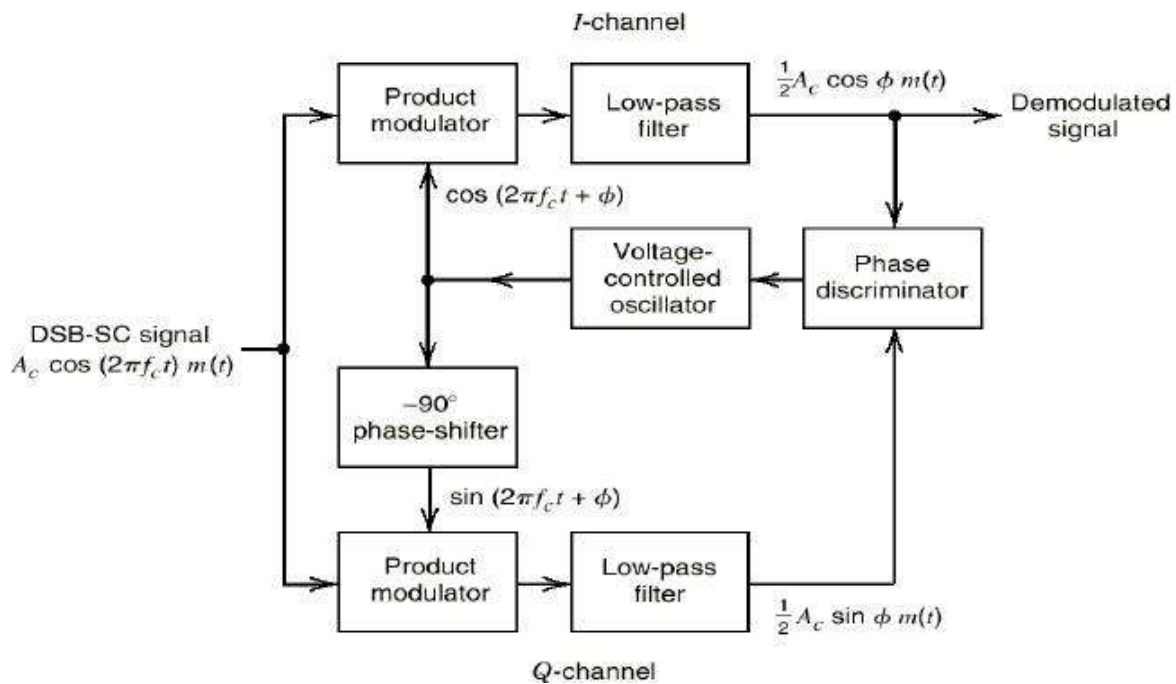


Fig.7. Costas Receiver

The frequency of the local oscillator is adjusted to be the same as the carrier frequency  $f_c$ . The detector in the upper path is referred to as the in-phase coherent detector or I-channel, and that in the lower path is referred to as the quadrature-phase coherent detector or Q-channel.

These two detectors are coupled together to form a negative feedback system designed in such a way as to maintain the local oscillator synchronous with the carrier wave. Suppose

the local oscillator signal is of the same phase as the carrier  $c(t) = A_c \cos(2\pi f_c t)$  wave used to generate the incoming DSBSC wave. Then we find that the I-channel output contains the desired demodulated signal  $m(t)$ , whereas the Q-channel output is zero due to quadrature null effect of the Q-channel. Suppose that the local oscillator phase drifts from its proper value by a small angle  $\phi$  radians. The I-channel output will remain essentially unchanged, but there will be some signal appearing at the Q-channel output, which is proportional to  $\sin(\phi) \approx \phi$  for small  $\phi$ .

This Q-channel output will have same polarity as the I-channel output for one direction of local oscillator phase drift and opposite polarity for the opposite direction of local oscillator phase drift. Thus by combining the I-channel and Q-channel outputs in a phase discriminator (which consists of a multiplier followed by a LPF), a dc control signal is obtained that automatically corrects for the local phase errors in the voltage-controlled oscillator.

## Radio Transmitters

There are two approaches in generating an AM signal. These are known as low and high level modulation. They're easy to identify: A low level AM transmitter performs the process of modulation near the beginning of the transmitter. A high level transmitter performs the modulation step last, at the last or "final" amplifier stage in the transmitter. Each method has advantages and disadvantages, and both are in common use.

### Low-Level AM Transmitter:

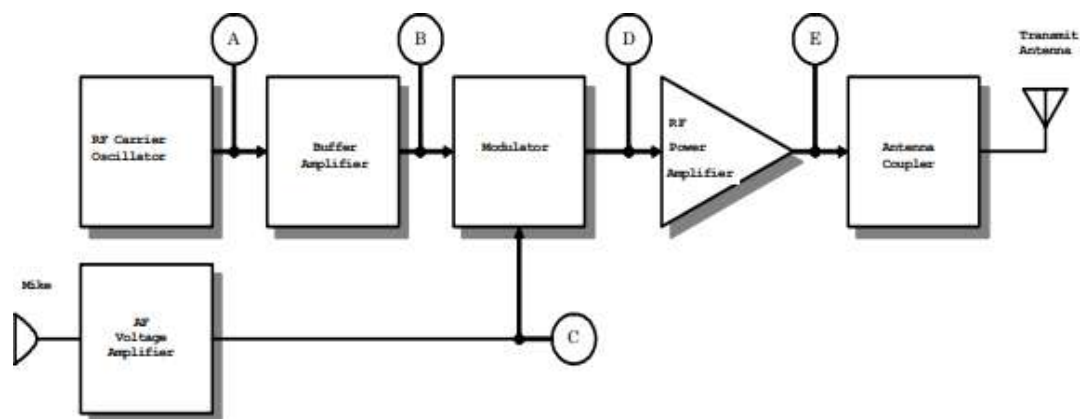


Fig.8. Low-Level AM Transmitter Block Diagram

There are two signal paths in the transmitter, audio frequency (AF) and radio frequency (RF). The RF signal is created in the RF carrier oscillator. At test point A the oscillator's output signal is present. The output of the carrier oscillator is a fairly small AC voltage, perhaps 200 to 400 mV RMS. The oscillator is a critical stage in any transmitter. It must produce an accurate and steady frequency. Every radio station is assigned a different carrier frequency. The dial (or display) of a receiver displays the carrier frequency. If the

oscillator drifts off frequency, the receiver will be unable to receive the transmitted signal without being readjusted. Worse yet, if the oscillator drifts onto the frequency being used by another radio station, interference will occur. Two circuit techniques are commonly used to stabilize the oscillator, buffering and voltage regulation.

The buffer amplifier has something to do with buffering or protecting the oscillator. An oscillator is a little like an engine (with the speed of the engine being similar to the oscillator's frequency). If the load on the engine is increased (the engine is asked to do more work), the engine will respond by slowing down. An oscillator acts in a very similar fashion. If the current drawn from the oscillator's output is increased or decreased, the oscillator may speed up or slow down slightly.

**Buffer amplifier** is a relatively low-gain amplifier that follows the oscillator. It has a constant input impedance (resistance). Therefore, it always draws the same amount of current from the oscillator. This helps to prevent "pulling" of the oscillator frequency. The buffer amplifier is needed because of what's happening "downstream" of the oscillator. Right after this stage is the modulator. Because the modulator is a nonlinear amplifier, it may not have a constant input resistance -- especially when information is passing into it. But since there is a buffer amplifier between the oscillator and modulator, the oscillator sees a steady load resistance, regardless of what the modulator stage is doing.

**Voltage Regulation:** An oscillator can also be pulled off frequency if its power supply voltage isn't held constant. In most transmitters, the supply voltage to the oscillator is regulated at a constant value. The regulated voltage value is often between 5 and 9 volts; zener diodes and three-terminal regulator ICs are commonly used voltage regulators. Voltage regulation is especially important when a transmitter is being powered by batteries or an automobile's electrical system. As a battery discharges, its terminal voltage falls. The DC supply voltage in a car can be anywhere between 12 and 16 volts, depending on engine RPM and other electrical load conditions within the vehicle.

**Modulator:** The stabilized RF carrier signal feeds one input of the modulator stage. The modulator is a variable-gain (nonlinear) amplifier. To work, it must have an RF carrier signal and an AF information signal. In a low-level transmitter, the power levels are low in the oscillator, buffer, and modulator stages; typically, the modulator output is around 10 mW (700 mV RMS into 50 ohms) or less.

**AF Voltage Amplifier:** In order for the modulator to function, it needs an information signal. A microphone is one way of developing the intelligence signal, however, it only produces a few millivolts of signal. This simply isn't enough to operate the modulator, so a voltage amplifier is used to boost the microphone's signal. The signal level at the output of the AF voltage amplifier is usually at least 1 volt RMS; it is highly dependent upon the transmitter's design. Notice that the AF amplifier in the transmitter is only providing a voltage gain, and not necessarily a current gain for the microphone's signal. The power levels are quite small at the output of this amplifier; a few mW at best.

**RF Power Amplifier:** At test point D the modulator has created an AM signal by impressing the information signal from test point C onto the stabilized carrier signal from test point B at the buffer amplifier output. This signal (test point D) is a complete AM signal, but has only a few milliwatts of power. The RF power amplifier is normally built with several stages. These stages increase both the voltage and current of the AM signal. We say that power amplification occurs when a circuit provides a current gain. In order to accurately amplify the tiny AM signal from the modulator, the RF power amplifier stages must be linear. You might recall that amplifiers are divided up into "classes," according to the conduction angle of the active device within. Class A and class B amplifiers are considered to be linear amplifiers, so the RF power amplifier stages will normally be constructed using one or both of these type of amplifiers. Therefore, the signal at test point E looks just like that of test point D; it's just much bigger in voltage and current.

**Antenna Coupler:** The antenna coupler is usually part of the last or final RF power amplifier, and as such, is not really a separate active stage. It performs no amplification, and has no active devices. It performs two important jobs: Impedance matching and filtering. For an RF power amplifier to function correctly, it must be supplied with a load resistance equal to that for which it was designed.

The antenna coupler also acts as a low-pass filter. This filtering reduces the amplitude of harmonic energies that may be present in the power amplifier's output. (All amplifiers generate harmonic distortion, even "linear" ones.) For example, the transmitter may be tuned to operate on 1000 kHz. Because of small nonlinearities in the amplifiers of the transmitter, the transmitter will also produce harmonic energies on 2000 kHz (2nd harmonic), 3000 kHz (3rd harmonic), and so on. Because a low-pass filter passes the fundamental frequency (1000 kHz) and rejects the harmonics, we say that harmonic attenuation has taken place.

### High-Level AM Transmitter:

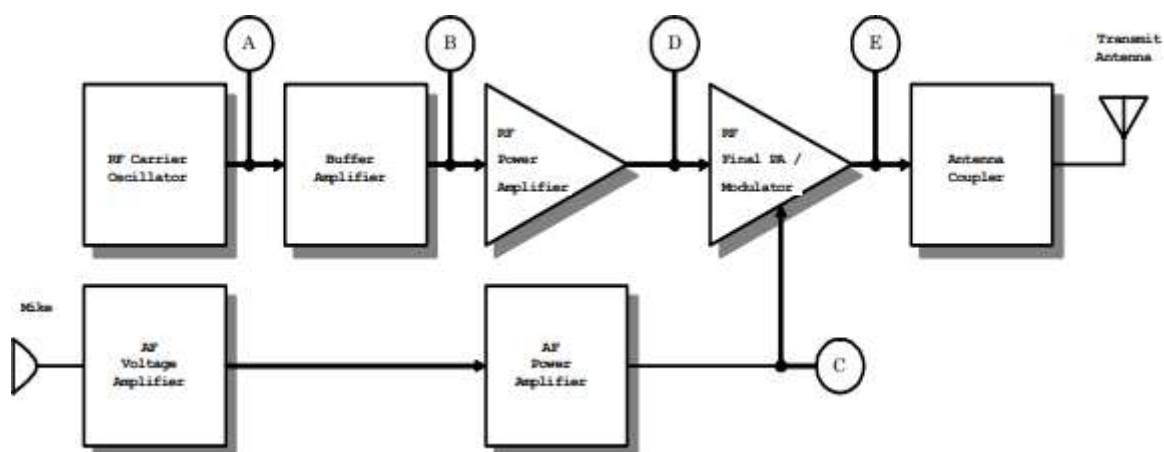


Fig.9. Low-Level AM Transmitter Block Diagram

The high-level transmitter of Figure 9 is very similar to the low-level unit. The RF section begins just like the low-level transmitter; there is an oscillator and buffer amplifier. The difference in the high level transmitter is where the modulation takes place. Instead of

adding modulation immediately after buffering, this type of transmitter amplifies the unmodulated RF carrier signal first. Thus, the signals at points A, B, and D in Figure 9 all look like unmodulated RF carrier waves. The only difference is that they become bigger in voltage and current as they approach test point D.

The modulation process in a high-level transmitter takes place in the last or final power amplifier. Because of this, an additional audio amplifier section is needed. In order to modulate an amplifier that is running at power levels of several watts (or more), comparable power levels of information are required. Thus, an audio power amplifier is required. The final power amplifier does double-duty in a high-level transmitter. First, it provides power gain for the RF carrier signal, just like the RF power amplifier did in the low-level transmitter. In addition to providing power gain, the final PA also performs the task of modulation. The final power amplifier in a high-level transmitter usually operates in class C, which is a highly nonlinear amplifier class.

### **Comparison:**

#### **Low Level Transmitters**

- Can produce any kind of modulation; AM, FM, or PM.
- Require linear RF power amplifiers, which reduce DC efficiency and increase production costs.

#### **High Level Transmitters**

- Have better DC efficiency than low-level transmitters, and are very well suited for battery operation.
- Are restricted to generating AM modulation only.

## UNIT-II

### Introduction of SSB-SC

Standard AM and DSBSC require transmission bandwidth equal to twice the message bandwidth. In both the cases spectrum contains two side bands of width  $W$  Hz, each. But the upper and lower sides are uniquely related to each other by the virtue of their symmetry about the carrier frequency. That is, given the amplitude and phase spectra of either side band, the other can be uniquely determined. Thus if only one side band is transmitted, and if both the carrier and the other side band are suppressed at the transmitter, no information is lost. This kind of modulation is called SSBSC and spectral comparison between DSBSC and SSBSC is shown in the figures 1 and 2.



Figure.1 : Spectrum of the DSBSC wave



Figure .2 : Spectrum of the SSBSC wave

### Frequency Domain Description

Consider a message signal  $m(t)$  with a spectrum  $M(f)$  band limited to the interval  $-w < f < w$  as shown in figure 3, the DSBSC wave obtained by multiplexing  $m(t)$  by the carrier wave  $c(t) = A_c \cos(2\pi f_c t)$  and is also shown, in figure 4. The upper side band is represented in duplicate by the frequencies above  $f_c$  and those below  $-f_c$ , and when only upper

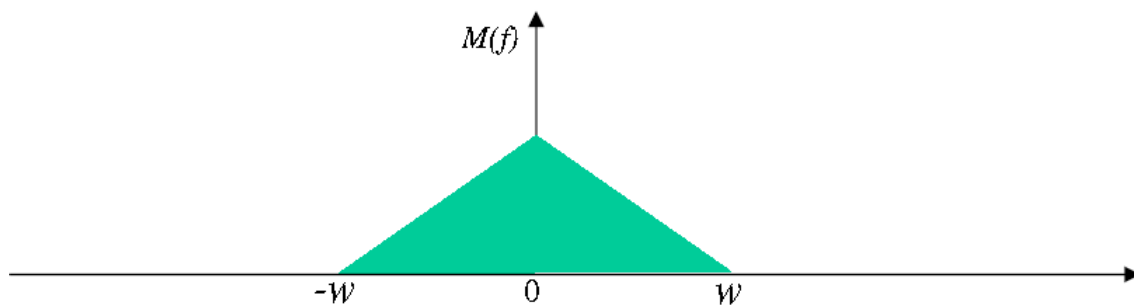


Figure 3. : Spectrum of message wave

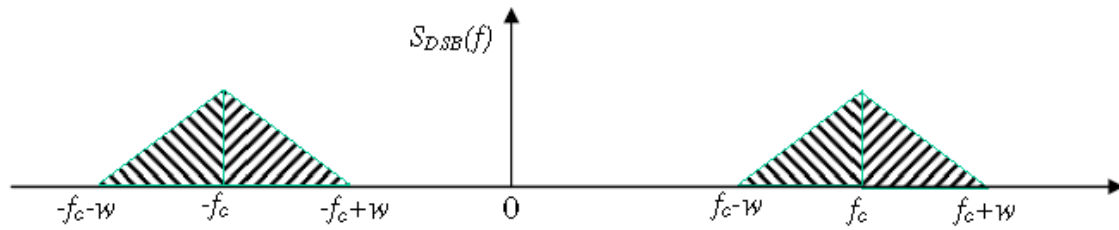


Figure .4 : Spectrum of DSBSC wave

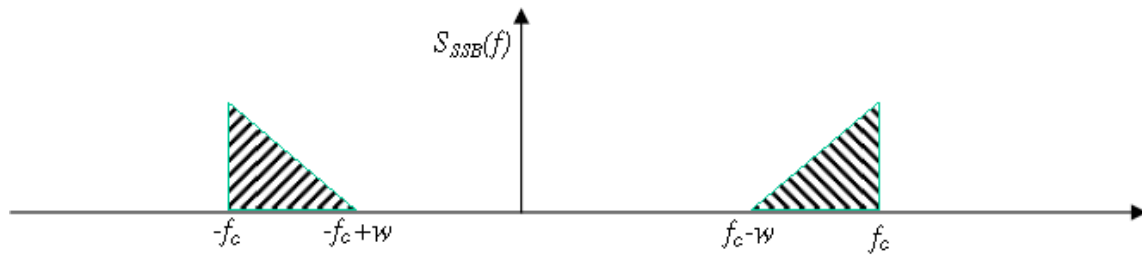


Figure.5 : Spectrum of SSBSC-LSB wave

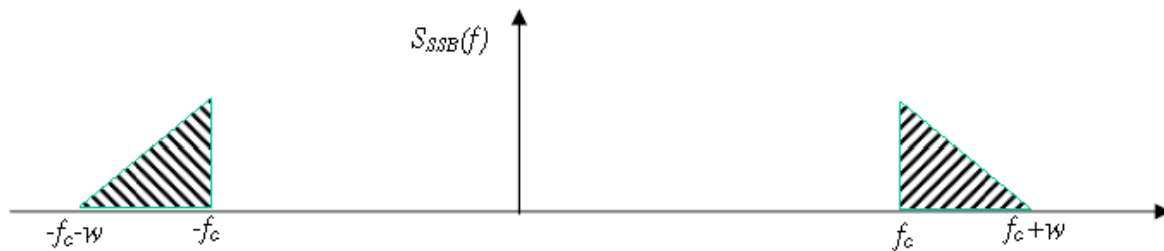


Figure .6 : Spectrum of SSBSC-USB wave

side band is transmitted; the resulting SSB modulated wave has the spectrum shown in figure 1. Similarly, the lower side band is represented in duplicate by the frequencies below  $f_c$  and those above  $-f_c$  and when only the lower side band is transmitted, the spectrum of the corresponding SSB modulated wave shown in figure 5. Thus the essential function of the SSB modulation is to translate the spectrum of the modulating wave, either with or without inversion, to a new location in the frequency domain. The advantage of SSB modulation is reduced bandwidth and the elimination of high power carrier wave. The main disadvantage is the cost and complexity of its implementation.

### Generation of SSB wave:

#### Frequency discrimination method

Consider the generation of SSB modulated signal containing the upper side band only. From a practical point of view, the most severe requirement of SSB generation arises from the unwanted sideband, the nearest component of which is separated from the desired side band by twice the lowest frequency component of the message signal. It implies that, for the generation of an SSB wave to be possible, the message spectrum must have an energy gap centered at the origin as shown in figure 7. This requirement is naturally satisfied by voice signals, whose energy gap is about 600Hz wide.

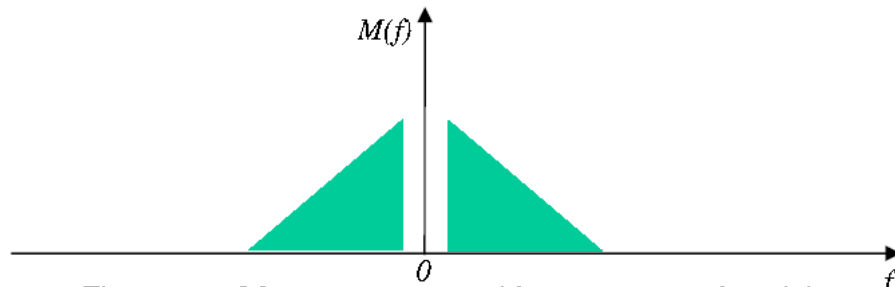


Figure .7 : Message spectrum with energy gap at the origin

The frequency discrimination or filter method of SSB generation consists of a product modulator, which produces DSBSC signal and a band-pass filter to extract the desired side band and reject the other and is shown in the figure 8.

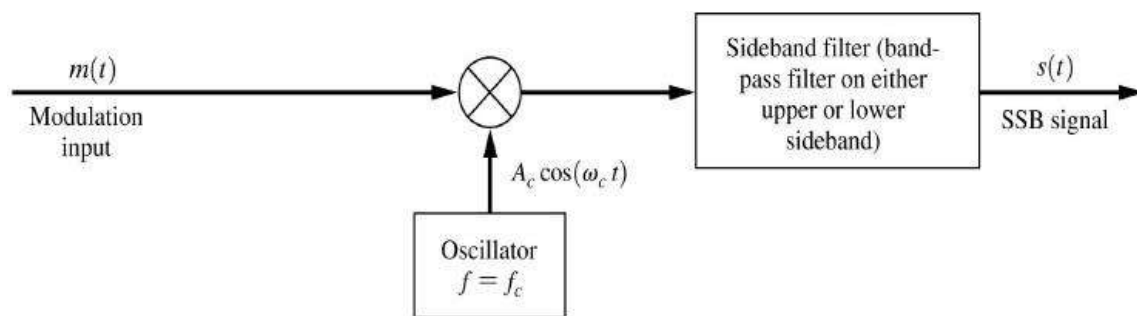


Figure .8 : Frequency discriminator to generate SSBSC wave

Application of this method requires that the message signal satisfies two conditions:

1. The message signal  $m(t)$  has no low-frequency content. Example: speech, audio, music.
2. The highest frequency component  $W$  of the message signal  $m(t)$  is much less than the carrier frequency  $f_c$ .

Then, under these conditions, the desired side band will appear in a non-overlapping interval in the spectrum in such a way that it may be selected by an appropriate filter.

In designing the band pass filter, the following requirements should be satisfied:

1. The pass band of the filter occupies the same frequency range as the spectrum of the desired SSB modulated wave.
2. The width of the guard band of the filter, separating the pass band from the stop band, where the unwanted sideband of the filter input lies, is twice the lowest frequency component of the message signal.

When it is necessary to generate an SSB modulated wave occupying a frequency band that is much higher than that of the message signal, it becomes very difficult to design an appropriate filter that will pass the desired side band and reject the other. In such a situation it is necessary to resort to a multiple-modulation process so as to ease the filtering

requirement. This approach is illustrated in the following figure 9 involving two stages of modulation.

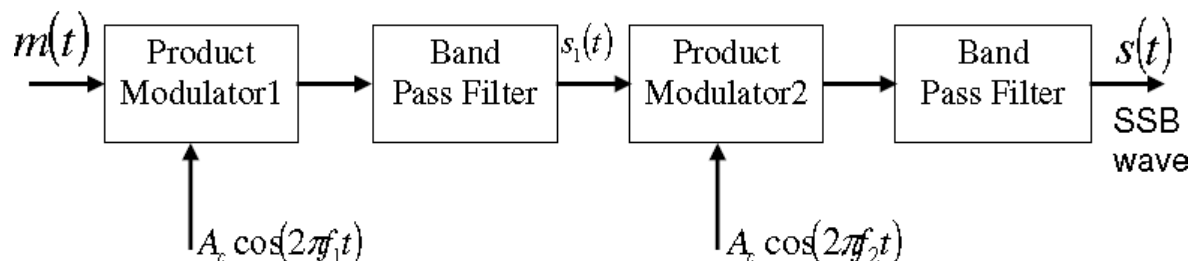


Figure .9 : Two stage frequency discriminator

The SSB modulated wave at the first filter output is used as the modulating wave for the second product modulator, which produces a DSBSC modulated wave with a spectrum that is symmetrically spaced about the second carrier frequency  $f_2$ . The frequency separation between the side bands of this DSBSC modulated wave is effectively twice the first carrier frequency  $f_1$ , thereby permitting the second filter to remove the unwanted side band.

### Hilbert Transform & its Properties:

The Fourier transform is useful for evaluating the frequency content of an energy signal, or in a limiting case that of a power signal. It provides mathematical basis for analyzing and designing the frequency selective filters for the separation of signals on the basis of their frequency content. Another method of separating the signals is based on phase selectivity, which uses phase shifts between the appropriate signals (components) to achieve the desired separation. In case of a sinusoidal signal, the simplest phase shift of  $180^\circ$  is obtained by “Ideal transformer” (polarity reversal). When the phase angles of all the components of a given signal are shifted by  $90^\circ$ , the resulting function of time is called the “Hilbert transform” of the signal.

Consider an LTI system with transfer function defined by equation 1

$$H(f) = \begin{cases} -j, & f > 0 \\ 0, & f = 0 \\ j, & f < 0 \end{cases} \quad \text{----- ( 1 )}$$

and the Signum function given by

$$\text{sgn}(f) = \begin{cases} 1, & f > 0 \\ 0, & f = 0 \\ -1, & f < 0 \end{cases}$$

The function  $H(f)$  can be expressed using Signum function as given by 2

$$H(f) = -j \text{sgn}(f) \quad \text{----- ( 2 )}$$

We know that  $1e^{-j\pi/2} = -j$ ,  $1e^{j\pi/2} = j$  and  $e^{\pm j\theta} = \cos(\theta) \pm j\sin(\theta)$

Therefore,

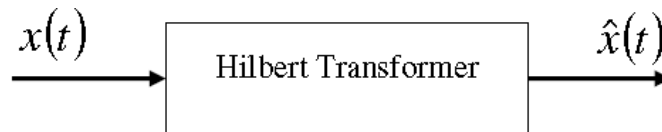
$$H(f) = \begin{cases} 1e^{-j\pi/2}, & f > 0 \\ 1e^{j\pi/2}, & f < 0 \end{cases}$$

Thus the magnitude  $|H(f)| = 1$ , for all  $f$ , and angle

$$\angle H(f) = \begin{cases} -\pi/2, & f > 0 \\ +\pi/2, & f < 0 \end{cases}$$

The device which possesses such a property is called Hilbert transformer. Whenever a signal is applied to the Hilbert transformer, the amplitudes of all frequency components of the input signal remain unaffected. It produces a phase shift of  $-90^\circ$  for all positive frequencies, while a phase shifts of  $90^\circ$  for all negative frequencies of the signal.

If  $x(t)$  is an input signal, then its Hilbert transformer is denoted by  $\hat{x}(t)$  and shown in the following diagram.



To find impulse response  $h(t)$  of Hilbert transformer with transfer function  $H(f)$ . Consider the relation between Signum function and the unit step function.

$$\text{sgn}(t) = 2u(t) - 1 = x(t),$$

Differentiating both sides with respect to  $t$ ,

$$\frac{d}{dt}\{x(t)\} = 2\delta(t)$$

Apply Fourier transform on both sides,

$$\text{sgn}(t) \leftrightarrow \frac{2}{j\omega} \quad \longrightarrow \quad \text{sgn}(t) \leftrightarrow \frac{1}{j\pi f}$$

Applying duality property of Fourier transform,

$$-Sgn(f) \leftrightarrow \frac{1}{j\pi t}$$

We have

$$H(f) = -j \operatorname{sgn}(f)$$

$$H(f) \leftrightarrow \frac{1}{\pi t}$$

Therefore the impulse response  $h(t)$  of an Hilbert transformer is given by the equation 3 ,

$$h(t) = \frac{1}{\pi t} \quad \text{----- ( 3 )}$$

Now consider any input  $x(t)$  to the Hilbert transformer, which is an LTI system. Let the impulse response of the Hilbert transformer is obtained by convolving the input  $x(t)$  and impulse response  $h(t)$  of the system.

$$\hat{x}(t) = x(t) * h(t)$$

$$\hat{x}(t) = x(t) * \frac{1}{\pi t}$$

$$\hat{x}(t) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{x(\tau)}{(t-\tau)} d\tau \quad \text{----- ( 4 )}$$

The equation 3.5 gives the Hilbert transform of  $x(t)$ .

The inverse Hilbert transform  $x(t)$  is given by

$$x(t) = \frac{-1}{\pi} \int_{-\infty}^{+\infty} \frac{\hat{x}(\tau)}{(t-\tau)} d\tau \quad \text{----- ( 5 )}$$

We have  $\hat{\hat{x}}(t) = x(t) * h(t)$

The Fourier transform  $\hat{X}(f)$  of  $\hat{x}(t)$  is given by

$$\hat{X}(f) = X(f)H(f)$$

$$\hat{X}(f) = -j \operatorname{sgn}(f)X(f) \quad \text{----- ( 6 )}$$

### Properties:

1. "A signal  $x(t)$  and its Hilbert transform  $\hat{x}(t)$  have the same amplitude spectrum".

The magnitude of  $-j\text{sgn}(f)$  is equal to 1 for all frequencies  $f$ . Therefore  $x(t)$  and  $\hat{x}(t)$  have the same amplitude spectrum.

That is 
$$|\hat{X}(f)| = |X(f)| \quad \text{for all } f$$

2. "If  $\hat{x}(t)$  is the Hilbert transform of  $x(t)$ , then the Hilbert transform of  $\hat{x}(t)$ , is  $-x(t)$ ".

To obtain its Hilbert transform of  $x(t)$ ,  $x(t)$  is passed through a LTI system with a transfer function equal to  $-j\text{sgn}(f)$ . A double Hilbert transformation is equivalent to passing  $x(t)$  through a cascade of two such devices. The overall transfer function of such a cascade is equal to

$$[-j\text{sgn}(f)]^2 = -1 \quad \text{for all } f$$

The resulting output is  $-x(t)$ . That is the Hilbert transform of  $\hat{x}(t)$  is equal to  $-x(t)$ .

### Time Domain Description:

The time domain description of an SSB wave  $s(t)$  in the canonical form is given by the equation 1.

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t) \quad \text{----- ( 1 )}$$

where  $S_I(t)$  is the in-phase component of the SSB wave and  $S_Q(t)$  is its quadrature component. The in-phase component  $S_I(t)$  except for a scaling factor, may be derived from  $S(t)$  by first multiplying  $S(t)$  by  $\cos(2\pi f_c t)$  and then passing the product through a low-pass filter. Similarly, the quadrature component  $S_Q(t)$ , except for a scaling factor, may be derived from  $s(t)$  by first multiplying  $s(t)$  by  $\sin(2\pi f_c t)$  and then passing the product through an identical filter.

The Fourier transformation of  $S_I(t)$  and  $S_Q(t)$  are related to that of SSB wave as follows, respectively.

$$S_I(f) = \begin{cases} S(f - f_c) + S(f + f_c), & -w \leq f \leq w \\ 0, & \text{elsewhere} \end{cases} \quad \text{----- ( 2 )}$$

$$S_Q(f) = \begin{cases} j[S(f - f_c) - S(f + f_c)], & -w \leq f \leq w \\ 0, & \text{elsewhere} \end{cases} \quad \text{----- ( 3 )}$$

where  $-w < f < w$  defines the frequency band occupied by the message signal  $m(t)$ .

Consider the SSB wave that is obtained by transmitting only the upper side band, shown in figure 10 . Two frequency shifted spectras  $s(f - f_c)$  and  $s(f + f_c)$  are shown in figure 11 and figure 12 respectively. Therefore, from equations 2 and 3 , it follows that the corresponding spectra of the in- phase component  $S_I(t)$  and the quadrature component  $S_Q(t)$  are as shown in figure 13 and 14 respectively.

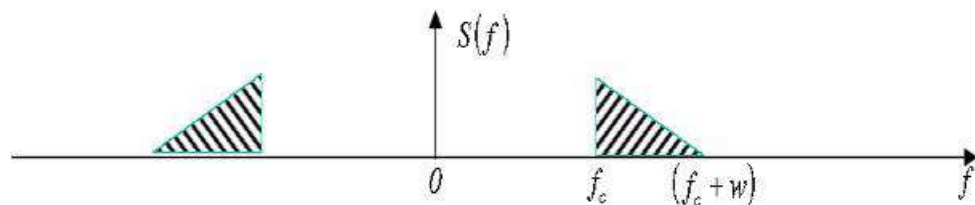


Figure 10 : Spectrum of SSBSC-USB

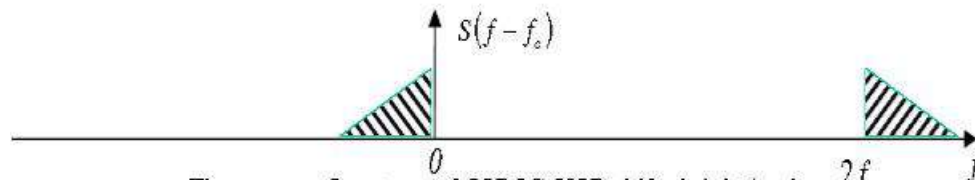


Figure 11 : Spectrum of SSBSC-USB shifted right by  $f_c$

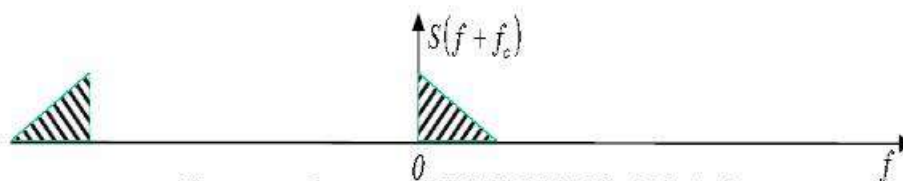


Figure 12 : Spectrum of SSBSC-USB shifted left by  $f_c$

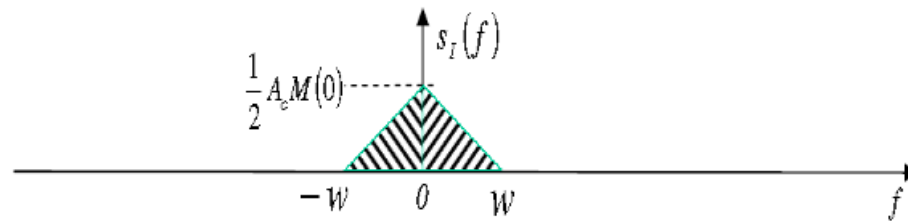


Figure 13 : Spectrum of in-phase component of SSBSC-USB

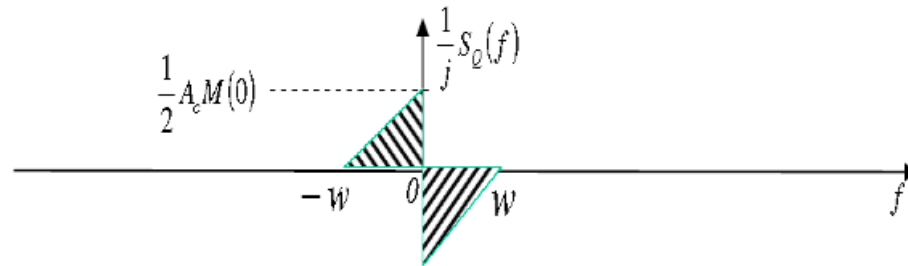


Figure 14 : Spectrum of quadrature component of SSBSC-USB

From the figure 13 , it is found that

$$S_I(f) = \frac{1}{2} A_c M(f)$$

where  $M(f)$  is the Fourier transform of the message signal  $m(t)$ . Accordingly in-phase component  $S_I(t)$  is defined by equation 4

$$s_I(t) = \frac{1}{2} A_c m(t) \quad \text{----- ( 4 )}$$

Now on the basis of figure14 , it is found that

$$S_Q(f) = \begin{cases} \frac{-j}{2} A_c M(f), & f > 0 \\ 0, & f = 0 \\ \frac{j}{2} A_c M(f), & f < 0 \end{cases}$$

$$S_Q(f) = \frac{-j}{2} A_c \text{sgn}(f) M(f) \quad \text{----- ( 5 )}$$

where  $\text{sgn}(f)$  is the Signum function.

But from the discussions on Hilbert transforms, it is shown that

$$-j \operatorname{sgn}(f) M(f) = \hat{M}(f) \quad \text{----- ( 6 )}$$

where  $\hat{M}(f)$  is the Fourier transform of the Hilbert transform of  $m(t)$ . Hence the substituting equation ( 6 ) in ( 5 ), we get

$$S_Q(f) = \frac{1}{2} A_c \hat{M}(f) \quad \text{----- ( 7 )}$$

Therefore quadrature component  $s_Q(t)$  is defined by equation 8

$$s_Q(t) = \frac{1}{2} A_c \hat{m}(t) \quad \text{----- ( 8 )}$$

Therefore substituting equations ( 4 ) and ( 8 ) in equation in ( 1 ), we find that canonical representation of an SSB wave  $s(t)$  obtained by transmitting only the upper side band is given by the equation 9

$$s_U(t) = \frac{1}{2} A_c m(t) \cos(2\pi f_c t) - \frac{1}{2} A_c \hat{m}(t) \sin(2\pi f_c t) \quad \text{----- ( 9 )}$$

Following the same procedure, we can find the canonical representation for an SSB wave

$s_L(t)$  obtained by transmitting only the lower side band is given by

$$s_L(t) = \frac{1}{2} A_c m(t) \cos(2\pi f_c t) + \frac{1}{2} A_c \hat{m}(t) \sin(2\pi f_c t) \quad \text{----- ( 10 )}$$

### Phase discrimination method for generating SSB wave:

Time domain description of SSB modulation leads to another method of SSB generation using the equations 9 or 10. The block diagram of phase discriminator is as shown in figure 15.

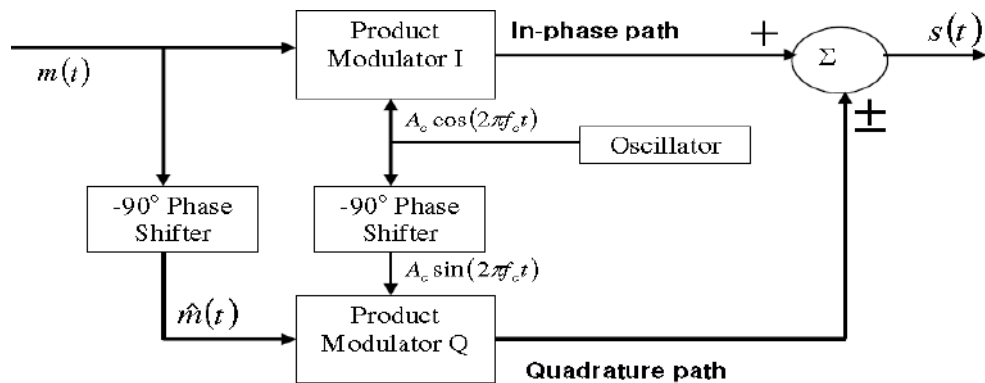


Figure 15 : Block diagram of phase discriminator

The phase discriminator consists of two product modulators I and Q, supplied with carrier waves in-phase quadrature to each other. The incoming base band signal  $m(t)$  is applied to product modulator I, producing a DSBSC modulated wave that contains reference phase sidebands symmetrically spaced about carrier frequency  $f_c$ .

The Hilbert transform  $\hat{m}(t)$  of  $m(t)$  is applied to product modulator Q, producing a DSBSC modulated that contains side bands having identical amplitude spectra to those of modulator I, but with phase spectra such that vector addition or subtraction of the two modulator outputs results in cancellation of one set of side bands and reinforcement of the other set.

The use of a plus sign at the summing junction yields an SSB wave with only the lower side band, whereas the use of a minus sign yields an SSB wave with only the upper side band. This modulator circuit is called Hartley modulator.

### Demodulation of SSB Waves:

Demodulation of SSBSC wave using coherent detection is as shown in 16 . The SSB wave  $s(t)$  together with a locally generated carrier  $c(t) = A_c^{-1} \cos(2\pi f_c t + \phi)$  is applied to a product modulator and then low-pass filtering of the modulator output yields the message signal.

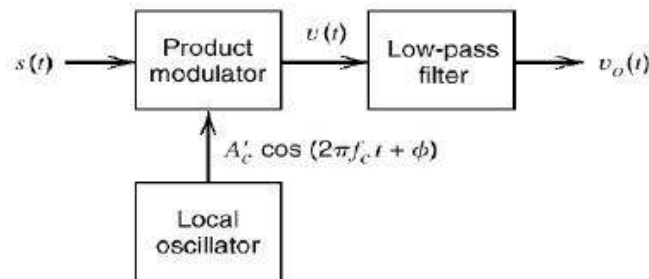


Figure 16 : Block diagram of coherent detector for SSBSC

The product modulator output  $v(t)$  is given by

$$v(t) = A_c^{-1} \cos(2\pi f_c t + \phi) s(t), \quad \text{Put } \phi = 0$$

$$v(t) = \frac{1}{2} A_c \cos(2\pi f_c t) [m(t) \cos(2\pi f_c t) \pm \hat{m}(t) \sin(2\pi f_c t)]$$

$$v(t) = \frac{1}{4} A_c m(t) + \frac{1}{4} A_c [m(t) \cos(4\pi f_c t) \pm \hat{m}(t) \sin(4\pi f_c t)] \quad \text{.....(1)}$$

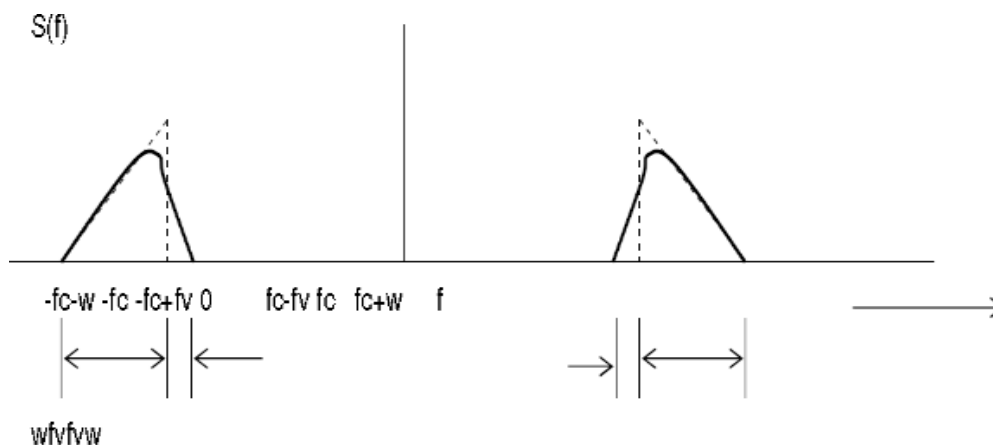
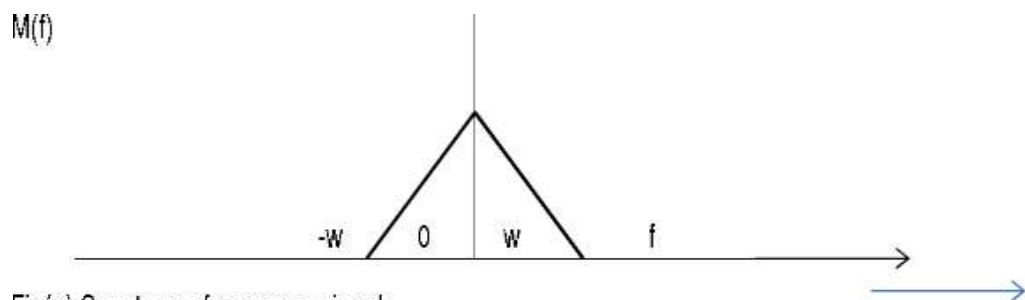
The first term in the above equation 1 is desired message signal. The other term represents an SSB wave with a carrier frequency of  $2f_c$  as such; it is an unwanted component, which is removed by low-pass filter.

## Introduction to Vestigial Side Band Modulation

Vestigial sideband is a type of Amplitude modulation in which one side band is completely passed along with trace or tail or vestige of the other side band. VSB is a compromise between SSB and DSBSC modulation. In SSB, we send only one side band, the Bandwidth required to send SSB wave is  $w$ . SSB is not appropriate way of modulation when the message signal contains significant components at extremely low frequencies. To overcome this VSB is used.

### Frequency Domain Description

The following Fig illustrates the spectrum of VSB modulated wave  $s(t)$  with respect to the message  $m(t)$  (band limited)



Assume that the Lower side band is modified into the vestigial side band. The vestige of the lower sideband compensates for the amount removed from the upper sideband. The bandwidth required to send VSB wave is

$$B = w + f_v$$

Where  $f_v$  is the width of the vestigial side band.

Similarly, if Upper side band is modified into the vestigial side band then,

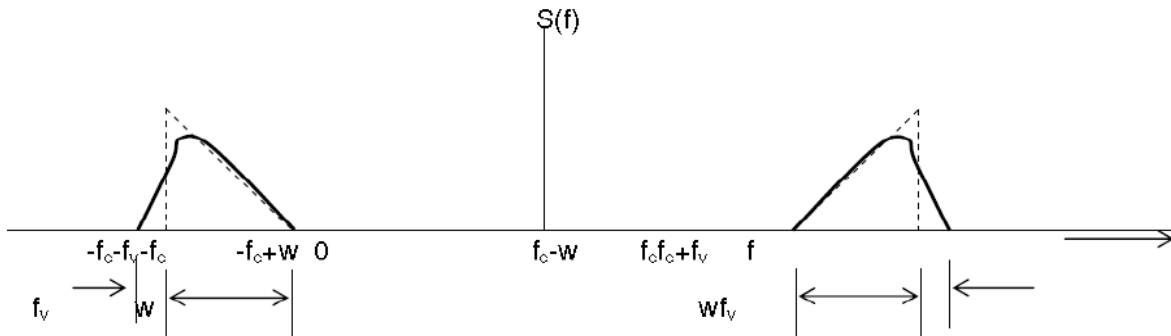


Fig (d) Spectrum of VSB wave containing vestige of the Upper side band

The vestige of the Upper sideband compensates for the amount removed from the Lower sideband. The bandwidth required to send VSB wave is  $B = w + f_v$ , where  $f_v$  is the width of the vestigial side band.

Therefore, VSB has the virtue of conserving bandwidth almost as efficiently as SSB modulation, while retaining the excellent low-frequency base band characteristics of DSBSC and it is standard for the transmission of TV signals.

### Generation of VSB Modulated Wave

VSB modulated wave is obtained by passing DSBSC through a sideband shaping filter as shown in fig below.

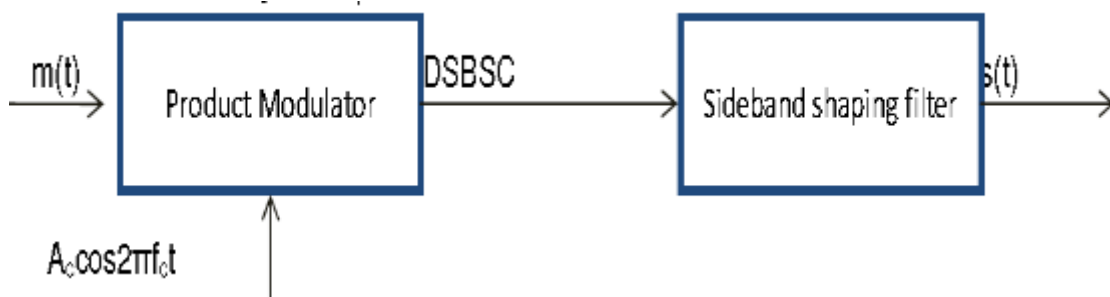


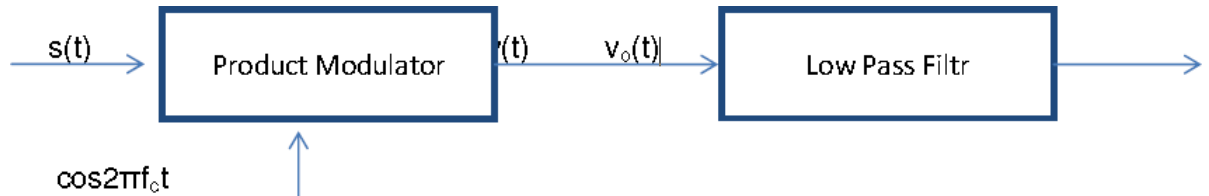
Fig.17. Block Diagram of VSB Modulator

The exact design of this filter depends on the spectrum of the VSB waves. The relation between filter transfer function  $H(f)$  and the spectrum of VSB waves is given by

$$S(f) = A_c / 2 [M(f - f_c) + M(f + f_c)]H(f) \text{ ----- (1)}$$

Where  $M(f)$  is the spectrum of Message Signal. Now, we have to determine the specification for the filter transfer function  $H(f)$ . It can be obtained by passing  $s(t)$  to a

coherent detector and determining the necessary condition for undistorted version of the message signal  $m(t)$ . Thus,  $s(t)$  is multiplied by a locally generated sinusoidal wave  $\cos(2\pi f_c t)$  which is synchronous with the carrier wave  $A_c \cos(2\pi f_c t)$  in both frequency and phase, as in fig below,



Fig(b). Block diagram of VSB Demodulator

$$\text{Then, } v(t) = s(t) \cdot \cos(2\pi f_c t) \text{-----(2)}$$

In frequency domain Eqn (2) becomes,

$$V(f) = \frac{1}{2} [S(f - f_c) + S(f + f_c)] \text{-----(3)}$$

Substitution of Eqn (1) in Eqn (3) gives

$$\begin{aligned} V(f) &= \frac{1}{2} [A_c / 2 [M(f - f_c - f_c) + M(f - f_c + f_c)] H(f - f_c) \\ &\quad + \frac{1}{2} [A_c / 2 [M(f + f_c - f_c) + M(f + f_c + f_c)] H(f + f_c) \\ V(f) &= \frac{1}{2} [A_c / 2 [M(f - 2f_c) + M(f)] H(f - f_c) \\ &\quad + \frac{1}{2} [A_c / 2 [M(f) + M(f + 2f_c)] H(f + f_c) \end{aligned}$$

$$\begin{aligned} V(f) &= A_c / 4 M(f) [H(f - f_c) + H(f + f_c)] \\ &\quad + A_c / 4 [M(f - 2f_c) H(f - f_c) + M(f + 2f_c) H(f + f_c)] \text{-----(4)} \end{aligned}$$

The spectrum of  $V(f)$  as shown in fig below,

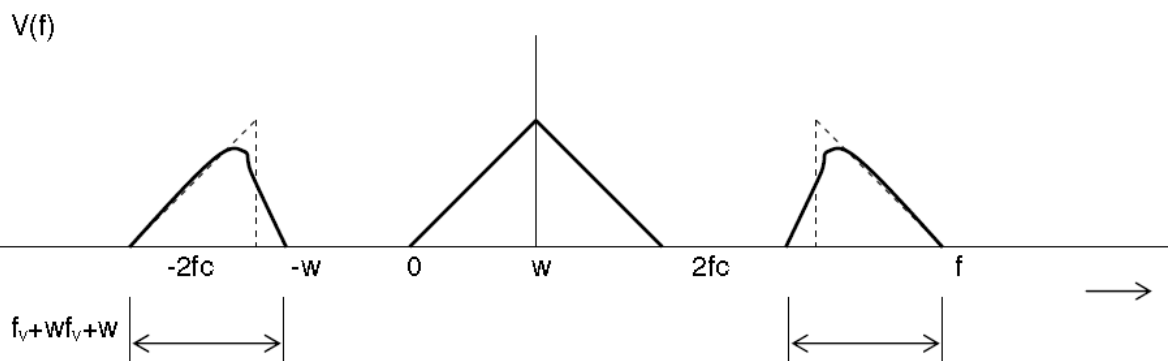


Fig ©. Spectrum of the product modulator output  $v(t)$

Pass  $v(t)$  to a Low pass filter to eliminate VSB wave corresponding to  $2f_c$ .

$$V_o(f) = A_c / 4 M(f) [H(f - f_c) + H(f + f_c)] \text{-----(5)}$$

The spectrum of  $V_o(f)$  is in fig below,

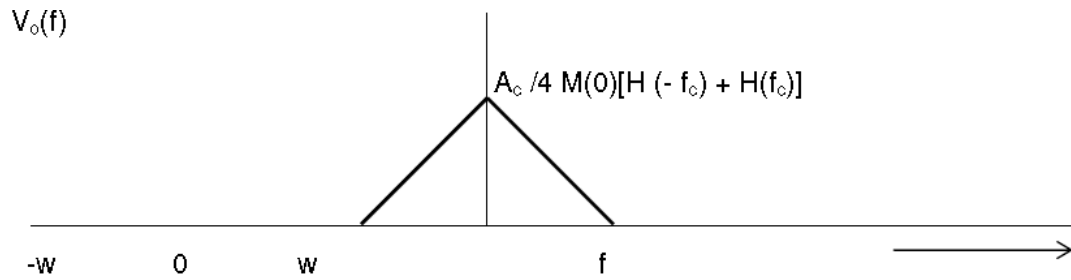


Fig (d). Spectrum of the demodulated Signal  $v_o(t)$ .

For a distortion less reproduction of the original signal  $m(t)$ ,  $V_o(f)$  to be a scaled version of  $M(f)$ . Therefore, the transfer function  $H(f)$  must satisfy the condition

$$H(f - f_c) + H(f + f_c) = 2H(f_c) \text{-----(6)}$$

Where  $H(f_c)$  is a constant

Since  $m(t)$  is a band limited signal, we need to satisfy eqn (6) in the interval  $-w \leq f \leq w$ . The requirement of eqn (6) is satisfied by using a filter whose transfer function is shown below

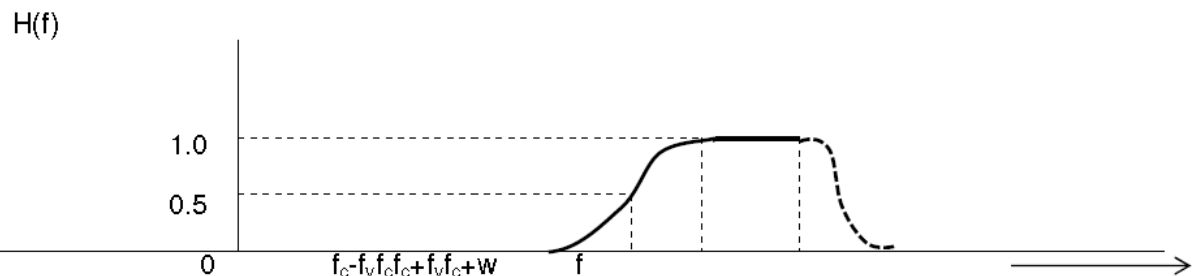


Fig (e) Frequency response of sideband shaping filter

Note:  $H(f)$  is Shown for positive frequencies only.

The Response is normalized so that  $H(f)$  at  $f_c$  is 0.5. Inside this interval  $f_c - f_v \leq f \leq f_c + f_v$  response exhibits odd symmetry. i.e., Sum of the values of  $H(f)$  at any two frequencies equally displaced above and below is Unity.

Similarly, the transfer function  $H(f)$  of the filter for sending Lower sideband along with the vestige of the Upper sideband is shown in fig below,

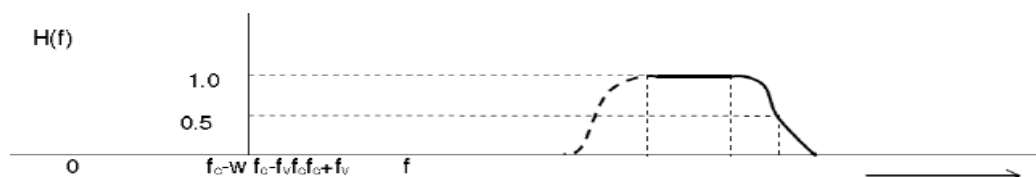


Fig (f) Frequency response of sideband shaping filter

Note:  $H(f)$  is Shown for positive frequencies only.

## Time Domain Description:

Time domain representation of VSB modulated wave, procedure is similar to SSB Modulated waves. Let  $s(t)$  denote a VSB modulated wave and assuming that  $s(t)$  containing Upper sideband along with the Vestige of the Lower sideband. VSB modulated wave  $s(t)$  is the output from Sideband shaping filter, whose input is DSBSC wave. The filter transfer function  $H(f)$  is of the form as in fig below,

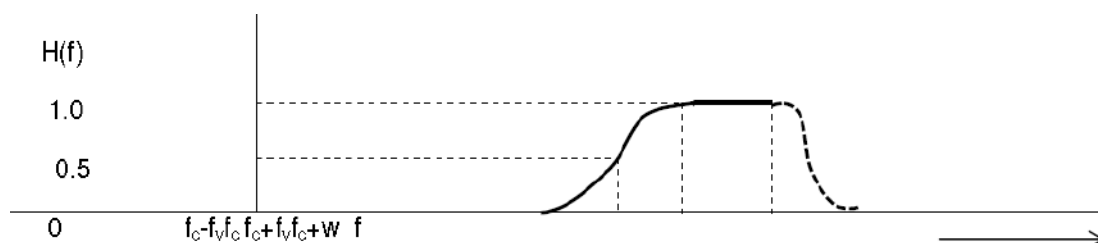


Fig (1)  $H(f)$  of sideband shaping filter

The DSBSC Modulated wave is

$$S_{\text{DSBSC}}(t) = A_c m(t) \cos 2\pi f_c t \text{ -----(1)}$$

It is a band pass signal and has in-phase component only. Its low pass complex envelope is given by

$$\tilde{S}_{\text{DSBSC}}(t) = A_c m(t) \text{ -----(2)}$$

The VSB modulated wave is a band pass signal.

Let the low pass signal  $\tilde{s}(t)$  denote the complex envelope of VSB wave  $s(t)$ , then

$$s(t) = \text{Re}[\tilde{s}(t) \exp(j2\pi f_c t)] \text{ -----(3)}$$

To determine  $\tilde{s}(t)$  we proceed as follows

1. The side band shaping filter transfer function  $H(f)$  is replaced by its equivalent complex low pass transfer function denoted by  $\tilde{H}(f)$  as shown in fig below

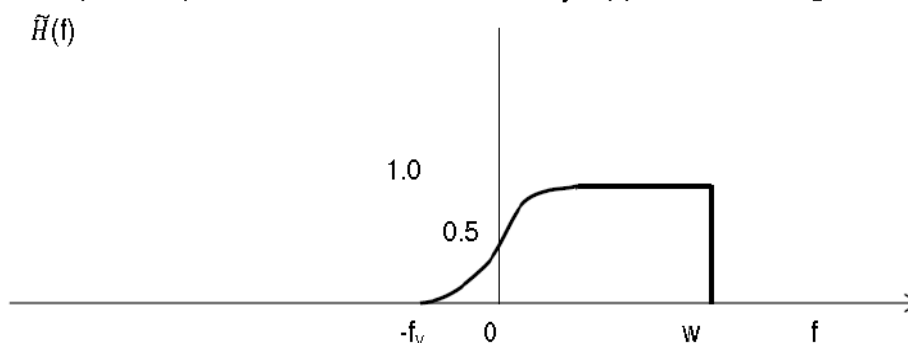


Fig (2) Low pass equivalent to  $H(f)$

We may express  $\tilde{H}(f)$  as the difference between two components  $\tilde{H}_u(f)$  and  $\tilde{H}_v(f)$  as

$$\tilde{H}(f) = \tilde{H}_u(f) - \tilde{H}_v(f) \text{ -----(4)}$$

These two components are considered individually as follows

i). The transfer function  $\tilde{H}_u(f)$  pertains to a complex low pass filter equivalent to a band pass filter design to reject the lower side band completely as

$\tilde{H}_u(f)$

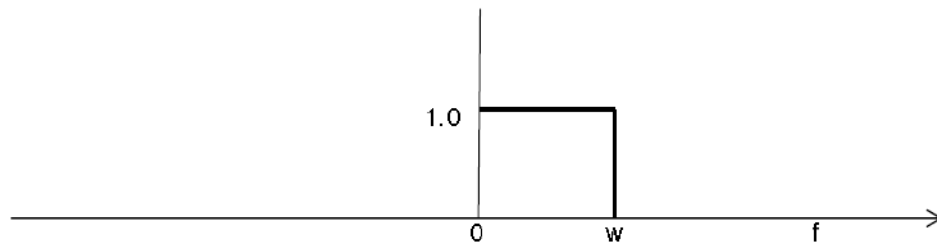


Fig (3) First component of  $\tilde{H}(f)$

$$\tilde{H}_u(f) = \begin{cases} \frac{1}{2} [1 + \text{sgn}(f)], & 0 < f < w \\ 0, & \text{otherwise} \end{cases} \text{ -----(5)}$$

ii). The transfer function  $\tilde{H}_v(f)$  accounts for the generation of vestige and removal of a corresponding portion from the upper side band.

$\tilde{H}_v(f)$

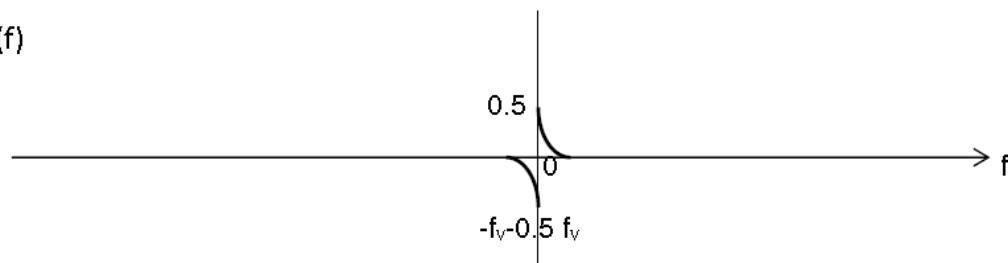


Fig (4) Second component of  $\tilde{H}(f)$

Substitute eqn(5) in eqn (4) we get,

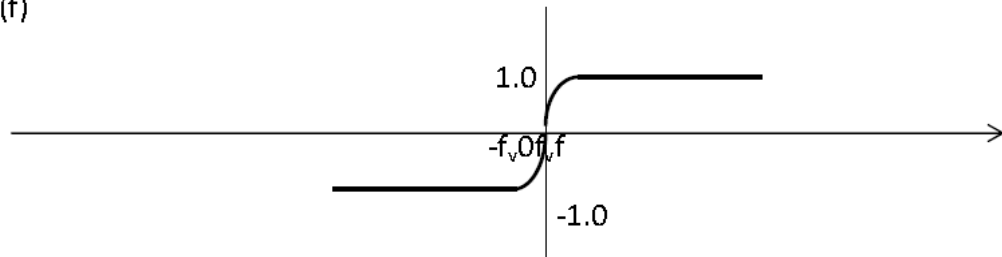
$$\tilde{H}(f) = \begin{cases} \frac{1}{2} [1 + \text{sgn}(f) - 2\tilde{H}_v(f)], & f_v < f < w \\ 0, & \text{otherwise} \end{cases} \text{ -----(6)}$$

The  $\text{sgn}(f)$  and  $\tilde{H}_v(f)$  are both odd functions of frequency, Hence, both they have purely imaginary Inverse Fourier Transform (FT). Accordingly, we may introduce a new transfer function

$$H_Q(f) = 1/j[\text{sgn}(f) - 2\tilde{H}_v(f)] \text{ -----(7)}$$

It has purely Inverse FT and  $h_Q(t)$  denote IFT of  $H_Q(f)$

$jH_Q(f)$



Fig(5) Transfer function of the filter  $jH_Q(f)$

Rewrite eqn(6) in terms of  $H_Q(f)$  as

$$\tilde{H}(f) = \begin{cases} \frac{1}{2} [1 + jH_Q(f)], & f_v < f < w \\ 0, & \text{otherwise} \end{cases} \text{ -----(8)}$$

- The DSBSC modulated wave is replaced by its complex envelope as

$$\tilde{S}_{\text{DSBSC}}(f) = A_c M(f) \text{ -----(9)}$$

- The desired complex envelope  $\tilde{S}(t)$  is determined by evaluating IFT of the product  $\tilde{H}(f)\tilde{S}_{\text{DSBSC}}(f)$ .

$$\text{i.e., } \tilde{S}(f) = \tilde{H}(f)\tilde{S}_{\text{DSBSC}}(f) \text{ -----(10)}$$

$$\tilde{S}(f) = A_c/2 [1 + jH_Q(f)] M(f) \text{ -----(11)}$$

Take IFT of eqn(11) we get,

$$\tilde{S}(t) = A_c/2 [m(t) + jm_Q(t)] \text{ -----(12)}$$

Where  $m_Q(t)$  is the response produced by passing the message through a low pass filter of impulse response  $h_Q(t)$ .

Substitute eqn(12) in eqn(3) and simplify, we get

$$S(t) = A_c/2 m(t) \cos 2\pi f_c t - A_c/2 m_Q(t) \sin 2\pi f_c t \text{ -----(13)}$$

Where  $A_c/2 m(t)$  ----- In-phase component

$A_c/2 m_Q(t)$  ----- Quadrature component

Note:

- If vestigial side band is increased to full side band, VSB becomes DSCSB ,i.e.,  $m_Q(t) = 0$ .
- If vestigial side band is reduced to Zero, VSB becomes SSB.  
i.e.,  $m_Q(t) = \hat{m}(t)$   
Where  $\hat{m}(t)$  is the Hilbert transform of  $m(t)$ .

Similarly If VSB containing a vestige of the Upper sideband, then  $s(t)$  is given by

$$S(t) = A_c/2 m(t) \cos 2\pi f_c t + A_c/2 m_Q(t) \sin 2\pi f_c t \text{ -----(14)}$$

### Envelope detection of a VSB Wave plus Carrier

To make demodulation of VSB wave possible by an envelope detector at the receiving end it is necessary to transmit a sizeable carrier together with the modulated wave. The scaled expression of VSB wave by factor  $k_a$  with the carrier component  $A_c \cos(2\pi f_c t)$  can be given by

$$\begin{aligned} s(t) &= A_c \cos(2\pi f_c t) + \frac{A_c}{2} k_a m(t) \cos(2\pi f_c t) \\ &\quad - \frac{A_c}{2} k_a m_Q(t) \sin(2\pi f_c t) \\ &= A_c \left[ 1 + \frac{k_a}{2} m(t) \right] \cos(2\pi f_c t) - \frac{A_c k_a}{2} m_Q(t) \sin(2\pi f_c t) \text{ .....(1)} \end{aligned}$$

where  $k_a$  is the modulation index; it determines the percentage modulation.

When above signal  $s(t)$  is passed through the envelope detector, the detector output is given by,

$$\begin{aligned} a(t) &= A_c \left[ \left( 1 + \frac{k_a}{2} m(t) \right)^2 + \left( \frac{k_a}{2} m_Q(t) \right)^2 \right]^{1/2} \\ &= A_c \left[ 1 + \frac{k_a}{2} m(t) \right] \left( 1 + \frac{\left[ \frac{1}{2} k_a m_Q(t) \right]^2}{\left[ 1 + \frac{1}{2} k_a m(t) \right]^2} \right)^{1/2} \text{ .....(2)} \end{aligned}$$

The detector output is distorted by the quadrature component  $m_Q(t)$  as indicated by equation (2).

### Methods to reduce distortion

- Distortion can be reduced by reducing percentage modulation,  $k_a$ .
- Distortion can be reduced by reducing  $m_Q(t)$  by increasing the width of the vestigial sideband.

### Comparison of AM Techniques:

Sr. No.	Parameter	Standard AM	SSB	DSBSC	VSB
1	Power	High	Less	Medium	Less than DSBSC but greater than SSB
2	Bandwidth	$2 f_m$	$f_m$	$2 f_m$	$f_m < B_w < 2 f_m$
3	Carrier suppression	No	Yes	Yes	No
4	Receiver complexity	Simple	Complex	Complex	Simple
5	Application	Radio communication	Point to point communication preferred for long distance transmission.	Point to point communication	Television broadcasting
6	Modulation type	Non linear	Linear	Linear	Linear
7	Sideband suppression	No	One sided completely	No	One sideband suppressed partly
8	Transmission efficiency	Minimum	Maximum	Moderate	Moderate

### Applications of different AM systems:

- Amplitude Modulation: AM radio, Short wave radio broadcast
- DSB-SC: Data Modems, Color TV's color signals.
- SSB: Telephone
- VSB: TV picture signals

# **UNIT II**

## **ANGLE MODULATION**

- **Basic concepts**
- **Frequency Modulation**
- **Single tone frequency modulation**
- **Spectrum Analysis of Sinusoidal FM Wave**
- **Narrow band FM, Wide band FM, Constant Average Power**
- **Transmission bandwidth of FM Wave**
- **Generation of FM Waves:**
  - **Indirect FM, Direct FM: Varactor Diode and Reactance Modulator**
- **Detection of FM Waves:**
  - **Balanced Frequency discriminator, Zero crossing detector, Phase locked loop**
- **Comparison of FM & AM**
- **Pre-emphasis & de-emphasis**
- **FM Transmitter block diagram and explanation of each block**

## Instantaneous Frequency

The frequency of a cosine function  $x(t)$  that is given by

$$x(t) = \cos(\omega_c t + \theta_0)$$

is equal to  $\omega_c$  since it is a constant with respect to  $t$ , and the phase of the cosine is the constant  $\theta_0$ . The angle of the cosine  $\theta(t) = \omega_c t + \theta_0$  is a linear relationship with respect to  $t$  (a straight line with slope of  $\omega_c$  and y-intercept of  $\theta_0$ ). However, for other sinusoidal functions, the frequency may itself be a function of time, and therefore, we should not think in terms of the constant frequency of the sinusoid but in terms of the INSTANTANEOUS frequency of the sinusoid since it is not constant for all  $t$ . Consider for example the following sinusoid

$$y(t) = \cos[\theta(t)],$$

where  $\theta(t)$  is a function of time. The frequency of  $y(t)$  in this case depends on the function of  $\theta(t)$  and may itself be a function of time. The instantaneous frequency of  $y(t)$  given above is defined as

$$\omega_i(t) = \frac{d\theta(t)}{dt}.$$

As a checkup for this definition, we know that the instantaneous frequency of  $x(t)$  is equal to its frequency at all times (since the instantaneous frequency for that function is constant) and is equal to  $\omega_c$ . Clearly this satisfies the definition of the instantaneous frequency since  $\theta(t) = \omega_c t + \theta_0$  and therefore  $\omega_i(t) = \omega_c$ .

If we know the instantaneous frequency of some sinusoid from  $-\infty$  to sometime  $t$ , we can find the angle of that sinusoid at time  $t$  using

$$\theta(t) = \int_{-\infty}^t \omega_i(\alpha) d\alpha.$$

Changing the angle  $\theta(t)$  of some sinusoid is the bases for the two types of angle modulation: Phase and Frequency modulation techniques.

## Phase Modulation (PM)

In this type of modulation, the phase of the carrier signal is directly changed by the message signal. The phase modulated signal will have the form

$$g_{PM}(t) = A \cdot \cos \left[ \omega_c t + k_p m(t) \right],$$

where  $A$  is a constant,  $\omega_c$  is the carrier frequency,  $m(t)$  is the message signal, and  $k_p$  is a parameter that specifies how much change in the angle occurs for every unit of change of  $m(t)$ . The phase and instantaneous frequency of this signal are

$$\begin{aligned} \theta_{PM}(t) &= \omega_c t + k_p m(t), \\ \omega_i(t) &= \omega_c + k_p \frac{dm(t)}{dt} = \omega_c + k_p \dot{m}(t). \end{aligned}$$

So, the frequency of a PM signal is proportional to the derivative of the message signal.

### Frequency Modulation (FM)

This type of modulation changes the frequency of the carrier (not the phase as in PM) directly with the message signal. The FM modulated signal is

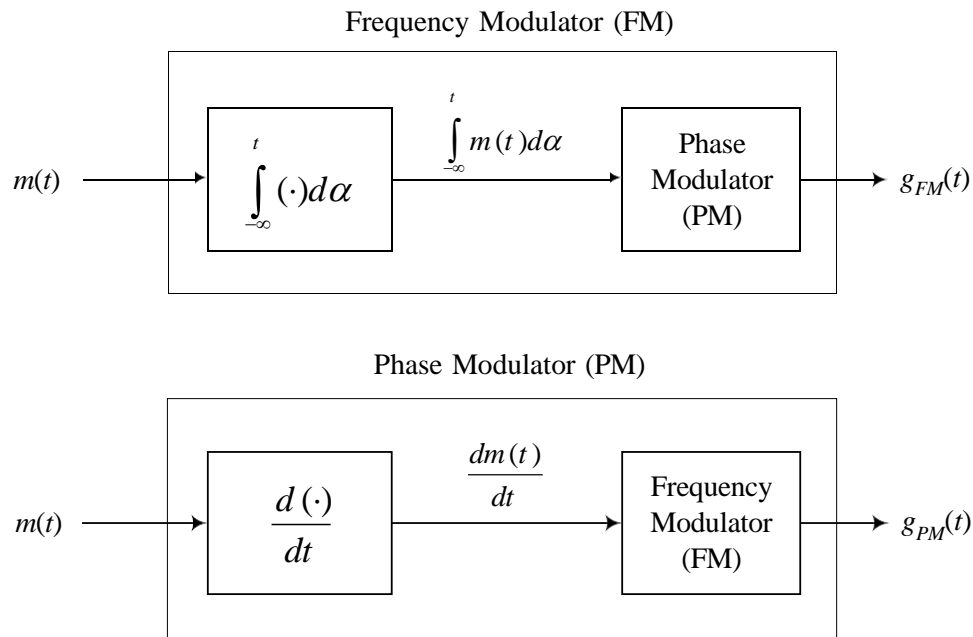
$$g_{FM}(t) = A \cdot \cos \left[ \omega_c t + k_f \int_{-\infty}^t m(\alpha) d\alpha \right],$$

where  $k_f$  is a parameter that specifies how much change in the frequency occurs for every unit change of  $m(t)$ . The phase and instantaneous frequency of this FM are

$$\begin{aligned} \theta_{FM}(t) &= \omega_c t + k_f \int_{-\infty}^t m(\alpha) d\alpha, \\ \omega_i(t) &= \omega_c + k_f \frac{d}{dt} \left[ \int_{-\infty}^t m(\alpha) d\alpha \right] = \omega_c + k_f m(t). \end{aligned}$$

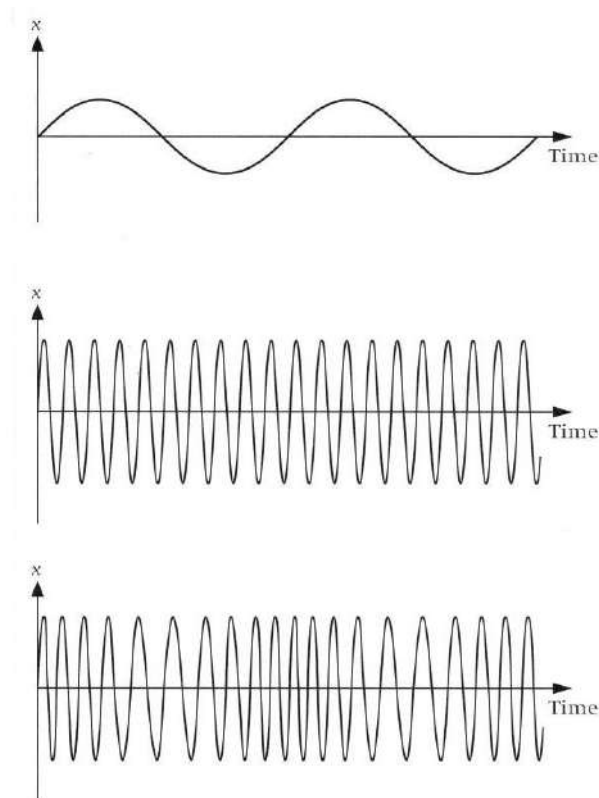
### Relation between PM and FM

PM and FM are tightly related to each other. We see from the phase and frequency relations for PM and FM given above that replacing  $m(t)$  in the PM signal with  $\int_{-\infty}^t m(\alpha) d\alpha$  gives an FM signal and replacing  $m(t)$  in the FM signal with  $\frac{dm(t)}{dt}$  gives a PM signal. This is illustrated in the following block diagrams.



## Frequency Modulation

In **Frequency Modulation (FM)** the instantaneous value of the information signal controls the frequency of the carrier wave. This is illustrated in the following diagrams.



Notice that as the information signal increases, the frequency of the carrier increases, and as the information signal decreases, the frequency of the carrier decreases.

The frequency  $f_i$  of the information signal controls the rate at which the carrier frequency increases and decreases. As with AM,  $f_i$  must be less than  $f_c$ . The amplitude of the carrier remains constant throughout this process.

When the information voltage reaches its maximum value then the change in frequency of the carrier will have also reached its maximum deviation above the nominal value. Similarly when the information reaches a minimum the carrier will be at its lowest frequency below the nominal carrier frequency value. When the information signal is zero, then no deviation of the carrier will occur.

The maximum change that can occur to the carrier from its base value  $f_c$  is called the frequency deviation, and is given the symbol  $\Delta f_c$ . This sets the dynamic range (i.e. voltage range) of the transmission. The dynamic range is the ratio of the largest and smallest analogue information signals that can be transmitted.

### **Bandwidth of FM and PM Signals**

The bandwidth of the different AM modulation techniques ranges from the bandwidth of the message signal (for SSB) to twice the bandwidth of the message signal (for DSBSC and Full AM). When FM signals were first proposed, it was thought that their bandwidth can be reduced to an arbitrarily small value. Compared to the bandwidth of different AM modulation techniques, this would in theory be a big advantage. It was assumed that a signal with an instantaneous frequency that changes over of range of  $\Delta f$  Hz would have a bandwidth of  $\Delta f$  Hz. When experiments were done, it was discovered that this was not the case. It was discovered that the bandwidth of FM signals for a specific message signal was at least equal to the bandwidth of the corresponding AM signal. In fact, FM signals can be classified into two types: Narrowband and Wideband FM signals depending on the bandwidth of each of these signals

### **Narrowband FM and PM**

The general form of an FM signal that results when modulating a signals  $m(t)$  is

$$g_{FM}(t) = A \cdot \cos \left[ \omega_c t + k_f \int_{-\infty}^t m(\alpha) d\alpha \right].$$

A narrow band FM or PM signal satisfies the condition

$$|k_f a(t)| \ll 1$$

For FM and

$$|k_p \cdot m(t)| \ll 1$$

For PM, where

$$a(t) = \int_{-\infty}^t m(\alpha) d\alpha,$$

such that a change in the message signal does not results in a lot of change in the instantaneous frequency of the FM signal.

Now, we can write the above as

$$g_{FM}(t) = A \cdot \cos[\omega_c t + k_f a(t)].$$

Starting with FM, to evaluate the bandwidth of this signal, we need to expand it using a power series expansion. So, we will define a slightly different signal

$$\hat{g}_{FM}(t) = A \cdot e^{j\{\omega_c t + k_f a(t)\}} = A \cdot e^{j\omega_c t} \cdot e^{jk_f a(t)}.$$

Remember that

$$\hat{g}_{FM}(t) = A \cdot e^{j\{\omega_c t + k_f a(t)\}} = A \cdot \cos[\omega_c t + k_f a(t)] + jA \cdot \sin[\omega_c t + k_f a(t)],$$

so

$$g_{FM}(t) = \text{Re}\{\hat{g}_{FM}(t)\}.$$

Now we can expand the term  $e^{jk_f a(t)}$  in  $\hat{g}_{FM}(t)$ , which gives

$$\begin{aligned} \hat{g}_{FM}(t) &= A \cdot e^{j\omega_c t} \cdot \left[ 1 + jk_f a(t) + \frac{j^2 k_f^2 a^2(t)}{2!} + \frac{j^3 k_f^3 a^3(t)}{3!} + \frac{j^4 k_f^4 a^4(t)}{4!} + \dots \right] \\ &= A \cdot \left[ e^{j\omega_c t} + jk_f a(t) e^{j\omega_c t} - \frac{k_f^2 a^2(t)}{2!} e^{j\omega_c t} - \frac{jk_f^3 a^3(t)}{3!} e^{j\omega_c t} + \frac{k_f^4 a^4(t)}{4!} e^{j\omega_c t} + \dots \right] \end{aligned}$$

Since  $k_f$  and  $a(t)$  are real ( $a(t)$  is real because it is the integral of a real function  $m(t)$ ), and since  $\text{Re}\{e^{j\omega_c t}\} = \cos(\omega_c t)$  and  $\text{Re}\{je^{j\omega_c t}\} = -\sin(\omega_c t)$ , then

$$g_{FM}(t) = \text{Re}\{\hat{g}_{FM}(t)\}$$

$$= A \cdot \left[ \cos(\omega_c t) - k_f a(t) \sin(\omega_c t) - \frac{k_f^2 a^2(t)}{2!} \cos(\omega_c t) + \frac{k_f^3 a^3(t)}{3!} \sin(\omega_c t) + \frac{k_f^4 a^4(t)}{4!} \cos(\omega_c t) + \dots \right]$$

The assumption we made for narrowband FM is ( $|k_f a(t)| \ll 1$ ). This assumption will result in making all the terms with powers of  $k_f a(t)$  greater than 1 to be small compared to the first two terms. So, the following is a reasonable approximation for  $g_{FM}(t)$

$$g_{FM(Narrowband)}(t) \approx A \cdot [\cos(\omega_c t) - k_f a(t) \sin(\omega_c t)], \quad \text{when } |k_f a(t)| \ll 1.$$

It must be stressed that the above approximation is only valid for narrowband FM signals that satisfy the condition ( $|k_f a(t)| \ll 1$ ). The above signal is simply the addition (or actually the subtraction) of a cosine (the carrier) with a DSBSC signal (but using a sine as the carrier). The message signal that modulates the DSBSC signal is not  $m(t)$  but its integration  $a(t)$ . One of the properties of the Fourier transform informs us that the bandwidth of a signal  $m(t)$  and its integration  $a(t)$  (and its derivative too) are the same (verify this). Therefore, the bandwidth of the narrowband FM signal is

$$BW_{FM(Narrowband)} = BW_{DSBSC} = 2 \cdot BW_{m(t)} =$$

We will see later that when the condition ( $k_f \ll 1$ ) is not satisfied, the bandwidth of the FM signal becomes higher than twice the bandwidth of the message signal. Similar relationships hold for PM signals. That is

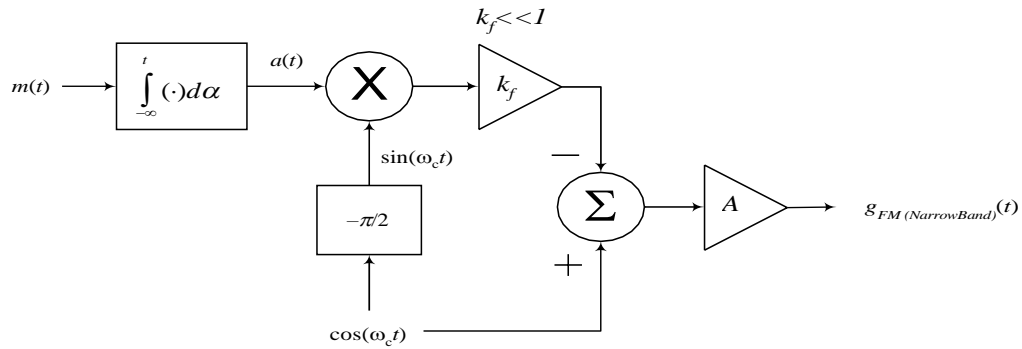
$$g_{PM(Narrowband)}(t) \approx A \cdot [\cos(\omega_c t) - k_p m(t) \sin(\omega_c t)], \quad \text{when } |k_p \cdot m(t)| \ll 1,$$

and

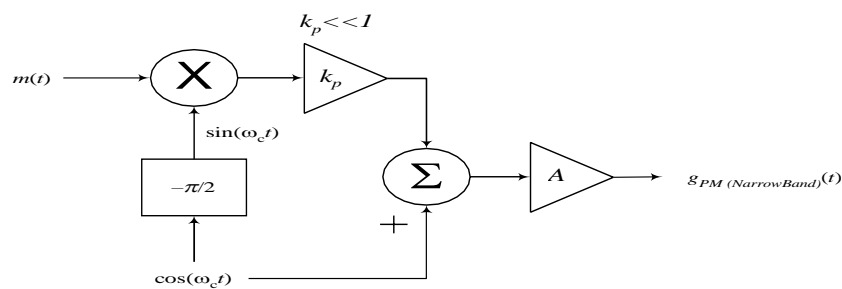
$$BW_{PM(Narrowband)} = BW_{DSBSC} = 2 \cdot BW_{m(t)} =$$

### Construction of Narrowband Frequency and Phase Modulators

The above approximations for narrowband FM and PM can be easily used to construct modulators for both types of signals



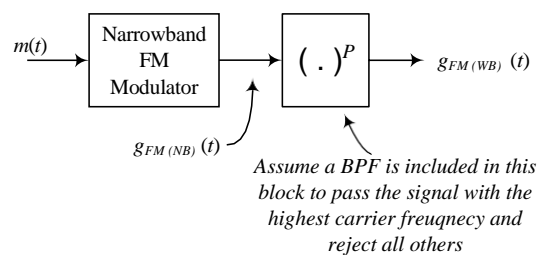
Narrowband FM Modulator



Narrowband PM Modulator

## Generation of Wideband FM Signals

Consider the following block diagram



A narrowband FM signal can be generated easily using the block diagram of the narrowband FM modulator that was described in a previous lecture. The narrowband FM modulator generates a narrowband FM signal using simple components such as an integrator (an OpAmp), oscillators, multipliers, and adders. The generated narrowband FM signal can be converted to a wideband FM signal by simply passing it through a non-linear device with power  $P$ . Both the carrier frequency and the frequency deviation  $\Delta f$  of the narrowband signal

are increased by a factor  $P$ . Sometimes, the desired increase in the carrier frequency and the desired increase in  $\Delta f$  are different. In this case, we increase  $\Delta f$  to the desired value and use a frequency shifter (multiplication by a sinusoid followed by a BPF) to change the carrier frequency to the desired value.

## SINGLE-TONE FREQUENCY MODULATION

### Time-Domain Expression

Since the FM wave is a nonlinear function of the modulating wave, the frequency modulation is a nonlinear process. The analysis of nonlinear process is the difficult task. In this section, we will study single-tone frequency modulation in detail to simplify the analysis and to get thorough understanding about FM.

Let us consider a single-tone sinusoidal message signal defined by

$$n(t) = A_n \cos(2\pi f_n t) \quad (5.13)$$

The instantaneous frequency from Eq. (5.8) is then

$$f(t) = f_c + k_f A_n \cos(2\pi f_n t) = f_c + \Delta f \cos(2\pi f_n t) \quad (5.14)$$

where

$$\Delta f = k_f A_n$$

$$\begin{aligned} \theta(t) &= 2\pi f_c t + 2\pi k_f \int_0^t A_m \cos(2\pi f_m t) dt \\ &= 2\pi f_c t + 2\pi k_f \frac{A_m}{2\pi f_m} \sin(2\pi f_m t) \\ &= 2\pi f_c t + \frac{k_f A_m}{f_m} \sin(2\pi f_m t) \\ &= 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t) \end{aligned}$$

$$\therefore \theta(t) = 2\pi f_c t + \beta_f \sin(2\pi f_m t)$$

Where

$$\beta_f = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m}$$

is the modulation index of the FM wave. Therefore, the single-tone FM wave is expressed by

$$s_{FM}(t) = A_c \cos[2\pi f_c t + \beta_f \sin(2\pi f_m t)] \quad (5.18)$$

This is the desired time-domain expression of the single-tone FM wave

Similarly, **single-tone phase modulated wave** may be determined from Eq.as

$$\begin{aligned} s_{PM}(t) &= A_c \cos[2\pi f_c t + k_p A_m \cos(2\pi f_m t)] \\ \text{or, } s_{PM}(t) &= A_c \cos[2\pi f_c t + \beta_p \cos(2\pi f_m t)] \end{aligned} \quad (5.19)$$

where

$$\beta_p = k_p A_m \quad (5.20)$$

is the modulation index of the single-tone phase modulated wave.

The frequency deviation of the single-tone PM wave is

$$s_{FM}(t) = A_c \cos[2\pi f_c t + \beta_f \sin(2\pi f_m t)]$$

### **Spectral Analysis of Single-Tone FM Wave**

The above Eq. can be rewritten as

$$s_{FM}(t) = \text{Re}\{A_c e^{j2\pi f_c t} e^{j\beta \sin(2\pi f_m t)}\}$$

For simplicity, the modulation index of FM has been considered as  $\beta$  instead of  $\beta_f$  afterward. Since  $\sin(2\pi f_m t)$  is periodic with fundamental period  $T = 1/f_m$ , the complex exponential  $e^{j\beta \sin(2\pi f_m t)}$  is also periodic with the same fundamental period. Therefore, this complex exponential can be expanded in Fourier series representation as

$$e^{j\beta \sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_m t}$$

where the Fourier series coefficients  $c_n$  are obtained as

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} e^{j\beta \sin(2\pi f_m t)} e^{-j2\pi n f_m t} dt \quad (5.24)$$

Let  $2\pi f_m t = x$ , then Eq. (5.24) reduces to

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j\beta \sin(x)} e^{-jnx} dx = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin(x) - nx)} dx \quad (5.25)$$

The integral on the right-hand side is known as the  $n^{\text{th}}$  order Bessel function of the first kind and is denoted by  $J_n(\beta)$ . Therefore,  $c_n = J_n(\beta)$  and Eq. (4.23) can be written as

$$e^{j\beta \sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t} \quad (5.26)$$

By substituting Eq. (5.26) in Eq. (5.22), we get

$$\begin{aligned} s_{FM}(t) &= \text{Re} \left\{ A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t} e^{j2\pi f_c t} \right\} \\ &= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + n f_m)t] \end{aligned} \quad (5.27)$$

Taking Fourier transform of Eq. (5.27), we get

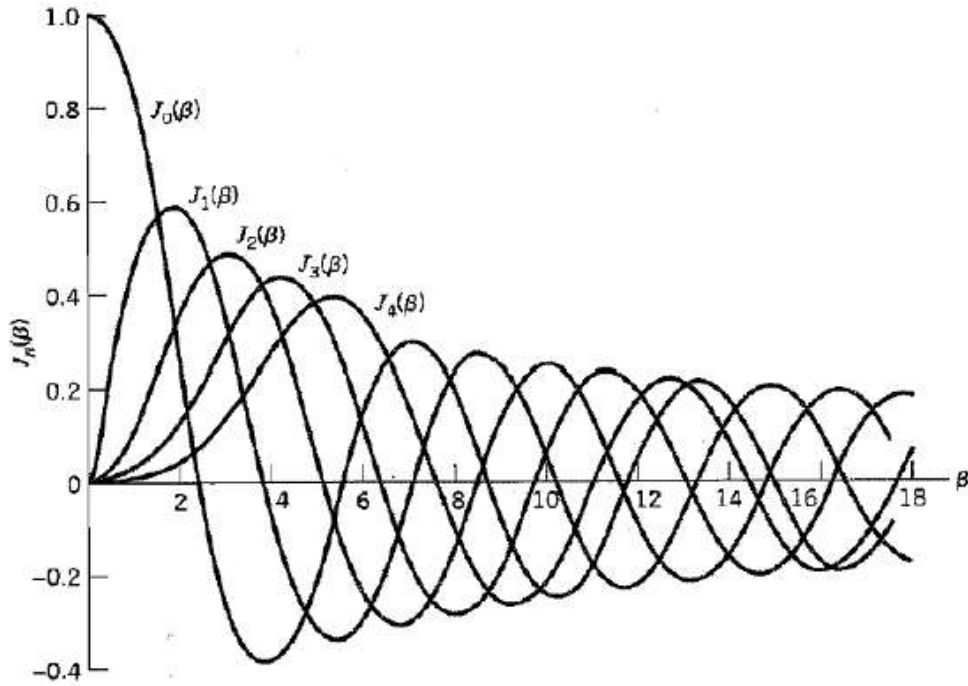
$$S(f) = \frac{1}{2} A_c \sum_{n=-\infty}^{\infty} J_n(\beta) [\delta(f - f_c - n f_m) + \delta(f + f_c + n f_m)] \quad (5.28)$$

From the spectral analysis we see that there is a carrier component and a number of side-frequencies around the carrier frequency at  $\pm n f_m$ .

**The Bessel function** may be expanded in a power series given by

$$J_n(\beta) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{1}{2}\beta\right)^{n+2k}}{k! (k+n)!} \quad (5.29)$$

Plots of Bessel function  $J_n(\beta)$  versus modulation index  $\beta$  for  $n = 0, 1, 2, 3, 4$  are shown in Figure 5.3.



**Figure 5.3** Plot of Bessel function as a function of modulation index.

Figure 5.3 shows that for any fixed value of  $n$ , the magnitude of  $J_n(\beta)$  decreases as  $\beta$  increases. One property of Bessel function is that

$$J_{-n}(\beta) = \begin{cases} J_n(\beta), & n \text{ even} \\ -J_n(\beta), & n \text{ odd} \end{cases} \quad (5.30)$$

One more property of Bessel function is that

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1 \quad (5.31)$$

- (iii) The average power of the FM wave remains constant. To prove this, let us determine the average power of Eq. (5.27) which is equal to

$$P = \frac{1}{2} A_c^2 \sum_{n=-\infty}^{\infty} J_n^2(\beta)$$

Using Eq. (5.31), the average power  $P$  is now

$$P = \frac{1}{2} A_c^2$$

## TRANSMISSION BANDWIDTH OF FM WAVE

The transmission bandwidth of an FM wave depends on the modulation index  $\beta$ . The modulation index, on the other hand, depends on the modulating amplitude and modulating frequency. It is almost impossible to determine the exact bandwidth of the FM wave. Rather, we use a rule-of-thumb expression for determining the FM bandwidth.

For single-tone frequency modulation, the approximated bandwidth is determined by the expression

$$B = 2(\Delta f + f_m) = 2(\beta + 1)f_m = 2\Delta f \left(1 + \frac{1}{\beta}\right)$$

This expression is regarded as the Carson's rule. The FM bandwidth determined by this rule accommodates at least 98 % of the total power.

For an arbitrary message signal  $n(t)$  with bandwidth or maximum frequency  $W$ , the bandwidth of the corresponding FM wave may be determined by Carson's rule as

$$B = 2(\Delta f + W) = 2(D + 1)W = 2\Delta f \left(1 + \frac{1}{D}\right)$$

## GENERATION OF FM WAVES

FM waves are normally generated by two methods: indirect method and direct method.

### Indirect Method (Armstrong Method) of FM Generation

In this method, narrow-band FM wave is generated first by using phase modulator and then the wideband FM with desired frequency deviation is obtained by using frequency multipliers.

$$s(t) = A_c \cos \left[ 2\pi f_c t + 2\pi k_f \int_0^t m(t) dt \right]$$

$$\text{or, } s(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

$$\phi(t) = 2\pi k_f \int_0^t m(t) dt$$

$$s(t) = A_c \cos(2\pi f_c t) \cos[\phi(t)] - A_c \sin(2\pi f_c t) \sin[\phi(t)]$$

The above eq is the expression for narrow band FM wave

In this case  $\cos[\phi(t)] \approx 1$  and  $\sin[\phi(t)] \approx \phi(t)$

$$s(t) = A_c \cos(2\pi f_c t) - A_c \sin(2\pi f_c t) \phi(t)$$

$$\text{or, } s(t) = A_c \cos(2\pi f_c t) - 2\pi A_c k_f \sin(2\pi f_c t) \int_0^t m(t) dt$$

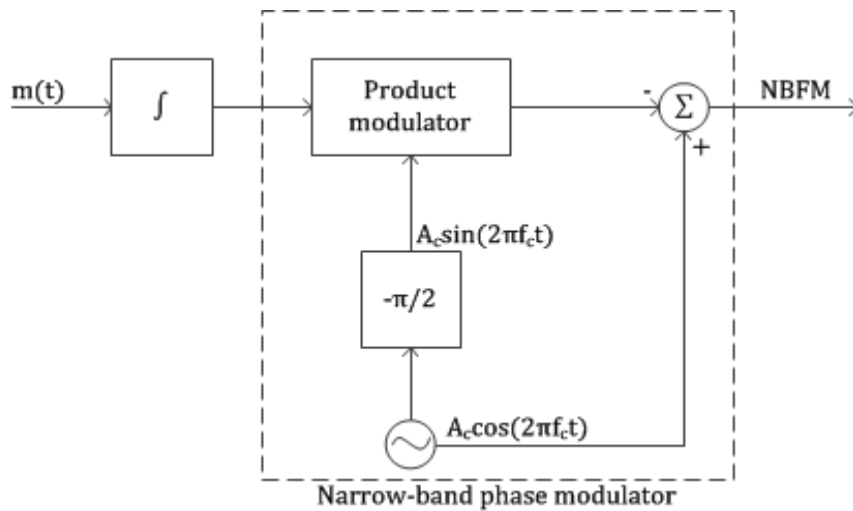


Fig: Narrowband FM Generator

The frequency deviation  $\Delta f$  is very small in narrow-band FM wave. To produce wideband FM, we have to increase the value of  $\Delta f$  to a desired level. This is achieved by means of one or multiple frequency multipliers. A frequency multiplier consists of a nonlinear device and a bandpass filter. The  $n^{\text{th}}$  order nonlinear device produces a dc component and  $n$  number of frequency modulated waves with carrier frequencies  $f_c, 2f_c, \dots, nf_c$  and frequency deviations  $\Delta f, 2\Delta f, \dots, n\Delta f$ , respectively. If we want an FM wave with frequency deviation of  $6\Delta f$ , then we may use a  $6^{\text{th}}$  order nonlinear device or one  $2^{\text{nd}}$  order and one  $3^{\text{rd}}$  order nonlinear devices in cascade followed by a bandpass filter centered at  $6f_c$ . Normally, we may require very high value of frequency deviation. This automatically increases the carrier frequency by the same factor which may be higher than the required carrier frequency. We may shift the carrier frequency to the desired level by using mixer which does not change the frequency deviation.

The narrowband FM has some distortion due to the approximation made in deriving the expression of narrowband FM from the general expression. This produces some amplitude modulation in the narrowband FM which is removed by using a limiter in frequency multiplier.

## Direct Method of FM Generation

In this method, the instantaneous frequency  $f(t)$  of the carrier signal  $c(t)$  is varied directly with the instantaneous value of the modulating signal  $n(t)$ . For this, an oscillator is used in which any one of the reactive components (either C or L) of the resonant network of the oscillator is varied linearly with  $n(t)$ . We can use a varactor diode or a varicap as a voltage-variable capacitor whose capacitance solely depends on the reverse-bias voltage applied across it. To vary such capacitance linearly with  $n(t)$ , we have to reverse-bias the diode by the fixed DC voltage and operate within a small linear portion of the capacitance-voltage characteristic curve. The unmodulated fixed capacitance  $C_0$  is linearly varied by  $n(t)$  such that the resultant capacitance becomes

$$C(t) = C_0 - kn(t)$$

where the constant  $k$  is the sensitivity of the varactor diode (measured in capacitance per volt).

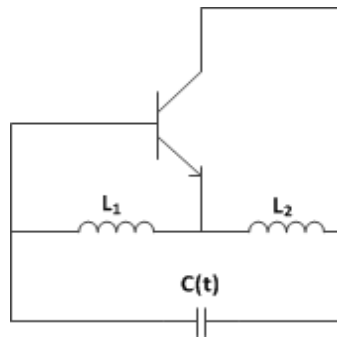


fig: Hartley oscillator for FM generation

The above figure shows the simplified diagram of the Hartley oscillator in which is implemented the above discussed scheme. The frequency of oscillation for such an oscillator is given

$$f(t) = \frac{1}{2\pi\sqrt{(L_1 + L_2)C(t)}}$$

$$f(t) = \frac{1}{2\pi\sqrt{(L_1 + L_2)(C_0 - km(t))}}$$

$$= \frac{1}{2\pi\sqrt{(L_1 + L_2)C_0}} \frac{1}{\sqrt{1 - \frac{km(t)}{C_0}}}$$

or,  $f(t) = f_c \left(1 - \frac{km(t)}{C_0}\right)^{-1/2}$

where  $f_c$  is the unmodulated frequency of oscillation. Assuming,

$$\frac{km(t)}{C_0} \ll 1$$

we have from binomial expansion,

$$\begin{aligned} \left(1 - \frac{km(t)}{C_0}\right)^{-1/2} &\approx 1 + \frac{km(t)}{2C_0} \\ f(t) &\approx f_c \left(1 + \frac{km(t)}{2C_0}\right) \\ &= f_c + \frac{kf_c m(t)}{2C_0} \\ \text{or, } f(t) &= f_c + k_f m(t) \end{aligned}$$

$$k_f = \frac{kf_c}{2C_0}$$

is the frequency sensitivity of the modulator. The Eq. (5.42) is the required expression for the instantaneous frequency of an FM wave. In this way, we can generate an FM wave by direct method.

Direct FM may be generated also by a device in which the inductance of the resonant circuit is linearly varied by a modulating signal  $n(t)$ ; in this case the modulating signal being the current.

The main advantage of the direct method is that it produces sufficiently high frequency deviation, thus requiring little frequency multiplication. But, it has poor frequency stability. A feedback scheme is used to stabilize the frequency in which the output frequency is compared with the constant frequency generated by highly stable crystal oscillator and the error signal is feedback to stabilize the frequency.

## DEMODULATION OF FM WAVES

The process to extract the message signal from a frequency modulated wave is known as frequency demodulation. As the information in an FM wave is contained in its instantaneous frequency, the frequency demodulator has the task of changing frequency variations to amplitude variations. Frequency demodulation method is generally categorized into two types: direct method and indirect method. Under direct method category, we will discuss about limiter discriminator method and under indirect method, phase-locked loop (PLL) will be discussed.

## Limiter Discriminator Method

Recalling the expression of FM signal,

$$s(t) = A_c \cos \left[ 2\pi f_c t + 2\pi k_f \int_0^t m(t) dt \right]$$

In this method, extraction of  $n(t)$  from the above equation involves the three steps: amplitude limit, discrimination, and envelope detection.

### A. Amplitude Limit

During propagation of the FM signal from transmitter to receiver the amplitude of the FM wave (supposed to be constant) may undergo changes due to fading and noise. Therefore, before further processing, the amplitude of the FM signal is limited to reduce the effect of fading and noise by using limiter as discussed in the section 5.9. The amplitude limitation will not affect the message signal as the amplitude of FM does not carry any information of the message signal.

### B. Discrimination/ Differentiation

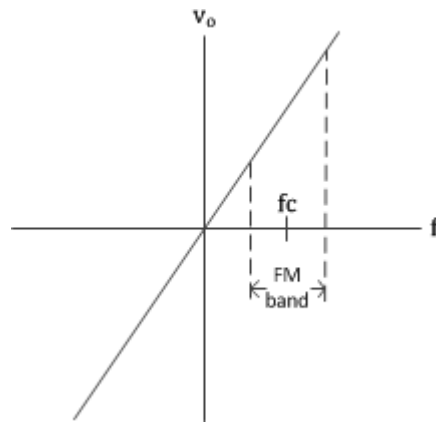
In this step we differentiate the FM signal as given by

$$\begin{aligned} \frac{ds(t)}{dt} &= \frac{d}{dt} \left\{ A_c \cos \left[ 2\pi f_c t + 2\pi k_f \int_0^t m(t) dt \right] \right\} \\ &= \frac{d \left\{ A_c \cos \left[ 2\pi f_c t + 2\pi k_f \int_0^t m(t) dt \right] \right\}}{d \left\{ 2\pi f_c t + 2\pi k_f \int_0^t m(t) dt \right\}} \frac{d \left\{ 2\pi f_c t + 2\pi k_f \int_0^t m(t) dt \right\}}{dt} \\ &= -A_c \left[ 2\pi f_c + 2\pi k_f m(t) \right] \sin \left[ 2\pi f_c t + 2\pi k_f \int_0^t m(t) dt \right] \end{aligned}$$

Here both the amplitude and frequency of this signal are modulated.

In this case, the differentiator is nothing but a circuit that converts change in frequency into corresponding change in voltage or current as shown in Fig. 5.11. The ideal differentiator has transfer function

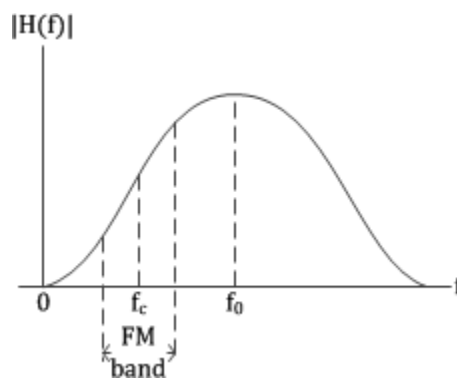
$$H(j\omega) = j2\pi f$$



**Figure :** Transfer function of ideal differentiator.

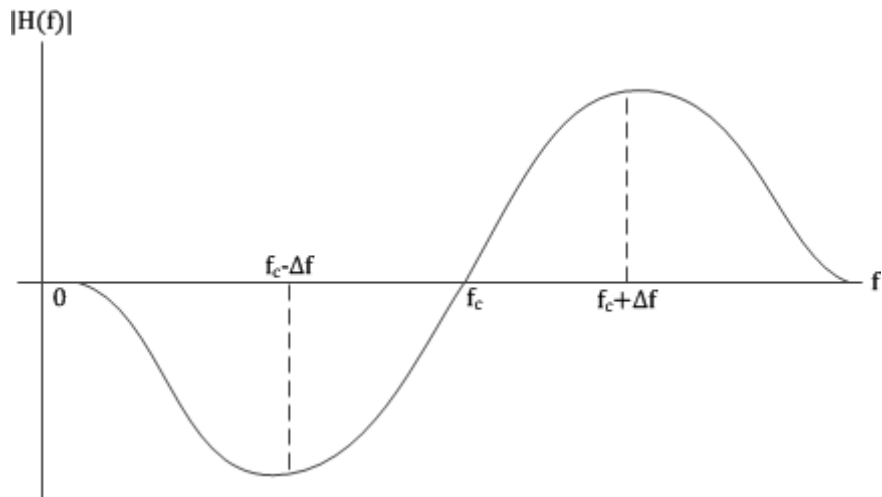
Instead of ideal differentiator, any circuit can be used whose frequency response is linear for some band in positive slope. This method is known as slope detection. For this, linear segment with positive slope of RC high pass filter or LC tank circuit can be used. Figure 5.13 shows the use of an LC circuit as a differentiator. The drawback is the limited linear portion in the

slope of the tank circuit. This is not suitable for wideband FM where the peak frequency deviation is high.



**Figure :** Use of LC tank circuit as a differentiator.

A better solution is the ratio or balanced slope detector in which two tank circuits tuned at  $f_c + \Delta f$  and  $f_c - \Delta f$  are used to extend the linear portion as shown in below figure.



**Figure :** Frequency response of balanced slope detector.

Another detector called Foster-seely discriminator eliminates two tank circuits but still offer the same linear as the ratio detector.

### C. Envelope Detection

The third step is to send the differentiated signal to the envelope detector to recover the message signal.

### Phase-Locked Loop (PLL) as FM Demodulator

A PLL consists of a multiplier, a loop filter, and a VCO connected together to form a feedback loop as shown in Fig. 5.15. Let the input signal be an FM wave as defined by

$$s(t) = A_c \cos[2\pi f_c t + \phi_1(t)]$$

$$\phi_1(t) = 2\pi k_f \int_0^t m(t) dt$$

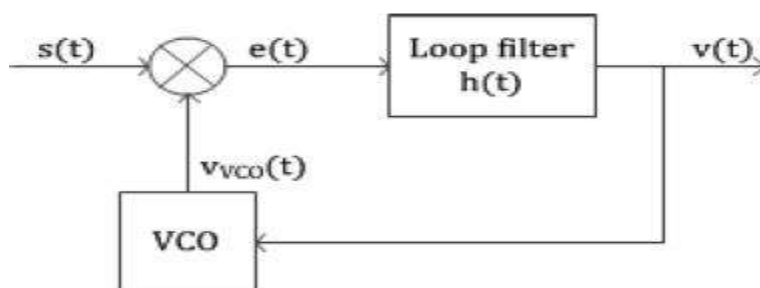


Fig: PLL Demodulator

Let the VCO output be defined by

$$v_{VCO}(t) = A_v \sin[2\pi f_c t + \phi_2(t)]$$

where

$$\phi_2(t) = 2\pi k_v \int_0^t v(t) dt$$

0

Here  $k_v$  is the frequency sensitivity of the VCO measured in hertz per volt. The multiplication of  $s(t)$  and  $v_{VCO}(t)$  results

$$\begin{aligned} s(t)v_{VCO}(t) &= A_c \cos[2\pi f_c t + \phi_1(t)] A_v \sin[2\pi f_c t + \phi_2(t)] \\ &= \frac{A_c A_v}{2} \sin[4\pi f_c t + \phi_1(t) + \phi_2(t)] + \frac{A_c A_v}{2} \sin[\phi_2(t) - \phi_1(t)] \end{aligned}$$

The high-frequency component is removed by the low-pass filtering of the loop filter. Therefore, the input signal to the loop filter can be considered as

$$e(t) = \frac{A_c A_v}{2} \sin[\phi_2(t) - \phi_1(t)]$$

The difference  $\phi_2(t) - \phi_1(t) = \phi_e(t)$  constitutes the phase error. Let us assume that the PLL is in phase lock so that the phase error is very small. Then,

$$\sin[\phi_2(t) - \phi_1(t)] \approx \phi_2(t) - \phi_1(t)$$

$$\phi_e(t) = 2\pi k_v \int_0^t v(t) dt - \phi_1(t)$$

$$e(t) = \frac{A_c A_v}{2} \phi_e(t)$$

Differentiating Eq. (5.48) with respect to time, we get

$$\frac{d\phi_e(t)}{dt} = 2\pi k_v v(t) - \frac{d\phi_1(t)}{dt}$$

Since

$$v(t) = e(t) * h(t) = \frac{A_c A_v}{2} [\phi_e(t) * h(t)]$$

Eq. (5.50) becomes

$$\begin{aligned} \frac{d\phi_e(t)}{dt} &= 2\pi k_v \frac{A_c A_v}{2} [\phi_e(t) * h(t)] - \frac{d\phi_1(t)}{dt} \\ \text{or, } \pi k_v A_c A_v [\phi_e(t) * h(t)] - \frac{d\phi_e(t)}{dt} &= \frac{d\phi_1(t)}{dt} \end{aligned}$$

Taking Fourier transform of Eq. (5.52), we get

$$\begin{aligned} \pi k_v A_c A_v \phi_e(f) H(f) - j2\pi f \phi_e(f) &= j2\pi f \phi_1(f) \\ \text{or, } \phi_e(f) &= \frac{j2\pi f}{\pi k_v A_c A_v H(f) - j2\pi f} \phi_1(f) \\ \text{or, } \phi_e(f) &= \frac{1}{\frac{\pi k_v A_c A_v}{j2\pi f} H(f) - 1} \phi_1(f) \end{aligned}$$

Fourier transform of Eq. (5.51) is

$$V(f) = \frac{A_c A_v}{2} \phi_e(f) H(f)$$

Substituting Eq. (5.53) into (5.54), we get

$$V(f) = \frac{A_c A_v}{2} \frac{1}{\frac{\pi k_v A_c A_v}{j2\pi f} H(f) - 1} \phi_1(f) H(f)$$

We design  $H(f)$  such that

$$\left| \frac{\pi k_v A_c A_v}{j2\pi f} H(f) \right| \gg 1$$

in the frequency band  $|f| < W$  of the message signal.

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$$\begin{aligned} \therefore V(f) &= \frac{A_c A_v}{2} \frac{1}{\frac{\pi k_v A_c A_v}{j2\pi f} H(f)} \phi_1(f) H(f) \\ \text{or, } V(f) &= \frac{1}{2\pi k_v} j2\pi f \phi_1(f) \end{aligned}$$

Taking inverse Fourier transform of Eq. (4.56), we get

$$\begin{aligned} v(t) &= \frac{1}{2\pi k_v} \frac{d\phi_1(t)}{dt} \\ &= \frac{1}{2\pi k_v} \frac{d}{dt} \left\{ 2\pi k_f \int_0^t m(t) dt \right\} \\ &= \frac{1}{2\pi k_v} 2\pi k_f m(t) \\ \therefore v(t) &= \frac{k_f}{k_v} m(t) \end{aligned}$$

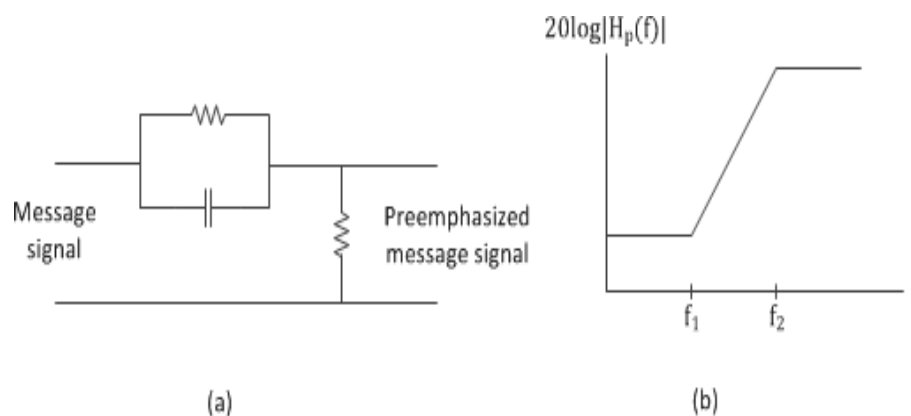
Since the control voltage of the VCO is proportional to the message signal,  $v(t)$  is the demodulated signal.

We observe that the output of the loop filter with frequency response  $H(f)$  is the desired message signal. Hence the bandwidth of  $H(f)$  should be the same as the bandwidth  $W$  of the message signal. Consequently, the noise at the output of the loop filter is also limited to the bandwidth  $W$ . On the other hand, the output from the VCO is a wideband FM signal with an instantaneous frequency that follows the instantaneous frequency of the received FM signal.

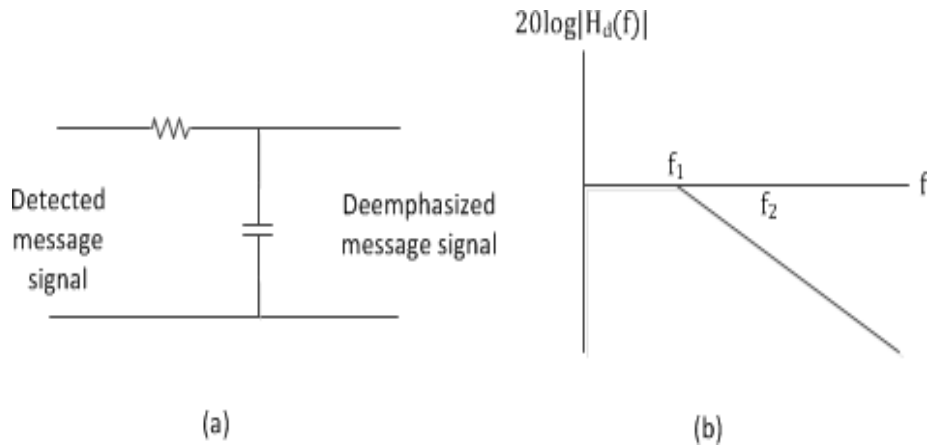
## PREEMPHASIS AND DEEMPHASIS NETWORKS

In FM, the noise increases linearly with frequency. By this, the higher frequency components of message signal are badly affected by the noise. To solve this problem, we can use a preemphasis filter of transfer function  $H_p(f)$  at the transmitter to boost the higher frequency components before modulation. Similarly, at the receiver, the deemphasis filter of transfer function  $H_d(f)$  can be used after demodulator to attenuate the higher frequency components thereby restoring the original message signal.

The preemphasis network and its frequency response are shown in Figure 5.19 (a) and (b) respectively. Similarly, the counter part for deemphasis network is shown in Figure 5.20.



**Figure ;**(a) Preemphasis network. (b) Frequency response of preemphasis network.



**Figure** (a) Deemphasis network. (b) Frequency response of Deemphasis network.

In FM broadcasting,  $f_1$  and  $f_2$  are normally chosen to be 2.1 kHz and 30 kHz respectively.

The frequency response of preemphasis network is

$$H_p(f) = \left(\frac{w_2}{w_1}\right) \frac{jw + w_1}{jw + w_2}$$

Here,  $w = 2\pi f$  and  $w_1 = 2\pi f_1$ . For  $w \ll w_1$ ,

$$H_p(f) \approx 1$$

And for  $w_1 \ll w \ll w_2$ ,

$$H_p(f) \approx \frac{j2\pi f}{w_1}$$

So, the amplitude of frequency components less than 2.1 kHz are left unchanged and greater than that are increased proportional to  $f$ .

The frequency response of deemphasis network is

$$H_d(f) = \frac{w_1}{j2\pi f + w_1}$$

For  $w \ll w_2$ ,

$$H_p(f) \approx \frac{j2\pi f + w_1}{w_1}$$

such that

$$H_p(f)H_d(f) \approx 1$$

over the baseband of 0 to 15 KHz.

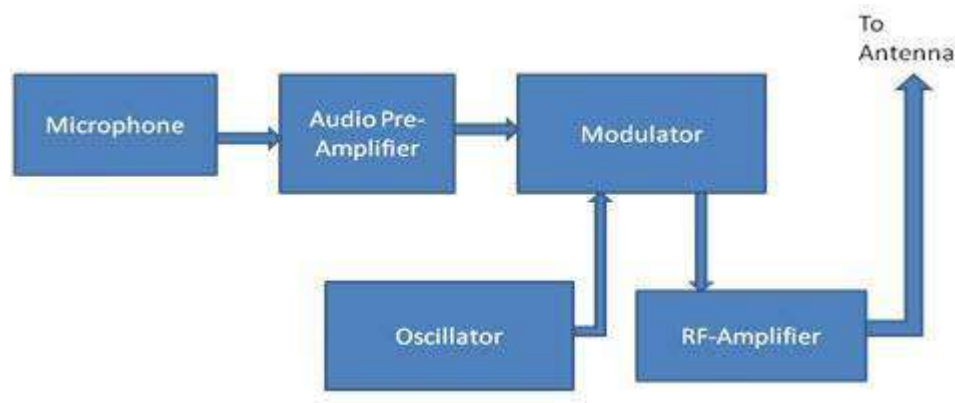
## Comparison of AM and FM:

S.NO	AMPLITUDE MODULATION	FREQUENCY MODULATION
1.	Band width is very small which is one of the biggest advantage	It requires much wider channel ( 7 to 15 times ) as compared to AM.
2.	The amplitude of AM signal varies depending on modulation index.	The amplitude of FM signal is constant and independent of depth of the modulation.
3.	Area of reception is large	The are of reception is small since it is limited to line of sight.
4.	Transmitters are relatively simple & cheap.	Transmitters are complex and hence expensive.
5.	The average power in modulated wave is greater than carrier power. This added power is provided by modulating source.	The average power in frequency modulated wave is same as contained in un-modulated wave.
6.	More susceptible to noise interference and has low signal to noise ratio, it is more difficult to eliminate effects of noise.	Noise can be easily minimized amplitude variations can be eliminated by using limiter.
7.	it is not possible to operate without interference.	it is possible to operate several independent transmitters on same frequency.
8.	The maximum value of modulation index = 1, other wise over-modulation would result in distortions.	No restriction is placed on modulation index.

## FM Transmitter

The FM transmitter is a single transistor circuit. In the telecommunication, the frequency modulation (FM) transfers the information by varying the frequency of carrier wave according to the message signal. Generally, the FM transmitter uses VHF radio frequencies of 87.5 to 108.0 MHz to transmit & receive the FM signal. This transmitter accomplishes the most excellent range with less power. The performance and working of the wireless audio transmitter circuit is depends on the induction coil & variable capacitor. This article will explain about the working of the FM transmitter circuit with its applications.

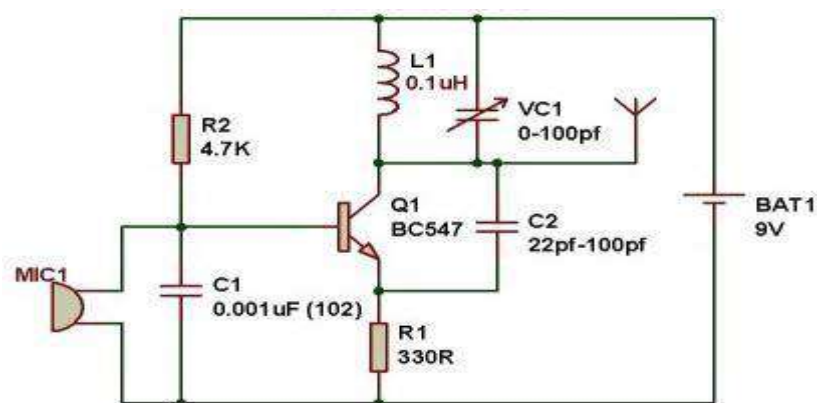
The FM transmitter is a low power transmitter and it uses FM waves for transmitting the sound, this transmitter transmits the audio signals through the carrier wave by the difference of frequency. The carrier wave frequency is equivalent to the audio signal of the amplitude and the FM transmitter produce VHF band of 88 to 108MHZ. Please follow the below link for: [Know all About Power Amplifiers for FM Transmitter](#)



Block Diagram of FM Transmitter

### Working of FM Transmitter Circuit

The following circuit diagram shows the FM transmitter circuit and the required electrical and electronic components for this circuit is the power supply of 9V, resistor, capacitor, trimmer capacitor, inductor, mic, transmitter, and antenna. Let us consider the microphone to understand the sound signals and inside the mic there is a presence of capacitive sensor. It produces according to the vibration to the change of air pressure and the AC signal.



FM Transmitter circuit

The formation of the oscillating tank circuit can be done through the transistor of 2N3904 by using the inductor and variable capacitor. The transistor used in this circuit is an NPN transistor used for general purpose amplification. If the current is passed at the inductor L1 and variable capacitor then the tank circuit will oscillate at the resonant carrier frequency of the FM modulation. The negative feedback will be the capacitor C2 to the oscillating tank circuit.

To generate the radio frequency carrier waves the FM transmitter circuit requires an oscillator. The tank circuit is derived from the LC circuit to store the energy for oscillations.

The input audio signal from the mic penetrated to the base of the transistor, which modulates the LC tank circuit carrier frequency in FM format. The variable capacitor is used to change the resonant frequency for fine modification to the FM frequency band. The modulated signal from the antenna is radiated as radio waves at the FM frequency band and the antenna is nothing but copper wire of 20cm long and 24 gauge. In this circuit the length of the antenna should be significant and here you can use the 25-27 inches long copper wire of the antenna.

### **Application of Fm Transmitter**

- The FM transmitters are used in the homes like sound systems in halls to fill the sound with the audio source.
- These are also used in the cars and fitness centers.
- The correctional facilities have used in the FM transmitters to reduce the prison noise in common areas.

### **Advantages of the FM Transmitters**

- The FM transmitters are easy to use and the price is low
- The efficiency of the transmitter is very high
- It has a large operating range
- This transmitter will reject the noise signal from an amplitude variation.

# UNIT III

## NOISE

- Noise in communication System,
- White Noise
- Narrowband Noise –In phase and Quadrature phase components
- Noise Bandwidth
- Noise Figure
- Noise Temperature
- Noise in DSB& SSB System
- Noise in AM System
- Noise in Angle Modulation System
- Threshold effect in Angle Modulation System

## Noise in communication system

A signal may be contaminated along the path by noise. Noise may be defined as any unwanted introduction of energy into the desired signal. In radio receivers, noise may produce “hiss” in the loudspeaker output. Noise is random and unpredictable.

Noise is produced both external and internal to the system. External noise includes atmospheric noise (e.g., from lightning), galactic noise (thermal radiation from cosmic bodies), and industrial noise (e.g., from motors, ignition). We can minimize or eliminate external noise by proper system design. On the other hand, internal noise is generated inside the system. It is resulted due to random motion of charged particles in resistors, conductors, and electronic devices. With proper system design, it can be minimized but never can be eliminated. This is the major constraint in the rate of telecommunications.

- Noise is unwanted signal that affects wanted signal
- Noise is random signal that exists in communication systems

Effect of noise

- Degrades system performance (Analog and digital)
- Receiver cannot distinguish signal from noise
- Efficiency of communication system reduces

Types of noise

- Thermal noise/white noise/Johnson noise or fluctuation noise
- Shot noise
- Noise temperature
- Quantization noise

### Noise temperature

Equivalent noise temperature is not the physical temperature of amplifier, but a theoretical construct, that is an equivalent temperature that produces that amount of noise power

$$T_e = (F - 1)$$

## White noise

One of the very important random processes is the *white noise* process. Noises in many practical situations are approximated by the white noise process. Most importantly, the white noise plays an important role in modelling of WSS signals.

A white noise process  $\{W(t)\}$  is a random process that has constant power spectral density at all frequencies. Thus

$$S_W(\omega) = \frac{N_0}{2} \quad -\infty < \omega < \infty$$

where  $N_0$  is a real constant and called the *intensity* of the white noise. The corresponding autocorrelation function is given by

$$R_W(\tau) = \frac{N_0}{2} \delta(\tau) \quad \text{where } \delta(\tau) \text{ is the Dirac delta.}$$

The average power of white noise

$$P_{avg} = E W^2(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{N_0}{2} d\omega \rightarrow \infty$$

The autocorrelation function and the PSD of a white noise process is shown in Figure 1 below.

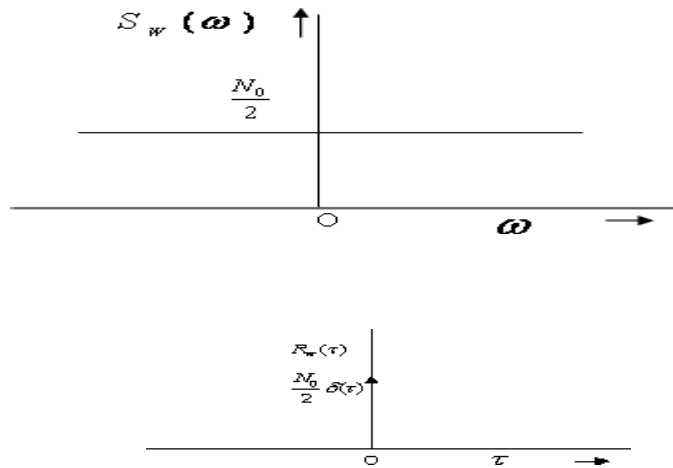


fig: auto correlation and psd of white noise

### NARROWBAND NOISE (NBN)

In most communication systems, we are often dealing with band-pass filtering of signals. Wideband noise will be shaped into band limited noise. If the bandwidth of the band limited noise is relatively small compared to the carrier frequency, we refer to this as *narrowband noise*.

the narrowband noise is expressed as as

$$n(t) = x(t) \cos 2\pi f_c t - y(t) \sin 2\pi f_c t$$

where  $f_c$  is the carrier frequency within the band occupied by the noise.  $x(t)$  and  $y(t)$  are known as the *quadrature components* of the noise  $n(t)$ . The Hilbert transform of

$n(t)$  is

*Proof.*

The Fourier transform of  $n(t)$  is

$$N(f) = \frac{1}{2} X(f - f_c) + \frac{1}{2} X(f + f_c) + \frac{1}{2} jY(f - f_c) - \frac{1}{2} jY(f + f_c)$$

Let  $\hat{N}(f)$  be the Fourier transform of  $\hat{n}(t)$ . In the frequency domain,  $\hat{N}(f) = N(f)[-j \operatorname{sgn}(f)]$ . We simply multiply all positive frequency components of  $N(f)$  by  $-j$  and all negative frequency components of  $N(f)$  by  $j$ . Thus

$$\hat{n}(t) = H[n(t)] = x(t) \sin 2\pi f_c t + y(t) \cos 2\pi f_c t$$

$$\hat{N}(f) = -j \frac{1}{2} X(f - f_c) + j \frac{1}{2} X(f + f_c) - j \frac{1}{2} jY(f - f_c) - j \frac{1}{2} jY(f + f_c)$$

$$\hat{N}(f) = -j \frac{1}{2} X(f - f_c) + j \frac{1}{2} X(f + f_c) + \frac{1}{2} Y(f - f_c) + \frac{1}{2} Y(f + f_c)$$

and the inverse Fourier transform of  $\hat{N}(f)$  is

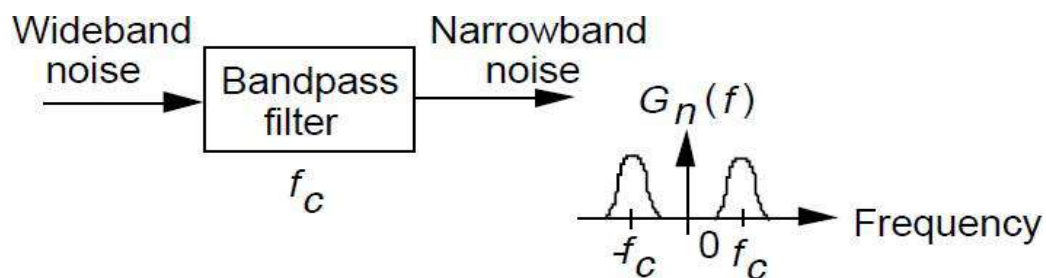
$$\hat{n}(t) = x(t) \sin 2\pi f_c t + y(t) \cos 2\pi f_c t$$

The quadrature components  $x(t)$  and  $y(t)$  can now be derived from equations

$$x(t) = n(t) \cos 2\pi f_c t + \hat{n}(t) \sin 2\pi f_c t$$

and

$$y(t) = n(t) \sin 2\pi f_c t - \hat{n}(t) \cos 2\pi f_c t$$



**Fig: generation of narrow band noise**

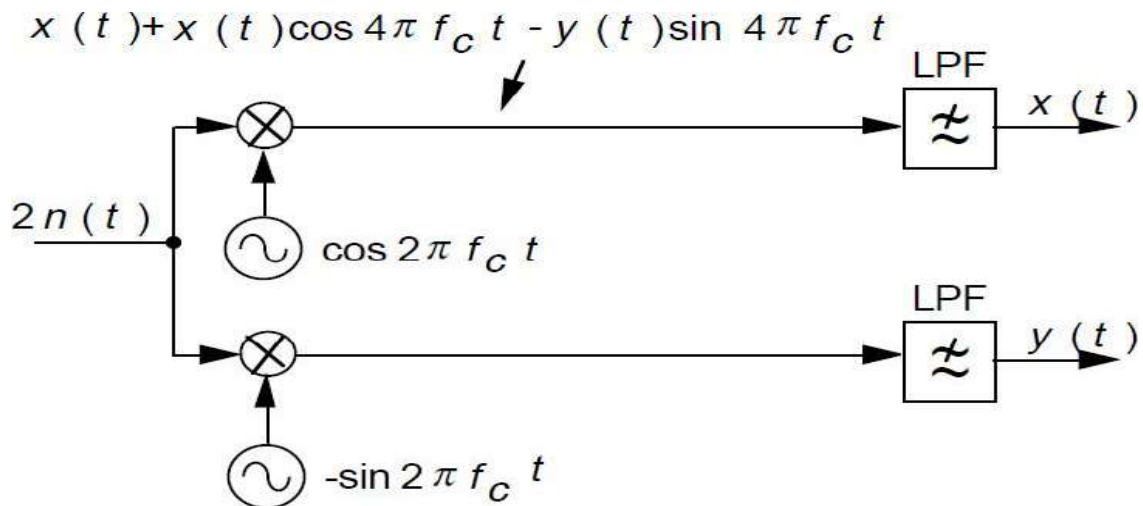


Fig: Generation of quadrature components of  $n(t)$ .

- Filters at the receiver have enough bandwidth to pass the desired signal but not too big to pass excess noise.
- Narrowband (NB)  $f_c$  center frequency is much bigger than the bandwidth.
- Noise at the output of such filters is called narrowband noise (NBN).
- NBN has spectral concentrated about some mid-band frequency  $f_c$
- The sample function of such NBN  $n(t)$  appears as a sine wave of frequency  $f_c$  which modulates slowly in amplitude and phase

**Input signal-to-noise ratio ( $SNR_I$ ):** is the ratio of the average power of modulated signal  $s(t)$  to the average power of the filtered noise.

**Output signal-to-noise ratio ( $SNR_O$ ):** is the ratio of the average power of demodulated message to the average power of the noise, both measured at the receiver output.

**Channel signal-to-noise ratio ( $SNR_C$ ):** is the ratio of the average power of modulated signal  $s(t)$  to the average power of the noise in the message bandwidth, both measured at the receiver input.

## Noise figure

The Noise figure is the amount of noise power added by the electronic circuitry in the receiver to the thermal noise power from the input of the receiver. The thermal noise at the input to the receiver passes through to the demodulator. This noise is present in the receive channel and cannot be removed. The noise figure of circuits in the receiver such as amplifiers and mixers, adds additional noise to the receive channel. This raises the noise floor at the demodulator.

$$\text{Noise Figure} = \frac{\text{Signal to noise ratio at input}}{\text{Signal to noise ratio at output}}$$

## Noise Bandwidth

A filter's equivalent noise bandwidth (ENBW) is defined as the bandwidth of a perfect rectangular filter that passes the same amount of power as the cumulative bandwidth of the channel selective filters in the receiver. At this point we would like to know the noise floor in our receiver, i.e. the noise power in the receiver intermediate frequency (IF) filter bandwidth that comes from  $kTB$ . Since the units of  $kTB$  are Watts/ Hz, calculate the noise floor in the channel bandwidth by multiplying the noise power in a 1 Hz bandwidth by the overall equivalent noise bandwidth in Hz.

## NOISE IN DSB-SC SYSTEM:

Let the transmitted signal is

$$u(t) = A_c m(t) \cos(2\pi f_c t)$$

The received signal at the output of the receiver noise- limiting filter : Sum of this signal and filtered noise .A filtered noise process can be expressed in terms of its in-phase and quadrature components as

$$\begin{aligned} n(t) &= A(t) \cos[2\pi f_c t + \theta(t)] = A(t) \cos \theta(t) \cos(2\pi f_c t) - A(t) \sin \theta(t) \sin(2\pi f_c t) \\ &= n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t) \end{aligned}$$

where  $n_c(t)$  is in-phase component and  $n_s(t)$  is quadrature component

Received signal (Adding the filtered noise to the modulated signal)

$$\begin{aligned} r(t) &= u(t) + n(t) \\ &= A_c m(t) \cos(2\pi f_c t) + n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t) \end{aligned}$$

Demodulate the received signal by first multiplying  $r(t)$  by a locally generated sinusoid  $\cos(2\pi f_c t + \phi)$ , where  $\phi$  is the phase of the sinusoid. Then passing the product signal through an ideal lowpass filter having a bandwidth  $W$ .

The multiplication of  $r(t)$  with  $\cos(2\pi f_c t + \phi)$  yields

$$\begin{aligned} r(t) \cos(2\pi f_c t + \phi) &= u(t) \cos(2\pi f_c t + \phi) + n(t) \cos(2\pi f_c t + \phi) \\ &= A_c m(t) \cos(2\pi f_c t) \cos(2\pi f_c t + \phi) \\ &\quad + n_c(t) \cos(2\pi f_c t) \cos(2\pi f_c t + \phi) - n_s(t) \sin(2\pi f_c t) \cos(2\pi f_c t + \phi) \\ &= \frac{1}{2} A_c m(t) \cos(\phi) + \frac{1}{2} A_c m(t) \cos(4\pi f_c t + \phi) \\ &\quad + \frac{1}{2} [n_c(t) \cos(\phi) + n_s(t) \sin(\phi)] + \frac{1}{2} [n_c(t) \cos(4\pi f_c t + \phi) - n_s(t) \sin(4\pi f_c t + \phi)] \end{aligned}$$

The low pass filter rejects the double frequency components and passes only the low pass components.

$$y(t) = \frac{1}{2} A_c m(t) \cos(\phi) + \frac{1}{2} [n_c(t) \cos(\phi) + n_s(t) \sin(\phi)]$$

the effect of a phase difference between the received carrier and a locally generated carrier at the receiver is a drop equal to  $\cos(\phi)$  in the received signal power.

Phase-locked loop

The effect of a phase-locked loop is to generate phase of the received carrier at the receiver.

If a phase-locked loop is employed, then  $\phi = 0$  and the demodulator is called a coherent or synchronous demodulator.

In our analysis in this section, we assume that we are employing a coherent demodulator.

With this assumption, we assume that  $\phi = 0$

$$y(t) = \frac{1}{2} [A_c m(t) + n_c(t)]$$

Therefore, at the receiver output, the message signal and the noise components are additive and we are able to define a meaningful SNR. The message signal power is given by

$$P_o = \frac{A_c^2}{4} P_M$$

Power  $P_M$  is the content of the messagesignal

The noise power is given by

$$P_{n_0} = \frac{1}{4} P_{n_c} = \frac{1}{4} P_n$$

The power content of  $n(t)$  can be found by noting that it is the result of passing  $n_w(t)$  through a filter with bandwidth  $B_c$ . Therefore, the power spectral density of  $n(t)$  is given by

$$S_n(f) = \begin{cases} \frac{N_0}{2} & |f - f_c| < W \\ 0 & \text{otherwise} \end{cases}$$

The noise power is

$$P_n = \int_{-\infty}^{\infty} S_n(f) df = \frac{N_0}{2} \times 4W = 2WN_0$$

Now we can find the output SNR as

$$\left( \frac{S}{N} \right)_0 = \frac{P_o}{P_{n_0}} = \frac{\frac{A_c^2}{4} P_M}{\frac{1}{4} 2WN_0} = \frac{A_c^2 P_M}{2WN_0}$$

In this case, the received signal power, given by

$$P_R = A_c^2 P_M / 2.$$

The output SNR for DSB-SC AM may be expressed as

$$\left( \frac{S}{N} \right)_{0_{DSB}} = \frac{P_R}{N_0 W}$$

which is identical to baseband SNR.

In DSB-SC AM, the output SNR is the same as the SNR for a baseband system. DSB-SC AM does not provide any SNR improvement over a simple baseband communication system.

## NOISE IN SSB-SC SYSTEM:

Let SSB modulated signal is

$$u(t) = A_c m(t) \cos(2\pi f_c t) \mp A_c \hat{m}(t) \sin(2\pi f_c t)$$

Input to the demodulator

$$\begin{aligned}
 r(t) &= A_c m(t) \cos(2\pi f_c t) \mp A_c \hat{m}(t) \sin(2\pi f_c t) + n(t) \\
 &= A_c m(t) \cos(2\pi f_c t) \mp A_c \hat{m}(t) \sin(2\pi f_c t) + n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t) \\
 &= [A_c m(t) + n_c(t)] \cos(2\pi f_c t) + [\mp A_c \hat{m}(t) - n_s(t)] \sin(2\pi f_c t)
 \end{aligned}$$

Assumption : Demodulation with an ideal phase reference.

Hence, the output of the lowpass filter is the in-phase component (with a coefficient of  $1/2$ ) of the preceding signal.

Parallel to our discussion of DSB, we have

$$\begin{aligned}
 P_o &= \frac{A_c^2}{4} P_M \\
 P_{n_0} &= \frac{1}{4} P_{n_c} = \frac{1}{4} P_n \\
 P_n &= \int_{-\infty}^{\infty} S_n(f) df = \frac{N_0}{2} \times 2W = WN_0
 \end{aligned}$$

$\left( \frac{S}{N} \right)_0 = \frac{P_o}{P_{n_0}} = \frac{A_c^2 P_M}{WN_0}$

$P_R = P_U = A_c^2 P_M$

$\left( \frac{S}{N} \right)_{0SSB} = \frac{P_R}{N_0 W} = \left( \frac{S}{N} \right)_b$

The signal-to-noise ratio in an SSB system is equivalent to that of a DSB system.

## Noise in Conventional AM

DSB AM signal :  $u(t) = A_c [1 + am_n(t)] \cos(2\pi f_c t)$

Received signal at the input to the demodulator

$$\begin{aligned}
 r(t) &= A_c [1 + am_n(t)] \cos(2\pi f_c t) + n(t) \\
 &= A_c [1 + am_n(t)] \cos(2\pi f_c t) + n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t) \\
 &= [A_c [1 + am_n(t)] + n_c(t)] \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t)
 \end{aligned}$$

Where

$a$  is the modulation index

$m_n(t)$  is normalized so that its minimum value is -1

If a synchronous demodulator is employed, the situation is basically similar to the DSB case, except that we have  $1 + am_n(t)$  instead of  $m(t)$ .

$$y(t) = \frac{1}{2} [A_c a m_n(t) + n_c(t)]$$

Received signal power

$$P_R = \frac{A_c^2}{2} [1 + a^2 P_{M_n}]$$

□ Assumed that the message process is zero mean.

Now we can derive the output SNR as

$$\begin{aligned} \left( \frac{S}{N} \right)_{0_{AM}} &= \frac{\frac{1}{4} A_c^2 a^2 P_{M_n}}{\frac{1}{4} P_{n_c}} = \frac{A_c^2 a^2 P_{M_n}}{2 N_0 W} = \frac{a^2 P_{M_n}}{1 + a^2 P_{M_n}} \frac{\frac{A_c^2}{2} [1 + a^2 P_{M_n}]}{N_0 W} \\ &= \frac{a^2 P_{M_n}}{1 + a^2 P_{M_n}} \frac{P_R}{N_0 W} = \frac{a^2 P_{M_n}}{1 + a^2 P_{M_n}} \left( \frac{S}{N} \right)_b = \eta \left( \frac{S}{N} \right)_b \end{aligned}$$

□  $\eta$  denotes the modulation efficiency

□ Since  $a^2 P_{M_n} < 1 + a^2 P_{M_n}$ , the SNR in conventional AM is always smaller than the SNR in a baseband system.

- In practical applications, the modulation index  $a$  is in the range of 0.8-0.9.
- Power content of the normalized message process depends on the message source.
- Speech signals : Large dynamic range,  $P_M$  is about 0.1.
- The overall loss in SNR, when compared to a baseband system, is a factor of 0.075 or equivalent to a loss of 11 dB.

**The reason for this loss** is that a large part of the transmitter power is used to send the carrier component of the modulated signal and not the desired signal. To analyze the envelope-detector performance in the presence of noise, we must use certain approximations.

This is a result of the nonlinear structure of an envelope detector, which makes an exact analysis difficult

In this case, the demodulator detects the envelope of the received signal and the noise process.

The input to the envelope detector is

$$r(t) = [A_c [1 + a m_n(t)] + n_c(t)] \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t)$$

Therefore, the envelope of  $r(t)$  is given by

$$V_r(t) = \sqrt{[A_c [1 + a m_n(t)] + n_c(t)]^2 + n_s^2(t)}$$

Now we assume that the signal component in  $r(t)$  is much stronger than the noise component. Then

$$P(n_c(t) \ll A_c[1 + am_n(t)]) \approx 1$$

Therefore, we have a high probability that

$$V_r(t) \approx A_c[1 + am_n(t)] + n_c(t)$$

After removing the DC component, we obtain

$$y(t) = A_c am_n(t) + n_c(t)$$

which is basically the same as  $y(t)$  for the synchronous demodulation without the  $\frac{1}{2}$  coefficient.

This coefficient, of course, has no effect on the final SNR. So we conclude that, under the assumption of high SNR at the receiver input, the performance of synchronous and envelope demodulators is the same.

However, if the preceding assumption is not true, that is, if we assume that, at the receiver input, the noise power is much stronger than the signal power, Then

$$\begin{aligned} V_r(t) &= \sqrt{[A_c[1 + am_n(t)] + n_c(t)]^2 + n_s^2(t)} \\ &= \sqrt{A_c^2[1 + am_n(t)]^2 + n_c^2(t) + n_s^2(t) + 2A_cn_c(t)[1 + am_n(t)]} \\ &\xrightarrow{a} \sqrt{(n_c^2(t) + n_s^2(t)) \left[ 1 + \frac{2A_cn_c(t)}{n_c^2(t) + n_s^2(t)} (1 + am_n(t)) \right]} \\ &\xrightarrow{b} V_n(t) \left[ 1 + \frac{A_cn_c(t)}{V_n^2(t)} (1 + am_n(t)) \right] \\ &= V_n(t) + \frac{A_cn_c(t)}{V_n(t)} (1 + am_n(t)) \end{aligned}$$

(a) :  $A_c^2[1 + am_n(t)]^2$  is small compared with the other components

(b) :  $\sqrt{n_c^2(t) + n_s^2(t)} = V_n(t)$  ; the envelope of the noise process

Use the approximation

$$\sqrt{1 + \varepsilon} \approx 1 + \frac{\varepsilon}{2}, \text{ for small } \varepsilon, \text{ where } \varepsilon = \frac{2A_cn_c(t)}{n_c^2(t) + n_s^2(t)} (1 + am_n(t))$$

Then

$$V_r(t) = V_n(t) + \frac{A_c n_c(t)}{V_n(t)} (1 + \alpha m_n(t))$$

We observe that, at the demodulator output, the signal and the noise components are no longer additive. In fact, the signal component is multiplied by noise and is no longer distinguishable. In this case, no meaningful SNR can be defined. We say that this system is operating below the threshold. The subject of threshold and its effect on the performance of a communication system will be covered in more detail when we discuss the noise performance in angle modulation.

### Effect of threshold in angle modulation system:

**FM THRESHOLD EFFECT** FM threshold is usually defined as a Carrier-to-Noise ratio at which demodulated Signal-to-Noise ratio falls 1dB below the linear relationship. This is the effect produced in an FM receiver when noise limits the desired information signal. It occurs at about 10 dB, as earlier stated in 5 the introduction, which is at a point where the FM signal-to-Noise improvement is measured. Below the FM threshold point, the noise signal (whose amplitude and phase are randomly varying) may instantaneously have amplitude greater than that of the wanted signal. When this happens, the noise will produce a sudden change in the phase of the FM demodulator output. In an audio system, this sudden phase change makes a “click”. In video applications the term “click noise” is used to describe short horizontal black and white lines that appear randomly over a picture

An important aspect of analogue FM satellite systems is FM threshold effect. In FM systems where the signal level is well above noise received carrier-to-noise ratio and demodulated signal-to-noise ratio are related by:

$$S/N = 3 \beta^2 C/N \quad \text{Eqn 9}$$

where

S/N	=	signal-to-noise ratio at output of FM demodulator
$\beta$	=	FM deviation ratio or modulation index = $(\Delta f/B)$
C/N	=	carrier-to-noise ratio at input of FM demodulator
$\Delta f$	=	peak deviation
B	=	basebandwidth of signal being modulated

The expression however does not apply when the carrier-to-noise ratio decreases below a certain point. Below this critical point the signal-to-noise ratio decreases significantly. This is

known as the FM threshold effect (FM threshold is usually defined as the carrier-to-noise ratio at which the demodulated signal-to-noise ratio fall 1 dB below the linear relationship given in Eqn 9. It generally is considered to occur at about 10 dB).

Below the FM threshold point the noise signal (whose amplitude and phase are randomly varying), may instantaneously have an amplitude greater than that of the wanted signal. When this happens the noise will produce a sudden change in the phase of the FM demodulator output. In an audio system this sudden phase change makes a "click". In video applications the term "click noise" is used to describe short horizontal black and white lines that appear randomly over a picture, because satellite communications systems are power limited they usually operate with only a small design margin above the FM threshold point (perhaps a few dB). Because of this circuit designers have tried to devise techniques to delay the onset of the FM threshold effect. These devices are generally known as FM threshold extension demodulators. Techniques such as FM feedback, phase locked loops and frequency locked loops are used to achieve this effect. By such techniques the onset of FM threshold effects can be delayed till the C/N ratio is around 7 dB.

### Noise in Angle Modulated Systems

Like AM, noise performance of angle modulated systems is characterized by parameter  $\gamma$

$$\gamma_{FM} = \frac{3}{2} \beta^2$$

If it is compared with AM

$$\frac{\gamma_{FM}}{\gamma_{AM}} = \frac{1}{2} \left( \frac{\omega_{FM}}{\omega_{AM}} \right)^2$$

Note: if bandwidth ratio is increased by a factor 2. Then  $\frac{\gamma_{FM}}{\gamma_{AM}}$  increases by a factor 4

This exchange of bandwidth and noise performance is an important feature of FM

$$\text{Figure of merit } (\gamma) = \frac{SNR_O}{SNR_C}$$

CW- Modulation System	$SNR_O$	$SNR_C$	Figure of merit	Figure of merit (single tone)
DSB-SC	$\frac{C^2 A_c^2 P}{2WN_0}$	$\frac{C^2 A_c^2 P}{2WN_0}$	1	1
SSB	$\frac{C^2 A_c^2 P}{4WN_0}$	$\frac{C^2 A_c^2 P}{4WN_0}$	1	1
AM	$\frac{A_c^2 k_a^2 P}{2WN_0}$	$\frac{A_c^2 (1 + k_a^2 P)}{2WN_0}$	$\approx \frac{k_a^2 P}{1 + k_a^2 P} < 1$	$\frac{\mu^2}{2 + \mu^2}$
FM	$\frac{3A_c^2 k_f^2 P}{2N_0 W^3}$	$\frac{A_c^2}{2WN_0}$	$\frac{3k_f^2 P}{W^2}$	$\frac{3}{2} \beta^2$

$P$  is the average power of the message signal.

$C^2$  is a constant that ensures that the ration is dimensionless.

$W$  is the message bandwidth.

$A_c$  is the amplitude of the carrier signal.

$k_a$  is the amplitude sensitivity of the modulator.

$\mu = k_a A_m$  and  $A_m$  is the amplitude sinusoidal wave

$\beta = \frac{\Delta f}{W}$  is the modulation index.

$k_f$  is the frequency sensitivity of the modulator.

$\Delta f$  is the frequency deviation.

## UNIT-V

### Receivers

#### Introduction to Radio Receivers:

In radio communications, a **radio receiver** (**receiver** or simply **radio**) is an electronic device that receives **radio** waves and converts the information carried by them to a usable form.

#### Types of Receivers:

The TRF (Tuned Radio Frequency ) Receiver and Superheterodyne Receiver are the two main configurations of the receivers, they have real practical or commercial significance. Most of the present day receivers use superheterodyne configuration. But the TRF receivers are simple and easy to understand.

#### Tuned Radio Frequency Receiver:

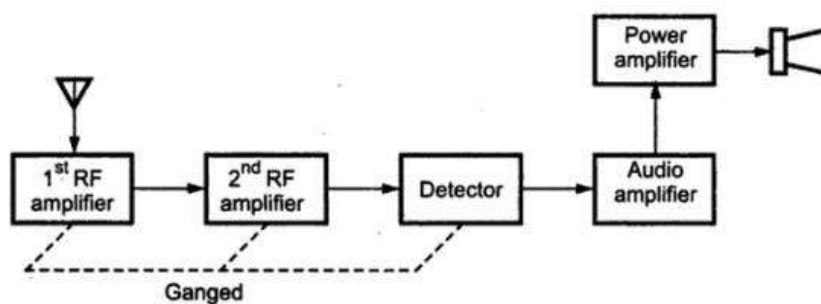


Fig. 1 shows the block diagram of TRF receiver.

It consists of two or three stages of RF amplifiers, detector, audio amplifier and

Fig.1. TRF Receiver

power amplifier. The RF amplifier stages placed between the antenna and detector are used to increase the strength of the received signal before it is applied to the detector. These RF amplifiers are tuned to fix frequency, amplify the desired band of frequencies. Therefore, they provide amplification for selected band of frequencies and rejection for all others. As selection and amplification process is carried out in two or three stages and each stage must amplify the same band of frequencies, the ganged tuning is provided.

The amplified signal is then demodulated using detector to recover the modulating signal. The recovered signal is amplified further by the audio amplifier followed by power amplifier which provides sufficient gain to operate a loudspeaker. The TRF receivers suffered from number of annoying problems. These are listed in the next section.

## Problems in TRF Receivers:

### 1. Tracking of Tuned Circuit

In a receiver, tuned circuits are made variable so that they can be set to the frequency of the desired signal. In most of the receivers, the capacitors in the tuned circuits are made variable. These capacitors are 'ganged' between the stages so that they all can be changed simultaneously when the tuning knob is rotated. To have perfect tuning the capacitor values between the stages must be exactly same but this is not the case. The differences in the capacitors cause the resonant frequency of each tuned circuit to be slightly different, thereby increasing the pass band.

### 2. Instability

As high gain is achieved at one frequency by a multistage amplifier, there are more chances of positive feedback (of getting back the small part of output of the last stage at the input to the first with the correct polarity) through some stray path, resulting in oscillations. These oscillations are unavoidable at high frequencies.

### 3. Variable Bandwidth

TRF receivers suffer from a variation in bandwidth over the tuning range. Consider a medium wave receiver required to tune over 535 kHz to 1640 kHz and it provides the necessary bandwidth of 10 kHz at 535 kHz. Let us calculate Q of this circuit.

$$Q = \frac{f}{\text{Bandwidth}} = \frac{535 \text{ kHz}}{10 \text{ kHz}} = 53.5$$

Now consider the frequency at the other end of the broadcast band, i.e. 1640 kHz. At 1640 kHz, Q of the coil should be 164 (1640 kHz / 10 kHz). However, in practice due to various losses depending on frequency, we will not get so large increase in Q. Let us assume that at 1640 kHz frequency Q is increased to value 100 instead of 164. With this Q of the tuned circuit bandwidth can be calculated as follows

$$\text{Bandwidth} = \frac{f}{Q} = \frac{1640 \text{ kHz}}{100} = 16.4 \text{ kHz}$$

We know, necessary bandwidth is 10 kHz. This increase in bandwidth of tuned circuit, pick up the adjacent stations along with station it is tuned for, providing insufficient adjacent frequency rejection. In other words we can say that in TRF receivers the bandwidth of the tuned circuit varies over the frequency range, resulting in poor selectivity of the receiver.

Because of the problems of tracking, instability and bandwidth variation, the TRF receivers have almost been replaced by superheterodyne receivers.

## Superheterodyne Receivers

To solve basic problems of TRF receivers, in these receivers, first all the incoming RF frequencies are converted to a fix lower frequency called **intermediate frequency (IF)**. Then this fix intermediate frequency is amplified and detected to reproduce the original information. Since the characteristics of the IF amplifier are independent of the frequency to which the receiver is tuned, the selectivity and sensitivity of superheterodyne receivers are fairly uniform throughout its tuning range.

Mixer circuit is used to produce the frequency translation of the incoming signal down to the IF. The incoming signals are mixed with the local oscillator frequency signal in such a way that a constant frequency difference is maintained between the local oscillator and the incoming signals. This is achieved by using ganged tuning capacitors.

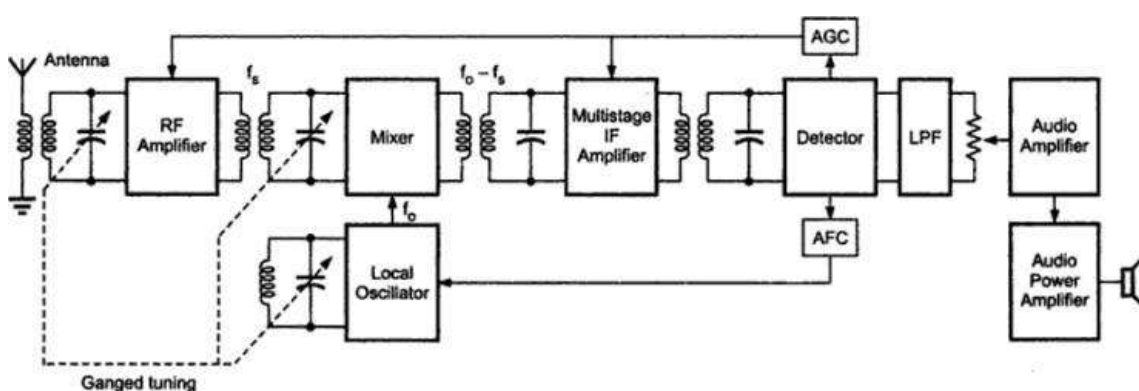


Fig.2. Block diagram of Super heterodyne Receiver.

Fig. 2 shows the block diagram of superheterodyne receiver. As shown in the Fig. 2 antenna picks up the weak radio signal and feeds it to the RF amplifier. The RF amplifier provides some initial gain and selectivity. The output of the RF amplifier is applied to the input of the mixer. The mixer also receives an input from local oscillator.

The output of the mixer circuit is difference frequency ( $f_o - f_s$ ) commonly known as IF (Intermediate Frequency). The signal at this intermediate frequency contains the same modulation as the original carrier. This signal is amplified by one or more IF amplifier stages, and most of the receiver gain is obtained in these IF stages.

The highly amplified IF signal is applied to detector circuits to recover the original modulating information. Finally, the output of detector circuit is fed to audio and power amplifier which provides a sufficient gain to operate a speaker.

Another important circuit in the superheterodyne receiver are AGC and AFC circuit. AGC is used to maintain a constant output voltage level over a wide range of RF input signal levels.

It derives the dc bias voltage from the output of detector which is proportional to the amplitude of the received signal. This dc bias voltage is feedback to the IF amplifiers, and sometimes to the RF amplifier, to control the gain of the receiver. As a result, it provides a constant output voltage level over a wide range of RF input signal levels. AFC circuit generates AFC signal which is used to adjust and stabilize the frequency of the local oscillator.

### Characteristics of Radio Receiver:

The performance of the radio receiver can be measured in terms of following receiver characteristics

- Selectivity
- Sensitivity
- Fidelity
- Image frequency and its rejection
- Double spotting

#### Selectivity

Selectivity refers to the ability of a receiver to select a signal of a desired frequency while reject all others. Selectivity in a receiver is obtained by using tuned circuits. These are LC circuits tuned to resonate at a desired signal frequency. The Q of these tuned circuits determines the selectivity. Selectivity shows the attenuation that the receiver offers to signals at frequencies near to the one to which it is tuned. A good receiver isolates the desired signal in the RF spectrum and eliminate all other signals.

Recall that Q is the ratio of inductive reactance to resistance ( $Q = X_L / R$ ), and we know that bandwidth of the tuned circuit is given by

$$B_w = \frac{f_r}{Q}$$

where  $f_r$  is the resonant frequency. The bandwidth of a tuned circuit is measure of the selectivity. Narrower the bandwidth the better selectivity. To have narrower bandwidth and better selectivity the Q of the tuned circuit must be high.

## Sensitivity

The sensitivity of a communication receiver refers to the receiver's ability to pick up weak signals, and amplify it. It is often defined in terms of the voltage that must be applied to the receiver input terminals to give a standard output power, measured at the output terminals. The more gain that a receiver has, the smaller the input signal necessary to produce desired output power. Therefore, sensitivity is a primary function of the overall receiver gain. It is often expressed in microvolts or in decibels. The sensitivity of a receiver mostly depends on the gain of the IF amplifiers. Good communication receiver has sensitivity of 0.2 to 1  $\mu\text{V}$ .

## Fidelity

Fidelity refers to the ability of the receiver to reproduce all the modulating frequencies equally. Fig. 3 shows the typical fidelity curve for a radio receiver.

The fidelity at the lower modulating frequencies is determined by the low frequency response of the IF amplifier and the fidelity at the higher modulating frequencies is determined by the high frequency response of the IF amplifier. Fidelity is difficult to obtain in AM receiver because good fidelity requires more bandwidth of IF amplifier resulting in poor selectivity.

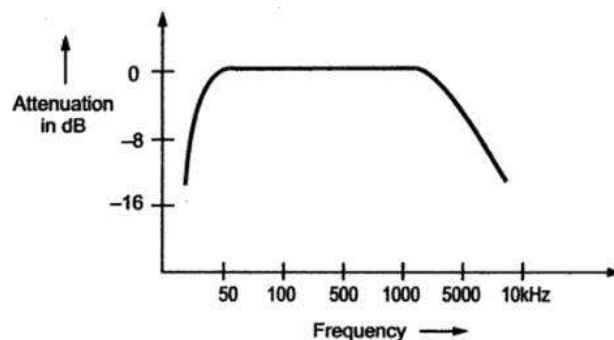


Fig.3. Typical Fidelity curve

## Image Frequency and its Rejection

In standard broadcast receiver the local oscillator frequency is made higher than the signal frequency by an amount equal to intermediate frequency (IF). Therefore  $f_o = f_s + f_i$ . When  $f_o$  and  $f_s$  are mixed, the difference frequency, which is one of the by products, is equal to  $f_i$  only  $f_i$  is passed and amplified by the IF stage.

If a frequency  $f_{si}$  ( $f_o + f_i$ ), i.e.  $f_{si} = f_s + 2 f_i$ , appears at the input of the mixer then it will produce the sum and difference frequencies regardless of the inputs. Therefore, the mixer output will be the difference frequency at the IF value. The term  $f_{si}$  is called the **image frequency** and is defined as the signal frequency plus twice the intermediate frequency. Unfortunately, this image frequency signal is also amplified by the IF amplifiers resulting in interference. This has the effect of two stations being received simultaneously and is naturally undesirable.

The rejection of an image frequency by a single tuned circuit is the ratio of the gain at the signal frequency to the gain at the image frequency. It is given by

$$\alpha = \sqrt{1 + Q^2 \rho^2}$$

where  $\rho = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}}$

and  $Q = \text{loaded } Q \text{ of tuned circuit}$

If the receiver has an RF stage, then there are two tuned circuits, both tuned to  $f_s$ ; the rejection of each will be calculated by the same formula, and the total rejection will be the product of two.

The image rejection depends on the selectivity of the RF amplifier and tuned circuits and must be achieved before the IF stage. Once the spurious frequency enters the first IF amplifier, it becomes impossible to remove it from the wanted signal.

### **Double Spotting**

The phenomenon of double spotting occurs at higher frequencies due to poor front end selectivity of the receivers. In this, receiver picks up same short-wave station at two nearby points on the receiver dial.

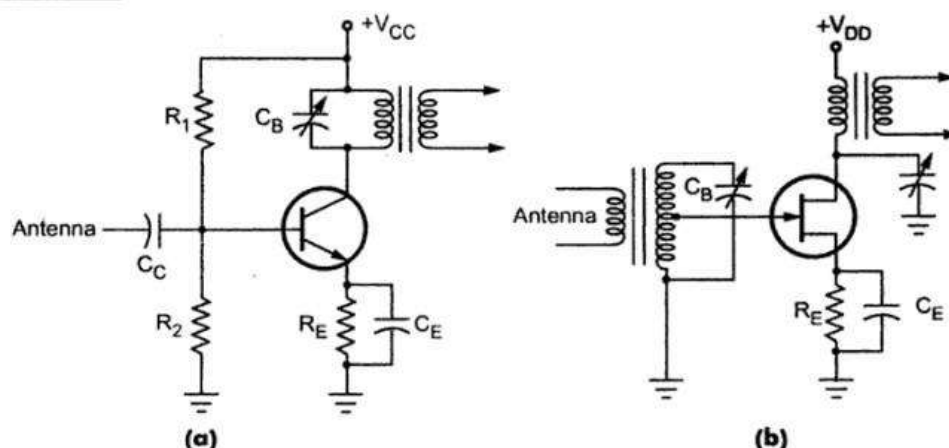
When the receiver is tuned across the band, a strong signal appears to be at two different frequencies, once at the desired frequency and again when the receiver is tuned to 2 times IF below the desired frequency. In this second case, the signal becomes the image, reduced in strength by the image rejection, thus making it appear that the signal is located at two frequencies in the band.

### **Blocks in Super heterodyne Receiver:**

- **Basic principle**
  - Mixing
  - Intermediate frequency of 455 KHz
  - Ganged tuning
- **RF section**
  - Tuning circuits – reject interference and reduce noise figure
  - Wide band RF amplifier
- **Local Oscillator**
  - 995 KHz to 2105 KHz
  - Tracking
- **IF amplifier**
  - Very narrow band width Class A amplifier – selects 455 KHz only
  - Provides much of the gain
  - Double tuned circuits
- **Detector**
  - RF is filtered to ground

## 1. RF Amplifier:

RF amplifier provides initial gain and selectivity. Fig. 4 shows the RF amplifier circuits. It is a tuned circuit followed by an amplifier. The RF amplifier is usually a simple class A circuit. A typical bipolar circuit is shown in Fig. 4. (a), and a typical FET circuit is shown in Fig. 4. (b).



The values of resistors  $R_1$  and  $R_2$  in the bipolar circuit are adjusted such that the amplifier works as class A amplifier. The antenna is connected through coupling capacitor to the base of the transistor. This makes the circuit very broad band as the transistor will amplify virtually any signal picked up by the antenna. However the collector is tuned with a parallel resonant circuit to provide the initial selectivity for the mixer input.

The FET circuit shown in Fig. 4 (b) is more effective than the transistor circuit. Their high input impedance minimizes the loading on tuned circuits, thereby permitting the Q of the circuit to be higher and selectivity to be sharper.

The receiver having an RF amplifier stage has following advantages :

1. It provides greater gain, i.e. better sensitivity.
2. It improves image-frequency rejection.
3. It improves signal to noise ratio.
4. It improves rejection of adjacent unwanted signals, providing better selectivity.
5. It provides better coupling of the receiver to the antenna.
6. It prevents spurious frequencies from entering the mixer and heterodyning there to produce an interfering frequency equal to the IF from the desired signal.
7. It also prevents reradiation of the local oscillator through the antenna of the receiver.

## 2. Mixer

The frequency converter is a nonlinear resistance having two sets of input terminals and one set of output terminal. The two inputs to the frequency converter are the input signal along with any modulation and the input from a local oscillator (LO). The output contains several frequencies including the difference between the input frequencies. The difference frequency is called intermediate frequency and output circuit of the mixer is tuned for the intermediate frequency.

### Separately Excited Mixer:

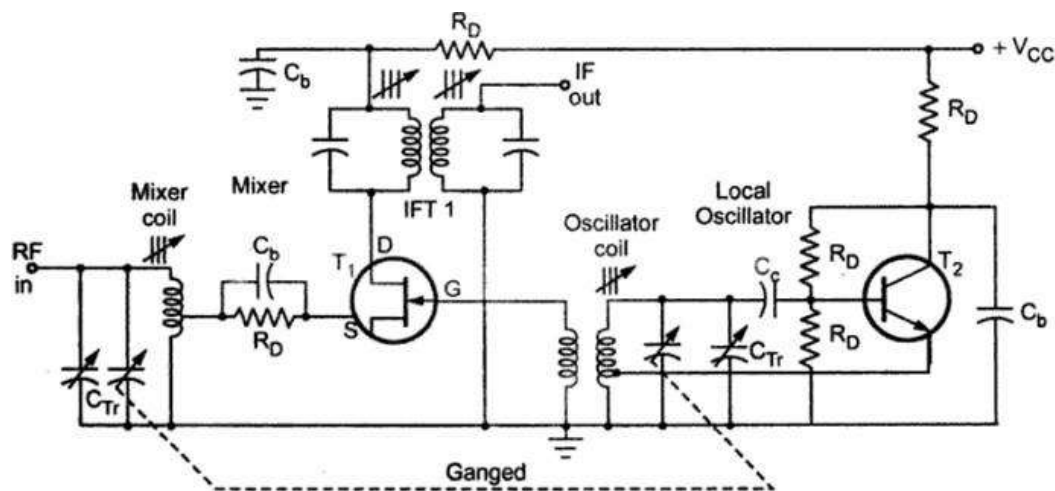


Fig.5 Separately Excited FET Mixer

Fig. 5 shows the separately excited mixer using FET. Here, one device acts as a mixer while the other supplies the necessary oscillations. The bipolar transistor  $T_2$ , forms the Hartley oscillator circuit. It oscillates with local frequency ( $f_o$ ). FET  $T_1$ , is a mixer, whose gate is fed with the output of local oscillator and its bias is adjusted such that it operates in a nonlinear portion of its characteristic. The local oscillator varies the gate bias of the FET to vary its transconductance in a nonlinear manner, resulting intermediate frequency (IF) at the output. The output is taken through double tuned transformer in the drain of the mixer and fed to the IF amplifier. The ganged tuning capacitor allows simultaneous tuning of mixer and local oscillator.

The  $C_{Tr}$ , a small trimmer capacitors across each of the tuning capacitors are used for fine adjustments.

### Self Excited Mixer:

It is possible to combine the function of the mixer and local oscillator in one circuit. The circuit is commonly known as self excited mixer. Fig. 6 shows self excited bipolar transistor mixer. The circuit oscillates and the transconductance of the transistor is varied in a nonlinear manner at the local oscillator rate. This variable transconductance ( $g_m$ ) is used by the transistor to amplify the incoming RF signal.

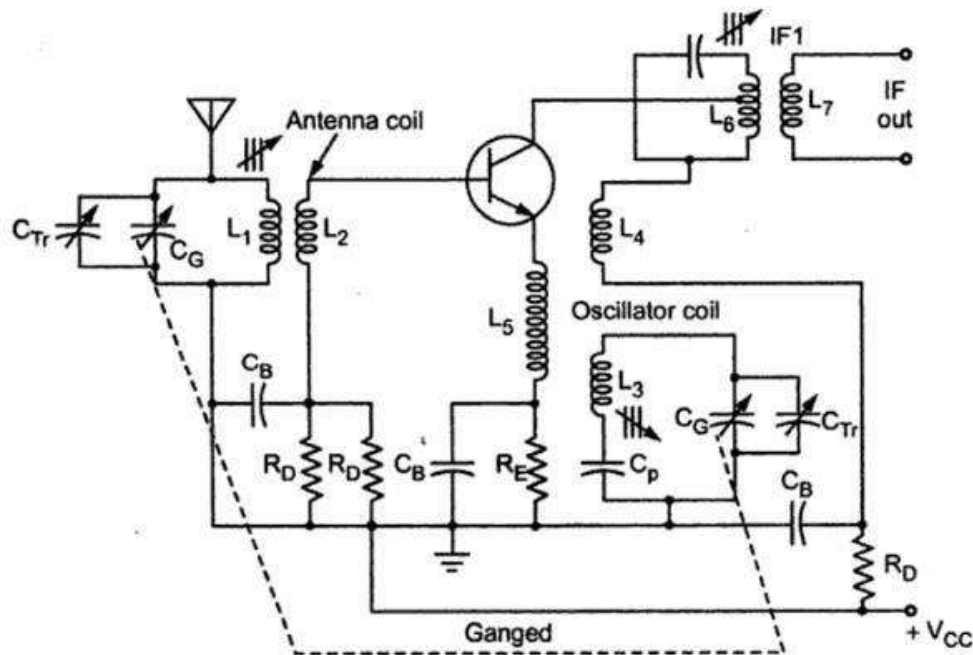


Fig.6. Self Excited Mixer

### 3. Tracking

The superheterodyne receiver has number of tunable circuits which must all be tuned correctly if any given station is to be received. The ganged tuning is employed to do this work, which mechanically couples all tuning circuits so that only one tuning control or dial is required. Usually, there are three tuned circuits : Antenna or RF tuned circuit, mixer tuned circuit and local oscillator tuned circuit. All these circuits must be tuned to get proper RF input and to get IF frequency at the output of mixer. The process of tuning circuits to get the desired output is called **Tracking**. Any error that exists in the frequency difference will result in an incorrect frequency being fed to the IF amplifier. Such errors are known as '**Tracking Errors**' and these must be avoided.

To avoid tracking errors standard capacitors are not used, and ganged capacitors with identical sections are used. A different value of inductance and special extra capacitors called trimmers and padders are used to adjust the capacitance of the oscillator to the proper range. There are three common methods used for tracking. These are

- Padder tracking
- Trimmer tracking

#### 4. Local Oscillator

In shortwave broadcasting, the operating limit for receivers is 36 MHz. For such operating limit local oscillators such as Armstrong, Hartley, Colpitts, Clapp or ultra-audion are used. The Colpitts, Clapp and ultra-audion oscillators are used at the top of the operating limit, whereas Hartley oscillator is used for frequencies below 120 MHz. All these oscillators are LC oscillators and each employs only one tuned circuit to determine its frequency. When higher frequency stability of local oscillator is required, the circuits like AFC (Automatic Frequency Control) are used.

#### 5. IF Amplifier

IF amplifiers are tuned voltage amplifiers tuned for the fixed frequency. Its important function is to amplify only tuned frequency signal and reject all others. As we know, most of the receiver gain is provided by the IF amplifiers, to obtain required gain, usually two or more stages of IF amplifiers are required.

Fig. 7 shows the two stage IF amplifier. Two stages are transformer coupled and all IF transformers are single tuned, i.e. tuned for single frequency.

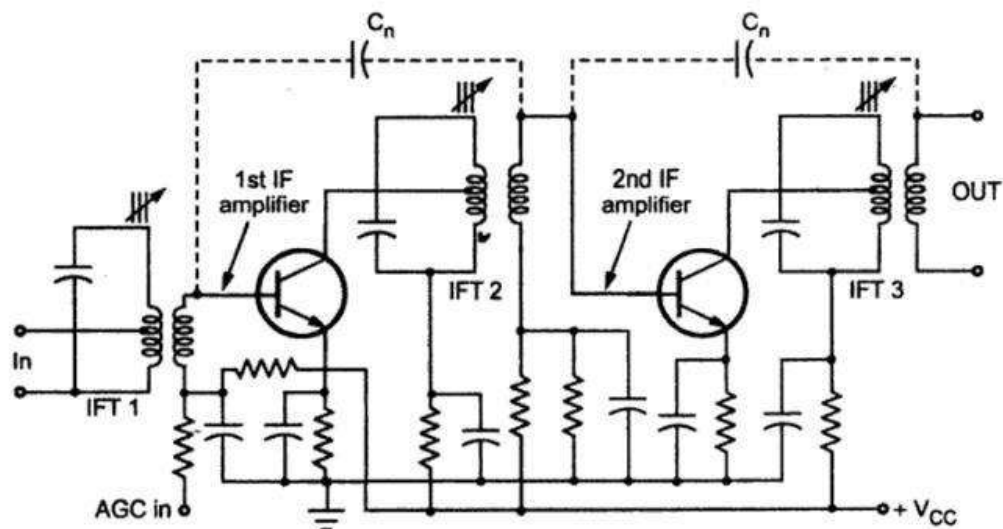


Fig.7 Two Stage IF Amplifier

## Choice of Intermediate Frequency:

Selection of the intermediate frequency depends on various factors. While choosing the intermediate frequency it is necessary to consider following factors.

1. Very high intermediate frequency will result in poor selectivity and poor adjacent channel rejection.
2. A high value of intermediate frequency increases tracking difficulties.
3. At low values of intermediate frequency, image frequency rejection is poor.
4. At very low values of intermediate frequency, selectivity is too sharp. Cutting off the sidebands.
5. At very low IF, the frequency stability of the local oscillator must be correspondingly high because any frequency drift is now a larger proportion of the low IF than of a high IF.
6. The IF must not fall in the tuning range of the receiver, otherwise instability will occur and heterodyne whistles will be heard, making it impossible to tune to the frequency band immediately adjacent to the intermediate frequency.

With the above considerations the standard broadcast AM receivers [tuning to 540 to 1650 kHz] use an IF within the 438 kHz to 465 kHz range. The 465 kHz IF is most commonly used.

## 6. Automatic Gain Control

Automatic Gain Control is a system by means of which the overall gain of a radio receiver is varied automatically with the variations in the strength of the receiver signal, to maintain the output substantially constant. AGC circuitry derives the dc bias voltage from the output of the detector. It applies this derived dc bias voltage to a selected number of RF, IF and mixer stages to control their gains. When the average signal level increases, the size of the AGC bias increases, and the gain of the controlled stages decreases. When there is no signal, there is a minimum AGC bias, and the amplifiers produce maximum gain. There are two types of AGC circuits in use : Simple AGC and Delayed AGC

### Simple AGC

In simple AGC receivers the AGC bias starts to increase as soon as the received signal level exceeds the background noise level. As a result receiver gain starts falling down, reducing the sensitivity of the receiver.

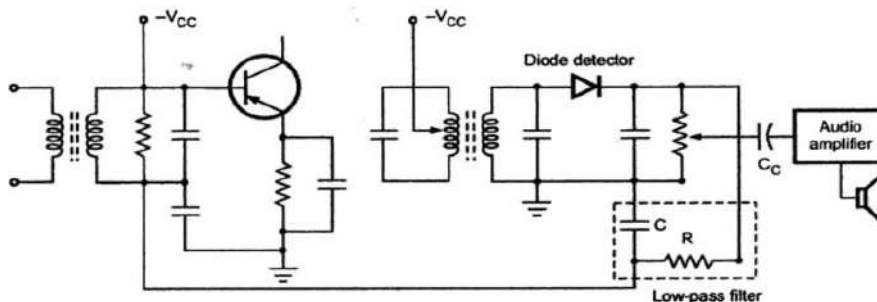


Fig.8. Simple AGC circuit

Fig. 8 shows the simple AGC circuit. In this circuit, dc bias produced by half wave rectifier as a AM detector, is used to control the gain of RF or IF amplifier. Before application of this voltage to the base of the RF and / or IF stage amplifier the audio signal is removed by the lowpass filter. The time constant of the filter is kept at least 10 times longer than the period of the lowest modulation frequency received. If the time constant is kept longer, it will give better filtering, but it will cause an annoying delay in the application of the AGC control when tuning from one signal to another. The recovered signal is then passed through  $C_C$  to remove the dc. The resulting ac signal is further amplified and applied to the loudspeaker.

### Delayed AGC

Simple AGC is clearly an improvement over no AGC at all. Unfortunately, in simple AGC circuit, the unwanted weak signals(noise signals) are amplified with high gain. To

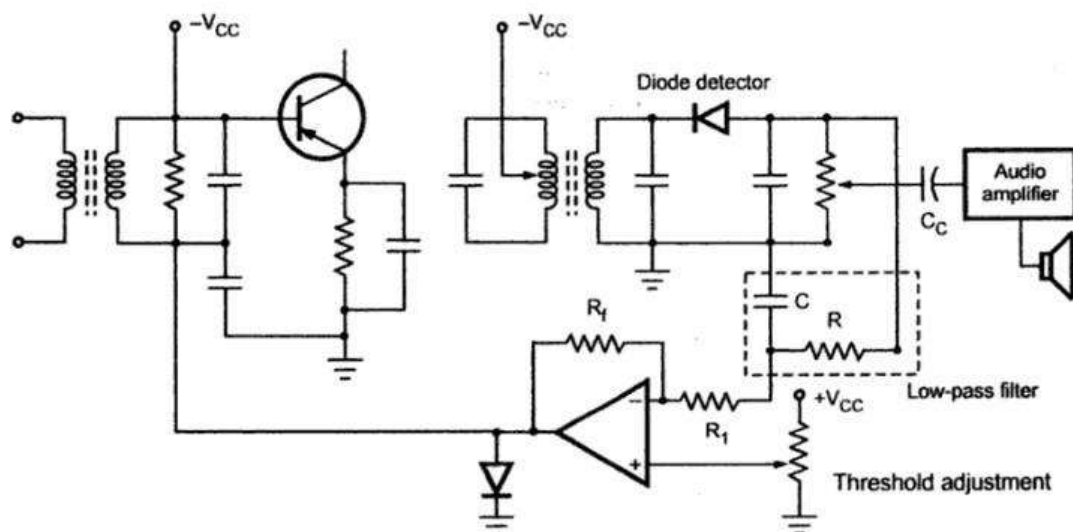


Fig.9. Delayed AGC circuit

avoid this, in delayed AGC circuits, AGC bias is not applied to amplifiers until signal strength has reached a predetermined level, after which AGC bias is applied as with simple AGC, but more strongly.

Here, AGC output is applied to the difference amplifier. It gives negative dc AGC only when AGC output generated by diode detector is above certain dc threshold voltage. This threshold voltage can be adjusted by adjusting the voltage at the positive input of the operational amplifier.

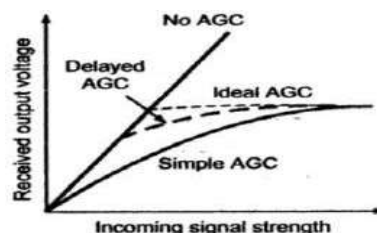


Fig.10. Response of receiver with various AGC circuits.

### FM Receiver:

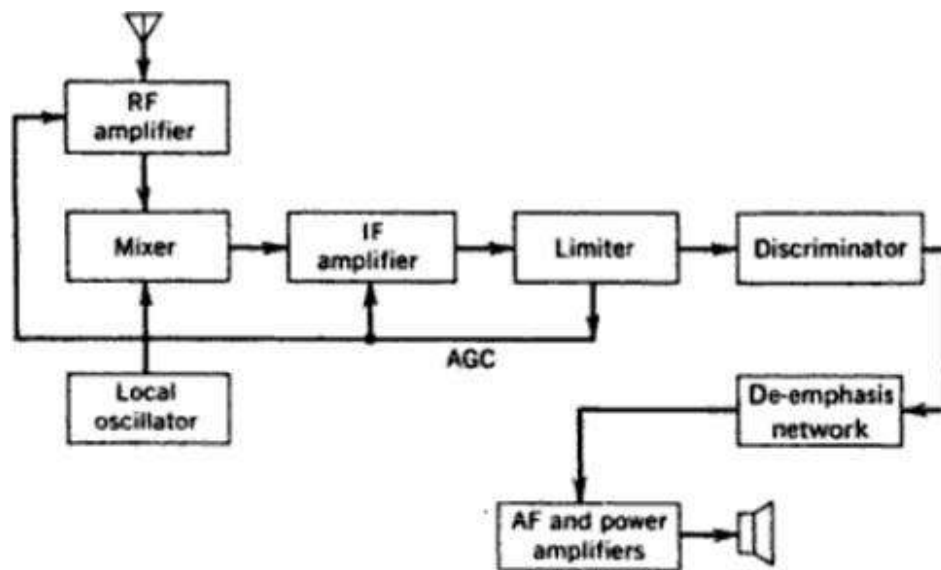


Fig.11. FM Receiver Block diagram

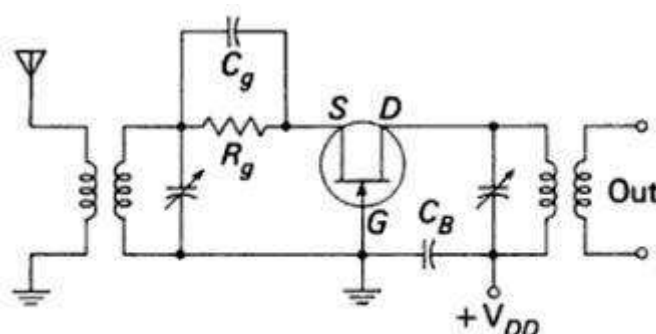
The FM receiver is a superheterodyne receiver, and the block diagram of Figure 11 shows just how similar it is to an AM receiver. The basic differences are as follows:

1. Generally much higher operating frequencies in FM
2. Need for limiting and de-emphasis in FM
3. Totally different methods of demodulation
4. Different methods of obtaining AGC

## Comparisons with AM Receivers

A number of sections of the FM receiver correspond exactly to those of other receivers already discussed. The same criteria apply in the selection of the intermediate frequency, and IF amplifiers are basically similar. A number of concepts have very similar meanings so that only the differences and special applications need be pointed out.

**RF amplifiers** An RF amplifier is always used in an FM receiver. Its main purpose is to reduce the noise figure, which could otherwise be a problem because of the large bandwidths needed for FM. It is also required to match the input impedance of the receiver to that of the antenna. To meet the second requirement, grounded gate (or base) or cascode amplifiers are employed. Both types have the property of low input impedance and matching the antenna, while neither requires neutralization. This is because the input electrode is grounded on either type of amplifier, effectively isolating input from output. A typical FET grounded-gate RF amplifier is shown in Figure It has all the good points mentioned and the added features of low distortion and simple operation.



**FIGURE** Grounded-gate FET RF amplifier.

**Oscillators and mixers** The oscillator circuit takes any of the usual forms, with the Colpitts and Clapp predominant, being suited to VHF operation. Tracking is not normally much of a problem in FM broadcast receivers. This is because the tuning frequency range is only 1.25:1, much less than in AM broadcasting.

A very satisfactory arrangement for the front end of an FM receiver consists of FETs for the RF amplifier and mixer, and a bipolar transistor oscillator. As implied by this statement, separately excited oscillators are normally used

**Intermediate frequency and IF amplifiers** Again, the types and operation do not differ much from their AM counterparts. It is worth noting, however, that the intermediate frequency and the bandwidth required are far higher than in AM broadcast receivers. Typical figures for receivers operating in the 88- to 108-MHz band are an IF of 10.7 MHz and a bandwidth of 200 kHz. As a consequence of the large bandwidth, gain per stage may be low. Two IF amplifier stages are often provided, in which case the shrinkage of bandwidth as stages are cascaded must be taken into account.

#### *Double limiter*

A double limiter consists of two amplitude limiters in cascade, an arrangement that increases the limiting range very satisfactorily. Numerical values given to illustrate limiter performance showed an output voltage (all values peak-to-peak, as before) of 5 V for any input within the 0.4- to 4-V range, above which output gradually decreases. It is quite possible that an output of 0.6 V is not reached until the input to the first limiter is about 20 V. If the range of the second limiter is 0.6 to 6 V, it follows that all voltages between 0.4 and 20 V fed to the double limiter will be limited. The use of the double limiter is seen to have increased the limiting range quite considerably.

#### *Automatic gain control (AGC)*

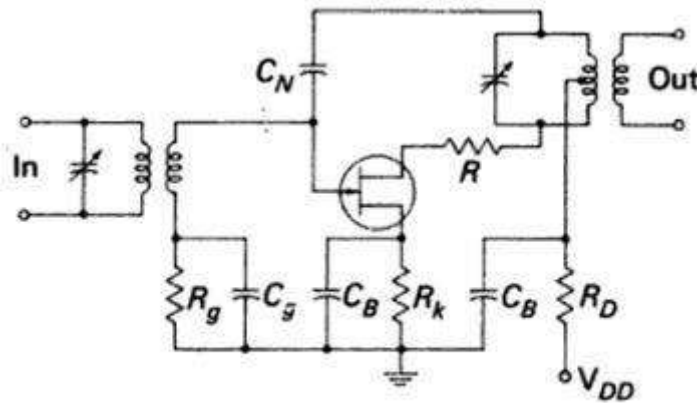
A suitable alternative to the second limiter is automatic gain control. This is to ensure that the signal fed to the limiter is within its limiting range, regardless of the input signal strength, and also to prevent overloading of the last IF amplifier. If the limiter used has leak-type bias, then this bias voltage will vary in proportion to the input voltage (as shown in Figure 6-31) and may therefore be used for AGC. Sometimes a separate AGC detector is used, which takes part of the output of the last IF amplifier and rectifies and filters it in the usual manner.

#### **Amplitude Limiter:**

In order to make full use of the advantages offered by FM, a demodulator must be preceded by an amplitude limiter, on the grounds that any amplitude changes in the signal fed to the FM demodulator are spurious

They must therefore be removed if distortion is to be avoided. The point is significant, since most FM demodulators react to amplitude changes as well as frequency changes. The limiter is a form of clipping device, a circuit whose output tends to remain constant despite changes in the input signal. Most limiters behave in this fashion, provided that the input voltage remains within a certain range. The common type of limiter uses two separate electrical effects to provide a relatively constant output. There are leak-type bias and early (collector) saturation.

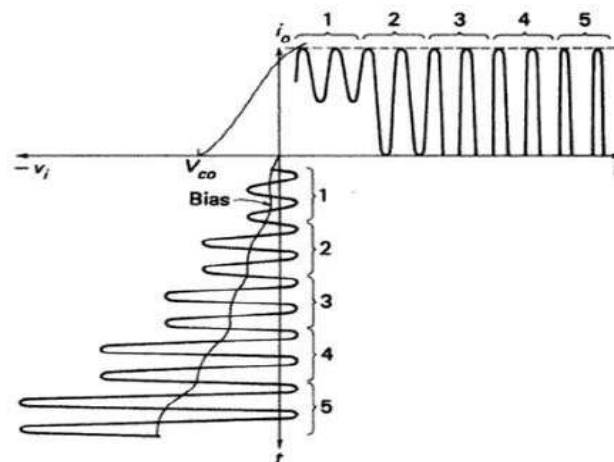
**Operation of the amplitude limiter** Figure shows a typical FET amplitude limiter. Examination of the dc conditions shows that the drain supply voltage has been dropped through resistor  $R_D$ . Also, the bias on the gate is leak-type bias supplied by the parallel  $R_g - C_g$  combination. Finally, the FET is shown neutralized by means of capacitor  $C_N$ , in consideration of the high frequency of operation.



**FIGURE** Amplitude limiter.

Leak-type bias provides limiting, as shown in Figure . When input signal voltage rises, current flows in the  $R_g - C_g$  bias circuit, and a negative voltage is developed across the capacitor. It is seen that the bias on the FET is increased in proportion to the size of the input voltage. As a result, the gain of the amplifier is lowered, and the output voltage tends to remain constant.

Although some limiting is achieved by this process, it is insufficient by itself, the action just described would occur only with rather large input voltages. To overcome this, early saturation of the output current is used, achieved by means of a low drain supply voltage. This is the reason for the drain dropping resistor of Figure . The supply voltage for a limiter is typically one-half of the normal dc drain voltage. The result of early saturation is to ensure limiting for conveniently low input voltages.



**FIGURE** Amplitude limiter transfer characteristic.

It is possible for the gate-drain section to become forward-biased under saturation conditions, causing a short circuit between input and output. To avert this, a resistance of a few hundred ohms is placed between the drain and its tank. This is  $R$  of Figure

## UNIT-4

### PULSE MODULATION

#### Introduction:

#### Pulse Modulation

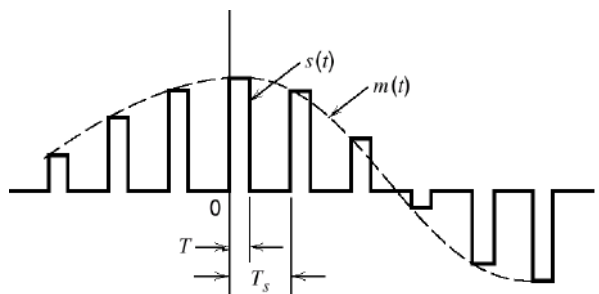
- Carrier is a train of pulses
- Example: Pulse Amplitude Modulation (PAM), Pulse width modulation (PWM) , Pulse Position Modulation (PPM)

#### Types of Pulse Modulation:

- The immediate result of sampling is a pulse-amplitude modulation (PAM) signal
- PAM is an analog scheme in which the amplitude of the pulse is proportional to the amplitude of the signal at the instant of sampling
- Another analog pulse-forming technique is known as **pulse-duration modulation (PDM)**. This is also known as **pulse-width modulation (PWM)**
- **Pulse-position modulation** is closely related to PDM

#### Pulse Amplitude Modulation:

In PAM, amplitude of pulses is varied in accordance with instantaneous value of modulating signal.



#### PAM Generation:

The carrier is in the form of narrow pulses having frequency  $f_c$ . The uniform sampling takes place in multiplier to generate PAM signal. Samples are placed  $T_s$  sec away from each other.

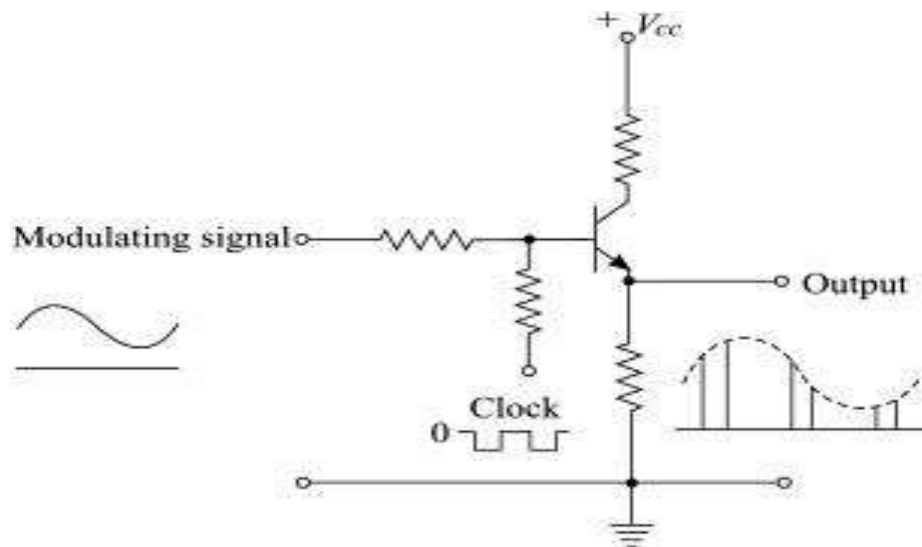


Fig.12. PAM Modulator

- The circuit is simple emitter follower.
- In the absence of the clock signal, the output follows input.
- The modulating signal is applied as the input signal.
- Another input to the base of the transistor is the clock signal.
- The frequency of the clock signal is made equal to the desired carrier pulse train frequency.
- The amplitude of the clock signal is chosen the high level is at ground level(0v) and low level at some negative voltage sufficient to bring the transistor in cutoff region.
- When clock is high, circuit operates as emitter follower and the output follows in the input modulating signal.
- When clock signal is low, transistor is cutoff and output is zero.
- Thus the output is the desired PAM signal.

#### PAM Demodulator:

- The PAM demodulator circuit which is just an envelope detector followed by a second order op-amp low pass filter (to have good filtering characteristics) is as shown below

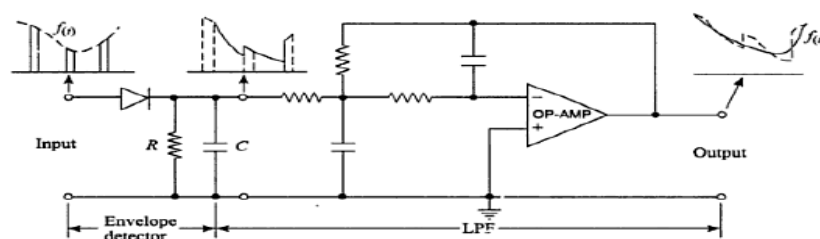
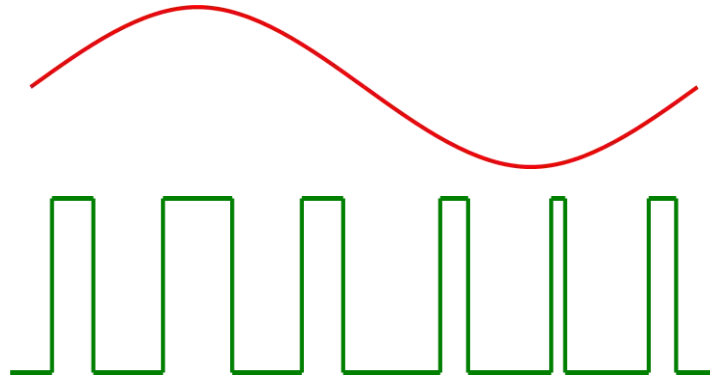


Fig.13. PAM Demodulator

### Pulse Width Modulation:

- In this type, the amplitude is maintained constant but the width of each pulse is varied in accordance with instantaneous value of the analog signal.



- In PWM information is contained in width variation. This is similar to FM.
- In pulse width modulation (PWM), the width of each pulse is made directly proportional to the amplitude of the information signal.

### Pulse Position Modulation:

- In this type, the sampled waveform has fixed amplitude and width whereas the position of each pulse is varied as per instantaneous value of the analog signal.
- PPM signal is further modification of a PWM signal.

### PPM & PWM Modulator:

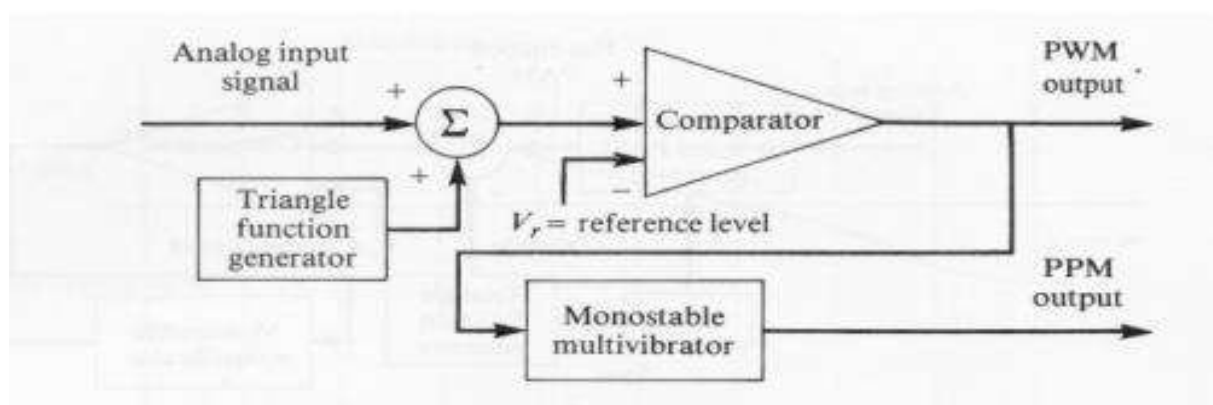


Fig.14. PWM & PPM Modulator

- The PPM signal can be generated from PWM signal.
- The PWM pulses obtained at the comparator output are applied to a mono stable multi vibrator which is negative edge triggered.

- Hence for each trailing edge of PWM signal, the monostable output goes high. It remains high for a fixed time decided by its RC components.
- Thus as the trailing edges of the PWM signal keeps shifting in proportion with the modulating signal, the PPM pulses also keep shifting.
- Therefore all the PPM pulses have the same amplitude and width. The information is conveyed via changing position of pulses.

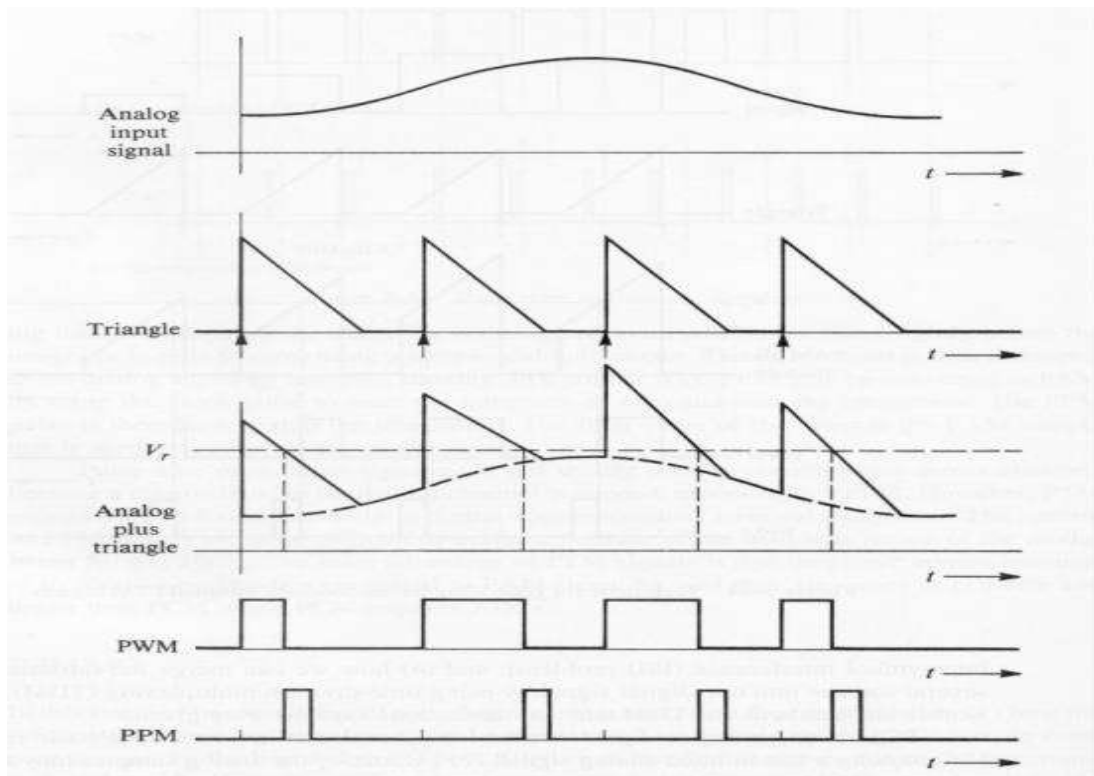


Fig.15. PWM & PPM Modulation waveforms

### PWM Demodulator:

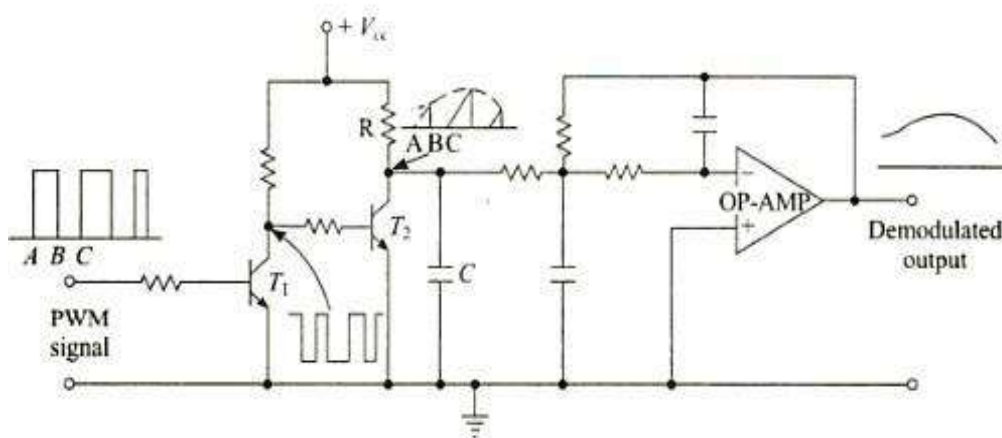


Fig.16. PWM Demodulator

- Transistor T1 works as an inverter.
- During time interval A-B when the PWM signal is high the input to transistor T2 is low.
- Therefore, during this time interval T2 is cut-off and capacitor C is charged through an R-C combination.
- During time interval B-C when PWM signal is low, the input to transistor T2 is high, and it gets saturated.
- The capacitor C discharges rapidly through T2. The collector voltage of T2 during B-C is low.
- Thus, the waveform at the collector of T2 is similar to saw-tooth waveform whose envelope is the modulating signal.
- Passing it through 2<sup>nd</sup> order op-amp Low Pass Filter, gives demodulated signal.

#### PPM Demodulator:

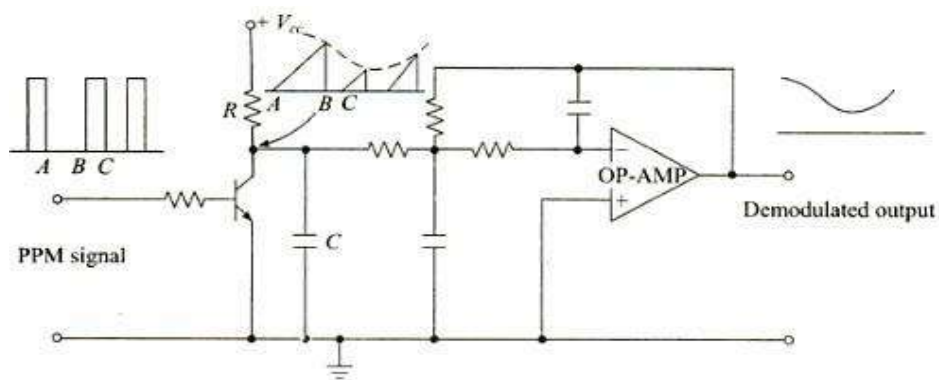


Fig.17. PPM Demodulator

- The gaps between the pulses of a PPM signal contain the information regarding the modulating signal.
- During gap A-B between the pulses the transistor is cut-off and the capacitor C gets charged through R-C combination.
- During the pulse duration B-C the capacitor discharges through transistor and the collector voltage becomes low.
- Thus, waveform across collector is saw-tooth waveform whose envelope is the modulating signal.
- Passing it through 2<sup>nd</sup> order op-amp Low Pass Filter, gives demodulated signal.

## Unit - V

### Information And Channel Capacity

" Introduction, Information content of message, Entropy, Entropy of symbols in long independent and dependent sequences, Entropy and information rate of Markoff sources, Shannon's Encoding theorem, Discrete communication channels, Rate of information over a discrete channel, Capacity of discrete memoryless channels, Discrete channels with memory, Shannon-Hartley Theorem and its implications, Illustrative problems."

x ————— x ————— x ————— x —————

Introduction: In the analysis and design of Communication Systems, we need to answer two questions.

- 1) Given an information source, how do we evaluate the rate at which the source is emitting information.
- 2) Given a noisy comm. channel, how do we evaluate the max. rate at which reliable information transmission can take place over the channel.

We can develop answers for the above two questions based on probabilistic models for information sources and communication channels.

Information sources can be classified into two categories.

- Analog (Continuous) source (Microphone actuated by voice signal)
- Discrete sources [The o/p of discrete source such as  
• Teletype consists of sequence of letters or symbols]

A discrete information source consists of a discrete set of letters or alphabet of symbols. In general, any message emitted by the source consists of a string or sequence of symbols. Every message coming out of the source contains some information, but some messages convey more information than others.

### Measure of Information:

Information content of a message: The output of a discrete information source is a message that consists of sequence of symbols. The actual message that is emitted by the source during a message interval is selected at random from a set of possible messages. The communication system is designed to reproduce at the receiver either exactly or approximately the message emitted by the source.

Some messages produced by the information source contain more information than other messages. So in order to compare the information content of various messages produced by the source, it is necessary to measure the information content of a message quantitatively. The amount of information is a relative measure of uncertainty relevant to the occurrence of each message in the message ensemble.

"The principle of improbability" If dog bites a man, it's no news, but if man bites a dog, it's a news"

The probability of a dog biting a man is quite high. So this is not a news, i.e. very little amount of information is communicated by a message "a dog bites a man". On the other hand the probability of a man biting a dog is extremely small, so this is a news. i.e. quite an amount of information is communicated by the message "a man bites a dog".

Thus we see that there should be a sort of inverse relationship between the probability of an event and the amount of information associated with it. The more the probability of an event (Certainty is more), the less is the amount of information associated with it and vice-versa, Thus,

$$I(x_i) = f\left(\frac{1}{P(x_i)}\right) \rightarrow \textcircled{1}$$

where  $x_i$  is an event with a probability  $P(x_i)$

and  $I(x_i)$  is amount of information associated with  $x_i$

Let there be another event  $y_j$  such that  $x_i$  and  $y_j$  are independent. Hence the probability of the joint event is  $P(x_i, y_j) = P(x_i) \cdot P(y_j)$  with the associated information content as

$$I(x_i, y_j) = f\left(\frac{1}{P(x_i, y_j)}\right) = f\left(\frac{1}{P(x_i) \cdot P(y_j)}\right) \rightarrow \textcircled{2}$$

The total information  $I(x_i, y_j)$  must be equal to the sum of individual informations  $I(x_i)$ ,  $I(y_j)$ ,

where 
$$I(y_j) = f\left(\frac{1}{P(y_j)}\right) \rightarrow \textcircled{3}$$

$$\therefore I(x_i, y_j) = I(x_i) + I(y_j)$$

Thus it can be seen that the function on R.H.S of eqn  $\textcircled{2}$  must be a function which converts multiplication into addition. Logarithm is one such function.

$$\begin{aligned} \therefore I(x_i, y_j) &= \log\left(\frac{1}{P(x_i) \cdot P(y_j)}\right) \\ &= \log\left(\frac{1}{P(x_i)}\right) + \log\left(\frac{1}{P(y_j)}\right) \\ &= I(x_i) + I(y_j) \end{aligned}$$

Hence the basic equation defining the amount of information (or self information) is

$$I(x_i) = \log\left(\frac{1}{P(x_i)}\right) = -\log P(x_i). \rightarrow \textcircled{A}$$

Different units of information can be defined for different bases of logarithms. When the base is 2 the units are "bits", when the base is  $e$  the units are "nats" and when the base is 10 the units are "decit" or "Hartley". The base 2 or binary system is more important and when no base is mentioned, the units of information are "bits".

Conversion of information units:

<u>unit</u>	Bits (2)	Nats (e)	Decit (10)
Bits (Base = 2)	—	$1 \text{ bit} = \frac{1}{\log_2 e}$ $= 0.6932 \text{ nats}$	$1 \text{ bit} = \frac{1}{\log_2 10}$ $= 0.3010 \text{ Decit}$
Nats (Base = e)	$1 \text{ nat} = \frac{1}{\log_e 2}$ $= \frac{1}{\ln 2} = 1.4426 \text{ bits}$	—	$1 \text{ nat} = \frac{1}{\ln 10}$ $= 0.4342 \text{ decit}$
Decits (Base = 10)	$1 \text{ decit} = \frac{1}{\log_{10} 2} = 3.3219 \text{ bits}$	$1 \text{ decit} = \frac{1}{\log_{10} e}$ $= 2.3026 \text{ nats}$	—

Entropy : (Average Amount of Information).

A communication system is not only meant to deal with a single message, but with all possible messages. Hence, although, the instantaneous information flows corresponding to individual messages from the source may be unpredictable we may describe the source in terms of average information per individual message known as Entropy of the source.

Let there be  $M'$  different messages <sup>( $m_1, m_2, \dots, m_M$ )</sup> with their respective probabilities of occurrences  $P_1, P_2, \dots, P_M$ . Let us assume that in a long time interval, ' $L$ ' messages have been generated. Let ' $L$ ' be very large so that  $L \gg M$ . Then the no. of messages  $m_i = P_i \cdot L$ . The amount of information in message  $m_i$  is  $\log \frac{1}{P_i}$ . Thus the total amount of information in all  $m_i$  messages will be  $P_i L \cdot \log \frac{1}{P_i}$ .

The total amount of information in all  $M'$  no. of messages will be

$$I_{\text{tot}} = P_1 L \log \frac{1}{P_1} + P_2 L \log \frac{1}{P_2} + \dots + P_M L \log \frac{1}{P_M}$$

$\therefore$  The Average amount of information per message or the Entropy, will then be given as

$$H = \frac{I_{\text{tot}}}{L} = P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2} + \dots + P_M \log \frac{1}{P_M}$$

$$H = \sum_{k=1}^M P_k \log \frac{1}{P_k} = - \sum_{k=1}^M P_k \log P_k.$$

→ If there is only a single possible message i.e.  $M=1$ ,  $P_k = P_1 = 1$  then,

$$H = P_1 \log \frac{1}{P_1} = \log 1 = 0$$

$\therefore$  This implies that in case of a single possible message, the reception of that message conveys no information.

→ Let there be only one message out of  $M'$  messages having a probability '1' and all others '0', then,

$$\begin{aligned} H &= \sum_{k=1}^M P_k \log \frac{1}{P_k} \\ &= P_1 \log \frac{1}{P_1} + \lim_{P \rightarrow 0} [P \log \frac{1}{P} + P \log \frac{1}{P} + \dots] \\ &= 1 \log \frac{1}{1} + 0 = 0 \end{aligned}$$

Thus if all probabilities are zero except for one, the entropy is '0'. In all other cases the Entropy will be greater than '0'.

→ For a Binary ( $M=2$ ) System, the entropy is given by

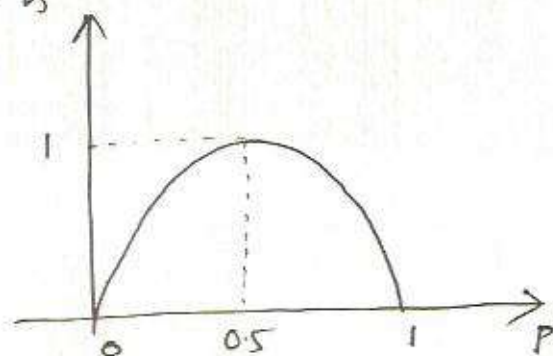
$$H = P_1 \log \frac{1}{P_1} + P_2 \log \frac{1}{P_2}$$

$$\text{Let } P_1 = p \text{ and } P_2 = q, \quad P_1 + P_2 = 1 \Rightarrow q = 1-p$$

$$\therefore H = p \log \frac{1}{p} + q \log \frac{1}{q}$$

$$= p \log \frac{1}{p} + (1-p) \log \frac{1}{(1-p)} = H(p) = H(q)$$

A plot of 'H' as a function of 'p' is as shown.



→ The Condition for Max. Entropy and its Value can be found as follows.

$$H = p \log \frac{1}{p} + (1-p) \log \frac{1}{(1-p)}$$

Differentiating Above w.r.t.  $p$  and equating  $\frac{dH}{dp} = 0$

gives,

$$\frac{dH}{dp} = \frac{d}{dp} \left( -p \log p - (1-p) \log (1-p) \right)$$

$$= - \left[ p \left( \frac{1}{p} \right) + \log p \right] - \left[ (1-p) \left( \frac{1}{1-p} \right) (-1) + \log (1-p) (-1) \right]$$

$$= -1 - \log p + 1 + \log (1-p)$$

$$= \log (1-p) - \log p = 0$$

$$\therefore \log p = \log (1-p)$$

$$\text{OR} \\ p = 1-p \Rightarrow p = \frac{1}{2}, q = \frac{1}{2}$$

$\therefore H$  is maximum when  $P = \frac{1}{2}$

$$\therefore H_{\max} = \frac{1}{2} \log \frac{1}{(\frac{1}{2})} + \frac{1}{2} \log \frac{1}{(\frac{1}{2})}$$

$$H_{\max} = \log_2 2 = 1 \text{ bit/message.}$$

Thus we have seen that for the binary case ( $M=2$ ), the entropy is maximum when  $P = \frac{1}{2}$  i.e. when <sup>both</sup> the messages are equally likely. Similarly it can be shown that for an  $M$ -ary case, the entropy is maximum when all the messages are equally likely. In this case the max. entropy is given as

$$H_{\max} = \sum P_k \log \frac{1}{P_k}$$
$$= \sum \frac{1}{M} \log M, \quad P_k = \frac{1}{M}$$

$$H_{\max} = \log_2 M \text{ bits/messages}$$

Since there are 'M' terms in the summation, the important properties of entropy can now be summarised as follows.

(i)  $\log M \geq H(x) \geq 0$

(ii)  $H(x) = 0$  if all probabilities are zero except for one which must be unity.

(iii)  $H(x) = \log_2 M$  if all the probabilities are equal so that  $P(x_i) = P_i = \frac{1}{M}$  for all  $i$ 's.

Let us examine 'H' under different conditions for  $m=2$ .

Case (i) :  $P_1 = 0.01, P_2 = 0.09, H = 0.08$

Case (ii) :  $P_1 = 0.4, P_2 = 0.6, H = 0.97$

Case (iii) :  $P_1 = 0.5, P_2 = 0.5, H = 1.$

$\underbrace{\hspace{1cm}}_{m_1} \quad \underbrace{\hspace{1cm}}_{m_2}$

In case (i), it is very easy to guess whether the message  $m_1$  with probability (0.01) will occur or the message  $m_2$  (with probability 0.99) will occur. Most of the time  $m_2$  will occur. Thus in this case the uncertainty is less. In case (ii) it is somewhat difficult to guess whether  $m_1$  will occur or  $m_2$  will occur, because their probabilities are nearly equal. Thus in this case the uncertainty is more. In case (iii) it is extremely difficult to guess whether  $m_1$  or  $m_2$  will occur as their probabilities are equal. Thus in this case the uncertainty is maximum. Here we can see that the entropy is less when the uncertainty is less and it is more when the uncertainty is more. Thus we can say that

" Entropy is a Measure of Uncertainty".

Rate of Information: (R):

Suppose that if a message source generates messages at a rate of ' $\gamma$ ' messages per second, then the Rate of information ' $R$ ' is defined as the average number of bits of information per second and is given as

$$R = \gamma \cdot H \text{ bits/sec,}$$

where ' $H$ ' is the average no. of bits of information per message or Entropy.

Let us consider two sources of equal Entropy ' $H$ ', generating ' $\gamma_1$ ' and ' $\gamma_2$ ' messages per second respectively. The first source will transmit information at a rate  $R_1 = \gamma_1 \cdot H$  and the second source will transmit the information at a rate  $R_2 = \gamma_2 \cdot H$ . Now if  $\gamma_1 > \gamma_2$  then  $R_1 > R_2$ . Thus in a given period more information is transmitted from the first source than second source, hence the source is not only described by its entropy but also by its rate of information.

Some times ' $R$ ' is called as bits/sec Entropy  $\xi$

$H$  is referred to as bits/message entropy.

Prob: An Event has six possible outcomes with probabilities  $P_1 = \frac{1}{2}$ ,  $P_2 = \frac{1}{4}$ ,  $P_3 = \frac{1}{8}$ ,  $P_4 = \frac{1}{k}$ ,  $P_5 = \frac{1}{32}$ ,  $P_6 = \frac{1}{32}$ . Find the Entropy of the System. Also find the rate of information if there are 16 outcomes per second.

Solw: The Entropy  $H = \sum_{k=1}^6 P_k \log \frac{1}{P_k} = \frac{31}{16} \text{ bits/msg.}$

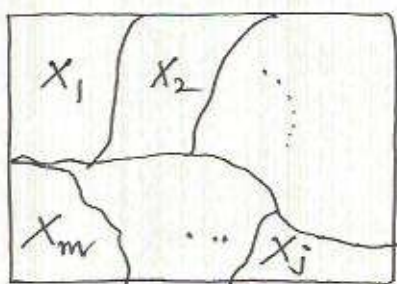
$$\gamma = 16,$$

$$\text{Rate of Information, } R = \gamma \cdot H = 16 \cdot \frac{31}{16} = 31 \text{ bits/sec.}$$

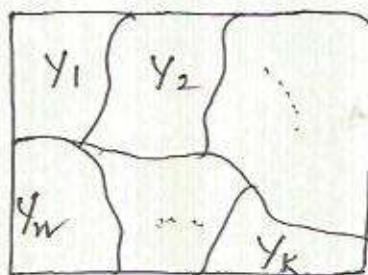
### Joint Entropy And Conditional Entropy:

One dimensional probability Scheme and its associated entropy can be used to analyze the behaviour of either transmitter or the Receiver. But to study the behaviour of a Communication System, we must simultaneously study the behaviour of the transmitter and receiver. This gives rise to the concept of two dimensional probability Scheme.

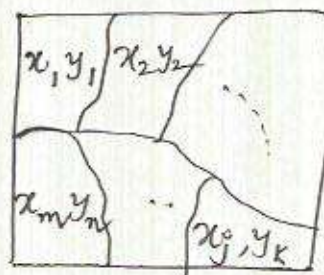
Let there be two finite discrete Sample Space  $S_1$  and  $S_2$  and let their product space be  $S = S_1 \cdot S_2$  as shown in fig. below.



(a) Sample Space  $S_1$



(b) Sample Space  $S_2$



(c) Sample Space  $S = S_1 S_2$

let  $[X] = [X_1, X_2 \dots X_m]$  and

$[Y] = [Y_1, Y_2 \dots Y_n]$  be the sets of

events in  $S_1$  &  $S_2$  respectively. Each event  $X_j$  in  $S_1$  may occur in conjunction with any event  $Y_k$  in  $S_2$ . Hence

The complete set of events in  $S = S_1 S_2$  is given as

$$[XY] = \begin{bmatrix} x_1 y_1 & x_1 y_2 & \dots & x_1 y_n \\ x_2 y_1 & x_2 y_2 & \dots & x_2 y_n \\ \vdots & \vdots & \ddots & \vdots \\ x_m y_1 & x_m y_2 & \dots & x_m y_n \end{bmatrix}$$

Thus we have, three sets of Complete probability Schemes,

$$P[X] = [P(x_j)]$$

$$P[Y] = [P(y_k)]$$

$$P[XY] = [P(x_j, y_k)]$$

Note: A probability Scheme  $P(x_j)$  is said to be Complete if

$$\sum_j P(x_j) = 1.$$

Since we have three Complete probability Schemes, we have its associated entropies given as

$$H[X] = - \sum_{j=1}^m P(x_j) \log P(x_j) \rightarrow (1)$$

$$\text{Where } P(x_j) = \sum_{k=1}^n P(x_j, y_k)$$

$$H[Y] = - \sum_{k=1}^n P(y_k) \log P(y_k) \rightarrow (2)$$

$$\text{Where, } P(y_k) = \sum_{j=1}^m P(x_j, y_k) \text{ and}$$

$$H[XY] = - \sum_{j=1}^m \sum_{k=1}^n P(x_j, y_k) \log P(x_j, y_k) \rightarrow (3)$$

$H[X]$  and  $H[Y]$  are marginal entropies of  $X$  &  $Y$  respectively and  $H[XY]$  is the joint entropy of the  $\hat{X}$  and  $\hat{Y}$ .

The conditional probability  $P(X/Y)$  is given as

$$P(X/Y) = \frac{P(X,Y)}{P(Y)}$$

we know that  $y_k$  may occur in conjunction with  $x_1, x_2, \dots, x_m$ , thus,

$$[X/y_k] = \left[ \frac{x_1}{y_k}, \frac{x_2}{y_k}, \dots, \frac{x_m}{y_k} \right] \text{ and the}$$

associated probability scheme is

$$\begin{aligned} P[X/y_k] &= [P(x_1/y_k), P(x_2/y_k), \dots, P(x_m/y_k)] \\ &= \left[ \frac{P(x_1, y_k)}{P(y_k)}, \frac{P(x_2, y_k)}{P(y_k)}, \dots, \frac{P(x_m, y_k)}{P(y_k)} \right] \end{aligned} \rightarrow \textcircled{4}$$

we have,  $P(x_1, y_k) + P(x_2, y_k) + \dots + P(x_m, y_k) = P(y_k)$

Thus the Sum of elements of matrix given by Eqn  $\textcircled{4}$  is 1 and hence is a complete probability scheme. Thus the entropy associated with it is given by,

$$\begin{aligned} H[X/y_k] &= - \sum_{j=1}^m \frac{P(x_j, y_k)}{P(y_k)} \log \left( \frac{P(x_j, y_k)}{P(y_k)} \right) \\ &= - \sum_{j=1}^m P(x_j/y_k) \log P(x_j/y_k) \end{aligned}$$

we may take the average of this conditional entropy for all admissible values of  $y_k$  in order to obtain a measure of an average conditional entropy of the system

$$\begin{aligned} H(X/Y) &= \overline{H(X/y_k)} \\ &= \sum_{k=1}^n P(y_k) \cdot H(X/y_k) \\ &= - \sum_{k=1}^n P(y_k) \sum_{j=1}^m P(x_j/y_k) \log P(x_j/y_k) \end{aligned}$$

$$H(X/Y) = - \sum_{j=1}^m \sum_{k=1}^n P(y_k) \cdot P(x_j/y_k) \log P(x_j/y_k)$$

$$H(X/Y) = - \sum_{j=1}^m \sum_{k=1}^n P(x_j, y_k) \log P(x_j/y_k) \rightarrow (5)$$

Similarly it can be shown that,

$$H(Y/X) = - \sum_{j=1}^m \sum_{k=1}^n P(x_j, y_k) \log P(y_k/x_j) \rightarrow (6)$$

$H(X/Y)$  and  $H(Y/X)$  are average conditional entropies or simply Conditional entropies.

Thus there are five entropies associated with a two dimensional probability scheme. They are,  $H(X)$ ,  $H(Y)$ ,  $H(XY)$ ,  $H(X/Y)$  and  $H(Y/X)$ .

Let 'X' represents a transmitter and 'Y' be a Receiver, the following interpretations of the different entropies for a two port Communication System can be derived.

- $H(X)$ : Average information per character at the transmitter or the entropy of the transmitter.
- $H(Y)$ : Average information per character at the Receiver or the entropy of the receiver.
- $H(X, Y)$ : Average information per pair of transmitted & Received Characters or the Average uncertainty of a Comm. System.
- $H(X/Y)$ : A measure of information about the Tx, where it is known that 'Y' is received. It indicates how well one can recover the Txed Symbols from the Rxed Symbols.
- $H(Y/X)$ : A measure of information about the Rx, where it is known that 'X' is Txed. It indicates how well one can recover the Rxed Symbols from the Txed Symbols. It gives a measure of noise or error.

## Relationship b/w different Entropies:

we know that the joint Entropy  $H(XY)$  is given by

$$\begin{aligned}
 H(XY) &= - \sum_{j=1}^m \sum_{k=1}^n P(x_j, y_k) \cdot \log P(x_j, y_k) \\
 &= - \sum_{j=1}^m \sum_{k=1}^n P(x_j, y_k) \cdot \log (P(x_j/y_k) \cdot P(y_k)) \\
 &= - \sum_{j=1}^m \sum_{k=1}^n P(x_j, y_k) [\log P(x_j/y_k) + \log P(y_k)] \\
 &= - \sum_{j=1}^m \sum_{k=1}^n P(x_j, y_k) \log P(x_j/y_k) - \sum_{j=1}^m \sum_{k=1}^n P(x_j, y_k) \cdot P(y_k) \\
 &= H(X/Y) - \sum_{k=1}^n P(y_k) \cdot \log P(y_k) \quad \because \sum_{j=1}^m P(x_j, y_k) = P(y_k) \\
 H(XY) &= H(X/Y) + H(Y) \rightarrow \textcircled{6}
 \end{aligned}$$

Similarly it can be shown that,

$$H(XY) = H(Y/X) + H(X) \rightarrow \textcircled{7}$$

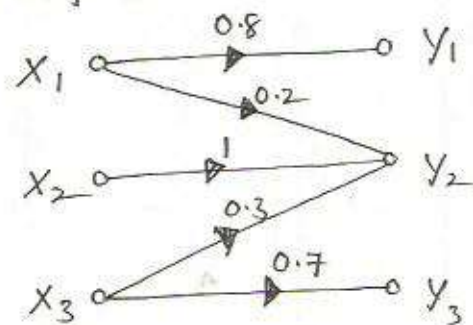
prob: ① A Discrete Source transmits messages  $x_1, x_2$  and  $x_3$  with the probabilities 0.3, 0.4 and 0.3. The Source is connected to the Channel Shown in fig. Calculate all the entropies.

Solw: Given Data,

$$P[X] = [0.3, 0.4, 0.3]$$

The Channel Shown in fig. gives the Conditional probability matrix  $P(Y/X)$  as,

$$P(Y/X) = \begin{matrix} & \begin{matrix} y_1 & y_2 & y_3 \end{matrix} \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} & \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0 & 1 & 0 \\ 0 & 0.3 & 0.7 \end{bmatrix} \end{matrix}$$



The joint probability matrix  $P(XY)$  can be obtained by multiplying the rows of  $P(Y/X)$  by  $P(x_1)$ ,  $P(x_2)$  and  $P(x_3)$  respectively. Thus,

$$P(XY) = \begin{bmatrix} 0.8 \times 0.3 & 0.2 \times 0.3 & 0 \\ 0 & 1 \times 0.4 & 0 \\ 0 & 0.3 \times 0.3 & 0.7 \times 0.3 \end{bmatrix}$$

$$\therefore P(XY) = \begin{matrix} & y_1 & y_2 & y_3 \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} & \begin{bmatrix} 0.24 & 0.06 & 0 \\ 0 & 0.4 & 0 \\ 0 & 0.09 & 0.21 \end{bmatrix} \end{matrix} \quad \because \sum_i \sum_j P(x_i, y_j) = 1.$$

The probabilities  $P(y_1)$ ,  $P(y_2)$  and  $P(y_3)$  can be obtained by adding the Column of  $P(X, Y)$  giving

$$P(y_1) = 0.24, P(y_2) = 0.55, P(y_3) = 0.21$$

$$\therefore P[Y] = [0.24, 0.55, 0.21]$$

The Conditional probability matrix  $P(X/Y)$  can be obtained by dividing the Column of  $P(X, Y)$  by,  $P(y_1)$ ,  $P(y_2)$  and  $P(y_3)$  respectively, Thus.

$$P(X/Y) = \begin{bmatrix} \frac{0.24}{0.24} & \frac{0.06}{0.55} & 0 \\ 0 & \frac{0.4}{0.55} & 0 \\ 0 & \frac{0.09}{0.55} & \frac{0.21}{0.21} \end{bmatrix} = \begin{matrix} & y_1 & y_2 & y_3 \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} & \begin{bmatrix} 1 & 0.109 & 0 \\ 0 & 0.727 & 0 \\ 0 & 0.164 & 1 \end{bmatrix} \end{matrix}$$

The Entropies now can be Calculated as

$$\begin{aligned} \rightarrow H(X) &= - \sum_{i=1}^3 P(x_i) \log P(x_i) \\ &= - (0.3 \log 0.3 + 0.4 \log 0.4 + 0.3 \log 0.3) = 1.571 \text{ bits/msgs.} \end{aligned}$$

$$\begin{aligned} \rightarrow H(Y) &= - \sum_{j=1}^3 P(y_j) \log P(y_j) \\ &= - (0.24 \log 0.24 + 0.55 \log 0.55 + 0.21 \log 0.21) = 1.441 \text{ bits/msgs.} \end{aligned}$$

$$\begin{aligned} \rightarrow H(X, Y) &= - \sum_{i=1}^3 \sum_{j=1}^3 P(x_i, y_j) \log P(x_i, y_j) \\ &= - [0.24 \log 0.24 + 0.06 \log 0.06 + 0.4 \log 0.4 + 0.09 \log 0.09 + 0.21 \log 0.21] \\ &= 2.053 \text{ bits/msgs.} \end{aligned}$$

$$\begin{aligned} \rightarrow H(X/Y) &= \sum_{j=1}^3 \sum_{k=1}^3 P(x_j, y_k) \cdot P(x_j/y_k) \\ &= - [0.24 \log 1 + 0.06 \log 0.109 + 0.4 \log 0.727 + 0.09 \log 0.164 + 0.21 \log 1] \\ &= 0.612 \text{ bits/msg.} \end{aligned}$$

$$H(Y/X) = - \sum_{j=1}^3 \sum_{k=1}^3 P(x_j, y_k) \cdot \log P(y_j/x_k)$$

$$= - [0.24 \log 0.8 + 0.06 \log 0.2 + 0.4 \log 1 + 0.09 \log 0.3 + 0.21 \log 0.7]$$

$$H(Y/X) = 0.482 \text{ bits/msg.}$$

We can also find Conditional Entropies by using the following

$$H(X/Y) = H(XY) - H(Y)$$

$$H(Y/X) = H(XY) - H(X).$$

Prob. ② A transmitter has an alphabet of four letters  $[x_1, x_2, x_3, x_4]$  and the receiver has an alphabet of three letters  $[y_1, y_2, y_3]$ . The joint probability matrix is

Calculate all the Entropies.

$$P(X, Y) = \begin{matrix} & y_1 & y_2 & y_3 \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix} & \begin{bmatrix} 0.3 & 0.05 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0.15 & 0.05 \\ 0 & 0.05 & 0.15 \end{bmatrix} \end{matrix}$$

Soln:

The probabilities of Tx Symbols are found by the Summation of rows and the probabilities of Rx Symbols are found by the Summation of Columns of the joint prob. matrix and are given as

$$P(X) = [0.35 \quad 0.25 \quad 0.2 \quad 0.2]$$

$$P(Y) = [0.3 \quad 0.5 \quad 0.2]$$

The entropies now can be Calculated as follows,

$$H(X) = - \sum_{j=1}^4 P(x_j) \log P(x_j) = - (0.35 \log 0.35 + 0.25 \log 0.25 + 2(0.2) \log 0.2) = 1.96 \text{ bits/message.}$$

$$H(Y) = - \sum_{k=1}^3 P(y_k) \log P(y_k) = - (0.3 \log 0.3 + 0.5 \log 0.5 + 0.2 \log 0.2) = 1.49 \text{ bits/message.}$$

$$H(X, Y) = - \sum_{j=1}^4 \sum_{k=1}^3 P(x_j, y_k) \log P(x_j, y_k) = - (0.3 \log 0.3 + 0.05 \log 0.05 + 0.25 \log 0.25 + 0.15 \log 0.15 + 0.05 \log 0.05 + 0.15 \log 0.15) = 2.49 \text{ bits/message.}$$

$$\therefore H(X/Y) = H(X, Y) - H(Y) = 2.49 - 1.49 = 1 \text{ bit/message}$$

$$H(Y/X) = H(X, Y) - H(X) = 2.49 - 1.96 = 0.53 \text{ bits/message.}$$

Mutual Information: Communication involves the transfer of information from a transmitter through a channel to a receiver. Prior to the reception of a message, the state of knowledge at the receiver about a fixed signal  $x_j$  is the probability that  $x_j$  would be selected for transmission. This is known as a-priori probability  $P(x_j)$ . After the reception and selection of the symbol  $y_k$ , the state of knowledge concerning  $x_j$  is the conditional probability  $P(x_j/y_k)$  which is also known as a-posteriori probability.

Thus before  $y_k$  is received the uncertainty becomes  $-\log P(x_j)$  and After  $y_k$  is received, the uncertainty becomes  $-\log P(x_j/y_k)$ .

" The information gained about  $x_j$  by the reception of  $y_k$  is the net reduction in its uncertainty and is also known as mutual information  $I(x_j, y_k)$ ."

Thus  $I(x_j, y_k) = \text{initial uncertainty} - \text{final uncertainty}$

$$= -\log P(x_j) - (-\log P(x_j/y_k))$$

$$= \log \left[ \frac{P(x_j/y_k)}{P(x_j)} \right]$$

$$= \log \frac{P(x_j, y_k)}{P(x_j) \cdot P(y_k)}$$

$$= \log \left( \frac{P(y_k/x_j)}{P(y_k)} \right)$$

$$I(x_j; y_k) = I(y_k; x_j)$$

From the Above it is clear that the mutual information is symmetrical in  $x_j$  and  $y_k$ .

Note: When  $y_k = x_j$ ,  $I(x_j; x_j) = \log \frac{P(x_j/x_j)}{P(x_j)}$

$$= \log \frac{1}{P(x_j)}$$

$$= I(x_j)$$

∴ Self information may be considered as a mutual information when  $y_k = x_j$ .

The Average of mutual information, i.e. the entropy corresponding to the mutual information is given as,

$$\begin{aligned}
 I(X;Y) &= \overline{I(x_j; y_k)} \\
 &= \sum_{j=1}^m \sum_{k=1}^n P(x_j, y_k) I(x_j; y_k) \\
 &= \sum_{j=1}^m \sum_{k=1}^n P(x_j, y_k) \log \frac{P(x_j/y_k)}{P(x_j)} \\
 &= \sum_{j=1}^m \sum_{k=1}^n P(x_j, y_k) [\log P(x_j/y_k) - \log P(x_j)] \\
 &= - \sum_{j=1}^m \sum_{k=1}^n P(x_j, y_k) \log P(x_j) - \left[ - \sum_{j=1}^m \sum_{k=1}^n P(x_j, y_k) \log P(x_j/y_k) \right] \\
 &= - \sum_{j=1}^m \left( \sum_{k=1}^n P(x_j, y_k) \right) \log P(x_j) - H(X/Y) \\
 &= - \sum_{j=1}^m P(x_j) \cdot \log P(x_j) - H(X/Y)
 \end{aligned}$$

$$I(X;Y) = H(X) - H(X/Y) \rightarrow \textcircled{1}$$

Similarly it can be shown that

$$I(X;Y) = H(Y) - H(Y/X) \xrightarrow{\text{Also}} \textcircled{2}$$

$$I(X;Y) = H(X) + H(Y) - H(X,Y) \rightarrow \textcircled{3}$$

$$\therefore H(XY) = H(Y/X) + H(X)$$

$I(X;Y)$  indicates a measure of the information transferred through the channel, it is also known as transferred information or transformation of the channel.

Equations ① and ② explain the meaning of conditional entropies  $H(X/Y)$  and  $H(Y/X)$  respectively. Equation ① states that the mutual information  $I(X;Y)$  is equal to the average source information minus the Average uncertainty that still remains about the messages. In other words,  $H(X/Y)$  is the average additional information needed at the Rx, after reception in order to completely specify the message sent. Thus  $H(X/Y)$  gives the information lost in the channel. Equation ② states that, the transferred information  $I(X;Y)$  is equal to the receiver entropy minus that part of receiver entropy which is not the information about the source. Thus  $H(Y/X)$  can be considered as the noise entropy added in the channel. Thus  $H(Y/X)$  is a measure of noise or error due to the channel.

prob ③ Find the Mutual information given in prob ① & prob ②

Soln: prob ① 
$$I(X;Y) = H(X) - H(X/Y)$$
$$= 1.571 - 0.612 = 0.959 \text{ bits/msg}$$

(or)

$$I(X;Y) = H(Y) - H(Y/X)$$
$$= 1.441 - 0.482 = 0.959 \text{ bits/msg.}$$

prob ② 
$$I(X;Y) = H(X) - H(X/Y)$$
$$= 1.96 - 1.00 = 0.96 \text{ bits/msg.}$$

(or)

$$I(X;Y) = H(Y) - H(Y/X)$$
$$= 1.99 - 0.53 = 0.96 \text{ bits/msg.}$$

## Entropy And Information Rate of Markoff Sources:

Let us assume that the source can be modeled as a discrete finite state Markoff process. Also assume that the process to be ergodic so that the time averages can be applied. The ergodic assumption implies that the process is stationary and hence,

$$P_i(k) = P_i(k+j) \text{ for any values of } k \text{ and } j.$$

In other words, the probability of being in state 'i' at the beginning of the first symbol interval is same as the probability of being in state 'i' at the beginning of the second symbol interval. and so on. The probability of going from state i to j also does not depend on time.

We define the entropy of the source as a weighted average of the entropy of the symbols emitted each state. The entropy of state 'i' denoted by  $H_i$  is defined as the average information content of the symbols emitted from the  $i^{\text{th}}$  state.

$$\therefore H_i = \sum_{j=1}^N P_{ij} \log_2 \left( \frac{1}{P_{ij}} \right) \text{ bits/symbol.}$$

The entropy of the source is then the average of the entropy of each state. i.e

$$\begin{aligned} H &= \sum_{i=1}^N P_i H_i \\ &= \sum_{i=1}^N P_i \left( \sum_{j=1}^N P_{ij} \log_2 \left( \frac{1}{P_{ij}} \right) \right) \text{ bits/symbol.} \end{aligned}$$

where  $P_i \rightarrow$  the probability that the source is in state 'i'.

The Average information rate for the Source is defined as,

$$R = V_s \cdot H \text{ bits/Sec.}$$

Where  $V_s$  is the number of State Transitions per Second or the Symbol rate of the Source.

Encoding of The Source Output :

Source encoding is the process by which the output of an information source is converted into a binary sequence. The functional block that performs this task in a communication system is called the Source encoder. The input to the Source encoder is the Symbol Sequence emitted by the information source. The Source encoder assigns variable length binary code words to blocks of symbols and produces a binary sequence as its output.

The performance of the encoder is usually measured in terms of the coding efficiency that is defined as the ratio of the Source information rate and the average output bit rate of the encoder. There are many algorithms available for designing a Source encoder.

Shannon's Encoding Algorithm: The design of Source encoder can be formulated as follows.

The input to the Source encoder consists of one of ' $q$ ' possible messages, each message containing ' $N$ ' symbols. Let,  $P_1, P_2, \dots, P_q$  be the probabilities of these messages. Here, we would like to code (or replace) the  $i$ th message using a unique binary codeword  $C_i$  of length  $n_i$  bits, where  $n_i$  is an integer. Our objective is to find  $n_i$

and  $c_i$  for  $i = 1, 2, \dots, q$ , such that the average number of bits per symbol  $\hat{H}_N$  used in the coding scheme is as close to  $G_N$  as possible. where,

$$G_N = -\frac{1}{N} \sum_i P(m_i) \log_2 P(m_i)$$

In other words

$$\hat{H}_N = \frac{1}{N} \sum_{i=1}^q n_i P_i \rightarrow \frac{1}{N} \sum_{i=1}^q P(m_i) \log \frac{1}{P(m_i)} \rightarrow (1)$$

Several solutions have been proposed to the above problem and the algorithm given by Shannon (and Fano) is stated below.

" Suppose the 'q' messages  $m_1, m_2, \dots, m_q$  are arranged in order of decreasing probability such that  $P_1 \geq P_2 \geq \dots \geq P_q$ . let  $F_i = \sum_{k=1}^{i-1} P_k$  with  $F_1 = 0$ . let  $n_i$  be an integer such that,

$$\log_2 \left( \frac{1}{P_i} \right) \leq n_i < 1 + \log_2 \left( \frac{1}{P_i} \right) \rightarrow (2)$$

for the message  $m_i$  is the binary expansion of the fraction  $F_i$  upto  $n_i$  bits i.e

$$C_i = (F_i)_{\text{binary}}, n_i \text{ bits.} \rightarrow (3)$$

This algorithm gives a source encoding procedure that has the following properties.

- Messages of high probability are represented by short code words and those of low probability are represented by long code words. This is clear for Eqn (2)
- The code word for  $m_i$  will differ from all succeeding code words in one or more places and hence it is possible to decode messages uniquely from their code words. This can be proved by rewriting Eqn (2) as

$$\frac{1}{2^{n_i}} \leq P_i < \frac{1}{2^{n_i-1}}$$

→ The Average no. of bits per symbol used by the encoder is bounded by  $G_N \leq \hat{H}_N \leq G_N + \frac{1}{N}$ .

The rate efficiency 'e' of the encoder using blocks of 'n' symbols is defined as,

$$e = \frac{H}{\hat{H}_N}$$

SHANON - FANO Coding: let  $[X]$  be the ensemble of the messages to be transmitted and  $[P]$  be their corresponding probabilities. The Sequence  $C_k$  of binary numbers of length  $n_k$  associated to each message  $x_k$  should fulfill the following conditions.

- No Sequences of employed binary numbers  $C_k$  can be obtained from each other by adding one or more binary digits to the shorter Sequence
- The transmission of an encoded message is reasonably efficient i.e 1 and 0 appear independently, with almost equal probabilities.

The procedure for constructing Shannon-Fano Coding is as follows.

- The message were first arranged in the order of non-increasing probabilities.
- Divide the messages Set into two most equi probable Subsets  $[X_1]$  and  $[X_2]$ .
- Assign a Zero ('0') to each message in one subset and a One ('1') to each message contained in the other subset.
- Repeat Above two steps for the subsets of  $[X_1]$  &  $[X_2]$  i.e  $[X_1]$  is divided into  $[X_{11}]$  and  $[X_{12}]$  and  $[X_2]$  is divided into  $[X_{21}]$  and  $[X_{22}]$
- The Code words in  $[X_{11}]$  Starts with 00,  $[X_{12}]$  will start with 01,  $[X_{21}]$  will start with 10 and  $[X_{22}]$  will start with 11.
- The procedure is continued untill each subset contains only one message.

The coding efficiency is thus calculated as

$$\eta = \frac{\bar{H}}{\bar{L} \log M}$$

where  $\bar{H}$  is the Source Entropy given by

$$\bar{H} = - \sum_{k=1}^N P_k \log P_k \text{ bits/message.}$$

$\bar{L}$  = The Average length per code word given as

$$\bar{L} = \sum_{i=1}^n P_i w_i \text{ letters/message.}$$

$M \Rightarrow$  The no. of Symbols in the encoding Alphabet.

prob: Apply Shannon - Fano Coding procedure for the following message ensemble.

$$[X] = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7 \ x_8]$$

$$[P] = [\frac{1}{4} \ \frac{1}{8} \ \frac{1}{16} \ \frac{1}{16} \ \frac{1}{16} \ \frac{1}{4} \ \frac{1}{16} \ \frac{1}{8}]$$

Take  $M=2$ .

Solu: Arrange the messages in the non increasing probability order

Messages	probability	$C_k$ Encoded Message	Length ( $w_i$ )
$x_1$	0.25	0 0	2
$x_6$	0.25	0 1	2
$x_2$	0.125	1 0 0	3
$x_8$	0.125	1 0 1	3
$x_3$	0.0625	1 1 0 0	4
$x_4$	0.0625	1 1 0 1	4
$x_5$	0.0625	1 1 1 0	4
$x_7$	0.0625	1 1 1 1	4

The Average Length per Code  $\bar{L}$  is given by

$$\bar{L} = \sum_{k=1}^8 P_k n_k = \frac{1}{4}(2) + \frac{1}{4}(2) + \frac{1}{8}(3) + \frac{1}{8}(3) + \frac{1}{16}(4) + \frac{1}{16}(4) + \frac{1}{16}(4) + \frac{1}{16}(4)$$

$$\bar{L} = 2.75 \text{ letters/message.}$$

The Entropy of the Source is given by

$$\begin{aligned} H(X) &= - \sum_{k=1}^8 P_k \log P_k \\ &= - \left( \frac{1}{4} \log \frac{1}{4} + \frac{1}{8} \log \frac{1}{8} + \frac{1}{16} \log \frac{1}{16} + \frac{1}{16} \log \frac{1}{16} + \frac{1}{16} \log \frac{1}{16} + \frac{1}{16} \log \frac{1}{16} + \frac{1}{16} \log \frac{1}{16} + \frac{1}{16} \log \frac{1}{16} \right) \\ &= 2.75 \text{ bits/message.} \end{aligned}$$

$$M = 2, \Rightarrow \log_2 M = \log_2 2 = 1$$

$\therefore$  The Coding Efficiency is thus given by,

$$\eta = \frac{H(X)}{\bar{L} \log M} = \frac{2.75}{2.75 \times 1} = 100\%$$

prob: A Discrete memoryless Source has five Symbols  $X_1, X_2, X_3, X_4$  and  $X_5$  with respective probabilities 0.2, 0.15, 0.05, 0.1 & 0.5. Construct a Shannon fano code for  $X'$  and Calculate the Code efficiency.

Soln: Arrange the messages in the non increasing probabilities order, as follows.

Messages	probabilities	$(C_k)$ Encoded Message	length( $n_i$ )
$X_5$	0.5	0	1
$X_1$	0.2	1 0	2
$X_2$	0.15	1 1 0	3
$X_4$	0.1	1 1 1 0	4
$X_3$	0.05	1 1 1 1	4

$\therefore$  The Average Length per Code  $\bar{L}$  is given by

$$\begin{aligned} \bar{L} &= \sum_{k=1}^5 P_k n_k = 0.5(1) + 0.2(2) + 0.15(3) + 0.1(4) + 0.05(4) \\ &= 1.95 \text{ letters/message} \end{aligned}$$

The entropy of the Source is given by

$$H(X) = -\sum_{k=1}^5 P_k \log P_k = [0.2 \log 0.2 + 0.15 \log 0.15 + 0.05 \log 0.05 + 0.1 \log 0.1 + 0.5 \log 0.5]$$

$$H(X) = 1.923 \text{ bits/message}$$

$$\text{let } M=2 \Rightarrow \log_2 M = 1$$

∴ The Coding Efficiency is given by

$$\eta = \frac{H(X)}{\bar{L} \log_2 M} = \frac{1.923}{1.95} = 98.61\%$$

prob: Consider an alphabet of a discrete memoryless Source having seven Source Symbols with their respective probabilities given below.

$$[S_k] = [S_0, S_1, S_2, S_3, S_4, S_5, S_6]$$

$$[P_k] = [0.40, 0.20, 0.12, 0.08, 0.08, 0.08, 0.04]$$

Suppose there are 3 numbers of Symbols in an encoding alphabet.

- Create a Shannon Fano Source Code word for each Symbol. Compute the respective length of the code words for each of the given Source Symbols.
- Determine the average code word length.
- Determine the entropy of the Specified discrete memoryless Source
- Determine the Coding efficiency.

Soln: Messages	probabilities	Encoded Message	Length
(a) $S_0$	0.40	-1	1
$S_1$	0.20	0 -1	2
$S_2$	0.12	0 0	2
$S_3$	0.08	1 -1	2
$S_4$	0.08	1 0	2
$S_5$	0.08	1 1 -1	3
$S_6$	0.04	1 1 0	3

$$(b) \bar{L} = \sum_{k=1}^7 P_k \log P_k = 0.4(1) + 0.2(2) + 0.12(2) + 2(0.08)2 + 0.08(3) + 0.04(3) = 1.72 \text{ letters/message}$$

$$(c) H(X) = -\sum_{k=1}^7 P_k \log P_k = -(0.4 \log 0.4 + 0.2 \log 0.2 + 0.12 \log 0.12 + 3(0.08) \log 0.08 + 0.04 \log 0.04) = 2.42 \text{ bits/message}$$

$$(d) \eta = \frac{H(X)}{\bar{L} \log_2 M} = \frac{2.42}{1.72 \log_2 3} = 88.7\%$$

## Discrete Communication Channels:

A practical Communication System is divided into a transmitter, a physical Channel or Txion medium and a receiver. The transmitter consists of an encoder and a modulator, while the receiver consists of a demodulator and a decoder.

The term Communication Channel carries different meanings and characterizations depending on its terminal points and functionality.

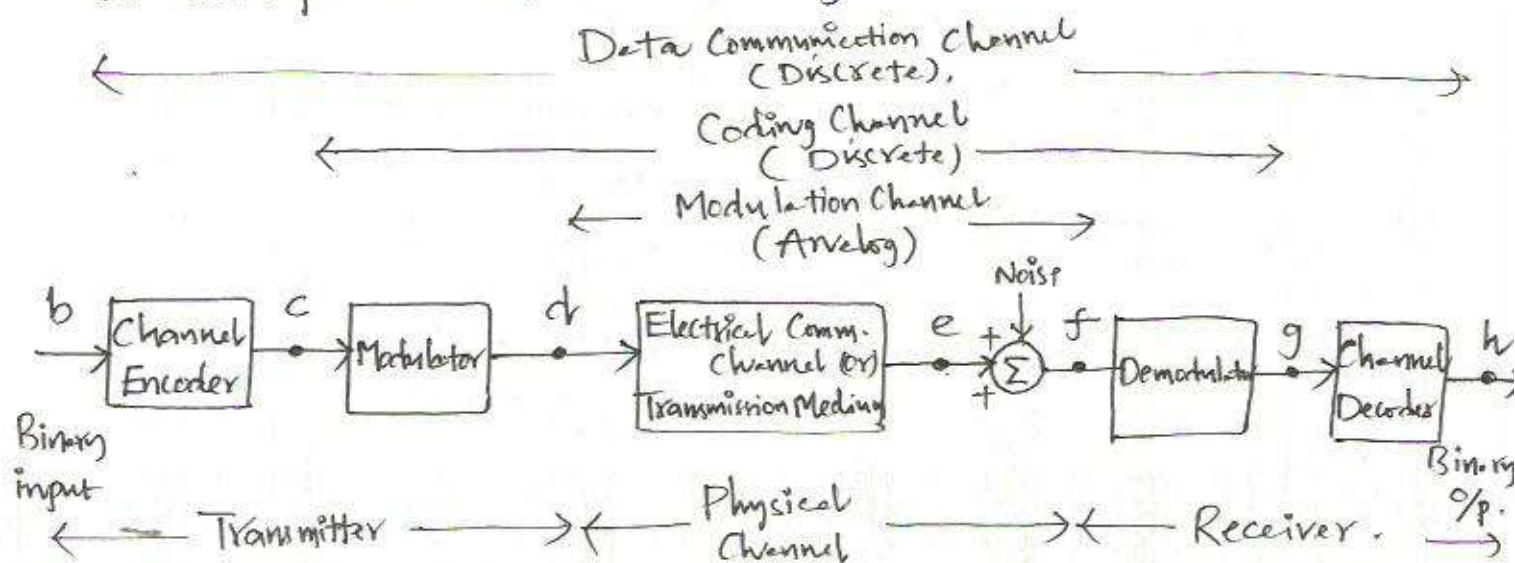


Fig: Characterization of a binary Comm. Channel.

Between points 'c' and 'g' in the System shown above, we have a discrete channel often referred to as a coding channel, that accepts a sequence of symbols at its input and produces a sequence of symbols at its output. This channel is completely characterized by a set of transition probabilities  $P_{ij}$ , where  $P_{ij}$  is the probability that the  $i$ th symbol is received as the  $j$ th symbol of the alphabet at the output of the channel. These probabilities will depend on the parameters of the modulator, transmission media, noise and demodulator.

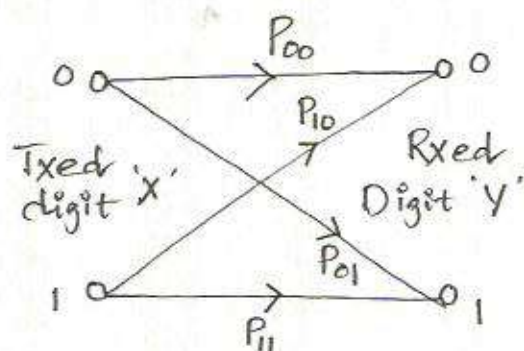
The Comm. Channel b/w points 'd' and 'f' in the System provides the electrical connection b/w the Tx and Rx.

The input and o/p are analog electrical waveforms. This portion of the channel is often called a continuous or modulation channel.

The Comm. Channel between points 'c' and 'g' shown in fig. is discrete in nature. In general, the input to the channel is a Symbol belonging to an alphabet of  $M'$  ~~input~~ Symbols. The o/p of the Channel is a symbol belonging to the same alphabet of  $M'$  input Symbols. Due to errors in the channel, the output Symbol may be different from the input symbol during some Symbol intervals. Errors are mainly due to the noise in the analog portion of the Comm. channel. The discrete Channel is completely modelled by a Set of probabilities  $P_i^t$  ( $i=1,2,\dots,M$ ) and  $P_{ij}$  ( $i,j=1,2,\dots,M$ ).  $P_i^t$  is the probability that the input to the channel is the  $i^{\text{th}}$  Symbol of the alphabet and  $P_{ij}$  is the probability that, the  $i^{\text{th}}$  Symbol is received as  $j^{\text{th}}$  Symbol of the alphabet at the output of the channel. The channels are designed to transmit and receive one of  $M'$  possible Symbols and are called  $M$ -ary Channels ( $M \geq 2$ ).

When  $M=2$ , the Channel is called Binary Channel. The Statistical Model of a binary digital Channel is as follows.

The input to the Channel is a binary Valued discrete Random variable 'X', and the two nodes on the left hand side of the graph in fig. represent the values '0' and '1' of the random variable 'X'.



$$P_{ij} = P(Y=j/X=i)$$

$$P_{00} + P_{01} = 1$$

$$P_{11} + P_{10} = 1$$

$$P(X=0) = P_0^t, \quad P(X=1) = P_1^t$$

$$P(Y=0) = P_0^r, \quad P(Y=1) = P_1^r$$

The op of the channel is also a binary valued random variable 'Y' and its values are marked on the R.H.s of the fig. Four paths connect the input nodes to the output nodes. The path on the top portion of the graph represents an input '0' and a correct output '0'. The diagonal path from 0 to 1 represents an input bit '0' appearing incorrectly as '1' at the output of the channel due to noise. Errors occur in a random fashion and we can statistically model the occurrence of errors by assigning probabilities to the paths shown in fig.

To simplify the analysis, we assume that the occurrence of an error during a bit interval, does not effect the behaviour of the system during other bit intervals. (i.e channel is assumed to be memory less).

$$\text{Let } P(X=0) = P_0^t \quad P(X=1) = P_1^t$$

$$P(Y=0) = P_0^r, \quad P(Y=1) = P_1^r \text{ we have.}$$

$$P(\text{error}) = P_e = P(X \neq Y)$$

$$= P(X=0, Y=1) + P(X=1, Y=0)$$

$$= P(Y=1/X=0) + P(Y=0/X=1) \text{ or}$$

$$P_e = P_0^t P_{01} + P_1^t P_{10} \quad \text{Also}$$

$P_0^r$  and  $P_1^r$  can be expressed as,

$$P_0^r = P_0^t P_{00} + P_1^t P_{10}$$

$$P_1^r = P_0^t P_{01} + P_1^t P_{11}$$

This channel is called a binary Symmetric Channel (BSC) if  $P_{00} = P_{11} = P$ . To characterize a BSC we need only one parameter i.e 'P'.

In a discrete memoryless channel, there are two statistical processes at work; the input to the channel and noise. Thus there are no. of entropies or information contents to be calculated.

The Entropy of the input 'X' is defined as

$$H(X) = - \sum_{i=1}^M P_i^x \log_2(P_i^x) \text{ bits/symbol.}$$

where  $P_i^x$  is the probability that the  $i^{\text{th}}$  symbol of the alphabet is transmitted.

Also, we can define the entropy of the output 'Y' as

$$H(Y) = - \sum_{i=1}^M P_i^y \log_2(P_i^y) \text{ bits/symbol.}$$

where  $P_i^y$  is the prob. that the output of the channel is the  $i^{\text{th}}$  symbol of the alphabet.

Similarly the conditional entropies are defined as

$$H(X/Y) = - \sum_{i=1}^M \sum_{j=1}^M P(x=i, y=j) \log_2 P(x=i/y=j)$$

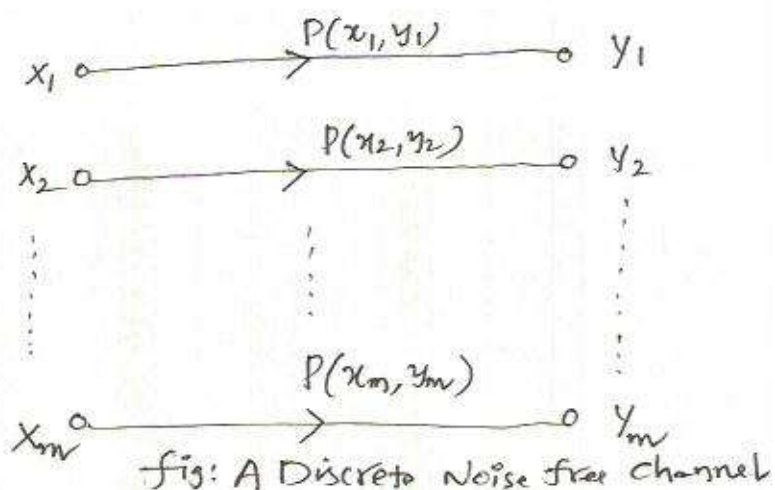
$$H(Y/X) = - \sum_{i=1}^M \sum_{j=1}^M P(x=i, y=j) \log_2 P(y=j/x=i)$$

The joint entropy is given by,

$$H(XY) = - \sum_{i=1}^M \sum_{j=1}^M P(x=i, y=j) \log_2 P(x=i, y=j).$$

Noise free Channel: Let us consider the communication channel as shown. It is also known as Noise free channel.

For a discrete noise free channel, there is a one to one correspondence between input and output i.e. each input symbol is received as one and only one output symbol. Also we have  $m = n$ .



The joint probability matrix  $P(X, Y)$  of a noise free channel is of the diagonal form, as

$$[P(X, Y)] = \begin{matrix} & \begin{matrix} y_1 & y_2 & \dots & y_m \end{matrix} \\ \begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{matrix} & \begin{bmatrix} P(x_1, y_1) & 0 & \dots & 0 \\ 0 & P(x_2, y_2) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & P(x_m, y_m) \end{bmatrix} \end{matrix}$$

and the channel probability matrices  $P(Y/X)$  and  $P(X/Y)$  are unit diagonal matrices, given by

$$P(Y/X) = P(X/Y) = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

The joint entropy  $H(X, Y)$  is given by

$$H(X, Y) = - \sum_{j=1}^m \sum_{k=1}^n P(x_j, y_k) \log P(x_j, y_k)$$

Since  $P(x_j, y_k) = 0, j \neq k$ .

we have

$$H(X, Y) = - \sum_{j=1}^m P(x_j, y_j) \log P(x_j, y_j)$$

Also from joint probability matrix, we have

$$P(x_j, y_j) = P(x_j) = P(y_j)$$

Hence we have,  $H(X, Y) = H(X) = H(Y)$ .

From the channel probability matrix, we have,

$$H(X/Y) = H(Y/X) = 0$$

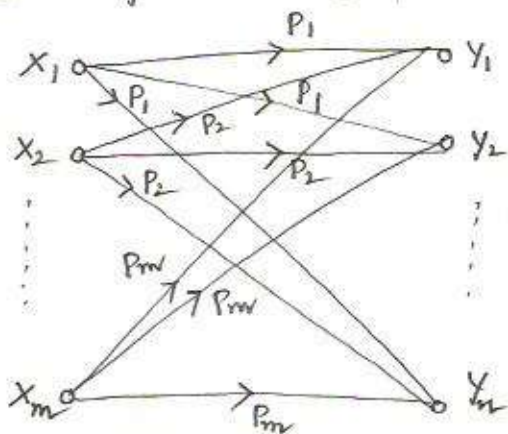
∴ Thus for a noise free channel,

$$\begin{aligned} I(X:Y) &= H(X) - H(X/Y) = H(X) = H(X, Y) \\ &= H(Y) - H(Y/X) = H(Y) = H(X, Y) \end{aligned}$$

Channels with independent input and output:

In this type of channels, there is no correlation between input and output symbols.

Let us consider the channel as shown in fig. The joint probability matrix in this case is



$$P(x, y) = \begin{matrix} & y_1 & y_2 & \dots & y_n \\ \begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{matrix} & \begin{bmatrix} P_1 & P_1 & \dots & P_1 \\ P_2 & P_2 & \dots & P_2 \\ \vdots & \vdots & \ddots & \vdots \\ P_m & P_m & \dots & P_m \end{bmatrix} \end{matrix}$$

It can be seen from the Above matrix that

$$\sum_{j=1}^m P_j = P_1 + P_2 + \dots + P_m = \frac{1}{n}$$

$$P(x_j) = n P_j, \quad j = 1, 2, \dots, m$$

$$P(y_k) = \frac{1}{n}, \quad k = 1, 2, \dots, n$$

$$P(x_j, y_k) = P_j$$

$$P(x_j, y_k) = P(x_j) \cdot P(y_k)$$

Above equation Shows that  $x_j$  and  $y_k$  are independent for all  $j$  and  $k$ . i.e input and output are independent for the channel shown in fig.

Also we have,

$$P(x_j/y_k) = \frac{P(x_j, y_k)}{P(y_k)} = P(x_j) \text{ and}$$

$$P(y_k/x_j) = \frac{P(x_j, y_k)}{P(x_j)} = P(y_k)$$

Now the conditional Entropies becomes

$$H(Y/X) = - \sum_{j=1}^m \sum_{k=1}^n P(x_j, y_k) \cdot \log P(y_k/x_j)$$

$$= - \sum_{j=1}^m \sum_{k=1}^n P(x_j) P(y_k) \log P(y_k)$$

$$= - \sum_{k=1}^n \left( \sum_{j=1}^m P(x_j) \right) \cdot P(y_k) \log P(y_k)$$

$$= - \sum_{k=1}^n P(y_k) \log P(y_k) \quad \because \sum_{j=1}^m P(x_j) = 1$$

$$H(Y/X) = H(Y).$$

Similarly it can be shown that,

$$H(X/Y) = H(X).$$

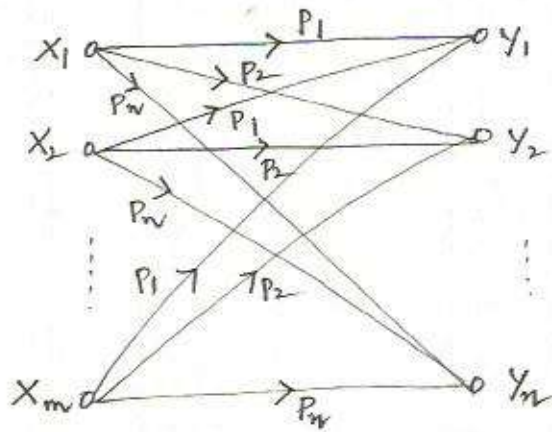
∴ For the given channel, the mutual information  $I(X;Y)$  is given by,

$$I(X;Y) = H(X) - H(X/Y) = H(X) - H(X) = 0$$

$$I(X;Y) = H(Y) - H(Y/X) = H(Y) - H(Y) = 0$$

Above <sup>(equation)</sup> information Shows that, no information is transmitted through the channel as shown.

Another similar channel with an independent input and output is as shown.



$$P(X, Y) = \begin{matrix} & \begin{matrix} y_1 & y_2 & y_3 & \dots & y_n \end{matrix} \\ \begin{matrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{matrix} & \begin{bmatrix} P_1 & P_2 & P_3 & \dots & P_n \\ P_1 & P_2 & P_3 & \dots & P_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_1 & P_2 & P_3 & \dots & P_n \end{bmatrix} \end{matrix}$$

Note: A Channel is said to be with an independent input and output when the joint probability matrix satisfies atleast one of the following conditions.

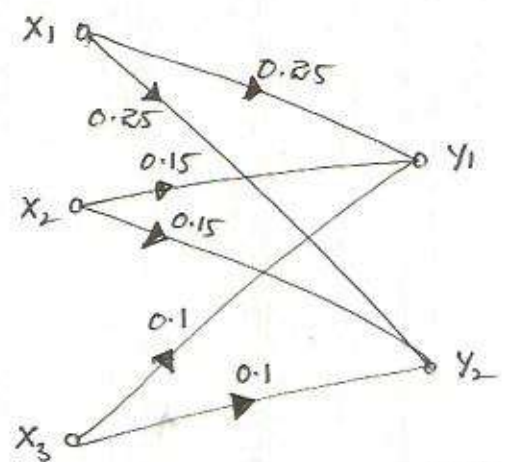
- Each row consists the same element.
- Each column consists the same element.

Prob: Find the mutual information for the Channel shown in fig.

Soln: The joint prob.  
The Channel Matrix for the given channel is given by

$$P(X, Y) = \begin{matrix} & \begin{matrix} y_1 & y_2 \end{matrix} \\ \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} & \begin{bmatrix} 0.25 & 0.25 \\ 0.15 & 0.15 \\ 0.1 & 0.1 \end{bmatrix} \end{matrix}$$

$$\begin{aligned} P(x_1) &= 0.25 + 0.25 = 0.5 \\ P(x_2) &= 0.15 + 0.15 = 0.3 \\ P(x_3) &= 0.1 + 0.1 = 0.2 \end{aligned}$$



$$\begin{aligned} P(y_1) &= 0.25 + 0.15 + 0.1 = 0.5 \\ P(y_2) &= 0.25 + 0.15 + 0.1 = 0.5 \end{aligned}$$

$$\therefore H(X) = - \sum_{k=1}^3 P_k \log P_k = - (0.5 \log 0.5 + 0.3 \log 0.3 + 0.2 \log 0.2) = 1.485 \text{ bits/message}$$

$$H(Y) = - \sum_{k=1}^2 P(y_k) \log P(y_k) = - (0.5 \log 0.5 + 0.5 \log 0.5) = 1 \text{ bit/message}$$

$$\begin{aligned}
 H(X;Y) &= - \sum_{j=1}^3 \sum_{k=1}^2 P(x_j, y_k) \log P(x_j, y_k) \\
 &= - (0.25 \log 0.25 + 0.25 \log 0.25 + 0.15 \log 0.15 + 0.15 \log 0.15 \\
 &\quad + 0.1 \log 0.1 + 0.1 \log 0.1) \\
 &= 2.485 \text{ bits/message.}
 \end{aligned}$$

$\therefore$  The mutual information is thus given by,

$$\begin{aligned}
 I(X;Y) &= H(X) + H(Y) - H(X,Y) \\
 &= 1.485 + 1 - 2.485 = 0
 \end{aligned}$$

Since the given channel is having independent input and output, the mutual information is always zero i.e

Channel Capacity: The mutual information  $I(X;Y)$  indicates a measure of the average information per Symbol fixed in the system. Shannon has introduced a significant concept of Channel Capacity defined as the maximum of mutual information. Thus the Channel Capacity 'C' is given by,

$$C = \text{Max}(I(X;Y)) = \text{Max}(H(X) - H(X/Y))$$

The transmission efficiency or the channel efficiency is defined as,

$$\eta = \frac{\text{Actual Transinformation}}{\text{Maximum Transinformation}}$$

$$\therefore \eta = \frac{I(X;Y)}{\text{Max}(I(X;Y))} = \frac{I(X;Y)}{C}$$

The Redundancy of the Channel is defined as

$$R = 1 - \eta = 1 - \frac{I(X;Y)}{C}$$

$$R = \frac{C - I(X;Y)}{C}$$

for a noise free Channel, the mutual information is given by,

$$I(X;Y) = H(X)$$

$\therefore$  The Channel Capacity is given by,

$$C = \text{Max}(I(X;Y)) = \text{Max}(H(X))$$

we know that

$$\text{Max}(H(X)) = \log M \text{ bits/message.}$$

where 'M' is the total no. of messages.

Hence for a noise free Channel,

$$C = \log_2 M \text{ bits/message}$$

Symmetric Channel: A Symmetric channel is defined as the one for which

- (i)  $H(Y/X_j)$  is independent of 'j' i.e the entropy corresponding to each row of  $P(Y/X)$  is the same. and
- (ii) The Sum of all the columns of  $P(Y/X)$  is the same i.e  $\sum_{j=1}^m P(Y_k/X_j)$  is independent of 'k'.

It can be seen that a channel is Symmetric if the rows and columns of the Channel matrix  $D = P(Y/X)$  are independently identical except for permutations. If  $D$  is a Square matrix, then for a Symmetric Channel the rows and columns are identical except for permutations.

Ex:

(a)  $P(Y/X) = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/4 & 1/2 & 1/4 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$  Symmetric Since Rows and Columns are identical

$$(b) P(Y/X) = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/4 & 1/4 & 1/2 \\ 1/2 & 1/4 & 1/4 \end{bmatrix} \quad \text{Not Symmetric, Since row are identical but Columns are not}$$

$$(c) P(Y/X) = \begin{bmatrix} 1/3 & 1/6 & 1/3 & 1/6 \\ 1/6 & 1/3 & 1/6 & 1/3 \end{bmatrix} \quad \text{Symmetric Since rows \& Columns are identical.}$$

Note, Since the rows of the above matrices are Complete Probability Schemes, the sum of each row in each matrix is unity.

For a Symmetric Channel,

$$\begin{aligned} I(X;Y) &= H(Y) - H(Y/X) \\ &= H(Y) - \sum_{j=1}^m H(Y/x_j) \cdot P(x_j) \\ &= H(Y) - H(Y/x_j) \cdot \sum_{j=1}^m P(x_j) \end{aligned}$$

$\therefore H(Y/x_j) = A$  is independent of 'j'

$$\therefore I(X;Y) = H(Y) - A \cdot \sum_{j=1}^m P(x_j),$$

$$\text{Also we have, } \sum_{j=1}^m P(x_j) = 1$$

$$I(X;Y) = H(Y) - A$$

$\therefore$  The Channel Capacity is thus given as

$$\begin{aligned} C &= \text{Max}(I(X;Y)) \\ &= \text{Max}(H(Y) - A) \\ &= \text{Max}(H(Y)) - A \end{aligned}$$

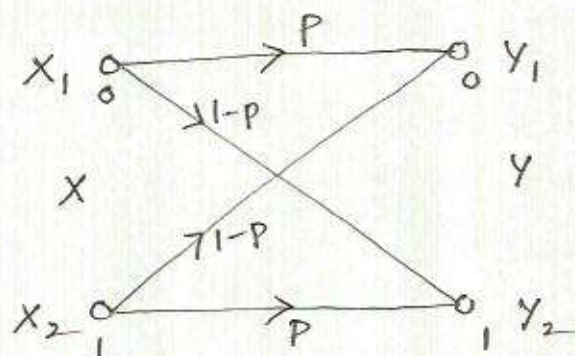
$$C = \log n - A$$

Where 'n' is the total no of receiver Symbols.

Binary Symmetric Channel: The most important case of a Symmetric channel is the binary Symmetric Channel (BSC). In this case  $m = n = 2$  and the channel matrix 'D' is given by,

$$D = P(Y/X) = \begin{bmatrix} P & 1-P \\ 1-P & P \end{bmatrix} = \begin{bmatrix} P & q \\ q & P \end{bmatrix}$$

BSC can be represented graphically as shown in fig.



prob.: For a binary Symmetric Channel shown above find the channel capacity for (i)  $P = 0.9$ , (ii)  $P = 0.6$ .

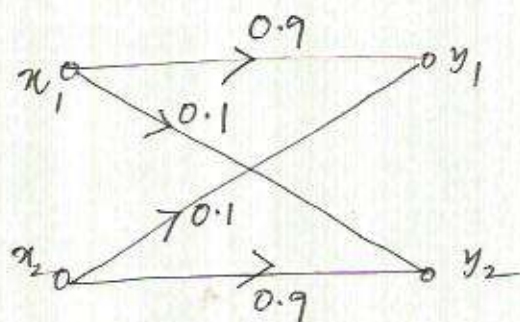
Soln.:

(i)

$$C = \log n - A$$

$$n = 2,$$

$$A = H(Y/X_j)$$



$$C = \log 2 - [H(Y/X_j)]$$

$$= \log 2 - \left[ - \sum_{k=1}^2 P(y_k/x_j) \log P(y_k/x_j) \right]$$

$$= \log 2 + P \log P + (1-P) \log (1-P)$$

$$= \log 2 + P \log P + q \log q.$$

$$= 1 - (-P \log P - q \log q) = 1 - H(P)$$

$$= 1 - H(q)$$

$$P = 0.9,$$

$$C = 1 + 0.9 \log 0.9 + 0.1 \log 0.1$$

$$= 0.531 \text{ bits/message}$$

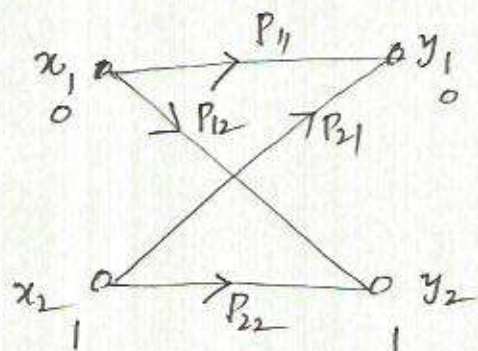
$$P = 0.6,$$

$$C = 1 + 0.6 \log 0.6 + 0.4 \log 0.4$$

$$= 0.029 \text{ bits/message.}$$

Binary Channel: Although it is easy to analyse a BSC in practice, we come across binary channels with non symmetric structures. A binary channel is as shown whose channel matrix is given by.

$$D = [P[Y/X]] = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$



To find the Channel Capacity of a binary channel, the auxiliary variables  $Q_1$  &  $Q_2$  are defined by,

$$[P][Q] = -[H]$$

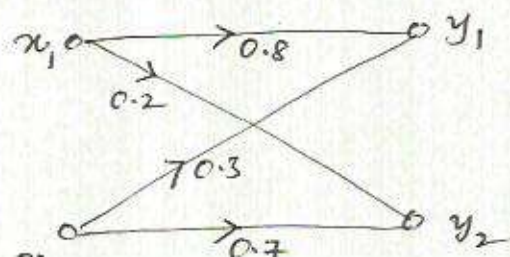
$$\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} P_{11} \log P_{11} + P_{12} \log P_{12} \\ P_{21} \log P_{21} + P_{22} \log P_{22} \end{bmatrix}$$

The Channel Capacity is thus given by.

$$C = \log_2 (2^{Q_1} + 2^{Q_2})$$

prob.: Find the mutual information and channel Capacity of the channel shown in fig. Given  $P(x_1) = 0.6$ ,  $P(x_2) = 0.4$ .

Soln.



The channel matrix for the given channel is given by,

$$D = [P(Y/X)] = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$$

$$P(x_1) = 0.6, \quad P(x_2) = 0.4$$

The joint probability matrix is obtained by multiplying the rows of 'D' by  $P(x_1)$  and  $P(x_2)$  respectively.

$$\therefore [P(X,Y)] = \begin{bmatrix} 0.8 \times 0.6 & 0.2 \times 0.6 \\ 0.3 \times 0.4 & 0.7 \times 0.4 \end{bmatrix}$$

$$= \begin{bmatrix} 0.48 & 0.12 \\ 0.12 & 0.28 \end{bmatrix}$$

$$\therefore P(y_1) = 0.48 + 0.12 = 0.6$$

$$P(y_2) = 0.12 + 0.28 = 0.4$$

$$\therefore P(X/Y) = \begin{bmatrix} \frac{0.48}{0.6} & \frac{0.12}{0.4} \\ \frac{0.12}{0.6} & \frac{0.28}{0.4} \end{bmatrix} = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$$

$$\therefore H(X) = -\sum_{j=1}^2 P(x_j) \log P(x_j) = -(0.6 \log 0.6 + 0.4 \log 0.4) = 0.971 \text{ bits/message.}$$

$$\begin{aligned} H(X/Y) &= -\sum_{j=1}^2 \sum_{k=1}^2 P(x_j, y_k) \log P(x_j/y_k) \\ &= -(0.48 \log 0.8 + 0.12 \log 0.3 + 0.12 \log 0.2 + 0.28 \log 0.7) \\ &= 0.786 \text{ bits/message.} \end{aligned}$$

$$\therefore I(X;Y) = H(X) - H(X/Y) = 0.971 - 0.786 = 0.185 \text{ bits/message.}$$

$$\rightarrow P_{11} = 0.8, \quad P_{12} = 0.2, \quad P_{21} = 0.3, \quad P_{22} = 0.7$$

$$\begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} 0.8 \log 0.8 + 0.2 \log 0.2 \\ 0.3 \log 0.3 + 0.7 \log 0.7 \end{bmatrix}$$

$$\text{Solving, } Q_1 = -0.6568, \quad Q_2 = -0.9769$$

$$\therefore C = \log_2 \left( 2^{-0.6568} + 2^{-0.9769} \right)$$

$$= \log(0.633 + 0.513) = \log 1.146 = 0.2 \text{ bits/message. } \textcircled{20}$$

Continuous channel: A number of Comm. Systems use Continuous Sources and thus use the Channel Continuously. AM, FM & PM are examples of Systems using Continuous channel.

The information theory concept of discrete Channels can be extended to Continuous Channels. If  $p(x)$  is the probability density function associated with the signal  $x(t)$ , then, the entropy of the Source is given by,

$$H(x) = E(-\log p(x)) = - \int_{-\infty}^{\infty} p(x) \log p(x) dx$$

Similarly, the different entropies associated with two dimensional random variable with a joint density  $P(x,y)$  and marginal densities  $P_1(x)$  and  $P_2(y)$  may be defined as,

$$H(x) = E(-\log P_1(x)) = - \int_{-\infty}^{\infty} P_1(x) \log P_1(x) dx$$

$$H(y) = E(-\log P_2(y)) = - \int_{-\infty}^{\infty} P_2(y) \log P_2(y) dy$$

$$H(x,y) = E(-\log P(x,y)) = - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(x,y) \log P(x,y) dx dy$$

$$H(x/y) = E(-\log P(x/y)) = - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(x,y) \log P(x/y) dx dy$$

$$H(y/x) = E(-\log P(y/x)) = - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(x,y) \log P(y/x) dx dy$$

→ In discrete case, all the entropies involved are positive quantities because the probability of occurrence of an event is always positive and less than 1.

In continuous case, however,

$$\int_{-\infty}^{\infty} P(x) dx = 1 \text{ and}$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(x,y) dx dy = 1$$

## Capacity of a Gaussian Channel - SHANNON HARTLEY Theorem:

Statement: The channel capacity of a white bandlimited gaussian channel is given by,

$$C = W \log\left(1 + \frac{S}{N}\right) \text{ bits/sec.}$$

where 'W' is the channel bandwidth

'S' is average signal power

'N' is the average noise power.

if  $\eta/2$  is the two sided psd of noise in Watts/Hz

then

$$N = \eta_{\text{Hz}}(2W) = \eta W$$

$$\therefore C = W \log\left(1 + \frac{S}{\eta W}\right)$$

Proof:

The channels with gaussian noise characteristics are known as gaussian channels. The study of gaussian channel is very important because the results obtained for a gaussian channel often provide a lower bound on the performance of a system with the non-gaussian channel.

For a gaussian channel,

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/2\sigma^2} \rightarrow (1).$$

Hence, we have,

$$H(x) = - \int_{-\infty}^{\infty} P(x) \log P(x) dx$$

$$\therefore H(x) = - \int_{-\infty}^{\infty} P(x) \cdot \log\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/2\sigma^2}\right) dx$$

$$H(x) = \int_{-\infty}^{\infty} P(x) \log \sqrt{2\pi\sigma^2} dx + \int_{-\infty}^{\infty} P(x) \log e^{x^2/2\sigma^2} dx$$

After Simplifying we get,

$$H(x) = \log \sqrt{2\pi e \sigma^2} \text{ bits/message} \rightarrow (2)$$

Now if the signal is band limited to ' $\omega$ ' Hz, then it may be uniquely specified by taking  $2\omega$  Samples/Sec. Hence the rate of information transmission is

$$\begin{aligned} R(x) &= 2\omega \cdot H(x) \\ &= 2\omega \log \sqrt{2\pi e \sigma^2} \\ &= \omega \log \left[ \sqrt{2\pi e \sigma^2} \right]^2 \end{aligned}$$

$$\therefore R(x) = \omega \log(2\pi e \sigma^2) \rightarrow (3)$$

If  $P(x)$  is a band limited gaussian noise with an average noise power ' $N$ ', then we have ( $\sigma^2 = N$ )

$$\therefore R(x) = R(x) = \omega \log(2\pi e N) \rightarrow (A)$$

Now Consider a continuous source transmitting information over a noisy channel. If the received signal is composed of a transmitted signal ' $x$ ' and a noise ' $n$ ' then the joint entropy of the source [bits/sec basis] ~~is~~ and noise is given by,

$$R(x, n) = R(x) + R(n) \rightarrow (4)$$

Assuming that the Txed signal and the noise are independent,  $R(n/x) = R(n)$

$$\therefore R(x, n) = R(x) + R(n) \rightarrow (5)$$

Since the received signal ' $y$ ' is the sum of the txed signal and the noise we may equate.

$$H(x, y) = H(x, n)$$

(or)

$$H(y) + H(x/y) = H(x) + H(n) \quad \text{or}$$

$$R(y) + R(x/y) = R(x) + R(n) \rightarrow \textcircled{6}$$

The rate at which the information is received from a noisy channel is

$$R = R(x) - R(x/y)$$

$$\therefore \textcircled{6} \Rightarrow R = R(y) - R(n) \text{ bits/sec.}$$

$\therefore$  The Channel Capacity in bits/sec is given by

$$C = \text{Max}(R)$$

$$= \text{Max}(R(y) - R(n)) \text{ bits/sec} \rightarrow \textcircled{7}$$

Since  $R(n)$  is assumed to be independent of  $x(t)$ , maximizing 'R' requires maximizing  $R(y)$ .

Let a Txed signal be limited to an average power 'S' and the noise on the channel be white gaussian with an average power 'N' within the band width 'W' of the channel. The Rxed signal will now have an average power  $(S+N)$ .

$R(y)$  is maximum when  $y(t)$  is also gaussian random process because noise is assumed to be gaussian.

Thus the entropy from equ. (A) is given as

$$R(y) = W \log(2\pi e(S+N)) \text{ bits/s and}$$

the entropy of the noise is given by.

$$R(n) = W \log(2\pi e N).$$

$$\therefore C = W \log 2\pi e(S+N) - W \log(2\pi e N)$$

$$= W \log \frac{2\pi e(S+N)}{2\pi e N}$$

$$C = W \log \left(1 + \frac{S}{N}\right) \text{ bits/sec.} \rightarrow \textcircled{8}$$

Above is called Shannon Hartley Theorem, applied to a gaussian noise channel.

Bandwidth / SNR Trade off :

$$C = W \log \left( 1 + \frac{S}{N} \right) \rightarrow (1)$$

Above equation shows that a noiseless channel ( $\frac{S}{N} = \infty$ ) has an infinite capacity. On the other hand the channel capacity does not become infinity as the B.W approaches infinity because, with an increase in bandwidth the noise power also increases. Thus for a fixed signal power, and in the presence of white gaussian noise, the channel capacity approaches an upper limit with increasing bandwidth. This is as given below.

$$(1) \Rightarrow C = W \log \left( 1 + \frac{S}{\eta W} \right) \quad N = \frac{\eta}{2} (2W)$$

$$= \frac{S}{\eta} \cdot \frac{\eta W}{S} \log \left( 1 + \frac{S}{\eta W} \right)$$

$$= \frac{S}{\eta} \log \left( 1 + \frac{S}{\eta W} \right)^{\eta W/S}$$

$$\text{Now, } \lim_{x \rightarrow 0} \log(1+x)^{1/x} = e$$

$$\text{let } x = \frac{S}{\eta W}, \text{ as } W \rightarrow \infty, x \rightarrow 0.$$

$$\therefore \lim_{W \rightarrow \infty} C = \lim_{W \rightarrow \infty} \frac{S}{\eta} \log \left( 1 + \frac{S}{\eta W} \right)^{\eta W/S}$$

$$\lim_{W \rightarrow \infty} C = \frac{S}{\eta} \cdot \log e = 1.44 \frac{S}{\eta} = R_{max} \rightarrow (2)$$

Let us now consider the trade off b/w B.W &  $S/N$  ratio

$$\text{let } \frac{S}{N} = 15 \text{ and } W = 5 \text{ KHz}$$

$$C = W \log \left( 1 + \frac{S}{N} \right) = 5 \text{ K} \log(1+15)$$

$$C = 5K \log 16 = 20K \text{ bits/sec.}$$

If the SNR is increased to 31, the B.W for the same Channel Capacity can be found from,

$$\begin{aligned} 20K &= W \log(1+31) \\ &= W \log 32 = 5W \end{aligned}$$

$$\text{or } W = 4KHz.$$

Thus to decrease the bandwidth, the signal power has to be increased. Similarly it can be shown that to decrease the signal power, the B.W must be increased.

Prob: A gaussian channel has 1MHz B.W. Calculate the channel Capacity if the signal power to noise spectral density ratio ( $S/N$ ) is  $10^5$  Hz. Also find the max. information rate.

Soln:

$$W = 1 \text{ MHz}$$

$$S/N = 10^5$$

$$\begin{aligned} C &= W \log \left( 1 + \frac{S}{N W} \right) \\ &= 1M \log \left( 1 + \frac{10^5}{10^6} \right) \\ &= 10^6 \log \left( 1 + \frac{1}{10} \right) \\ &= 13,800 \text{ bits/sec.} \end{aligned}$$

$$\begin{aligned} R_{\max} &= 1.44 \cdot \frac{S}{N} \\ &= 1.44 \times 10^5 = 14,4000 \text{ bits/sec.} \end{aligned}$$

Prob: Calculate the capacity of a low pass channel with a usable B.W. of 3000 Hz and  $\frac{S}{N} = 10^3$  at the channel o/p. Assume the Channel Noise to be gaussian & white.

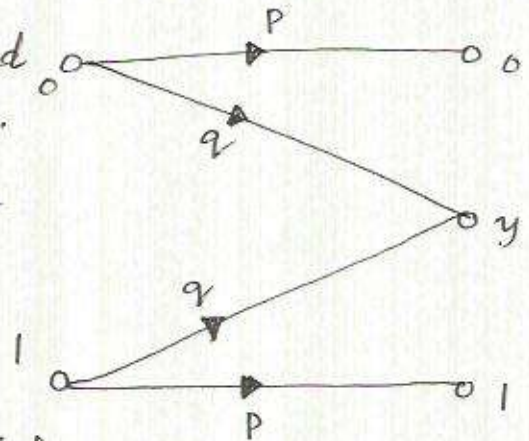
Soln:  $W = 3KHz; \quad \frac{S}{N} = 10^3$

$$C = W \log \left( 1 + \frac{S}{N} \right) = 3K \log(1+10^3) = 30,000 \text{ bits/sec.}$$

Binary Erasure Channel: A binary erasure channel (BEC) has two inputs (0, 1) and three outputs (0, y, 1) as shown.

Here, 0 and 1 are transmitted and they are received as 0, y & 1.

The Symbol 'y' indicates that due to noise, no deterministic decision can be made as to whether the received Symbol is '0' or '1'. In other words the symbol 'y' indicates that the output is erased. Hence the name binary erasure channel.



For BEC, the Channel matrix is

$$D = P(Y/X) = \begin{bmatrix} P & q & 0 \\ 0 & q & P \end{bmatrix}$$

Let us assume that  $P(0) = \alpha$  &  $P(1) = 1 - \alpha$  at the transmitter. Hence,

$$H(X) = \alpha \log \frac{1}{\alpha} + (1 - \alpha) \log \frac{1}{(1 - \alpha)}$$

Now, Since  $P(x_1) = P(0) = \alpha$  &  $P(x_2) = P(1) = (1 - \alpha)$ , the joint probability matrix  $P(X; Y)$  can be obtained from  $P(Y/X)$ , by multiplying rows of  $P(Y/X)$  with  $P(x_1)$  &  $P(x_2)$  respectively.

$$\text{i.e. } P(X; Y) = \begin{bmatrix} P\alpha & q\alpha & 0 \\ 0 & (1 - \alpha)q & (1 - \alpha)P \end{bmatrix}$$

$$\therefore P(y_1) = P\alpha, \quad P(y_2) = \alpha q + (1 - \alpha)q = q \text{ and} \\ P(y_3) = (1 - \alpha)P.$$

The Conditional probability matrix  $P(X/Y)$  can be obtained by dividing the columns of  $P(X, Y)$  with  $P(y_1)$ ,  $P(y_2)$  &  $P(y_3)$  respectively. (24)

$$\therefore P(x/y) = \begin{bmatrix} \frac{\alpha p}{\alpha p} & \frac{\alpha q}{q} & \frac{0}{(1-\alpha)p} \\ \frac{0}{\alpha p} & \frac{(1-\alpha)q}{q} & \frac{(1-\alpha)p}{(1-\alpha)p} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \alpha & 0 \\ 0 & 1-\alpha & 1 \end{bmatrix}$$

$$\begin{aligned} \therefore H(x/y) &= - \sum_{j=1}^2 \sum_{k=1}^3 P(x_j, y_k) \log P(x_j/y_k) \\ &= - \left( \alpha p \log 1 + \alpha q \log \alpha + (1-\alpha)q \log (1-\alpha) + (1-\alpha)p \log 1 \right) \\ &= -q \left( \alpha \log \alpha + (1-\alpha) \log (1-\alpha) \right) \\ &= +q \cdot H(x). \quad q = 1-p \\ &= (1-p) H(x). \end{aligned}$$

$\therefore$  The mutual information is thus given by,

$$\begin{aligned} I(x; y) &= H(x) - H(x/y) \\ &= H(x) - (1-p) H(x) \\ I(x; y) &= p \cdot H(x). \end{aligned}$$

$\therefore$  Channel Capacity is given by

$$\begin{aligned} C &= \text{Max}(I(x; y)) \\ &= \text{Max}(p \cdot H(x)) \\ &= p \cdot \log_2 2 \\ &= p \end{aligned}$$

Since  $\text{Max}(H(x)) = 1$

prob: An analog signal is band limited to 4KHz and is sampled at its Nyquist rate. The samples are quantized into 4 levels. Find the information rate of the source if (i) The prob. of occurrence of inner two levels are three times that of extreme two levels.  
(ii) All the levels are equally likely.

Soln:

An analog signal is band limited to 4KHz

$$\therefore \text{The sampling rate } f_s = \gamma_s = 2 \times 4\text{KHz} \\ = 8000 \text{ Samples/Sec}$$

The samples are quantized into four levels. Let them be  $q_1, q_2, q_3$  and  $q_4$ .

(i) Let the prob. of occurrence of  $q_1$  and  $q_4$  be 'P'.  
Thus the prob. of occurrence of  $q_2$  and  $q_3$  be  $3P$ .

$$\therefore P_1 + P_2 + P_3 + P_4 = 1 \\ P + 3P + 3P + P = 1 \Rightarrow 8P = 1 \\ P = \frac{1}{8}$$

$\therefore$  The probabilities of occurrence of  $q_1, q_2, q_3$  &  $q_4$  be,  $1/8, 3/8, 3/8, 1/8$  respectively.

$$\therefore H = - \sum_k P_k \log P_k \\ = - \left( \frac{1}{8} \log \frac{1}{8} + \frac{3}{8} \log \frac{3}{8} + \frac{3}{8} \log \frac{3}{8} + \frac{1}{8} \log \frac{1}{8} \right) \\ = 1.811 \text{ bits/message (Sample)}$$

$\therefore$  The information rate of the source is thus given by  $R = \gamma_s \cdot H$

$$= 8000 \times 1.811 \\ = 14.488 \text{ Kbits/Sec.}$$

(ii) All the quantized levels are equally likely i.e. the probabilities of occurrence are given by  $\frac{1}{4}, \frac{1}{4}, \frac{1}{4}$  and  $\frac{1}{4}$ .

$$\begin{aligned}\therefore H &= -\sum_k P_k \log P_k \\ &= -\left(\frac{1}{4} \log \frac{1}{4} + \frac{1}{4} \log \frac{1}{4} + \frac{1}{4} \log \frac{1}{4} + \frac{1}{4} \log \frac{1}{4}\right) \\ &= 2 \text{ bits/sample}\end{aligned}$$

$$\begin{aligned}R &= \gamma_s \cdot H \\ &= 8000 \times 2 = 16,000 \text{ bits/sec.} \\ &= 16 \text{ K bits/sec.}\end{aligned}$$

prob.: A discrete source emits one of five symbols once every millisecond. The symbol probabilities are  $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$  and  $\frac{1}{16}$  respectively. Find the source entropy and information rate.

Soln.: A source emits five symbols for every millisecond.  $\therefore \gamma_s = 1000 \text{ messages/sec.}$

The symbol probabilities are  $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$  &  $\frac{1}{16}$ .

$$\begin{aligned}\therefore H(x) &= -\sum_i P_i \log P_i \\ &= -\left(\frac{1}{2} \log \frac{1}{2} + \frac{1}{4} \log \frac{1}{4} + \frac{1}{8} \log \frac{1}{8} + \frac{1}{16} \log \frac{1}{16} + \frac{1}{16} \log \frac{1}{16}\right) \\ &= 1.875 \text{ bits/Symbol (message)}\end{aligned}$$

$$\begin{aligned}\therefore R &= \gamma_s \cdot H \\ &= 1000 \times 1.875 \\ &= 1875 \text{ bits/sec}\end{aligned}$$