MICROWAVE TRANSMISSION LINES

INTRODUCTION:

* The term "Microwaves" usually refer to electromagnetic waves with wavelengths ranging from about one meter (1m) to one millimetre (1mm).
* The frequency range of microwaves in EM wave spectrum is from 300MHz - 300GHz.

HISTORY:

* In 1845, Michel Faraday observed the effect of magnetic field on the line propogation. In the year 1864, James C. Maxwell developed the four basic equations of electromagnetic theory of light.
* In 1893, Heinrich Hertz subsequently verified to show that a parabolic antenna fed by an element i.e., dipole on excitation by a spark discharge, sends a signal to an identical receiving arrangement at a distance.
* G. Marconi transmitted wireless signals across the Atlantic Ocean successfully.
* William Thompson developed the waveguide theory and the mode properties of propogation through hollow mettalic wave-guide.
* The recongnition of Microwave Engineering as a major field within electrical engineering resulted in creation of IRE group MITT in 1952.

MICROWAVE SPECTRUM AND BANDS:

* Microwaves usually corresponds to frequencies between 300MHz and 300 GHz (wavelengths between 1mm to 1m).

* In the Electromagnetic spectrum, microwaves occupy the frequencies above ordinary radio waves and below infrared light.

SL.NO	DESIGNATION	FREQUENCY BAND	WAVELENGTH
1.	VERY LOW FREQUENCY (VLF)	0 - 30KHZ	$\infty - 10^4 m$
2.	LOW FREQUENCY (LF)	30 KHZ - 300KHZ	$10^4 m - 10^3 m$
3.	MEDIUM FREQUENCY (MF)	300KHZ - 3000KHZ	$10^3 m - 10^2 m$
4.	HIGH FREQUENCY (HF)	3MHZ - 30 MHZ	$10^2 m - 10 m$
5.	VERY HIGH FREQUENCY (VHF)	30MHZ - 300MHZ	10m - 1m
6.	ULTRA HIGH FREQUENCY (UHF)	300MHZ - 3000MHZ	1m - 0.1m
7.	SUPER HIGH FREQUENCY (SHF)	3GHZ - 30GHZ	0.1m - 10mm
8.	EXTREME HIGH FREQUENCY (EHF)	30GHZ - 300GHZ	10mm - 1mm

TABLES

* The above table shows the portion of microwave bands in EM spectrum.

* The microwave frequency range is further subdivided into several bands.

Microwave Frequency Bands [IEEE]:

Designation	Frequency Range
UHF	0.3GHz - 1GHz
L-Band	1GHz - 2GHz
S-Band	2GHz - 4GHz
C-Band	4GHz - 8GHz
X-Band	8GHz - 12GHz
Ku-Band	12GHz - 18GHz
K-Band	18GHz - 27GHz
Ka-Band	27GHz - 40GHz
V-Band	40GHz - 75GHz
W-Band	75GHz - 110GHz
Millimeter wave	30GHz - 300GHz
Sub-millimeter waves	300GHz - 3000GHz

Advantages of microwaves over low frequencies:

1. DIRECTIVITY:-

* The first main characteristic of microwave is the directivity. We know that as frequency is increased, the directivity increases and Beam width decreases.

* Beam width $\propto \frac{\lambda}{D}$

$$\text{Beam width } \theta_B = \frac{140^\circ \times \lambda}{D}$$

Where, θ_B = Beam width (Degrees), λ = Wavelength, D = Diameter of antenna

* At 30GHz, $\lambda = 1$ cm for 1° Beam width:

$$\theta_B = \frac{140 \times 1\,cm}{D}$$

$$1^\circ = \frac{140 \times 1\,cm}{D}$$

$$\boxed{D = 140\,cm}$$

* At 300MHz, $\lambda = 1$m, for 1° Beam width

$$\theta_B = \frac{140 \times 1\,m}{D}$$

$$1^\circ = \frac{140 \times 100\,cm}{D}$$

$$D = 140 \times 100\,cm$$

$$\boxed{D = 4000\,cm}$$

* From above example it is clear that antenna size is small for microwave frequencies.

* Power Radiated is given by, $\mu_0 \pi^2 I_0^2 \left(\frac{l}{\lambda}\right)^2$

Where l = length I_0 = AC current carried.

* As frequency increases, λ decreases hence power radiated and gain increases.

2. Increased Bandwidth Availability:

* Microwaves have large bandwidths (1GHz – 103GHz) Compared to the common bands. The advantage of large bandwidth is that the frequency range of information channels will be a small percentage of the carrier frequency and more information can be transmitted in microwave frequency ranges.

3. Fading effect and Realiability:

* Fading effect due to variation in the transmission medium is more effective at low frequency.

* Due to line of sight (LOS) propogation and high frequencies there is less fading effect and hence microwave communication is more reliable.

4. Power Requirements:

* Transmitter / Receiver power requirements are pretty low at microwave frequencies compared to that at short wave bands

5. Transperency property of Microwaves:

* Microwave frequency band ranging from 300MHZ – 10GHZ are capable of freely propogating through the ionized layers surrounding the earth as well as through the atmosphere.

* The presence of such a transparent window in a microwave band facilitates the study of microwave radiation from the sun and stars in radio astronomical research of space.

APPLICATIONS:

1. Broad Casting
2. Communication
3. Radar
4. Remote sensing applications
5. Microwave heating
6. Industrial and Domestic applications
7. Biomedical applications

MAXWELL EQUATIONS:

1. Ampere's Law:

Basic form $\oint H \cdot dl = I$

Maxwell's 1st equation $\nabla \times H = J + \frac{\partial D}{\partial t}$

2. Faraday's Law:

Basic form $V = -\frac{d\phi}{dt}$

Maxwell 2nd equation $\nabla \times E = -\frac{\partial B}{\partial t}$

3. Gauss Law

Basic form $\int_S D \cdot dS = \int_V \rho \cdot dv$

Maxwell 3rd equation $\nabla \cdot D = \rho$

4. Maxwell 4th equation $\nabla \cdot B = 0$

WAVE GUIDES:-

* At frequencies higher than 3GHz, transmission of electromagnetic waves along transmission lines and cables becomes difficult mainly due to losses that occur both in the solid dielectric needed to support the conductor and in the conductor themselves.

* A hollow metallic tube of uniform cross section for transmitting electromagnetic waves by successive reflections from the inner walls of the tube is called a "waveguide."

* There are various types of waveguides available.

1) Rectangular waveguides
2) Cylindrical waveguides
3) Elliptical waveguides
4) Ridged waveguides.

RECTANGULAR WAVEGUIDES

* A Rectangular waveguide is a hollow metallic tube with a rectangular cross section.

* The conducting walls of the guide confine the electromagnetic fields and thereby guide the electromagnetic wave.

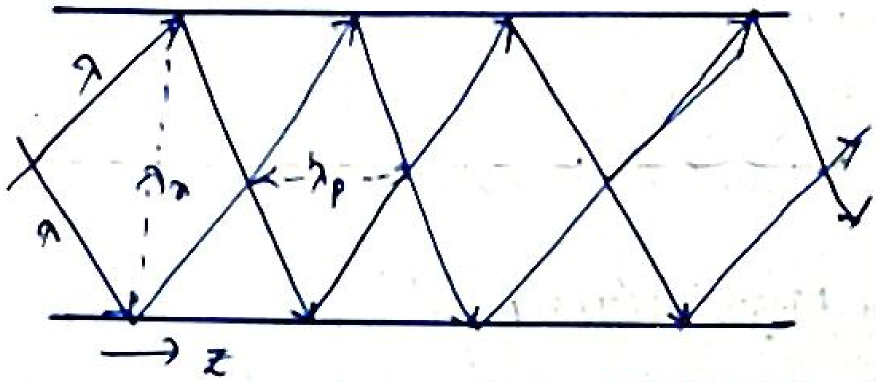


FIG: PLANE WAVE REFLECTED IN A WAVEGUIDE

* It is clear that when the wavelength λ is in the direction of propagation of the incident wave, there will be one component λ_n in the direction normal to the reflecting plane and another λ_p parallel to the plane.

* These components are $\lambda_n = \frac{\lambda}{\cos\theta}$ and $\lambda_p = \frac{\lambda}{\sin\theta}$

MODES OF WAVEGUIDES:

* In waveguide propogation, we have infinity number of modes patterns, known as modes.

* For the existence of mode patterns in the waveguide, it should obey certain physical laws of waveguides.

1. To have propogation in the waveguide, the electric field component must always be perpendicular to the surface of the conductor not

parallel to surface of the Conductor.

2) In other conditions, the magnetic field components must always be parallel to the surface of the conductor not perpendicular to the surface of the conductor.

* In general there are two kinds of modes in a Waveguide.

1. TRANSVERSE ELECTRIC (TE) MODE:

* Electric field is always transverse to the direction of propogation then it is called TE Mode.

* If 'z' is the direction of propogation then $\boxed{E_z = 0 \text{ but } H_z \neq 0}$

2. TRANSVERSE MAGNETIC (TM) MODE:

* Magnetic field is always transverse to the direction of propogation then it is called as TM Mode.

* If 'z' is the direction of propogation then $\boxed{H_z = 0 \text{ but } E_z \neq 0.}$

3. TEM MODE:

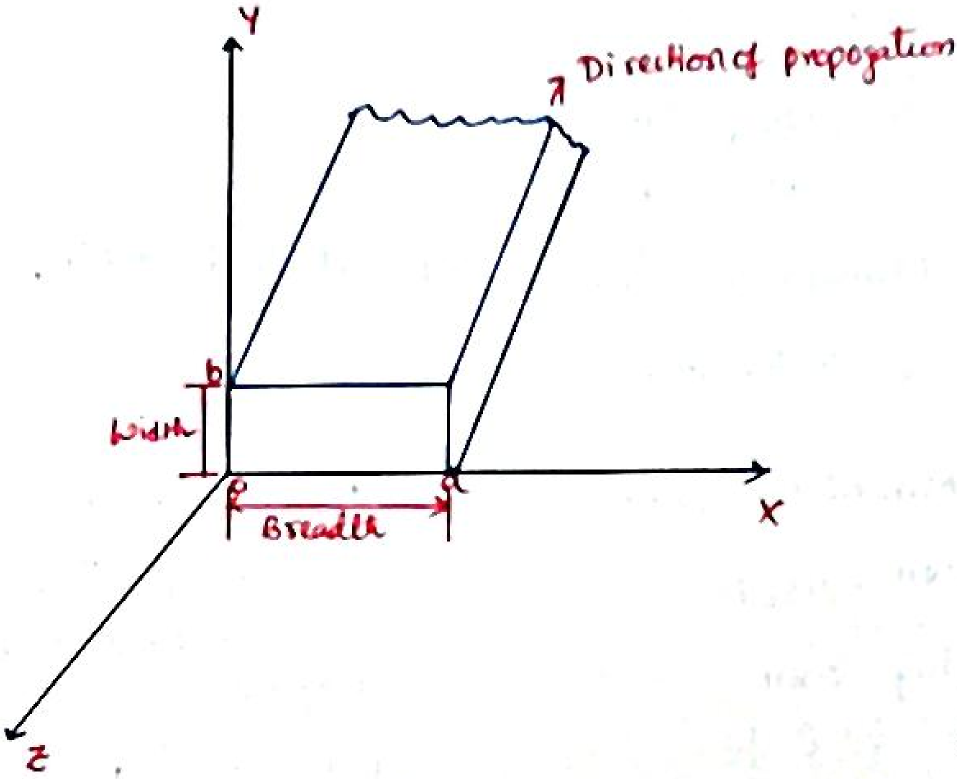
* If both Electric field and magnetic field are transverse to the direction of propogation then it called as TEM Mode.

* If 'z' is the direction of propogation then $\boxed{E_z = 0 \ \& \ H_z = 0}$

* All the field components along x and y directions E_x, E_y, H_x, H_y vanish and hence a TEM mode cannot exist inside a Waveguide.

SOLUTION OF WAVE EQUATION IN RECTANGULAR CO ORDINATES

* Consider a rectangular waveguide situated in the rectangular Co-ordinate system with its breadth along x-axis, width along y-axis and the wave is assumed to propogate along the z-direction.



* The wave equations for TE and TM waves are given by,

$$\nabla^2 H_z = -\omega^2 \mu \varepsilon H_z \quad \text{for TE wave } (E_z = 0)$$

$$\nabla^2 E_z = -\omega^2 \mu \varepsilon E_z \quad \text{for TM wave } (H_z = 0)$$

* Expanding $\nabla^2 E_z$ in rectangular co-ordinate system

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = -\omega^2 \mu \varepsilon E_z \rightarrow (1)$$

* Since the wave is propogating in z-direction we have the operator

$$\frac{\partial^2}{\partial z^2} = \gamma^2$$

then eq (1) becomes $\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z = -\omega^2 \mu \varepsilon E_z \rightarrow (2)$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z + \omega^2 \mu \varepsilon E_z = 0$$

$$\frac{\partial^2 E_z}{\partial x^2}+\frac{\partial^2 E_z}{\partial y^2}+(\gamma^2+\omega^2\mu\varepsilon)E_z=0 \rightarrow (3)$$

Let $\gamma^2+\omega^2\mu\varepsilon=h^2$ then eq (3) becomes

$$\frac{\partial^2 E_z}{\partial x^2}+\frac{\partial^2 E_z}{\partial y^2}+h^2E_z=0 \quad \text{for TM wave} \rightarrow (4)$$

Similarly $\frac{\partial^2 H_z}{\partial x^2}+\frac{\partial^2 H_z}{\partial y^2}+h^2H_z=0$ for TE wave $\rightarrow$ (5)

* Using Maxwells equations, it is possible to find the various components along x and y directions.

From Maxwell 1st equation

$$\nabla\times H=j\omega\varepsilon E$$

Expanding $\nabla\times H$,

i.e, $$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ H_x & H_y & H_z\end{vmatrix}=j\omega\varepsilon(\hat{i}E_x+\hat{j}E_y+\hat{k}E_z)$$

replacing $\frac{\partial}{\partial z}=-\gamma$ we get,

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma\\ H_x & H_y & H_z\end{vmatrix}=j\omega\varepsilon[\hat{i}E_x+\hat{j}E_y+\hat{k}E_z]$$

Expanding and equating coefficient of $\hat{i},\hat{j},\hat{k}$, we get

$$\frac{\partial H_z}{\partial y}+\gamma H_y=j\omega\varepsilon E_x \rightarrow (6)$$

$$\frac{\partial H_z}{\partial x}+\gamma H_x=-j\omega\varepsilon E_y \rightarrow (7)$$

$$\frac{\partial H_y}{\partial x}-\frac{\partial H_x}{\partial y}=j\omega\varepsilon E_z \rightarrow (8)$$

From Maxwell 2nd equation, we

$$\nabla\times E=-j\omega\mu H$$

Expanding $\nabla\times E$

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ E_x & E_y & E_z\end{vmatrix}=-j\omega\mu[\hat{i}H_x+\hat{j}H_y+\hat{k}H_z]$$

Replacing $\frac{\partial}{\partial z}=-\gamma$

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & -\gamma\\ E_x & E_y & E_z\end{vmatrix}=-j\omega\mu[\hat{i}H_x+\hat{j}H_y+\hat{k}H_z]$$

$$\frac{\partial E_z}{\partial y}+\gamma E_y=-j\omega\mu H_x \rightarrow (9)$$

$$\frac{\partial E_z}{\partial x}+\gamma E_x=j\omega\mu H_y \rightarrow (10)$$

$$\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}=-j\omega\mu H_z \rightarrow (11)$$

Combining eq ⑥ and eq ⑩ then

$$H_y = \frac{1}{j\omega\mu}\left[\frac{\partial}{\partial x}\{E_z + \gamma E_x\}\right]$$

Substitute H_y in eq ⑥

$$\frac{\partial H_z}{\partial y} + \gamma\left[\frac{1}{j\omega\mu}\left[\frac{\partial E_z}{\partial x} + \gamma E_x\right]\right] = j\omega\varepsilon E_x$$

$$\frac{\partial H_z}{\partial y} + \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial x} + \frac{\gamma^2}{j\omega\mu}E_x = j\omega\varepsilon E_x$$

$$\frac{\partial H_z}{\partial y} + \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial x} = j\omega\varepsilon E_x - \frac{\gamma^2}{j\omega\mu}E_x$$

$$\frac{\partial H_z}{\partial y} + \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial x} = E_x\left[j\omega\varepsilon - \frac{\gamma^2}{j\omega\mu}\right]$$

Multiply with $j\omega\mu$ on both sides

$$j\omega\mu\left[\frac{\partial H_z}{\partial y} + \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial x}\right] = j\omega\mu E_x\left[j\omega\varepsilon - \frac{\gamma^2}{j\omega\mu}\right]$$

$$j\omega\mu\frac{\partial H_z}{\partial y} + j\omega\mu \times \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial x} = j^2\omega^2\mu\varepsilon E_x - j\omega\mu E_x \times \frac{\gamma^2}{j\omega\mu}$$

$$j\omega\mu\frac{\partial H_z}{\partial y} + \gamma\frac{\partial E_z}{\partial x} = -E_x\omega^2\mu\varepsilon - E_x\gamma^2$$

$$j\omega\mu\frac{\partial H_z}{\partial y} + \gamma\cdot\frac{\partial E_z}{\partial x} = -E_x\left[\omega^2\mu\varepsilon + \gamma^2\right] \qquad \left(\because \gamma^2 + \omega^2\mu\varepsilon = h^2\right)$$

$$j\omega\mu\frac{\partial H_z}{\partial y} + \gamma\cdot\frac{\partial E_z}{\partial x} = -h^2 E_x$$

divide with $-h^2$, then

$$-\frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} - \frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} = \frac{-h^2}{-h^2}E_x$$

$$E_x = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} \rightarrow (12)$$

Similarly $$E_y = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} \rightarrow (13)$$

$$H_x = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y} \rightarrow (14)$$

$$H_y = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} \rightarrow (15)$$

* These equations gives a general relationship for field components within a waveguide. Therefore, when E_z and H_z are known, we can obtain the solution for E_x, E_y, H_x and H_y.

TM MODE ANALYSIS:-

* For TM wave $H_z = 0$, $E_z \neq 0$
* The wave equation of TM wave is given by,

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0 \rightarrow (1)$$

* This is a partial differential equation which can be solved to get the different field components E_x, E_y, H_x, H_y by "Separation of variables method.

Let us assume $E_z = XY$

where, X is a pure function of 'x' only

Y is a pure function of 'y' only

Since x and y are independent variables

$$\frac{\partial^2 E_z}{\partial x^2} = \frac{\partial^2 (XY)}{\partial x^2} = Y \cdot \frac{\partial^2 X}{\partial x^2} \qquad (\because E_z = XY)$$

$$\frac{\partial^2 E_z}{\partial y^2} = \frac{\partial^2 (XY)}{\partial y^2} = X \cdot \frac{\partial^2 Y}{\partial y^2}$$

then eq ① becomes

$$Y \cdot \frac{\partial^2 X}{\partial x^2} + X \cdot \frac{\partial^2 Y}{\partial y^2} + h^2 (XY) = 0$$

divide with XY on bothsides

$$\frac{1}{\cancel{XY}} \cdot \cancel{Y} \cdot \frac{\partial^2 X}{\partial x^2} + \frac{1}{XY} \cdot \cancel{X} \cdot \frac{\partial^2 Y}{\partial y^2} + \frac{1}{\cancel{XY}} h^2 (\cancel{XY}) = 0$$

$$\frac{1}{X} \frac{\partial^2 X}{\partial x^2} + \frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} + h^2 = 0 \rightarrow ②$$

Let $\frac{1}{X} \frac{\partial^2 X}{\partial x^2} = -B^2 \rightarrow ③$

$$\frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} = -A^2 \rightarrow ④$$

then eq ② becomes

$$-B^2 - A^2 + h^2 = 0$$

$$h^2 = A^2 + B^2$$

* The eq's ③ and ④ are ordinary 2nd order differential equations, the solutions of which are given by,

$$X = C_1 \cos Bx + C_2 \sin Bx \rightarrow ⑤$$

$$Y = C_3 \cos Ay + C_4 \sin Ay \rightarrow ⑥$$

* Where C_1, C_2, C_3 and C_4 are Constants which can be solved by applying the Boundary Conditions.

* Substitute eq (5) and (6) in $E_z = XY$

$$E_z = [C_1 \cos Bx + C_2 \sin Bx][C_3 \cos Ay + C_4 \sin Ay] \rightarrow (7)$$

BOUNDARY CONDITIONS:

* Since there are four walls, there are four boundary conditions.

1st Boundary Condition. [Bottom plane or bottom wall]

* We know that $E_z = 0$ all along the bottom wall

$$\boxed{E_z = 0 \text{ at } y = 0} \quad \forall x \rightarrow 0 \text{ to } a$$

then $E_z = [C_1 \cos Bx + C_2 \sin Bx][C_3 \cos Ay + C_4 \sin Ay]$

$$0 = [C_1 \cos Bx + C_2 \sin Bx][C_3 \cos 0 + C_4 \sin 0]$$

$$0 = [C_1 \cos Bx + C_2 \sin Bx] \cdot C_3$$

Here $C_1 \cos Bx + C_2 \sin Bx \neq 0 \quad C_3 = 0$

$$\boxed{\therefore E_z = [C_1 \cos Bx + C_2 \sin Bx][C_4 \sin Ay]} \rightarrow (8)$$

2nd Boundary Condition: [Left plane or Left side wall]

* Here $E_z = 0$ at $x = 0 \ \forall y \rightarrow 0$ to b

Substitute 2nd boundary in eq (8).

$$E_z = [C_1 \cos Bx + C_2 \sin Bx][C_3 \cos Ay + C_4 \sin Ay]$$

$$0 = [C_1 \cos 0 + C_2 \sin 0][C_3 \cos Ay + C_4 \sin Ay]$$

$$0 = [C_1] \quad C_1 C_4 \sin Ay = 0$$

$\because \sin Ay \neq 0$ and $C_4 \neq 0$

$C_1 = 0$

then $\boxed{E_z = C_2 C_4 \sin Bx \sin Ay}$ $\rightarrow$ (9)

3rd Boundary Condition: [Top plane]

* Here $E_z = 0$ at $y = b$ $\forall x \rightarrow 0$ to a

Substitute 3rd boundary in eq (9)

$E_z = C_2 C_4 \sin Bx \sin Ay$

$= 0$ $C_2 C_4 \sin Bx \sin A(b)$

$\because \sin Bx \neq 0$, $C_4 \neq 0$, $C_2 \neq 0$ other wise there is no solution

$\sin Ab = 0$

$Ab = n\pi$ (a multiple of $\pi = n\pi$)

$n = 0, 1, 2 \ldots$

$\boxed{A = \frac{n\pi}{b}}$ $\rightarrow$ (10)

4th Boundary Condition: [RIGHT PLANE]

* Here $E_z = 0$ at $x = a$ $\forall$ $y \rightarrow 0$ to b

Substitute 4th boundary in eq (9)

$E_z = C_2 C_4 \sin Bx \sin Ay$

$0 = C_2 C_4 \sin Ba \cdot \sin Ay$

$\sin Ay, C_2, C_4 \neq 0$

$\sin Ba = 0$

$\beta a = m\pi$

$$\boxed{\beta = \frac{m\pi}{a}} \rightarrow (14)$$

Now eq (9) becomes $E_z = C_2 C_4 \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-\gamma z} e^{j\omega t}$

where $e^{-\gamma z}$ = propogation along z-direction

$e^{j\omega t}$ = sinusoidal variation w.r.t 't'.

Let $C_2 C_4 = C$

then $E_z = C \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cdot e^{-\gamma z + j\omega t}$

* Since E_z is known and $H_z = 0$ for TM wave then E_x, E_y, H_x, H_y are as follows.

$\rightarrow$ $E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \cancel{\frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}}^{=0}$ $(\because H_z = 0)$

$$E_x = \frac{-\gamma}{h^2}\frac{\partial}{\partial x}\left[C \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y\, e^{j\omega t - \gamma z}\right]$$

$$E_x = \frac{-\gamma}{h^2} C \left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}$$

$\rightarrow$ $E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \cancel{\frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}}^{0}$

$$E_y = \frac{-\gamma}{h^2}\frac{\partial}{\partial y}\left[C \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}\right]$$

$$E_y = \frac{-\gamma}{h^2} C \left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y\, e^{j\omega t - \gamma z}$$

$\Rightarrow$ $H_x = \cancel{\frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x}}^{=0} + \frac{j\omega\epsilon}{h^2}\frac{\partial E_z}{\partial y}$

$$H_x = \frac{j\omega\varepsilon}{h^2}\frac{\partial}{\partial y}\left[C \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \cdot e^{j\omega t - \gamma z}\right]$$

$$H_x = \frac{j\omega\varepsilon}{h^2} C\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \, e^{j\omega t - \gamma z}$$

$$\Rightarrow H_y = \frac{-\gamma}{h^2}\cancel{\frac{\partial H_z}{\partial y}}^{0} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

$$H_y = \frac{j\omega\varepsilon}{h^2}\frac{\partial}{\partial x}\left[C \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \, e^{j\omega t - \gamma z}\right]$$

$$H_y = \frac{j\omega\varepsilon}{h^2} C\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \, e^{j\omega t - \gamma z}.$$

TE MODE ANALYSIS

4.3.9 Propagation of TE Waves in a Rectangular Waveguide

The TE_{mn} modes in a rectangular waveguide are characterised by $E_z = 0$. In other words the 'z' component of the magnetic field, H_z, must exist in order to have energy transmission in the guide.

The wave equation (Helmhottz equation) for TE wave is given by

$$\Delta^2 H_z = -\omega^2 \mu\varepsilon H_z$$

i.e.,
$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} = -\omega^2 \mu\varepsilon H_z$$

or
$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z + \omega^2 \mu\varepsilon H_z = 0 \qquad \left[\because \frac{\partial^2}{\partial z^2} = \gamma^2\right]$$

or
$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} = (\gamma^2 + \omega^2 \mu\varepsilon) H_z = 0$$

or
$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0 \qquad ...(4.74)$$

This is a *partial differential equation whose solution can be assumed.*

Assume a solution $H_z = XY$. Where

X is a pure function of 'x' only.

Y is a pure function of 'y' only.

Substituting for H_z in Eq. 4.70, we get

$$Y\frac{d^2X}{dx^2} + X\frac{d^2Y}{dy^2} + h^2XY = 0$$

Dividing throughout by XY, we get

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + h^2 = 0$$

Here $\frac{1}{X}\frac{d^2X}{dx^2}$ is purely a function of x,

and $\frac{1}{Y}\frac{d^2Y}{dx^2}$ is purely a function of y,

Equating each of these items to a constant, we get

$$\frac{1}{X}\frac{d^2X}{dx^2} = -B^2$$

and

$$\frac{1}{Y}\frac{d^2Y}{dy^2} = -A^2$$

where $-B^2$ and $-A^2$ are constants.

Substituting these in Eq. 4.75 above, we get

$$-B^2 - A^2 + h^2 = 0$$

$$\therefore \qquad h^2 = A^2 + B^2$$

Solving fox X and Y by separation of variable method.

$$X = C_1 \cos Bx + C_2 \sin Bx$$

$$X = C_3 \cos Ay + C_4 \sin Ay$$

Therefore the complete solution is, $H_z = XY$

i.e.,

$$H_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay)$$

where C_1, C_2, C_3 and C_4 are constants which can be evaluated by applying boundary conditions.

Boundary Conditions

As in case of TM waves, we have four boundaries for TE waves also, as shown in Fig. 4.40.

Here since we are considering a TE wave.

$E_z = 0$ but we have components along x and y direction.

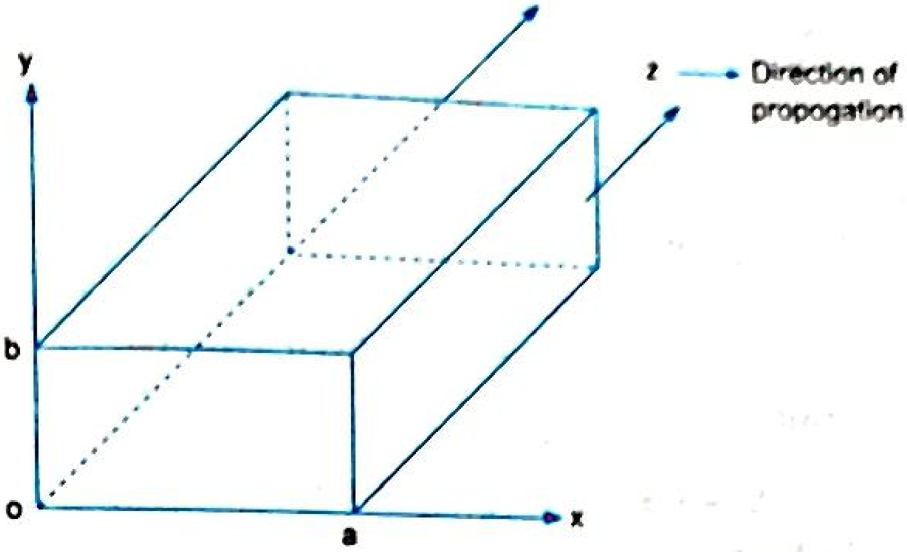


Fig. 4.40

$E_x = 0$ all along bottom and top walls of the waveguide.

$E_y = 0$ all along left and right walls of the waveguide.

1st Boundary Condition :

$$E_x = 0 \text{ at } y = \forall x \longrightarrow 0 \text{ to } a \text{ (bottom wall)}$$

2nd Boundary Condition :

$$E_x = 0 \text{ at } y = b \; \forall x \longrightarrow 0 \text{ to } a \text{ (top wall)}$$

3rd Boundary Condition :

$$E_y = 0 \text{ at } x = 0 \; \forall y \longrightarrow 0 \text{ to } b \text{ (left side wall)}$$

4th Boundary Condition :

$$E_y = 0 \text{ at } x = a \; \forall y \longrightarrow 0 \text{ to } b \text{ (right side wall)}$$

(i) Substituting 1st Boundary Condition in Eq. 4.77.

Since, 1st Boundary Condition is

$E_x = 0$ at $y = 0 \; \forall x \longrightarrow 0$ to a, let us write E_x in terms of H_z.

From Eq. 4.31, we have

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}.$$

Since $E_z = 0$, the 1st term = 0.

$$\therefore \qquad E_x = \frac{-j\omega\mu}{h^2}\frac{\partial}{\partial y}\left[(C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay)\right]$$

i.e.,
$$E_x = \frac{-j\omega\mu}{h^2}(C_1 \cos Bx + C_2 \sin Bx)(-AC_3 \sin Ay + AC_4 \cos Ay)$$

Substituting 1st Boundary condition in the above equation we get

$$0 = \frac{-j\omega\mu}{h^2}(C_1 \cos Bx + C_2 \sin Bx)(0 + AC_4)$$

Since $(C_1 \cos Bx + C_2 \sin Bx) \neq 0$, $A \neq 0$.

$$C_4 = 0$$

Substituting the value of C_4 in Eq. 4.77, the solution reduces to,

$$H_z = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay) \quad ...(4.78)$$

(ii) 3rd Boundary condition :

$$E_y = 0 \text{ at } x = 0 \;\forall\; y \longrightarrow 0 \text{ to } b.$$

From Eq. 4.32 We have,

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{-j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}.$$

Since $E_z = 0$ and substituting the value of H_z from Eq. 4.78, we get

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial}{\partial x}[(C_1 \cos Bx + C_2 \sin Bx)\, C_3 \cos Ay]$$

i.e.,

$$E_y = \frac{j\omega\mu}{h^2}[(-BC_1 \sin Bx + BC_2 \cos Bx)\, C_3 \cos Ay]$$

Substituting the 3rd boundary condition,

$$x = 0, \;\forall\; y \longrightarrow 0 \text{ to } b \text{ in the above equation.}$$

$$0 = \frac{j\omega\mu}{h^2}(0 + BC_2)\, C_3 \cos Ay.$$

Since, $\cos Ay \neq 0$, $B \neq 0$, $C_3 \neq 0$.

$$C_2 = 0$$

Substituting the value of C_2 in Eq. 4.78, the solution now reduces to,

$$H_z = C_1 C_3 \cos Bx \cos Ay \quad ...(4.79)$$

(iii) 2nd Boundary Condition :

$$E_x = 0 \text{ at } y = b \;\forall\; x \longrightarrow 0 \text{ to } a.$$

From Eq. 4.31, we have

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$= \frac{-j\omega\mu}{h^2}\frac{\partial}{\partial y}[C_1 C_3 \cos Bx \cos Ay] \quad (\because E_z = 0)$$

$$= \frac{+j\omega\mu}{h^2} C_1 C_3 A \cos Bx \sin Ay.$$

Substituting 2nd Boundary condition, in the above equation, we get

$$0 = \frac{j\omega\mu}{h^2} C_1C_3 A \cos Bx \sin Ab$$

$$\cos Bx \neq 0,\ C_1, C_3 \neq 0$$

$\therefore$ $\sin Ab = 0$ or $Ab = n\pi$

where $n = 0, 1, 2, \ldots, \infty$.

or $$A = \frac{n\pi}{b} \qquad \text{...(4.80)}$$

(*iv*) 4th Boundary condition :

$$E_y = 0 \text{ at } x = a \ \forall\ y \longrightarrow 0 \text{ to } b.$$

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial}{\partial x}[C_1C_3 \cos Bx \cos Ay]$$

($\because$ $E_z = 0$ and $H_z = C_1C_3 \cos Bx \cos Ay$)

i.e., $$E_y = \frac{-j\omega\mu}{h^2} C_1C_3 B \sin Bx \cos Ay.$$

Substituting the boundary condition

$$0 = \frac{-j\omega\mu}{h^2} C_1C_3 B \sin Bx \cos Ay \Rightarrow \forall\ y \longrightarrow 0 \text{ to } b.$$

$$\cos Ay \neq 0,\ C_1, C_3 \neq 0$$

$\therefore$ $\sin Ba = 0$

$\therefore$ $Ba = m\pi$ where $m = 0, 1, 2, \ldots, \infty$

$\therefore$ $$B = \frac{m\pi}{a} \qquad \text{...(4.81)}$$

The complete solution is (as per Eq. 4.79),

$$H_z = C_1C_3 A \cos Bx \cos Ay$$

Substituting for A and B from Eqns. 4.80 and 4.81, we get.

$$H_z = C_1C_3 \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y.$$

Let $C_1C_3 = C$ (another constant)

$\therefore$ $$H_z = C \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y \cdot e^{(j\omega t - \gamma z)} \qquad \text{...(4.82)}$$

Thus it can be seen that for a TM wave E_z has sine-sine components (as per Eq. 4.52) and for a TE wave H_z has *cosine-cosine* components (as per Eq. 4.82).

Field Components

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

Here 1st term = 0 since $E_z = 0$ for TM wave

i.e.,
$$E_x = \frac{j\omega\mu}{h^2}\cdot C\cdot\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi}{a}\right)x\,\sin\left(\frac{n\pi}{b}\right)y\cdot e^{(j\omega t-\gamma z)}$$

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

Again, 1st term = 0 since $E_z = 0$ for TM wave.

$\therefore$
$$E_y = \frac{-j\omega\mu}{h^2}C\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)y\cdot e^{(j\omega t-\gamma z)}$$

Similarly,

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$$

$\therefore$
$$H_x = \frac{+\gamma}{h^2}C\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)y\cdot e^{(j\omega t-\gamma z)}$$

and
$$H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

$\therefore$
$$H_y = \frac{-\gamma}{h^2}C\cdot\left(\frac{n\pi}{b}\right)^2\cos\left(\frac{m\pi}{a}\right)x\cdot\sin\left(\frac{n\pi}{b}\right)y\cdot e^{j\omega t-\gamma z}$$

Field Patterns

Figure 4.28 shows the field pattern for a TE wave. Solid lines depict electric field lines or voltage lines and dotted lines depict magnetic field lines.

We use subscript for designating a particular mode. TE_{mn} or TM_{mn} where 'm' indicates the number of half wave variations of the electric field (or magnetic field in a TM) across the wider dimension a, of the waveguide and 'n' indicates the number across the narrow dimension b. Referring to TE pattern shown in Fig. 4.28 it can be seen that the voltage varies from '0' to maximum and maximum to '0' across the wide dimension a. This is half variation. Hence m = 1. Across the narrow dimension, there is no variation in voltage v. Hence n = 0. Therefore this mode TE_{10} mode. The mode having the *highest cutoff wavelength* is known as dominant mode of the waveguide and all other modes are called higher modes. For example TE_{10} is the dominant mode for TE waves. It is the mode which is used for practically all electromagnetic transmission in a rectangular waveguide. Dominant mode is almost always a low-loss, distortion less transmission and higher modes result in a significant loss of power and also undesirable harmonic distortion. The radiation pattern for TE mode is shown in Fig. 4.29. Sketches of some higher order TE modes are shown in Fig. 4.30.

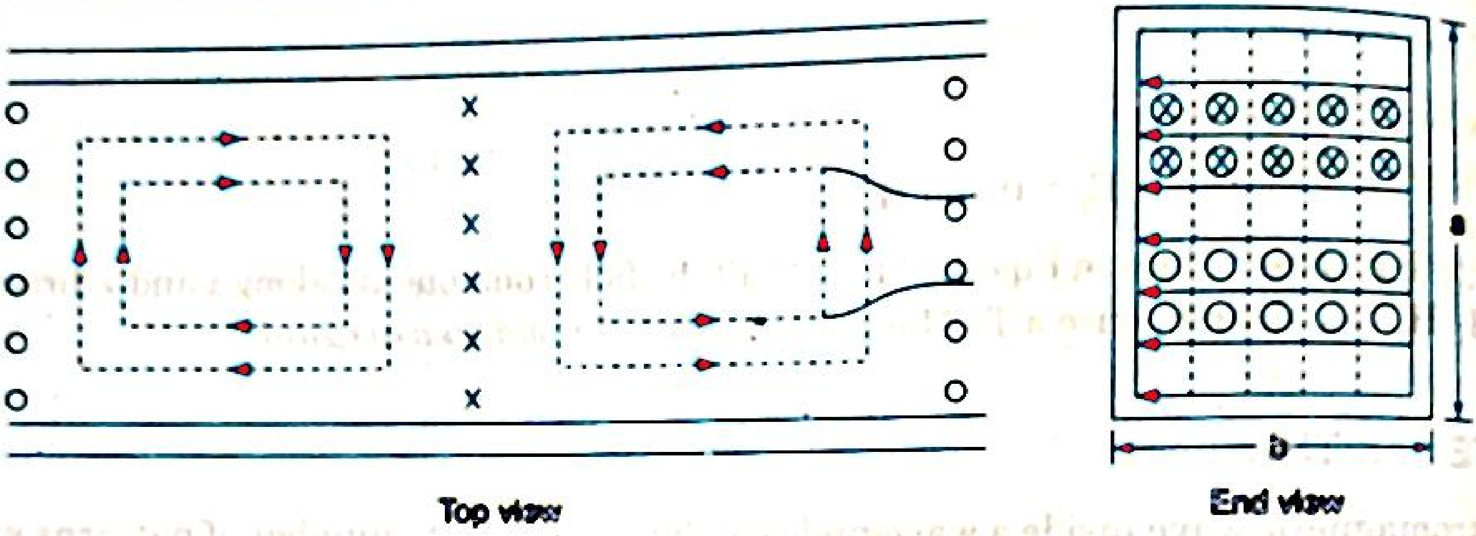


Fig. 4.28 Field pattern of a TE wave

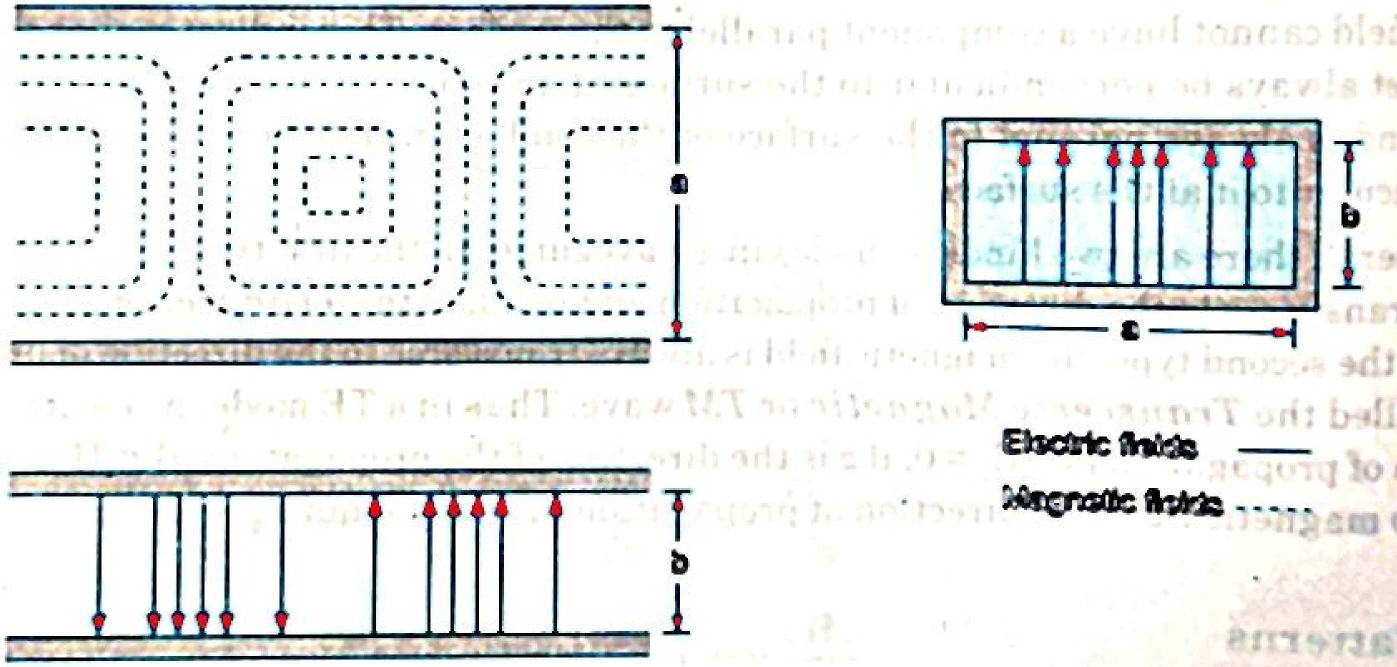


Fig. 4.29 Radiation pattern for TE_{10} mode.

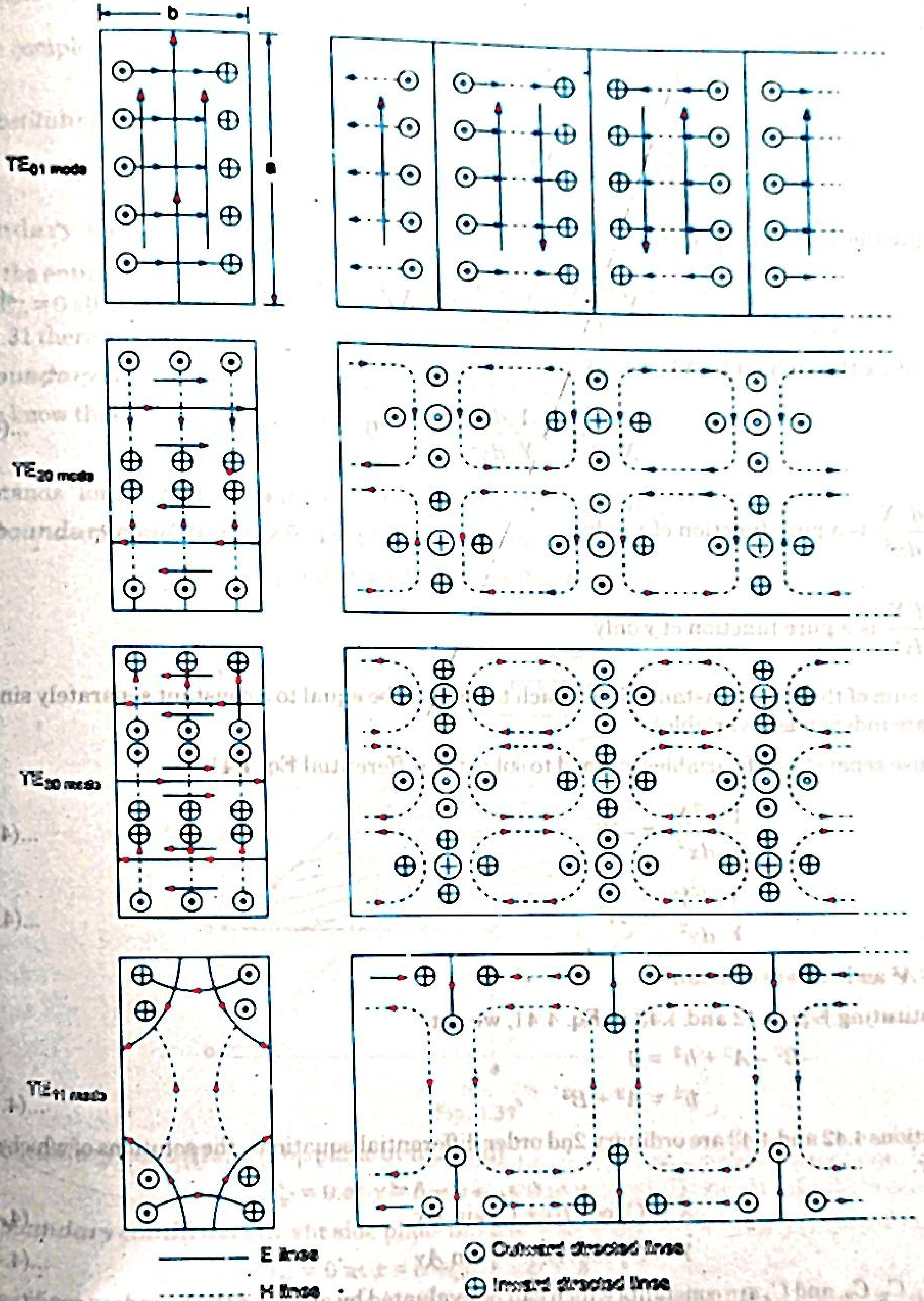


Fig. 4.30 Field pattern of higher order modes.

CUT-OFF FREQUENCY:-

* The frequency above which the waveguide offers minimum attenuation to the propagation of wave is called Cut-off frequency (f_c).

From $h^2 = \gamma^2 + \omega^2 \mu \varepsilon$

We know that $h^2 = A^2 + B^2$, $A = \frac{m\pi}{a}$, $B = \frac{n\pi}{b}$

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \gamma^2 + \omega^2 \mu \varepsilon$$

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \varepsilon$$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \varepsilon}$$

Since $\gamma = \alpha + j\beta$

$$\alpha + j\beta = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \varepsilon}$$

At Low frequencies:

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 > \omega^2 \mu \varepsilon$$

$$\gamma = \alpha + j\beta = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$\gamma = \alpha = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

* γ becomes Real and positive and equal to α. The wave is Completely attenuated and there is no phase Change. Hence wave Cannot propogate.

At High Frequencies:-

$$\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2 < \omega^2\mu\varepsilon$$

$$\gamma = \alpha + j\beta = \sqrt{-\omega^2\mu\varepsilon}$$

$$= \sqrt{j^2\omega^2\mu\varepsilon}$$

$$= j\sqrt{\omega^2\mu\varepsilon}$$

$$\gamma = \beta = \sqrt{\omega^2\mu\varepsilon}$$

* γ becomes imaginary. There will be phase change β hence the wave propogates.

At Cut-off frequency:

$$\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2 = \omega^2\mu\varepsilon$$

$$\gamma = \alpha + j\beta = \sqrt{\omega^2\mu\varepsilon - \omega^2\mu\varepsilon}$$

$$\boxed{\gamma = 0}$$

* The frequency at which $\gamma = 0$ is defined as cut-off frequency (f_c).

At $\gamma = 0$, $\omega = \omega_c$

$$\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2 = \omega_c^2\mu\varepsilon$$

$$\omega_c^2 = \frac{1}{\mu\varepsilon}\left[\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2\right]$$

$$\omega_c = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$$

$$2\pi f_c = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\pi^2\left(\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2\right)}$$

$$\boxed{f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}} \qquad \left(\because c = \frac{1}{\sqrt{\mu\varepsilon}}\right)$$

Cut-off wavelength:

$$\lambda_c = \frac{c}{f_c}$$

$$\lambda_c = \frac{c}{\frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}}$$

$$\boxed{\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}}}$$

* Cut-off wavelength is the wavelength of the signal below which propagation of wave occurs above which there is no propogation.

* Therefore, operating frequency must be above cut-off frequency in order to propogate the wave in waveguide.

GUIDE WAVELENGTH (λ_g):

* The distance travelled by the wave in order to undergo a phase shift of 2π radians is called Guide wavelength (λ_g)

$$\boxed{\lambda_g = \frac{2\pi}{\beta}}$$

* The wavelength in the waveguide is differed from the wavelength in freespace.

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda_0^2} - \frac{1}{\lambda_c^2}$$

$$\frac{1}{\lambda_g^2} = \frac{\lambda_c^2 - \lambda_0^2}{\lambda_0^2 \lambda_c^2}$$

$$\lambda_g^2 = \frac{\lambda_0^2 \lambda_c^2}{\lambda_c^2 - \lambda_0^2}$$

$$\lambda_g^2 = \frac{\lambda_0^2}{\frac{\lambda_c^2}{\lambda_c^2} - \frac{\lambda_0^2}{\lambda_c^2}}$$

$$\lambda_g^2 = \frac{\lambda_0^2}{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

$$\boxed{\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}}$$

PHASE VELOCITY:

* The wave propogates in the waveguide, when guide wavelength λ_g is greater than the free space wavelength (λ_0).
* Since the velocity of propogation is the product of λ and f it follows that in a waveguide

$$V_p = \lambda_g \cdot f$$

where V_p is the phase velocity

* The rate at which the wave changes its phase in terms of the guide wavelength is called phase velocity (V_p).

$$\therefore V_p = \frac{\lambda_g}{\text{unit time}}$$

$$V_p = \lambda_g \cdot f$$

$$= \frac{\lambda_g \, 2\pi f}{2\pi}$$

$$= \frac{\lambda_g \, 2\pi f}{2\pi/\lambda_g} \qquad \left(\because \lambda_g = \frac{2\pi}{\beta} \quad \beta = \frac{2\pi}{\lambda_g}\right)$$

$$\omega = 2\pi f$$

$$\boxed{V_p = \frac{\omega}{\beta}}$$

Expression for V_p :

$$h^2 = \gamma^2 + \omega^2 \mu \varepsilon = A^2 + B^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\gamma = \alpha + j\beta$$

For wave propogation $\gamma = j\beta$

$$\gamma^2 = (j\beta)^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \varepsilon \rightarrow (1)$$

At $f = f_c$, $\omega = \omega_c$, $\gamma = 0$

$$0 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega_c^2 \mu \varepsilon$$

$$\omega_c^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \rightarrow (2)$$

sub eq (2) in eq (1)

$$\gamma^2 = (j\beta)^2 = \omega_c^2 \mu \varepsilon - \omega^2 \mu \varepsilon$$

$$\beta^2 = -\mu \varepsilon (\omega_c^2 - \omega^2)$$

$$\beta = \sqrt{-\mu\varepsilon(\omega_c^2 - \omega^2)}$$

$$\beta = \sqrt{\mu\varepsilon(\omega^2 - \omega_c^2)}$$

$$V_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\mu\varepsilon}\sqrt{\omega^2 - \omega_c^2}}$$

$$V_p = \frac{c \cdot \omega}{\sqrt{\omega^2 - \omega_c^2}} \qquad \left(\because c = \frac{1}{\sqrt{\mu\varepsilon}}\right)$$

$$V_p = \frac{c \cdot \omega}{\sqrt{\omega^2\left(1 - \left(\frac{\omega_c}{\omega}\right)^2\right)}} = \frac{c}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}$$

$$\boxed{V_p = \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}}$$

we know that $f = \frac{c}{\lambda_0}$ $\quad f_c = \frac{c}{\lambda_c}$

$$\frac{f_c}{f} = \frac{c}{\lambda_c} \cdot \frac{\lambda_0}{c} = \frac{\lambda_0}{\lambda_c}$$

$$\boxed{V_p = \frac{c}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}}$$

GROUP VELOCITY (V_g)

* When a modulated Carrier travels through a waveguide, the modulation enevelope travels with a velocity much less than that of the Carrier.
* The velocity of modulation enevelope is called "Group velocity"

$$\boxed{V_g = \frac{\lambda_0}{\lambda_g} \cdot c}$$

* It is defined as rate at which the wave propogates in through the waveguide.

$$\boxed{V_g = \frac{d\omega}{d\beta}}$$

Expression for V_g:

$$V_g = \frac{d\omega}{d\beta}, \quad \beta = \sqrt{\mu\varepsilon(\omega^2 - \omega_c^2)}$$

Now differentiate β w.r.t ω, we get

$$\frac{d\beta}{d\omega} = \frac{1}{\cancel{2}\sqrt{(\omega^2-\omega_c^2)\mu\varepsilon}} \cdot \cancel{2}\omega\mu\varepsilon$$

$$\frac{d\beta}{d\omega} = \frac{\omega\mu\varepsilon}{\sqrt{\omega^2\left(1-\left(\frac{\omega_c}{\omega}\right)^2\right)}} \cdot \frac{1}{\sqrt{\mu\varepsilon}} = \frac{\omega\sqrt{\mu\varepsilon}\,\cancel{\sqrt{\mu\varepsilon}}}{\sqrt{\left(1-\left(\frac{\omega_c}{\omega}\right)^2\right)}} \cdot \frac{1}{\cancel{\sqrt{\mu\varepsilon}}}$$

$$\frac{d\beta}{d\omega} = \frac{\sqrt{\mu\varepsilon}}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}} = \frac{\sqrt{\mu\varepsilon}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$V_g = \frac{d\omega}{d\beta}$$

$$V_g = \frac{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}{\sqrt{\mu\varepsilon}} \qquad \left(\because c = \frac{1}{\sqrt{\mu\varepsilon}}\right)$$

$$\boxed{V_g = c\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \qquad V_g = c\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

Consider the product of V_p & V_g

$$V_p V_g = \frac{c}{\cancel{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}} \cdot c\cancel{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$\boxed{V_p V_g = c^2}$$

CUT-OFF WAVELENGTHS FOR DIFFERENT MODES:

- We know that $\lambda_c = \dfrac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$

TE Modes:

$$TE_{01} \Rightarrow \lambda_c = \frac{2}{\sqrt{0+\left(\frac{1}{b}\right)^2}} = \frac{2}{\sqrt{1/b^2}} = 2b$$

$$TE_{10} \Rightarrow \lambda_c = \frac{2}{\sqrt{\left(\frac{1}{a}\right)^2 + 0}} = \frac{2}{\sqrt{1/a^2}} = 2a$$

$$TE_{11} \Rightarrow \lambda_c = \frac{2}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}} = \frac{2}{\sqrt{\frac{b^2+a^2}{a^2b^2}}} = \frac{2ab}{\sqrt{a^2+b^2}}$$

TM modes

$$TM_{11} \Rightarrow \lambda_c = \frac{2ab}{\sqrt{a^2+b^2}}$$

$TM_{12} \Rightarrow \lambda_c = \dfrac{2ab}{\sqrt{4a^2+b^2}}$

$TM_{21} \Rightarrow \lambda_c = \dfrac{2ab}{\sqrt{a^2+4b^2}}$

DOMINENT MODE:

* The modes for both TE and TM which offers highest cut-off wavelength & lowest cut-off frequency (f_c) in a particular waveguide is called "Dominent mode".

For TE_{mn} modes TE_{10} mode is the Dominent mode

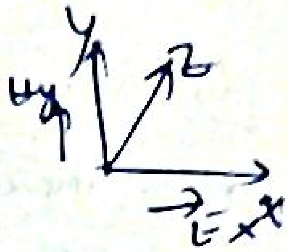
For TM_{mn} modes TM_{11} mode is the Dominent mode

DEGENERATE MODE:

* The higher order modes which are having the same cut-off frequency there are called Degenerate modes.
* For rectangular waveguides, TE_{mn}/TM_{mn} for which both $m \neq 0$ and $n \neq 0$ are always degenerate.
* Waveguide dimensions are therefore selected such that higher order modes are not supported in the operating band and thus only desired mode propogates through the waveguide.

Wave Impedance (or) Characteristic Impedance:

- The ratio of the strength of electric field in one traverse direction to the strength of magnetic field along the other traverse direction is called as Wave impedance.



$$Z_w = \frac{E_x}{H_y} = \frac{-E_y}{H_x}$$

For TM wave:

$$Z_w = Z_{TM} = \frac{E_x}{H_y}$$

$$= \frac{-\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}}{-\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}}$$

For TM wave $H_z = 0$ and for wave to propogate $\gamma = j\beta$

$$Z_{TM} = \frac{-\frac{(j\beta)}{\cancel{h^2}}\cancel{\frac{\partial E_z}{\partial x}} - 0}{0 - \frac{\cancel{j}\omega\varepsilon}{\cancel{h^2}}\cancel{\frac{\partial E_z}{\partial x}}}$$

$$Z_{TM} = \frac{\beta}{\omega\varepsilon}$$

$$Z_{TM} = \frac{\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}}{\omega\varepsilon} \qquad (\because \beta = \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$Z_{TM} = \frac{\sqrt{\omega^2\mu\varepsilon\left(1-\left(\frac{\omega_c}{\omega}\right)^2\right)}}{\sqrt{\omega^2\varepsilon^2}} = \frac{\cancel{\omega}\sqrt{\mu\varepsilon}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}{\cancel{\omega}\varepsilon}$$

$$Z_{TM} = \sqrt{\frac{\mu}{\varepsilon}} \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$$

$$Z_{TM} = \sqrt{\frac{\mu}{\varepsilon}} \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$$

$$Z_{TM} = \sqrt{\frac{\mu}{\varepsilon}} \sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}$$

Here $\sqrt{\frac{\mu}{\varepsilon}} = \eta \rightarrow$ Intrinsic Impedance

$$\boxed{Z_{TM} = \eta \cdot \sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

For TE mode:

$$Z_{TE} = \frac{E_x}{H_y} = \frac{\frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}}{\frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}}$$

For TE wave $E_z = 0$ and $\gamma = j\beta$

$$Z_{TE} = \frac{\frac{-j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}}{\frac{-j\beta}{h^2}\frac{\partial H_z}{\partial y}} = \frac{\omega\mu}{\beta}$$

$$= \frac{\omega\mu}{\sqrt{\mu\varepsilon(\omega^2 - \omega_c^2)}} = \frac{\omega\mu}{\sqrt{\mu\varepsilon\,\omega^2\left(1 - \left(\frac{\omega_c}{\omega}\right)^2\right)}}$$

$$= \frac{\sqrt{\mu/\varepsilon}}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}} = \frac{\eta}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}$$

$$Z_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$Z_{TE} \cdot Z_{TM} = \frac{\eta}{\cancel{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}} \cdot \eta \cdot \cancel{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$\boxed{Z_{TE} \cdot Z_{TM} = \eta^2}$$

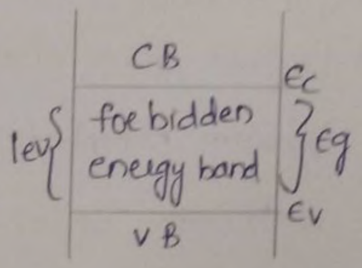
MWE ①

Microwave solid state Devices

Introduction:-

→ Semiconductors are a group of substance having electrical conductivit-ies that are intermediate between metals and insulators

→ The energy bands of a semiconductor play a major role in their electrical behaviour



ex: Si, Ge, GaAs, GaP, & InP, ZnO etc.

At 300K. Si - 1.1eV ; GaAs - 1.43eV

Ge - 0.8eV ; GaP - 2.26eV

→ To achieve two noise, high frequency, greater band width lesser switching time and other improvement in the performance characteris-tics we need solid state devices.

→ Such as bipolar and field effect transistors and 2 terminal devices such as (Gunn diodes, LSA diode) transferred electron devices, avalanche transit time devices (IMPATT, TRAPATT, BARITT, Parametric devices; tunnel diodes, varactors, quantum electronic devices such as MASERS, Semiconductor lasers

Classification:

Microwave solid state devices are becoming increasingly popular at microwave frequency. These are brodly classified into 4 groups

→ microwave transistor

* Microwave BJT
* Hetero junction Bipolar transistor (HBT)
* Tunnel diode.

→ field effect transistors

* junction FETS
* MOSFET

* MESFETS
* High electron mobility transistors (HEMTs)
* NMOS, PMOS, CMOS
* Memories
* Charge coupled devices

-> Transfered electron devices (TEDs)
* Gunn diode
* limited space charge Accumulation diode (LSA diode)
* Indium phosphide diode (In P)
* cadmium Telloride diode (CdTe)

-> Avalanche transit time devices:
* Read diode
* IMPATT diode (IMPact ionization Avalanche transit time devices)
* TRAPATT diode (Trapped plasma Avalanche triggered transit time diodes
* BARITT diode (Barrier Injected transit time devices)

Transfer electron Devices

The application of two terminal semiconductor devices at microwave frequencies has been increased usage during the past decades. The CW, average, and peak power outputs of these devices at higher microwave frequencies are much larger than those obtainable with the best power transistor. The common characteristic of all active two-terminal Solid-state devices is their negative resistance. The real part of their impedance is negative over a range of frequencies. In a positive resistance the current through the resistance and the voltage across it are in phase. The voltage drop across a positive resistance is positive and a power of (I^2R) is ~~dis~~ dissipated in the resistance.

In a negative resistance, however, the current and voltage are out of phase by 180. The voltage drop across a negative resistance is negative, and a power of $(-I^2R)$ is generated by the power supply associated with the negative resistance. In positive resistance absorb power whereas negative resistances generate power (active devices). In this chapter the transferred electron devices (TEDs) are analyzed. The differences between microwave transistors and transferred electron devices (TEDs) are fundamental. Transistors operate with either junctions or gates, but TEDs bulk devices having no junctions or gates. The majority of transistors are fabricated from elemental semiconductors such as silicon or germanium, whereas TEDs are fabricated from compound semiconductors such as gallium arsenide (GaAS) Indium phosphide (Inp) or Cadmium telluride (CdTe) Transistors operate As "warm" electrons whose energy is not much greater than the thermal energy 0.026eV at room temperature of electrons in the semiconductors

Applications:

low-noise local oscillators for mixers (2 to 140 GHz). low-power transmitters and wide band tunable sources.

Continuous-wave (CW) power levels up to several hundred mill watts can be obtained in the X- ku-, and ka-bands. A power output of 30mw can be achieved from commercially available devices at 94 GHz.

Higher power can be achieved by combining several devices in a power combiner

Gunn oscillators exhibit very low dc-to-RF efficiency of 1 to 4%.

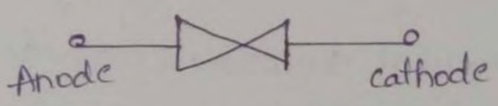
Gunn also discovered that the threshold electric field E_{th} varied with the length and type of material. He developed an elabrote capacitive

Probe for plotting the electric field distribution within a specimen of n-type GaAs of length $l = 210\,\mu m$ and cross-sectional area $3.5 \times 10^{-3}\,cm^2$ with a low field resistance of 16Ω.

current instabilities occured at specimen voltage above 59v, which means that the threshold field is

$$E_{th} = \frac{V}{L} = \frac{59}{210 \times 10^{-6} \times 10^{2}} = 2810 \text{ volts/cm}$$

Gunn diode:-

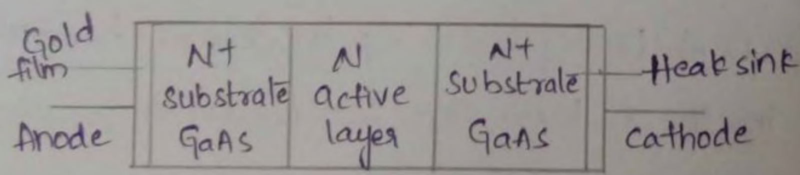


symbol of Gunn diode.

Gunn diode is also known as transferred electron device is a form of diode with negative resistance used in high frequency electronic. It is based on Gunn effect. It is made up of GaAs semiconductor Discovered in 1962 by J.B Gunn.

It is based on N-type material. In these electrons are the majority charge carriers.

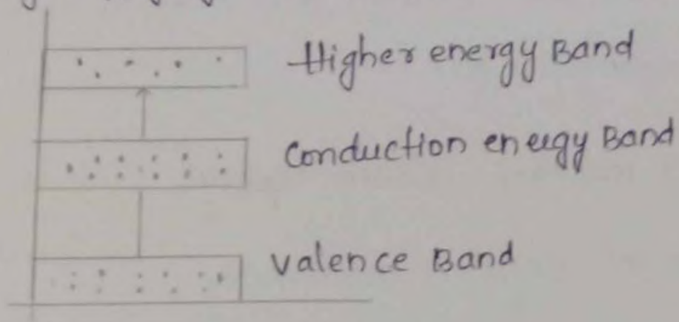
Construction:-



In Gunn diode there are three layer device in which a lightly doped N-type semiconductor is placed between two highly doped N-type material.

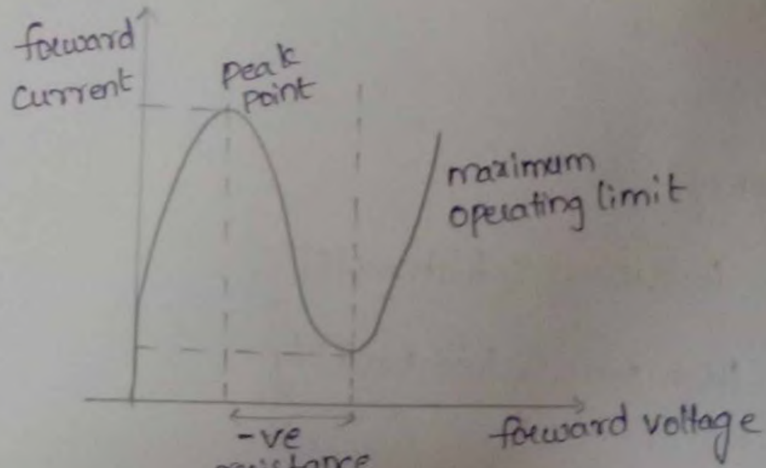
Working:

In material like GaAs electron present low mass as well as high mass so the electron present in low mass state are fo forced to high mass state by applying external potential



Band structure Consists three energy levels on applying external potential to the gunn diode the electron present in the valence band moves to conduction band as we increase external potential then electron present in Conduction Band moves to higher energy state. due to these transferring of electron these device also known as transferred electron device. The higher energy state the electrons with less mobilized so with increasing potential at paraticular point of time current through it start decreasing this will cause negative resistance inside the device. after certain Voltage electron at higher energy state gain has sufficient energy & electrons comes back to conduction band there by increase current by increasing voltage.

Characteristics:-

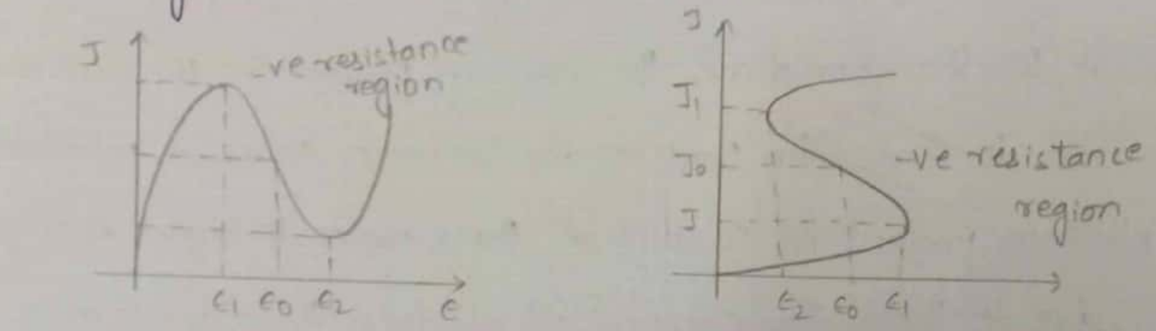


Initially with increasing voltage current through the device increases after the peak point the current devices start decrease these show the negative resistance characteristic perform by the device after reaching valley point current through device again start increasing upto its maximum operating limit.

RWH Theory (Ridley-watkins-Hilsum theory)

There are two modes of negative resistance

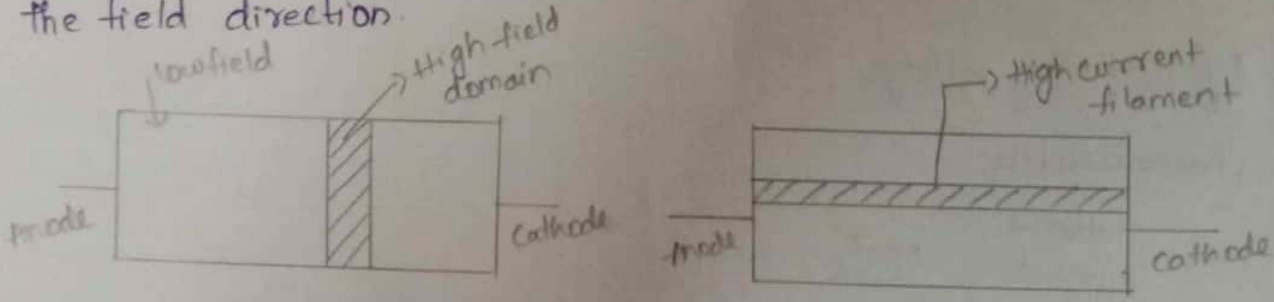
(a) voltage controlled mode (b) current controlled mode.



In voltage controlled mode the current density can be multivalued current controlled mode voltage can be multivalued.

In voltage controlled mode, high field domains are formed seperating two low field regions.

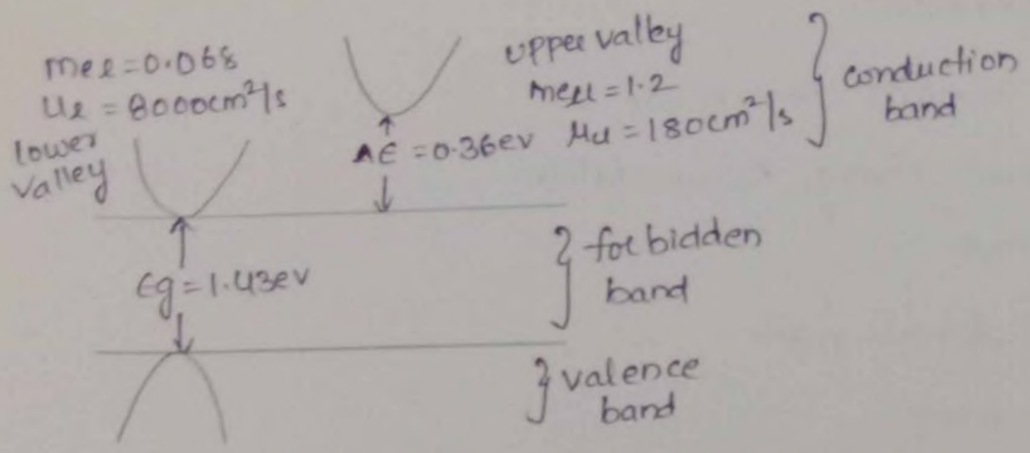
In current controlled mode, high current filaments running along the field direction.



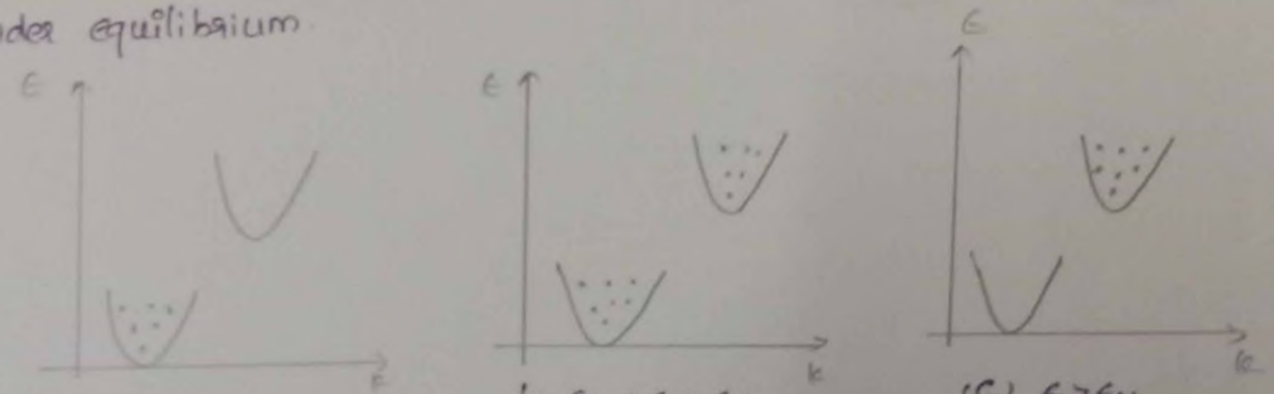
Two valley model theory:-

Lower Valley :- low mass & high mobility

Upper valley : High mass & low mobility.



Electron densities in the low valley & upper Valley remain the same under equilibrium.



(a) $E < E_l$ (b) $E_l < E < E_u$ (c) $E > E_u$

When the applied electric field is lower than electric field of the lower valley ($E < E_l$) the electron will transfer to the upper valley as shown in (a).

When the applied electric field is $E_l < E < E_u$ electron will begin to transfer to the upper valley as shown in (b)

When the applied electric field is higher than that & upper valley $E > E_u$ all electrons will transfer to the upper valley as shown in (c)

If electrons density in the lower valley and upper valley were n_l & n_u respectively. The conductivity of n-type GaAs is

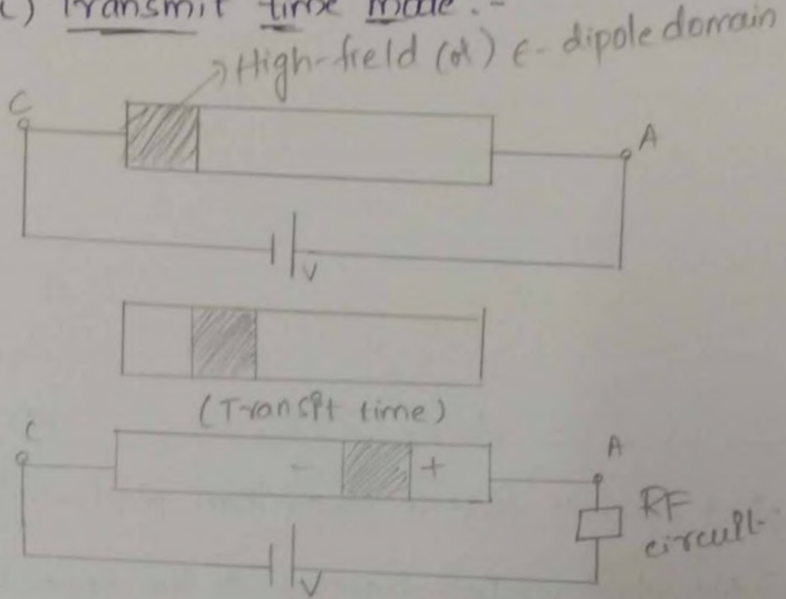
$$\boxed{\sigma = e(\mu_l n_l + \mu_u n_u)}$$

e = electron charge

Modes of operation:-

1. Gunn (or) Transmit time mode
2. Limited space charge Accumulation (LSA) mode.
3. Quenched domain mode
4. delayed mode.

1. Gunn (or) Transmit time mode:-



* When the voltage applied across the diode exceeds a thresold voltage, electrons are transferred from low energy to high energy band

* The time taken by the high field domain from the cathode to an anode is the transist time of the device.

* The movement of ~~dip~~ dipole domains results in a pulse of current at the output.

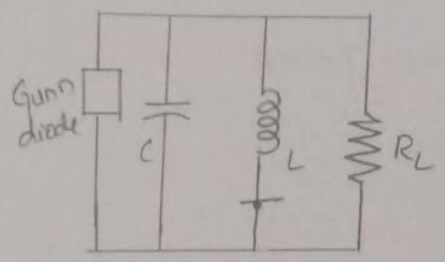
* These current fluctuations occur at microwave frequencies.

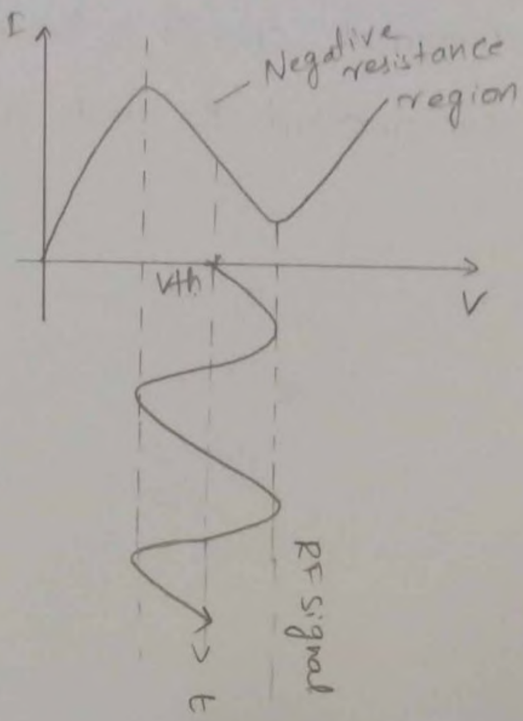
Drawback:

→ low frequency

→ frequency cannot be controlled by the external circuit.

2. Limited - Space charge Accumulation (LSA) mode:-





-> In this mode, the gunn diode works as a part of a resonant circuit.

-> The resonant circuit is ~~based~~ tuned to a frequency several time greater than TT

-> LSA mode can produce several watts of power with 20% efficiency

-> 1W at 10GHz
1mW at 100GHz.

-> The circuit operates as a negative resistance oscillator when the dc voltage is adjusted ~~to~~ a value greater than the Vth is nearly at the mid point.

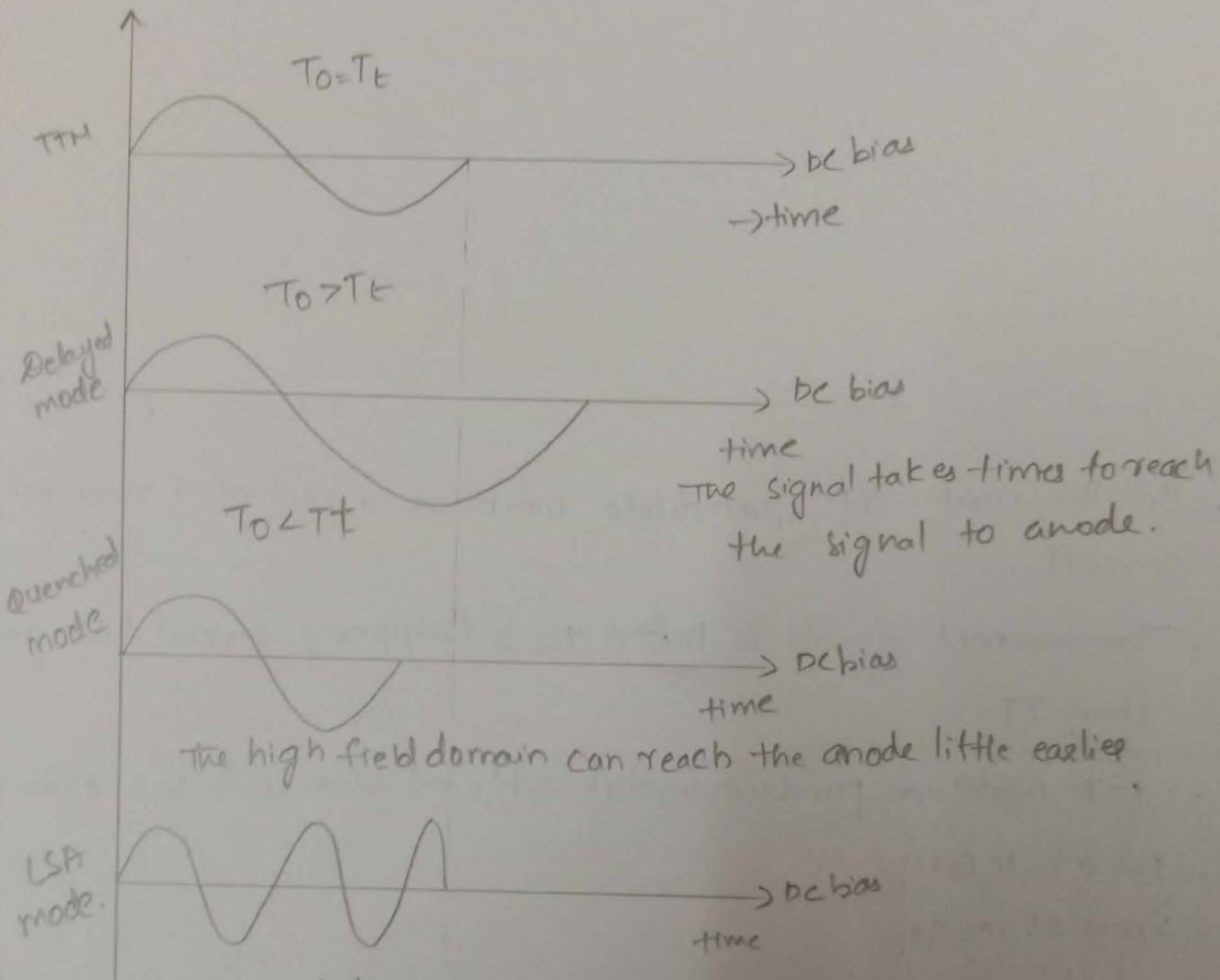
-> The peak-to-peak amplitude is equal to the voltage range in the negative ~~reg.~~ resistance region.

3. Quenched domain mode:- (>TT)

If the resonant circuit is tuned to a value greater that TT mode the dipole domain will be quenched before arriving the anode.

Delayed mode :- (< TT)

The resonant circuit is tuned to a value lesser then the TT mode that can reached the anode after a time period



Several time greater than the gun mode.

Advantages
- High frequency Stability
- Compact in size

Disadvantages:-
- less efficient (<10GHz)
- poor temperature stability

Application:-
- low microwave power oscillator
- Used as sensors in microwave system
- microwave delay data link

Parametric amplifier:

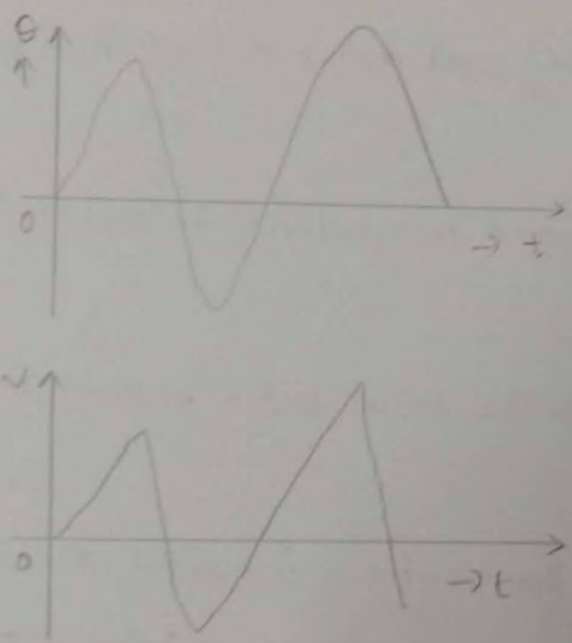
* A parametric amplifiers is one that uses a non-linear reactance (capacitive) or inductive (or) a time varying rectance for it's amplification (rather than resistance as in normal amplifier).

* In fact, parametric device basically depend on the possibility of increasing the energy of the signal at one frequency by supply energy at some other frequency.

* Consider this simple tank circuit with separate the plate of capacitor.

* Assume that prior to the time $t=0$, the circuit has been energized so that voltage 'v' and charge on capacitance are varying sinusoidally

$$V = \frac{Q}{C} \quad \text{Where } C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$



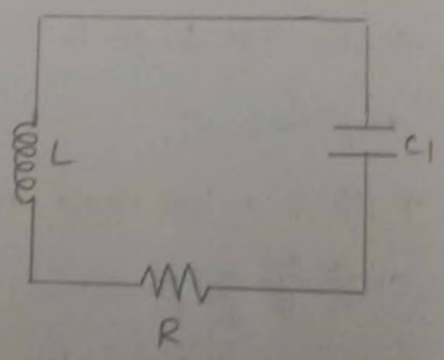


fig: Tank circuit.

* To obtain amplification, capacitor plates are pulled a part when the charge and the voltage are at their maximum.

* Because of the electric field between the plates it requires an expenditure of energy (mechanical energy) to pull the plates a part.

* This mechanical energy appears as additional electric energy stored in capacitance and itself increase in the voltage.
* The voltage and charge continue the oscillations towards zero.
* At zero voltage, the capacitance plates are brought back to their original separation and this requires no expenditure of energy as the electric field is also zero now.
* The voltage and charge now swing to their wave maximum at which plates are pulled a-part once again and the process can be continued at each maximum and minimum of voltage and hence a signal builds up.
* for each time the plates are pulled apart, energy is added to the signal.

Note:-

* varactor diode is the most widely used active element in a parametric amplifier.
* It is a low noise amplifier because no resistance is involved in the amplifying process.
* There will be no thermal noise as the active device involved is reactive (capacitive).
* Amplification is obtained if the reactance is varied at some frequency. higher than the frequency of the signal being amplified

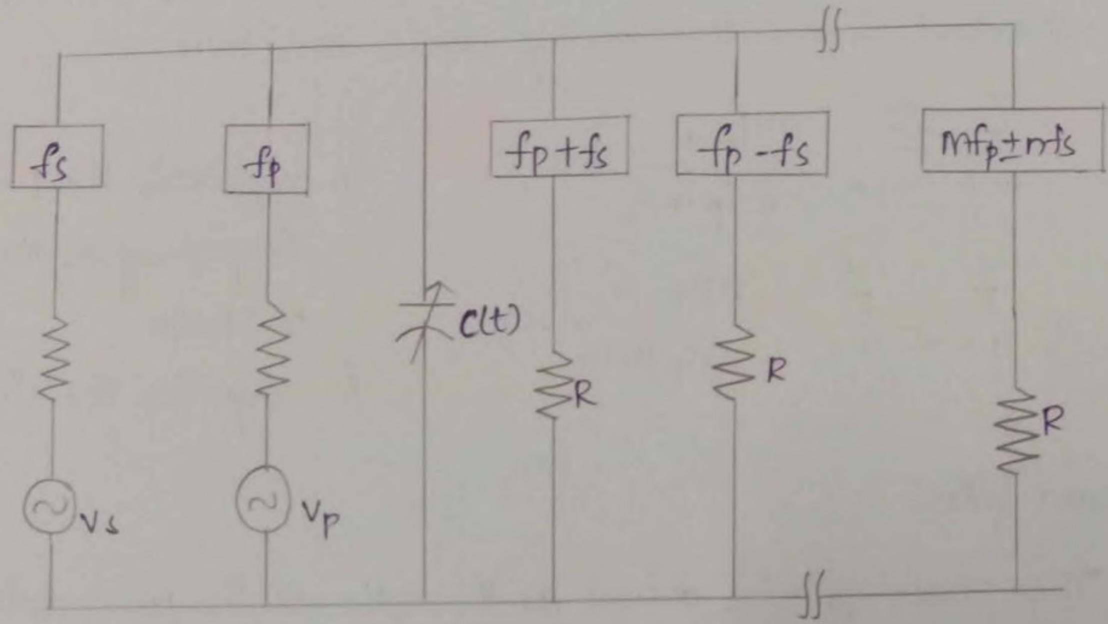
Manley - Rowe Relations

* Manley and Rowe have derived a set of general equations relating to power flowing into and out of an ideal non-linear reactance.
* These relations are powerfull tool.

→ In predicting whether power gain is possible in a paramplifier.

→ In understanding the principles of varactor applications.

→ In determining the maximum gain conversion efficiency and other performance parameters.



* signal generator and pump generator with frequencies fs and fp, series resistance and bandpass filters are applied to a non-linear capacitance c (t).
* These resonating circuits of filters are useful in rejecting power at all frequencies.
* The two frequencies fs and fp generate an infinite number of resonant frequencies given by mfp±nfs are generated.
* The manley - Rowe relations for any single valued, non-linear loss less reactance are given by two independent equations.

$$\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{mP_{m,n}}{m\omega_p+n\omega_s} = 0$$

$$\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{nP_{m,n}}{m\omega_p+n\omega_s}$$

Where m & n are integers varying from 0 to ∞

$P_{m,n}$ is Avg power flowing into the non-linear reactance at frequencies $\pm (mf_p + nf_s)$

W_p and W_s are respec replaced by f_p and f_s respectively giving the standard forms for manley - Rowe relations.

$$\sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} \frac{mP_{m,n}}{mf_p + nf_s} = 0$$

$$\sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} \frac{nP_{m,n}}{mf_p + nf_s} = 0$$

f_p - represents the pump frequency of the pumping oscillator

f_s - signifies the frequency

Power Gain:-

The power gain is defined as the ratio of the power delivered by capacitor at a frequency of $f_p + f_s$ to that absorbed by the capacitor at a frequency of f_s.

$$\text{Power gain} = \frac{f_p + f_s}{f_s} = \frac{f_0}{f_s} \quad \text{(for modulator)}$$

Where $f_p + f_s = f_0$ $\qquad$ $f_0 > f_p > f_s$

If signal frequency is the sum of pump frequency and the output frequency then power gain is

$$\text{Power gain} = \frac{f_s}{f_p + f_s} \quad \text{[for demodulator]}$$

Where $f_s = f_p + f_0$

$f_0 = f_s - f_p$

Amplification mechanism of a parametric amplifier:-

* In parametric amplifier the pump generator act as local oscillator and varactor diode c(t) as mixer.

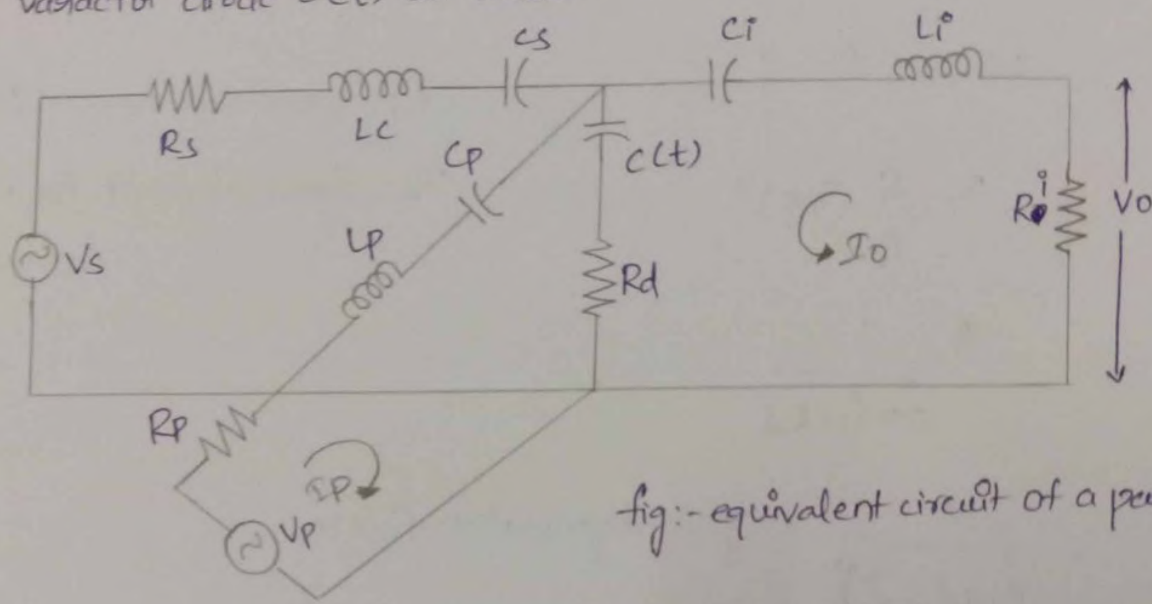


fig:- equivalent circuit of a par-amp.

* The signal frequency f_s and pump frequency f_p are mixed in non-linear capacitor c(t) to generate voltages sum and difference frequencies $mf_p \pm nf_s$ across c(t).
* The output circuit which does not require external excitation is called Idler circuit.
* The output frequencies f_0 is given by

$f_0 = mf_p + nf_s$ m and n are positive integers from 0 to ∞.

If $f_0 > f_s$, the device is called parametric up converter

If $f_0 < f_s$ the device is called parametric down converter

Parametric up converter (puc)

In a puc, the output frequency is equals to sum of f_s & f_p there is no power flow in the parametric at frequency other than the signal, pump and output frequencies.

Power Gain:-

$$\text{Max power Gain} = \frac{f_0}{f_s} = \frac{x}{[1+\sqrt{1+x}]^2}$$

Where $f_0 = f_p + f_s$.

R_d = series resistance of P-n junction

γ_Q = figure of merit for non-linear Capacitance

$$x = \frac{f_s}{f_0}(\gamma Q)^2$$

$$Q = \frac{1}{2\pi f_s C R_d}$$

$$\frac{x}{[1+\sqrt{1+x}]^2} = \text{gain degradation factor}$$

* $R_d = 0$, $\gamma Q = \infty$ and gain degration factor is unity
* In typical microwave diode $\gamma Q = 10$, if $f_0/f_s = 15$, the max gain works out to 7.3 dB.

Noise figure (F)

T_d = diode Temp in K

T_0 = ambient temp (300K)

$$F = 1 + \frac{2T_d}{T_0}\left[\frac{1}{\gamma Q} + \frac{1}{(\gamma Q)^2}\right]$$

In typical microwave diode $\gamma Q = 10$, $f_0/f_s = 10$ and $T_d = 300k$

The min noise figure is 0.90 dB

Band width (BW)

$$BW = 2\gamma\sqrt{\frac{f_0}{f_s}}$$

for typical microwave diode $f_0/f_s = 10$ and $\gamma = 0.2$ and $BW = 1.264$.

Parametric down converter (PDC)

For PDC, input power must feed into the idler circuit and the output power must move out from the signal circuit

$$\text{Gain} = \frac{f_s}{f_0} = \frac{x}{(1+\sqrt{1+x})^2}$$

Power gain is actually loss

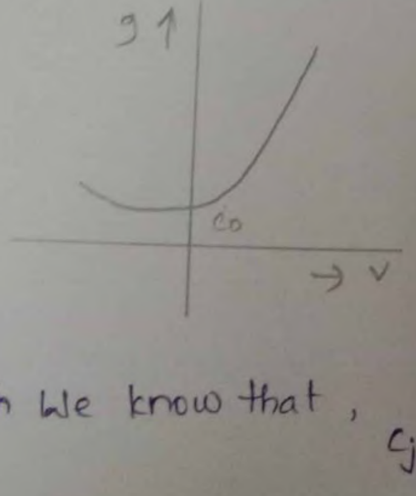
Varactor diode :-

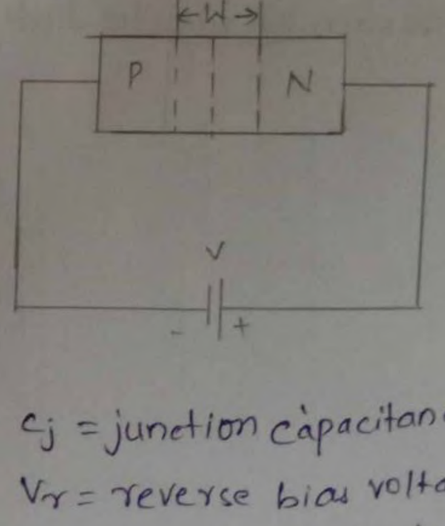
→ The term varactor is a shortened form of variable reactor refering to the voltage variable capacitance of a reverse biased junction. They have non-linearity of capacitance which is fast enough to follow microwaves.

→ Varactor diode is a semiconductor devices in which the junction capacitance can be varied as a function of reverse voltage of the diode.

→ Losses in this non linear element will be a almost negligible.

The junction capacitance depends on the applied voltage and junction design.





We know that, $C_j \propto V_r^{-n}$

C_j = junction capacitance

V_r = reverse bias voltage

n = a parameter that decides the type of junction

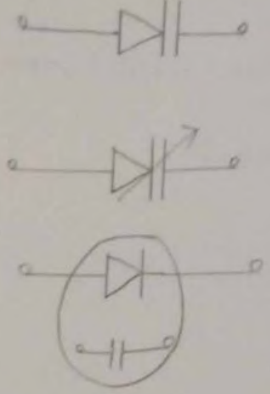


fig:- Symbols

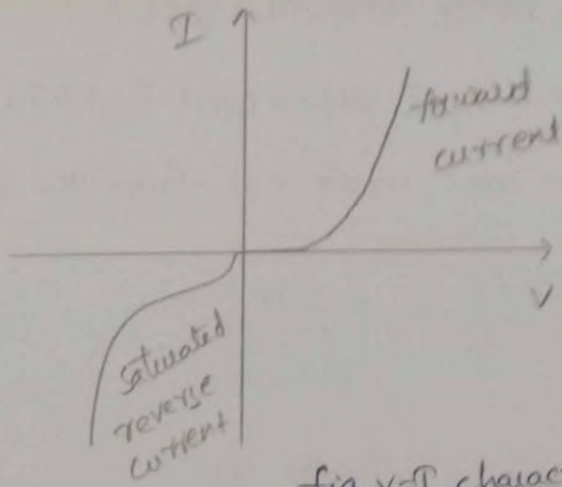
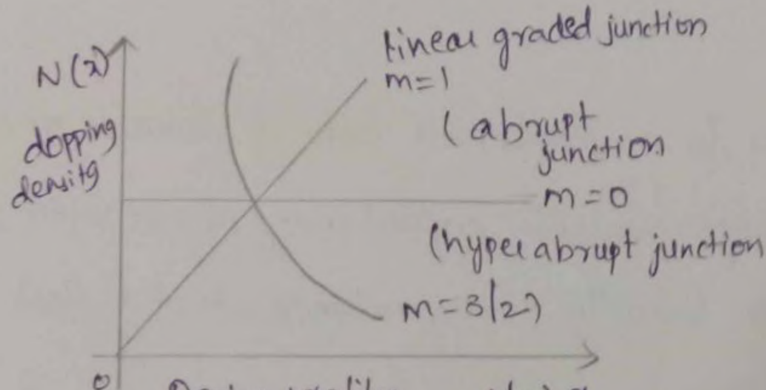


fig V-I characteristics

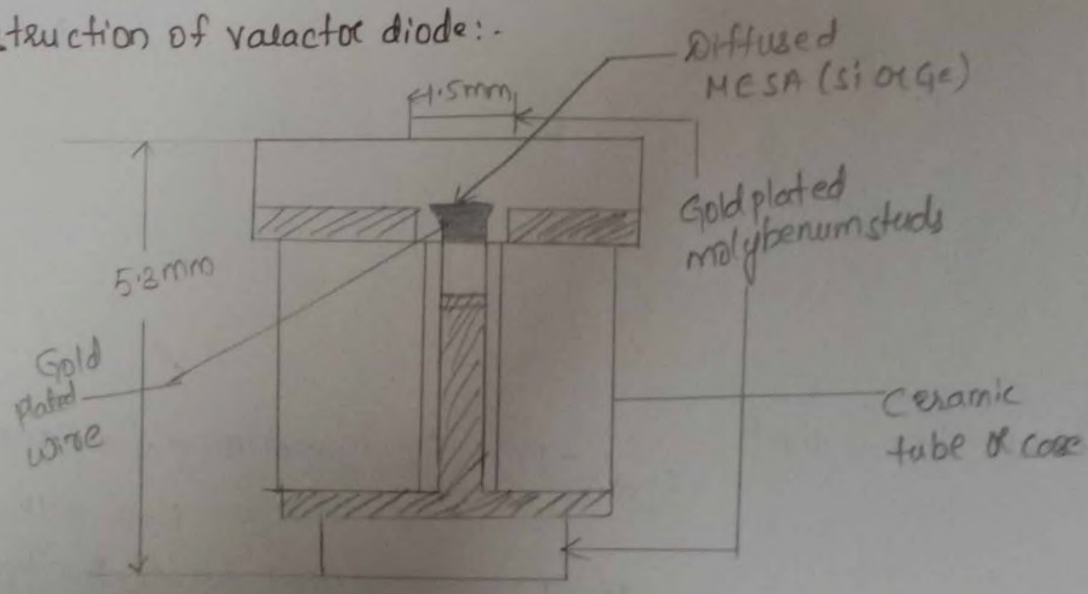


Doping profiles

If the P-n junction is abrupt, the capacitance varies a square root of the reverse bias 'V_r' i.e $n = 1/2$. If the P-n junction is a linear graded one, n is $1/3$ i.e, voltage sensitivity of C_j is greater for an abrupt junction than for a linear graded junction.

For a hyper abrupt junction $n > 1/2$; In general, $c_j \propto V_r^{-1/m+2}$

Construction of varactor diode:-



→ The diode encapsulation contains electrical leads attached to the semiconductor wafer and a lead to the ceramic case.

→ Diffused junction MESA Si diodes are widely used at microwave frequencies

→ They are capable of handling larger powers and large reverse break down voltages and have low noise

→ frequency limit of Si diodes is upto 25GHz. varactors made of GaAs have high operating frequency (over 90 GHz) and better functioning at the lowest temperature.

Equivalent circuit:-

C_j → junction capacitance

R_j → junction resistance

R_s → series resistances (including resistance of the wafer & the resistance of the ohmic electrical leads)

C_c = capacitance of ceramic case

C_f = fringe capacitance

L_s = lead inductance

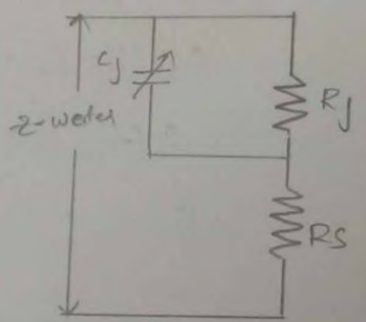


fig:- electrical equivalent circuit for varactor diode.

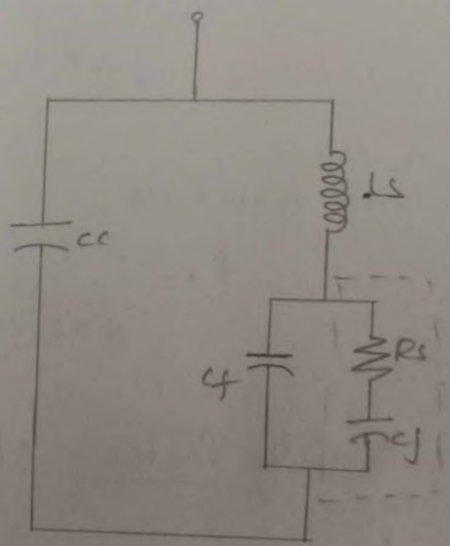


fig:- find equivalent circuit of varactor diode.

At microwave frequence R_j is of the order of 10 MΩ and may be neglected compared to capacitive reactance.

- Although variation in junction capacitance is the most important characteristic of a varactor diode there are parasitic resistance, capacitances and conductances associated with every practical associated diode.

The diode encapsulation contains electrical leads attached to the wafer and low loss ceramic cases as a mechanism support to the wafer. The parasities should be kept as low as possible. for many applications there should be a large capacitance variation and small value of minimum capacitance and series resistance.

figure of merit of varactor :-

Static figure of merit :-

→ Cut-off frequency:- At specific reverse bias v is given by

$$f_{cv} = \frac{1}{2\pi R_s C_{jv}}$$

C_{jv} = junction capacitance at voltage

R_s = series resistance of diode

f_{cv} for Si = 250 GHz ; GaAs = 900 GHz

at zero bias

$$f_{co} = \frac{1}{2\pi R_s C_{jo}}$$

$f_s(\text{pract}) = f_c/10$

for Si f_c = 25 GHz ; GaAs ; GaAs f_c = 90 GHz

Quality factor :- At specific voltage v and frequency f is defined by

$$Q_v = \frac{f_{cv}}{f}$$

Q = quality factor at a bias voltage 'v'

f_{cv} = cut-off freq at a bias voltage 'v'

f = any frequency of interest at which Q_v is ~~increase~~ measured

Dynamic figure of merit :-

Cut-off freq :- It is the cutoff freq at which the device is operated b/w bias extremes and is given by

$$f_c = \left(\frac{1}{C_{jmin}} - \frac{1}{C_{jo}}\right)\frac{1}{2\pi R_s}$$

C_{jmin} = capacitance of device near the reverse breakdown voltage.

C_{jo} = junction capacitance correspondin to zero bias.

Dynamic quality factor :-

$$Q = \frac{S_1}{\omega R_s}$$

$S_1 = 1^{st}$ fourier component of the time depend

$\omega = 2\pi f$

Where $0.17 < \gamma < 0.25$ fa most varactor junction

Also $Q = \gamma \frac{f_{cv}}{f}$

$\gamma = 0.17$ for graded junction

$= 0.25$ fa step junction

$$S_1 = \frac{1}{C_1} = \frac{\gamma}{C_{jv}}$$

Note that when the varactor is under dynamic condition ie when the junction capacitance varies because of the applied voltage and frequency $f = \omega/2\pi$, the capacitance value varies as the instantaneous value of the signal and hence it is taken as the time dependent non-linear capacitance.

Applications:-

- Harmonic generation
- microwave freq multiplication
- lownoise amplification
- pulse generation and pulse shaping
- Tuning stage of a radio receiver
- Active filters
- switching circuits and modulation of a microwave signal

Introduction to Avalanche transit time devices:

It is possible to make a microwave diode exhibit negative resistance by having a delay between voltage and current in an avalanche together with transit time through the material.
Such devices are called Avalanche transit time devices.
» They use carrier impact ionization and drift in the high field region of a semiconductor junction to produce negative resistance at microwave frequencies.
There are three distinct modes of avalanche oscillators.
1. IMPATT: Impact Ionization Avalanche transit time device
2. TRAPATT: Trapped Plasma Avalanche transit time device.
3. BARITT: Barrier Injected Transit time device

IMPATT Diode:

* This is a high-power semiconductor diode, used in high frequency microwave applications. The full form IMPATT is IMPact ionization Avalanche Transit time diode.
* A voltage gradient when applied to the IMPATT diode, results in a high current. A normal diode will eventually breakdown by this. However, IMPATT diode is developed to withstand all this. A high potential gradient is applied to back bias the diode and hence minority carriers flow across the junction.
* Application of a RF AC voltage if superimposed on a high DC voltage, the increased velocity of holes and electrons results in additional holes and electrons by thrashing them out of the crystal structure by Impact ionization.
* If the original DC field applied was at the threshold of developing this situation, then it leads to the avalanche current multiplication and this process continues. This can be understood by the following figure

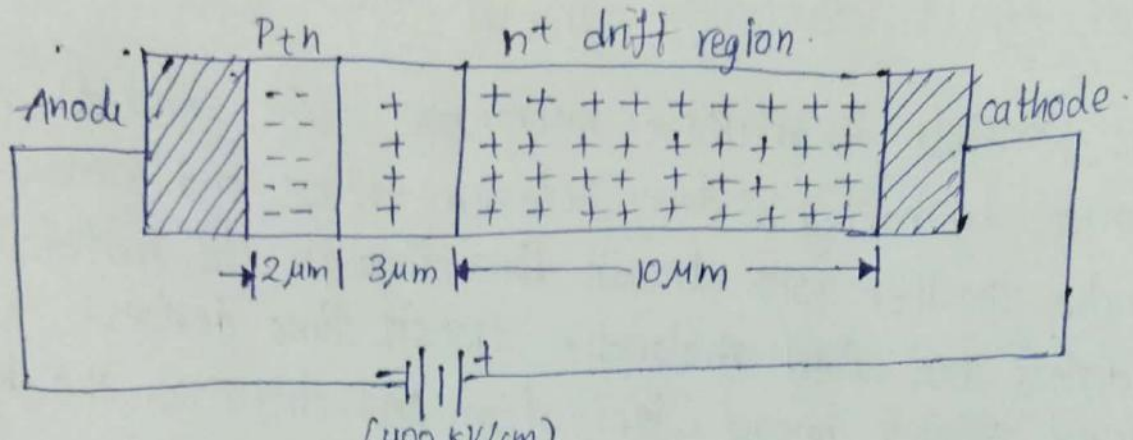


fig: IMPATT diode Schematic

Due to this effect, the current pulse takes a phase shift of 90°. However, instead of being there, it moves towards cathode due to the reverse bias applied. The time (tim) taken for the pulse to reach cathode depends upon the thickness of n+ layer, which is adjusted to make it 90° phase shift. Now, a dynamic RF negative resistance is proved to exist. Hence, IMPATT diode acts both as an oscillator and an amplifier.

The following figure shows the constructional details of an IMPATT diode.

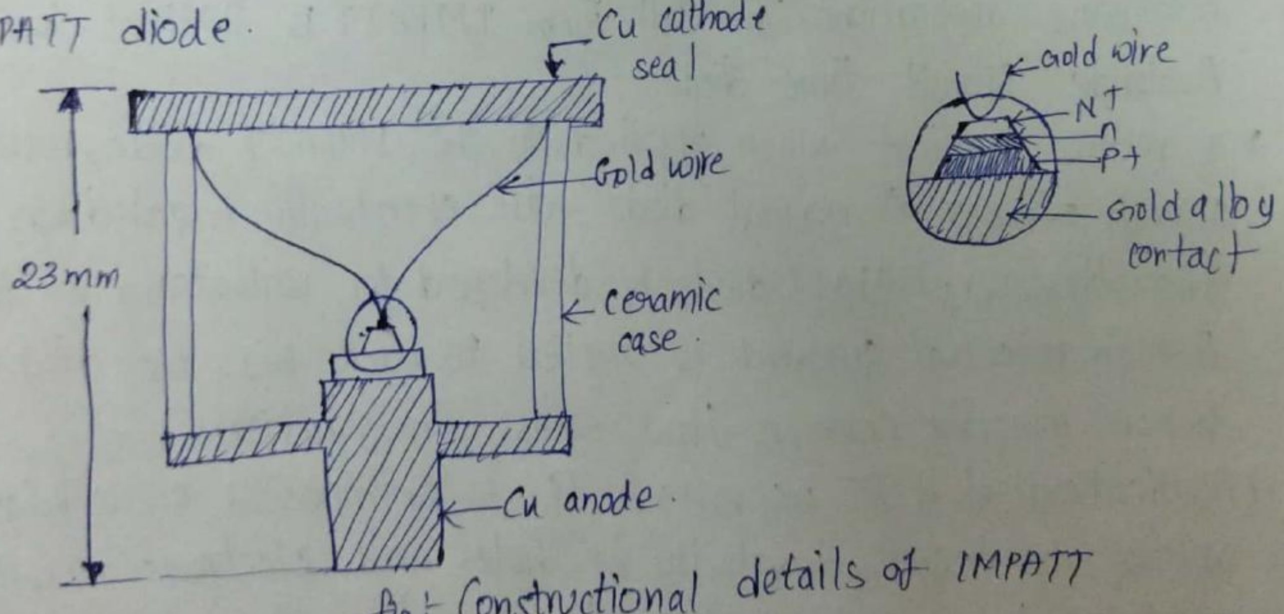


fig: Constructional details of IMPATT

The efficiency of IMPATT diode is represented as

$$\eta = \left[\frac{P_{ac}}{P_{dc}}\right] = \frac{V_a}{V_d}\left[\frac{I_a}{I_d}\right]$$

where, P_{ac} = AC power ; V_a & I_a = AC voltage & current

P_{dc} = DC power ; V_d & I_d = DC voltage & current

Disadvantages:-

» It is noisy as avalanche is a noisy process

» Tuning range is not as good as in Gunn diodes.

Applications:-

» Microwave oscillator
» Microwave generators
» Modulated output oscillator
» Receiver local oscillator
» Negative Resistance amplifications.
» Intrusion alarm networks high Q IMPATT
» Police radar high Q IMPATT
» Low power microwave Tx high Q IMPATT
» CW Dopple radar Tx low Q IMPATT

TRAPATT Diode

» The full form of TRAPATT diode is TRApped Plasma Avalanche Triggered Transit diode. A microwave generator which operates between hundreds of MHz to GHz. These are high peak power diodes usually n+- p-p+ or p+-n-n+ structures with n-type depletion region, width varying from 2.5 to 1.25 Aμm.

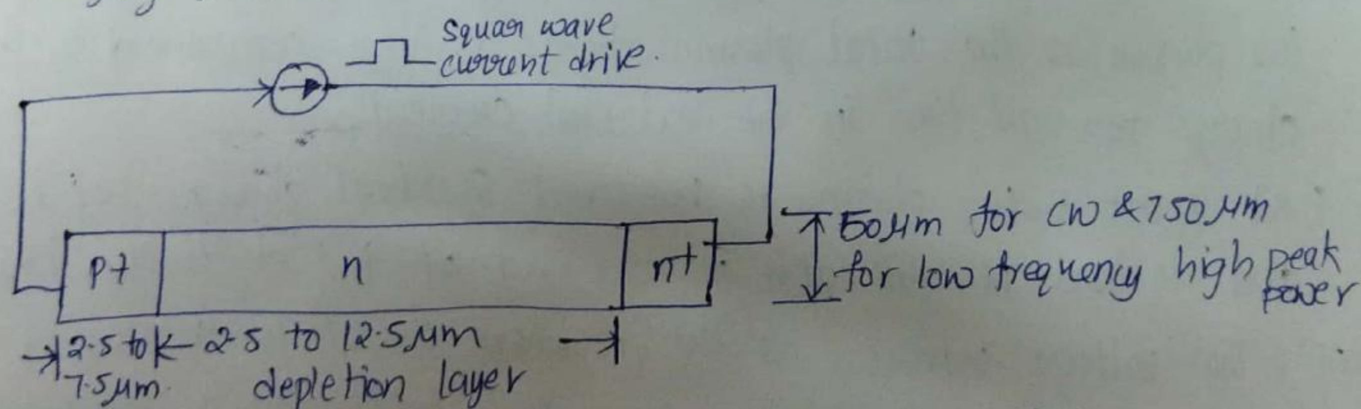
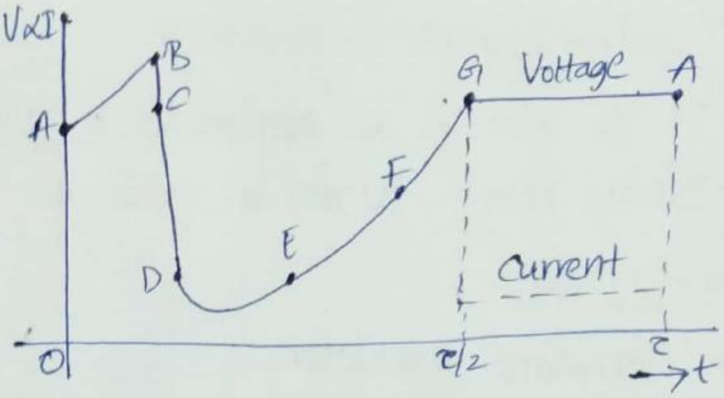


fig:- Arrangement in TRAPATT diode.

» The electrons and holes trapped in low field region behind the zone, are made to fill the depletion region in the diode. This is done by a high field avalanche region which propagates through the diode.

» The following figure shows a graph in which AB shows charging, BC shows plasma formation, DE shows plasma extraction, EF shows residual extraction, and FG shows charging

Let us see what happens at each of the points.



A:- The voltage at point A is not suffient for the avalanche break-down to occur. At A, charge carriers due to thermal generation results in charging of the diode like a linear capacitance.

A-B :- At this point, the magnitude of the electric field increases. When a sufficient no. of carriers (~~due to thermal gene~~) are generated, the electric field is depressed throughout the depletion region causing the voltage to decrease from B to C.

C :- This charge helps the avalanche to continue and a dense plasma of electrons and holes is created. The field is further depressed so as not to let the electrons or holes out of the depletion layer, and traps the remaining plasma.

D :- The voltage decreases at point D. A long time is required to clear the plasma as the total plasma charge is large compared to the charge per unit time in the external current.

E :- At point E, the plasma is removed. Residual charges of holes and electrons remain each at one end of the deflection layer

E to F :- The voltage increases as the residual charge is removed

F :- At point F, all the charge generated internally is removed

F to G : The diode charges like a capacitor

G : At point G, the diode current comes to zero for half a period. The Voltage remains constant as shown in the graph above. This state continues until the current comes back on and the cycle repeats.

The avalanche zone velocity V_s is represented as

$$V_s = \frac{dx}{dt} = \frac{J}{qN_A}$$

where, J = current density

q = electron charge 1.6×10^{-19}

N_A = Doping concentration.

The avalanche zone will quickly sweep across most of the diode and the transit time of the carriers is represented as

$$\tau_s = \frac{L}{V_s}$$

where, V_s = Saturated carrier drift velocity

L = length of the specimen.

The transit time calculated here is the time between the injection and the collection. The repeated action increases the output to make it an amplifier, whereas a microwave low pass filter connected in shunt with the circuit can make it work as an oscillator.

Applications:-

>> Low power Doppler radars

>> Local oscillator for radars

>> Microwave beacon landing system

>> Radio altimeter

>> Phased array radar, etc.

BARITT Diode

* The full form of BARITT Diode is BARrier Injection Transit Time diode. These are the latest invention in this family.
* Though these diodes have long drift regions like IMPATT diodes the carrier injection in BARITT diodes is caused by forward biased junctions, but not from the plasma of an avalanche region as in them.

* In IMPATT diodes, the carrier injection is quite noisy due to the impact ionization. In BARITT diodes, to avoid noise, carrier injection is provided by punch through of the depletion region.
* The negative resistance in a BARITT diode is obtained on account of the drift of the injected holes to the collector end of the diode, made of p-type material.

The following figure shows the constructional details of a BARITT diode.

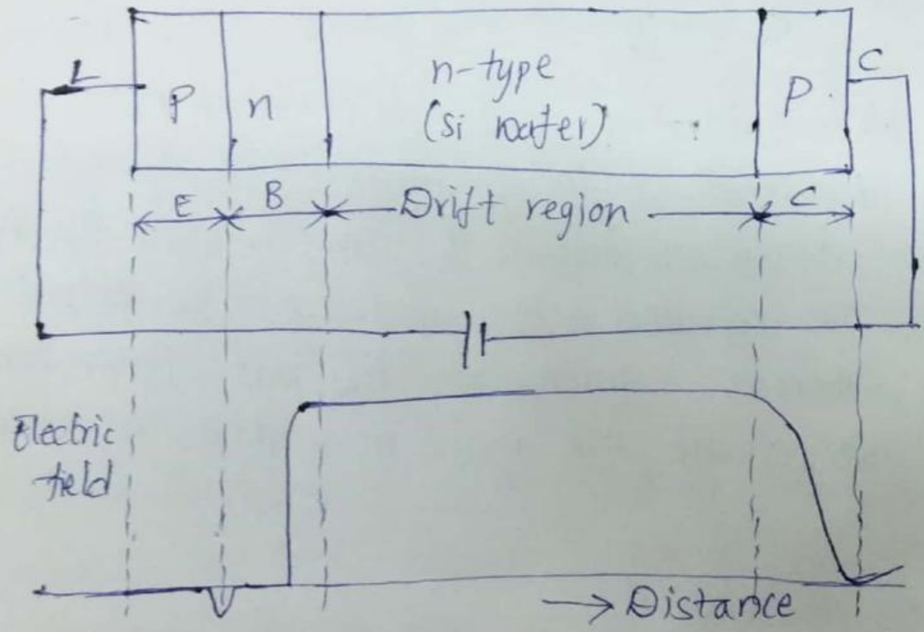


fig:- Constroction of BARITT diode.

For a m-n-m BARITT diode, Ps-Si Schottky barrier contacts metals with n-type Si wafer in between. A rapid increase in current with applied voltage above 30v is due to the thermionic hole injection into the semiconductor.

The critical voltage (V_c) depends on the doping constant (N), length of the semiconductor (L) and the semiconductor dielectric permittivity (ϵS) represented as

Voltage breakdown; breakdown Electric field

$$V_c = \frac{qNL^2}{2\epsilon S} \; ; \; V_{bd} = 2V_c = \frac{qNL^2}{\epsilon_s} \; ; E_{bd} = \frac{V_{bd}}{L} = \frac{qNL}{\epsilon_s}$$

* BARITTs are primarily used for amplifiers rather than oscillators because of lower efficiences.

COUPLING MECHANISMS:

* Coupling probes and loops are common techniques for coupling microwave signal to the waveguide.

PROBES:-

* When a small probe is inserted into a waveguide and supplied with microwave energy. It will radiate and if it is placed correctly, the wanted mode will set up.
* The probe is placed at a distance of $\lambda_g/4$ from the shorted end of the waveguide and at the center of wider dimension of the waveguide because at that point electric field is maximum.
* This probe will now act as an antenna which is polarized in the plane parallel to that of electric field.

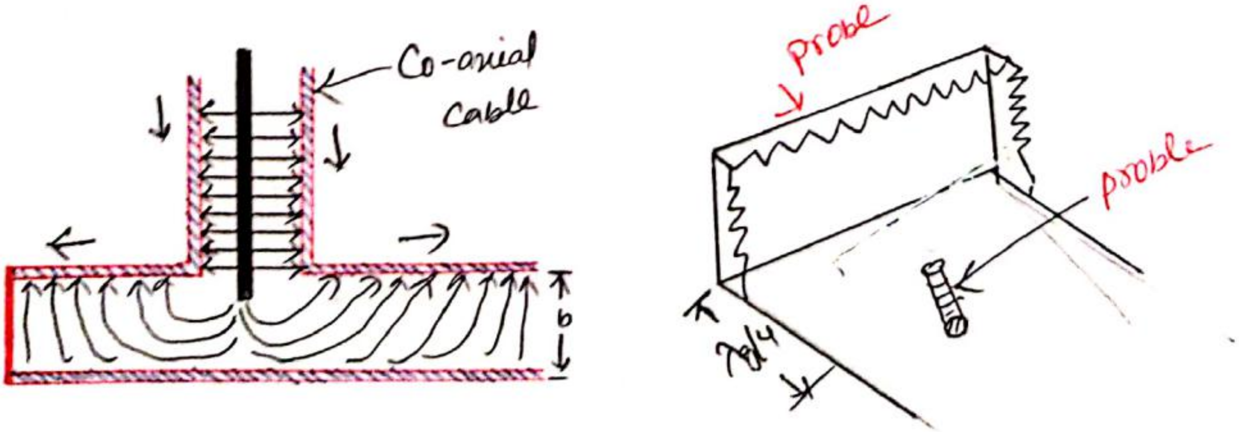


FIG: COUPLING PROBE

Loops:

* The Coupling loop placed at the centre of shorted end plate of the waveguide i.e., Coupling is achieved by means of a loop antenna located in a plane perpendicular to the plane of probe.

* The loop can be mounted in the middle of top (or) bottom wall at a distance $\left(\frac{n\lambda}{2}\right)$ where n is an integer.

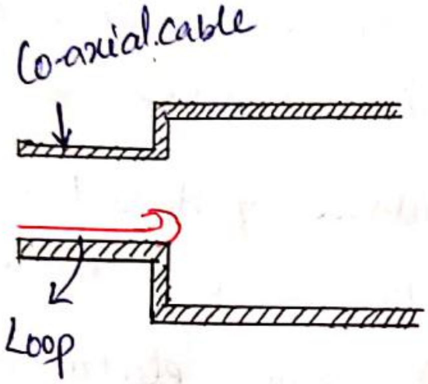


FIG: COUPLING LOOP

* The loop is placed where the magnetic field is maximum.

WAVE GUIDE DISCONTINUITIES:

WAVEGUIDE WINDOWS:

* In any waveguide system, when there is a mismatch there will be reflections
* Any susceptance appearing across the guide, causing mismatch needs to be cancelled by introducing another susceptance of the same magnitude but of opposite nature.
* There are three types of windows are available.

  1. Inductive window
  2. Capacitive window
  3. Resonant window

1. INDUCTIVE WINDOW:

* An inductive window allows a current to flow where none flowed before.
* This window is placed in a position where the magnetic field is strong.
* Since the plane of polarization of electric field is parallel to the plane of window, the current flow due to window causes a magnetic field to be set up.

2. CAPACITIVE WINDOW:

* In capacitive window, the potential which existed between the top and bottom walls of the waveguide now exists between surfaces which are closer.

* Therefore, the capacitance effect is increased at that point of waveguide.

* The capacitive window is placed where electric field is strong

3. RESONANT WINDOW:

* The inductive and capacitive windows if combined suitably the inductive and capacitive reactances introduced will be equal and the window becomes parallel resonant window.
* For the dominant mode, the window presents a high impedance and the shunting effect for this mode will be negligible.
* other modes are completely attenuated and the resonant window acts as a Bandpass filter.

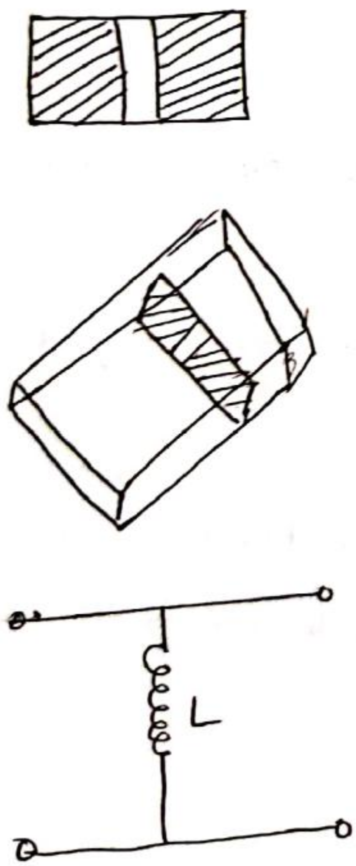


FIG:- Inductive window

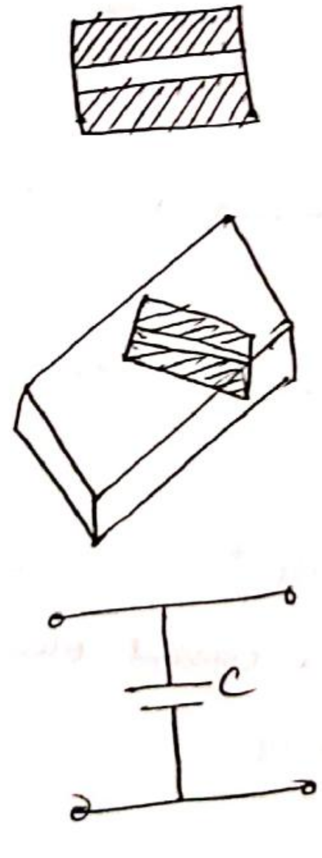


FIG:- Capacitive window

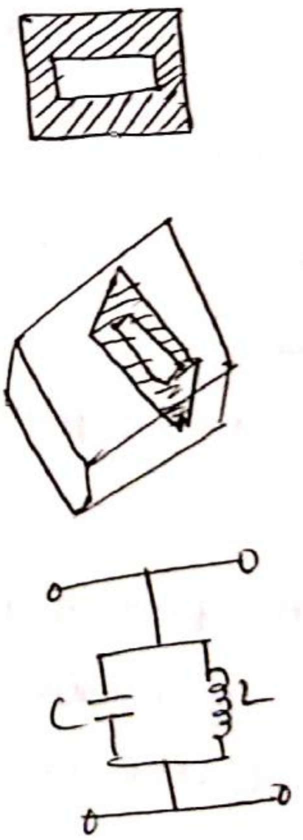


FIG: Parallel Resonant window

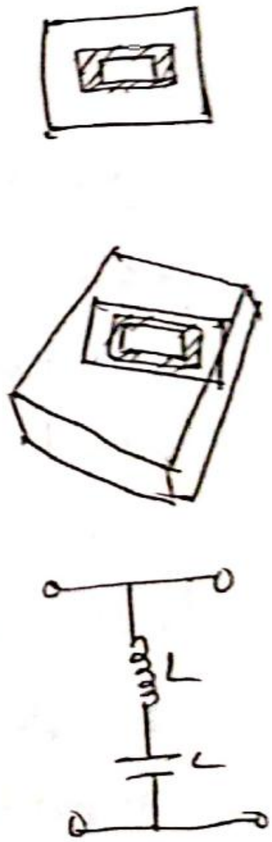


FIG: Series Resonant window.

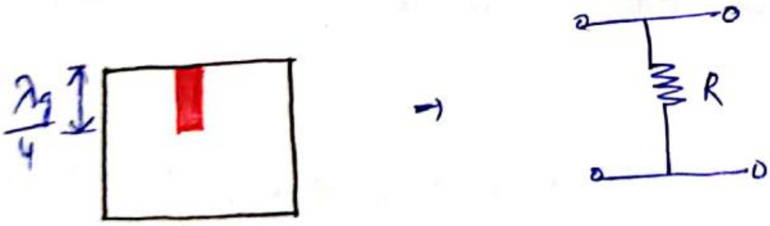
Posts & Tuning Screws:-

Posts:-

* When a metallic cylindrical post is introduced into the wider side of the waveguide, it ~~indro~~ introduces same effect as an Iris ie providing lumped reactance at that point

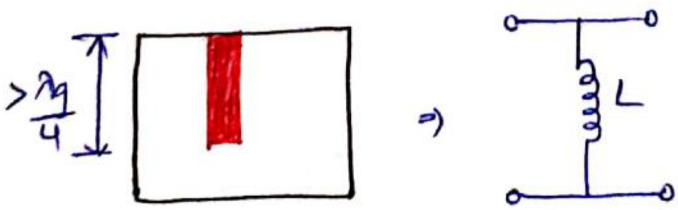
* If the post extends less than $\lambda g/4$ into the waveguide it behaves capacitively and this susceptance increases with depth of penetration.

* If depth of post is equal to $\lambda g/4$, it acts as a series resonant circuit.

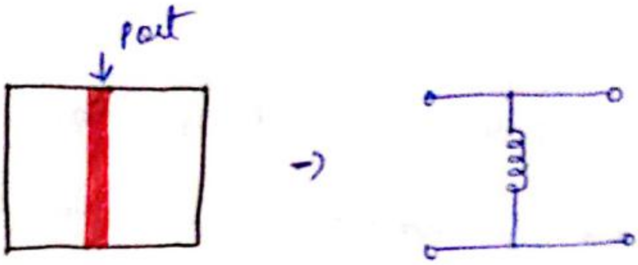


* Since the inductive and capacitive reactance gets cancelled out and ~~so~~ post behaves like resistive.

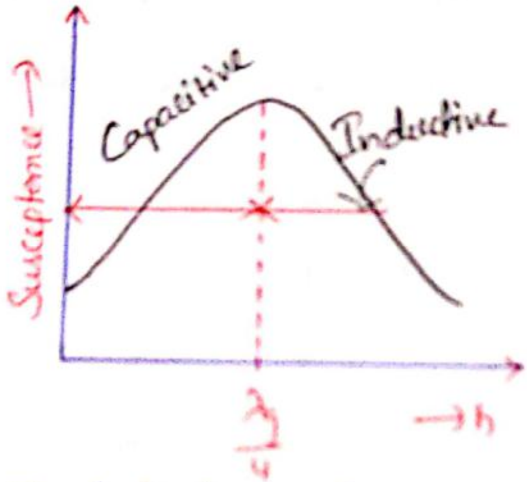
* If depth of post is greater than $\lambda g/4$, it behaves inductively and this inductive susceptance decreases ~~as~~ as depth of penetration increase.



* When the post is Completely extended, the post becomes Inductive.



* The Susceptance Vs penetration (h) Characteristic is shown below.



* The amount of susceptance decreases as the diameter of post reduces.

* When the post extends from zero to $\lambda_g/4$ the Capacitive susceptances increases

* When the post extends greater than $\lambda_g/4$ the post behaves inductive and its susceptance decreases.

* If the post is made thicker, effective Q will be lowered and act as a bandpass filter similar to an iris.

TUNING SCREWS:

* An adjustable post is known as a screw or slug. As in case of posts depending upon the depth of penetration, the tuning screw may introduce inductive or capacitive susceptance.

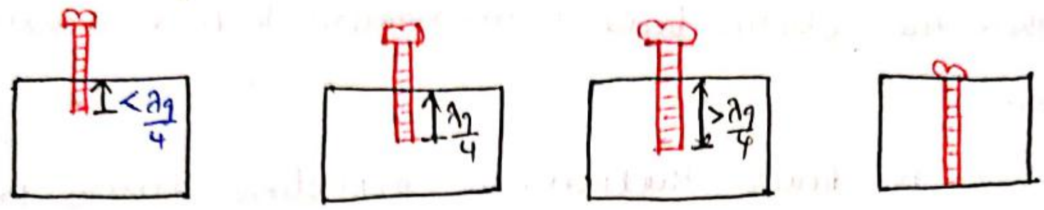
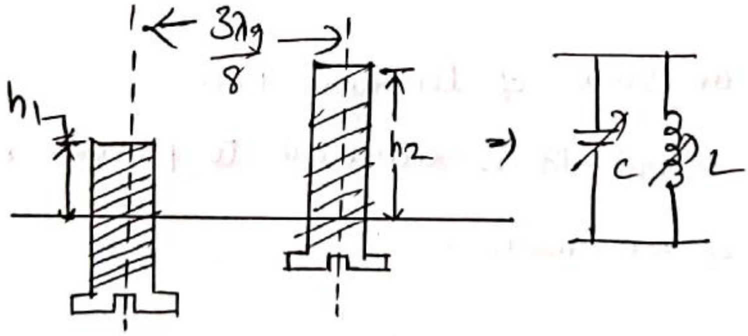


FIG: TUNING SCREWS

* A combination of two screws $\frac{\lambda_g}{4}$ apart can be used to match a waveguide to its load similar to use of two fixed stubs in a transmission line.
* A very effective waveguide matcher can be realized when two tuning screws are placed in close proximity separated by $3\lambda_g/8$.



MATCHED LOADS:

* Matched load is a device which absorbs the incident power completely with no reflections. It is a one port device.
* The impedance of matched load is equal to characteristic impedance of transmission line. It can be constructed by mounting a absorbing card in the space near the closed end of a waveguide section

as shown in figure. If a card consists of powdered iron & carbon mixed with a binder and deposited on a dielectric strip.

* The reflections arising from the end are minimised by tapered the card.

* The card is placed parallel to the dominant TE_{10} mode at a place where the electric field is maximum to have maximum attenuation.

* As the card has finite thickness, the reflections arising from it cannot be ruled out.

* To avoid this pad is kept closer to the side walls and its length is increased.

* Tunable matched termination as shown in figure.

Features:

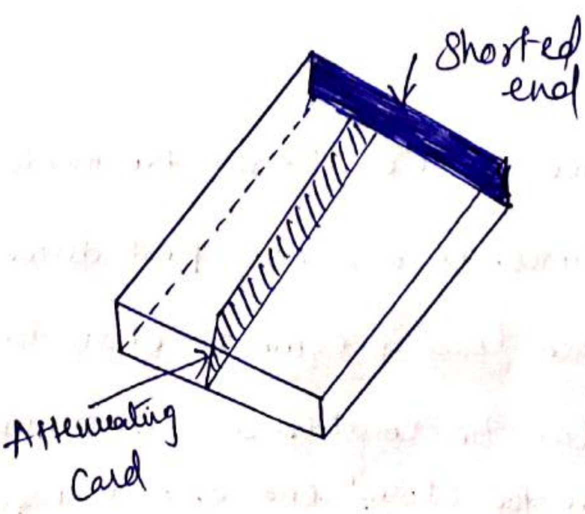
* It is equivalent to terminating the line in its characteristic impedance.

* Reflections are eliminated by tapering the lossy material into a wedge.

* It provides an SWR of less than 1.01.

* A length of about 1λ is sufficient to provide a matched load.

* It is a type of termination.



WAVEGUIDE ATTENUATORS:-

* The passive elements used to Control the amount of microwave power transferred from one point to another on a microwave transmission line is called Attenuators.
* It may reflects the energy or absorbs the energy in some dissipative elements.
* They are fixed or variable types of attenuators are there.

RESISTIVE CARD ATTENUATOR:

FIXED RESISTIVE CARD:

* In fixed resistive card attenuator, the resistive card is bounded to the waveguide.
* The card is tappered at both ends in order to maintain a low i/p and o/p SWR over useful microwave band.

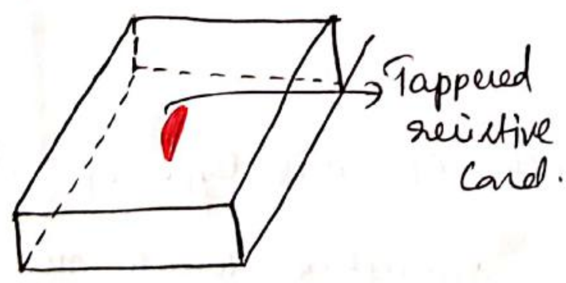


FIG: FIXED RESISTIVE CARD.

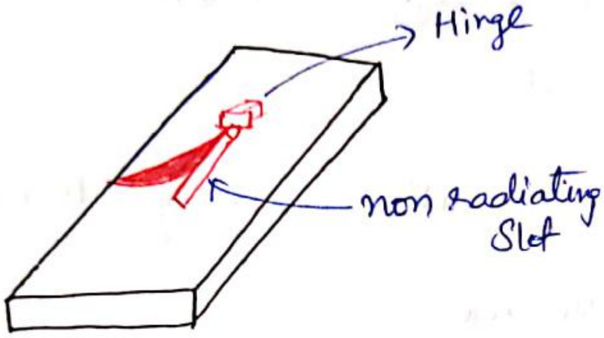
* The maximum attenuation is achieved by having the card parallel to electric field and at the center of waveguide where the electric field is maximum.
* The Conductivity and size of the card are adjusted by trail and error in order to obtain desired attenuation value.

Scanned with CamScanner

* In order to absorb high power, Ceramic type absorbing materials are used instead of resistive Cards.

VARIABLE RESISTIVE CARD:

* The variable type of resistive Card attenuator is also known as "Flap attenuator".



* The Card enters the waveguide through the non-radiating slot in wider wall, and there by intersecting the and absorbing a portion of TE_{10} mode wave.
* The hinge arrangement allows, the Card penetration and hence attenuation in the range of 0 to 3 dB.
* None of the TE_{10} wave is radiated through a slot since it is non-radiating.
* The main drawback of this type of attenuator is, the attenuator is frequency sensitive which makes it inconvinient to use as a Calibrated attenuator.

ROTARY VANE ATTENUATOR :

* A rotary vane attenuator provides precision attenuation with an energy of ±2.1% of indicated attenuation over the operating frequency range.

STRUCTURE :

* This attenuator consists of 3 vanes in which two are fixed and one is variable rotary waveguide vane with resistive cards in it.
* It also includes input and output transitions from rectangular to circular and circular to rectangular cross sections.
* The two fixed circular waveguide sections are identical in all aspects each attached to a transition (i/p & o/p) and each consists of a circular waveguide with a resistive card lying horizontally in it.
* In middle exists a rotatable circular waveguide section with a resistive card which can be placed at any angle by rotating the waveguide section.

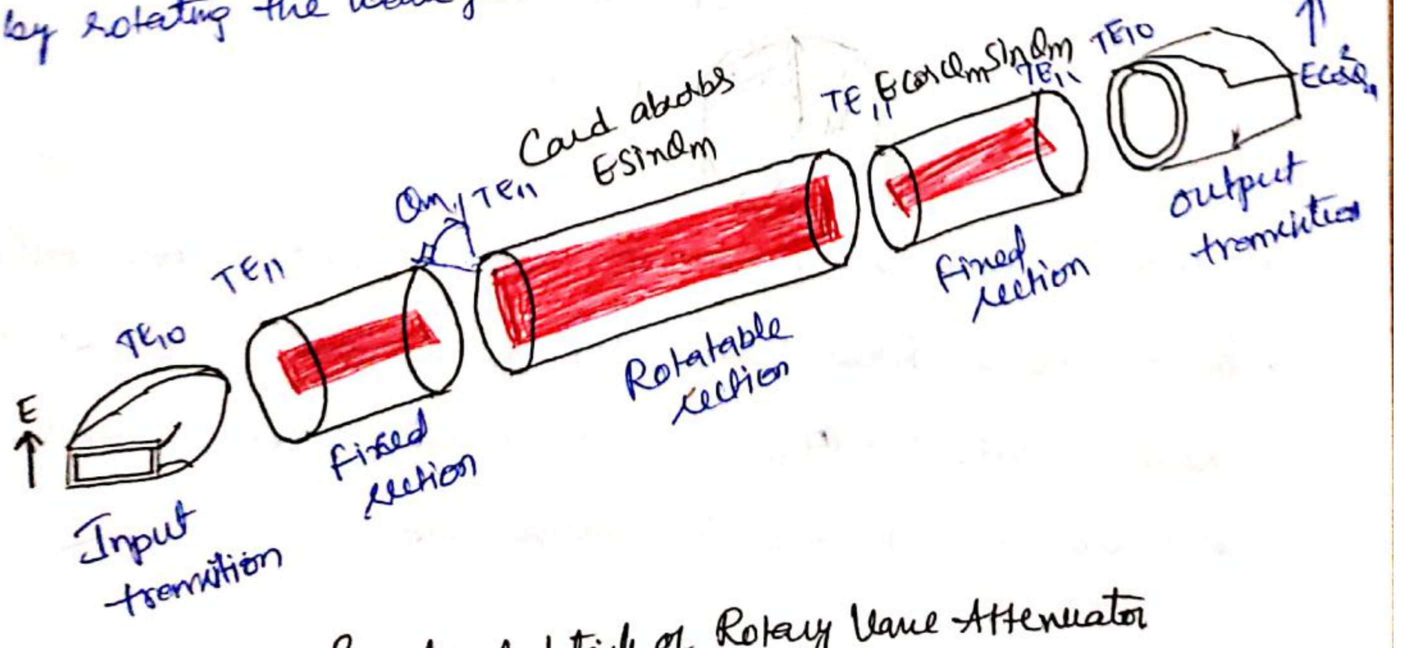
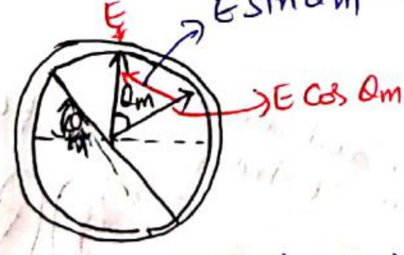


FIG: Structural details of Rotary Vane Attenuator

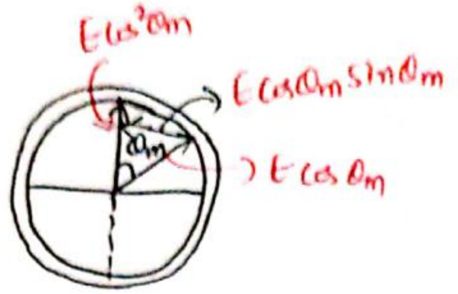
WORKING:

* The I/p transition converts the TE_{10} wave into vertically polarized TE_{11} wave in a circular waveguide
* With the i/p resistive card perpendicular to the electric field the wave propogates in first fixed section without any loss.

* When the card in the rotatable section in horizontal ($\theta_m = 0$), the wave passes through it and O/p section without loss, thus for $\theta_m = 0$, total loss is 0 dB.
* For any other angle, the component of electric field parallel to rotatable card is absorbed and perpendicular component arrives.
* The electric field here is resolved into two components $E\cos\theta_M$ (perpendicular) and $E\sin\theta_M$ (parallel).



* $E\cos\theta_m$ component is arrived with its vertical polarization
* At the second fixed section, the electric field component $E\cos\theta_m$ is resolved into $E\cos^2\theta_m$ is resolved into $E\cos^2\theta_m$ perpendicular and $E\cos\theta_m \sin\theta_m$ parallel to resistive card.



* Since $E\cos\theta_m \sin\theta_m$ is parallel to card so it is absorbed and $E\cos^2\theta_m$ passes through the card.

* Since the power flow is proportional to the square of electric field, the fraction of incident power delivered to a matched load is "$\cos^4\theta_m$".

* Therefore, the attenuation in 'dB' of Rotary vane attenuator is

$$\boxed{A = 10 \log_{10}\left(\frac{1}{\cos^4\theta_m}\right)}$$

* The fraction of incident power absorbed at 3 resistive cards are 0, $E\sin^2\theta_m$, $E\cos^2\theta_m \sin^2\theta_m$ respectively.

* The attenuation of this rotary vane attenuator is greater than 80 dB.

* In this attenuator, the phase of the o/p signal is independent of attenuation arrangement.

WAVE GUIDE PHASE SHIFTERS :-

* A Waveguide phase shifter is a two port device which produces a fixed or a variable phase change of the incoming microwave signal.
* The phase shift provided by a Waveguide of length 'l' is given by,

$$\beta_l = \frac{2\pi}{\lambda_g} * l$$

* Since phase Constant β_l is inversely proportional to guide wavelength. The phase shift Can be variable by the changing the magnitude of λ_g.
* The guide wavelength λ_g Can be varied either by 'ε_r' or by reducing wider dimension (a) of a rectangular waveguide

DIELECTRIC PHASE SHIFTER:

* Dielectric phase shifter Consists of a dielectric slab specially shaped to minimize reflection effects.
* The insertion of dielectric slab into waveguide at a point where electric field is maximum, which increases the effective dielectric constant along the wider dimension of a waveguide.

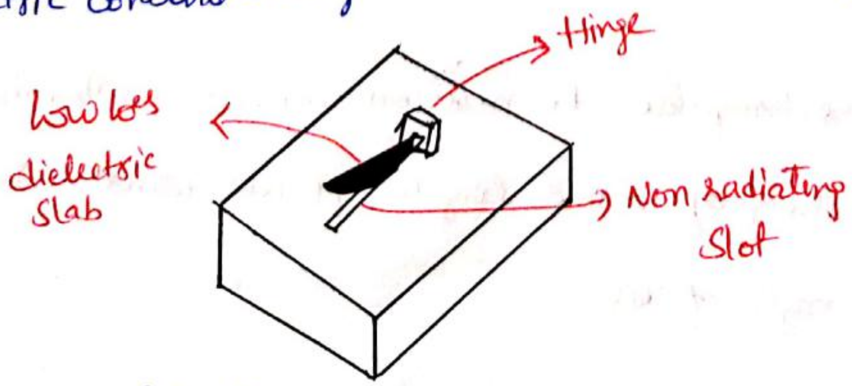


FIG: DIELECTRIC PHASE SHIFTER.

* This causes 'λ_g' to decreases which increases phase shift through the fixed length of waveguide section.
* The tappered section of dielectric slab minimizes the reflections.
* If the dielectric slab is inserted deeper, there is more change in the medium and there is a greater phase shift.
* The amount of phase shift is maximum when the slab is at the center and minimum when it is adjacent to the wall of the waveguide.
* If the dielectric slab is placed such that the slab inside dimension is parallel to the direction of electric flux lines.

2. ROTARY VANE PHASE SHIFTERS:

* The rotary phase shifter consists of a circular waveguide containing a lossless dielectric slabs of length $2l$ called "Half wave section.
* A section of rectangular to circular transistion containing a lossless dielectric slab of length 'l' called quarter wave section oriented at an angle of $45°$ to the wider dimension of a waveguide.
* A circular to rectangular transition again containing a lossless dielectric slab of same length (quarter wave section) oriented at an angle of $45°$.

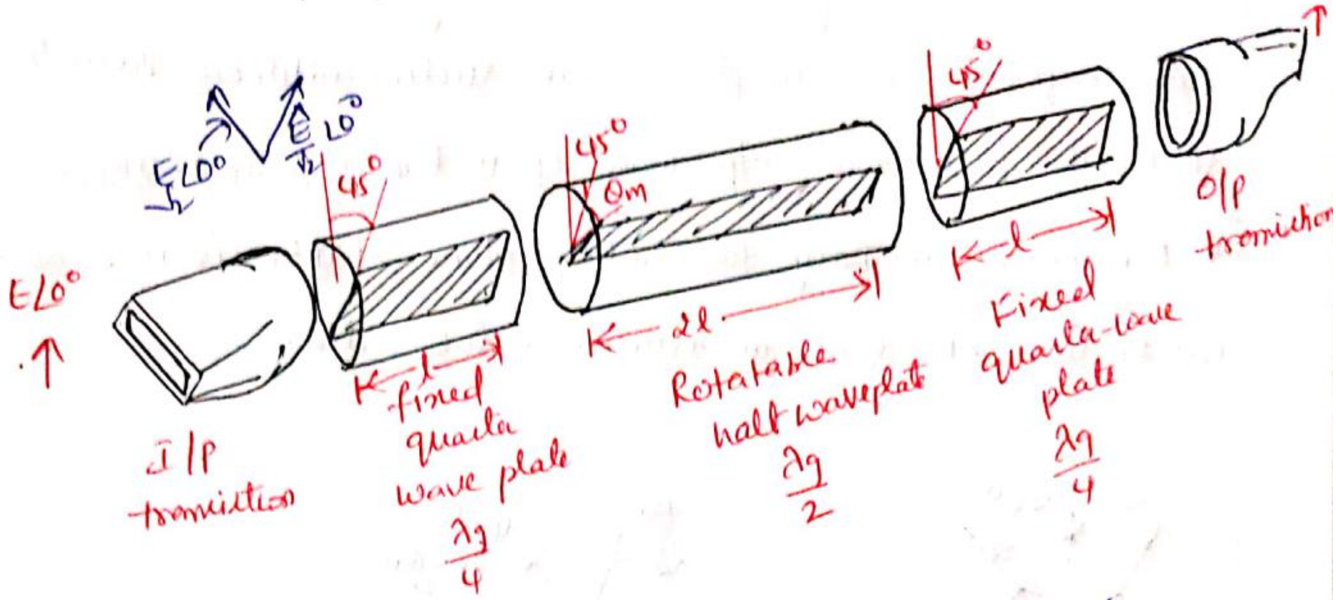


FIG: ROTARY VANE PHASESHIFTER.

Operation:

* The vector phasor $E\angle 0^\circ$ represents the vertically polarised input wave. It is resolved into two Components, parallel and perpendicular to the dielectric slab of the i/p quarter wave plate the value of each Component is $\frac{E}{\sqrt{2}}\angle 0^\circ$.

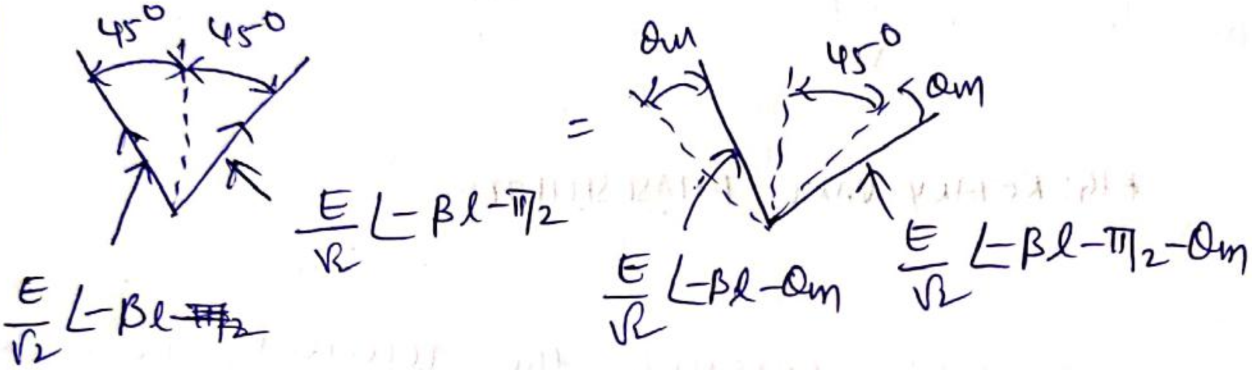
* The effect of i/p quarter wave plate is to delay perpendicular Component by 'βl' and parallel component by '$\beta l + \pi/2$'. which results in a clockwise circularly polarised wave at the i/p of rotatable section.

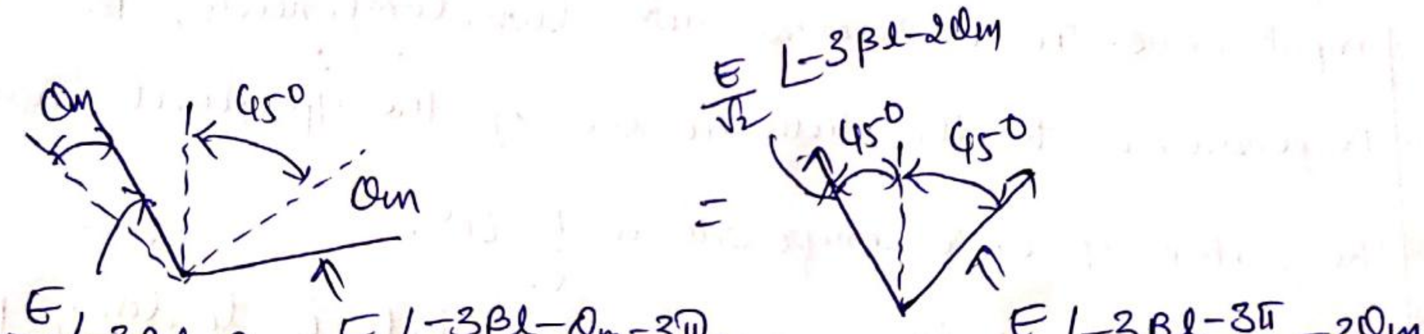
* With the length of half plate is equal to '$2l$', the perpendicular and parallel Components are further delayed by $2\beta l$ and $2\beta l + \pi$ respectively as shown.

* The o/p quarter wave plate delays these Components by an additional 'βl' and $\beta l + \pi/2$ respectively. As a result the o/p

components are '$\frac{E}{\sqrt{2}} \angle -4\beta l - 2\theta_m$' & $\frac{E}{\sqrt{2}} \angle -4\beta l - 2\pi - 2\theta_m$. These two components are in phase, the vector addition results in a vertically polarized o/p wave of value $E \angle -4\beta l - 2\theta_m$.

* Because of accuracy, the rotary phase shifter is used as a calibration standard in microwave laboratories.





FERRITES:

* Ferrite is a device, that is composed of materials that causes it to have useful magnetic properties.

* They are non-metallic materials with resistivities (ρ) nearly 10^{14} times greater than metals and with dielectric constants (ϵ_r) around 10-15 and relative permeabilities of order of 1000.

* They are oxide based compounds having general composition of form $MeO \cdot Fe_2O_3$.

* They are obtained by firing powdered oxides of materials at 1100°c or more and pressing them into different shapes.

* This processing gives them the added characteristics of ceramic insulators so that they can be used at microwave frequencies.

Characteristics:

* Ferrites have atoms with large number of spinning electrons resulting in strong magnetic properties.

* These magnetic properties are due to the magnetic dipole moment associated with the electron spin.

* Because of this properties, they can be used in microwave devices to reduce reflected power, for modulation purposes and in switching circuits.

* Because of high resistivity they can be used upto 100 GHz

* Ferrites have one more peculiar property i.e., the nonreciprocal

property which is useful at microwave frequencies.

FARADAY ROTATION IN FERRITES:

* Consider an infinite lossless medium. A static magnetic field (B_0) is applied along the z-direction.

* A plane TEM wave that is linearly polarized along the x-axis at $Z=0$ is made to propogate through the ferrite in the z-direction.

* The plane of polarization of this wave will rotate with distance, this phenomenon is known as "Faraday Rotation".

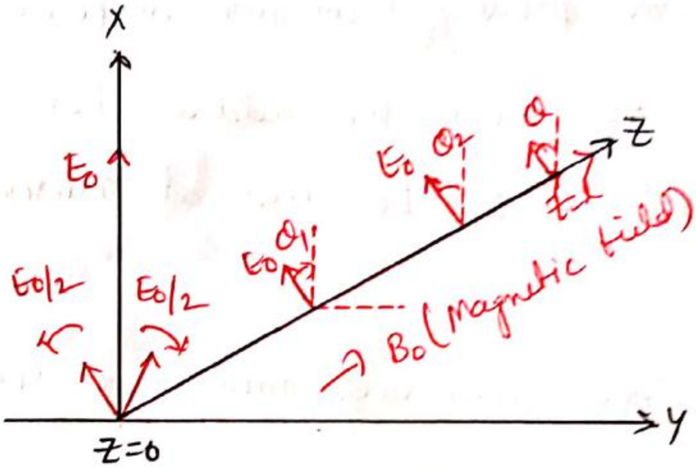


FIG: Faraday Rotation

* Any linearly polarised wave can be regarded as the vector sum of two counter circularly polarised waves.

* The ferrite material offers different characteristics to these waves, with the result that the phase change for one wave is larger than the other wave resulting in rotation θ of the linearly polarised wave at $z=l$.

* If the direction of propogation is reversed, the plane of polarization continues to rotate in the same direction i.e., from $z=l$ to $z=0$, the wave will arrive back at $z=0$ polarised at an angle 2θ relative to x-axis.

* Angle of rotation 'θ' is given by,

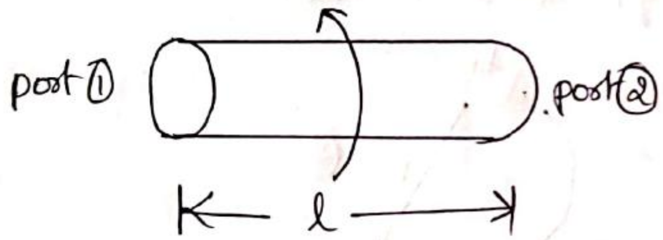
$$\theta = \frac{l}{2}(\beta_+ - \beta_-)$$

Where l = length of the ferrite rod

β_+ = phase shift for the right circularly polarized wave

β_- = phase shift for the left circularly polarized wave.

* A two port ferrite device is as shown,



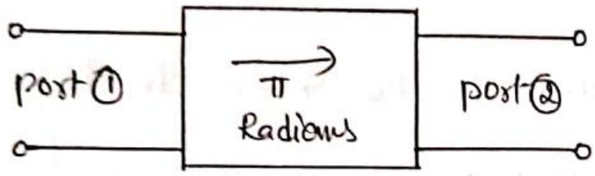
* When a wave is transmitted from port ① to port ②, it undergoes a rotation in the anticlockwise direction

* Even if the same wave is allowed to propogate from port ② to port ①, it will undergo rotation in the same direction i.e., anticlockwise direction.

* The Ferrite Components are 1) Gyrator
2) Isolator
3) Circulator

1. GYRATOR :-

* Gyrator is a two port device that has a relative phase difference of 180° for transmission from port ① to port ② and 'no' phase shift (0° phase shift) for transmission from port ② to port ①.



CONSTRUCTION:

* The Construction of a gyrator is as shown.

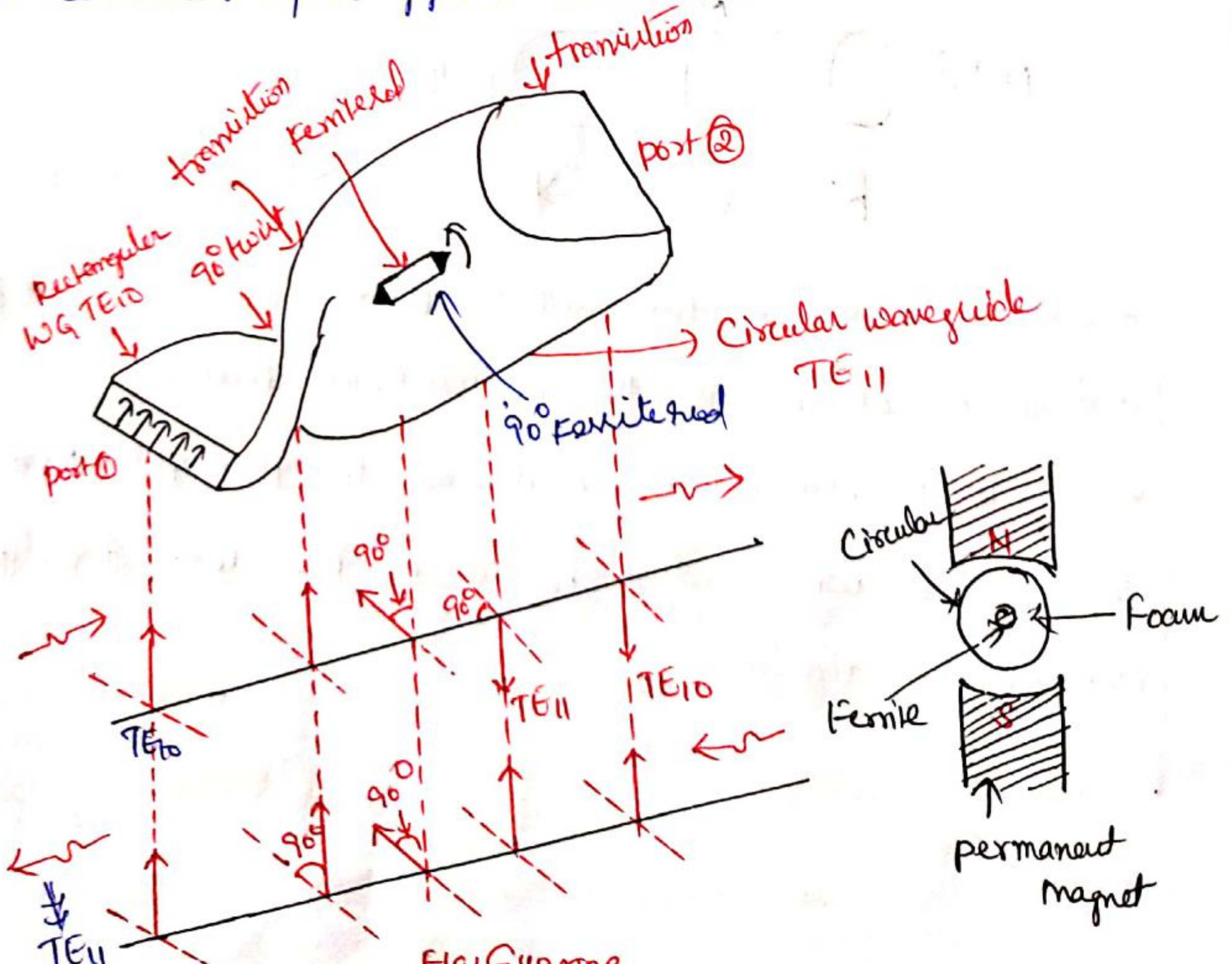


FIG: GYRATOR

* It consists of a piece of circular waveguide carrying the dominant mode TE_{11} with transitions to a standard rectangular waveguide with dominant mode (TE_{10}) at both ends.

* A thin ferrite rod tapered at both ends is located inside the circular waveguide supported by polyfoam and the waveguide is surrounded by a permanent magnet which generates dc magnetic field for proper operation of ferrite.

* At the input end, a 90° twisted rectangular waveguide is connected.

* The ferrite rod is tapered at both ends to reduce the attenuation and also for smooth rotation of polarized wave.

OPERATION:

* When a wave enters port ① its plane of polarization rotates by 90° because of the twist in the waveguide in anticlockwise direction.
* It again undergoes Faraday rotation through 90° because of ferrite rod and the wave which comes out of port ② will have a phase shift of 180° compared to the wave entering at port ①.

* When the same wave (TE_{10} mode signal) enters port ②, it undergoes faraday rotation through 90° in the same anticlockwise direction

* Because of the twist, this wave gets rotated back by 90° Comes out of port ① with 0° phase shift.

* Hence a wave at port ① undergoes a phase shift of π radians (180°) but a wave fed from port ② doesn't change its phase in a gyrator.

ISOLATOR:-

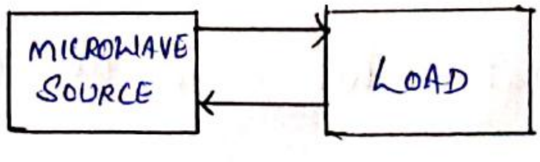
* An Isolator is a two port device which provides very small amount of attenuation for transmission from port ① to port ② but provides maximum attenuation for transmission from port ② to port ①.

* This requirement is very much useful when we want to match a source with a variable load.

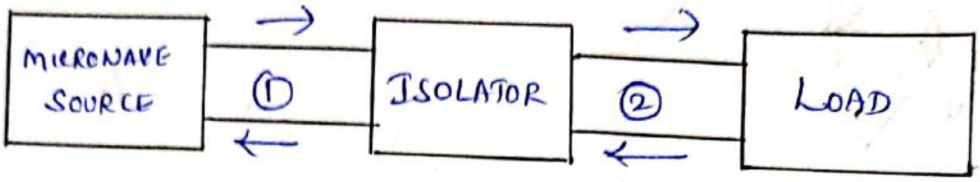
* In most microwave generators, the output amplitude and frequency tend to fluctuate very significantly with changes in load impedance.

* This is due to mismatch of generator output to the load resulting in reflected wave from load.

* But these reflected waves should not be allowed to reach the microwave generator, which will cause amplitude and frequency instability of the microwave generator.



* When Isolator is inserted between generator and load, the generator is coupled to the load with zero attenuation and reflection if any from the load side are completely absorbed by the isolator without affecting the generator o/p.



CONSTRUCTION:

* The Construction of Isolator is similar to gyrator except that an isolator make use of 45° twisted rectangular waveguide (instead of 90° twist)

* It Consists of 45° Faraday rotation ferrite rod instead of 90° in gyrator.

* A resistive card is placed along the larger dimension of the rectangular waveguide, so as to absorb any wave whose plane of polarization is parallel to it.

* It does not absorbs the waves whose plane of polarization is perpendicular to it.

* The Constructional details of Isolator is as shown ~~blt~~ below.

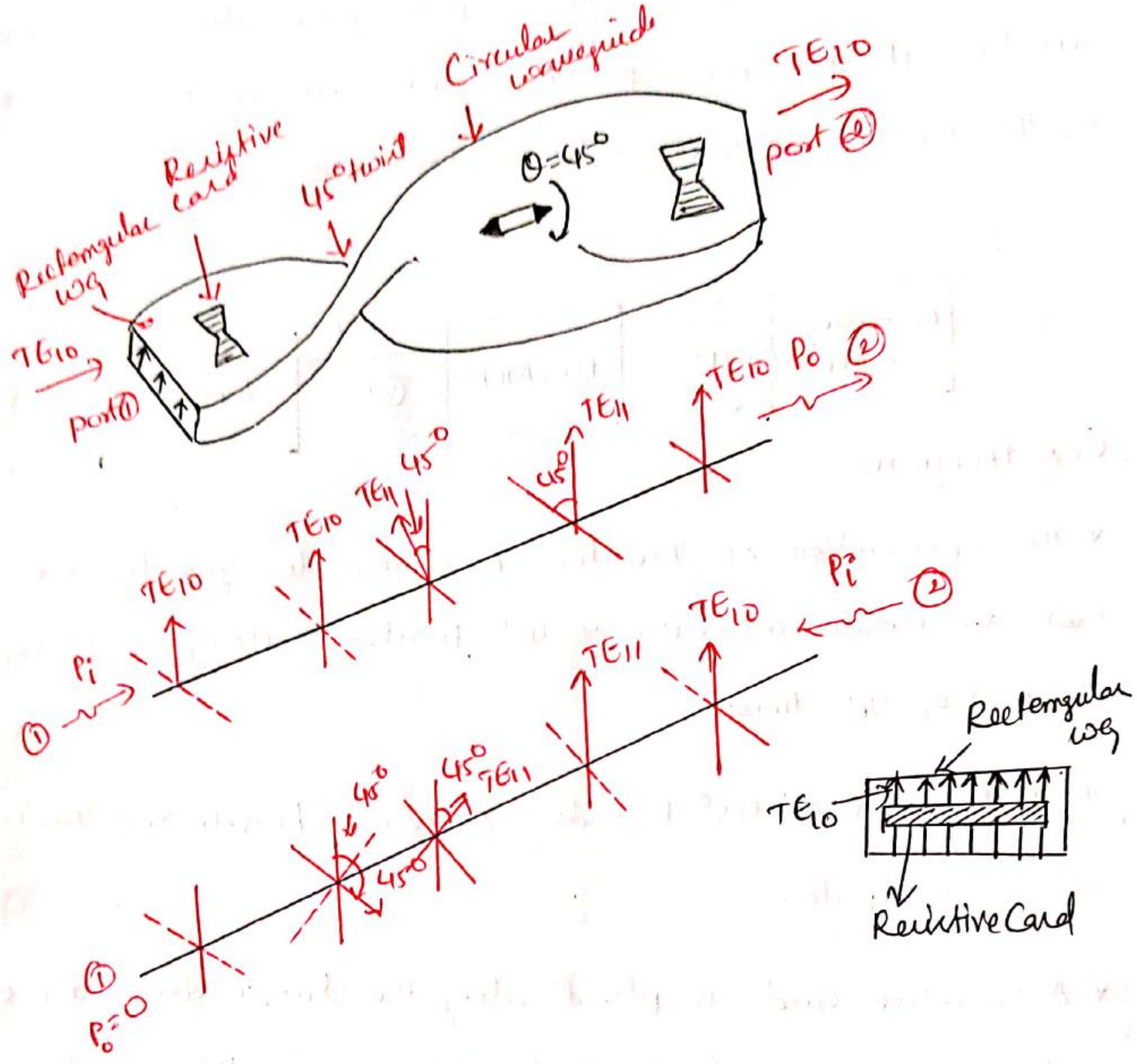


FIG: ISOLATOR.

OPERATION:

* A TE_{10} wave is passing from port ① through the resistive card and is not attenuated.

* After coming out of the card, the wave gets shifted by 45° because of the twist in anticlockwise direction.

* It is then shifted by 45° because of the ferrite rod in clockwise direction. and

* Hence it comes out of port ② with the same polarization as at port ① without any attenuation.

* If a TE_{10} wave is fed ~~at~~ from port ②, it gets passes from the resistive card placed near the port ② since the plane of polarization is perpendicular to the plane of the resistive card.

* Then the wave gets rotated by 45° due to Faraday rotation in clockwise direction.

* It further gets rotated by 45° in anticlockwise direction due to twist in the waveguide

* Now plane of polarization of wave will be parallel with that of resistive card and the wave will be completely absorbed by the resistive card.

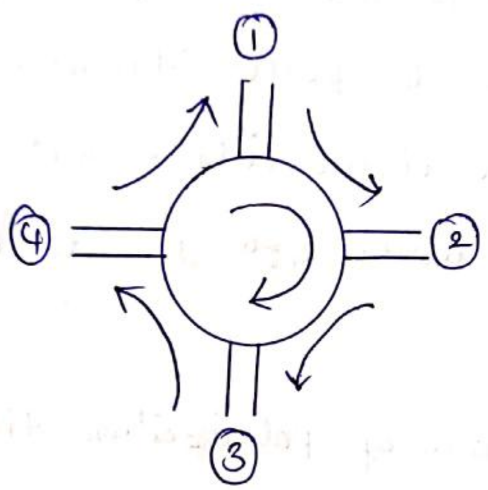
* The o/p at port ① will be zero. This power is dissipated as heat in the resistive card.

* In practice 20 to 30dB isolation is obtained from transmission from port ② to port ①.

3 CIRCULATOR:-

* A circulator is a four port microwave device which has a peculiar property that each terminal is connected only to the next clockwise terminal i.e., port ① is connected only to the next port ② and not to port ③ and port ④.

* Similarly port ② is connected to port ③ but not to port ④ and port ① etc.



* They are useful in parametric amplifiers, tunnel diode amplifiers and as duplexer in radars.

~~CONSTRUCTION~~ OPERATION:

* The power entering port ① is TE_{10} mode and is converted to TE_{11} mode because of gradual rectangular to circular transition.

* This power passes port ③ unaffected since the electric field is not significantly cut and is rotated through 45° due to the ferrite, passes port ④ unaffected and finally emerges out of port ②.

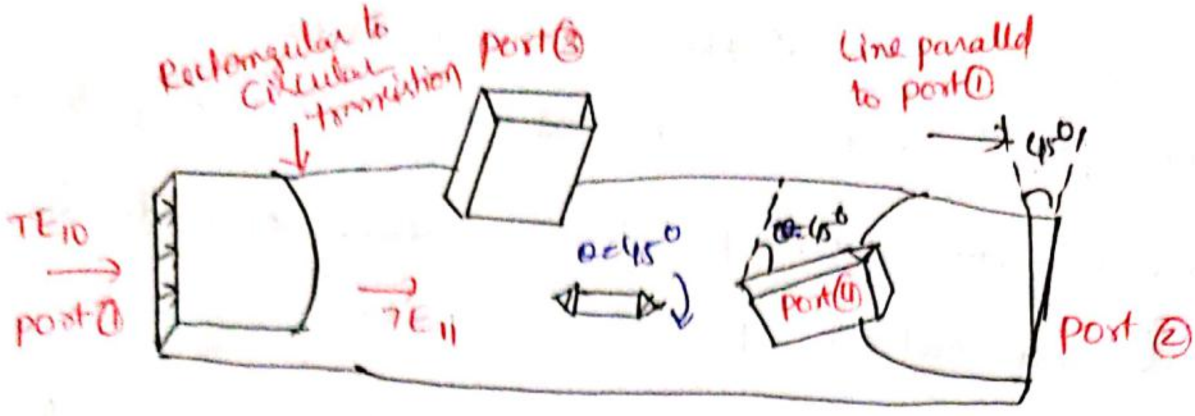


FIG: FOUR PORT CIRCULATOR.

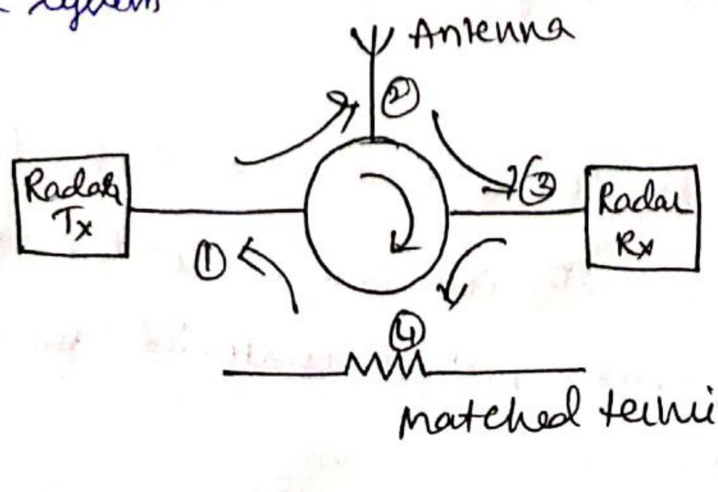
* The wave entering port ② will have plane of polarization already tilted 45° with respect to port ①. This wave passes port ④ unaffected because the electric field is not significantly cut. This wave gets rotated another 45° due to ferrite rod in the clock wise direction

* This wave whose plane of polarization tilted by 90° finds port 3 suitably aligned and emerges out of it.

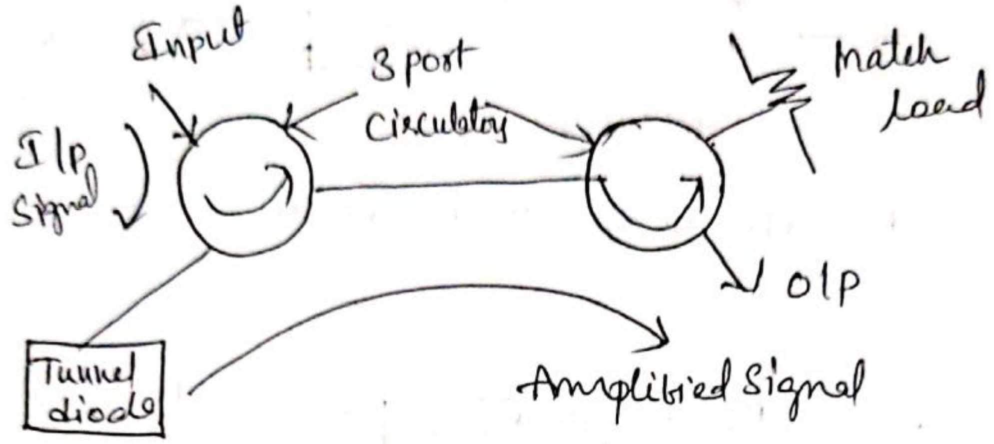
* Similarly port ③ is coupled to port ④ and port ④ to port ①.

Applications:

* A circulator can be used as a duplexer for a radar antenna system.



* we can have 3 port circulators, strip line circulators that can have several applications. Two port circulators can be used in tunnel diode & parametric amplifiers.



* Circulators can be used as low power devices as they can handle low power only.

DIRECTIONAL COUPLERS

* A Directional Coupler is a device that samples a small amount of microwave power for measurement purposes. The power measurements include incident power, reflected power, VSWR values etc
* Directional Coupler is used to couple the microwave power which may be unidirectional or bi-directional.
* Unidirectional it will measure only incident power whereas bidirectional measures both incident and reflected powers.
* Basically, Directional Coupler is a 4-port waveguide junction consisting of a primary main waveguide and a secondary auxillary waveguide.

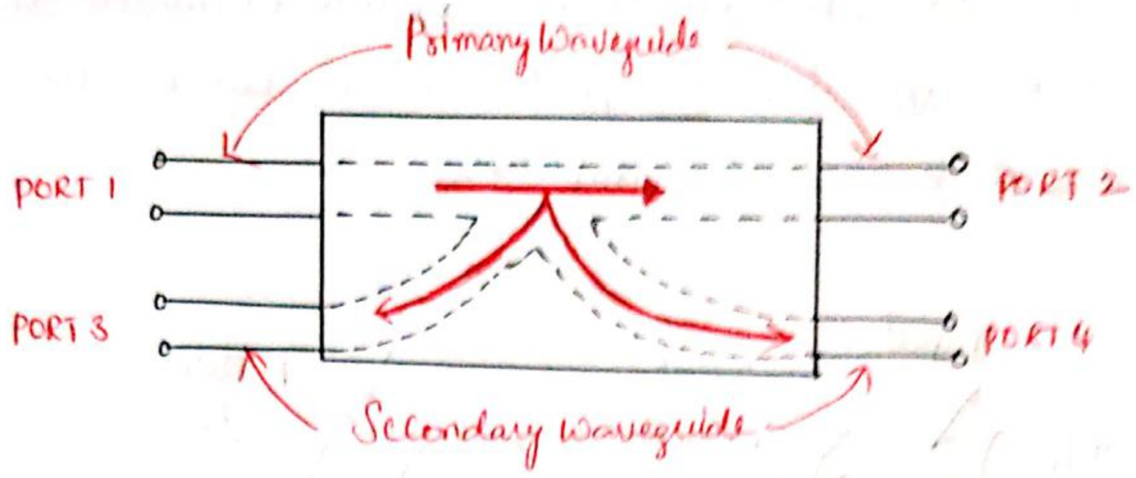


FIG: Schematic of Directional Coupler

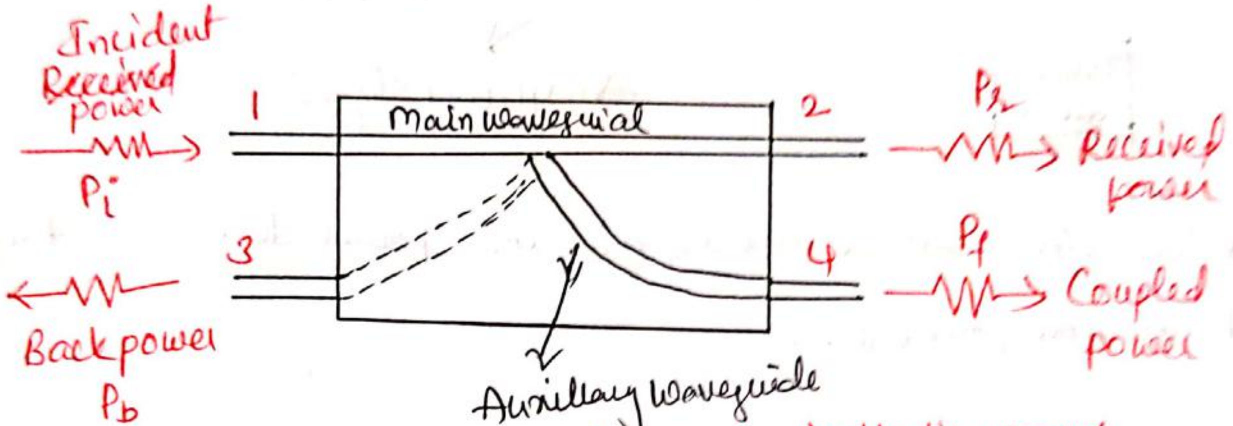


FIG: Directional Coupler indicating powers

PROPERTIES:

* The properties of an ideal directional couplers are as follows.

1. All the terminations are matched to the ports.
2. When the power travels from port ① to port 2, some portion of it gets Coupled to port 4 but not to port 3.
3. In bidirectional coupler, when the power travels from port 2 to port ①, some portion of its gets Coupled to port 3 but not to port 4.
4. If the power is incident through port 3, a portion of it Coupled to port 2 but not to port 1.
5. If the power is incident through port 4, a portion of its is Coupled to port 1 but not to port 2.

6. port ① and port ③ are decoupled as are port 2 and port 4.

* Ideally, the o/p of port ③ should be zero. However practically a small amount of power called Back power is observed at port 3.

where P_i = Incident power at port 1.

P_r = Received power at port 2

P_f = Forward Coupled power at port 4

P_b = Back power at port 3.

* The performance of a directional Coupler is usually defined in terms of following parameters.

Coupling factor (C):-

* The Coupling factor (C) of a directional Coupler is defined as the ratio of the incident power P_i to the forward power P_f measured in dB.

$$\boxed{C = 10 \log_{10} \frac{P_i}{P_f} \text{ dB}}$$

DIRECTIVITY (D):

* The directivity of a directional Coupler is defined as the ratio of forward power P_f to the back power P_b.

$$\boxed{D = 10 \log_{10} \frac{P_f}{P_b} \text{ dB}}$$

⇒ For a typical directional Coupler $C = 20$ dB, $D = 60$ dB

$\frac{P_i}{P}$ $C = 10\log_{10}\frac{P_i}{P_f}$ $D = 10\log_{10}\frac{P_f}{P_b}$

$20 = 10\log_{10}\frac{P_i}{P_f}$ $\cancel{60}\,6 = \cancel{10}\log_{10}\frac{P_f}{P_b}$

$\frac{P_i}{P_f} = 10^2 = 100$ $\frac{P_f}{P_b} = 10^6$

$P_f = \frac{P_i}{100}$ $P_b = \frac{P_f}{10^6}$

$$P_b = \frac{P_i}{100} \times \frac{1}{10^6} = \frac{P_i}{10^8}$$

* Since P_b is very small $\left(\frac{1}{10^8}\right)P_i$, the power coming out of port 3 can be neglected.
* The coupling factor is a measure of the incident power is being sampled while directivity is a measure of how well the directional coupler distinguishes between the forward and reverse travelling powers.

ISOLATION:

* It is defined as the ratio of the incident power P_i to the back power P_b.

$$\boxed{I = 10\log_{10}\frac{P_i}{P_b}\ dB}$$

* It may be noted that isolation in dB equals coupling factor plus directivity.

TWO-HOLE DIRECTIONAL COUPLER:

* A two hole directional Coupler is a device in which two Connected waveguides have 2 holes present between them.
* One waveguide is known as primary waveguide and the ~~other~~ other one is a Auxillary waveguide.
* The two holes are placed at a distance of $\lambda_g/4$.

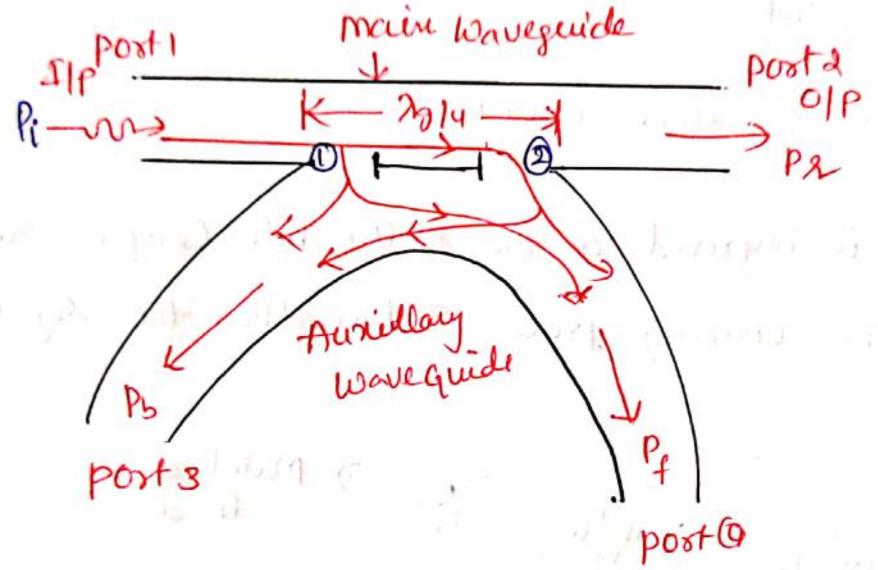


FIG: TWO HOLE DIRECTIONAL COUPLER

* Suppose a microwave signal is to be transmitted from port ① to port ②. When the signal is transmitting inside the main waveguide then on passing through first hole a part of energy gets radiated towards the auxillary waveguide while the rest proceeds through 2nd hole.
* The two leakages out of holes ① and ② both are in phase at the position of 2nd hole and hence they added up contributing to P_f.
* But the two leakages are out of phase by 180° at the position of first hole and therefore they cancel each other

making $P_b = 0$ (ideally).

* The magnitude of the power coming out of 2 holes depend upon the dimension of two holes.

* Since the distance between two holes is $\lambda g/4$, P_b is made '0' because the incident power will have to travel a distance of $\lambda g/4 + \lambda g/4$ when it comes back from hole 2 resulting in 180° phase shift.

BETHE (or) SINGLE HOLE COUPLER:

* Directivity is improved as the Bethe hole coupler relies on a single hole for coupling process rather than the separation between two holes.

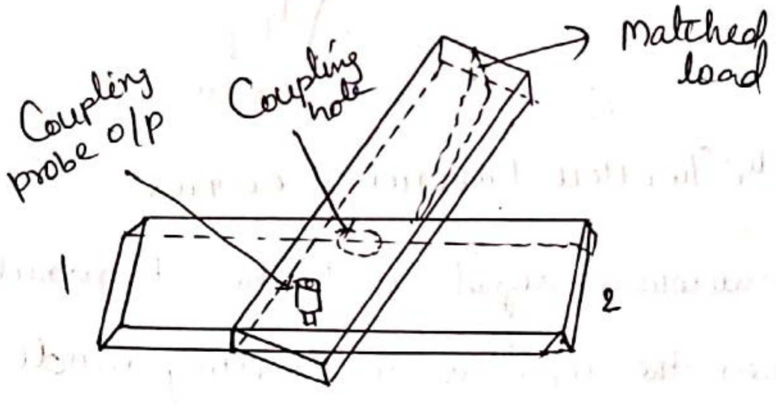


FIG: BETHE (or) SINGLE-HOLE COUPLER

* The power entering port① is coupled to the co-axial probe o/p and the power entering port② is absorbed by the matched load.

* The auxiliary guide is placed at an angle such that the magnitude of magnetically excited wave is made equal to that of the electrically excited wave for improved directivity.

* In this coupler, the waves in the auxillary guide are generated through a single hole which includes both electric and magnetic fields.

* Because of the phase relationships involved in the coupling process, the signals generated by the two types of coupling cancel in forward direction and reinforce in the reverse direction.

Introduction for waveguide Components:-

Microwave systems normally consist of several microwave Components including the source and the load being connected to eachother by waveguide or coaxial or transmission line system. All the components with low standing wave ratios, lower attenuation lower insertion losses and other desirable characteristics to achieve the desired transmission of microwave signal. In rectangular waveguide, cavity resonator etc that were discussed in previous chapter are also microwave components. Other components like waveguide junctions, posts, screws, ferrite devices, phase shifters, directional coupler etc.

Waveguide / Microwave Junctions:-

Junction may be used to combine two or more signals. A microwave junction is an interconnection of two or more microwave components as shown in fig(a). This junction has four ports similar to low frequency two port networks. fig(b) shows a microwave source at port ① and microwave loads at ports ②, ③ and ④.

The microwave junction is analogous to a traffic junction where a number of roads meet on which vehicles enter and leave the traffic Junction. In similar manner, when input from microwave source is applied at port ① a part of it comes out of port ② another part out of port ③ some part out of port ④ and remaining part may come out of port①

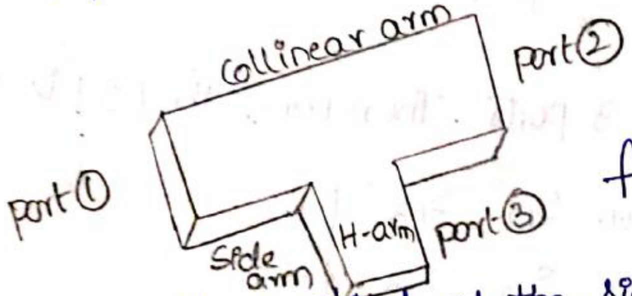
Microwave T-Junction / Multiport Junction:-

T-Junction is an intersection of three waveguides in the form of english alphabet 'T'. ~~Five~~ four types of T-Junction They are

1. H-plane Tee Junction.
2. E-Plane Tee Junction.
3. E-H plane Tee Junction (Hybrid T Junction)
4. Magic Tee junction.
5. Rat race Junction.

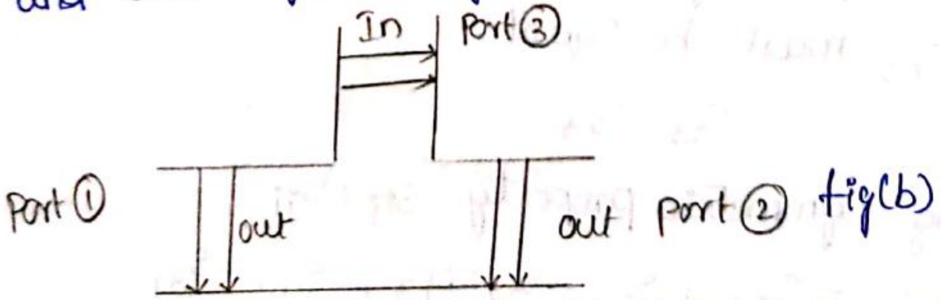
1. H-plane Tee Junction (Shunt tee):-

An H-Plane Tee is a waveguide tee in which the axis of its side arm is parallel to the 'H' field of the main guide as shown in figure(a). is known as H-plane Tee. H-plane Tee



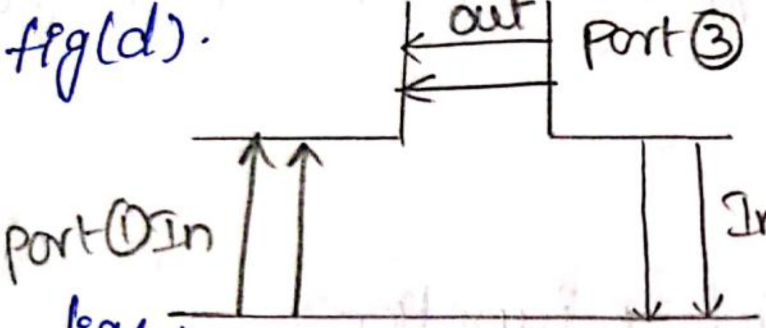
fig(a)

when the input is applied at the side arm i.e at port ③, the outputs are obtained from collinear arms i.e port ① and Port ② and are Equal magnitude as shown in Figure (b).

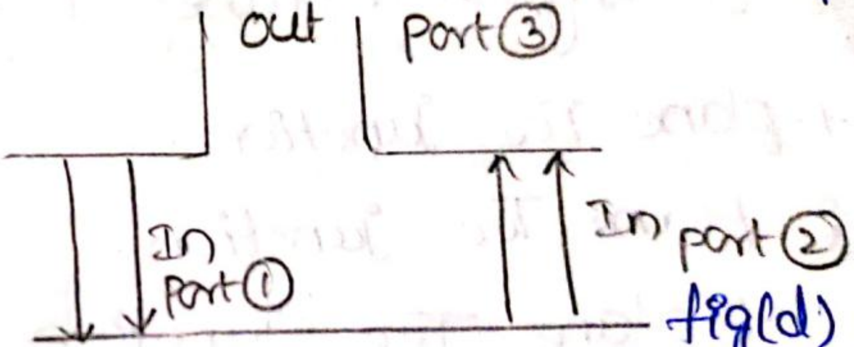


fig(b)

when the input is applied at collinear arms i.e at port ① and ②, the o/p obtained from the side arm depends on the phase of the inputs applied at collinear arms i.e., if phased inputs are applied at port ① and port ② then maximum pow obtained at port ③ and if a 180° phase shift is applied port ① and ② then the o/p at port ③ is zero as shown in fig (c) & fig(d).

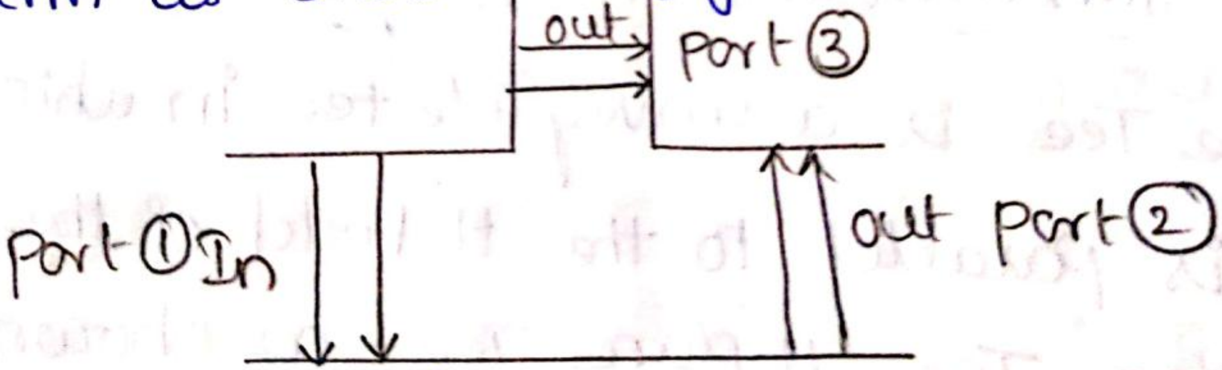


fig(c)



fig(d)

when the input is applied at one of collinear arms i.e por or port ②, the respective outputs are obtained at port ② or port ② and side arm as shown in fig (e).

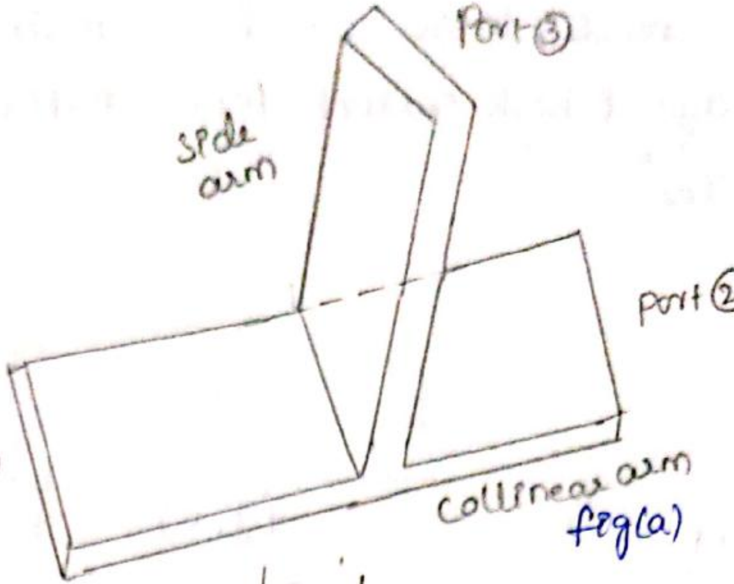


fig(e).

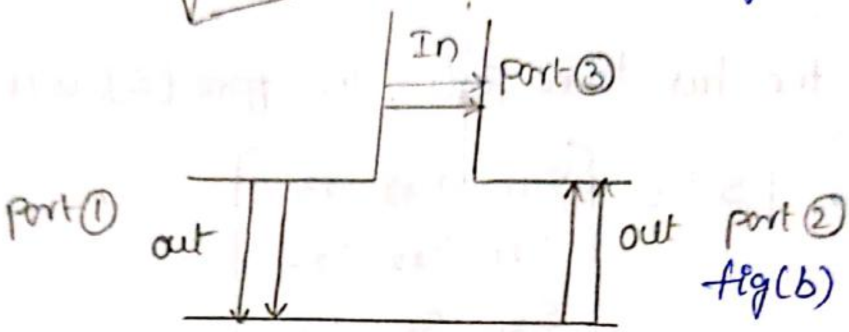
2. E-plane Tee junction:-

An E-plane Tee is a waveguide tee in which the axis of its side arm is parallel to the 'E' field of the mainguide is known as E-plane Tee. E-plane Tee as shown in figure(a).

when the input is applied at the side arm i.e at port③, the outputs obtained from collinear arms ie port ① and port② are of equal magnitude and opposite phase as shown in fig(b)

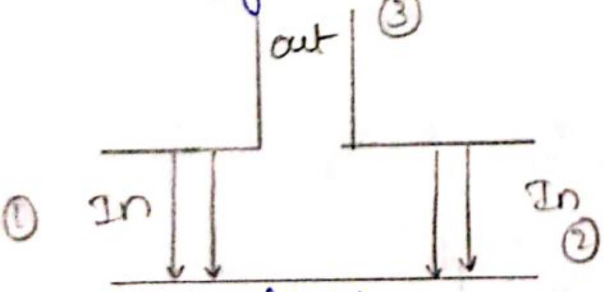


fig(a)

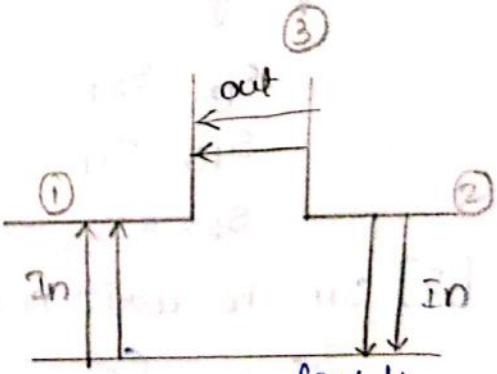


fig(b)

when the input is applied at collinear arms i.e at port① and port② then the output obtained from the side arm depends on the phase of the inputs applied at collinear arms i.e if phased inputs are applied at port① and port② then output at port③ is zero and if a 180° phase shift is applied between port① and port② then the output at Port③ is maximum as shown in fig (c) & fig (d)



fig(c)



fig(d)

when the input is applied at one of the collinear arms i.e at port① or port②, the respective outputs are obtained at port② or port① and side arm as shown in fig(a)

In the E-plane Tee junction, high amount of energy is delivered to an auxiliary guide connected to a tx line, if the auxiliary guide is connected in series with the main line at a point low voltage & high current. Hence E-plane tee is also "Series Tee".

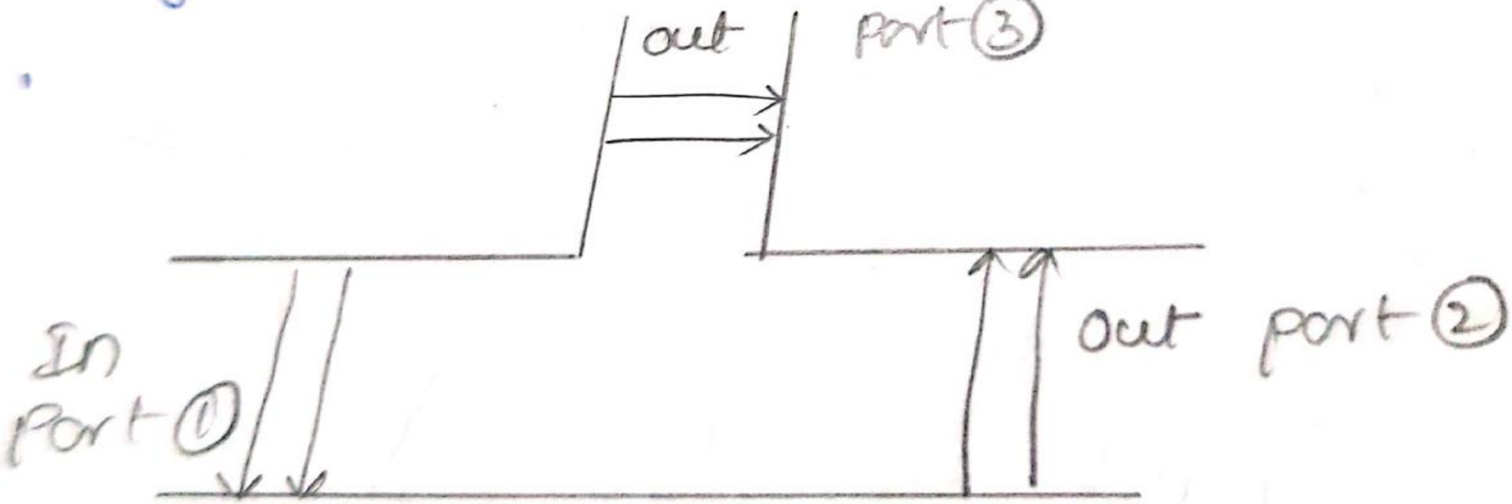


fig (e)

3. E-H plane Tee junction (Hybrid or Magic):-

A waveguide tee which is obtained by cutting the rectangular slots along both the length and breadth of a long waveguide and side arm are attached as shown in fig (a) is known as Magic tee. It is a Combination of both E-plane Tee and H-plane tee. port ① and ② are collinear arms, Port ③ is the H-arm and port ④ is the E-arm. It is a four port hybrid Tee junction Combines the power dividing properties of both H-plane tee and E-plane Tee as shown in fig (b) and advantage of completely matched at all its ports.

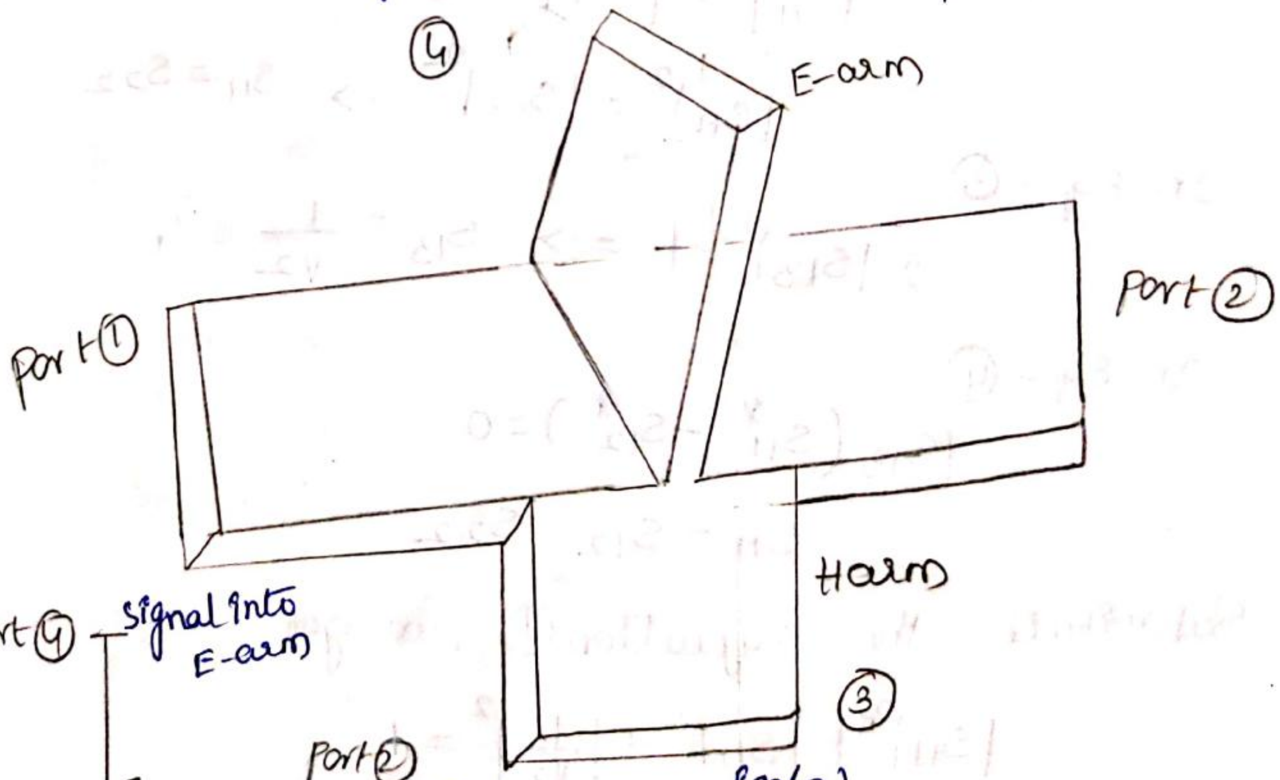


fig (a)

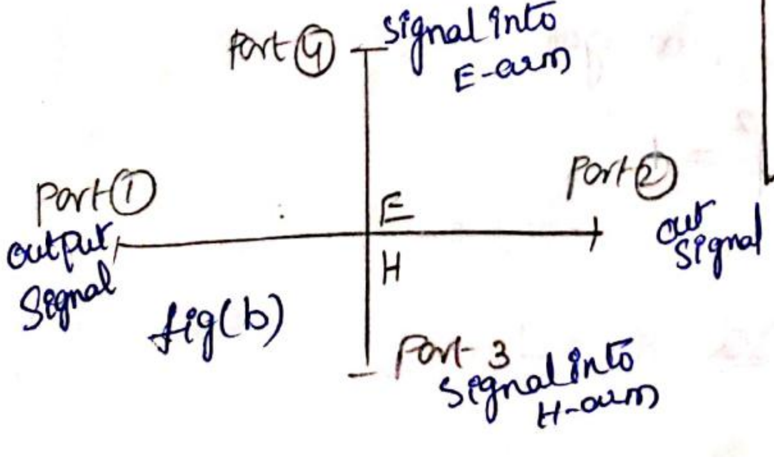


fig (b)

Microwave Measurements

Scattering Matrix - significance, formulation and properties. S-Matrix calculation for 2-port junction, E-plane and H-plane Tees, Magic Tee, Directional coupler, circulator and isolator, problems.

Description of Microwave bench - different blocks and their features errors and precautions; Microwave measurement - Bolometers, Measurements of attenuation, frequency standing wave measurements - Measurement of low and High VSWR, Cavity - Q, impedance measurements.

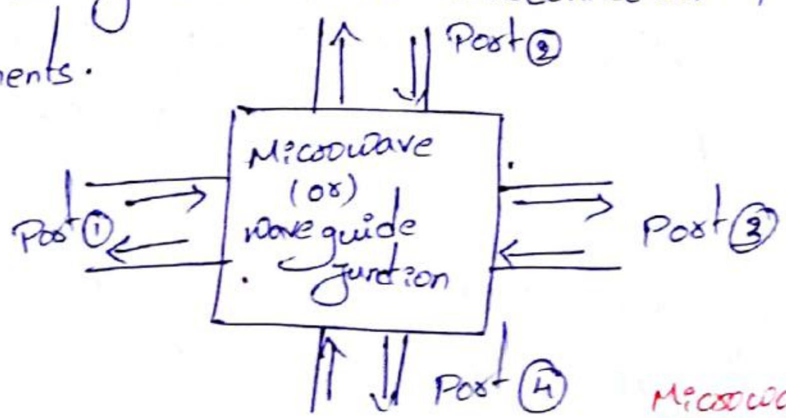
Microwave systems normally consists of several microwave components including the source and the load being connected to each other by waveguide or co-axial or transmission line s/ms. All these components must built with low standing wave ratios, lower attenuation, lower insertion losses and other desirable characteristics to achieve the desired fxion of μw signals.

Waveguide Microwave Junctions:-

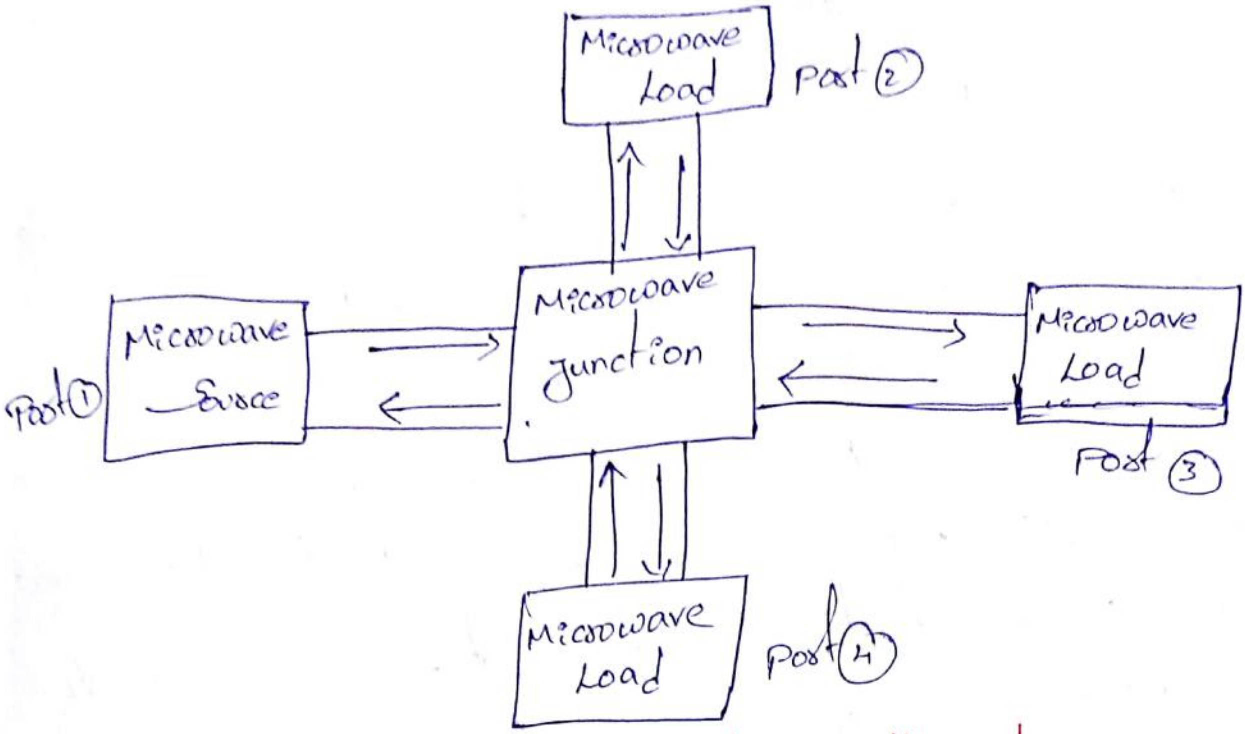
In a waveguides, it is necessary to split all or part of the μw energy into a particular direction.

This is achieved by waveguides or in general by μw junctions.

In general, a μw junction is an interconnection of two or more microwave components.



Microwave Junction

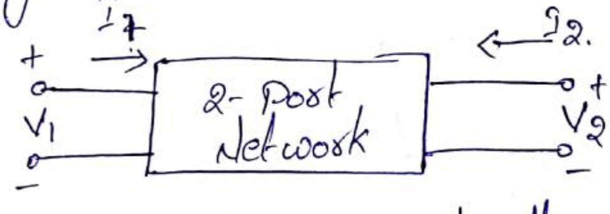


Microwave junction with 4 Ports

Scattering (or) S-Parameters :-

→ Low frequency circuits can be described by two port networks and their parameters such as Z, Y, H, ABCD etc., as per network theory.

→ In Network theory, the parameters (Z, Y, H, - - -) relates the total voltages and total currents in the circuit (or) N/w.



→ lly, At microwave frequencies, travelling waves with associated Powers instead of voltages and currents and the microwave junctions can be defined by so called S-parameters (or) Scattering parameters (lly to H, Z, Y - - -).

→ Let us consider, for a four port n/w, if the i/p is applied at all the ports, then we will has 16 combinations, which are represented in a matrix form and that matrix is known to be as Scattering matrix. ②

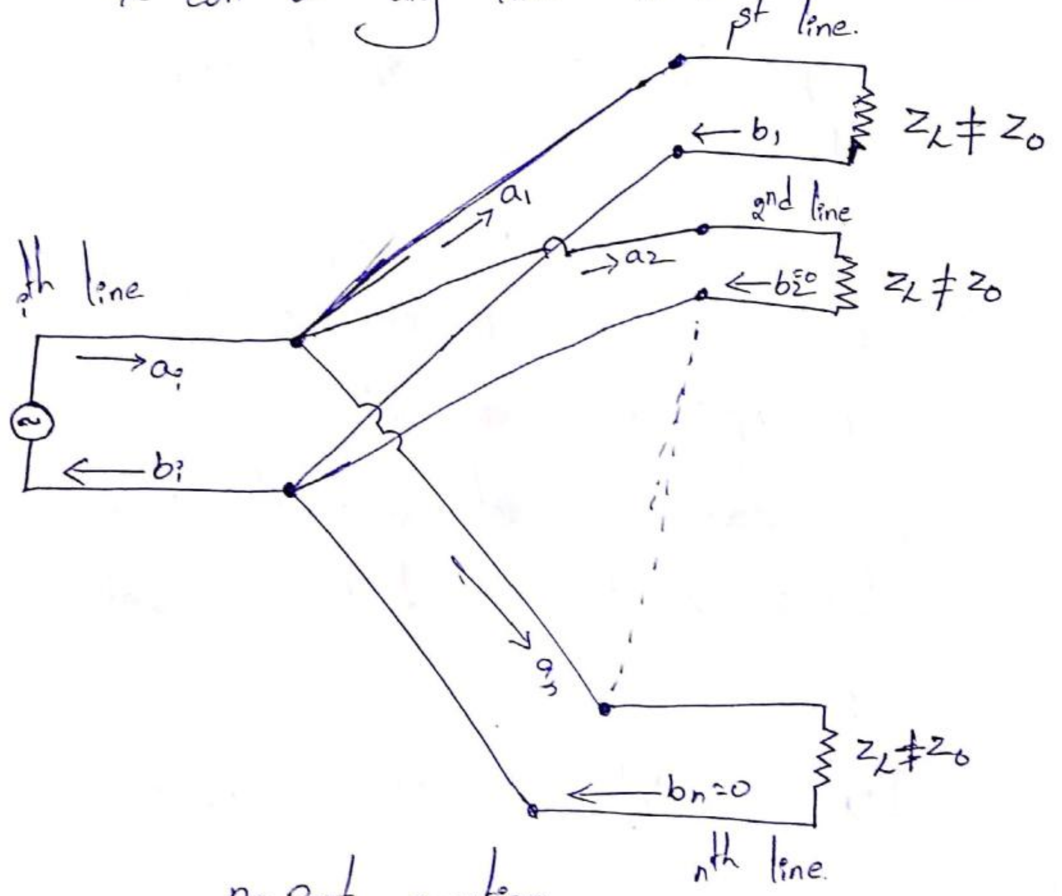
→ It is a square matrix which gives all the combinations of power relationships b/w various input and output ports of a μw junction.

→ The elements of this matrix are called scattering co-efficients or Scattering (S) Parameters.

Relation b/w S-matrix and i/p/o/p powers at different ports :-

Let us consider a junction of "n" number of txion lines where in the i^{th} line is terminated in a source.

i = can be any line in between 1 to n^{th} lines.



n- Port junction

Case (i) :-

→ Let the first line be terminated in an impedance other than the characteristic impedance ($Z_L \neq Z_0$) and all the remaining lines (from 2nd to n^{th} line) in an impedance equal to Z_0 ($Z_L = Z_0$).

→ If " a_i " be the incident wave at junction due to source at the i^{th} line, then it divides itself among $(n-1)$ number of lines as $a_1, a_2, a_3, \ldots \ldots a_n$.

→ There will be no reflections from 2^{nd} to n^{th} line and the incident waves are absorbed since their impedances are equal to characteristic impedance (Z_0).

i.e, $b_2, b_3, \ldots \ldots b_n = 0$ $\because S_{i2} = S_{i3} = S_{i4} \cdots S_{in} = 0$

→ But, there is a mismatch at the 1^{st} line and hence there will be reflected wave " b_1 " going back into the junction.

b_1 is related to a_1 by.

$$b_1 = (\text{reflection co-efficient}) \cdot a_1$$

$$\boxed{b_1 = S_{i1} \cdot a_1}$$

where S_{i1} : reflection co-efficient of 1^{st} line.

1 : reflection from 1^{st} line.

i : source connected at i^{th} line.

Hence, the contribution to the outward travelling wave in the i^{th} line is given by.

$$\boxed{b_i = S_{i1} \cdot a_1} \qquad b_1 = b_i \;;\; \because b_2 = b_3 = \cdots b_n = 0,$$

Case (ii) :-

Let all the $(n-1)$ lines be terminated in an impedance other than Z_0 (i.e., $Z_L \neq Z_0$ for all the lines).

then, there will be reflections into the junction from every line (i) and hence the total contribution to the outward travelling wave in the ith line is given by

$$b_i = S_{i1} \cdot a_1 + S_{i2} \cdot a_2 + S_{i3} a_3 + \cdots + S_{in} a_n \longrightarrow (1)$$

$i = 1$ to n since i can be any line from 1 to n

$$\therefore \left.\begin{aligned} b_1 &= S_{11} a_1 + S_{12} a_2 + S_{13} a_3 + \cdots + S_{1n} a_n \\ b_2 &= S_{21} a_1 + S_{22} a_2 + S_{23} a_3 + \cdots + S_{2n} a_n \\ b_3 &= S_{31} a_1 + S_{32} a_2 + S_{33} a_3 + \cdots + S_{3n} a_n \\ &\vdots \\ b_n &= S_{n1} a_1 + S_{n2} a_2 + S_{n3} a_3 + \cdots + S_{nn} a_n. \end{aligned}\right\} \rightarrow (2)$$

In matrix form,

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & \cdots & S_{1n} \\ S_{21} & S_{22} & S_{23} & \cdots & S_{2n} \\ S_{31} & S_{32} & S_{33} & \cdots & S_{3n} \\ \vdots & & & & \\ S_{n1} & S_{n2} & S_{n3} & \cdots & S_{nn} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{bmatrix} \longrightarrow (3)$$

Column matrix [b] Corresponding to Reflected waves or output

Scattering matrix [S] of order n×n.

Column matrix [a] Corresponding to incident waves or i/ps.

$$\boxed{[b] = [S][a]} \longrightarrow (4)$$

when a junction of n-number of waveguides are considered,

→ a's represent inputs to particular ports

→ b's represent outputs out of various ports

→ S_{ij} corresponds to scattering co-efficients resulting due to i/p at i^{th} port and output taken out of j^{th} port.

→ S_{ii} denotes how much power is reflected back from the junction into the i^{th} port when input power is applied at the i^{th} port itself.

Properties of [S] - matrix :-

1. [S] is always a square matrix of order $n \times n$.

2. (a) [S] is a symmetric matrix.

$$\boxed{S_{ij} = S_{ji}}$$

(b) The scattering coefficient

$$\boxed{S_{23} = S_{13}}$$

Since, the outputs at ports ① & ② are inphase with an i/p at port ③.

3. [S] is a unitary matrix

$$\boxed{[S][S]^{*} = [I]}$$

where, $[S]^{*}$ = Complex conjugate of [S]

[I] = unit matrix or identity matrix of same order as that of [S].

4. The sum of the product of each term of any row (or column) multiplied by the complex conjugate of the corresponding terms of any other row (or column) is zero.

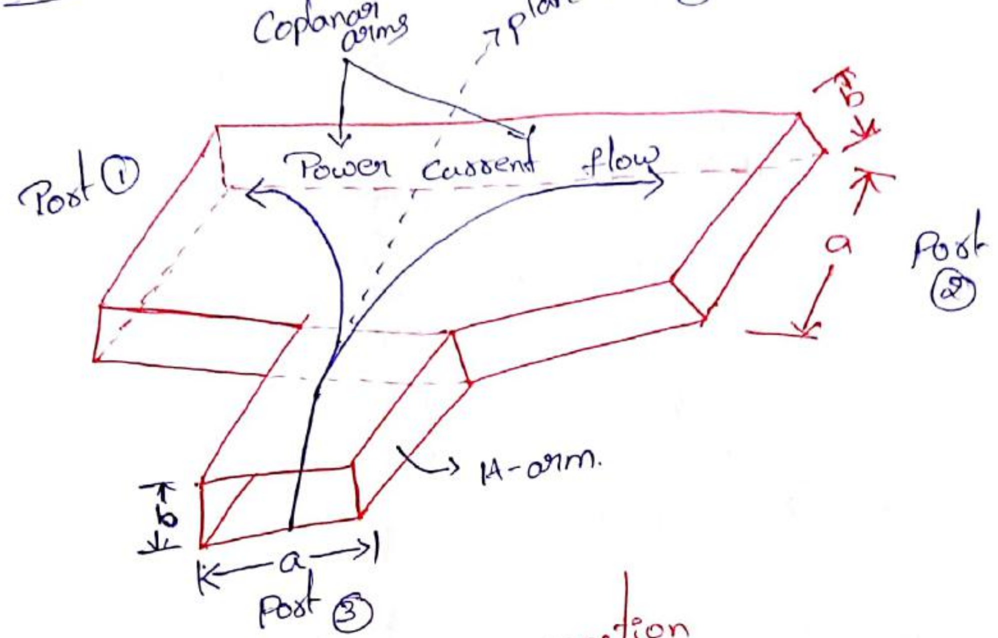
$$\boxed{\sum_{i=1}^{n} S_{ik} S_{ij}^{*} = 0 \quad k \neq j} \quad \left\{ \begin{array}{l} k = 1, 2, \ldots n \\ j = 1, 2, 3, \ldots n \end{array} \right.$$

5. If any of the terminal or reference planes (say k^{th} port) are moved away from the junction by an electric distance $\beta_k l_k$, each of the co-efficients S_{ij} involving "k" will be multiplied by a

factor $e^{-j\beta_K l_K}$.

H-plane Tee Junction:

A H-plane Tee junction is formed by cutting a rectangular slot along the width of a main waveguide and attaching another waveguide. the side arm called the H-arm.



H-plane Tee junction

→ The port ① and port ② of the main waveguide are called collinear ports and port ③ in the H-arm (side arm).

→ H-plane Tee is so called bcoz the axis of the side arm is parallel to the planes of the main txion line.

As all three arms of H-plane Tee lie in the plane of magnetic field, the magnetic field divides itself into the arms. Therefore this is also called a current junction.

→ The properties of a H-plane Tee can be completely defined by its [S]. matrix.

The order of scattering matrix is 3×3. Since, there are

3 Possible inputs and 3 possible outputs.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \longrightarrow (1)$$

Determination of S-parameters S_{ij} $i = 1,2,3$ & $j = 1,2,3$ by making use of the properties of $[S]$:- (S-matrix)

1. Because of the plane of symmetry of the junction, scattering coefficients S_{13} and S_{23} must be equal.

$$\boxed{S_{13} = S_{23}}$$

2. From the symmetric property, $S_{ij} = S_{ji}$.

$$\boxed{S_{12} = S_{21}}, \quad \boxed{S_{23} = S_{32} = S_{13}}, \quad \boxed{S_{13} = S_{31}}$$

3. Since port 3 is perfectly matched to the junction.

$$\boxed{S_{33} = 0}$$

By Applying these Properties to eqn (1), it can be written as.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \longrightarrow (2)$$

There are four unknowns. $(S_{11}, S_{12}, S_{13}, S_{22})$.

4. From the unitary property (5)

$$[S]\cdot[S]^* = [I]$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On multiplying, we have

$R_1 \cdot C_1 \Rightarrow S_{11}\cdot S_{11}^* + S_{12}\cdot S_{12}^* + S_{13}\cdot S_{13}^* = 1$

$$\Rightarrow |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \longrightarrow (3)$$

lly

$R_2 \cdot C_2 \Rightarrow |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \longrightarrow (4)$

$R_3\, C_3 \Rightarrow |S_{13}|^2 + |S_{13}|^2 + 0 = 1$

$$2|S_{13}|^2 = 1$$

$$\boxed{|S_{13}|^2 = 1/2.} \quad \text{or} \quad \boxed{|S_{13}| = 1/\sqrt{2}} \longrightarrow (5)$$

$R_3 C_1 \Rightarrow S_{13}\cdot S_{11}^* + S_{13} S_{12}^* + 0 = 0$

$$S_{13}\left[S_{11}^* + S_{12}^*\right] = 0$$

$$S_{13} \neq 0 \quad \because \; S_{11}^* + S_{12}^* = 0$$

$$\Rightarrow S_{11}^* = -S_{12}^* \quad (\text{or}) \quad \boxed{\begin{array}{c} S_{11} = -S_{12} \\ S_{12} = -S_{11} \end{array}} \longrightarrow (6)$$

By comparing eqn (3) and (4)

$$|S_{11}|^2 + \cancel{|S_{12}|^2} + \cancel{|S_{13}|^2} = 1$$

$$|S_{22}|^2 + \cancel{|S_{12}|^2} + \cancel{|S_{13}|^2} = 1$$

$$- \quad - \quad - \quad (-)$$

$$|S_{11}|^2 - |S_{22}|^2 = 0$$

$$|S_{11}|^2 = |S_{22}|^2$$

$$\boxed{S_{11} = S_{22}} \longrightarrow (7)$$

Now from eqn (3)

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1$$

$$|S_{11}|^2 + |-S_{11}|^2 + |S_{13}|^2 = 1$$

$$2S_{11}^2 + \frac{1}{2} = 1$$

$$2S_{11}^2 = 1 - \frac{1}{2} = \frac{1}{2}$$

$$S_{11}^2 = \frac{1}{4} \Rightarrow \boxed{S_{11} = \frac{1}{2}} \longrightarrow (8)$$

From eqn (7)

$$\boxed{S_{22} = S_{11} = \frac{1}{2}} \longrightarrow (9)$$

from eqn (6) $\boxed{S_{12} = -S_{11} = -\frac{1}{2}} \longrightarrow (10)$

The four unknowns

$S_{11} = \frac{1}{2}$ $\quad S_{12} = -\frac{1}{2}$ $\quad S_{13} = \frac{1}{\sqrt{2}}$ $\quad S_{22} = \frac{1}{2}$

On substituting all these values in eqn ⑧, the S-matrix ⑥ becomes

$$[S] = \begin{bmatrix} 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} \longrightarrow (11)$$

The outputs are related to inputs by making use of scattering parameters as

$$[b] = [S][a]$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \longrightarrow (12)$$

Now,

$$b_1 = \frac{1}{2}a_1 + -\frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3 \longrightarrow (13)$$

$$b_2 = -\frac{1}{2}a_1 + \frac{1}{2}a_2 + \frac{1}{\sqrt{2}}a_3 \longrightarrow (14)$$

$$b_3 = \frac{1}{\sqrt{2}}a_1 + \frac{1}{\sqrt{2}}a_3 \longrightarrow (15)$$

Case (i) :- $a_3 \neq 0$, $a_1 = 0$, $a_2 = 0$

Input is given at port ③ and no inputs at port ① and port ②.

Scanned by CamScanner

By substituting these in eqn (13), (14) & (15)

$$b_1 = 0 - 0 + \frac{1}{\sqrt{2}} a_3$$

$$\boxed{b_1 = a_3/\sqrt{2}}$$

$$b_2 = -0 + 0 + \frac{1}{\sqrt{2}} a_3 \Rightarrow \boxed{b_2 = a_3/\sqrt{2}}$$

$$\boxed{b_3 = 0}$$

Let us consider P_3 be the power input at port (3).
(Corresponding to a_3)

Then this divides equally between port (1) and port (2) inphase
i.e., $P_1 = P_2$.

$$\boxed{P_3 = P_1 + P_2 = 2P_1 = 2P_2}$$

The total amount of power coming out of port (1) or port (2) due to input at port (3).

$$= 10 \log_{10} P_1/P_3 = 10 \log_{10} P_1/2P_1 = 10 \log_{10} (1/2) = 10 \log_{10} 2^{-1}$$

$$= -10 \log_{10} 2 = -10 (0.3010)$$

$$= -3 \text{ dB}$$

Hence the power coming out of port (1) or port (2) is 3dB with respect to input power at port (3), Hence the H-plane Tee is called as 3-dB splitter.

when TE_{10} mode is allowed to propagate into port ③, the electric field lines donot change their direction when they come out of ports ① and ② hence called H-plane Tee. { the waves that come out of ports ① and ② are equal in magnitude and phase }.

Case (ii) :- $a_1 = a_2 = a$, $a_3 = 0$

the equations ⑬, ⑭ & ⑮ becomes

$$b_1 = \frac{a_1}{2} - \frac{a_2}{2} + \frac{1}{\sqrt{2}}a_3 = \frac{\cancel{a}}{\cancel{2}} - \frac{\cancel{a}}{\cancel{2}} + 0$$

$$\boxed{b_1 = 0}$$

$$b_2 = -\frac{a_1}{2} + \frac{a_2}{2} + \frac{1}{\sqrt{2}}a_3 = -\frac{a}{2} + \frac{a}{2} + 0$$

$$\boxed{b_2 = 0}$$

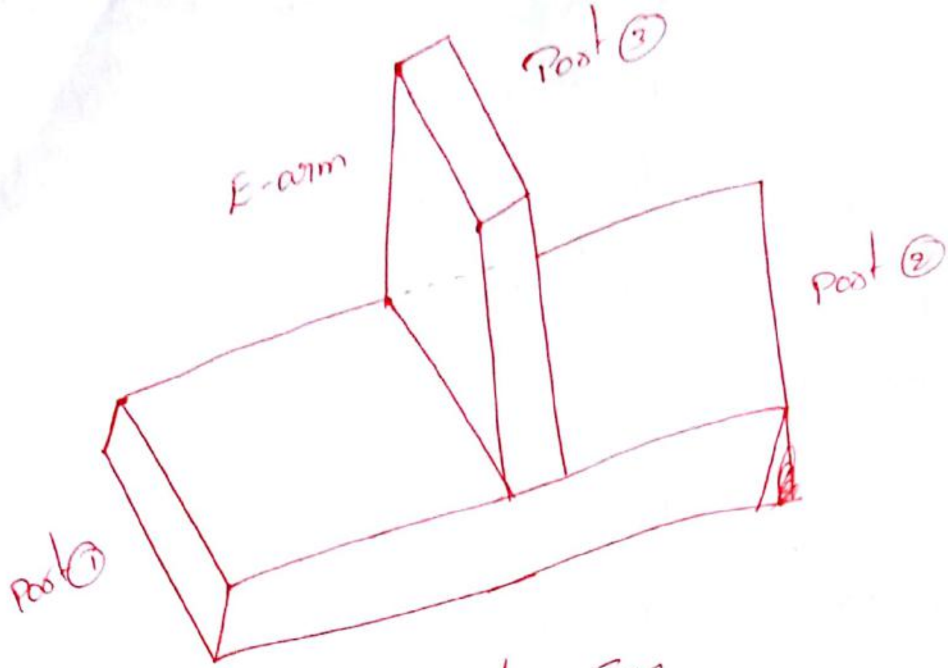
$$b_3 = \frac{a_1}{\sqrt{2}} + \frac{a_2}{\sqrt{2}} + 0 = \frac{a}{\sqrt{2}} + \frac{a}{\sqrt{2}} = \frac{2a}{\sqrt{2}} = \sqrt{2}\,a$$

the o/p at port ③ is addition of two i/ps at port ① & port ② and these are added in phase.

E-plane Tee :-

→ A rectangular slot is cut along the broader dimension of a long waveguide and a side arm is attached.

→ Port ① and port ② are the collinear arms and port ③ is the E-arm.



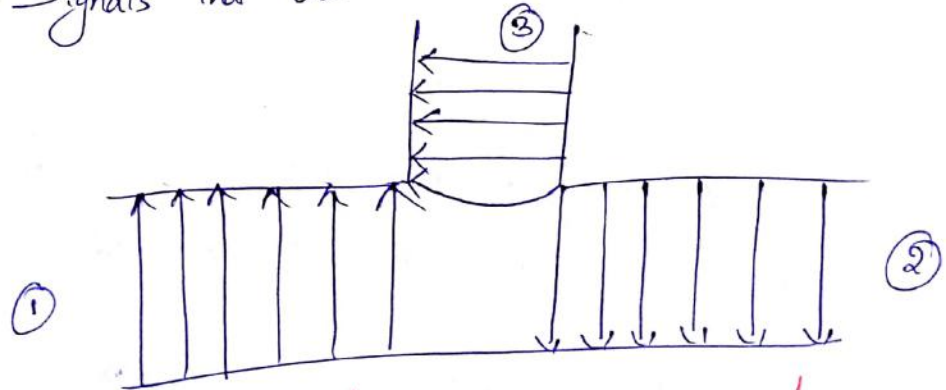
E-plane Tee

→ when TE_{10} mode is made to propagate into port ③, the two outputs at port ① and ② will have a phase shift of 180°.

Since the electric field line change their direction when they come out of port ① and ②, it is called E-plane Tee.

E-plane Tee is a voltage or series junction symmetrical about the central arm.

Hence any signals that is ~~fed to~~ to be split or any two signals that are to be combined will be fed from the E-arm.



Direction of wave propagation

The properties of E-plane Tee is completely defined by its S matrix. (3)

Scattering parameters or co-efficients of E-plane Tee :-

1. Since it is a 3 port junction. The order of the scattering matrix is 3×3.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \longrightarrow (1)$$

2. The scattering co-efficient

$$\boxed{S_{23} = -S_{13}} \longrightarrow (2)$$

Since o/p's at port (1) & port (2) are out of phase by 180° with an input at port (3).

3. If port (3) is perfectly matched to the junction.

$$\boxed{S_{33} = 0} \longrightarrow (3)$$

4. From the symmetric property $S_{ij} = S_{ji}$

$$\left.\begin{matrix} \boxed{S_{12} = S_{21}} \\ \boxed{S_{13} = S_{31}} \end{matrix} \quad \boxed{S_{23} = S_{32} = -S_{13}} \right\} \longrightarrow (4)$$

with these properties, the eqn (1) becomes

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \longrightarrow (5)$$

5. From unitary property

$$[S][S]^* = [I]$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* \\ S_{13}^* & -S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On multiplying.

$R_1 C_1 \Rightarrow S_{11} S_{11}^* + S_{12} \cdot S_{12}^* + S_{13} \cdot S_{13}^* = 1$

$$\boxed{|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1} \longrightarrow (6)$$

$R_2 C_2 \Rightarrow S_{12} \cdot S_{12}^* + S_{22} \cdot S_{22}^* + (-S_{13})(-S_{13}^*) = 1$

$$\Rightarrow \boxed{|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1} \longrightarrow (7)$$

$R_3 C_3 \Rightarrow S_{13} \cdot S_{13}^* + (-S_{13})(-S_{13}^*) + 0 = 1$

$|S_{13}|^2 + |S_{13}|^2 = 1 \Rightarrow 2|S_{13}|^2 = 1 \Rightarrow |S_{13}|^2 = 1/2$

$$\Rightarrow \boxed{S_{13} = 1/\sqrt{2}} \longrightarrow (8)$$

$R_3 C_1 \Rightarrow S_{13} \cdot S_{11}^* + (-S_{13})(S_{12}^*) + 0 = 0$

$S_{13} [S_{11}^* - S_{12}^*] = 0$

$\because S_{13} \neq 0 \quad S_{11}^* - S_{12}^* = 0 \Rightarrow \boxed{S_{11} = S_{12}} \longrightarrow (9)$

On solving eqn 6 & 7

$$|S_{11}|^2 + \cancel{|S_{12}|^2} + \cancel{|S_{13}|^2} = 1$$

$$|S_{22}|^2 + \cancel{|S_{12}|^2} + \cancel{|S_{13}|^2} = 1$$

(–) (–) (–) (–)

$$|S_{11}|^2 - |S_{22}|^2 = 0$$

$$\boxed{S_{11} = S_{22}} \longrightarrow (10)$$

Now from eqn 6

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1$$

By substituting eqn 9 and 8 in above equation.

$$|S_{11}|^2 + |S_{11}|^2 + 1/2 = 1$$

$$2|S_{11}|^2 = 1 - 1/2$$

$$2|S_{11}|^2 = 1/2$$

$$|S_{11}|^2 = 1/4 \Rightarrow \boxed{S_{11} = 1/2} \longrightarrow (11)$$

From the above calculations the unknowns of S-matrix is

$S_{11} = 1/2$ $S_{12} = 1/2$ $S_{13} = 1/\sqrt{2}$ $S_{22} = 1/2$

On substituting these values in eqn ⑤, the S-matrix becomes

$$[S] = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix} \longrightarrow (12)$$

the inputs and outputs can be related as

$$[b] = [S][a]$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \longrightarrow (13)$$

$$b_1 = \tfrac{1}{2} a_1 + \tfrac{1}{2} a_2 + \tfrac{1}{\sqrt{2}} a_3 \longrightarrow (14)$$

$$b_2 = \tfrac{1}{2} a_1 + \tfrac{1}{2} a_2 - \tfrac{1}{\sqrt{2}} a_3 \longrightarrow (15)$$

$$b_3 = \tfrac{1}{\sqrt{2}} a_1 - \tfrac{1}{\sqrt{2}} a_2 \longrightarrow (16)$$

Case (i) :- $a_1 = a_2 = 0, \quad a_3 \neq 0$

From eqns (14), (15) & (16)

$$b_1 = 0 + 0 + \frac{a_3}{\sqrt{2}} \Rightarrow \boxed{b_1 = \frac{a_3}{\sqrt{2}}}$$

$$b_2 = 0 + 0 - \frac{a_3}{\sqrt{2}} \Rightarrow \boxed{b_2 = -\frac{a_3}{\sqrt{2}}}$$

$$b_3 = 0 - 0 \Rightarrow \boxed{b_3 = 0}$$

An input at port ③ equally divides between ① and ② but introduces a phase shift of 180° b/w the two outputs. Hence E-plane Tee also acts as a 3dB splitter. (10)

Case(2) :- $a_1 = a_2 = a, \quad a_3 = 0$

Now eqn. (14), (15) & (16) becomes

$$b_1 = \frac{a}{2} + \frac{a}{2} + 0 = \frac{2a}{2} = a$$

$$b_2 = \frac{a}{2} + \frac{a}{2} - 0 = a$$

$$b_3 = \frac{a}{\sqrt{2}} - \frac{a}{\sqrt{2}} = 0$$

equal i/p's at port ① and port ② result in no output at port ③.

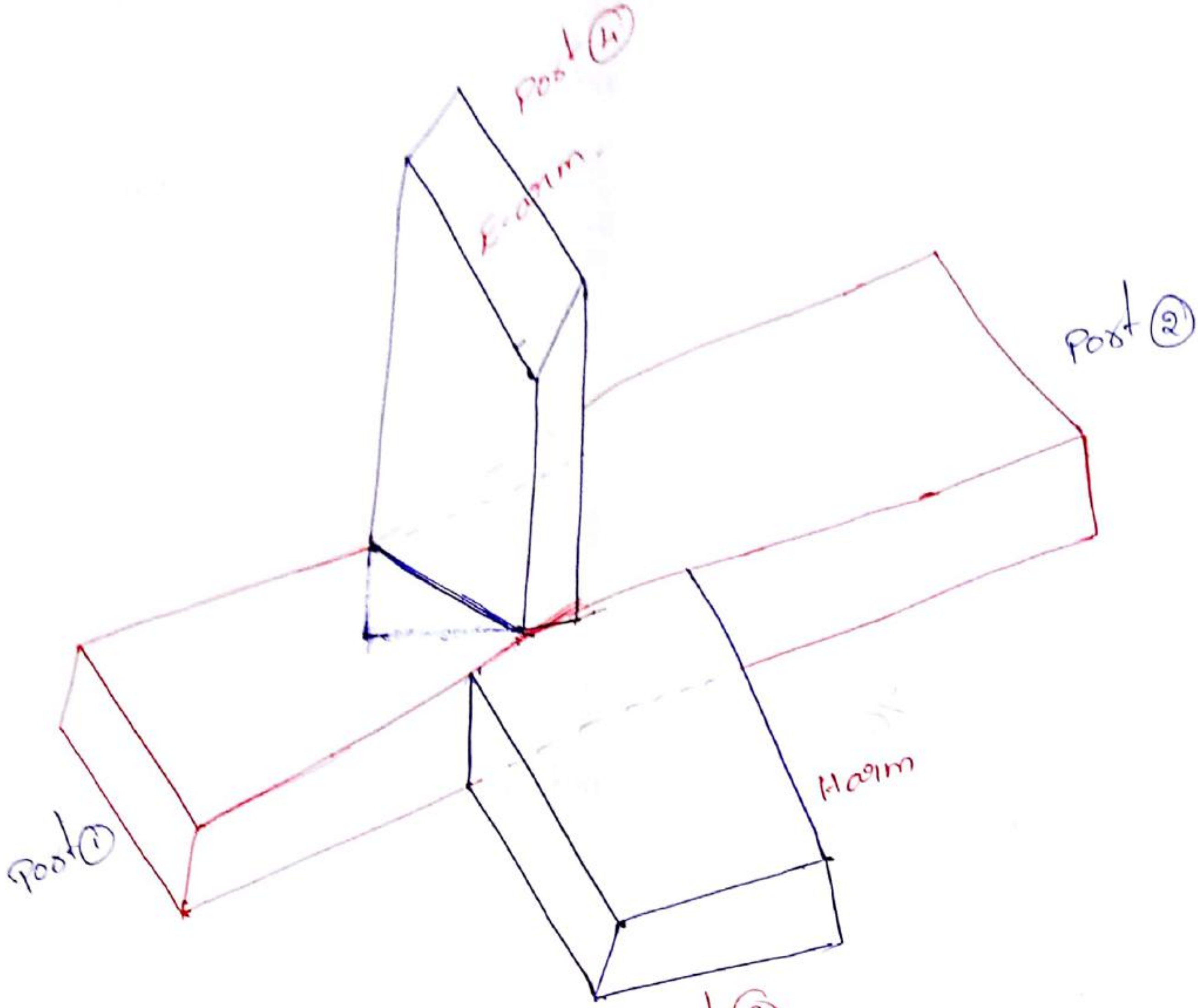
Case ③ :- $a_1 \neq 0, \ a_2 = 0, \ a_3 = 0$

$$b_1 = \frac{a_1}{2} \qquad b_2 = \frac{a_1}{2} \qquad b_3 = \frac{a_1}{\sqrt{2}}.$$

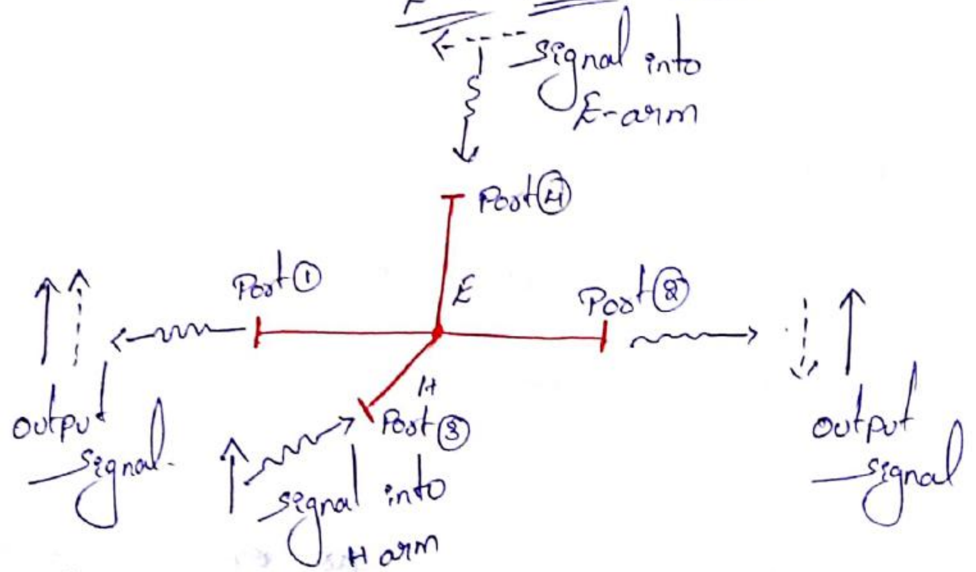
E-H plane (or) Magic Tee :-

-> Rectangular slots are cut both along the width and breadth of a long waveguide and side arms are attached.

-> Ports ① and ② are collinear arms, port ③ is the H-arm and port ④ is the E-arm.



E-H plane tee



→ The above fig., gives the four port hybrid tee junction combines the power dividing properties of both H-plane Tee & E-plane Tee & has the advantage of being completely matched at all its ports.

Using the properties of E-H plane tee, its scattering matrix can be obtained as follows.

(1) [S] is a 4×4 matrix, since there are 4-ports.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \longrightarrow (1)$$

(2) Because of H-plane section

$$\boxed{S_{23} = S_{13}} \longrightarrow (2)$$

(3) Because of E-plane Tee section

$$\boxed{S_{24} = -S_{14}} \longrightarrow (3)$$

(4) Because of geometry of the junction an input at port (3) cannot come out of port (4) since they are isolated ports & viceversa.

$$\boxed{S_{34} = S_{43} = 0} \longrightarrow (4)$$

(5) From the symmetric property, $S_{ij} = S_{ji}$.

$$\left.\begin{array}{l} \boxed{S_{12} = S_{21}}, \boxed{S_{13} = S_{31}}, \boxed{S_{14} = S_{41}} \\ \boxed{S_{23} = S_{32}} \ \boxed{S_{24} = S_{42}.} \ \boxed{S_{34} = S_{43}} \end{array}\right\} \longrightarrow (5)$$

(6) If ports (3) and (4) are perfectly matched to the junction.

$$\boxed{S_{33} = S_{44} = 0} \longrightarrow (6)$$

substituting all these properties from eqn (2) to (6) in eqn (1).

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \longrightarrow (7)$$

(7) From unitary property. $[S][S]^* = [I]$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

unknowns of S-matrix = $S_{11}, S_{12}, S_{13}, S_{14}, S_{22}$.

On multiplying.

$R_1C_1 \Rightarrow S_{11} \cdot S_{11}^* + S_{12} \cdot S_{12}^* + S_{13} \cdot S_{13}^* + S_{14} \cdot S_{14}^* = 1$

$$\Rightarrow \boxed{|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1} \longrightarrow (8)$$

$R_2C_2 \Rightarrow S_{12} \cdot S_{12}^* + S_{22} \cdot S_{22}^* + S_{13} \cdot S_{13}^* + (-S_{14})(-S_{14}^*) = 1$

$$\boxed{|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1} \longrightarrow (9)$$

$R_3C_3 \Rightarrow S_{13} \cdot S_{13}^* + S_{13} \cdot S_{13}^* = 1 \Rightarrow \boxed{\begin{matrix} |S_{13}|^2 + |S_{13}|^2 = 1 \\ S_{13} = 1/\sqrt{2} \end{matrix}} \longrightarrow (10)$

Scanned by CamScanner

$R_4 C_4 \Rightarrow S_{14} \cdot S_{14}^{*} + (-S_{14})(-S_{14}^{*}) = 1$

$$\Rightarrow \boxed{|S_{14}|^2 + |S_{14}|^2 = 1 \Rightarrow S_{14} = 1/\sqrt{2}} \longrightarrow (11)$$

On comparing equation (8) & (9)

$$|S_{11}|^2 + \cancel{|S_{12}|^2} + \cancel{|S_{13}|^2} + \cancel{|S_{14}|^2} = 1$$

$$|S_{22}|^2 + \cancel{|S_{12}|^2} + \cancel{|S_{13}|^2} + \cancel{|S_{14}|^2} = 1$$

$(-)\quad(-)\quad(-)\quad(-)\quad(-)$

$$|S_{11}|^2 - |S_{22}|^2 = 0$$

$$\boxed{S_{11} = S_{22}.} \longrightarrow (12)$$

On substituting eq (10) and (11) in eqn (8)

$$|S_{11}|^2 + |S_{12}|^2 + \frac{1}{2} + \frac{1}{2} = 1$$

$$|S_{11}|^2 + |S_{12}|^2 = 0$$

$$|S_{11}|^2 = -|S_{12}|^2$$

$$\therefore \boxed{S_{11} = S_{12} = 0} \longrightarrow (13)$$

$$\because \boxed{S_{22} = S_{11} = 0,} \longrightarrow (14)$$

this means ports (1) and (2) are also perfectly matched to the junction.

Hence in any 4-port junction, if any two ports are perfectly matched to the junction, then the remaining two ports are automatically matched to the junction.

Such a junction where in all the four ports are perfectly matched to the junction is called a "Magic Tee".

The scattering matrix "[S]" of magic Tee is obtained by substituting the scattering parameters.

$$[S] = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \end{bmatrix} \longrightarrow (15)$$

Now,

$$[b] = [S]\,[a]$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

On expansion.

Scanned by CamScanner

$$b_1 = 0 + 0 + \frac{a_3}{\sqrt{2}} + \frac{a_4}{\sqrt{2}}$$

$$\boxed{b_1 = (a_3 + a_4)\ 1/\sqrt{2}} \longrightarrow (16)$$

$$b_2 = 0 + 0 + \frac{a_3}{\sqrt{2}} - \frac{a_4}{\sqrt{2}} \Rightarrow \boxed{b_2 = \frac{1}{\sqrt{2}}(a_3 - a_4)} \longrightarrow (17)$$

$$b_3 = \frac{a_1}{\sqrt{2}} + \frac{a_2}{\sqrt{2}} + 0 + 0 \Rightarrow \boxed{b_3 = \frac{1}{\sqrt{2}}(a_1 + a_2)} \longrightarrow (18)$$

$$b_4 = \frac{a_1}{\sqrt{2}} - \frac{a_2}{\sqrt{2}} + 0 + 0 \Rightarrow \boxed{b_4 = \frac{1}{\sqrt{2}}(a_1 - a_2)} \longrightarrow (19)$$

Case (i) :- $a_3 \neq 0,\ a_1 = a_2 = a_4 = 0$

On substituting in eqn's (16), (17), (18) & (19)

$$b_1 = \frac{a_3}{\sqrt{2}} + 0 \Rightarrow \boxed{b_1 = \frac{a_3}{\sqrt{2}}}$$

$$b_2 = \frac{1}{\sqrt{2}}(a_3 - 0) \Rightarrow \boxed{b_2 = \frac{a_3}{\sqrt{2}}}$$

$$b_3 = 1/\sqrt{2}\,(0+0) \Rightarrow \boxed{b_3 = 0}$$

$$b_4 = 1/\sqrt{2}\,(0-0) \Rightarrow \boxed{b_4 = 0}$$

This is the property of H-plane tee.

Case (ii) :- $a_4 \neq 0,\ a_1 = a_2 = a_3 = 0$

$$\boxed{b_1 = \frac{a_4}{\sqrt{2}}} \quad \boxed{b_2 = \frac{-a_4}{\sqrt{2}}} \quad \boxed{b_3 = 0} \quad \boxed{b_4 = 0}$$

This is the property of E-plane Tee.

Case (iii) :- $a_1 \neq 0$, $a_2 = a_3 = a_4 = 0$

$\boxed{b_1 = 0}$ $\boxed{b_2 = 0}$ $\boxed{b_3 = \frac{a_1}{\sqrt{2}}}$ $\boxed{b_4 = \frac{a_1}{\sqrt{2}}}$

when the power is fed at Port ①, nothing comes out of Port ② even though they are collinear ports. Hence ports ① and ② are called isolated ports.

Ily an i/p at port ② cannot come out at port ①.

Ily E and H ports are isolated ports.

Case (iv) :- $a_3 = a_4$; $a_1 = a_2 = 0$

$$b_1 = \frac{1}{\sqrt{2}}(a_3 + a_4) \Rightarrow \boxed{b_1 = \frac{1}{\sqrt{2}}(2a_3)}$$

$$b_2 = \frac{1}{\sqrt{2}}(a_3 - a_4) \Rightarrow \boxed{b_2 = 0}$$

$\boxed{b_3 = 0}$ $\boxed{b_4 = 0}$

This is an additive property. Equal inputs at port ③ and port ④ result in an output at port ①.

Case (v) :- $a_1 = a_2$, $a_3 = a_4 = 0$

$\boxed{b_1 = 0}$, $\boxed{b_2 = 0}$ $b_3 = \frac{1}{\sqrt{2}}(a_1 + a_2)$ $b_4 = \frac{1}{\sqrt{2}}(a_1 - a_2)$

$\boxed{b_3 = \frac{1}{\sqrt{2}}(2a_1)}$ $\boxed{b_4 = 0}$

H-arm cancells in E arm.

Ily to case (iv).

Scattering Matrix of a Directional Coupler:-

By making use of the properties of directional coupler, the scattering parameters of a [S] matrix can be obtained.

(1) Directional coupler is a four port network. Hence [S] is of an order 4 X 4 matrix.

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \longrightarrow (1)$$

(2) In a directional coupler all four ports are perfectly matched to the junction. Hence all the diagonal elements are zero.

$$\boxed{S_{11} = S_{22} = S_{33} = S_{44} = 0} \longrightarrow (2)$$

(3) From symmetric property. $S_{ij} = S_{ji}$.

$$\left.\begin{array}{lll} S_{12} = S_{21} & & \\ S_{13} = S_{31}, & S_{23} = S_{32} & S_{34} = S_{43} \\ & S_{24} = S_{42} & \\ S_{14} = S_{41} & & \end{array}\right\} \longrightarrow (3)$$

Ideally back power is zero ($P_b = 0$). There is no coupling b/w port (1) and port (3)

$$\boxed{S_{13} = S_{31} = 0} \longrightarrow (4)$$

(4) Also there is no coupling b/w port (2) and port (4)

$$\boxed{S_{24} = S_{42} = 0} \longrightarrow (5)$$

On substituting the values of scattering parameters in eqn eqn 1

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \longrightarrow (6)$$

5 Since. $[S][S]^* = I$,

$$\begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14} & 0 & S_{34}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R_1C_1 : $|S_{12}|^2 + |S_{14}|^2 = 1 \longrightarrow (7)$

R_2C_2 : $|S_{12}|^2 + |S_{23}|^2 = 1 \longrightarrow (8)$

R_3C_3 : $|S_{23}|^2 + |S_{34}|^2 = 1 \longrightarrow (9)$

R_1C_3 : $S_{12}S_{23}^* + S_{14}S_{34}^* = 0 \longrightarrow (10)$

R_4C_4 : $|S_{14}|^2 + |S_{34}|^2 = 1 \longrightarrow (11)$

On comparing 7 & 8

$$|S_{12}|^2 + |S_{14}|^2 = 1$$
$$|S_{12}|^2 + |S_{23}|^2 = 1$$
$$(-) \quad (-) \quad (-)$$

$$|S_{14}|^2 - |S_{23}|^2 = 0$$

$$\boxed{S_{14} = S_{23}} \longrightarrow (12)$$

On comparing 9 and 11

$$|S_{23}|^2 + |S_{34}|^2 = 1$$

$$|S_{14}|^2 + |S_{34}|^2 = 1$$

$$(-) \quad (-) \quad (-)$$

$$\boxed{S_{23} = S_{14}} \longrightarrow 13$$

From eqn 10, 12 & 13

$$S_{12} \cdot S_{23}^* + S_{23} S_{34}^* = 0 \qquad S_{14}^* \text{ can be } (-S_{14})$$

$$S_{12} S_{14}^* = (-S_{14}) S_{34}$$

Let us assume that S_{12} is real & positive = "P"

$$S_{12} = S_{34} = P = S_{34}^*$$

Now eqn 10

$$P \cdot S_{23}^* + S_{23} P = 0$$

$$P[S_{23} + S_{23}^*] = 0$$

$$P \neq 0, \quad S_{23} = S_{23}^*$$

S_{23} is imaginary

$$S_{23} = jq \qquad \text{then } S_{23}^* = -jq$$

$$= S_{14}$$

$$\boxed{p^2 + q^2 = 1}$$

Now the s-matrix of directional coupler is

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} = \begin{bmatrix} 0 & P & 0 & jq \\ P & 0 & jq & 0 \\ 0 & jq & 0 & P \\ jq & 0 & P & 0 \end{bmatrix}$$

Circulator :- 3 Port Network :-

→ A lossless network results in unitary S-matrix.

→ When the S-matrix is non-reciprocal ($S_{ij} \neq S_{ji}$), (but the conditions of port match and lossless, the 3-port network is known as a circulator.)

→ A lossy 3-port network can be reciprocal and matched at all ports.

This type of network is useful as power divider, in addition it can be made to have isolation between its output ports

$$S_{11} = S_{22} = S_{33} = 0$$

→ S-matrix of a circulator is

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix} \longrightarrow A$$

Since it is a reciprocal

$$S_{ij} = S_{ji}$$

$$S_{12} = S_{21} \quad S_{13} = S_{31} \quad S_{23} = S_{32}$$

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

→ from unitary property.

$$[S][S]^* = [I]$$

$$\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & S_{12}^* & S_{13}^* \\ S_{12}^* & 0 & S_{23}^* \\ S_{13}^* & S_{23}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1C_1 \Rightarrow 0 + S_{12} \cdot S_{12}^* + S_{13} \cdot S_{13}^* = 1$

$$|S_{12}|^2 + |S_{13}|^2 = 1 \longrightarrow (1)$$

$R_2C_2 \Rightarrow$ lly $\quad |S_{12}|^2 + |S_{23}|^2 = 1 \longrightarrow (2)$

$R_3C_3 \Rightarrow \quad |S_{13}|^2 + |S_{23}|^2 = 1 \longrightarrow (3)$

(2) − (3)

$$|S_{12}|^2 + \cancel{|S_{23}|^2} = \cancel{1}$$
$$|S_{13}|^2 + \cancel{|S_{23}|^2} = \cancel{1}$$
$$(-) \quad (-) \quad (-)$$

$$|S_{12}|^2 - |S_{13}|^2 = 0$$

$$\boxed{S_{12} = S_{13}}$$

from eq (1)

$$|S_{12}|^2 + |S_{12}|^2 = 1$$

$$2|S_{12}|^2 = 1$$

$$\Rightarrow |S_{12}|^2 = 1/2 \Rightarrow \boxed{S_{12} = 1/\sqrt{2} = S_{13}}$$

$$\frac{1}{2} + |S_{23}|^2 = 1$$

$$|S_{23}|^2 = 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$

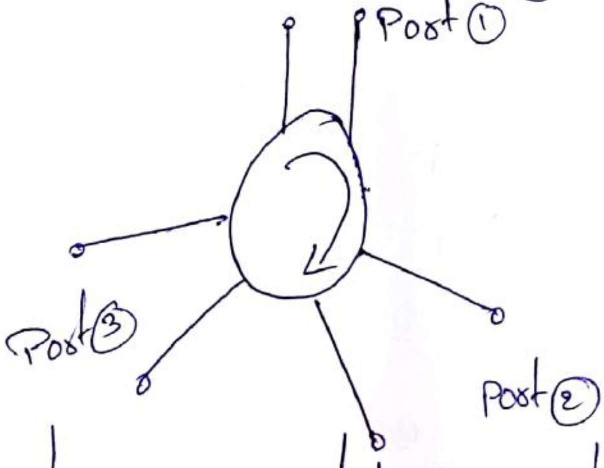
$$\boxed{S_{23} = 1/\sqrt{2}}$$

The S-matrix of a circulator 3-port is given by

$$[S] = \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

⇒ But,

Circulator is a device which has a peculiar property that each terminal is connected only to next clock wise terminal.



i.e., (i) Port 1 is connected to port ② not to Port ③

(ii) Port 2 is connected to port ③ not to Port ①

(iii) Port 3 is connected to port ① not to port ②

i.e., $S_{13} = 0$ $S_{21} = 0$ $S_{32} = 0$

$\downarrow$ i/p $\downarrow$ o/p

On substituting above values in eqn A

$$\text{A} \Rightarrow [S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 \\ 0 & 0 & S_{23} \\ S_{31} & 0 & 0 \end{bmatrix}$$

→ From unitary property,

$$[S][S]^* = [I]$$

$$\begin{bmatrix} 0 & S_{12} & 0 \\ 0 & 0 & S_{23} \\ S_{31} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & S_{12}^* & 0 \\ 0 & 0 & S_{23}^* \\ S_{31}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_i C_j$:

$$\left.\begin{array}{l} R_3 C_2 \Rightarrow S_{31} \cdot S_{12}^* = 0 \\ R_1 C_3 \Rightarrow S_{12} \cdot S_{23}^* = 0 \\ R_2 C_1 \Rightarrow S_{23} S_{31}^* = 0 \end{array}\right\}$$ this condition satifies if & only if any of the two components is zero.

lly for four port circulator.

Isolator:

$$[S][S]^* = [I] \Rightarrow \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & S_{13}^* \\ S_{21}^* & 0 & S_{23}^* \\ S_{31}^* & S_{32}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1C_1 \Rightarrow |S_{12}|^2 + |S_{13}|^2 = 1 \Rightarrow |S_{12}|^2 = 1 - |S_{13}|^2$$

$$|S_{21}|^2 + |S_{23}|^2 = 1 \Rightarrow |S_{23}|^2 = 1 - |S_{21}|^2$$

$$|S_{31}|^2 + |S_{32}|^2 = 1 \Rightarrow |S_{31}|^2 = 1 - |S_{32}|^2$$

Using zero property of S-matrix.

$R_1 C_2 \Rightarrow S_{13} S_{32}^* = 0$

$R_2 C_1 \Rightarrow S_{23} S_{13}^* = 0$ & using zero property

$R_3 C_2 \Rightarrow S_{21} \cdot S_{32}^* = 0$

$S_{32} = 0 \quad S_{21} = 0 \quad S_{13} = 0$

$|S_{12}| = 1 \quad \because S_{13} = 0$

$|S_{23}| = 1 \quad \because S_{21} = 0$

$|S_{31}| = 1 \quad \because S_{32} = 0$

$$\therefore [S] = \begin{bmatrix} 0 & S_{12} & 0 \\ 0 & 0 & S_{32} \\ S_{31} & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

for clock-wise 3 port circulator.

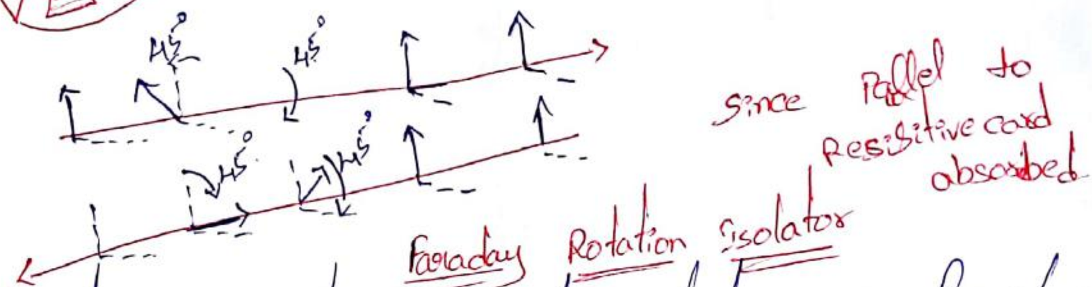
Scanned by CamScanner

Isolator:-

An isolator is a two port non-reciprocal device which produces a minimum attenuation to a wave propagation in one direction and very high attenuation in the opposite direction.

-> when inserted between a signal source and load almost all the signal power can be transmitted to the load and any reflected power from the load is not fed back to the generator output port.

This eliminates variation of source power output & frequency pulling due to changing loads.



Faraday Rotation isolator

-> The attenuation in ferrite for negative clockwise circular polarization is very small where as for positive counter clockwise circular polarization is very large.

-> Since the reverse power is absorbed in the ferrite & dissipated as heat, the maximum power handling capability of an isolator is limited.

For an ideal lossless matched isolator,

$$|S_{21}| = 1 ; \quad |S_{12}| = |S_{11}| = |S_{22}| = 0$$

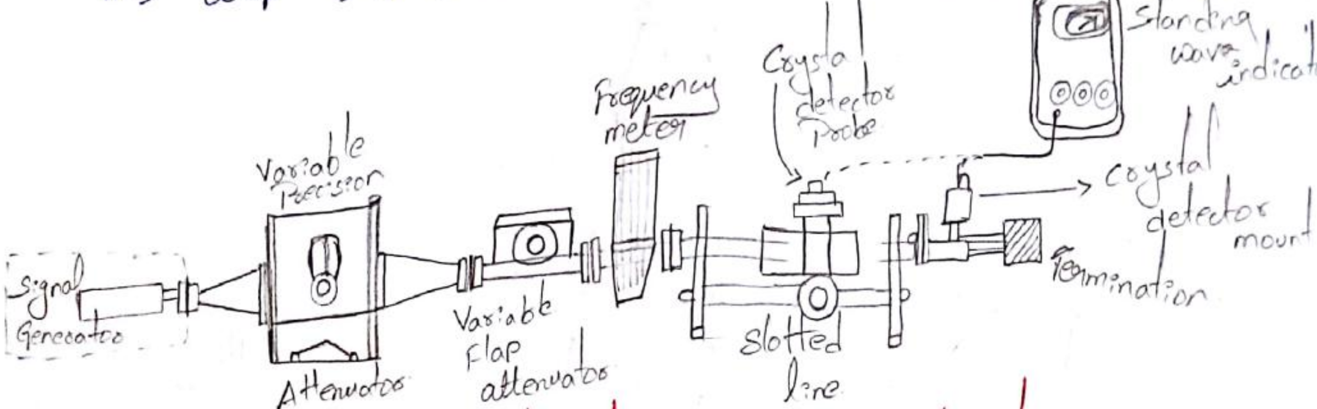
$$[S] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}.$$

Microwave Bench setup:-

Among the microwave measurements devices, a setup of microwave bench, which consists of microwave devices has a prominent place.

This whole setup, with few alternations, is able to measure many values like guide wavelength, free space wavelength, cutoff wavelength, impedance, frequency, VSWR, klystron characteristics, Gunn diode characteristics, Power measurements, etc...

This setup is a combination of different parts which as follows.



General setup of microwave bench

Signal Generator:-

It generates a microwave signal, in order of a few milliwatts. This uses velocity modulation technique to transfer continuous wave beam into milliwatt power.

A Gunn diode oscillator or a Reflex klystron tube could be an example for this microwave signal generator.

Precision Attenuator:-

This is the attenuator which selects the desired frequency and confines the output around 0 to 50db.

This is variable and can be adjusted according to the requirement.

Variable Attenuator:-

This attenuator sets the amount of attenuation. It can be understood as a fine adjustment of values, where the readings are checked against the values of precision Attenuator.

Isolator:-

This removes the signal that is not required to reach the detector mount.

Isolator allows the signal to pass through the waveguide only in one direction.

Frequency meter:-

This is the device which measures the frequency of the signal. With this frequency meter, the signal can be adjusted to its resonance frequency.

It also gives provision to couple the signal to waveguide.

Crystal detector:-

A crystal detector probe and crystal detector mount are indicated in microwave bench setup, where the detector is connected through a probe to the mount.

Standing wave indicator:-

The standing wave voltmeter provides the reading of standing wave ratio in dB.

The waveguide is slotted by some gap to adjust the clock cycles of the signal.

Signals transmitted by waveguide are forwarded through Bala cable to VSWR or CRO to measure its characteristics.

Slotted line :-

In a microwave txion line or waveguide, the electromagnetic field is considered as the sum of incident wave from the generator and the reflected wave to the generator.

The reflections indicate a mismatch or discontinuity. The magnitude and phase of the reflected wave depends upon the amplitude & phase of the reflecting impedance.

The standing waves obtained are measured to know the txion line imperfections which is necessary to have a knowledge on impedance mismatch for effective txion.

This slotted line helps in measuring the standing wave ratio of a microwave device.

Construction :-

The slotted line consists of a slotted section of a txion line, where the measurement has to be done.

It has a travelling probe carriage, to let the probe get connected wherever necessary, and the facility for attaching & detecting the instrument.

In a waveguide, a slot is made at the center of the broad side, axially. A movable probe connected to a crystal detector is inserted into the slot of the waveguide.

Operation :-

The output of the crystal detector is proportional to the square of the input voltage applied.

The movable probe permits convenient and accurate measurement at its position.

But, as the probe is moved along, its output is proportional to the standing wave pattern, which is formed inside the wave-guide.

A variable attenuator is employed here to obtain accurate result

~~Microwave Power measurements~~:-

Errors:-

Errors occurs due to the imperfect matched terminations leads to reflections hence the output power has errors.

whenever the individual components of microwave bench are loosely Coupled, leads to power leakage at each section. Hence the output Power measured is not accurate.

Precautions:-

-> Components should be connected tightly for avoiding power leakage.

-> Air cooling is required for Reflex klystron oscillator.

-> Microwave Power should not be measured directly as it effects the vision.

Measurement of Power:-

To measure microwave power, the existing methods based on their corresponding power levels are.

1. Measurement of low power (0.01mW - 10mW)
   - Bolometer technique.
2. Measurement of medium microwave power (10mW - 10W)
   - Calorimetric technique.
3. Measurement of high microwave power (>10W)
   - Calorimetric Watt meter.

Bolometer method:-

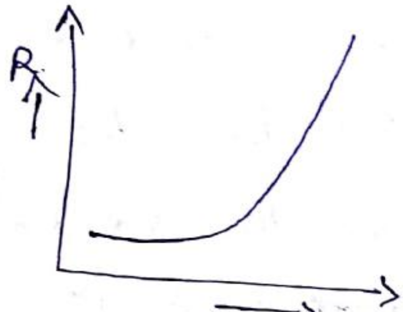
This method is used to measure low microwave power.

Bolometer is temperature sensitive device i.e, its resistance changes with temperature.

Bolometers are of two types.

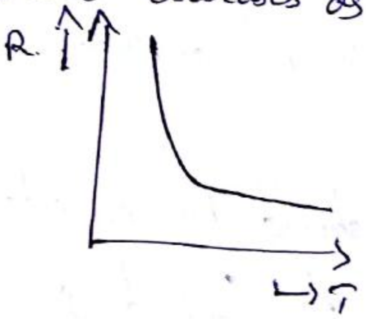
(i) positive temperature co-efficient Bolometer.
(ii) negative temperature co-efficient Bolometer.

→ In the first type the resistance increases as the temperature increases.



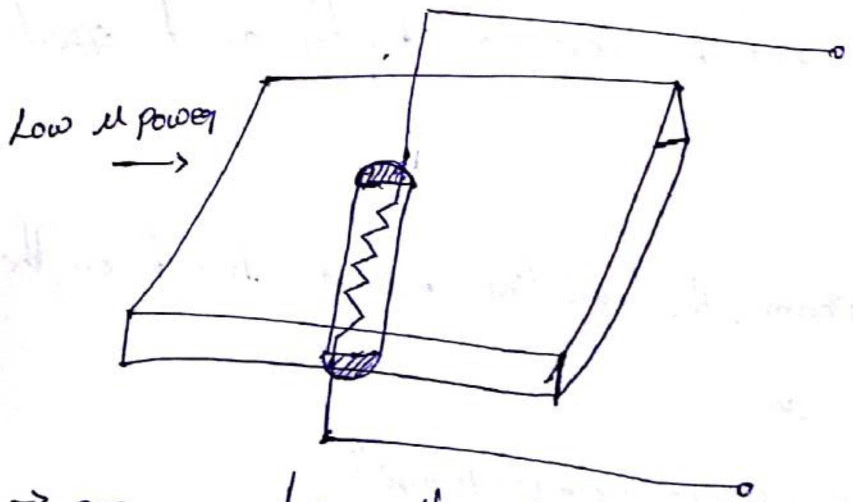
Ex:- Barreters.

→ In the second type the resistance decreases as temperature increases.



Ex:- Thermistors.

Operation:-



→ The bolometer is placed in a rectangular waveguide.

→ Before applying the low microwave power, the internal resistance of bolometer is measured and represented as "$R_1 \Omega$".

→ In the next step low microwave power is applied across the bolometer. Hence it absorbs certain power.

Scanned by CamScanner

-> The absorbed power is dissipated in the form of heat. Due to this process, the resistance of Bolometer changed to $R_2\ \Omega$.

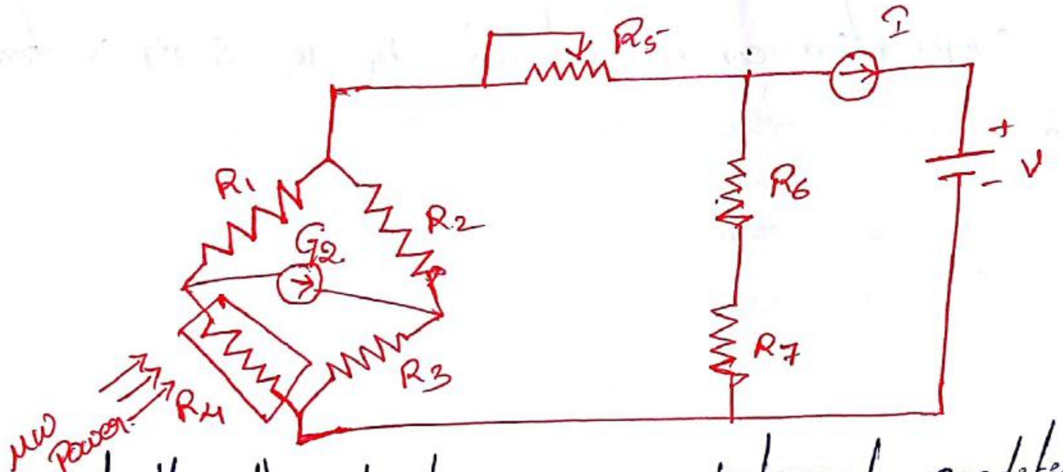
-> The difference in the resistance is $R_{12} = R_1 - R_2\ \Omega$.

-> R_{12} is proportional to the applied microwave power.

Limitations:-

Due to non-linear characteristics of bolometer, the obtained readings in this method has less accuracy.

To obtain accurate low power measurements. Balanced Bridge bolometer method is used.



Initially, the bridge is in balanced condition. On adjusting the resistor R_5 the bridge will be balanced condition.

-> The resistor R_4 indicates the internal resistance of bolometer.

-> Before applying low microwave power, the resistance of Bolometer should be measured.

Let the voltage of battery be "E_1" at balance.

-> Now low microwave power is applied across the bridge then the bolometer absorbs certain power and heats up.

This absorbed power is dissipated in the form of heat. Due to this process the resistance of bolometer changes.

So the bridge is in unbalanced condition & applied power changes to "E_2".

→ To get back the balanced condition, External power should be applied across the bridge. This is the required power and indicated by the Galvanometer. ($E_1 \sim E_2$)

Alternatively, the detector "G" can directly take the readings interms of microwave power when the bridge is unbalanced & balanced. Since, bolometers are temperature sensitive, some form of temperature compensation has to be used to avoid the errors.

The temperature compensation can be achieved by R_6 & R_7 resistors.

Measurement of medium power :-

Scanned by CamScanner

In this method at the input section, input load & (22)
input temperature guage exists in the arm ① and arm ② of the bridge.

On applying a medium microwave power, the input load absorbs certain power and heats up. so the temperature guage heats up and changes its temperature as well as resistance.

This leads to unbalancing condition of the Bridge.

To get back the balanced condition the external power is applied at the output.

So, the compensation load absorbs the power and dissipated absorbed power towards compensation temperature guage.

So, on absorbing certain power, the temperature guage heats up, which changes its temperature as well as resistance. Hence the bridge is balanced.

The required power to make the bridge balance is the medium power and its reading is provided by the external power meter.

Measurement of High Power:-

To measure high microwave powers, the calorimetric watt meters were used.

These are two types

(i) Dry type

(ii) Flow type.

-> In dry type, a co-axial cable filled by a dielectric was used.

→ In flow type, circulatting water or oil or any liquid was used in the design.

→ Before applying high microwave power, the temperature of circulating water (T_1 °c) was measured.

On applying high microwave power to the circulating water. it's temperature changes to T_2 °c.

→ The difference in the temperature is T_{12} c = T_1 c − T_2 °c.

→ This difference in temperature is proportional to the applied high microwave power.

→ On substituting T_1 and T_2 values in power measurements equations the required high power is to be measured.

The Power measurement equation is given by

$$\boxed{P = \frac{R \cdot K \cdot \rho (T_2 - T_1)}{4.18}}$$

where

P = measured Power in watts.

R = rate of flow in (cm³/s)

K = specific heat in cal/g.

ρ = Specific gravity in g/cm³

$T_2 - T_1$ = temperature difference in °C.

Attenuation Measurements :-

Microwave devices & component almost provide some degree of attenuation.

The attenuation is the ratio of input power to the output power & is normally expressed in decibels.

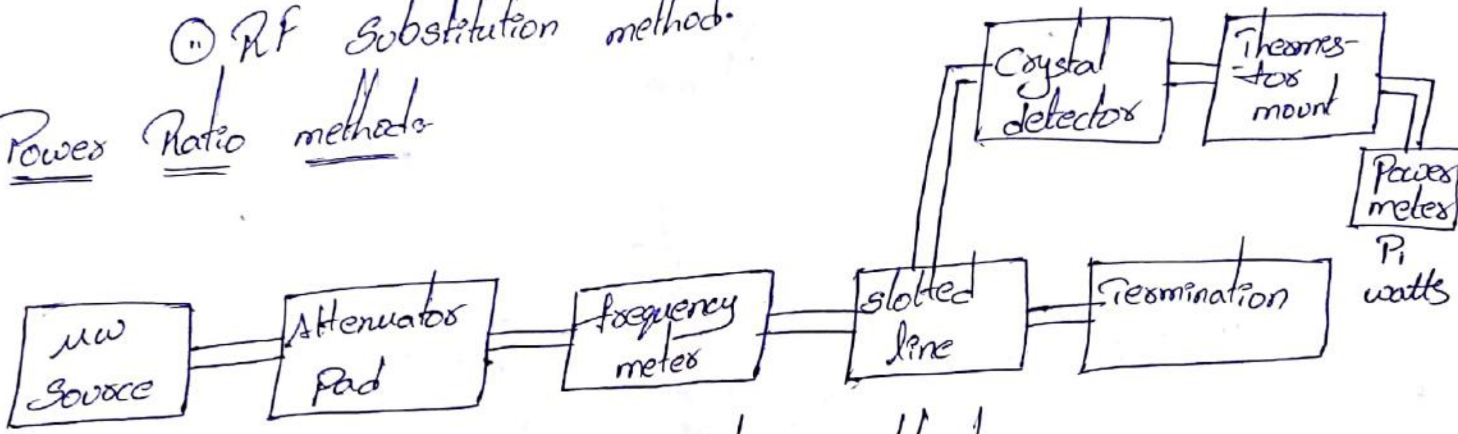
$$\text{Attenuation (in dBs)} = 10 \log_{10}\left(P_{in}/P_{out}\right)$$

The amount of attenuation can be measured by two methods.

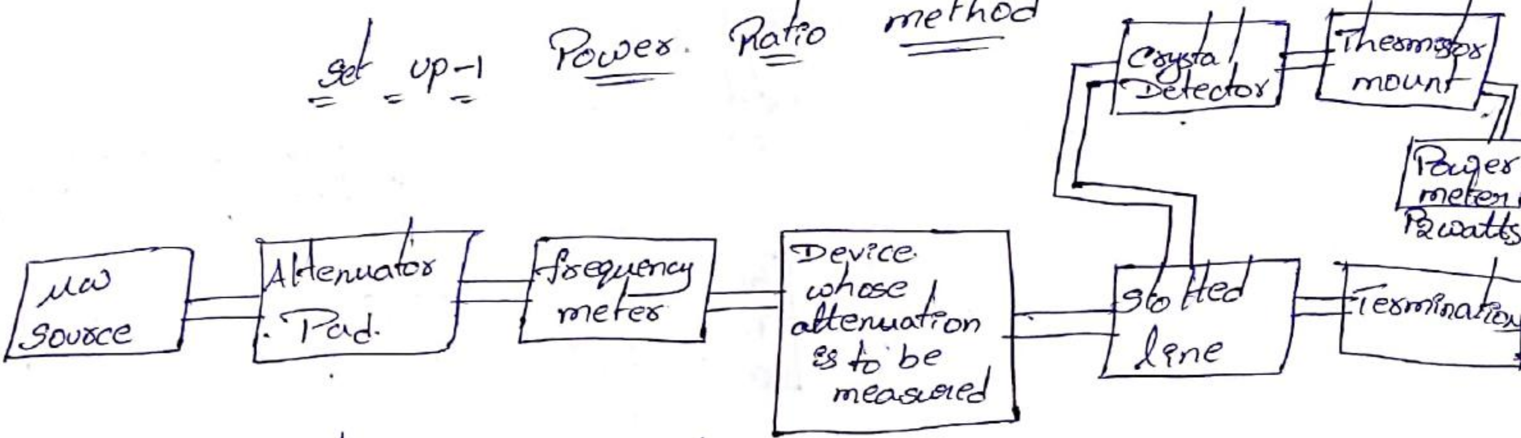
(i) Power Ratio method.

(ii) RF substitution method.

(i) Power Ratio method:-



Set up-1 Power Ratio method



Set up-2 Power ratio method.

-> Here we measure attenuation by using power Ratio method.

-> By using set up 1, measure power P_1.

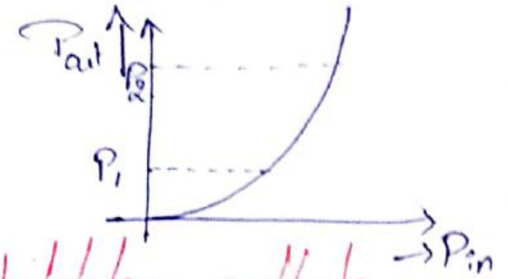
-> By using set up 2, with including device whose attenuation is to be measured, we can measure Power P_2.

-> The ratio Power P_1/P_2 expressed in decibels gives attenuation.

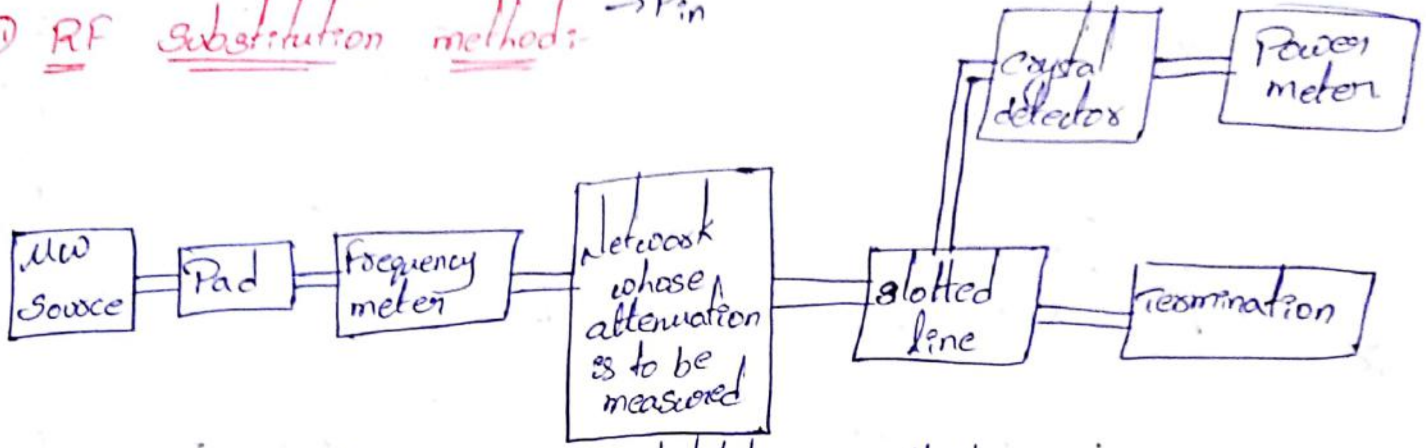
Drawback:-

Here attenuation corresponds two Power Position P_1 & P_2 on the Power meter.

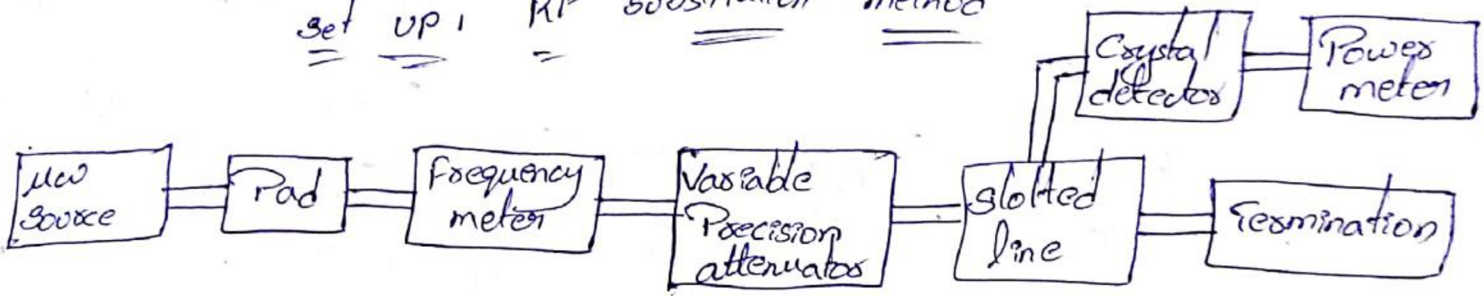
→ The attenuation calculated will not be accurate if the input Power is low, if the network is large, due to non-linear characteristics



(ii) RF substitution method:-



set up 1 RF substitution method



set up 2 RF substitution method

This method overcomes the drawback of Power ratio method. Since, here the attenuation is measured at a single Power Position.

→ The method consists of measuring the output power say "P" by including the network whose attenuation is to be measured in set up 1.

→ In set up 2 this network is replaced by precision attenuator, which can be adjusted to obtain the same Power "P" as measured in setup 1.

→ Under this condition the attenuation read on the precision attenuator could give attenuation of the network directly.

Measurement of Frequency :-

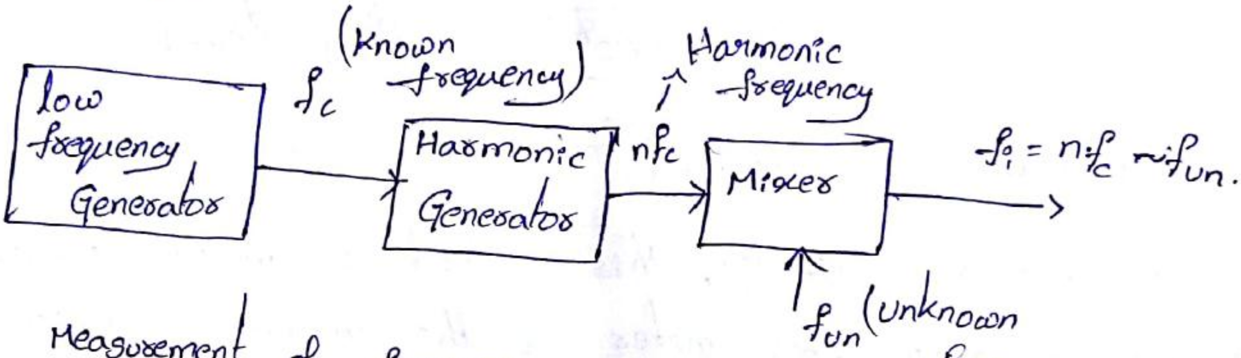
Microwave frequencies can be measured by following methods.

(i) electronic method.

(ii) slotted line method

(iii) Cavity wave meter method } → Mechanical method.

(i) Electronic method :-



Measurement of frequency by using electronic method.

→ This method is more accurate but expensive (high cost). Here unknown frequency is compared with Harmonics of a known lower frequency by the use of mixer & low frequency generator, harmonic generator.

(ii) slotted line method :-

In this method by the use of slotted line the frequency is measured.

There exists a relationship between λ_0, λ_g & λ_c

$$\frac{1}{\lambda_0^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2} \longrightarrow (1)$$

we know that

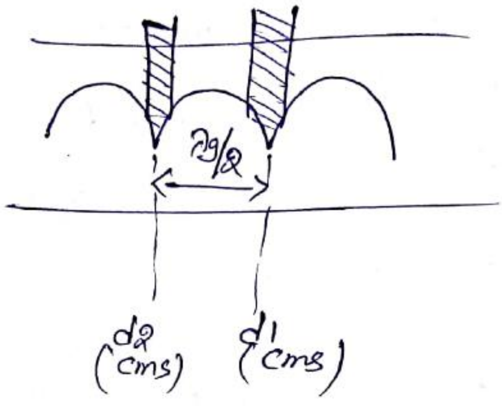
$$\lambda_c = 2a \longrightarrow (2) \text{ for } TE_{10} \text{ mode.}$$

where a: wider dimension of rectangular waveguide.

λ_g can be measured from the distance b/w maxima & minima of the slotted line,

$$\frac{\lambda_g}{2} = d_2 - d_1 \longrightarrow (3)$$

By substituting eqn (2) and (3) in eqn (1), we can find λ_0.



Hence the frequency is

$$f = c/\lambda_0$$

$$\boxed{f = \frac{3 \times 10^8}{\lambda_0}}$$

(iii) Cavity wave meter Method:-

To measure frequency, this method is mostly used. A resonant cavity wave meter is the microwave analog of tuned resonant circuit.

They are of two types.

(i) Transmissive cavities.

(ii) Absorption cavities.

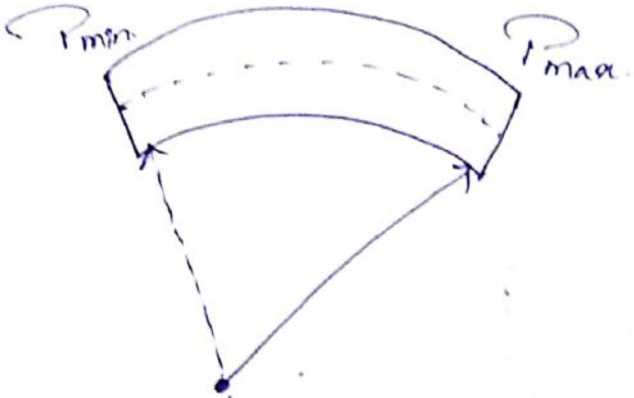
Transmissive cavities passes only the signal frequency to which they are tuned.

Absorption cavities which attenuate the signal frequency to which they are tuned. This type is preferred for laboratory frequency of measurement.

Cavity wave meters are simple, rugged and highly accurate. The frequency is determined by physical dimensions and it is given by.

$$f_0 = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{P}{d}\right)^2}$$

MW Source → Pad → Absorption cavity wave meter → Crystal Detector → Power meter



Using absorption type cavity wave meter

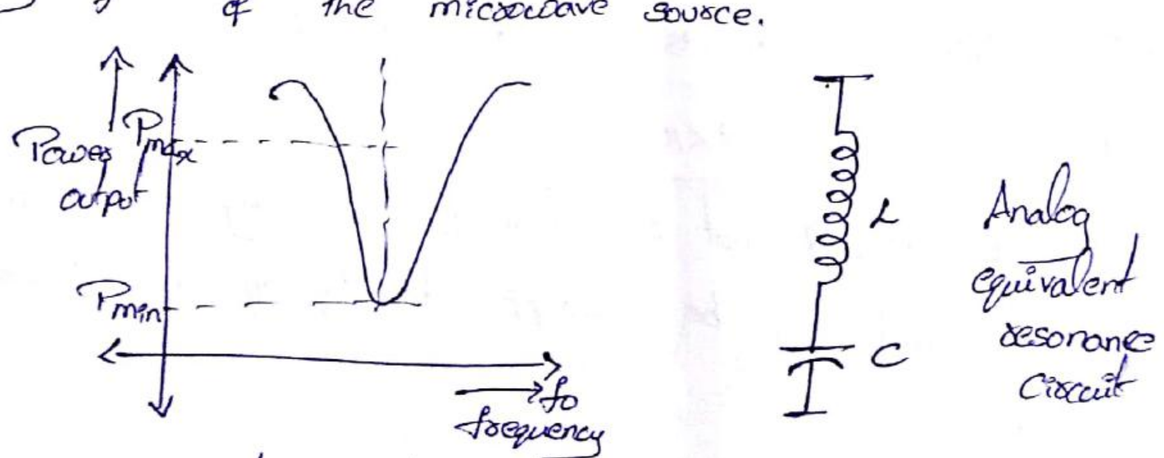
The SWR meter is replaced by a power meter.

→ Initially, the variable attenuator (Pad) is varied as to get full scale reading. Consider

f = be the frequency of microwave source.

f' = be the knob setting of the cavity.

→ Now the wave meter is adjusted from the "f" to new value until the reading on the power meter dips to a minimum (P_{min}) value. It indicates that the absorption cavity wave meter is now at resonance and the new value when this dip occurs will be the frequency "f" of the microwave source.

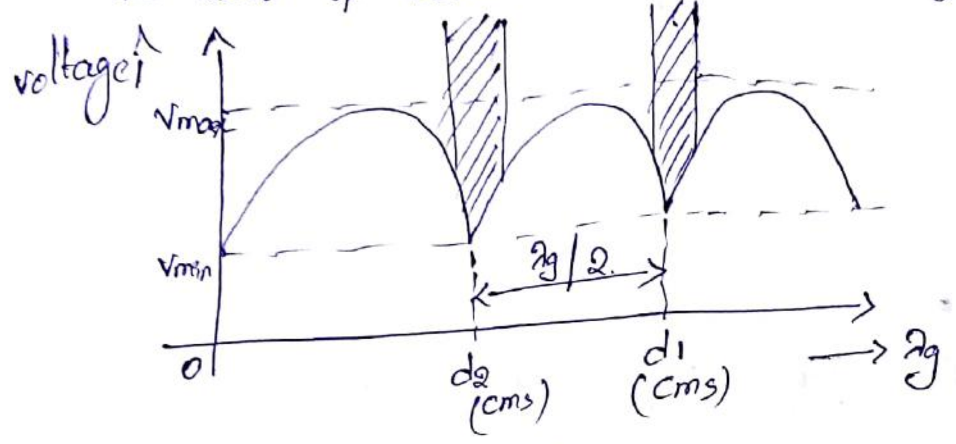


Absorption cavity characteristics and its Analog equivalent

Measurement of VSWR (voltage standing wave Ratio) :-

Any mismatched load leads to reflected waves resulting in standing waves along the length of the line.

The ratio of maximum to minimum voltage gives the VSWR.



$$VSWR = S = \frac{V_{max}}{V_{min}} = \frac{1+\rho}{1-\rho} \longrightarrow (1)$$

where ρ = reflection co-efficient = $\dfrac{P_{reflected}}{P_{incident}}$

S = varies from 1 to ∞

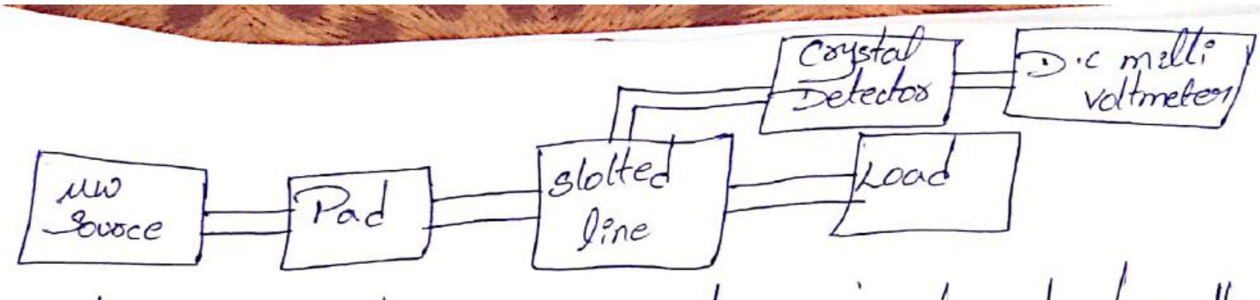
ρ = varies from 0 to ∞

Hence minimum value of S is unity.

1. If $S < 10$ then VSWR is called low VSWR.
2. If $S > 10$ then VSWR is called high VSWR.

Measurement of low VSWR ($S < 10$) :-

The values of VSWR not exceeding 10 are very easily measured with the setup and can be read off directly on the VSWR meter calibrated.



The measurement basically consists of simply adjusting the attenuator ① to give an adequate reading on the meter, which is a D.C milli-volt meter.

The probe on the slotted waveguide is moved to get maximum reading on the meter (V_{max}).

The attenuation is now adjusted to get full scale reading. & is noted down.

Next the probe on the slotted line is adjusted to get minimum reading on the meter (V_{min}).

The ratio of first reading to the second reading gives the VSWR.

$$\boxed{VSWR = S = \frac{V_{max}}{V_{min}}}$$

⇒ The meter will be congested and the measurement will not be accurate for $S > 10$.

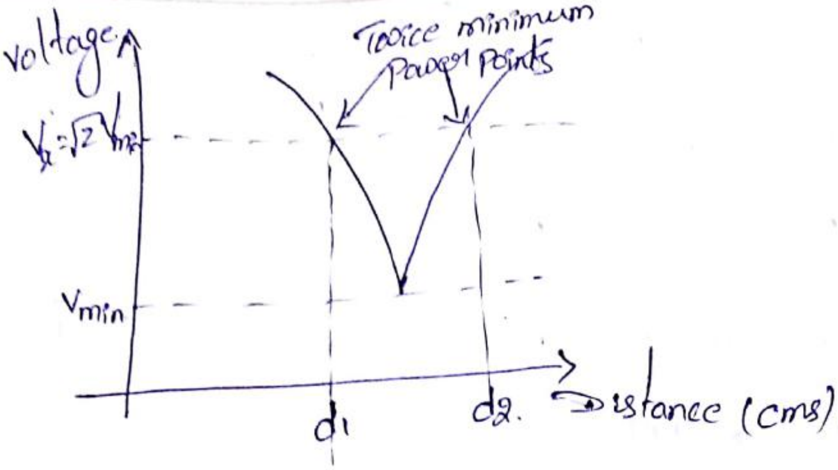
Measurement of High VSWR ($S>10$) :-

In this method, the probe is inserted to a depth where the minimum can be read without difficulty.

The probe is then moved to a point where the power is twice the minimum.

Let this position be denoted by "d_1".

The probe is then moved to twice the power point on the other side of the minimum "d_2".



$$P_{min} \propto V_{min}^2 \qquad V_{min} = \frac{V_x}{\sqrt{2}}$$

$$P_{min} \propto \frac{V_x^2}{2}$$

$$2P_{min} \propto V_x^2$$

$$\Rightarrow \frac{V_{min}^2}{V_x^2} = \frac{P_{min}}{2P_{min}}$$

$$\Rightarrow \frac{V_{min}^2}{V_x^2} = \frac{1}{2}$$

$$\Rightarrow V_x^2 = 2V_{min}^2 \Rightarrow V_x = \sqrt{2}\, V_{min}.$$

For TE_{10} mode, $\lambda_c = 2a$

$$\lambda_0 = c/f$$

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-(\lambda_0/\lambda_c)^2}}$$

then VSWR can be calculated using the empirical relation,

$$\boxed{VSWR = \frac{\lambda_g}{\pi(d_2 - d_1)}}$$

Measurement of Q of a Cavity Resonator:- (27)

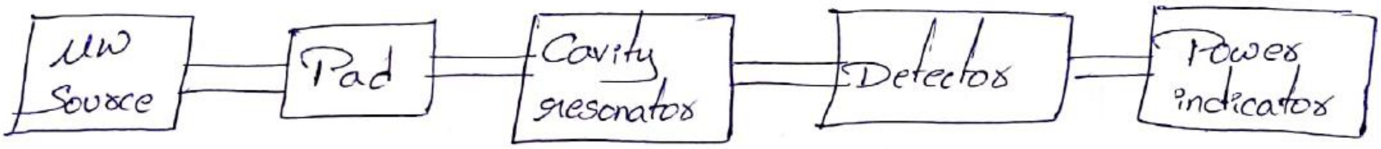
There are several methods for measuring the Q of a cavity resonator

① Transmission Method

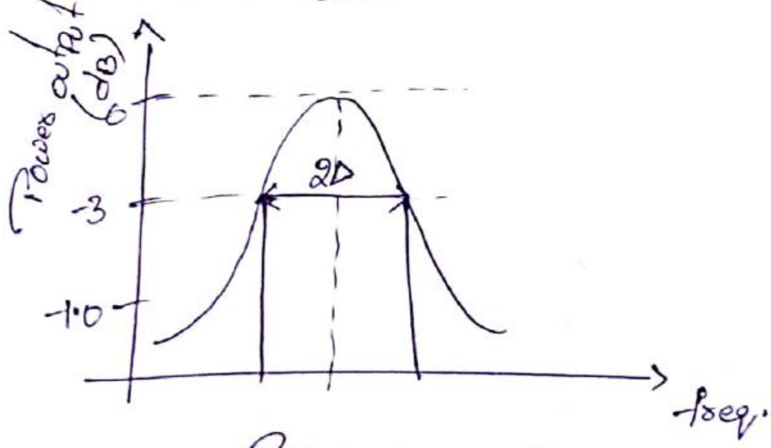
2. Impedance Measurement and

3. Transient decay or decrement method.

Among these, transmission method is the simplest and the setup for transmission method of measuring Q is.



In this method, the cavity resonator is used as a transmission device and the output signal is measured as a function of the frequency resulting in the resonance curve.



Resonance curve

By varying the frequency of microwave source and keeping signal level constant, the output power is measured.

Alternatively cavity can be tuned by keeping both signal level and frequency constant and output power measured.

From the resonance curve half power bandwidth (2Δ) can be obtained.

$$2\Delta = \pm \frac{1}{Q_L}$$

where,

Q_L = Loaded value

$$Q_L = \pm \frac{1}{2\Delta} = \pm \frac{\omega}{2(\omega - \omega_0)}$$

If the coupling b/w μw source and cavity and that b/w detector and cavity are neglected,

$$Q_L = Q_0 \text{ (unloaded Q)}$$

In case of very high "Q" systems due to narrow band of operation, the accuracy of this method is poor.

Measurement of impedance :-

Impedance at microwave frequencies can be measured using any of the following methods.

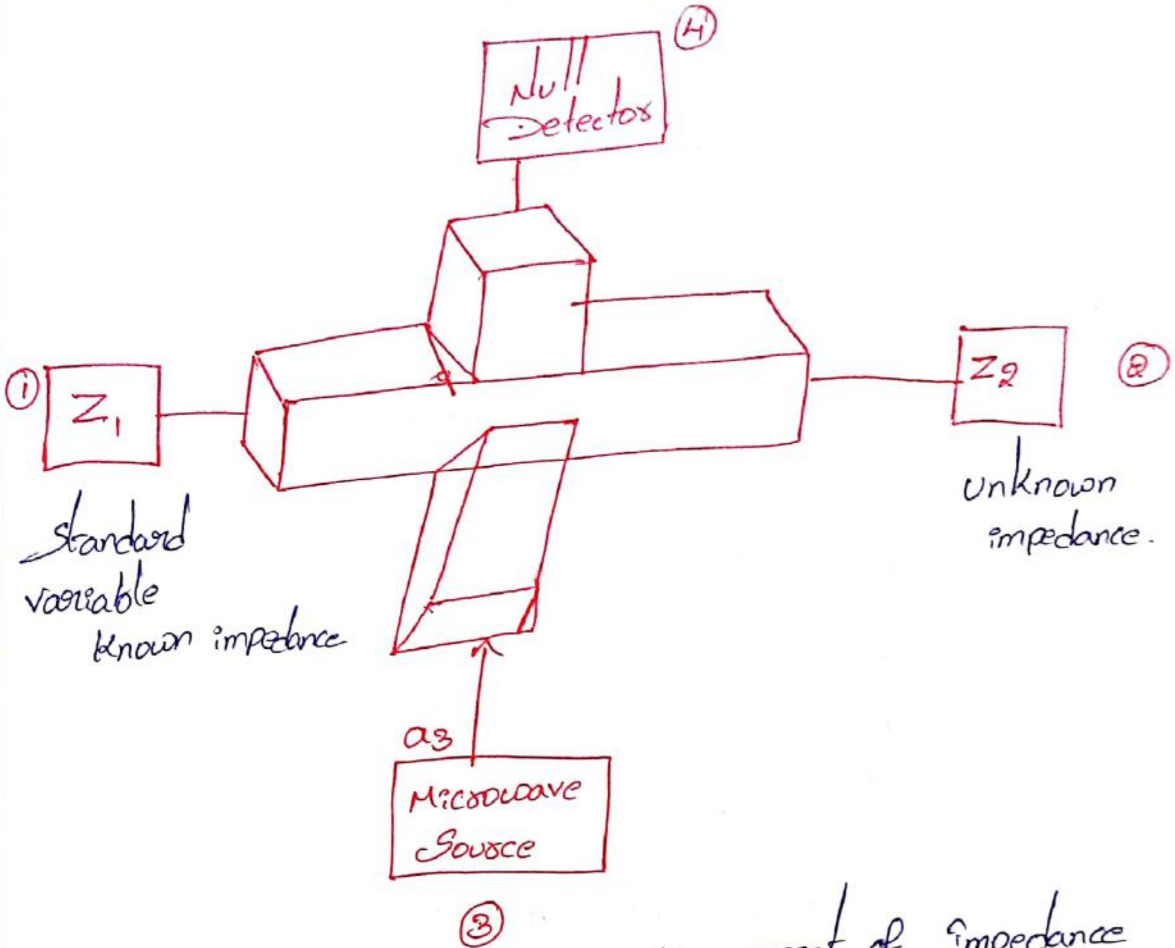
(i) Using ~~a~~ magic - Tee

(ii) Using slotted line. &

(iii) Using reflectometer.

Measurement of Impedance using magic Tee :-

A magic - Tee has been used in the form a bridge for measuring impedance.



Magic Tee Measurement of Impedance

Microwave Source is connected in arm ③ and a null detector in arm ④.

The unknown impedance is connected in arm ② and a standard variable known impedance in arm ①.

Using the properties of magic Tee, the power from microwave source (a_3) gets divided equally b/w arm ① and arm ② $\frac{a_3}{\sqrt{2}}$.

These impedances are not equal to characteristic impedance Z_0 & hence there will be reflections from arms ① and ②.

If ρ_1 and ρ_2 are the reflection co-efficients, $\frac{\rho_1 a_3}{\sqrt{2}}$ & $\frac{\rho_2 a_3}{\sqrt{2}}$ enters the magic T junction from arms ① and ② and the output from detector can be calculated.

The net wave reaching the null detector

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{2} a_3 \rho_1\right) - \frac{1}{2}\left(\frac{1}{\sqrt{2}} a_3 \rho_2\right)$$

$$= \frac{1}{2\sqrt{2}} a_3 (\rho_1 - \rho_2)$$

For perfect balancing of the bridge the above eqn is equated to zero.

$$\Rightarrow \frac{1}{2\sqrt{2}} a_3 (\rho_1 - \rho_2) = 0$$

$$\rho_1 - \rho_2 = 0$$

$$\Rightarrow \boxed{\rho_1 = \rho_2}$$

where ρ_1 = reflection co-efficient of z_1

$$\boxed{\rho_1 = \frac{Z_1 - Z_3}{Z_1 + Z_3}}$$

ρ_2 = reflection co-efficient of Z_2

$$\boxed{\rho_2 = \frac{Z_2 - Z_3}{Z_2 + Z_3}}$$

$$\frac{Z_1 - Z_3}{Z_1 + Z_3} = \frac{Z_2 - Z_3}{Z_2 + Z_3}$$

$$\Rightarrow \boxed{Z_1 = Z_2}$$

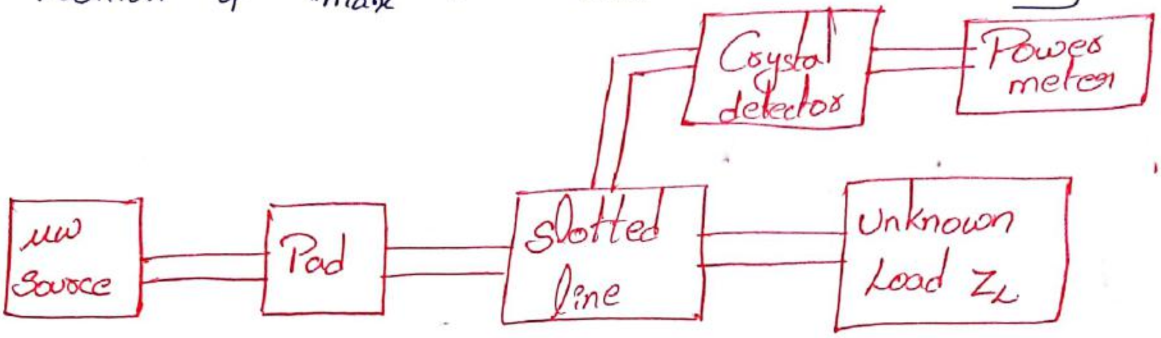
$$\boxed{R_1 + jX_1 = R_2 + jX_2} \Rightarrow \begin{array}{l} R_1 = R_2 \\ X_1 = X_2 \end{array}$$

Thus the unknown impedance can be measured by adjusting the standard variable impedance till the bridge is balanced and both the impedances become equal.

Measurement of Impedance using slotted lines

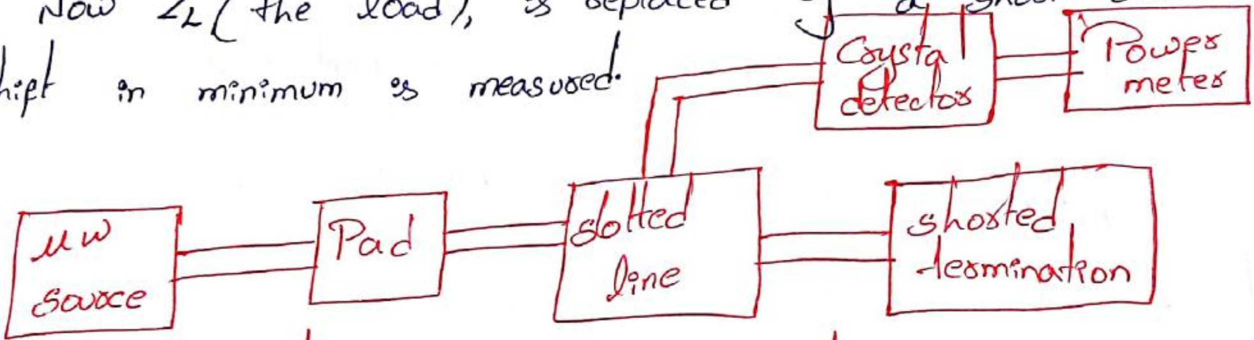
→ Incident and reflected waves present are proportional to the mismatch of the load under test resulting in standing waves.

Using slotted waveguide and with the load Z_L in the circuit, the position of V_{max} and V_{min} can be accurately determined.



setup1, Impedance Measurement using slotted line

Now Z_L (the load), is replaced by a short circuit and the shift in minimum is measured.

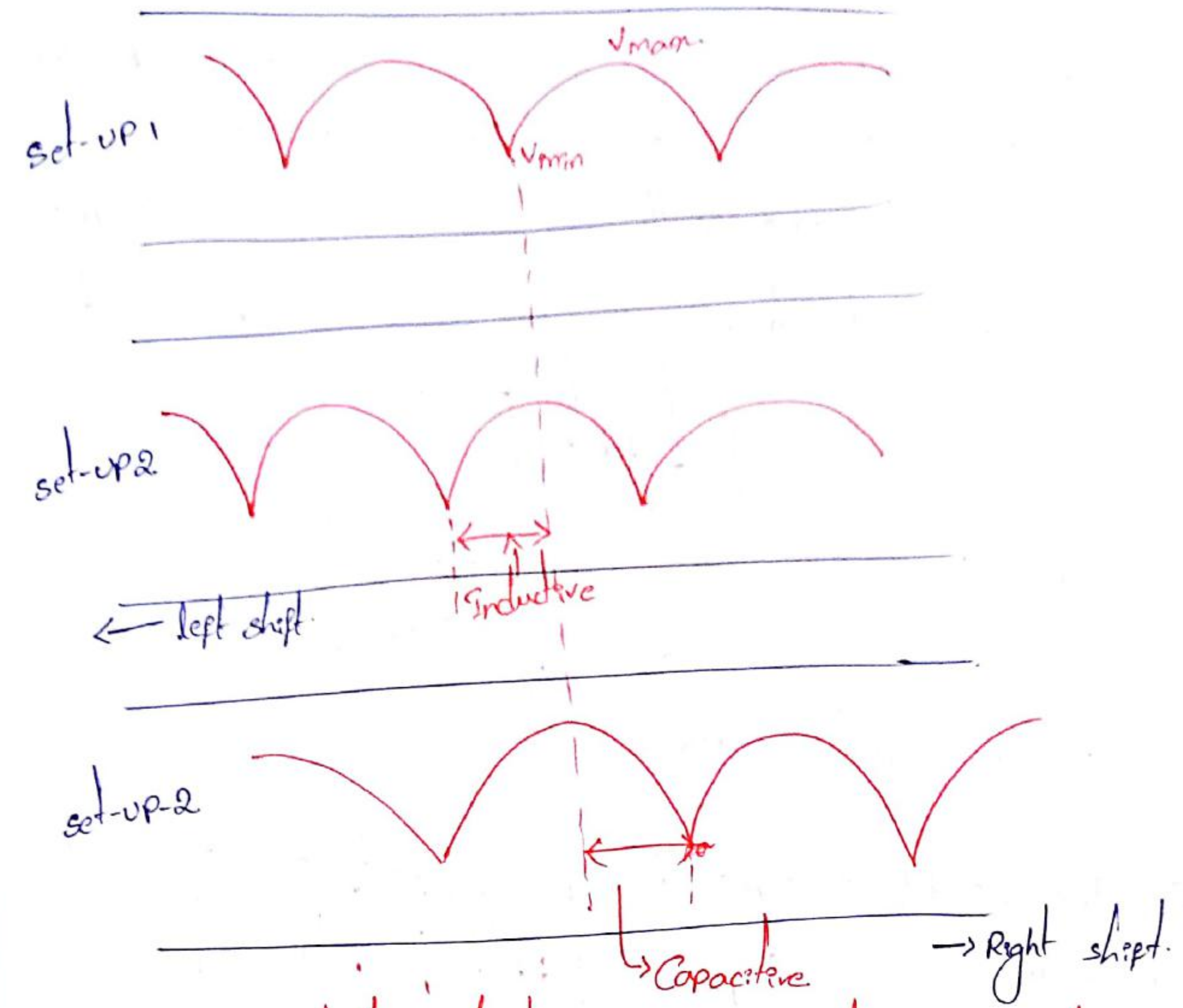


setup2, Impedance Measurement using slotted line

If the minimum is shifted to the left, then the impedan[ce is] inductive and if it shifts to right it is capacitive.

Unknown impedance can be obtained by usual methods usi[ng] the data recorded and a smith chart.

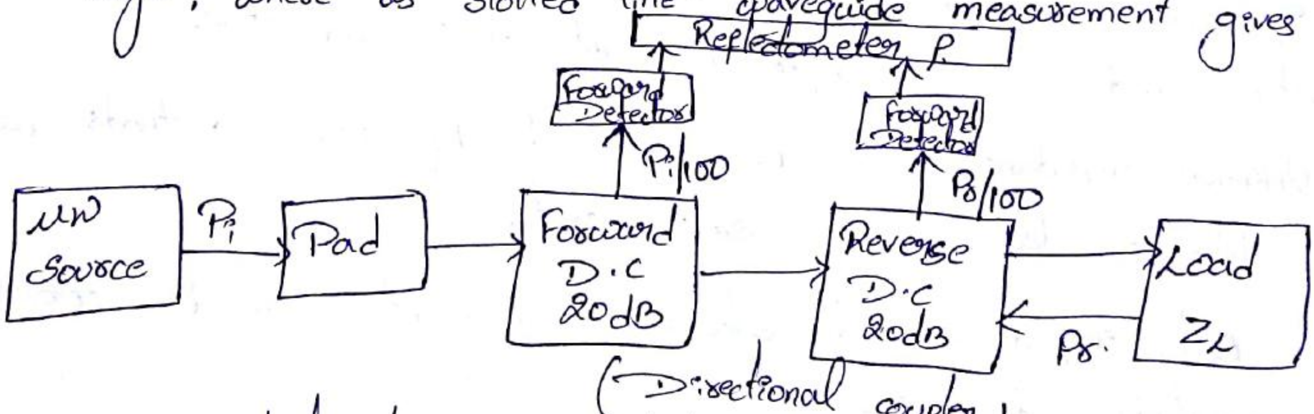
Both, impedance and reflection coefficient can be obtaine[d] in magnitude and phase.



Output standing waves of set up 1 & set up 2

Measurement of Impedance Using Reflectometer :-

The reflectometer shows the magnitude of impedance but not the phase angle, where as slotted line waveguide measurement gives both.



Reflectometer $\rho = \sqrt{\dfrac{\text{Reflected Power}}{\text{Incident Power}}}$, $\rho = \sqrt{\dfrac{P_r/100}{P_i/100}}$

Setup for measuring Impedance using Reflectometer

Here, two directional couplers are used to sample the incident Power P_i, & reflected power P_r, from the load.

Both the directional couplers are identical.

The magnitude of the reflection co-efficients can be directly obtained on the reflectometer from which impedance can be calculated.

From reflectometer reading

$$\rho = \sqrt{P_r/P_i}$$

Knowing ρ, we can calculate VSWR and impedance by using the relations.

$$S = \frac{1+\rho}{1-\rho} \quad \& \quad \frac{Z - Z_g}{Z + Z_g} = \rho$$

where

Z_g = waveguide impedance.

Z = Unknown impedance.

Due to directional Property of the couplers, there will be no interference b/w forward & Reverse waves.

The input power is kept at low level by means of pad.

The reflectometer accuracy is greatest at low VSWR. (i.e., low reflection co-efficient)

MICROWAVE TUBES

* At microwave frequencies, the size of electronic devices required for generation of microwave energy becomes smaller and smaller.
* This results in lower power handling capability and increased noise levels.
* Electronic devices such as tubes and transistors will be required even at microwave frequencies.
* Conventional triodes, tetrodes, and pentodes are useful only at low microwave frequencies. Special tubes would be required even at UHF frequencies (300 – 3000 MHZ) as Conventional tubes have certain limitations at microwave frequencies.

HIGH FREQUENCY LIMITATIONS OF CONVENTIONAL TUBES:

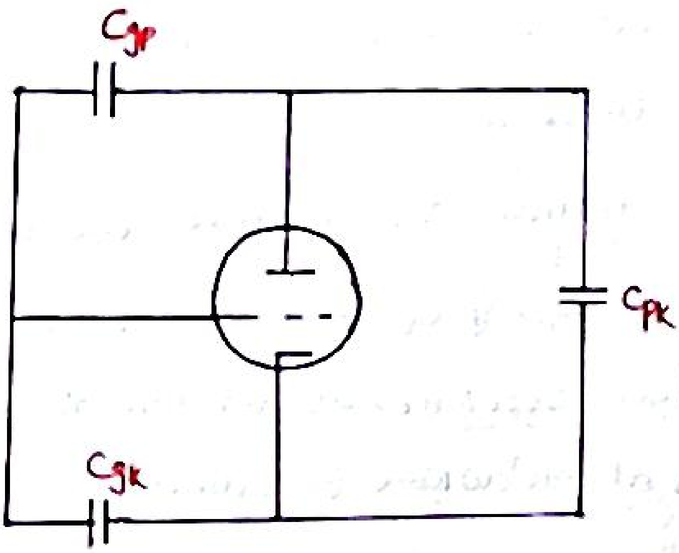
* The Conventional devices (tubes & transistors) cannot be used for frequencies > 100MHZ because of the following effects.

1. Inter electrode Capacitance effect
2. Lead Inductance effect
3. Transit time effect
4. Gain bandwidth limitation
5. Effect due to RF losses
6. Effect due to Radiation losses.

1. Inter Electrode Capacitance Effect:

* As frequency increases, the reactance $X_C = \frac{1}{2\pi f C}$, decreases and the output Voltage decreases due to shunting effect.

* Because at higher frequencies X_C becomes almost a short C_{gp}, C_{gk} and C_{pk} are the Inter electrode Capacitance's which Come into effect.



* The effect of Inter electrode Capacitance Can be reduced by decreasing the area of the electrodes.

2. Lead Inductance effect:

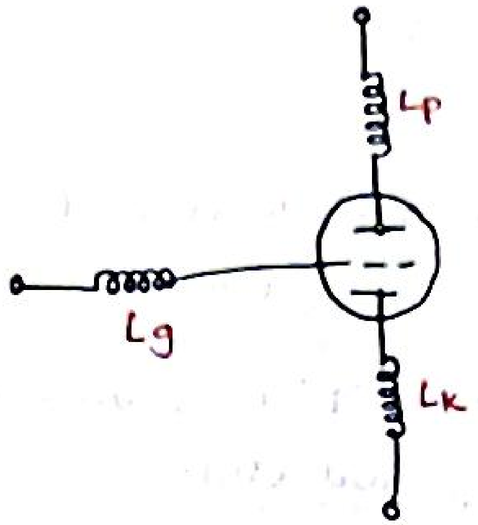
* As frequency increases, the reactance $X_L = 2\pi f L$ increases and hence the voltages appearing at the active electrodes are less than the voltages at the base pins.

* This results in reduced gain for the tube amplifier. L_k, L_p and L_g are the lead inductances that limit the performance of tube.

* The effect of Lead inductance can be minimized by decreasing L. Since L is proportional to reactance, L can be decreased by using large sized short leads without base pins

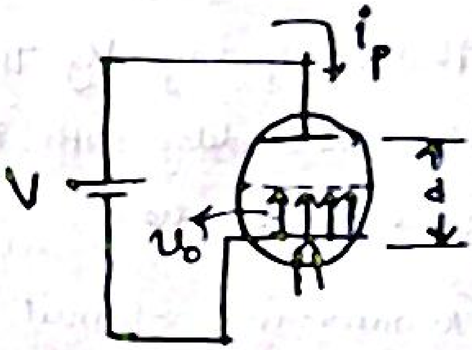
Scanned By Scanner Go

that is by increasing 'n' and decreasing l. This however reduces the power handling capability



3 TRANSIT TIME EFFECT:

* The time taken by electron to travel from cathode to anode is called Transit time.



Transit time $\tau = \frac{d}{v_0}$

Static energy of electrons $= eV$

kinetic energy $= \frac{1}{2}mv_0^2$

Under equilibrium, Static energy = kinetic energy

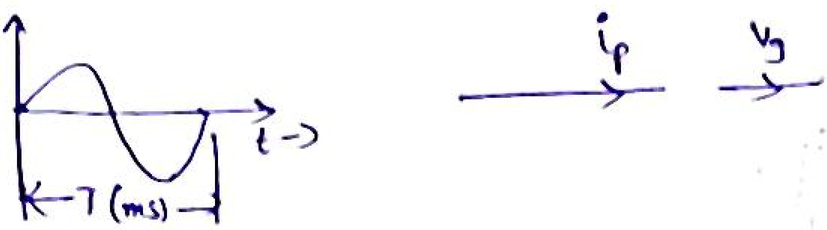
$$eV = \frac{1}{2}mv_0^2$$

$$v_0^2 = \frac{2eV}{m}$$

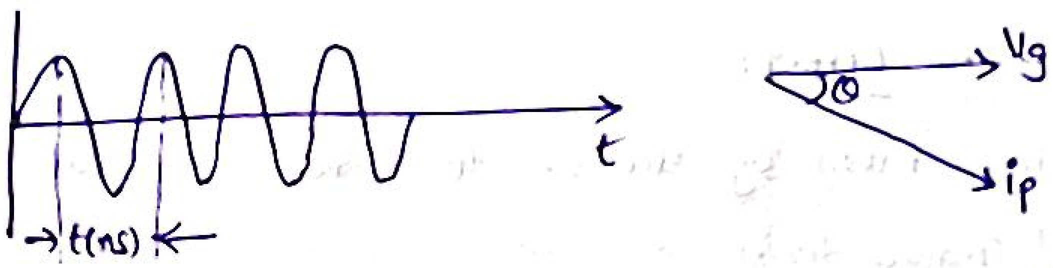
$$v_0 = \sqrt{\frac{2eV}{m}}$$

$$\tau = \frac{d}{v_0} = \frac{d}{\sqrt{\frac{2eV}{m}}}$$

* At low frequencies, transit time is negligible compared to the period of signal



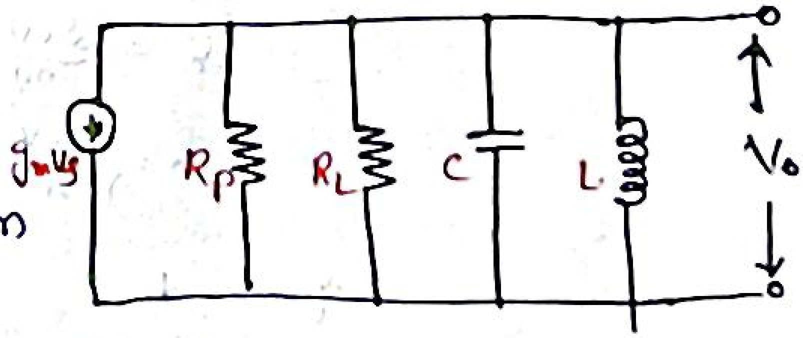
* That is, both V_g and i_p are in phase. Therefore the plate current i_p responds immediately to changes in grid voltage V_g.
* At high frequencies the transit time 'τ' is comparable with the period of the signal which is very small (ns).



* That is, I_p lags V_g. Therefore change in plate current occurs after finite delay with respect to change in grid voltage V_g, I_p and V_g are out of phase.
* To minimize transit time, the separation between electrodes can be decreased and the plate to cathode potential 'V' can be increased.

4. <u>Gain Bandwidth Limitation</u>:

* Maximum gain can be achieved when the tuned circuit is at resonance



$$\text{Gain } G = \frac{V_o(s)}{V_i(s)} = Z_o(s)$$

* Applying Laplace transform and replacing R_L and R_p by

$$R = \frac{1}{R_L} + \frac{1}{R_p}$$

$$\frac{1}{Z_e(S)} = Y_e(S) = CS + \frac{1}{LS} + \frac{1}{R} = \frac{S^2LCR + LS + R}{RLS}$$

$$Z_e(S) = \frac{S/C}{S^2 + \frac{S}{CR} + \frac{1}{LC}}$$

- The roots of the quadratic equation are ω_1, ω_2

$$\omega_1 = -\frac{G}{2C} - \sqrt{\left(\frac{G}{2C}\right)^2 - \frac{1}{LC}}$$

$$\omega_2 = -\frac{G}{2C} + \sqrt{\left(\frac{G}{2C}\right)^2 - \frac{1}{LC}}$$

where $G = \frac{1}{R}$

- Bandwidth, $BW = \omega_2 - \omega_1 = \frac{G}{C}$ for $\left(\frac{G}{2C}\right)^2 >> \frac{1}{LC}$

- The maximum gain at resonance $A_{max} = \frac{g_m}{G}$

∴ Gain Bandwidth product $= A_{max} \cdot B.W$

$$= \frac{g_m}{G} \cdot \frac{G}{C}$$

$$= g_m/C$$

- The gain bandwidth product is thus independent of frequency, higher gain can be achieved at the cost of bandwidth only. In microwave circuit this can be overcomed by use of reentrant cavities and slow wave tubes.

5. Effect due to RF losses.

a. Skin effect losses: These losses come into play at higher frequencies at which the current has the tendency to confine itself to a smaller cross-section of the conductor towards its outer surface.

$$\delta = \text{Skin depth} = \sqrt{\frac{2}{\omega\mu\sigma}}$$

$$\delta \propto \frac{1}{\sqrt{\omega}} \quad \text{and} \quad \delta \propto A_{eff}$$

where A_{eff} is the effective area over which current flows.

$$A_{eff} \propto \frac{1}{\sqrt{f}}$$

* As 'f' increases . R increases. Hence losses will increase at higher frequencies. These losses can be reduced by increasing the size of the conductor.

b. Dielectric losses: This occurs in various types of insulating materials used in the device i.e., spacers, glass envelope, silicon or plastic encapsulators etc.

* This loss in any of the material is given by,

$$P = \pi f \cdot V_0^2 \varepsilon_r \tan\theta$$

* As 'f' increases the power loss increases. The remedy for this is to eliminate the tube base and to reduce the surface area of glass.

6. RADIATION LOSSES:

* Whenever the dimensions of the wire approaches the wavelength $\lambda = c/f$, it will emit radiation that is, radiation losses increases with increase in frequency.

* The remedy for this is to use proper shielding of the tubes and its circuity.

Scanned By Scanner Go

KLYSTRONS:-

* A klystron is a vaccum tube that can be used either as a generator or as an amplifier at microwave frequencies.

* It is operated by the principle of Velocity modulation and Current modulation.

TWO CAVITY KLYSTRON AMPLIFIER:

* A two cavity klystron amplifier which is basically a velocity modulated tube.

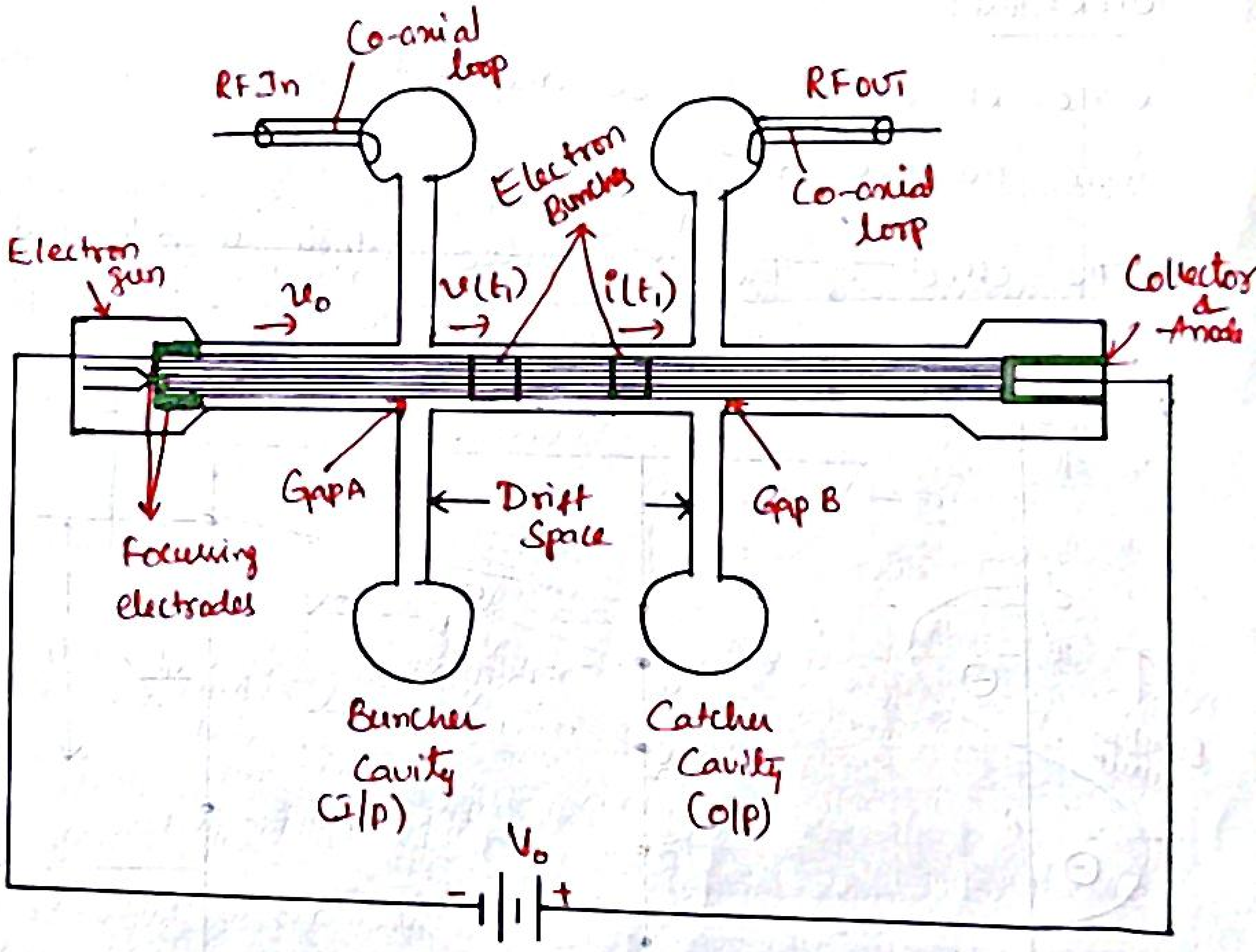


FIG: TWO CAVITY KLYSTRON AMPLIFIER

* Here a high velocity electron beam is formed focussed and sent down a glass tube through an input cavity (buncher), drift space and an catcher cavity to a collector electrode.
* The anode is kept at positive voltage with respect to cathode
* The electron beam passes through a gap 'A' consisting of two grids of the buncher cavity separated by a very small distance and through gap 'B'.
* The input and output are taken from the tube via resonant cavities with the aid of coupling loops.

OPERATION :-

* The RF signal to be amplified is used for exciting the input buncher cavity.
* The effect of the gap voltage which is developed due to RF signal will be explained with the help of Applegate diagram.

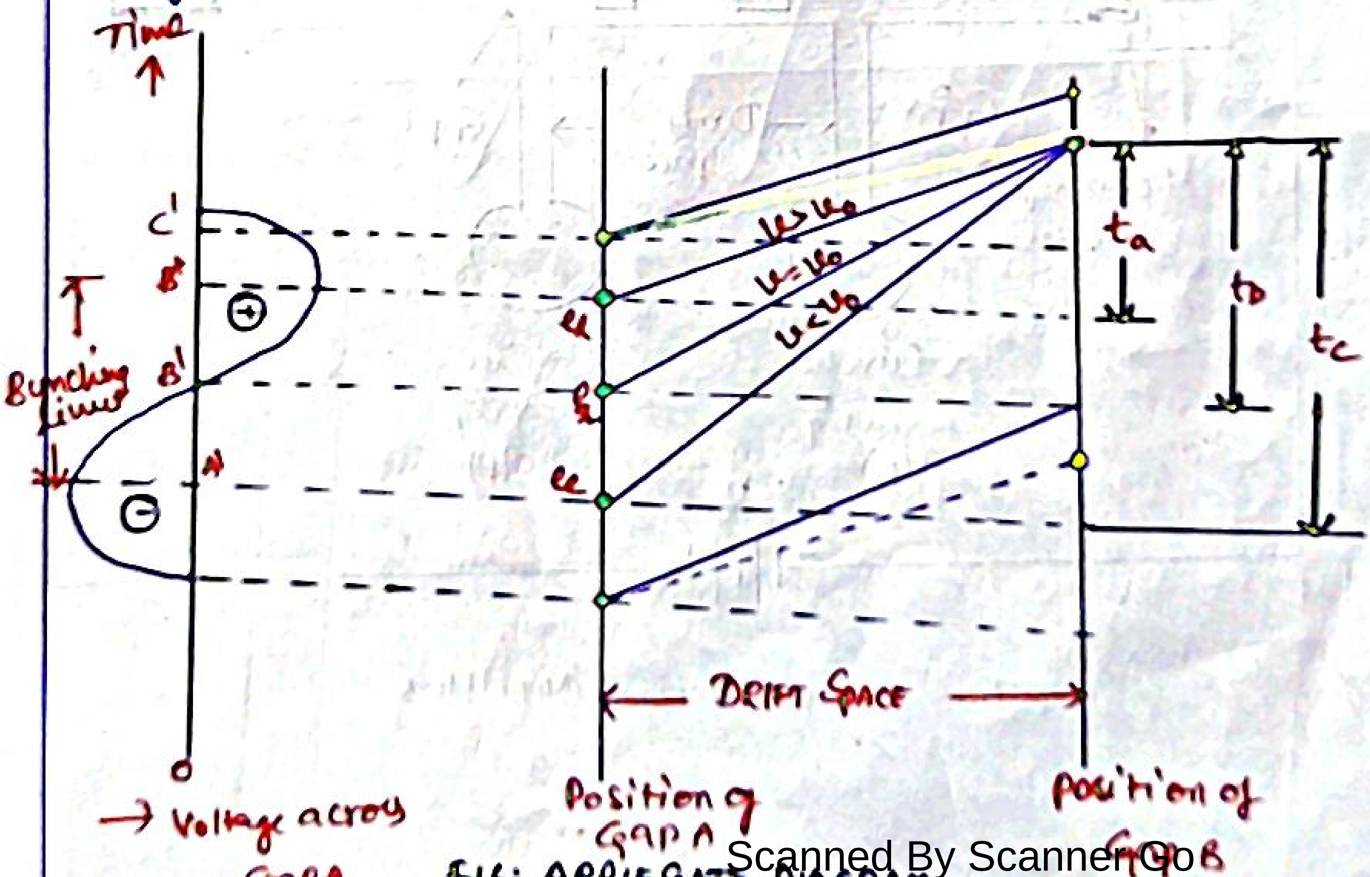


FIG: APPLEGATE DIAGRAM

At point B': ($V_s = 0$)

* At this point, the electric field across gap A is zero and an election which passes through gap A at this instant is unaffected by the RF signal. This 'election be called as Reference election e_R, which travels with unchanged velocity

At point C': (V_s is positive)

* At this point, the election which leaves gap A later than the reference election called the late election e_L is subjected to maximum positive RF voltage and gets accelerates. Hence travels towards gap B with an increased velocity ($v > v_0$).

At point A': (V_s is negative)

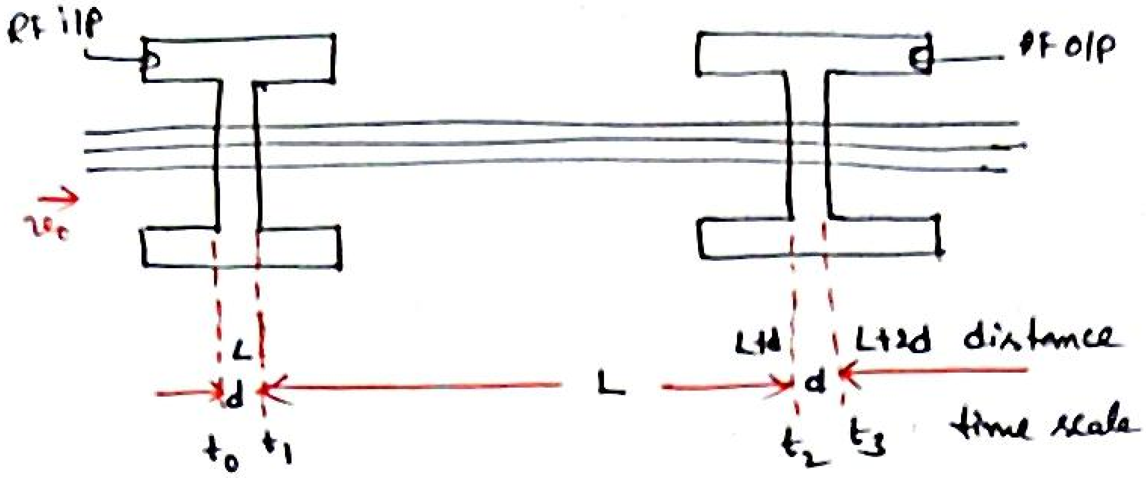
* Similarly, the election that passes the gap 'A' slightly before the reference election called Early election (e_E) is subjected to a maximum negative field.

* Hence this election is decelerated and travels with reduced velocity ($v < v_0$)

* Therefore, the velocity of election varies in accordance with RF input voltage, resulting in velocity modulation.

* As a result, the elections in the bunching limit gradually bunch together as they travel down the driftspace from gap A to gap B

VELOCITY MODULATION PROCESS :



* When electrons are first accelerated by the high dc voltage V_0 before entering the buncher grids,

$$v_0 = \sqrt{\frac{2eV_0}{m}} = 0.593 \times 10^6 \sqrt{V_0} \ m/s$$

* Let RF input $V_s = V_1 \sin \omega t$ given to buncher cavity

where $V_1 << V_0$.

* The average transit time through the buncher gap is

$$\tau = \frac{d}{v_0} = t_1 - t_0$$

* The average gap transit angle $\theta_g = \omega \tau = \omega (t_1 - t_0)$

* The average microwave voltage in the buncher gap during t_0 to t_1

$$\langle V_s \rangle = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} V_1 \sin \omega t \, dt$$

$$= \frac{V_1}{\tau} \int_{t_0}^{t_1} \sin \omega t \, dt$$

$$= \frac{V_1}{\tau} \left[\frac{-\cos \omega t}{\omega} \right]_{t_0}^{t_1}$$

$$\langle V_s \rangle = \frac{V_1}{\omega\tau}\left[-\cos\omega t_1 + \cos\omega t_0\right]$$

$\because t_1 = \tau + t_0$

$$= \frac{V_1}{\omega\tau}\left[\cos\omega t_0 - \cos\omega t_1\right]$$

$$= \frac{V_1}{\omega\tau}\left[\cos\omega t_0 - \cos\omega(\tau + t_0)\right]$$

$$= \frac{V_1}{\omega\tau}\left[\cos\omega t_0 - \cos(\omega t_0 + \omega\tau)\right]$$

$\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$$= \frac{-2V_1}{\omega\tau}\left[\sin\left(\frac{\omega t_0 + \omega t_0 + \omega\tau}{2}\right)\right]. \sin\left(\frac{\omega t_0 - \omega t_0 - \omega\tau}{2}\right)\Bigg]$$

$$= \frac{-2V_1}{\omega\tau}\sin\left(\frac{2\omega t_0}{2} + \frac{\omega\tau}{2}\right)\sin\left(-\frac{\omega\tau}{2}\right)$$

$$= \frac{2V_1}{\omega\tau}\sin\left(\omega t_0 + \frac{\omega\tau}{2}\right)\sin\left(\frac{\omega\tau}{2}\right)$$

$$= \frac{V_1}{\omega\tau/2}\sin\left(\omega t_0 + \frac{\omega\tau}{2}\right)\sin\left(\frac{\omega\tau}{2}\right)$$

$(\because \theta_g = \omega\tau)$

$$= \frac{V_1}{\theta_g/2}\sin(\omega t_0 + \theta_g/2)\sin(\theta_g/2)$$

$$= V_1 \frac{\sin(\theta_g/2)}{\theta_g/2}\sin(\omega t_0 + \theta_g/2)$$

$$V_s = V_1 \beta_i \sin(\omega t_0 + \theta_g/2)$$

$\therefore \beta_i = \dfrac{\sin(\theta_g/2)}{\theta_g/2}$ which is I/p Beam Coupling coefficient

∴ After velocity modulation, the velocity from the buncher gap is given by,

$$v(t_1) = \sqrt{\frac{2e(V_0+V_s)}{m}}$$

$$= \sqrt{\frac{2e}{m}\left[V_0 + V_1 \beta_i \sin(\omega t_0 + \theta_g/2)\right]}$$

$$= \sqrt{\frac{2eV_0}{m}\left[1 + \frac{V_1}{V_0}\beta_i \sin(\omega t_0 + \theta_g/2)\right]}$$

$$= \sqrt{\frac{2eV_0}{m}}\left[1 + \frac{\beta_i V_1}{V_0} \sin(\omega t_0 + \theta_g/2)\right]^{1/2}$$

$$v(t_1) = v_0\left[1 + \frac{\beta_i V_1}{V_0} \sin(\omega t_0 + \theta_g/2)\right]^{1/2}$$

According Binomial expansion $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + ----$

$$\boxed{\begin{aligned} v(t_1) &= v_0\left[1 + \frac{\beta_i V_1}{2V_0} \sin(\omega t_0 + \theta_g/2)\right] \\ v(t_1) &= v_0\left[1 + \frac{\beta_i V_1}{2V_0} \sin(\omega t_1 - \theta_g/2)\right] \end{aligned}}$$

BUNCHING PROCESS:

→ Once the electrons leave the buncher cavity, they drift with a velocity in the field free space between the two cavities.

→ The electrons that pass the buncher at $V_s=0$ travel with unchanged velocity v_0 and become the bunching centre.

→ The electrons that passes when V_s is positive half cycle travel faster than the electrons that passed during $V_s=0$.

* The electron that passes the buncher cavity during V_s is negative half cycle travels with slower velocity.

* At a distance of ΔL along the beam from the buncher cavity, the beam electrons have drifted into dense clusters.

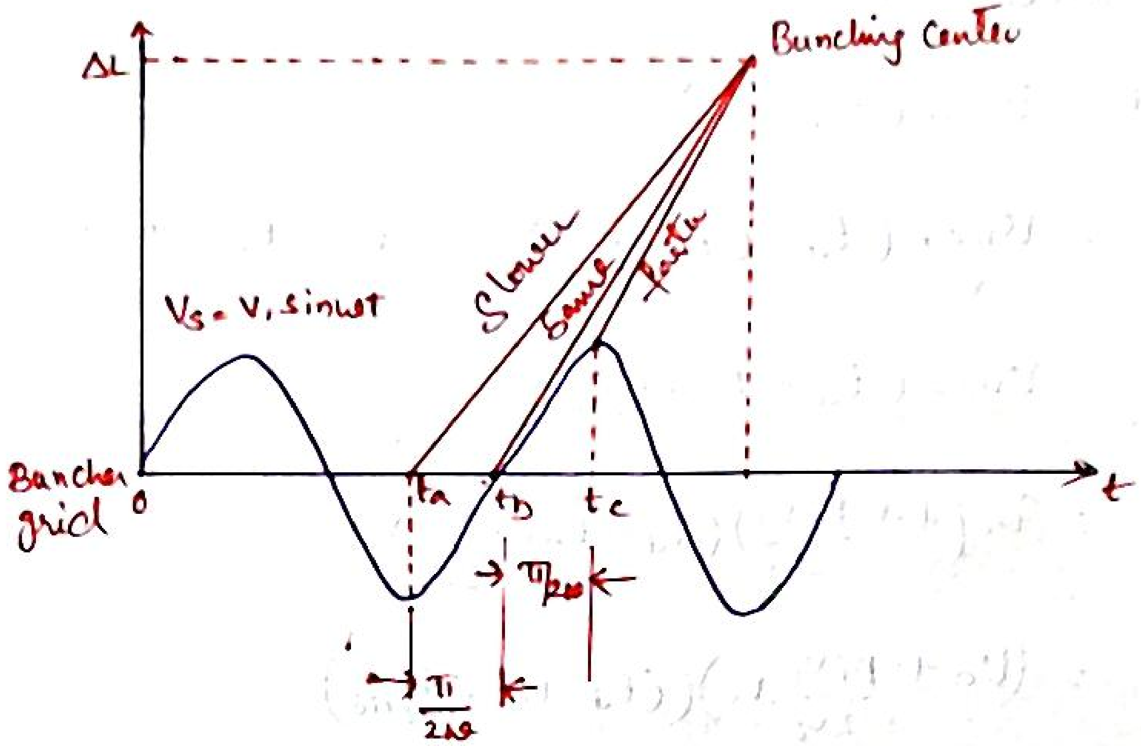


FIG: BUNCHING DISTANCE

* The distance from the buncher grid to the location of the dense electron bunching for electron at t_b is

$$\Delta L = v_0 (t_d - t_b) \longrightarrow (1)$$

* The distance of the electrons at t_a is

$$\left(\because \text{velocity} = \frac{\text{distance}}{\text{time}}\right)$$

$$\Delta L = v_{min} (t_d - t_a)$$

$$\left(\because t_a = t_b - \pi/2\omega\right)$$

$$= v_{min} (t_d - (t_b - \pi/2\omega))$$

$$\Delta L = v_{min} (t_d - t_b + \pi/2\omega)$$

$$\because v_{min} = v_0 \left(1 - \frac{\beta_i V_1}{2V_0}\right) \quad v_{max} = v_0 \left(1 + \frac{\beta_i V_1}{2V_0}\right)$$

$$\Delta L = v_0\left(1-\frac{\beta_i V_1}{2V_0}\right)\left(t_d - t_b + \pi/2\omega\right) = \left(v_0 - \frac{\beta_i V_1 v_0}{2V_0}\right)\left((t_d - t_b) + \pi/2\omega\right)$$

$$\Delta L = v_0(t_d - t_b) - v_0\frac{\beta_i V_1}{2V_0}(t_d - t_b) + \frac{\pi}{2\omega}v_0 - v_0\frac{\pi}{2\omega}\frac{\beta_i V_1}{2V_0} \rightarrow 5$$

* The distance of electrons at t_c

$$\Delta L = v_{max}(t_d - t_c)$$

$$= v_{max}\left(t_d - \left(t_b + \frac{\pi}{2\omega}\right)\right) \qquad (\because t_c = t_b + \pi/2\omega)$$

$$= v_{max}(t_d - t_b - \pi/2\omega)$$

$$= v_0\left(1 + \frac{\beta_i V_1}{2V_0}\right)\left((t_d - t_b) - \frac{\pi}{2\omega}\right)$$

$$= \left(v_0 + \frac{\beta_i V_1}{2V_0}.v_0\right)\left((t_d - t_b) - \pi/2\omega\right)$$

$$\Delta L = v_0(t_d - t_b) - \frac{\beta_i V_1}{2V_0}.v_0(t_d - t_b) + \frac{\pi}{2\omega}v_0 - v_0\frac{\beta_i V_1}{2V_0}\frac{\pi}{2\omega} \rightarrow 3$$

* The necessary and sufficient condition for three electrons at t_a, t_b, t_c to meet at the same distance ΔL is

$$-v_0\frac{\beta_i V_1}{2V_0}(t_d - t_b) + \frac{\pi}{2\omega}v_0 - v_0\frac{\pi}{2\omega}\frac{\beta_i V_1}{2V_0} = 0 \rightarrow 4$$

$$v_0\frac{\beta_i V_1}{2V_0}(t_d - t_b) - \frac{\pi}{2\omega}v_0 - v_0\frac{\pi}{2\omega}\frac{\beta_i V_1}{2V_0} = 0 \rightarrow 5$$

→ Let us consider equation 5,

$$v_0\frac{\beta_i V_1}{2V_0}(t_d - t_b) - \frac{\pi}{2\omega}v_0 - v_0\frac{\pi}{2\omega}\frac{\beta_i V_1}{2V_0} = 0$$

$$v_0 \frac{\beta_i V_1}{2V_0}(t_d - t_0) = \frac{\pi}{2\omega} v_0 + v_0 \frac{\pi}{2\omega} \frac{\beta_i V_1}{2V_0}$$

$$v_0 \frac{\beta_i V_1}{2V_0}(t_d - t_0) = v_0 \cdot \frac{\pi}{2\omega}\left[1 + \frac{\beta_i V_1}{2V_0}\right]$$

$(\because V_1 << V_0)$

$$\cancel{v_0} \frac{\beta_i V_1}{\cancel{2}V_0}(t_d - t_0) = \cancel{v_0} \frac{\pi}{\cancel{2}\omega}$$

$$(t_d - t_0) = \frac{\pi V_0}{\omega \beta_i V_1} \rightarrow (6)$$

Substitute eq (6) in eq (1)

$$\Delta L = v_0 (t_d - t_0)$$

$$\boxed{\Delta L = \frac{\pi V_0}{\omega \beta_i V_1} \cdot v_0}$$

Scanned By Scanner Go

Output Power:

* The maximum bunching should occur approximately midway between the catcher grids.
* The phase of the catcher gap voltage must be maintained in such a way that the bunched electrons, as they pass through the grids, encounter a retarding phase.
* When the bunched electron beam passes through the retarding phase, its kinetic energy is transferred to the field of the catcher cavity.
* When the electrons emerges from the catcher grids, they have reduced velocity and are finally collected by the collector.
* The current at the catcher cavity grid is given by,

$$i_2 = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t_2 + b_n \sin n\omega t_2 \qquad -\pi \le \omega t_2 \le \pi$$

Here, $a_0 = I_0$

$$a_n = 2 I_0 J_n(nx) \cos(n\theta_g + n\theta_0)$$

$$b_n = 2 I_0 J_n(nx) \cos(n\theta_g + n\theta_0)$$

where $J_n(nx)$ is the nth order Bessel's function of the first kind.

$$i_2 = I_0 + \sum_{n=1}^{\infty} 2 I_0 J_n(nx) \cos[n\omega(t_2 - \tau - T_0)$$

* The fundamental component of either induced current at catcher cavity grids

$$i_2(ind) = \beta_0 \, 2I_0 J_1(x) \cos[\omega(t_2 - \tau - T_0)]$$

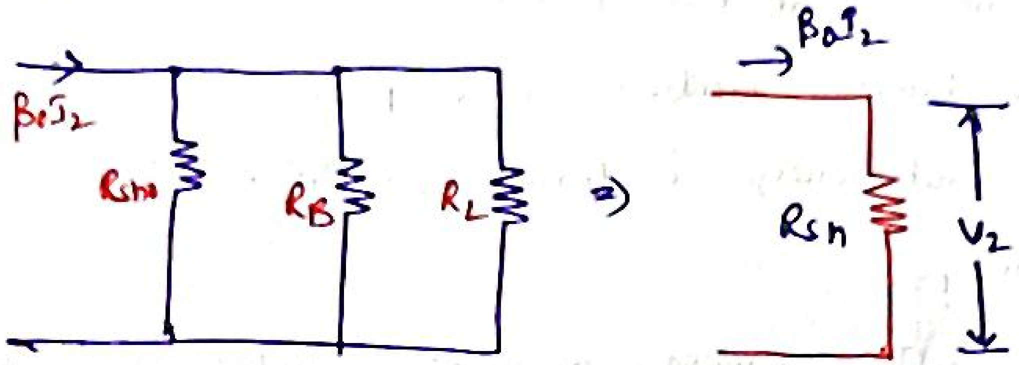
where β_0 is Beam Coupling coefficient of the cavity

* The amplitude of induced current into the catcher cavity gap is

$$I_2(ind) = \beta_0 2I_0 J_1(x)$$

$$I_2(ind) = \beta_0 i_2 \qquad (\because i_2 = 2I_0 J_1(x))$$

* The equivalent circuit of o/p cavity is,



where R_{sho} = Resistance of catcher cavity walls

R_B = Beam loading resistance

R_L = External load resistance.

R_{sh} = Effective shunt resistance.

* The output power delivered to the catcher cavity and the load is given by

$$(\because I_{rms} = \frac{\beta_0 I_2}{\sqrt{2}})$$

$$P_{out} = I_{rms}^2 R_{sh}$$

$$= \left(\frac{\beta_0 I_2}{\sqrt{2}}\right)^2 R_{sh}$$

$$= \frac{\beta_0 I_2}{2} \times (\beta_0 I_2 R_{sh})$$

$$(\because V_2 = \beta_0 I_2 R_{sh})$$

$$= \frac{\beta_0 I_2}{2} \times (V_2)$$

$$P_{out} = \frac{\beta_0 I_2 V_2}{2}$$

* Input power $P_{in} = V_0 I_0$

Efficiency:

* The efficiency of the two cavity klystron amplifier is,

$$\eta = \frac{P_{out}}{P_{in}}$$

$$\boxed{\eta = \frac{\beta_0 I_2 V_2}{2 I_0 V_0}}$$

* If the coupling is perfect $\beta_0 = 1$, the maximum Beam current reaches $I_{2max} = 2I_0(0.582)$ & $V_2 = V_0$, then

$$\eta = \frac{1 \cdot \cancel{2I_0}\, 0.582 \times \cancel{V_2}}{\cancel{2I_0 V_2}}$$

$$\eta = 0.582.$$

* Then the maximum efficiency is about 58%. In practice, the electronic efficiency of a klystron amplifier is in the range of 15 to 30%.

8.5.2 Multicavity Klystron

As explained in the previous section, gains of about 10 to 20 dB are typical with two-cavity tubes. A higher overall gain can be achieved by connecting several two cavity tubes in cascade, feeding the output of each of the tubes to the input of the succeeding one. Instead, multiple number of cavities can be used as in a multicavity klystron shown in Fig. 8.14.

Here, each of the intermediate cavities act as a buncher with the passing electron beam inducing an enhanced RF voltage than the previous cavity. With four cavities, power gains of around 50 dB can be easily achieved. The cavities can all be tuned to the same frequency (synchronous tuning) for narrow band operation. Bandwidth can be improved by stagger tuning of cavities upto about 80 MHz of course with reduction in gain (to about 45 dB). This stagger tuning is employed in UHF klystrons for TV transmitter output tubes and in satellite earth station transmitters as power amplifiers at 6 GHz.

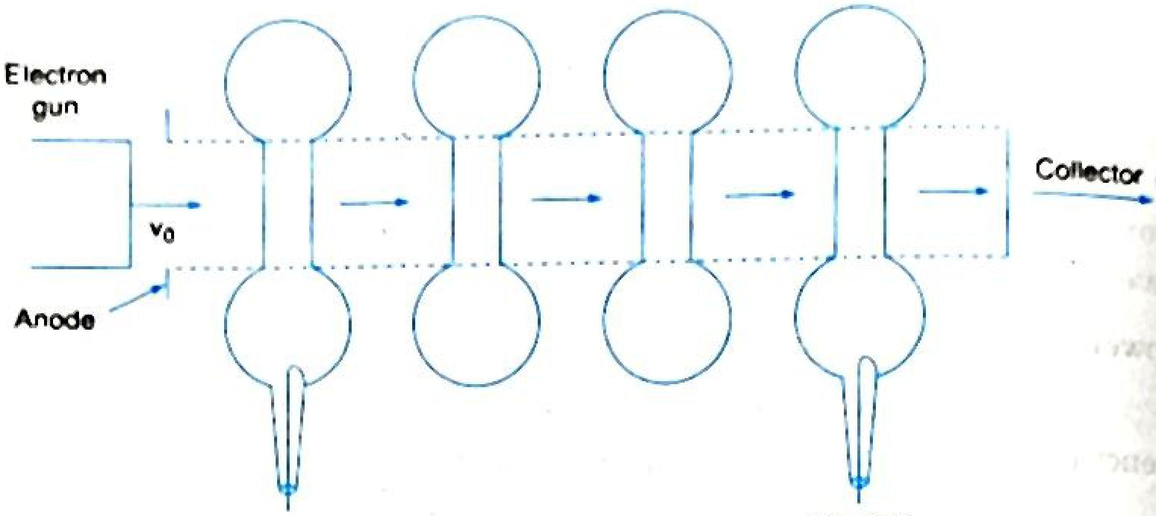


Fig. 8.14 Multicavity klystron (4 cavity)

8.5.3 Two Cavity Klystron Oscillator

A klystron amplifier can be converted into an oscillator by feeding back a part of the catcher output into the buncher in proper phase so as to satisfy Barkhausen criterion. The schematic is same as klystron amplifier (Fig. 8.7) except that a feedback loop needs to be added. The feedback must be so adjusted to give correct polarity and amplitude which basically depends on the cavity tuning and the various dc voltages. If θ is the phase shift in the resonators and the feedback cable, the criterion for oscillation is (since $A\beta = 1 \angle 2\pi n$ radians where n is any integer including zero.)

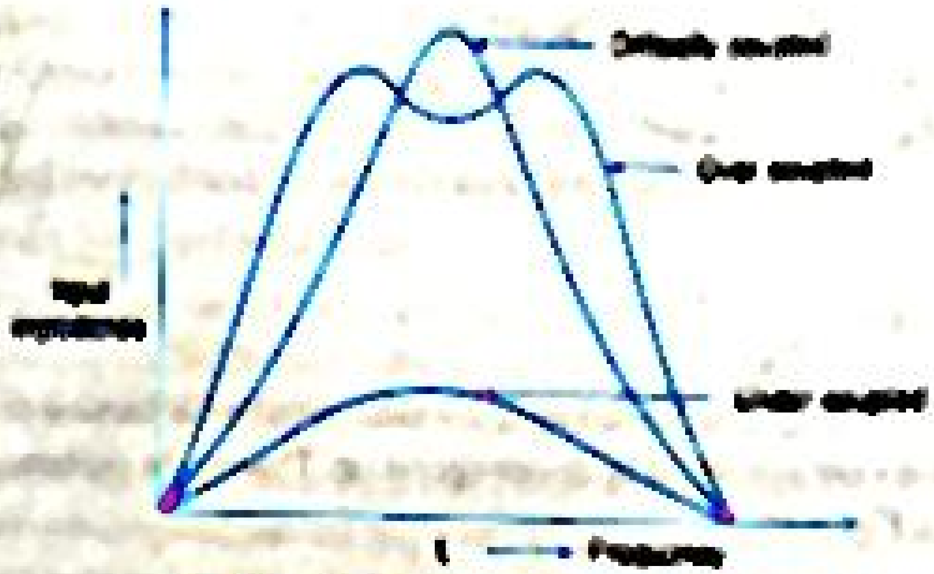
$$\theta + \alpha + \frac{\pi}{2} = 2\pi n \text{ radians} \qquad ...(8.39)$$

where $\alpha + \pi/2$ is the phase angle between the zeroes of buncher and catcher voltages. If the two resonators oscillate in same phase, then $\theta = 0$. Maximum power output is obtained by substituting for $\theta = 0$ in Eq. 8.39 for obtaining $\alpha = 2\pi n - (\pi/2)$. *i.e.*, If the two resonators oscillate in time phase, the condition that $\alpha = 2\pi n - (\pi/2)$ not only becomes a requirement for maximum power output but also for obtaining sustained oscillations. However, the two resonators in general need not have to oscillate in time phase.

Also, a small change in dc accelerating voltage causes a change in frequency since then the transit angle α varies. In that case the frequency of oscillation will shift in such a way as to yield a new value of θ so as to satisfy Eq. 8.39. Since two resonators having the same resonant frequency are coupled here, the input impedance looking into either resonator circuits will vary with frequency as shown in Fig. 8.15.

Oscillations can be obtained over a somewhat wide range if the resonators are over coupled. A critically coupled klystron oscillator has almost a linear variation in frequency with accelerating voltage making frequency modulation possible. High frequency stability of oscillator is obtained by controlling the temperature of the resonators and also by use of regulated power supplies.

Tuning of the oscillator is done by adjusting the grid voltage, dc accelerating voltage and the tuning of the two resonators.



Frequency vs impedance

REFLEX KLYSTRON :-

* The Reflex klystron is a single-cavity klystron that overcomes the disadvantages of the two cavity klystron oscillator.

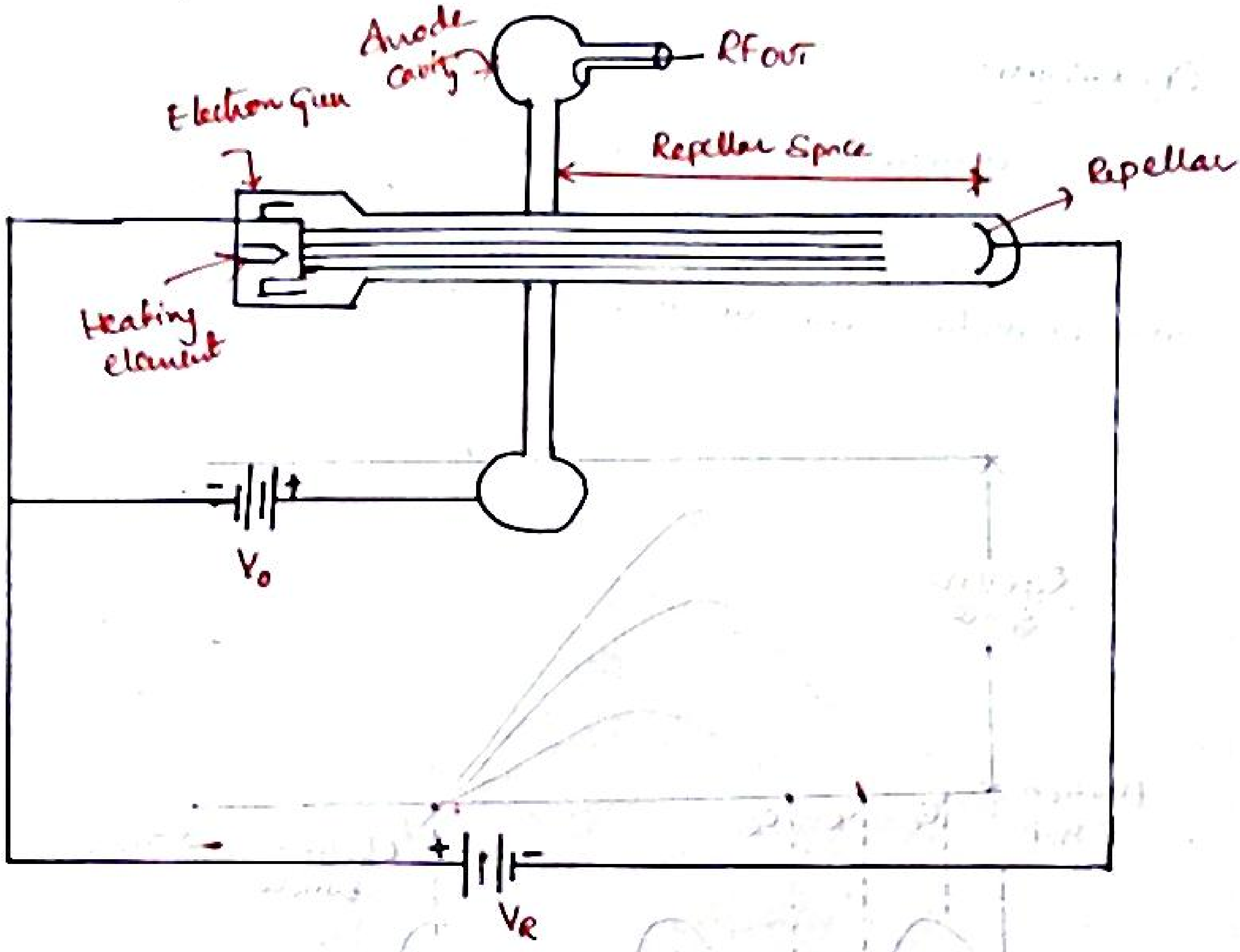


FIG: REFLEX KLYSTRON CONSTRUCTION

CONSTRUCTION:

* It consists of an electron gun, a filament surrounded by cathode and a focusing electrode at cathode potential.
* The electron gun is beam is accelerated towards the anode cavity at positive potential (V_0 is positive).
* After passing the cavity, electrons travels towards the repellar electrode which is at high negative potential V_R
* The electrons never reach the repellar because of the negative field and are returned back to the cavity.

* Under suitable conditions, the electrons kinetic energy is translated to the cavity guide and then oscillations are sustained.

OPERATION:

* It is assumed that the oscillation are set up in the tube initially due to noise & switching transients and these oscillations are sustained by device operation

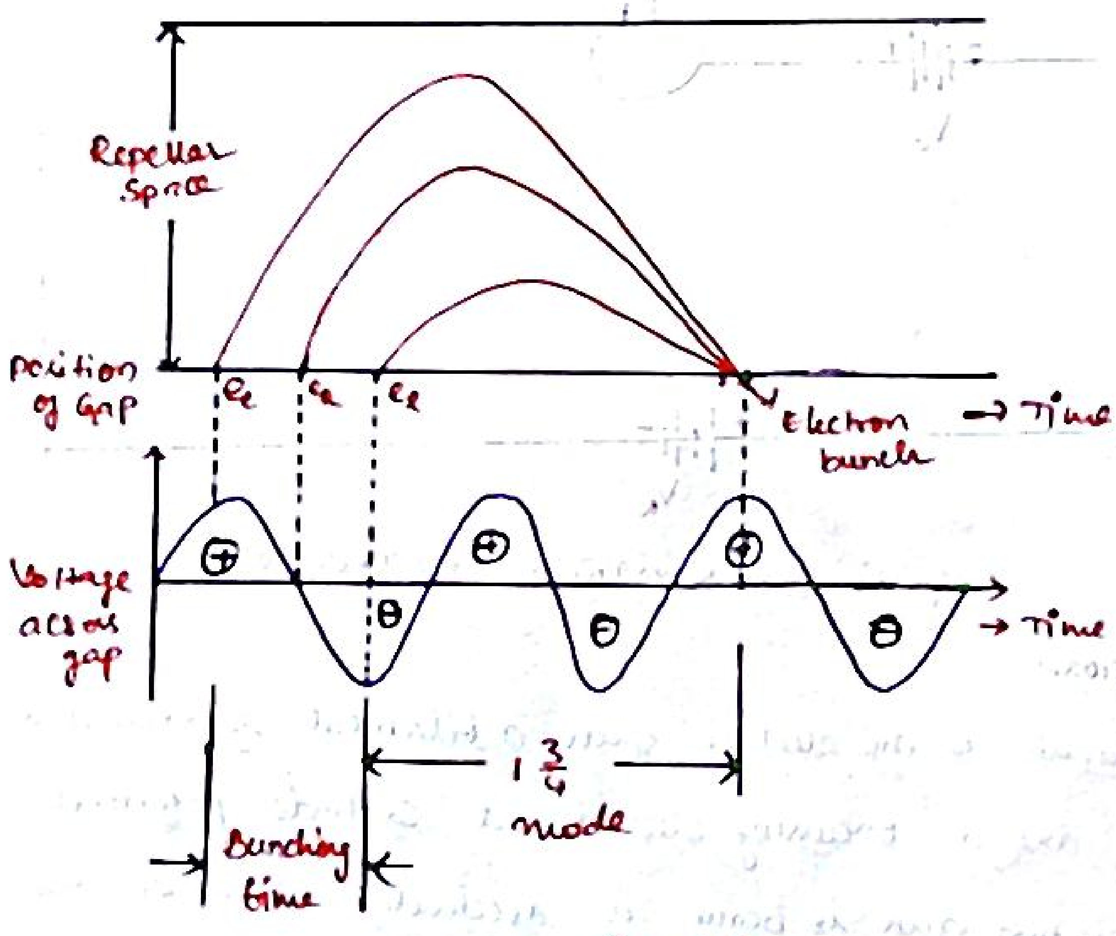


FIG: APPLEGATE DIAGRAM

* The RF voltage i.e, produced across the gap by the cavity oscillations act on the electron beam to cause velocity modulation

* The Reference electron e_R is unaffected by the gap voltage as it passes through the cavity gap when gap voltage V_s is zero.

* This moves towards the repellar and gets reflected by the negative voltage on the repellar.

* The early electron 'e_e' that passes through the gap before the reference electron e_R.

* The early electron experiences a maximum positive voltage as it passes when V_s is positive and this electron gets accelerated.

* It travels with greater velocity and penetrates deep into the repellar space. The return time for e_e is greater than the e_R as its depth of penetration into repellar space is more.

* The late electron 'e_l' that passes through the cavity gap when V_s is negative, it experiences a maximum negative voltage and electron gets decelerated.

* The return time is shorter as depth of penetration into repellar space is less and catches up with e_R and e_e to form a bunch.

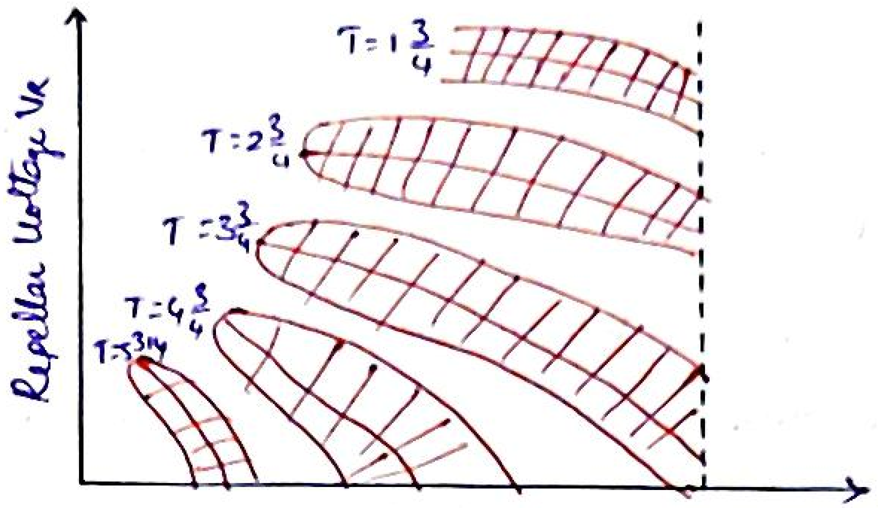
* For oscillations to be sustained, the time taken by the electrons to travel into the repellar space and back to the gap must have an optimum value.

* In general, the optimum transit time $T = n + \frac{3}{4}$

* Thus, maximum kinetic energy will be transferred to the cavity gap and thus oscillations gets sustained.

OPERATING CHARACTERISTICS:

1. VOLTAGE CHARACTERISTICS: Oscillations can be obtained only for specific combination of anode and repellar voltages. $T = n + \frac{3}{4}$



* Each value of $n = 1, 2, 3 \ldots$ is ~~said~~ said to correspond to a different mode for the reflex klystron oscillator. The earlier the mode the larger the o/p power. As a result the modes corresponding to $n = 2$ or 3 are most widely used.

2. POWER OUTPUT & FREQUENCY CHARACTERISTICS: The frequency of resonance of the cavity decides the frequency of oscillations. Variations of repellar voltage slightly changes with the frequency. This amounts to electronic tuning of reflex klystron. This makes it possible to use reflex klystron as a voltage tuned oscillator or frequency modulated oscillator.

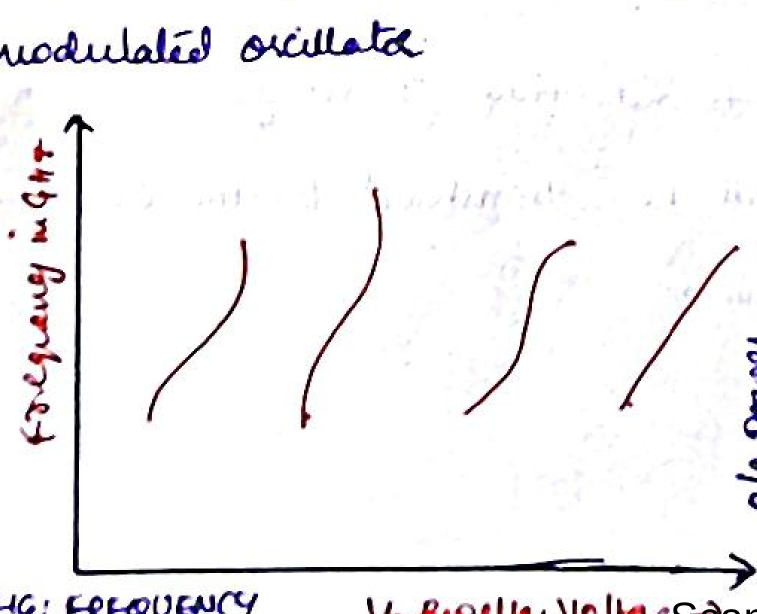


FIG: FREQUENCY CHARACTERISTICS

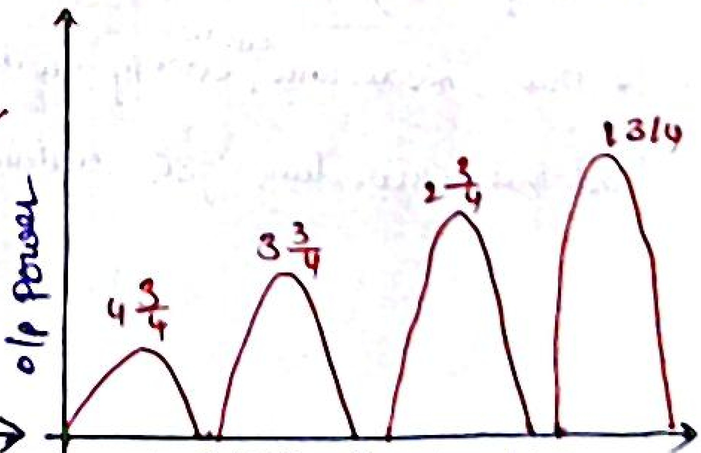
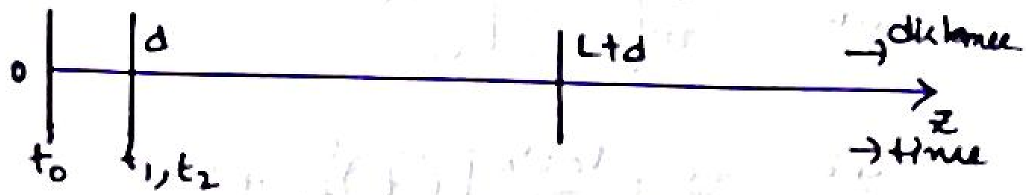


FIG: O/P POWER CHARACTERISTICS

VELOCITY MODULATION & BUNCHING PARAMETER:

* The analysis of reflex klystron is similar to that of a two-cavity klystron.

* The electron entering the cavity gap from the cathode at $z=0$ and time t_0 is assumed to have uniform velocity.



$$v_0 = \sqrt{\frac{2eV_0}{m}} = 0.593 \times 10^6 \sqrt{V_0} \text{ m/sec}$$

* The same electron leaves the cavity gap at $z=d$ at time t_1 with velocity

$$v(t_1) = v_0\left[1 + \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

* The same electron is forced back to the cavity $z=d$ and time t_2 by the retarding electric field E, which is given by

$$E = \frac{V}{d} = \frac{V_R + V_0 + V_1 \sin\omega t}{L}$$

* Since $V_0 + V_R \gg V_1 \sin\omega t$ then $E = \dfrac{V_0 + V_R}{L}$

* The force experienced by the electron due to retarding electric field

$$F = -eE$$

$$ma = -eE$$

$$m \frac{d^2z}{dt^2} = -e\left(\frac{V_0+V_R}{L}\right)$$

$$\frac{d^2z}{dt^2} = \frac{-e}{m}\left[\frac{V_0+V_R}{L}\right]$$

Integrate on both sides with respect to t.

$$\int \frac{d^2z}{dt^2} = \frac{-e}{m}\left[\frac{V_0+V_R}{L}\right]\int_{t_1}^{t} dt$$

$$\frac{dz}{dt} = -\frac{e}{m}\left[\frac{V_0+V_R}{L}\right](t)_{t_1}^{t} + k_1$$

$$\frac{dz}{dt} = \frac{-e}{m}\left[\frac{V_0+V_R}{L}\right](t-t_1)+k_1 \rightarrow ①$$

at $t=t_1$, $\frac{dz}{dt} = v(t_1)$ then eq ① becomes

$$dz\ v(t_1) = -\frac{e}{m}\left(\frac{V_0+V_R}{L}\right)(t_1-t_1)+k_1$$

$$v(t_1) = 0+k_1$$

$$k_1 = v(t_1) \rightarrow ②$$

Substitute eq ② in eq ①

$$\frac{dz}{dt} = -\frac{e}{m}\left[\frac{V_0+V_R}{L}\right](t-t_1)+v(t_1)$$

Integrate on both sides with respect to t

$$z = -\frac{e}{m}\left[\frac{V_0+V_R}{L}\right]\int_{t_1}^{t}(t-t_1)dt + v(t_1)\int_{t_1}^{t} dt$$

$$z = \frac{-e}{m}\left[\frac{V_0+V_R}{L}\right]\left[\left(\frac{t-t_1}{2}\right)^2\right]_{t_1}^{t} + v(t_1)(t)_{t_1}^{t} + k_2$$

$$z = \frac{-e}{m}\left[\frac{V_0+V_R}{L}\right]\left(\frac{t-t_1}{2} - \frac{t_1-t_1}{2}\right)^2 + v(t_1)(t-t_1) + k_2$$

$$z = \frac{-e}{m}\left[\frac{V_0+V_R}{L}\right]\left(\frac{(t-t_1)^2}{2}\right) + v(t_1)(t-t_1) + k_2 \,.$$

at $t = t_1$, $z = d$, then

$$d = \frac{-e}{m}\left(\frac{V_0+V_R}{L}\right)\left(\frac{t_1-t_1}{2}\right)^2 + v(t_1)(t_1-t_1) + k_2$$

$$\boxed{k_2 = d}$$

then, $$z = \frac{-e}{m}\left[\frac{V_0+V_R}{L}\right]\left(\frac{t-t_1}{2}\right)^2 + v(t_1)(t-t_1) + d \rightarrow (2)$$

* On the assumption that the electron leaves the Cavity gap at $z = d$ and time t_1 with a velocity of $v(t_1)$ and returns to the gap at $z = d$ and time t_2, then at $t = t_2$, $z = d$.

eq (2) becomes,

$$\cancel{d} = \frac{-e}{2m}\left[\frac{V_0+V_R}{L}\right]\left[\frac{t_2-t_1}{\cancel{2}}\right]^2 + v(t_1)(t_2-t_1) + \cancel{d}$$

$$v(t_1)\cancel{(t_2-t_1)} = \frac{+e}{2mL}(V_0+V_R)(t_2-t_1)^{\cancel{2}}$$

$$(t_2-t_1) = \frac{2mL}{e(V_0+V_R)}\, v(t_1)$$

* The round trip transit time in the repeller region is,

$$T = (t_2-t_1)$$

$$T = \frac{2mL}{e(V_0+V_R)} \cdot v(t_1) = \frac{2mL}{e(V_0+V_R)}\, v_0\left[1 + \frac{\beta_i V_1}{2V_0}\sin(\omega t_1 - \theta_g/2)\right]$$

$$T = \frac{2mL}{e(V_0+V_R)}\left\{v_0\left[1+\frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1-\frac{\theta_g}{2}\right)\right]\right.$$

$$\because T_0' = \frac{2mL v_0}{e(V_0+V_R)}$$

$$T = T_0'\left[1+\frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1-\frac{\theta_g}{2}\right)\right]$$

multiply with ω on both sides

$$\omega T = \omega T_0'\left[1+\frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1-\frac{\theta_g}{2}\right)\right]$$

Let $\theta_0' = \omega T_0'$ which is DC round trip transit angle.

$$\omega T = \omega T_0' + \omega T_0' \cdot \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1-\frac{\theta_g}{2}\right)$$

$$\omega T = \theta_0' + \theta_0' \cdot \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1-\frac{\theta_g}{2}\right)$$

Bunching parameter $\boxed{X' = \frac{\beta_i V_1}{2V_0}\,\theta_0'}$

$$\omega T = \theta_0' + X'\sin\left(\omega t_1-\frac{\theta_g}{2}\right)$$

OUTPUT POWER & EFFICIENCY:

* In order for the electron beam to generate a maximum amount of energy of oscillations, the retarding electron beam must cross the Cavity gap when the gap field is maximum retarding
* In this way maximum amount of kinetic energy can be transferred from the retarding electrons for the Cavity walls.

- Efficiency $\eta = \dfrac{P_{ac}}{P_{dc}}$

$$\eta = \frac{\beta_i V_1 I_0 J_1(x')}{V_0 I_0} \rightarrow (1)$$

From Bunching parameter $x' = \dfrac{\beta_i V_1}{2V_0} \theta_0'$

$$V_1 = \frac{2V_0 x'}{\beta_i \theta_0'} \rightarrow (2)$$

Substitute eq (2) in eq (1)

$$\eta = \frac{\cancel{\beta_i} \cancel{I_0} J_1(x')}{\cancel{V_0}\cancel{I_0}} \cdot \frac{2\cancel{V_0} x'}{\cancel{\beta_i} \theta_0'}$$

$$\eta = \frac{2x' J_1(x')}{\theta_0'} \qquad (\because \theta_0' = 2\pi n - \frac{\pi}{2})$$

$$= \frac{2x' J_1(x')}{2\pi n - \frac{\pi}{2}}$$

- $x' J_1(x')$ reaches maximum value of 1.252 at $x' = 2.408$,

$J_1(x') = 0.52$, $n = 2$

$$\eta_{max} = \frac{2(2.408)(0.52)}{2\pi(2) - \frac{\pi}{2}}$$

$$\boxed{\eta_{max} = 22.78\ \%}$$

- The maximum theoretical efficiency of a reflex klystron oscillator ranges from 20 to 30 %.

Scanned By Scanner Go

* The Condition for maximum kinetic energy transfer to the Cavity walls by electron bunch is given by

Round trip transit angle $\theta_0' = \omega T_0'$

$$= 2\pi N$$

$$\theta_0' = 2\pi\left(n - \frac{1}{4}\right) \quad \because N = n - \frac{1}{4}$$

$$\theta_0' = 2\pi n - \frac{\pi}{2}$$

* Here n is a integer, $N = 1\frac{3}{4}$ is a dominant mode where maximum efficiency occurs.

* Beam Current of a reflex klystron oscillator can be written as,

$$i_2 = -I_0 - \sum_{n=1}^{\infty} 2 I_0 J_n(nx') \cos\left[n(\omega t_2 - \theta_0' - \theta_g)\right]$$

* The fundamental component of the current induced in the cavity by the modulated electron beam is given by,

$$i_{2}(ind) = -\beta_i (2 I_0 J_1(x') \cos(\omega t_2 - \theta_0' - \theta_g)$$

Magnitude of current induced into the cavity

$$I_2(ind) = \beta_i 2 I_0 J_1(x')$$

* The dc power supplied by beam voltage V_0 is $P_{dc} = V_0 I_0$
* The ac power delivered to the load is given by,

$$P_{ac} = \frac{V_1}{\sqrt{2}} \cdot \frac{I_2(ind)}{\sqrt{2}}$$

$$= \frac{V_1 I_2(ind)}{2}$$

$$= \frac{V_1 (\beta_i 2 I_0 J_1(x'))}{2} = \beta_i V_1 I_0 J_1(x')$$

Scanned By Scanner Go

POWER OUTPUT IN TERMS OF REPELLAR VOLTAGE.

- $P_{out} = \beta_i V_1 I_0 J_1(X')$

$$= \cancel{\beta_i} I_0 J_1(X') \times \frac{2V_0 X'}{\beta_i \theta_0'}$$

$$= \frac{2V_0 I_0 X' J_1(X')}{\theta_0'}$$

$$\theta_0' = \omega T_0' = \frac{2\omega m L v_0}{e(V_0+V_R)}$$

$$P_{out} = 2V_0 I_0 X' J_1(X') \times \frac{e(V_0+V_R)}{2\omega m L v_0}$$

$$= 2\cancel{V_0} I_0 X' J_1(X') \frac{(V_0+V_R)}{\omega L} \times \frac{eV_0}{2m} = v_0^2$$

$$\boxed{P_{out} = \frac{2 I_0 X' J_1(X')(V_0+V_R)}{\omega L} \sqrt{\frac{e}{2mV_0}}}$$

HELIX TWTS

* The Travelling wave tube is a new type of tube which has displayed Considerable premise as a broad band amplifier

* The travelling wave tube (TWT) is an O-type, parallel-field, linear beam device.

* In Case of TWT, the microwave circuit is non resonant and the wave propogates with the same speed as the electrons in the beam.

KLYSTRONS	TWT
1. The interaction of electron beam and RF field occurs only at the gaps of a few resonant Cavities.	1. The interaction of electron beam and RF field occurs Continuous for entire length.
2. The Wave in the klystron is not propagating.	2. The wave in TWT is propagating.
3. In klystron each Cavity operates independently.	3. In the Coupled - Cavity TWT there is a Coupling effect between the Cavities.

Slow Wave Structures

* As the operating frequency is increased, both the inductance and the capacitance of the resonant circuit must be decreased in order to maintain resonance at the operating frequency.
* But the gain - bandwidth product is limited by the resonant circuit, the ordinary resonator cannot generate a large output.
* For producing large gain over a wide bandwidth, Slow-wave Structures are designed.
* Slow wave structures are special type circuits that are used in microwave tubes to reduce the wave velocity in a certain direction so that the electron beam and the signal wave can interact.
* The phase velocity of a wave in ordinary waveguides is greater than the velocity of light in a vaccum.
* In the operation of TWTs, the electron beam must keep in step with the microwave signal.
* Since the electron beam can be accelerated only to velocities that are about a fraction of the velocity of light.
* A Slow wave structure must be incorporated in the microwave devices so that the phase velocity of the microwave signal can keep pace with the electron beam for effective interactions.

TYPES:

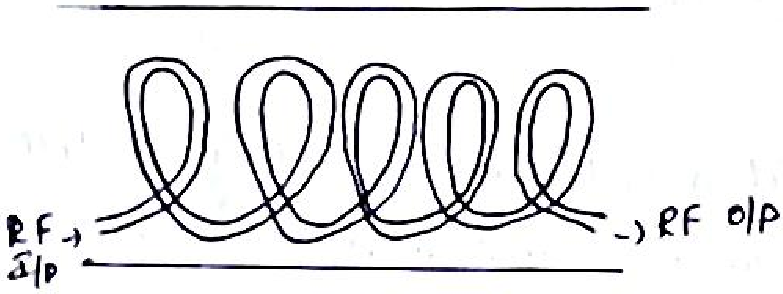


FIG: Helical line

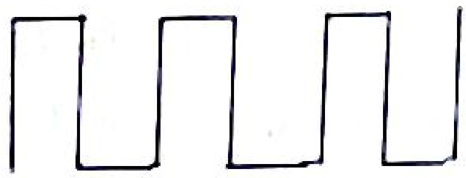


FIG: Folded-back line

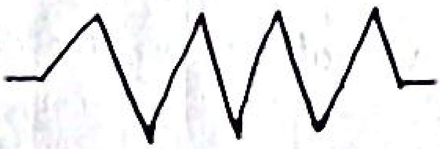


FIG: ZIG ZAG LINE

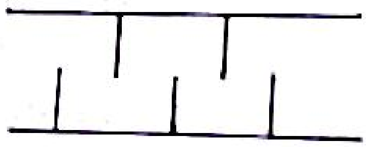


FIG: Inter digital line

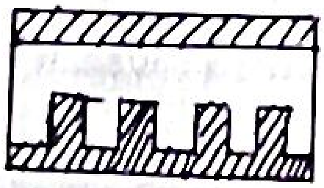


FIG: CORRUGATED WAVEGUIDE

HELIX TWT:

* The commonly used slow wave structure is a helical coil with a concentric conducting cylinder.
* The ratio of the phase velocity v_p along the pitch to the phase velocity along the coil is given by.

$$\frac{v_p}{c} = \frac{p}{\sqrt{p^2+(\pi d)^2}} = \sin \psi$$

where $c = 3\times10^8$ m/s is the velocity of light in freespace

P = helix pitch

d = diameter of the helix

ψ = Pitch angle.

* In general the helical coil may be within a dielectric-filled cylinder. The phase velocity in the axial direction is expressed as,

$$v_p = \frac{P}{\sqrt{\mu\varepsilon\left(P^2+(\pi d)^2\right)}}$$

* For a small pitch angle, the phase velocity along the coil in free space is,

$$v_p \simeq \frac{PC}{\pi d} = \frac{\omega}{\beta}$$

* The group velocity of the wave is, $v_g = \dfrac{\partial\omega}{\partial\beta}$.

AMPLIFICATION PROCESS OF HELIX TWTS:

* The schematic diagram of a helix type travelling wave tube is as shown in the figure.
* A helix travelling wave tube consists of an electron beam and a slow wave structure.
* The electron beam is focused by a magnetic field to prevent spreading of electron beam as it travels down the tube.
* The applied signal propogates around the turns of the helix and produces an "Axial electric field" at the centre of the helix, directed along the helix axis.

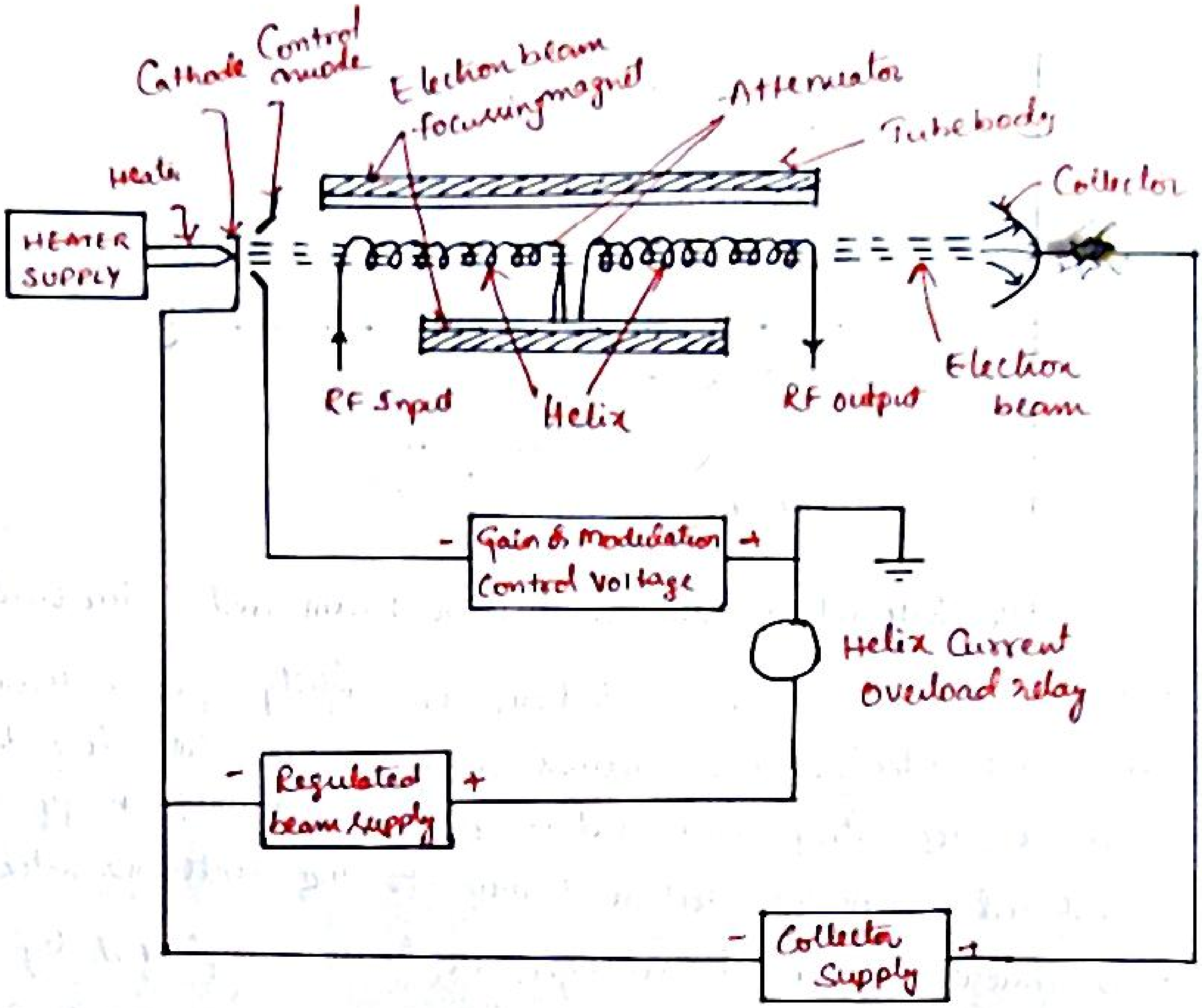


FIG. SCHEMATIC DIAGRAM OF HELIX TWT

* The axial electric field progresses with a velocity i.e, very close to velocity of light multiplied by the ratio of helix pitch to helix circumference.

$$\boxed{v_p = c\left(\frac{p}{\pi d}\right)}$$

* When the electron enters the helix tube, an interaction takes place between moving axial electric field and moving electrons. The electrons entering the retarding field are decelerated and those in accelerating field are accelerated.

* They begin forming a bunch centered around about those electrons that enter the helix during the zero field.

Scanned By Scanner Go

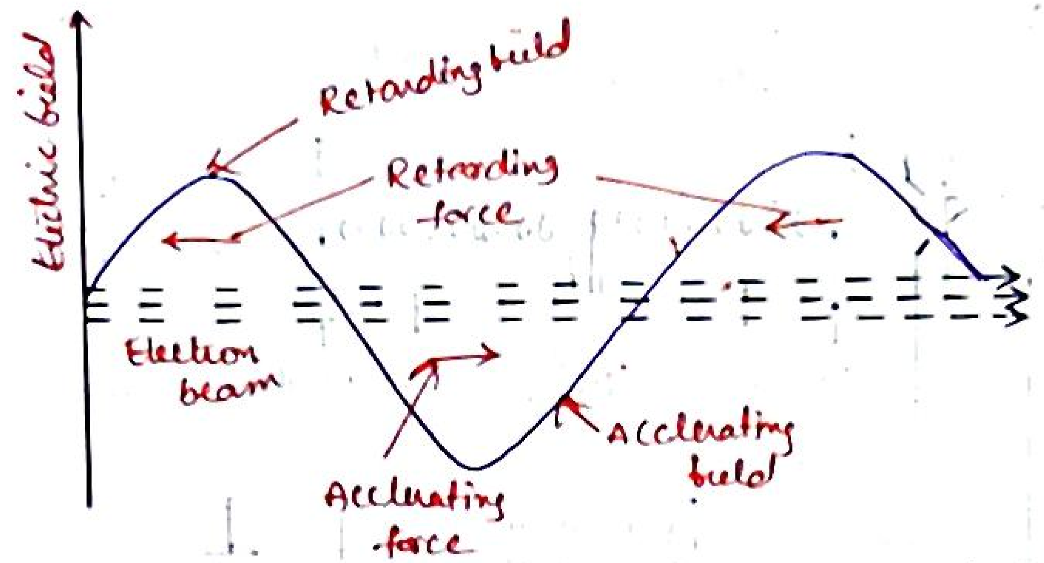


FIG: Interaction between electron beam and electric field.

* Since the dc velocity of electrons is slightly greater than the axial wave velocity, more electrons are in the retarding field than in accelerating field and a great amount of energy is transferred from the electron beam to the electromagnetic field.
* The microwave signal voltage, in turn amplified by the amplified field. The bunch continues to become more compact, and a larger amplification of the signal voltage occurs at the end of the helix.
* An attenuator placed near the center of tube reduces all reflections from mismatched loads to nearly zero.

CHARACTERISTICS:

* Frequency range: 3GHz and higher
* Bandwidth: about 0.8GHz
* Efficiency: 20 to 40%
* Power output: up to 10KW average
* Power gain: up to 60dB

Axial Electric Field :-

* The Convection Current in the election beam induces an electric field in the slow wave structure.
* This induced field adds to the field already present in the Circuit and Causes the Circuit power to increase with distance.
* The Coupling relationship between the election beam and the slow-wave helix is as shown in the figure.

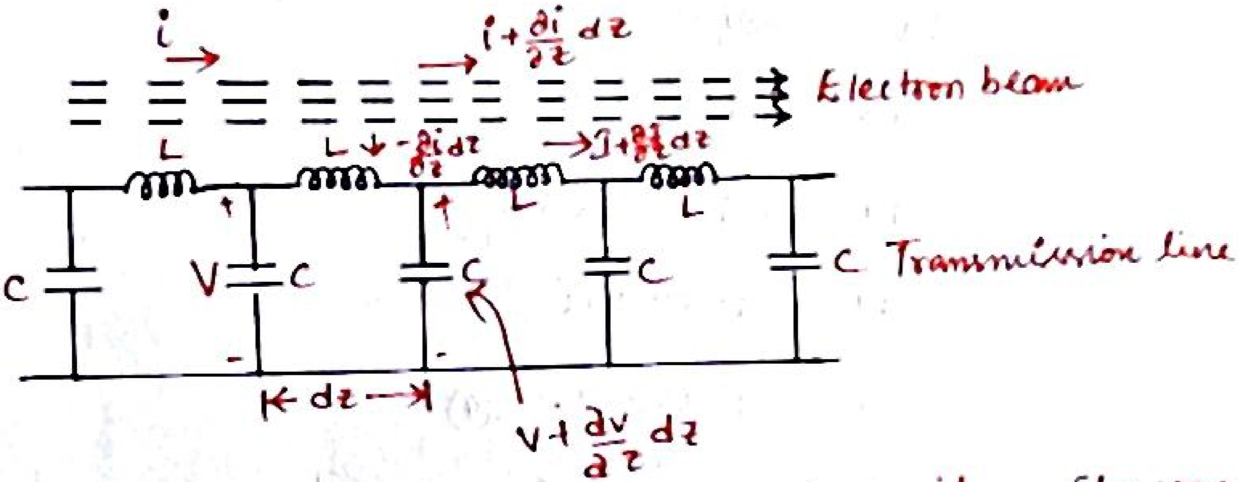


Fig: Election beam Coupled to equivalent Circuit of a Slow wave Circuit

* Helix travelling wave tube is assumed as a distributed loss less transmission line
* A current of $-\frac{\partial i}{\partial z}dz$ is injected from the election beam into transmission line.

L = Inductance per unit length

C = Capacitance per unit length.

I = alternating Current in transmission line

V = alternating voltage in transmission line

i = Convection Current

Apply KCL and KVL to the Circuit.

$$\frac{\partial v}{\partial z} = -L\frac{\partial i}{\partial t} \rightarrow (1)$$

$$\frac{\partial I}{\partial z} = -C\frac{\partial v}{\partial t} - \frac{\partial i}{\partial z} \rightarrow (2)$$

Let $\frac{\partial}{\partial z} = -\gamma$, $\frac{\partial}{\partial z} = j\omega$

eq (1) and (2) becomes

$$-\gamma V = -j\omega L I$$

$$\gamma V = j\omega L I \rightarrow (3)$$

$$-\gamma I = -Cj\omega V - (-\gamma)i$$

$$-\gamma I = -j\omega C V + \gamma i$$

$$I = \frac{j\omega C V - \gamma i}{\gamma} \rightarrow (4)$$

eq (3) becomes $\gamma V = j\omega L\left[\frac{j\omega C V - \gamma i}{\gamma}\right]$

$$\gamma^2 V = j^2\omega^2 CLV - j\omega L\gamma i$$

$$\gamma^2 V = -\omega^2 LCV - j\omega L\gamma i \rightarrow (5)$$

If Convection current $i = 0$, then $\gamma = \gamma_0$

$$\gamma_0^2 V = -\omega^2 LCV - j\omega L\gamma(0)$$

$$\gamma_0^2 \cancel{V} = -\omega^2 LC\cancel{V}$$

$$\gamma_0^2 = -\omega^2 LC \rightarrow \gamma_0^2 = j^2\omega^2 LC$$

$$\gamma_0 = j\omega\sqrt{LC}$$

Characteristic Impedance $Z_0 = \sqrt{\frac{L}{C}}$

$$\gamma_0 Z_0 = j\omega\sqrt{LC}\sqrt{\frac{L}{C}}$$

$$\gamma_0 Z_0 = j\omega L$$

Equation (5) becomes, $\gamma^2 V = -\omega^2 LCV - j\omega L \gamma i$

$$\gamma^2 V = \gamma_0^2 V - \gamma_0 Z_0 \gamma i$$

$$\gamma^2 V = \gamma_0^2 V - \gamma \gamma_0 Z_0 i$$

$$\gamma^2 V - \gamma_0^2 V = -\gamma \gamma_0 Z_0 i$$

$$V(\gamma^2 - \gamma_0^2) = -\gamma \gamma_0 Z_0 i$$

$$\boxed{V = \frac{-\gamma \gamma_0 Z_0 i}{\gamma^2 - \gamma_0^2}}$$

Since $E = -\nabla V$

* The axial electric field is given by,

$$E_1 = -\frac{\partial V}{\partial z} \qquad \left(\because \frac{\partial}{\partial z} = -\gamma\right)$$

$$E_1 = -(-\gamma)V$$

$$E_1 = \gamma V$$

$$E_1 = \gamma \left[\frac{-\gamma \gamma_0 Z_0 i}{\gamma^2 - \gamma_0^2}\right]$$

$$\boxed{E_1 = \frac{-\gamma^2 \gamma_0 Z_0 i}{\gamma^2 - \gamma_0^2}}$$

WAVE MODES [PROPOGATION CONSTANTS]

* The wave-modes of a helix type travelling wave tube can be determined by solving the electronic and circuit equations simultaneously for the propogation constants.
* Each solution for the propogation constants represents a mode of travelling wave in the tube.

* This means that there are four modes of travelling wave in the O-type travelling tube

* The axial electric field is given by, $E_1 = \frac{-\gamma^2 \gamma_0 Z_0}{\gamma^2 - \gamma_0^2} \cdot i$

* Convection of AC spatial current associated with the electron beam.

$$i = \frac{j\beta_e I_0}{2V_0(j\beta_e - \gamma^2)} \cdot E_1$$

$$i = \frac{j\beta_e I_0}{2V_0(j\beta_e - \gamma^2)} \cdot \frac{-\gamma^2 \gamma_0 Z_0}{\gamma^2 - \gamma_0^2} \cdot i$$

$$(\gamma^2 - \gamma_0^2)(j\beta_e - \gamma^2) = \frac{-j\gamma^2 \gamma_0 Z_0 I_0 \beta_e}{2V_0} \rightarrow (1)$$

* This is a fourth order equation in terms of propogation constant of axial waves. There will be four possible solutions for 'γ' which are called as wave modes.

* Its exact solutions can be obtained with numerical methods and a digital computer.

* The approximate solutions may be found by equating the dc electron beam velocity to the axial phase velocity of the travelling wave.

$$\gamma_0 = j\beta_e$$

then eq (1) becomes, $(\gamma^2 - (j\beta_e)^2)(j\beta_e - \gamma^2) = \frac{-j\gamma^2 (j\beta_e) Z_0 I_0 \beta_e}{2V_0}$

upon simplification

$$(\gamma - j\beta_e)^3(\gamma + j\beta_e) = 2\gamma^2 \beta_e^2 \left[\frac{I_0 Z_0}{4V_0}\right]$$

Let travelling wave gain parameter $C = \left[\frac{I_0 Z_0}{4V_0}\right]^{1/3}$

$$(\gamma - j\beta_e)^3(\gamma + j\beta_e) = 2\gamma^2\beta_e^2 C^3 \rightarrow ②$$

* from eq ②, that there are three forward travelling waves corresponding to $e^{-j\beta_e z}$ and one backward travelling wave corresponding to $e^{+j\beta_e z}$. Let the propogation constant of the three forward travelling waves be.

$$\gamma = j\beta_e - \beta_e C\delta \rightarrow ③$$

Substitute eq ③ in eq ②

$$(\cancel{j\beta_e} - \beta_e C\delta - \cancel{j\beta_e})^3(j\beta_e - \beta_e C\delta + j\beta_e) = 2C^3\beta_e^2[j\beta_e - \beta_e C\delta]^2$$

$$-\beta_e^3 C^3\delta^3(2j\beta_e - \beta_e C\delta) = 2C^3\beta_e^2[(j\beta_e)^2 - 2j\beta_e^2 C\delta + \beta_e^2 C^2\delta^2]$$

$\because C\delta << 1$ $\quad -\cancel{2}j\beta_e^4 C^3\delta^3 = -\cancel{2}C^3\beta_e^4$

$$j\delta^3 = 1$$

$$\delta^3 = \frac{1}{j}$$

$$\delta = (-j)^{1/3} = e^{-j[(\frac{\pi}{2} + 2n\pi)]^{1/3}} \quad (n = 0, 1, 2)$$

$\underline{n=0}$ $\quad \delta_1 = e^{-j(\frac{\pi}{6} + \frac{2(0)\pi}{3})}$

$$\delta_1 = e^{-j\pi/6} = \cos\frac{\pi}{6} - j\sin\frac{\pi}{6}$$

$$\delta_1 = \frac{\sqrt{3}}{2} - \frac{j}{2}$$

First root $\quad \gamma_1 = j\beta_e - \beta_e C\delta_1$

$$= j\beta_e - \beta_e C\left[\frac{\sqrt{3}}{2} - \frac{j}{2}\right] = j\beta_e - \frac{\sqrt{3}}{2}\beta_e C + \frac{j\beta_e C}{2}$$

$$\boxed{r_1 = \frac{-\sqrt{3}}{2}\beta_e + j\beta_e\left(1+\frac{c}{2}\right)}$$

n=1 $\quad \delta_2 = e^{-j\left(\frac{\pi}{6}+\frac{2(1)\pi}{3}\right)}$

$$= e^{-j\frac{5\pi}{6}}$$

$$= \cos 150^\circ - j\sin 150^\circ$$

$$\delta_2 = -\frac{\sqrt{3}}{2} - \frac{j}{2}$$

Second root $r_2 = j\beta_e - \beta_e c\,\delta_2$

$$= j\beta_e - \beta_e c\left[-\frac{\sqrt{3}}{2} - \frac{j}{2}\right]$$

$$= j\beta_e + \frac{\sqrt{3}}{2}\beta_e c + \frac{j\beta_e c}{2}$$

$$\boxed{r_2 = \frac{\sqrt{3}}{2}\beta_e c + j\beta_e\left(1+\frac{c}{2}\right)}$$

n=2 $\quad \delta_3 = e^{-j\left(\frac{\pi}{6}+\frac{2(2)\pi}{3}\right)}$

$$= e^{-j\frac{9\pi}{6}} = e^{-j\frac{3\pi}{2}}$$

$$= \cos 270^\circ - j\sin 270^\circ$$

$$\delta_3 = j$$

Third root, $r_3 = j\beta_e - \beta_e c\,\delta_3$

$$= j\beta_e - \beta_e c(j)$$

$$\boxed{r_3 = j\beta_e(1-c)}$$

* The Backward travelling wave, $r = -j\beta_e - \beta_e C \delta_4$

$$\delta_4 = -j\frac{C^2}{4}$$

Fourth root $r_4 = -j\beta_e - \beta_e C\left(-j\frac{C^2}{4}\right)$

$$= -j\beta_e + j\frac{\beta_e C^3}{4}$$

$$\boxed{r_4 = -j\beta_e\left(1 - \frac{C^3}{4}\right)}$$

Four roots, $r_1 = -\frac{\sqrt{3}}{2}\beta_e + j\beta_e\left(1+\frac{C}{2}\right)$

$$r_2 = \frac{\sqrt{3}}{2}\beta_e + j\beta_e\left(1+\frac{C}{2}\right)$$

$$r_3 = j\beta_e(1-C)$$

$$r_4 = -j\beta_e\left(1-\frac{C^3}{4}\right)$$

15A04701 OPTICAL FIBRE COMMUNICATION

UNIT-1 : Introduction to Optical Fibers :.

Introduction :.

Communication may be broadly defined as the transfer of information from one point to another. The information is to be conveyed over any distance a communication system is required. However communication may also be achieved using an electromagnetic carrier which is selected from the optical range of frequencies

Fiber-optic communication is a method of transmitting information from one place to another by sending pulses of light through an optical fiber. Fiber is preferred over electrical cabling when high bandwidth, long distance, or immunity to electromagnetic interference are required.

Optical fiber is used by many telecommunications companies to transmit telephone signals, internet communication, and cable television signals. Researchers at Bell Labs have reached internet speeds of over 100 petabits×kilometer per second using fiber-optic communication.

→ Communication is defined as transfer of information from one point to another.

→ In a communication system, the information is transferred to long distance by the process called modulation.

→ In modulation the information is superimposed on the carrier wave which is nothing but an EM wave (Electromagnetic wave)

→ Depending upon the type of carrier wave frequency, the communication is divided in to 3 types

i) RF communication
ii) μ-wave communication and
iii) Optical communication.

Optical Communication:

→ In optical communication, the light is used as carrier wave. Optical carrier wave is an electromagnetic (EM) wave of frequency ranges from 10^{14} to 10^{15} Hz.
In terms of wave length 10nm to 10^{6} nm.

→ The optical communication can be divided in to 2 types
i) Guided communication system
ii) un guided communication system.

→ In Guided communication system, the fiber is used as a chann between Transmitter & Receiver.

Evolution / History of Fiber optic system

In 1880 Alexander Graham Bell reported the (Transmission) of spe using a light beam. After 4 years, Bell proposed photo phone which giving speech Txion over a distance of 200 m.

But optical communication was limited to mobile, low-capac communication links due to lack of suitable light sources and the pr that light Txion in the atmosphere is restricted to line of sight is severly affected by disturbances such as rain, snow, fog, etc.

→ Later longer wavelength EM waves [radio & μ-wave's] us as a carriers for information transfer in the atmosphere with less affected by atmospheric conditions, but they are limited in information amount, and with considerable distances depending on their [EM-wave] wave length.

∴ Capacity ∝ Band width ∝ frequency of a carrier.

i.e as carrier frequency is high, Transmission Bandwidth is hig and hence information carrying capacity is high.

→ For these reasons, radio communication was developed to high frequencies (VHF & UHF) leading to the introduction of the even higher frequency μ-wave, lately millimeter wave transmission.

→ Optical communication was stimulated in the early 1960's with the invention of the laser.

→ Laser provided a powerful coherent light source, with low beam divergence. it enhanced free space optical transmission [a practical possibility] practically possible.

→ But it is also restricted to short distance applications due to atmospheric conditions again.

→ In 1966 Kao, Hockham and werts made proposals for optical communication via dielectric wave guides or optical fibers fabricated from glass to avoid degradation of optical signal by the atmosphere, which replaces co-axial cable.

→ Initially optical fibers exhibited very high attenuation (1000 dB/Km), which is not compatible with coaxial cable (5 to 10 dB/Km). Also fiber jointing becomes serious problem.

→ within 10 years, optical fiber losses were reduced to below 5dB/Km and suitable low-loss jointing technique were perfected.

→ Semiconductor optical sources [LASER, LED] & Detectors [photo diodes, pho transistors] compatible in size with optical fibers were designed and fabricated to enable successful implementation of the optical fiber system.

Since 1970s, 5 Generations of fiber-optic system were there

First generation: The first generation of light wave systems uses GaAs semiconductor laser and operating region was near 0.8μm. Other specifications of this generation are as under.

i) Bit rate 345 Mb/s

ii) Repeater spacing : 10 Km

Second generation :

i) Bit rate : 100 Mb/s to 1.7 Gb/s

ii) Repeater spacing : 50 km

iii) Operation wavelength : 1.3 μm

iv) Semiconductor : InGaAsP

Third Generation : In 3rd generation by the development of manufacturing technology to increase the purity of silica glasses, transmission capacity increased.

i) Bit rate : 10 Gb's

ii) Repeater spacing : 100 km

iii) Operating wavelength : 1.55 μm

Fourth Generation :

Fourth Generation uses WDM technique

Bit rate : 10 Tb/s

Repeater spacing : >10,000 km

Operating wavelength : 1.45 to 1.62 μm

Fifth Generation : uses erbium-doped fiber and the transmission capacity was increased.

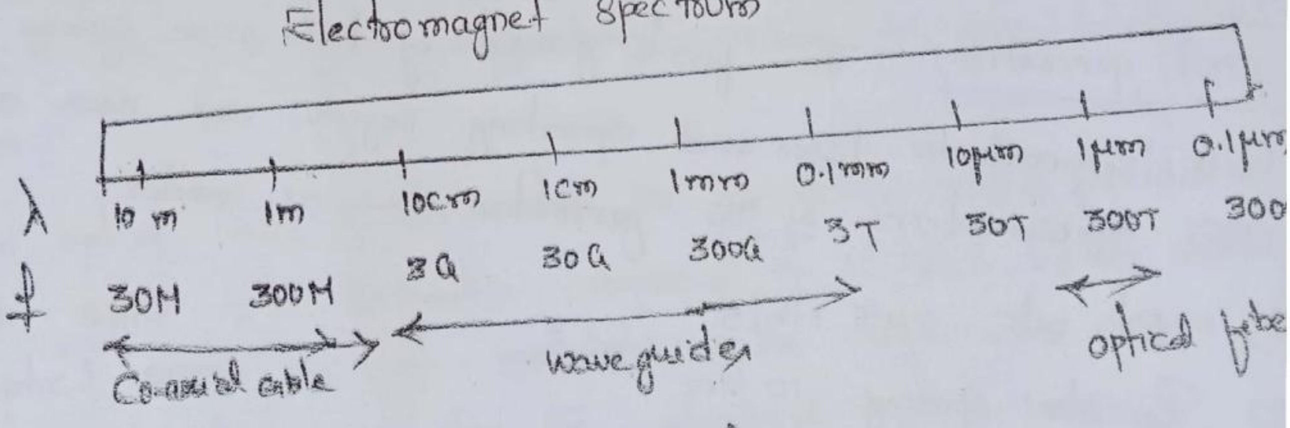
Fifth generation uses Raman amplification technique and optical solitons

Bit rate : 40 – 160 Gb/s

Repeater spacing : 24000 km – 35000 km

Operating wave length : 1.53 to 1.57 μm

Electromagnet spectrum



$$BW \propto f$$

The general system:-

An optical fiber communication system is similar in basic to any type of communication system. A block schematic of a general communication system is shown in fig 2(a). The function of which is to convey the sig from the information source over the transmission medium to the destination.

The communication system consists of a transmitter or modulator linked to the information source, the transmission medium, and a receiver or demodulator at the destination point. In electrical comm the information source provides an electrical signal which is not a electrical (e.g. sound) to a transmitter comprising electrical and electronic components which converts the signal in to a suitable form for propagation over the transmission medium.

The transmission medium consists of a pair of wires, a co-axial cable or a radio link through free space down which the signal is transmitted to the receiver where it is transformed in to the original electrical information signal (demodulated) before being passed to the destination. In any transmission medium the signal is attenuated, or suffers loss, and is subject to degradation due to contamination by random signal and noise, as well as possible distortion imposed by with in the medium itself. ∴ In any comm s/m there is a maximum permitted distance b/w the transmitter and the receiver beyond which the system effectively ceases to give intelligible communication

In optical fiber communication the information source provides an electrical signal to a transmitter comprising an electrical stage which drives an optical source to give modulation of the light wave carrier. The optical source which provides the electrical-optical conversion may be either a semiconductor laser or light-emitting diode (LED)

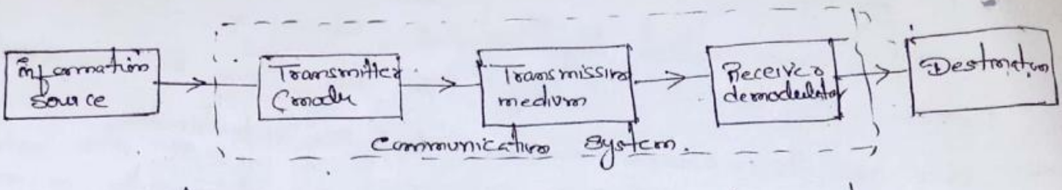


fig 2(a). The general communication system.

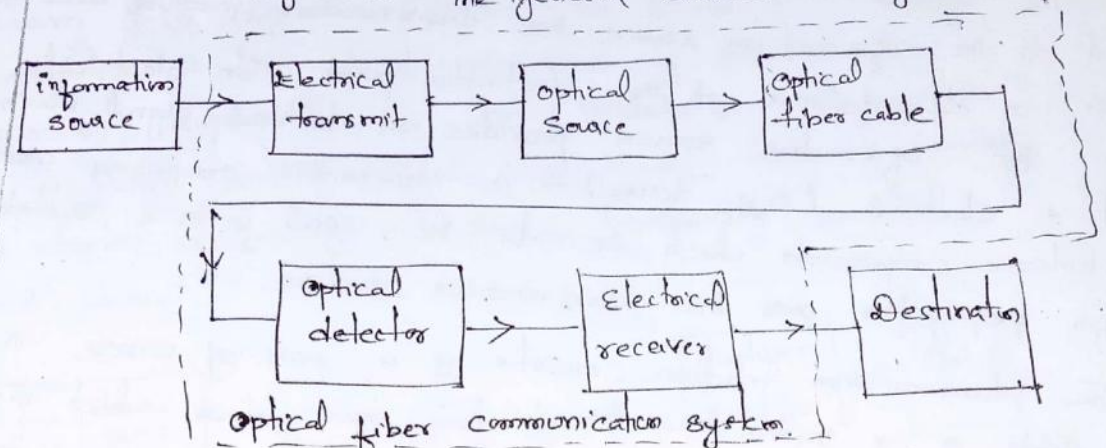
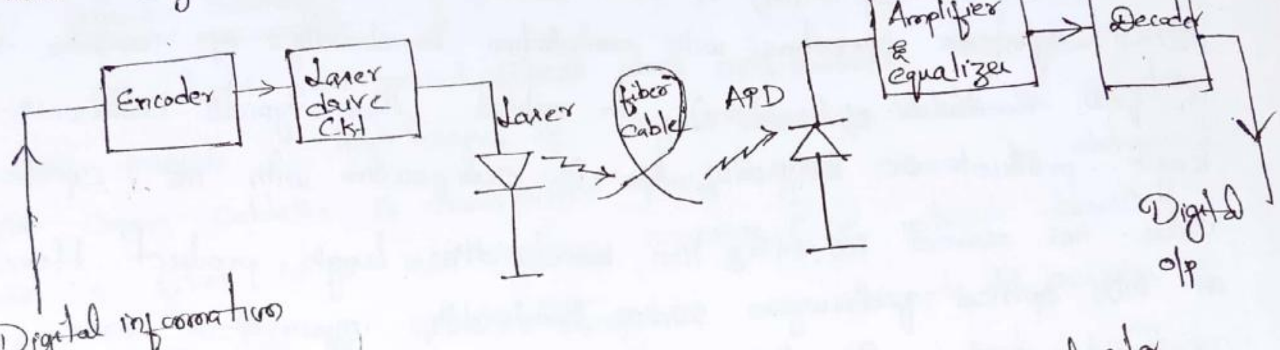


fig 2(b) The optical fibre communication system.

In optical fiber communication the information source provides an electrical sig to a transmitter comprising an electrical stage which drives an optical source to give modulation of the light wave carrier. The optical source which provides the electrical - optical conversion may be either a semiconductor laser or Light-emitting diode (LED).

The transmission medium consists of an optical fiber cable and the receiver consists of an optical detector which drives a further electrical stage and hence provides demodulation of the optical carrier. photo diodes (p-n, p-i-n (or) avalanche) and in some instances, photo transistors and photo conductors are utilized for the detection of the optical signal and the optical electrical conversion. Thus there is a requirement for electrical interfacing at either end of the optical link and at present the signal processing is usually performed electrically.

The optical carrier may be modulated using either an analog or digital information slg. From fig 2(b) analog modulation involves the variation of the light emitted from the optical source in a continuous manner. with digital modulation, discrete changes in the light intensity are obtained (i.e. on-off pulses). The also analog modulation with an optical fiber communication system is less efficient, requiring a far higher slg-to-noise ratio at the receiver than digital modulation. The linearity needed for analog modulation is not always provided by semiconductor optical sources, especially at high modulation frequencies for these reasons, analog optical fiber comm links are generally limited to shorter distances and lower bandwidth operation than digital links



fig(3) A digital optical fiber link using a semiconductor laser source and an avalanche photo diode (APD) detector.

The fig ③ shows a block schematic of a typical digital optical fiber link. The input digital slg from the information source is suitably encoded for optical transmission. The laser drive ckt directly modulates the intensity of the semiconductor laser with the encoded digital signal. Hence a digital optical signal is launched into the optical fiber cable. The avalanche photo diode (APD) detector is followed by a front-end amplifier and equalizer or filter to provide gain as well as linear signal processing and noise band width reduction. Finally the signal obtained is decoded to give the original digital informat

Advantages of optical fiber Communication:-

Communication using an optical carrier wave guided along a glass fiber has a no of extremely attractive features, it is useful to consider the merits and special features offered by optical fiber communications over more conventional electrical communications.

i) Enormous potential Bandwidth :-

The optical carrier frequency is the range 10^{13} to 10^{16} Hz (around 10^{14} Hz or 10^{5} GHz) a greater potential transmission bandwidth than metallic cable systems (i.e co-axial cable bandwidth is 20 MHz over distance of 10 km) or even millimeter wave radio systems (i.e systems operating with modulation bandwidths of 100 MHz over a few hundreds of meters). In optical fiber typical bandwidth length product is 5000 GHz km in comparision with the co-axial cable has around 100 MHz km bandwidth - length product. Hence at this optical fiber was 50,000 bandwidth improvement over - co-axial cable. So this was provided much longer transmission distance.

ii) Small size and weight :.

Optical fiber have very small diameters which are often no greater than the diameter of a human hair. The fibers are covered with protective coatings they are far smaller and much lighter than corresponding copper cables. An expansion of signal transmission within mobiles such as aircraft, satellites and even ships

iii) Electrical isolation :-

Optical fibers which are fabricated from glass or sometimes a plastic polymer, are electrical insulators, their metallic counter parts they do not exhibit earth loop and interface problems This property makes optical fiber transmission ideally suited for

Communication in electrically hazardous environments as the (iii) fibers create no arcing or spark hazard at abrasions or short circuits.

(iv) immunity to interference and crosstalk!.

Optical fibers form a dielectric waveguide and are therefore free from electromagnetic interface (EMI), radio-frequency interference (RFI), or switching transients giving electromagnetic pulses (EMPs). Hence the operation of an optical fiber communication system is unaffected by transmission through an electrically noisy environment and the fiber cable requires no shielding from EMI.

It is easy to ensure that there is no optical interference b/w fibers and hence comm using electrical conductor crosstalk is negligible, even when many fibers are cabled together.

(V) Signal Security!:

The light from optical fibers does not radiate significantly and ∴ they provide a high degree of signal security. Unlike the situation with copper cables, a transmitted optical signal cannot be obtained from a fiber in a noninvasive manner (ie without drawing optical power from the fiber). Therefore any attempt to acquire a message signal transmitted optically may be detected. This feature is obviously attractive for military, banking and general data transmission (i.e computer network) applications.

(Vi) Low transmission loss:-

The production of optical fiber cables which exhibit very low attenuation or transmission loss in comparision with the best copper conductors. Fibers have been fabricated with losses as low as 0.15 dB/km and this feature has become a major advantage of optical fiber communications. It facilitates the implementation of communication links with extremely wide optical repeater or amplifier spacings thus reducing both system cost and complexity

Together with the already proven modulation bandwidth capability of fiber cables, this property has provided of optical fiber communication and in the majority of long-haul telecommunication applications, replacing not only copper cables, but also satellite communications.

(vii) Ruggedness and flexibility:

Optical fibers are manufactured with very high tensile strengths. The fibers may also be bent to quite small radii or twisted without damage. Cable structures have been proved flexible, compact and extremely rugged. Taking the size and weight advantage into account, these optical fiber cables are generally superior in terms of storage, transportation, handling and installation to corresponding copper cables, while exhibiting at least comparable strength and durability.

(viii) System ~~reliability~~ reliability and ease of maintenance:

Optical fiber cables which reduces the requirement for intermediate repeaters or line amplifiers to boost the transmitted signal strength. Hence with fewer optical repeaters or amplifiers system reliability is generally enhanced in comparision with conventional electrical conductor systems. Furthermore, the reliability of the optical components is no longer a problem with predicted life-times of 20 to 30 years being quite common. Both these factors also tend to reduce maintenance time and costs.

(ix) potential low cost:

The optical fiber transmission medium is made from sand, so in comparision with copper conductors, optical fibers offer the potential for low-cost line communication. The optical fiber transmission medium which for bulk purchases has become competitive with copper wires (i.e twisted pairs)

Disadvantages of Optical Fiber Communications

i> High initial cost:- The initial cost of installation or setting up cost is very high compared to all other system.

ii> Maintenance and Repairing cost:- The maintanance and repairing of fiber optic systems is not only difficult but expensive also.

iii> Jointing and Test procedures: Since optical fibers are of very small size. The fibre joining process is very costly and requires skilled manpower.

iv> Tensile stress:- Optical fibers are more susceptible to buckling, bending and tensile stress than copper cables.

v> Short links:- Even though optical fiber cables are inexpensive, it is still not cost effective to replace every small conventional connectors

Applications of Optical Fiber Communications:

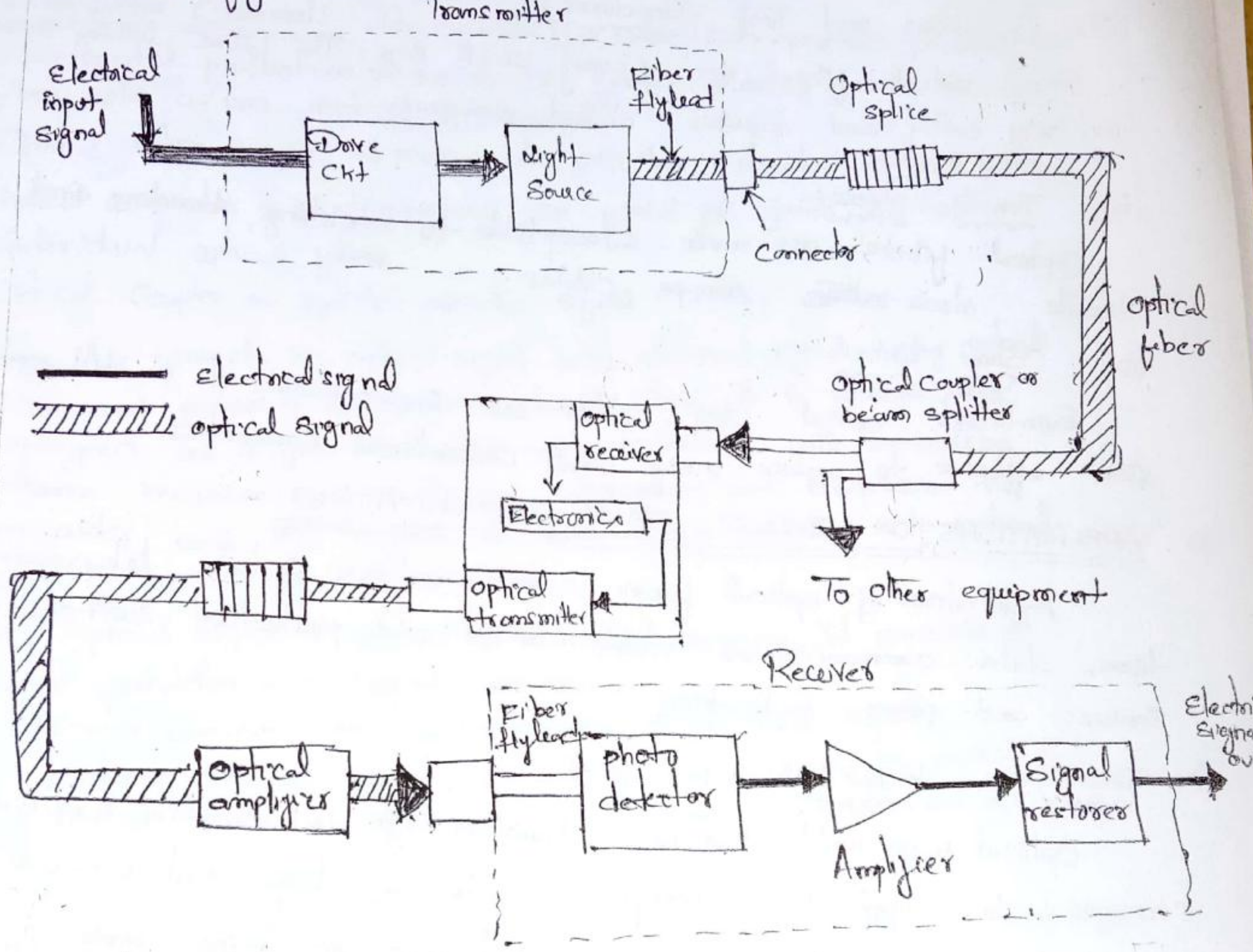
Applications of optical fiber communications include, telecommunications, data communications, video control and protection switching, sensors and power applications.

i> Telephone Networks:- optical wave guide has low attenuation, high transmission bandwidth compared to copper lines, therefore number of long haul co-axial trunks links between telephone exchanges are being replaced by optical fiber links.

ii> Urban broad band service networks:- Modern suburban communications involves video text, video conferencing, video telephony, switched broadband communication network. All these can be supplied over a single fiber optic link. Fiber optic cable is the solution to the many of today's high speed, high bandwidth data communication problems and will continue to play a large role in future telecom and data-com n/w

Elements of Optical fiber transmission link:-

An optical fiber transmission link comprises the elements shown in below fig.



Transmitter consisting of a light source and its associated drive circuitry, a cable offering mechanical and environmental protection to the optical fibers contained inside, and a receiver consisting of a photo detector plus amplification and signal restoring circuitry. Additional components include optical amplifiers, connectors, splices, couplers and regenerators.

The optical fiber consists of three main elements:

1. Transmitter:. An electric signal is applied to the optical transmitter. The optical transmitter consists of driver circuit, light source and fiber flylead.

* Driver circuit drives the light source
* Light source converts electrical signal to optical signal
* Fiber flylead is used to connect optical signal to optical fiber.

2. Transmission Channel:. It consists of a cable that provides mechanical and environmental protection to the optical fibers contained inside. Each optical fiber acts as an individual channel.

* Optical splice is used to permanently join two individual optical fibers
* Optical connector is for temporary non-fixed joint joints b/w two individual optical fibers
* Optical Coupler or splitter provides signal to other devices.
* Repeater converts the optical signal into electrical signal using optical receiver and passes it to electronic circuit where it is reshaped and amplified as it gets attenuated and distorted with increasing distance because of scattering, absorption and dispersion in waveguides and this signal is then again converted into optical signal by the optical transmitter.

3. Receiver: optical signal is applied to the optical receiver. It consists of photo detector, amplifier and signal restorer.

* Photo detector converts the optical signal to electrical signal.
* Signal restorer and amplifiers are used to improve signal to noise ratio of the signal as there are chances of noise to be introduced in the signal due to the use of photo detectors.

→ For short distance communication only main elements are required

Source - LED
Fiber - Multimode step index fiber
Detector - PIN detector.

→ For long distance communication along with the main elements there is need for couplers, beam splitters, repeaters, optical amplifiers.

Source - LASER diode
Fiber - Single mode fiber
Detector - Avalanche photo diode (APD)

The cable fiber is one of the most important element in an optical fiber link. In addition to protecting the glass fibers during installation and service, the cable may contain copper wires for powering optical amplifiers or signal regenerators for long distance links.

Analogous to copper cables, optical fiber cables can be installed either aerially, in ducts, undersea, or burried directly in the ground as shown in below figure.

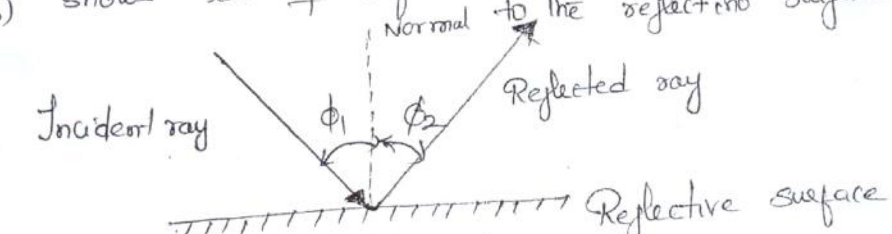
Individual cable lengths will range from several hundred meters to several kilometers. Size and cable weight determine the actual length of a single cable section. Shorter segments tend to be used when the cables are pulled through ducts. Longer lengths are used in aerial, direct - burial or under sea applications. Splicing together individual cable sections forms continuous transmission lines for these long distance links. For under sea installations, the splicing and repeater installation functions are carried out on board a specially designed cable - laying ship.

Ray optics:— (or) Ray optic Representation (or) Laws of optics (or) Ray Transmission Theory :-

Before studying how the light actually propagates through the fiber, laws governing the nature of light must be studied. These are called as laws of optics (Ray Theory).

In free space light travels at its maximum possible speed i.e 3×10^8 m/s. when light travels through a material, it exhibits certain behaviour explained by laws of reflection, refraction.

Reflection:- The law of reflection states that, when a light ray is incident upon a reflective surface at some incident angle ϕ_1 from imaginary perpendicular normal, the ray will be reflected from the surface at some angle ϕ_2 from normal which is equal to the angle of incidence fig 2(b) shows law of reflection.



law of reflection $\angle\phi_1 = \angle\phi_2$

fig 2(b): Reflection.

ii) Refraction:

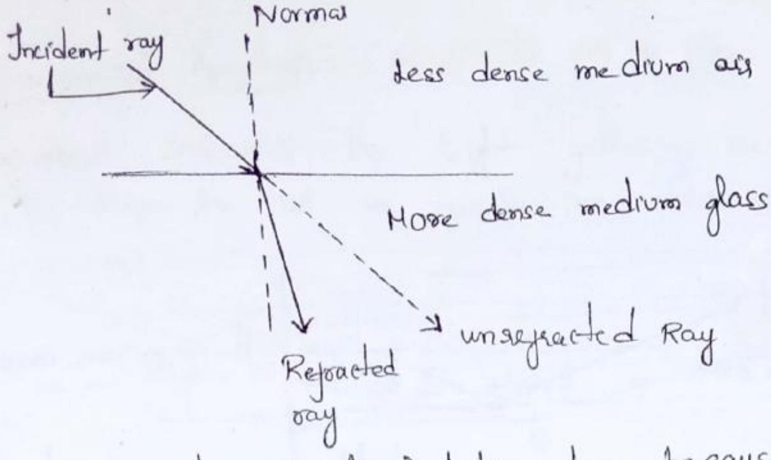
Refraction occurs, when light ray passes from one medium to another medium i.e the light ray changes its direction at interface.

Refraction occurs whenever density of medium changes.

The refraction can also be observed at air and glass interface.

when wave passes through less dense medium to more dense medium, the wave i.e - refracted (bent) towards the normal.

Fig 2(c) shows the refraction phenomena.



→ The refraction (bending) takes place because light travels at different speed in different mediums. The speed of light in free space is higher than in water or glass.

(iii) Refractive index:

→ The amount of refraction or bending that occurs at the interface of two materials of different densities is usually expressed as refractive index of two materials. Refractive index is also known as index of refraction and is denoted by n.

→ Based on material density, the refractive index is expressed as the ratio of the velocity of light in free space to the velocity of light of the dielectric material (substance).

$$\text{Refractive index } n = \frac{\text{Speed of light in air}}{\text{Speed of light in medium}} = \frac{c}{v}$$

The refractive index for vaccum and air is 1.0 for water it is 1.3 and for glass refractive index is 1.5

(iv) Total internal Reflection:-

Its an optical phenomenon, that occurs when a ray of light strike boundary at an angle larger than critical angle with respect to normal, all the light is reflected.

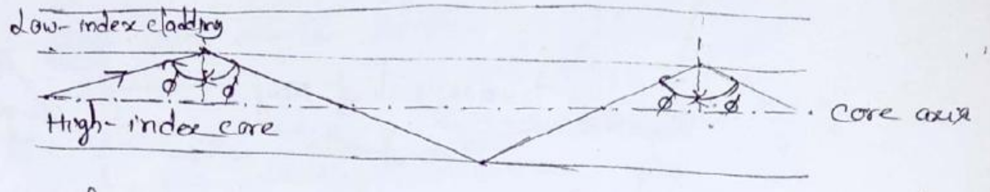


fig:- The transmission of a light ray in a perfect optical fiber

→ The figure illustrates transmission of light ray in an optical fiber via a series of total internal reflections at the interface of silica core & cladding.

→ The ray has an angle of incidence ϕ at the interface which is greater than critical angle and is reflected at the same angle to the normal.

(V) Acceptance angle: It is the angle at which light ray must enter the optical fiber to undergo total internal reflection (TIR)

→ The geometry concerned with launching the light ray is shown in the figure.

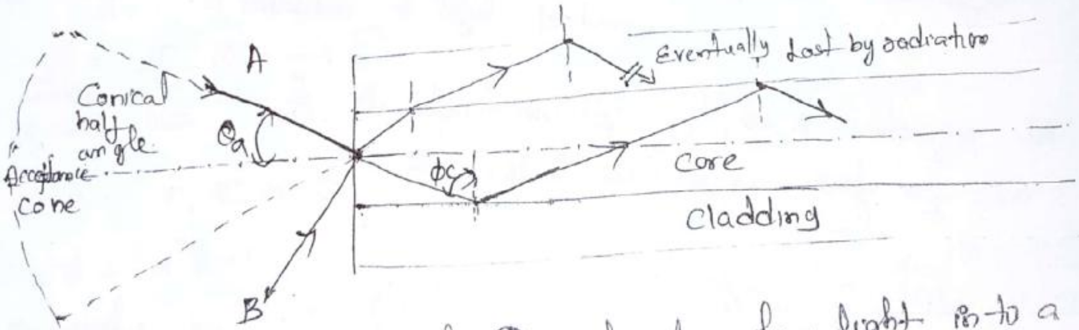


fig:- The acceptance angle θ_a when launching light in-to a optical fiber

→ fig illustrates a meridonial ray A at the critical angle ϕ_c which enters the fiber core at angle θ_a to the fiber axis & is refracted at the air-core interface before transmission to core-cladding interface at critical angle.

→ Also shows incident ray B at an angle greater than θ_a is refracted into cladding & lost by radiation.

(9) (11)

(vi) Numerical Aperture (NA):- The NA of fibre is a figure of merit. which represents NA indicates the light gathering capability of an optical fiber. Larger the NA, the greater the amount of light accepted by fiber

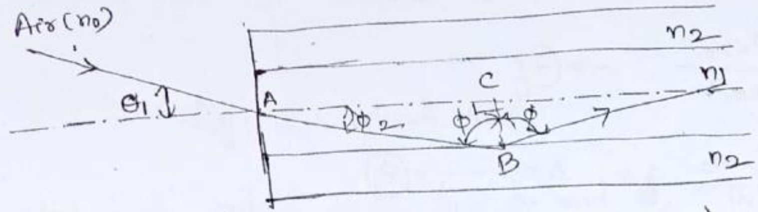


Fig:- The ray path for a meridional ray launched into an optical fiber in air at an input angle less than the acceptance angle for the fiber.

Figure shows a light ray incident on the fiber core at an angle θ_1 to the fiber axis that is less than acceptance angle θ_a.

→ A ray enters the fiber from a R.I. n_0 & the fiber core has R.I. n_1 slightly greater than cladding Refractive index R.I. n_2.

→ Consider refraction at the air-core interface & using Snell's law

$$n_0 \sin\theta_1 = n_1 \sin\theta_2 \rightarrow (1)$$

Consider the right angled triangle ABC in the above figure

$$\phi = \frac{\pi}{2} - \theta_2 \rightarrow (2)$$

where ϕ is greater than critical angle at core cladding interface.

$$n_0 \sin\theta_1 = n_1 \cos\phi \rightarrow (3)$$

$\phi = \pi/2 - \theta_2$
$-\theta_2 = \phi - \pi/2$
$\theta_2 = \pi/2 - \phi$
$n_0 \sin\theta_1 = n_1 \sin(\pi/2 - \phi)$
$n_0 \sin\theta_1 = n_1 \cos\phi$

$$\sin^2\phi + \cos^2\phi = 1$$

$$\Rightarrow n_0 \sin\phi_1 = n_1 (1 - \sin^2\phi)^{1/2} \rightarrow (4)$$

For TIR, ϕ becomes equal to critical angle. θ_1 become acceptance angle for fiber θ_a.

$$\therefore n_0 \sin\theta_a = (n_1^2 - n_2^2)^{1/2} \rightarrow (5)$$

$\therefore (1-\sin^2\phi)^{1/2}$
$\left(1 - \frac{n_2^2}{n_1^2}\right)^{1/2}$
$\left(\frac{n_1^2 - n_2^2}{n_1^2}\right)^{1/2}$
$(n_1^2 - n_2^2)^{1/2}$

NA is defined as:

$$NA = n_0 \sin\theta_a = (n_1^2 - n_2^2)^{1/2} \rightarrow (6)$$

from this acceptance angle

$\because n_0 = 1$ $\theta_a = \sin^{-1}(n_1^2 - n_2^2)^{1/2}$

NA can also be given in terms of relative refractive index difference Δ

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \rightarrow (7)$$

$$\simeq \frac{n_1 - n_2}{n_1} \text{ for } \Delta \leq 1 \longrightarrow (8)$$

$$NA \neq n_1 (2\Delta)^{1/2}$$

~~Critical angle:~~

Now squaring on both sides of equ $\rightarrow$ (6)

$$NA^2 = (n_1^2 - n_2^2) \longrightarrow (9)$$

Hence combining eq (7) & (9) we get

$$NA^2 = 2n_1^2\Delta$$

$$NA = \sqrt{2n_1^2\Delta}$$

$$\boxed{NA = n_1\sqrt{2\Delta} \quad \text{(or)} \quad NA = n_1(2\Delta)^{1/2}} \longrightarrow (10)$$

The relationship given in eq^n (10) for the numerical aperature are a very useful measure of the light controlling ability of a fiber.

~~Critical angle:~~

~~Problems:~~

Critical angle:

→ When the angle of incidence (ϕ_1) is progressively increased, there will be progressive increase of refractive angle (ϕ_2). At some condition (ϕ_1) the refractive angle (ϕ_2) becomes $90°$ to the normal. When this happens the refracted light ray travels along the interface. The angle of incidence (ϕ_1) at the point at which the refractive angle (ϕ_1) becomes $90°$ is called the critical angle. It is denoted by ϕ_c.

→ The critical angle is defined as the minimum angle of incidence (ϕ_1) at which the ray strikes the interface of two media and causes an angle of refraction (ϕ_2) equal to $90°$.

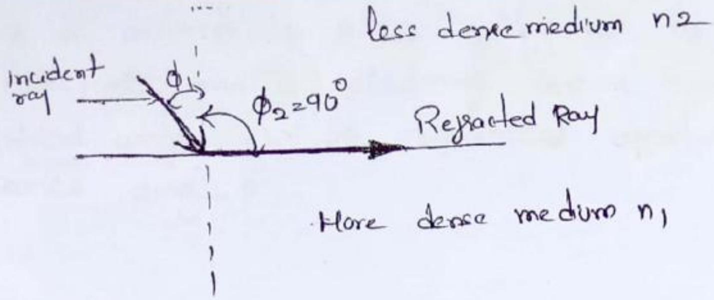


fig:. Critical angle.

Hence at critical angle $\phi_1 = \phi_c$ and $\phi_2 = 90^\circ$

Using Snell's law : $n_1 \sin\phi_1 = n_2 \sin\phi_2$

$$\sin\phi_c = \frac{n_2}{n_1} \sin 90^\circ$$

$$\sin 90^\circ = 1$$

$$\sin\phi_c = \frac{n_2}{n_1}$$

$$\boxed{\text{Critical angle } \phi_c = \sin^{-1}\left[\frac{n_2}{n_1}\right]}$$

Snell's Law:

→ Snell's Law states how light ray reacts when it meets the interface of two media having different indexes of refraction.

→ Let the two medias have refractive indexes n_1 and n_2 where $n_1 > n_2$

ϕ_1 and ϕ_2 be the angles of incidence and angle of refraction respectively. Then according to Snell's law, a relationship exists between the refractive index of both materials given by

$$n_1 \sin\phi_1 = n_2 \sin\phi_2$$

A refractive index model for Snell's law is shown in fig below

→ The refracted wave will be towards the normal when $n_1 < n_2$ and will away from it when $n_1 > n_2$

Problem A silica optical with a core diameter large enough to be considered by ray theory analysis has a core refractive index of 1.50 and a cladding refractive index of 1.47. Determine (a) The critical angle at the core-cladding interface. (b) The NA for the fiber (c) The acceptance angle in air for the fiber.

Sol: (a) The critical angle ϕ_c at the core-cladding interface is given by from eq (*)

$$\phi_c = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1}\frac{1.47}{1.50} = 78.5^\circ$$

(b) from eq

$$NA = (n_1^2 - n_2^2)^{1/2} = (1.50^2 - 1.47^2)^{1/2} = (2.25 - 2.16)^{1/2} = 0.30$$

(c) The acceptance angle in the air θ_a is given by

$$\theta_a = \sin^{-1} NA = \sin^{-1} 0.30 = 17.4^\circ$$

{ refractive index for air = 1
water = 1.33
Crown glass = 1.517
dense flint glass = 1.655
diamond = 2.4

Scanned with CamScanner

12

(1) Consider a multimode silica fiber that has a core refractive index $n_1 = 1.480$ and a cladding index $n_2 = 1.460$. Find (a) the critical angle, (b) the numerical aperture, and (c) the acceptance angle?

Solution

(a) from Critical angle is given by

$$\phi_c = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1}\frac{1.460}{1.480} = 80.5^0$$

(b) Numerical aperture is

$$NA = (n_1^2 - n_2^2)^{1/2} = \left[(1.480)^2 - (1.460)^2\right]^{1/2} = 0.242$$

(c) Acceptance angle in air $(n = 1.00)$ is

$$\Theta_A = \sin^{-1} NA = \sin^{-1} 0.242 = 14^0$$

(2) Consider a multimode fiber that has a core refractive index of 1.480 and a core-cladding index difference 2.0 percent ($\Delta = 0.020$). Find the (a) numerical aperture (b) the acceptance angle and (c) the critical angle.

$\Delta \rightarrow$ Core-cladding refractive index difference (or simply index difference)

Cladding index is $n_2 = n_1(1-\Delta) = 1.480(0.980) = 1.450$

(a) Numerical aperture $(N.A) = n_1\sqrt{2\Delta} = 1.480(0.04)^{1/2} = 0.296$

(b) Acceptance angle in air. $n = 1.00$

$$\Theta_A = \sin^{-1} NA = \sin^{-1} 0.296 = 17.2^0$$

(c) The critical angle at the core-cladding interface is

$$\phi_c = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1} 0.980 = 78.5^0$$

for Δ

$\Delta = \frac{n_1 - n_2}{n_1}$

$(0.020)(1.480) = n_1 - n_2$

$0.0296 = 1.480 - n_2$

$n_2 = 1.480 - 0.0296$

(or) $\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \simeq \Delta 2n_1^2 = n_1^2 - n_2^2$

$n_2^2 = n_1^2 - \Delta 2n_1^2$

$n_2 = \sqrt{n_1^2 - \Delta 2n_1^2} \Rightarrow n_2 = n_1 - (2\Delta)n_1 \Rightarrow n_2 = 1.480 - (2\times 0.020)^2 \times 1.480$

$n_2 = 1.480 - 0.0592$

One more formula to find n_2 value

① $\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}$

② $\Delta \approx \frac{n_1 - n_2}{n_1}$ for $\Delta \ll 1$

$\Delta = 0.020 \ll 1$ so we need to consider ② formula

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$\Delta n_1 = n_1 - n_2$$

$$n_2 = n_1 - \Delta n_1$$

$$n_2 = 1.480 - (0.020)(1.480)$$

$$n_2 = 1.480 - 0.0296$$

$$n_2 = 1.450$$

→ Repeated problem

③ A silica optical fiber with a core diameter large enough to be considered by ray theory analysis has a core refractive index of 1.50 and a cladding refractive index of 1.47. Determine: (a) the critical angle at the core-cladding interface (b) The N.A for the fiber (c) The acceptance angle in air for the fiber.

Solution (a): The critical angle ϕ_c at the core-cladding interface is given by

$$\phi_c = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1}\frac{1.47}{1.50} = 78.5^0$$

(b) $NA = (n_1^2 - n_2^2)^{1/2} = \sqrt{n_1^2 - n_2^2} = \sqrt{(1.50^2 - 1.47^2)} = \sqrt{(2.25 - 2.}$

$= 0.30$

(c) The acceptance angle in air θ_a is given by

$$\theta_a = \sin^{-1} NA = \sin^{-1} 0.30 = 17.4^0$$

Scanned with CamScanner

A typical refractive index difference for an optical fiber designed for long distance transmission is 1%. Estimate the N.A ~~in air~~ for the fiber when the core index is 1.46. Further calculate the critical angle at the core-cladding interface within the fiber. ~~It may be assumed that~~

Refractive index difference $\Delta = 1\% = \frac{1}{100} = 0.01$

$n_1 = 1.46$

Δ is ≤ 1 so

$\Delta = \frac{n_1 - n_2}{n_1}$

$\Delta n_1 = n_1 - n_2 \Rightarrow n_2 = n_1 - \Delta n_1 \Rightarrow n_2 = 1.46 - 0.01 \times 1.46$

$n_2 = 1.46 - 0.0146$

$n_2 = 1.4454$

Numerical aperture (NA) $= n_1 (2\Delta)^{1/2} = 1.46 (2 \times 0.01)^{1/2} = 0.21$

Critical angle $(\phi c) = \sin^{-1} \frac{n_2}{n_1} = \sin^{-1} \frac{1.4454}{1.46} = \sin^{-1}(0.99) = 81.9^\circ$

A light ray is incident from glass to air. Calculate critical angle (ϕc)

medium 1 glass $\rightarrow$ so refractive index value of glass $= 1.5 \Rightarrow n_1 = 1.5$

medium 2 air $\rightarrow$ " " " " " air $= 1.0$

according to Snell's Law

$n_1 \sin \theta_1 = n_2 \sin \theta_2$

By TIR, $\theta_1 = \theta_c$ & $\theta_2 = 90^\circ$

$\sin \theta_c = \frac{n_2}{n_1} \sin 90^\circ$

$\theta_c = \sin^{-1} \frac{n_2}{n_1} = \sin^{-1} \frac{1.0}{1.5}$ $(\because \sin 90^\circ = 1)$

$\theta_c = 41.8^\circ$

A light ray is incident from medium 1 to medium 2. If the refractive indices of medium 1 and medium 2 are 1.5 and 1.36 respectively then determine the angle of refraction for an angle of incidence of 30°

Given $n_1 = 1.5$

$n_2 = 1.36$

incidence angle $\phi_1 = 30°$

$\phi_2 = ?$

By using Snell's law

$$n_1 \sin\phi_1 = n_2 \sin\phi_2$$

$$1.5 \sin 30° = 1.36 \sin\phi_2$$

$$\sin\phi_2 = \frac{1.5 \sin 30°}{1.36}$$

$$\sin\phi_2 = 0.55$$

$$\phi_2 = \sin^{-1} 0.55$$

$$\phi_2 = 33.46°$$

$\therefore$ Refracted angle ϕ_2 w.r.to normal is $33.46°$

A light wave is travelling in a semiconductor medium (GaAs) of refractive index 3.6. It is incident on a different semiconductor medium (AlGaAs) of refractive index 3.4 and an angle of incidence is 80°. will this result in TIR

Given incidence angle $\phi_1 = 80°$, (GaAs) $n_1 = 3.6$ (AlGaAs) $n_2 = 3.4$

By Snell's law

$$n_1 \sin\phi_1 = n_2 \sin\phi_2$$

$$\phi_c = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1}\frac{3.4}{3.6} \Rightarrow \phi_c = 70.81°$$

$\phi_1 = 80°$ $\phi_c = 70.81°$ $\phi_1 > \phi_c$

$\therefore$ Yes TIR (Total internal reflection takes place bcz angle of incidence is greater than critical angle.

A multimode silica fiber has a core refractive index $n_1 = 1.48$ and cladding index $n_2 = 1.48$. Compute the numerical aperture. 13 14

Given

$n_1 = 1.48$ & $n_2 = 1.48$

$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{(1.48)^2 - (1.48)^2} = 0$

A typical ~~refractive~~ relative refractive index difference for an optical fiber designed for long distance transmission is 1%. Estimate the NA and the solid acceptance angle in air for the fiber when the core index is 1.46. Further calculate the critical angle at the core-cladding interface within the fiber.

[Dec/Jan-2011, 10M]

Given data $\Delta = 0.01$, $n_1 = 1.46$

$NA = n_1(2\Delta)^{1/2} = 1.46(2 \times 0.01)^{1/2} = 0.21$

$\boxed{\therefore NA = 0.21}$

for small angles, the solid acceptance angle in air ζ is given by

$$\zeta = \pi\theta_a^2 = \pi \sin^2\theta_a$$

$$\zeta = \pi(NA)^2 = \pi(0.21)^2 = 0.13 \text{ rad}$$

$$\therefore \boxed{\zeta = 0.13 \text{ rad}}$$

As wkt, for the relative refractive index difference Δ gives,

$$\Delta = \frac{n_1 - n_2}{n_1} \Rightarrow \Delta = 1 - \frac{n_2}{n_1}$$

Hence, $\frac{n_2}{n_1} = 1 - \Delta = 1 - 0.01 \Rightarrow \frac{n_2}{n_1} = 0.99$

Hence, the critical angle at core-cladding interface is

$$\phi_c = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1}(0.99)$$

$$\Rightarrow \boxed{\phi_c = 81.90^\circ}$$

Optical Fiber Modes and Configurations.

Fiber types:-

→ An optical fiber is a cylindrical dielectric waveguide capable of conveying electromagnetic waves at optical frequencies. The electromagnetic energy is in the form of light and propagates along the axis of the fiber. The structural of the fiber determines the transmission characteristics.

→ The propagation of light along the wave guide is decided by the modes of the wave guides, here mode means path: Each mode has distinct pattern of electric and magnetic field distributions along the fiber length.

→ When there is only one path for light to follow then it is called as single mode propagation. When there is more than one path then it is called multimode propagation.

Mode:- The mode of a fiber refers to the number of paths for the light ray within the cable.

Classification of Fibers:-

1. According to the modes optical fibres are classified into two types
(a) Single mode fibre:- Fiber allows propagation of light ray by only one path.
b) Multimode fibre:- Numerous light rays are carried simultaneously through the wave guide. The diameter of multimode fiber has a much larger diameter, compared to single mode fiber.

2. According to the refractive index profile
a) Step index fibre → Single mode step index fibre; → Multimode step index fibre
b) Graded index fibre → Single mode Graded index fibre; → Multimode step index fibre.

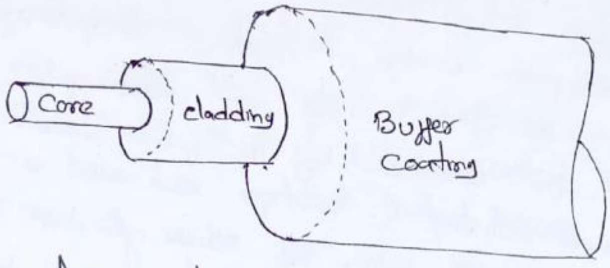


fig:- Schematic of single fiber structure.

The most widely accepted structure is the single solid dielectric cylinder of radius a and index of refraction n_1 for core of the fiber. The core is surrounded by a solid dielectric cladding which has a refractive index n_2. i.e $n_2 < n_1$.

A cladding is not necessary for light to propagate along the core of the fiber.

Uses of Cladding:- ① The cladding reduces scattering loss that results from dielectric discontinuities at the core surface.

2. It adds mechanical strength to the fiber.
3. It protects the core from absorbing surface contaminants with which it could come in contact.

Variations in the material composition of the core give rise to the two commonly used fiber types. i.e

Step index fiber: The refractive index of the core is uniform throughout and undergoes an abrupt change (or step) at the core-cladding interface. This is called step-index fiber.

Graded index fiber: The core refractive index is made to vary as a function of the radial distance from the center of the fiber. (or) The refractive index of the core is made to vary gradually such that it is maximum at the center of the core.

The larger core radii of multimode fibers make it easier to launch optical power into the fiber and facilitate the connecting together of similar fibers.

Another advantage is that light can be launched into a multimode fiber using a light emitting diode (LED) source, where as single mode fibers must generally be excited with Laser diodes. Although LEDs have less optical output power than laser diodes, they are easier to make, are less expensive, require less complex circuitory, and have longer lifetimes than laser diodes, thus making them more desirable in certain applications.

(1) Disadvantage of multimode fiber :

The fibers are suffer from intermodal dispersion. When an optical pulse is launched into a fiber, the optical power in the pulse is distributed over all of the modes of the fiber. Each of the modes that can propagate in a multimode fibers travels at a slightly different velocity. This means that the modes in a given optical pulse arrive at the fiber end at slightly different times, Thus causing the pulse to spreadout in time as it travels along the fiber. This effect is known as intermodal dispersion (or) modal delay, can be reduced by using a graded - index profile in a fiber core. This allows graded - index fibers to have much larger bandwidths (data rate transmission capabilities) than step-index fibers.

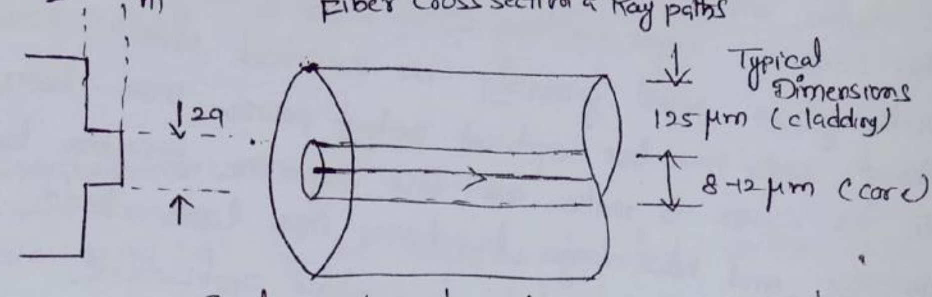
Step index fiber structure :-

To propagate a light in an optical wave guide by considering the step index fiber. In practical step-index fibers the core of radius 'a' has a refractive index n_1 which is typically equal to 1.48. This is surrounded by a ~~clad~~ cladding of lightly lower index n_2,

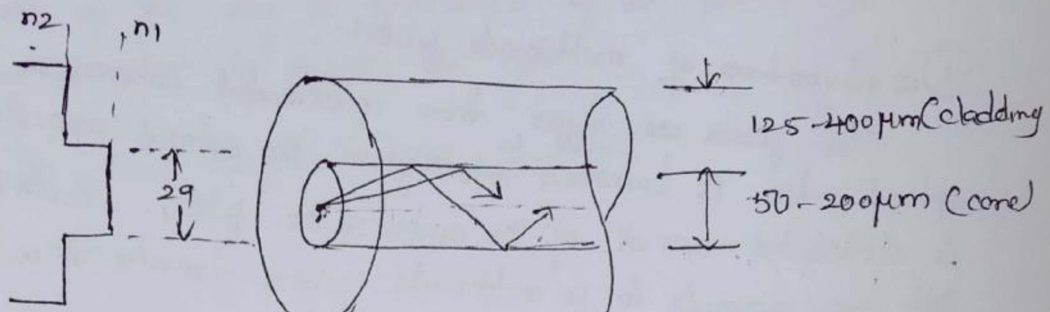
where $n_2 = n_1(1-\Delta)$ —(1)

index profile

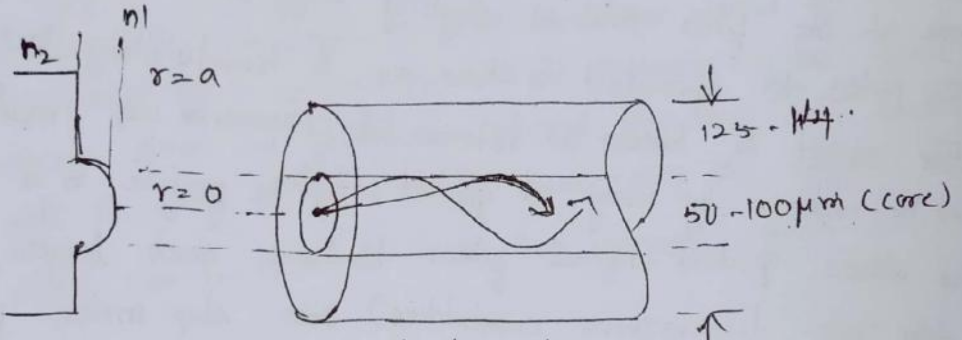
Fiber Cross section & Ray paths



Single mode step index fiber



Multimode step-index fiber



Multimode graded-index fiber

Both the step- and the graded index fibers can be further divided into single-mode and multimode classes. As the name implies, a single-mode fiber sustains only one mode of propagation, where as multimode fibers contains many hundreds of modes. A few typical size of single- and multimode fibers are to provide an idea of the dimensional scale. Multimode fibers offer several advantages compared with singl mode fibers.

Advantages of Multimode fibers

Multimode fibers offer several advantages compared with single mode fibres

The parameter 'Δ' is called the core-cladding index difference (or) the index difference values of n_2 are chosen such that Δ is nominally 0.01. Typical values range from 0.2 to 1.0 percent for single mode fibers. Since the core refractive index (n_1) is > than the cladding index n_2.

$\therefore n_1 > n_2$

Ray optics representation:-

We have seen that if the rays are launched within core of acceptance can be successfully propagated along the fiber. But the exact path of the ray is determined by the position and angle of ray at which it strikes the core.

There exists three different types of rays.

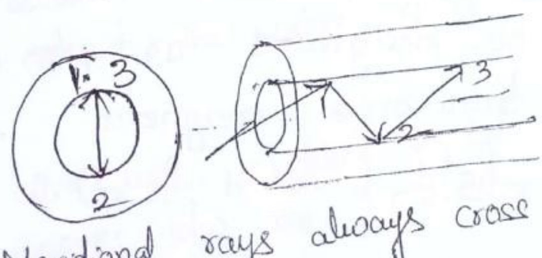
1) Meridional rays
2) Skew rays.
3) Axial rays.

(1) Meridional rays are confined to the meridian planes of the fiber, which are the planes that contain the axis of symmetry of the fiber (the core axis). Since a given meridional ray lies in a single plane, its path is easy to track as it travels along the fiber. Meridional rays can be divided in to two general classes.

(a) Bound rays
(b) Unbound rays

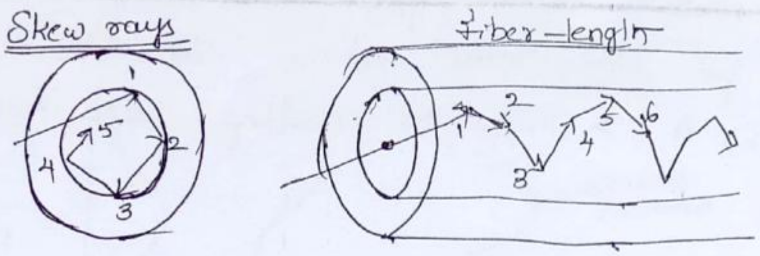
(a) Bound rays:- These rays are trapped in the core and propagate along the fiber axis according to the laws of geometric optics.

(b) Unbound rays:- These are refracted out of the fiber core.



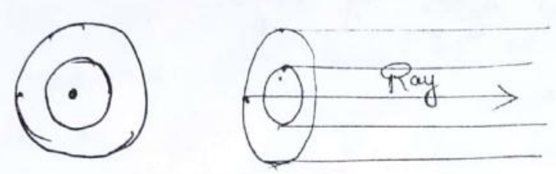
Meridional rays always cross the fibre axis

(2) Skew rays



The skew ray does not pass through the center. The skew rays reflects off from the core cladding boundaries and again bounces around the outside of the core. It takes some what similar shape of spiral or helical path.

(3) Axial Rays:-



The axial ray travels along the axis of the fiber and stays at the axis all the time.

Mode Theory for circular waveguides:-

→ The mode theory along with the ray theory is used to describe the propagation of light along an optical fibre.

The mode theory uses electromagnetic wave behavior to describe the propagation of light along a fiber.

When solving maxwell's equations for metallic waveguides, only transverse electric (TE) modes and transverse magnetic (TM) modes are found. In optical fibers the core-cladding boundary conditions lead to a coupling b/w the electric and magnetic field components. This gives rise to hybrid modes, which makes optical wave guide analysis more complex than metallic waveguide analysis. The hybrid modes are designated as HE or EH modes depending on whether the transverse magnetic field (The E-field or the transverse magnetic field (The H-field) is larger for that mode.

Overview of modes:

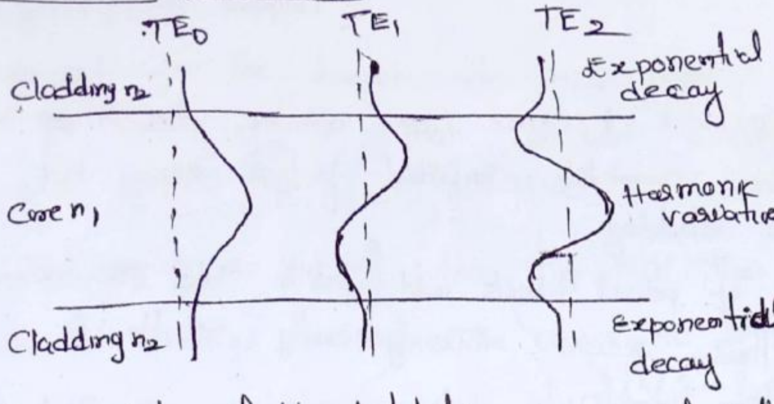


fig:. Electric field distributions for several of the lower order guided modes in a symmetrical slab wave guide.

-> The order of a mode is equal to the number of field zeros across the guide

-> The radiation field basically results from the optical power that is outside the fiber acceptance angle being refracted out of the core.

-> The plots show that the electric fields of the guided modes are not completely confined to the central dielectric slab. But instead they extent partially in to the cladding.

-> The fields vary harmonically in the guiding region of refractive index n1 and decay exponentially outside of this region.

-> For low-order modes the fields are tightly concentrated near the center of the slab (or the axis of an optical fiber), with little penetration in to the cladding region.

-> For higher-order modes the fields are distributed more towards the edges of the guide and penetrate further in to the cladding region.

-> Solving maxwell's equations a finite no. of guided modes are required.

-> The radiation field basically results from the optical power that is outside the fiber acceptance angle being refracted out of the core.

-> The core and cladding modes propagate along the fiber, mode coupling occurs b/w the cladding modes and the higher-order modes.

→ This coupling occurs because the electric fields of the guided core modes are not completely confined to the core but extend partially in to the cladding.

→ A diffusion of power back and forth b/w the core and cladding modes thus occurs; this generally results in a loss of power from the core modes.

→ In addition to bound and refracted modes, a third category of modes called leaky modes, is present in optical fibers.

→ This leaky modes are partially confined to the core region and attenuate by continuously radiating their power out of the core as they propagate along the fiber.

→ This power radiation out of the wave guide results from a quantum mechanical phenomenon known as the tunnel effect

→ A mode remains guided as long as β satisfies the condition

$$n_2 k < \beta < n_1 k .$$

→ where n_1 & n_2 are the refractive indices of the core and cladding respectively and $k = \frac{2\pi}{\lambda}$. The boundary b/w truely guided modes and leaky modes is defined by the cutoff condition $\beta = n_2 k$.

→ As soon as β becomes smaller than $n_2 k$, power leaks out of the core in to the cladding region.

→ leaky modes can carry significant amounts of optical power in short fibers.

<u>Summary Key Modal Concepts:</u>

An important parameter connected with the cutoff condition is the V number defined by

$$V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2} = \frac{2\pi a}{\lambda} NA \rightarrow (1)$$

→ This is dimensionless number that determines how many modes a fiber can support

→ except for the lowest order HE_{11} mode, each mode can exist only for values of 'V' that exceed a certain limiting value (each mode having a different V limit)

→ The modes are cut off when $\beta = n_2 k$. This occur when $V \leq 2.405$

→ The V number can also be used to express the no of modes 'M' in a multimode fiber when V is large. The total no. of modes supported in a fiber is

$$M = \frac{1}{2}\left(\frac{2\pi a}{\lambda}\right)^2 (n_1^2 - n_2^2) = \frac{V^2}{2} \longrightarrow ②$$

Since the field of a guided mode extends partly in to the cladding as shown in above fig.

→ As the 'V' number approaches cutoff for any particular mode, more of the power of that mode is in the cladding.

→ At cutoff point mode becomes radiative will all the optical power of the mode residing in the cladding.

→ For large values of V the fraction of the average optical power residing in the cladding can be estimated by

$$\frac{P_{clad}}{P} \simeq \frac{4}{3\sqrt{M}} \rightarrow ③$$

→ Where 'P' is the total optical power in the fiber.

→ M is proportional to V^2, the power flow in the cladding decreases as 'V' increases. This increase the no. of modes in the fiber. which is not desirable for a high band width capability.

Single Mode Fibers :-

Single mode fibers are constructed by the dimensions of the core diameter with few wave lengths (8–12) & having small index difference between the core & cladding. Single mode propagation is possible for large variation of core size 'a' and core-cladding index difference 'Δ'.

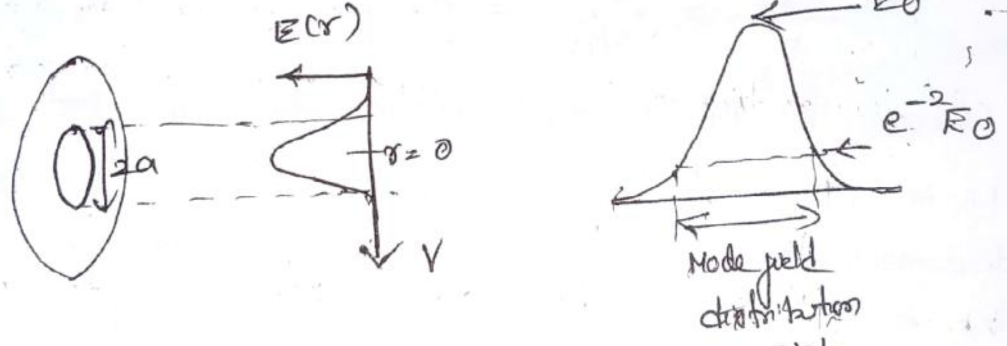
(a) Mode field diameter

For multimode fibers "core diameter" & "Numerical Aperture" are the key parameters, for the signal transmission.

The fundamental parameter for single mode fiber is the "mode-field diameter" [MFD]. which gives the performance characteristics of fiber.

This parameter can be determined from the mode field distribution of fundamental LP_{01} mode. (linearly polarized 01 mode)

→ The MFD is analogous to the core diameter in multimode fibers except that in single-mode fibers not all the light that propagates through the fiber is carried in the core.



→ Fig :- Distribution of light in a single mode fiber above its cutoff wavelength. For a gaussian distribution, the MFD is given by the $1/e^2$ width of the optical power.

→ The distribution of field in single mode fiber is gaussian

$$E(r) = E_0 \exp(-r^2/w_0^2) \rightarrow (1)$$

where 'r' is the radius, E_0 is the field at zero radius and w_0 is the width of the electric field distribution

NOTE :-

1. In multimode fibers all the light will propagate through the fiber is carried in the core.

2. In single mode fibers not all the light will propagate through the fiber is carried in the core.

Ex: If $V=2$, only 75% of optical power is confined to core.

∴ The percentage increases for large values of 'V' and is less for smaller 'V' values.

MFD is important because it determines fiber property such as splice loss, bending loss, cutoff wavelength, waveguide dispersion. Different models are proposed for characterizing & measuring MFD. These include far-field scanning, near field scanning, knife edge & mask methods.

These methods are useful to determine the optical power distribution

MFD is used to measure far-field intensity distribution $E^2(r)$ & then calculate MFD using the Petermann ~~II~~ equation.

$$MFD = 2W_0 = 2\left[\frac{2\int_0^\infty E^2(r)\, r^3\, dr}{\int_0^\infty r E^2(r)\, dr}\right]^{1/2}$$

$2W_0$ = spot size is the full width of dispersion exact field distribution & calculated by Gaussian function.

$$E(r) = E_0 \exp\left[-r^2/w_0^2\right]$$

where r = radius

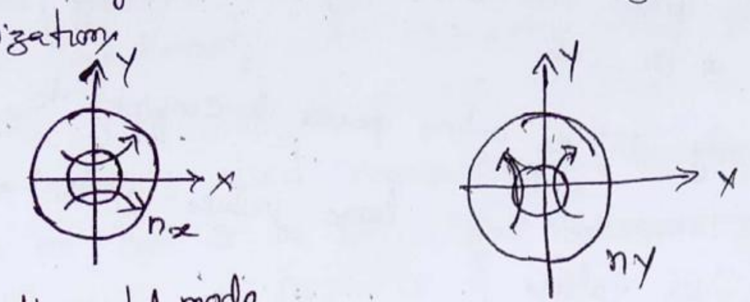
E_0 = field at zero radius

MFD = $1/e^2$ (width of optical power).

Propagation Modes in single mode fibers:

→ In an ordinary single mode fiber there are actually two independent degenerate propagation modes. These mode are similar but their polarization planes are orthogonal.

There may be chosen as the horizontal (H) and the vertical polarization.



Horizontal mode.

Vertical mode

→ Either one of these two polarization modes constitutes the fundamental HE_{11} mode.

→ In ideal fibers with perfect rotational symmetry, the two modes are degenerate with equal propagation constants ($k_x = k_y$)

→ In actual fibers, imperfections break the circular symmetry of the ideal fiber and lift the degeneracy of the two modes.

<u>NOTE</u>:- The modes propagate with different phase velocities & difference between their refractive indices is called the "<u>Fiber birefringence</u>"

$$B_f = n_y - n_x$$

Birefringence is also defined as

$$\beta = k_0 (n_y - n_x)$$

where $k_0 = \frac{2\pi}{\lambda}$ is the free space propagation constant.

If light is injected into the fiber. So that both modes are excited, then one will be delayed in phase relative to the other as they propagate. When this phase difference is an integral multiple of 2π, the two modes will beat at this point & the input polarization state will be reproduced. The ~~length~~ length over which this beating occurs is the "fiber beat length"

$$\boxed{L_p = \frac{2\pi}{\beta}}$$

Graded index fiber structure:-

In the graded-index fiber design the core refractive index decreases continuously with increasing radial distance r from the center of the fiber, but is generally constant in the cladding. The most commonly used construction for the refractive index variation in the core is the power law relationship.

$$n(r) = \begin{cases} n_1 \left[1 - 2\Delta \left[\frac{r}{a}\right]^{\alpha}\right]^{1/2} & \text{for } 0 \leq r \leq a \\ n_1 (1-2\Delta)^{1/2} \simeq n_1(1-\Delta) = n_2 & \text{for } r \geq a \end{cases}$$

Here r is the radial distance from the fibre axis, a is the core radius, n_1 is the refractive index at the core axis, n_2 is the refractive index of the cladding and the dimensionless parameter α defines the shape of the index profile. The index difference Δ for the graded-index fiber is given by

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \simeq \frac{n_1 - n_2}{n_1}$$

The approximation on the right hand side of the equation reduces the expression for Δ to that of the step-index fiber. Determining the NA for graded-index fibers is more complex than for step index fibers. Since it is a function of position across the core end face. This is in contrast to the step-index fiber where NA is constant across the core.

Base on geometrical optics consideration, light incident on the fiber core at position 'r' will propagate as a guided mode only if it is within. The local numerical aperture NA(r) at that point. The NA is defined as

$$NA(r) = \begin{cases} (n^2(r) - n_2^2)^{1/2} \simeq NA(0)\sqrt{1-(r/a)^{\alpha}} & \text{for } r \leq a \\ 0 & \text{for } r > a \end{cases}$$

where the axial numerical aperture is defined as

$$NA(0) = \left[n^2(0) - n_2^2\right]^{1/2} = \left(n_1^2 - n_2^2\right)^{1/2} \simeq n_1\sqrt{2\Delta}$$

→ Thus the NA of a graded-index fiber decreases from NA(0) to zero as r moves from the fibre axis to the core-cladding boundary

→ The no. of bound modes in a graded index fiber is

$$M = \frac{\alpha}{\alpha+2}\, a^2 k^2 n_1^2 \Delta$$

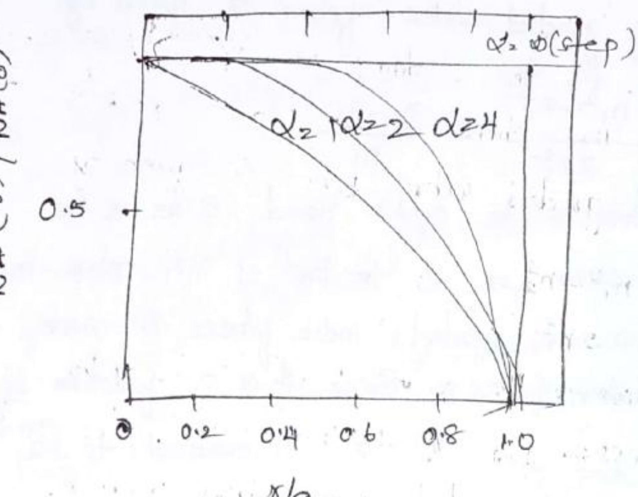


fig: A comparison of the N.A for fibers having various α profiles

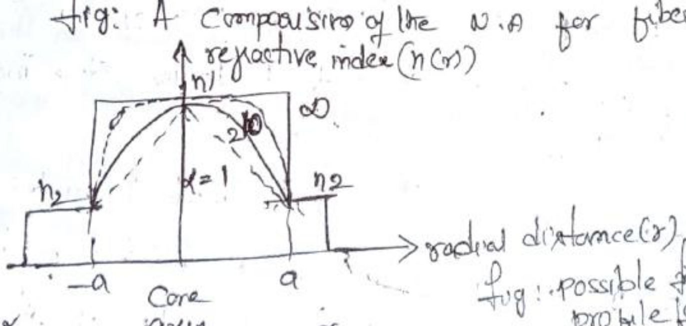


fig: possible fiber refractive index profile for different values of α

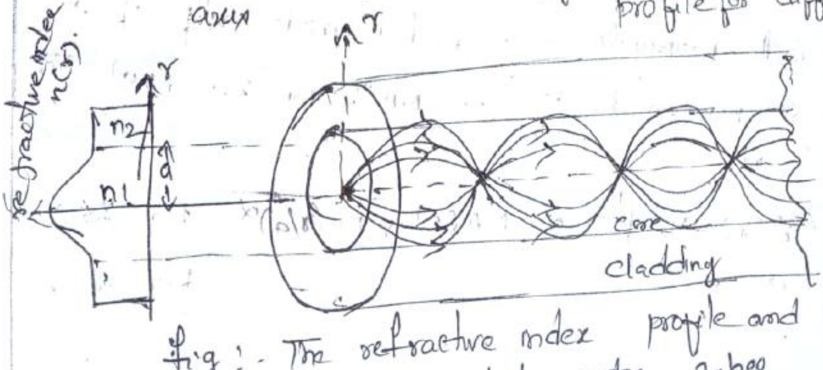


fig: The refractive index profile and ray transmission in a multimode graded index fiber.

Difference between step index fiber and graded index fiber

	STEP index Fiber	Graded Index fiber
1	The refractive index of the core is uniform throughout and undergoes an abrupt change at the core-cladding boundary	The refractive index of the core is made to vary gradually such that it is maximum at the center of the core.
2	The Diameter of the core is about 50-200μm in case of multimode fibers and 10μm in the case of single mode fiber.	→ The diameter of the core is about 50μm in the case of multi mode fiber.
3	The path of light propagation is Zig-Zag in manner	→ The path of light is helical in manner.
4	Attenuation is more for multimode step index fiber but for single mode fibre it is very less. Explanation: when a ray travels through the longer distances there will be some difference in reflected angles. Hence high angle rays arrive later than low angle rays causing dispersion resulting in distorted output	→ Attenuation is less. → explanation!. Here the light rays travel with different velocity in different paths because of their variation in their refractive indices. At the outer edge it travels faster than near the center. But almost all the rays reach the exit at the same time due to helical path. Thus there is no dispersion
5	The fiber has lower bandwidth	→ This fiber has higher bandwidth
6	The light ray propagation is in the form of meridional rays and it passes through the fiber axis	The light propagation is in the form of Skew rays and it will not cross fiber axis.
7	No of modes of propagation:- $N_{step} = 4.9\left[\frac{d \times NA}{\lambda}\right]^2 = \frac{V^2}{2}$	→ No of Modes of propagation: $N_{Graded} = \frac{4.9\left(\frac{d \times NA}{\lambda}\right)^2}{2} = \frac{V^2}{4}$

d →

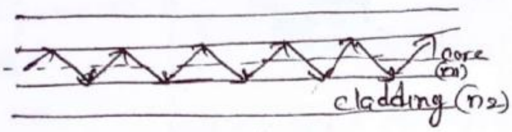
where d = diameter of the fiber core

λ = wave length

N.A. = Numerical Aperture

V — V-no. is less than or equal to 2.405 for single mode fibers and greater than 2.405 for multimode fibers

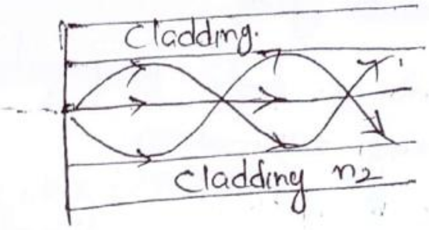
→ Ray path by total internal reflection



or

$$N_{graded} = \frac{N_{step}}{2}$$

Light ray travels in oscillatory fashion



A step index fiber has a normalized frequency $V = 26.6$ at 1300 nm wavelength. If the core radius is $25\mu m$, what is the numerical aperture? Given $V = 26.6$, $\lambda = 1300 nm$, $a = 25\mu m$, $NA = ?$ 22

$$V = \frac{2\pi a}{\lambda} \cdot NA \Rightarrow NA = V\frac{\lambda}{2\pi a} = 26.6 \times \frac{1.30 \times 10^{-6}}{2\pi \times 25 \times 10^{-6}} = 0.22$$

Consider a multimode step-index fiber with a $62.5\mu m$ core diameter and a core-cladding index difference of 1.5 %. If the core refractive index is 1.480, estimate the normalized frequency of the fiber and the total number of modes supported in the fiber at a wavelength of 850 nm.

$d = 62.5\mu m$

$\Delta = 1.5\% = 0.015$

$n_1 = 1.48$

$\lambda = 850 nm$

$a = \frac{d}{2} = \frac{62.5\mu m}{2}$

$a = 31.25\mu m$

$$V = \frac{2\pi a}{\lambda} \cdot NA$$

$$V = \frac{2\pi a}{\lambda} n_1 (2\Delta)^{1/2}$$

$$V = \frac{2\pi \times 31.25 \times 10^{-6}}{850 \times 10^{-9}} \times 1.48 \times (2 \times 0.015)^{1/2}$$

$$V = 59.2$$

$\boxed{\text{Normalized frequency } V = 59.2}$

Total no. of modes in step index fiber is

$$\boxed{M = \frac{V^2}{2} = \frac{(59.2)^2}{2} = 1752}$$

Suppose we have a multimode step-index optical fiber that has a core radius of $25\mu m$, a core index of 1.48, and an index difference $\Delta = 0.01$. What are the no. of modes in the fiber at wavelengths 860, 1310, and 1550 nm? Given $n_1 = 1.48$, $a = 25\mu m$, $\Delta = 0.01$

(a) At an operating wavelength of 860nm, the value of V is

$$V = \frac{2\pi a}{\lambda} n_1 (2\Delta)^{1/2} = \frac{2\pi \times 25\mu m \times 1.48}{0.86\mu m} (2 \times 0.01)^{1/2} = 38.2$$

$$M = \frac{V^2}{2} = \frac{(38.2)^2}{2} = 729$$

$$\boxed{M = 729}$$

Scanned with CamScanner

(b) $V = \frac{2\pi a}{\lambda} n_1 \sqrt{2\Delta}$

$$V = \frac{2\pi \times 25 \times 10^{-6}}{1310 \times 10^{-9}} \times 1.48 \times \sqrt{2 \times 0.01}$$

$$V = \frac{2\pi \times 25 \times 10^{3}}{1310} \times 1.48 \times \sqrt{2 \times 0.01}$$

$$\boxed{V = 25.1}$$

$$\boxed{M = \frac{V^2}{2} = \frac{(25.1)^2}{2} = 315}$$

(C) Finally at 1550 nm

$$V = \frac{2\pi \times 25 \times 10^{-6} \times 1.48}{1550 \times 10^{-9}} \times \sqrt{2 \times 0.015}$$

$$V = \frac{2\pi \times 25 \times 10^{3} \times 1.48}{1550} \times \sqrt{2 \times 0.015}$$

$$\boxed{V = 21.2}$$

$$\boxed{M = \frac{V^2}{2} = \frac{(21.2)^2}{2} = 224}$$

Suppose we have three multimode step-index optical fibers, each of which has a core index of 1.48 and an index difference $\Delta = 0.01$. Assume the three fibers have core diameters of 50, 62.5 and 100 μm. Assume the three fibers at a wavelengths of 1550 nm? what are the no. of modes in these fibers at a wavelengths of 1550 nm?

Given data

$n_1 = 1.48$

$\Delta = 0.01$

$d = 50,\ 62.5\ \&\ 100\,\mu m$

$\lambda = 1550\,nm.$

(a) $V = \frac{2\pi a}{\lambda} n_1 \sqrt{2\Delta} = \frac{2\pi \times 25\,\mu m \times 1.48}{1.55\,\mu m} \sqrt{2 \times 0.01}$

$$V = \frac{2\pi \times 25 \times 10^{-6} \times 1.48}{1.55 \times 10^{-9}} \times \sqrt{2 \times 0.01}$$

$V = 2\pi \times$ $\boxed{V = 21.2}$

$$M = \frac{V^2}{2} = 224$$

$$\boxed{M = 224}$$

UNIT – II

SIGNAL DEGRADATION IN OPTICAL FIBERS

Introduction:- Signal attenuation (also known as fiber loss or signal loss) is one of the most important properties of an optical fiber, because it largely determines the maximum unamplified or repeaterless seperation between a transmitter and a receiver. Since amplifiers and repeaterless are expensive to fabricate, install and maintain, the degree of attenuation in a fiber has large influence on system cost.

The distortion mechanisms in a fibre cause optical signal pulses to broaden as they travel along a fiber. If these pulses travel sufficiently far, they will eventually overlap with neighboring pulses, there by creating errors in the receiver output. This signal distortion mechanisms thus limit the information-carrying capacity of a fiber.

~~Attenuation!.~~

Signal Degradation!.

Signal degradation → Reduction in amplitude is called attenuation
→ Change in the shape of the signal called dispersion

→ When optical pulses travel along the fiber medium, the "light intensity or power decreases" (attenuation) over a distance and the width of the pulse broadens (dispersion).

Attenuation!-

→ Attenuation (Power loss) is a measure of decay of signal strength or loss of light power that occurs as light pulses propagate through the length of the fiber.

→ Attenuation of a light signal as it propagates along a fiber is an important consideration in the design of an optical communication system, since it plays a major role in determining the maximum transmission distance between a transmitter and a receiver or an in-line amplifier.

Attenuation Units:

As light travels along a fiber, its power decreases exponentially with distance. If $P(0)$ is the optical power in a fiber at the origin (at $z=0$), then the power $P(z)$ at a distance z further down the fiber is

$$P(z) = P(0)\, e^{-\alpha_p z}$$

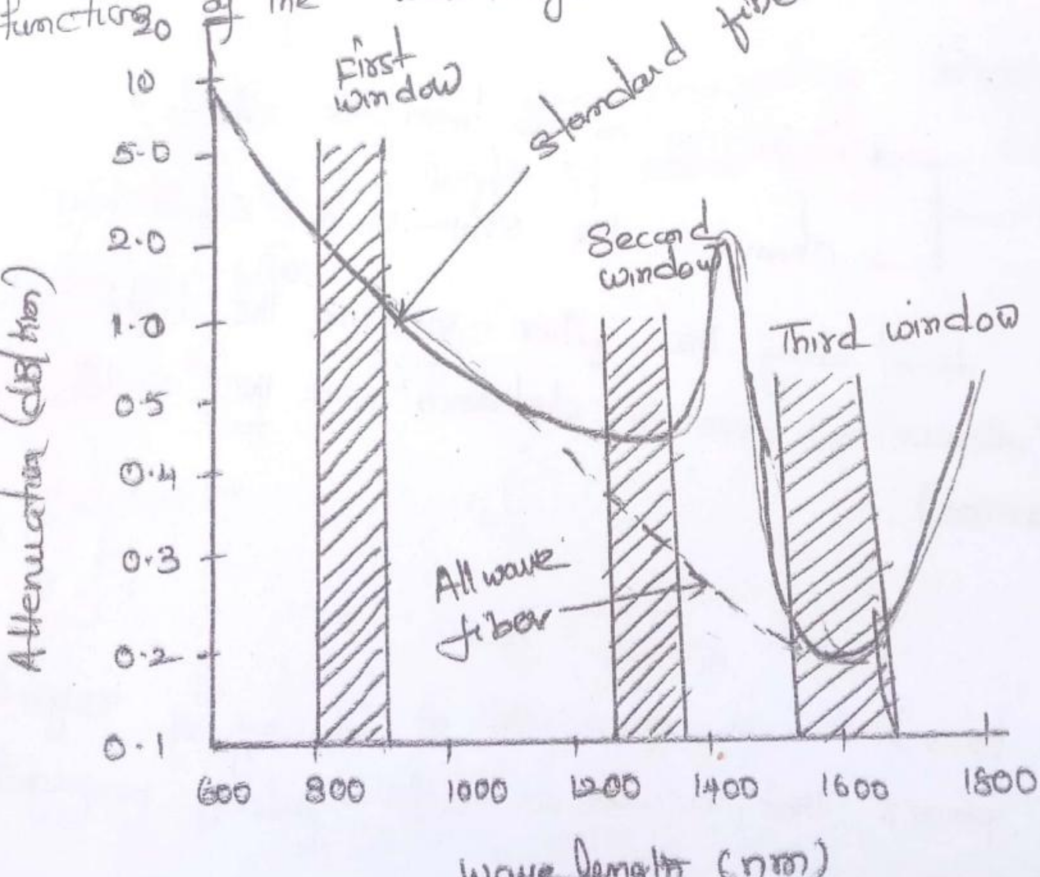
where α_p is the attenuation co-efficient it is given by

$$\alpha_p = \frac{1}{z} \ln\left[\frac{P(0)}{P(z)}\right]$$

Attenuation co-efficient in units of decibels per kilometer, denoted by dB/Km then

$$\alpha\,(dB/Km) = 10\,\frac{1}{z} \log\left[\frac{P(0)}{P(z)}\right] = 4.343\,\alpha_p (Km^{-1})$$

This parameter is generally referred to as the fiber loss or the fiber attenuation. It depends on several variables, & it is a function of the wavelength.



Problems

Q) Calculate the loss of an optical fiber in dB/Km where length is 100 m fed with an optical power of 10 μW and output power Pr 7.5 μW

Given

$z = 100\ m \xrightarrow{(or)} 100 \times 10^{-3}\ Km$

i/p power $P(0) = 10\ \mu W$

o/p power $P(z) = 7.5\ \mu W$

loss = ?

$$\alpha = 10 \frac{1}{z} \log \left[\frac{P(0)}{P(z)}\right] dB/Km$$

$$\alpha = 10 \cdot \frac{1}{100 \times 10^{-3}} \log \left[\frac{10 \times 10^{-6}}{7.5 \times 10^{-6}}\right] dB/Km$$

$$\alpha = 100 \log [1.3333]$$

$$\boxed{\alpha = 12.49 \simeq 12.5}$$

Q) An optical signal after propagating through a fiber has lossed 80% of its length of 600m of fiber. Calculate the loss in dB/Km of fibre.

Given : $\frac{P_{in}}{P_{out}} = 80\%$

$z = 600\ m$

$\alpha = ?$

$$\alpha = 10 \frac{1}{z} \log \left[\frac{P(0)}{P(z)}\right] dB/Km$$

$$\alpha = 10 \frac{1}{600 \times 10^{-3}} \log \left[\frac{80}{100}\right]$$

$$\boxed{\alpha = -1.615\ dB/Km}$$

Q) For a 30km long fiber attenuation 0.8 dB/Km at 1300nm If a 200 μW power is launched in to the fiber. find the output power.

$z = 30 \text{Km}$

$\alpha = 0.8 \text{ dB/Km}$

$P(0) = 200 \mu W$

attenuation in optical fiber is given by

$$\alpha = 10 \times \frac{1}{z} \log \left[\frac{P(0)}{P(z)}\right]$$

$$0.8 = 10 \times \frac{1}{30} \log \left[\frac{200 \mu W}{P(z)}\right]$$

$$2.4 = \log \left[\frac{200 \mu W}{P(z)}\right]$$

$$\boxed{P(z) = 0.7962 \mu W.}$$

Basic Attenuation Mechanism

1. Absorption (due to fiber material)
2. Scattering (due to fiber structural imperfections)
3. Radiative losses (due to fiber bending)

Absorption: The light is absorbed in the fiber by the materials of fiber optic. Thus light absorption is also known as material absorption.

→ Material absorption is caused by absorption of photons within the fiber.

→ Absorption is caused by three different mechanism:.

1. Absorption by atomic defects in the glass composition
2. Extrinsic absorption by impurity atoms in the glass material.
3. Intrinsic absorption by the basic constituent atoms of the fiber material.

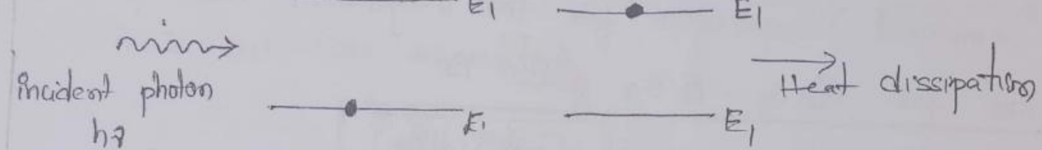
1. Absorption by atomic defects in the glass composition:. (3)

Atomic defects are imperfections in the atomic structure of the fiber material. Examples are missing molecules, high density clusters of atoms groups, or oxygen defects in the glass structure. Usually absorption losses arising from these defects are negligible compared with intrinsic and extrinsic absorption effects.

2. Intrinsic Absorption:.

Two types

a). Intrinsic absorption in UV region:- It is caused by electronic absorption bands. It occurs when a light particle (photon) interacts with a valence electron and excites it to a higher energy level.



→ it occurs when energy band gap of the material "E_g" is less than or equal to photon energy ($h\nu$) of light travelling along the fiber.

→ Intrinsic absorption is also caused due to absorption of photons in fiber medium which transforms to heat energy and dissipated out side the fiber medium.

→ The expression for loss in UV region is

$$\boxed{\alpha_{UV} = \frac{154.2x}{46.6x+60} \times 10^{-2}\, e\left[\frac{4.63}{\lambda}\right]}$$

where 'x' is the mole fraction of GeO_2 with pure silica

λ is the wavelength

(21)

b) Intrinsic absorption in IR region :- is caused by vibration frequency of atomic bonds. In silica glass, it is caused by Si-O bonds (Covalent bond)

→ The inherent infrared absorption absorption is associated with the characteristics vibration frequency of the particular chemical bond between the atoms of which the fiber is composed.

→ An interaction between the vibrating bond and the electromagnetic field of the optical signal results in a transfer of energy from the field to the bond, there by giving rise to absorption.

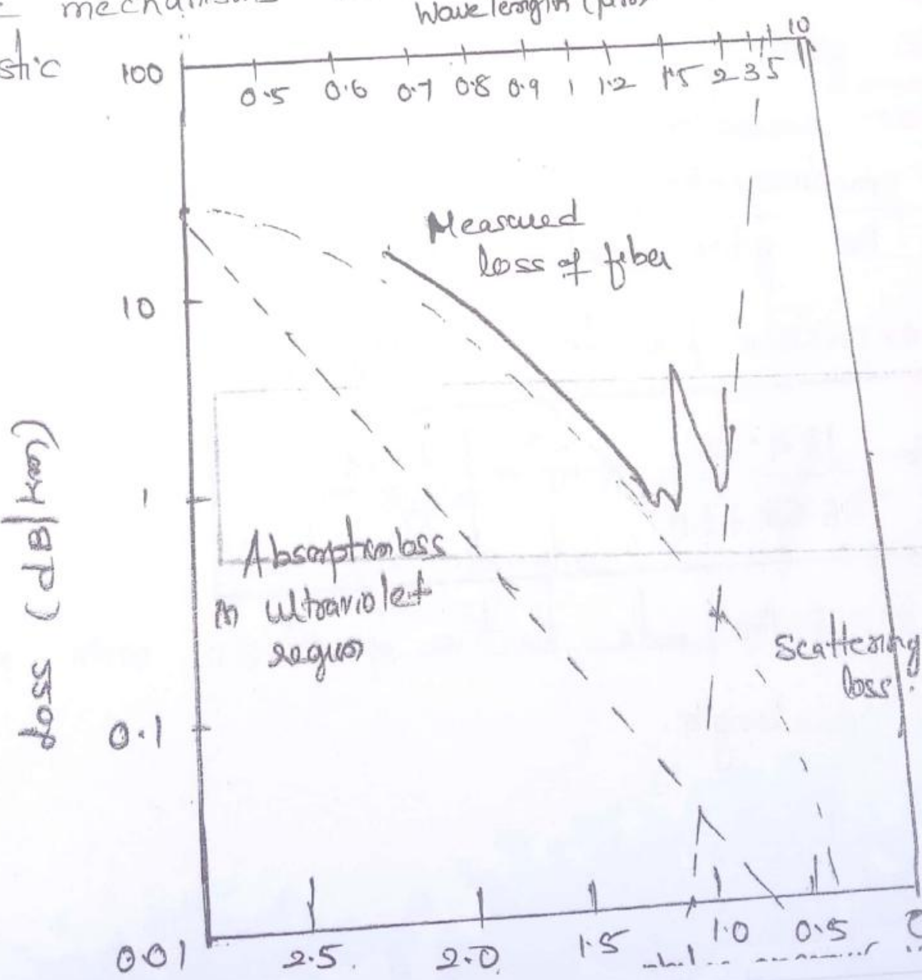
→ Covalent bond being weaker absorbs photon energy, vibrates and dissipates as heat energy.

→ This absorption is quite strong because of the many bonds present in the fiber.

→ An empirical expression for the infrared absorption loss in dB/km for $GeO_2 - SiO_2$ glass is

$$\boxed{\alpha_{IR} = 7.81 \times 10^{11} \times \exp\left(\frac{-48.48}{\lambda}\right)}$$

These mechanisms result in a wedge-shaped spectral-loss characteristic



(c) Extrinsic Absorption:

→ It results from the presence of impurities such as Fe, Co
Fe (Ferrum or iron), Co (carbon monoxide), Cu (copper)
Ni (Nickel), Mn (Manganese), Cr (chromium) in the silica
structure of the fiber cable.

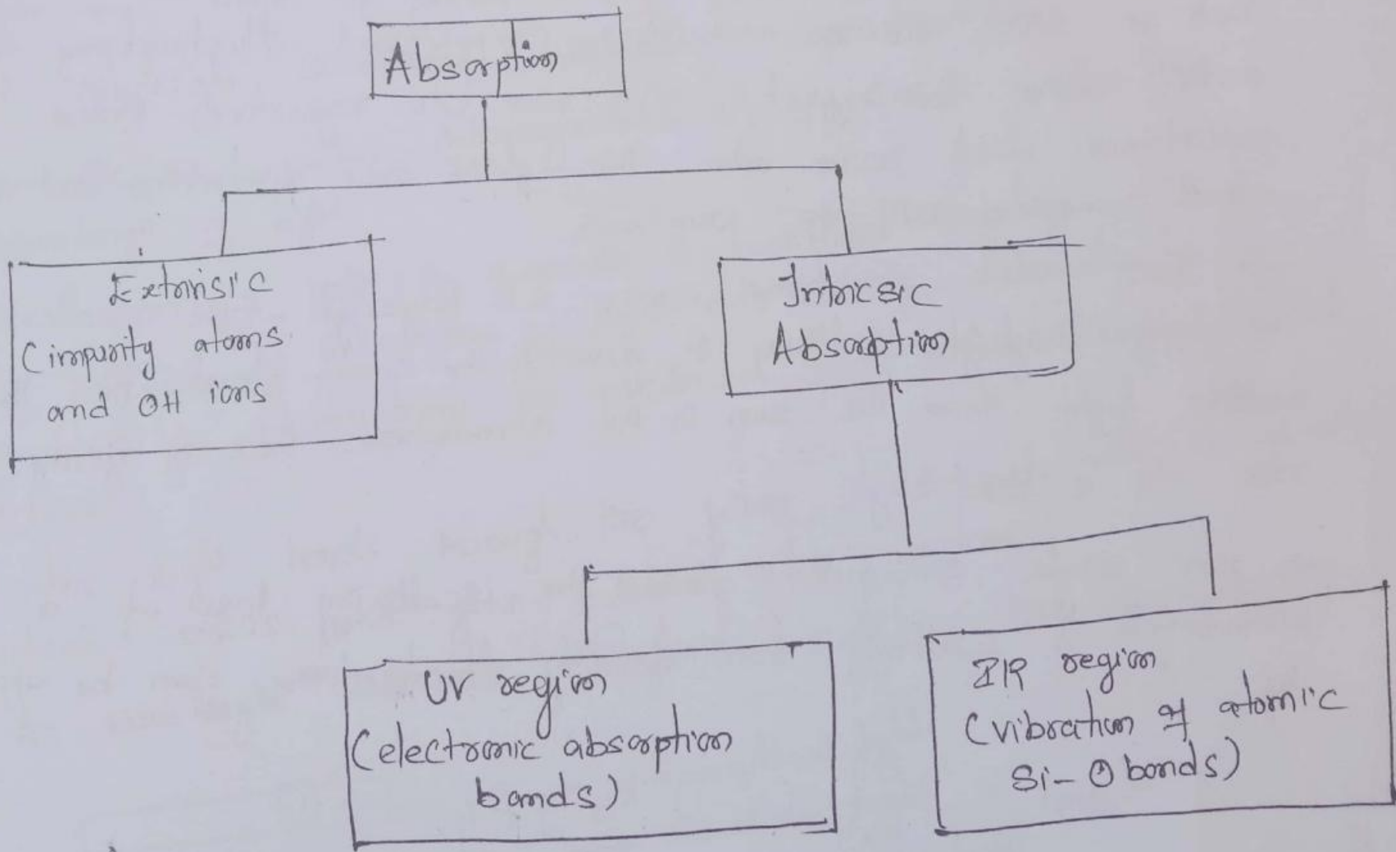
→ The raw material SiO_2 powder is placed in metallic crucibles and melted during manufacturing process.

→ The impurities (metallic ions) are added to the silica glass fiber during melting process.

→ It is also caused by the presence of Hydrogen ions (OH) in the silica glass fiber during manufacturing process.

→ The silica rod which is normally a blacky substance is brought back to glassy structure by passing water vapour through it.

→ Thus OH ions are added.



Scanned with CamScanner

2. Scattering Losses :-

→ Scattering losses in glass arise from microscopic variations in the material density, from compositional fluctuations, and from structural inhomogeneties or defects occuring during fiber manufacture.

Causes for Scattering losses :-

Scattering losses in fiber exists due

- Microscopic variations in density of fiber material.
- Compositional fluctuations
- Structural inhomogenities
- Structural defects in fiber

→ Glass is composed of a randomly connected network of molecules. Such a structure naturally contains regions in which the molecular density is either higher or lower than the average density in the glass. In addition, since glass is made up of several oxides, such as SiO_2, GeO_2, and P_2O_5 compositional fluctuations can occur. These two effects give rise to refractive-index variations which occur within the glass over distances that are small compared with the wavelength.

→ These index variations cause a Rayleigh-type scattering of the light. Rayleigh scattering in glass is the same phenomenon that scatters light from the sun in the atmosphere, there by giving rise to a blue sky.

→ For single-component glass the scattering loss at a wavelength λ resulting from density fluctuations can be approximated by

$$\alpha_{scat} = \frac{8\pi^3}{3\lambda^4} (n^2-1)^2 K_B T_f \beta_T \longrightarrow (1)$$

where n = refractive index

K_B = Boltzmann's constant

β_T = isothermal compressibility of the material (m^2/N)

$\lambda \rightarrow$ operating wave length

$p \rightarrow$ photo elastic co-efficient

$T_f \rightarrow$ fictive temperature, temperature at which Si changes from solid to semisolid state (1200 – 1400K)

Types of Scattering :-

Two types of scattering are

1. linear — Rayleigh & Mie scattering
2. Non linear — Stimulated Brillouin Scattering (SBS) and Stimulated Raman Scattering (SRS)

1. **Linear Scattering** :- occurs only at low power densities

→ The incident light frequency and scattered light frequency is same.

→ No frequency shift during scattering.

→ In linear scattering, attenuation occurs when optical power is transferred from one mode to another keeping frequency unaltered.

a) Rayleigh scattering (wave length dependent) :-

→ Occurs when inhomogenities size of fiber is smaller than wavelength of light.

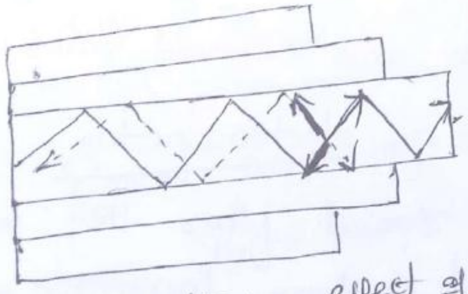
→ it occurs both in forward & backward direction.

→ Caused by interaction of light with density fluctuation.

→ Density fluctuations are produced during manufacturing of optical fibers.

→ when light travels through the fiber, it interacts with the density fluctuated areas and gets scattered in all directions.

→ As wave length increases, Rayleigh scattering loss decreases.



→ transmitted light

→ Scattered light

← Back scattered light

Back scattering effect of light transmission

The loss in Rayleigh scattering can be expressed as

$$\alpha_{scat} = \frac{8\pi^3}{3\lambda^4} n^8 p^2 K_B T_f \beta_T$$

where

K_B = Boltzmann constant (Joules/ kelvin)

T_f = fictive temperature, temperature at which S_i changes from solid to semisolid. State (1200 - 1400k)

n = Refractive index of Silica

λ = Operating wavelength (meters)

β_T = isothermal compressibility factor (m^2/N)

p = photo elastic co-efficient.

b) Mie - Scattering :- (Not strongly wavelength dependent)

→ This happens because of non perfect cylindrical structure of the wave guide & also due to fiber imperfections, irregularities in core cladding interface, core - cladding refractive index differences.

→ it occurs in forward direction.

→ it occurs when inhomogenities size of fiber is greater than one - tenth of wavelength of light.

How to minimize mie scattering :

→ Removing imperfections due to the glass manufacturing process.

→ Carefully controlled extrusion and coating of the fiber.

→ Increasing the fiber guidance by increasing the relative refractive index difference.

2. Non linear Scattering :-

(a) Stimulated Brillouin Scattering :- (SBS)

→ when an optical signals are launched (or coupled in to the optical fiber, it creates an acoustic wave (sound waves) in transmission medium through a process electrostriction.

→ when sound wave travels in optical fiber they produce compressions and rarefractions which in turn increases & decreases the refractive index of the fiber. This phenomina is called photo

→ This acoustic wave alters the optical properties like refractive index of fibre.

→ This fluctuations of refractive index scatters incident wave which propagates in opposite direction.

→ Backward process.

→ Threshold power level is less

→ Scattering process produces acoustic phonon as well as scattered photon.

→ Induce acoustic vibrations in the medium.

$$\boxed{P_B = (4.4 \times 10^{-3})\, d^2 \lambda^2 \alpha_{dB}\, B \text{ watts}}.$$

$$\boxed{\alpha_{dB} = \frac{P_B}{(4.4 \times 10^{-3})\, d^2 \lambda^2 B} \, dB}$$

$\alpha_{dB} \rightarrow$ Brillouin scattering

where
d is the diameter in μm
λ is the operating wavelength in nm
B is the source band width in GHz

(b) Stimulated Raman Scattering (SRS):

→ SRS was invented by C.V. Raman in 1928,

→ SRS may exist both in forward and backward directions in optical fibers.

→ The optical power threshold is three times higher than SBS threshold.

→ The threshold optical power for SRS is given by

$$\boxed{P_R = (5.9 \times 10^{-2})\, d^2 \lambda^2 \alpha_{dB} \text{ Watts}}$$

$$\boxed{\alpha_{dB} = \frac{P_R}{(5.9 \times 10^{-2})\, d^2 \lambda^2} \, dB}$$

↓
Raman scattering

where $P_R \rightarrow$ threshold power
$d \rightarrow$ Core diameter
$\lambda \rightarrow$ Wavelength of light

3 Bending losses (or) Radiative losses:.

→ Abrupt change in radius of curvature of the fiber causes bending loss.
→ Whenever optical fiber undergoes bends or curves on their paths, radiation losses occur which causes light energy to be radiated from the fiber.

→ Fiber can be subject to two types of bends:.

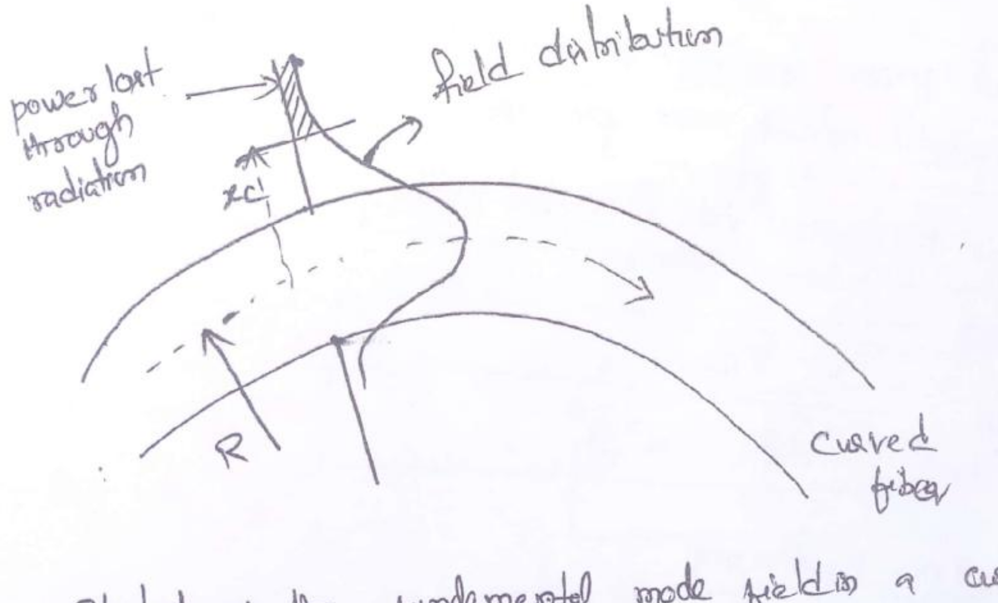
(a) Macroscopic bends →

(b) Microscopic bends

1. Macroscopic bends:. The macroscopic bends having radii that are large compared with the fiber diameter, i.e. Large curvature radiation losses (or) simply bending losses. For slight bends essentially unobservable the excess loss is small and is

→ The radius of curvature decreases, the loss increases exponentially until at a certain critical radius the curvature loss becomes observable.

→ When the fiber is bent, the field tail on the far side of the center of curvature must move faster to keep up with in the field in core, as shown in below fig) for the lowest-order fiber mode.



Sketch of the fundamental mode field in a curved optical wave guide.

Certain critical distance x_c from the center of the fiber, the field tail would have to move faster than the speed of light to keep up with the core field. Since this is not possible the optical energy in the field tail beyond x_c ~~radiates~~ are radiates away.

→ ~~The amount~~

→ Since higher-order modes are bound less tightly to the fiber core than lower-order modes, the higher order modes will radiate out of the fiber first

→ The effective no. of modes passing a bend fiber on the actual no. of mode which is received after bending is given by

$$\boxed{N_{eff} = N_{\infty}\left\{1 + \frac{\alpha+2}{2\alpha\Delta}\left[\frac{2a}{R} + \left(\frac{3}{2n_2kR}\right)^{2/3}\right]\right\}}$$

where

$$\boxed{N_{\infty} = \frac{\alpha}{\alpha+2}(n_1ka)^2\Delta}$$

where N_{∞} → Total no. of modes sent inside the fiber

α → graded index profile

a → core radius

Δ → Refractive index difference.

k → free space propagation constant $= \frac{2\pi}{\lambda}$

n_2 → cladding refractive index

R → Radius of curvature of the bend in fiber.

How to minimize Macro bending Losses:

1. By Designing fibers with large refractive index difference
2. Operating at shortest wavelength possible

The expression for Critical radius of curvature for macro-bending of fiber cable is $\boxed{R_c = \frac{3n_1^2\lambda}{4\pi(n_1^2-n_2^2)^{1/2}}}$.

where R_c is the critical radius of curvature of macro bending

n_1 is the refractive index of core,

n_2 is the refractive index of cladding.

NOTE!. Macro bend occurs when a fiber cable turns a corner and macroscopic bends having radius that are large compared with the fiber diameters.
→ it is also due to poor reeling.

<u>Microbends</u> :- Another form of radiation loss in optical wave guides results from mode coupling caused by random micro bends of the optical fiber. Microbends are repetitive small-scale fluctuations in the radius of curvature of the fiber axis,
→ In fiber optic transmission, micro bend is an imperfection in the optical fiber which was created during manufacturing.
→ Micro bending can cause extrinsic attenuation, a reduction of optical power in the glass.
→ unlike macro bending, the imperfection may not always be visible.
→ Micro bend loss refers to small scale "bends" in the fiber, often from pressure exerted on the fiber itself or lateral stresses along the length of the fiber when it is cabled.

" Micro bends are repetitive small scale fluctuations on the fiber access. These are caused by non-uniformities in manufacturing or by the lateral pressure created during cabling of fiber"

Increases in attenuation occurs from micro bending because it causes coupling of energy b/w guided modes & unguided modes.

How to minimize microbending losses ?

These can be reduced by using highly compressable Jacket. Microbending losses are minimized by extruding a compressible jacket over the fiber. When external forces are applied to this configuration, the jacket will be deformed but the fiber will tend to stay relatively straight as shown in the fig.

→ The micro bending loss α_m of a jacketed fiber is reduced from that of an unjacketed fiber by a factor

$$F(\alpha m) = \left[1 + \pi \Delta^2 \left(\frac{b}{a}\right)^4 \frac{E_f}{E_j}\right]^{-2}$$

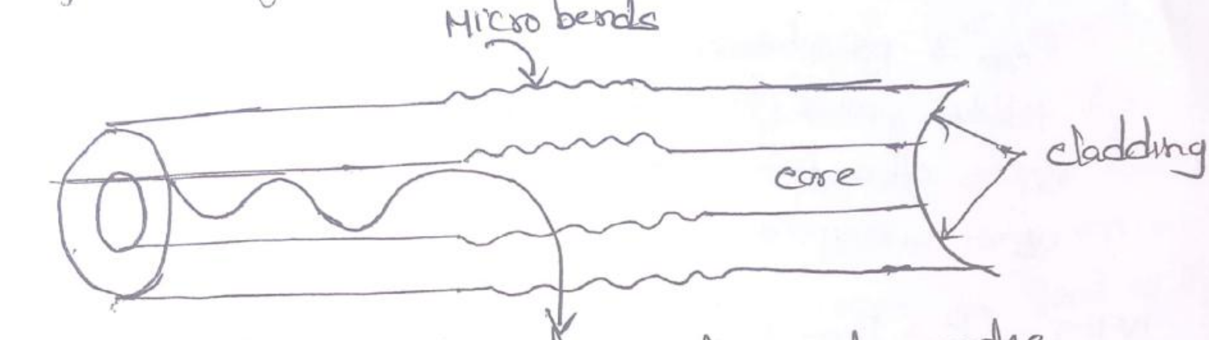
Δ → RI difference

a → core radius

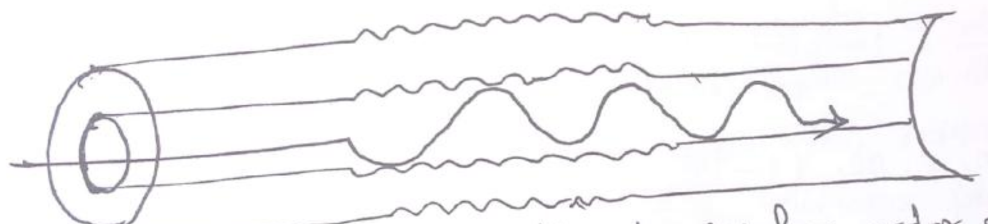
b → cladding radius

E_f → Young's modulus of Si fiber (65 G pascal)

E_j → Young's modulus of the jacket (20 - 500 mega pascals)



(a) power loss from higher - order modes.



(b) power coupling to higher order modes.

fig:- Small - scale fluctuations in the radius of curvature of the fiber axis leads to micro bending losses.

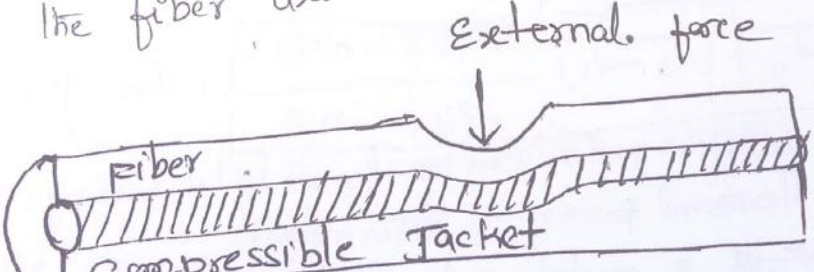


fig:. A compressible jacket extruded over a fiber reduces micro bending resulting from external force

Core and Cladding Losses:-

Upon measuring the propagation losses in an actual fiber, all the dissipative and scattering losses will be manifested simultaneously. Since the core and cladding have different indices of refraction and therefore differ in composition, the core and cladding generally have different attenuation co-efficients, denoted α_1 and α_2, respectively. If the influence of modal coupling is ignored, the loss for a mode of order (v, m) for a step index waveguide is

$$\alpha(vm) = \alpha_1 \frac{P_{core}}{P} + \alpha_2 \frac{P_{clad}}{P}, \quad P = P_{core} + P_{clad}$$

If P is the ~~total~~ power

$P_{core} \rightarrow$ power in core

$P_{clad} \rightarrow$ power in cladding

$\alpha_1 \rightarrow$ attenuation of core

$\alpha_2 \rightarrow$ attenuation of cladding

W.K.T $\quad P = P_{core} + P_{clad}$

$$1 = \frac{P_{core}}{P} + \frac{P_{clad}}{P}$$

$$\frac{P_{core}}{P} = 1 - \frac{P_{clad}}{P} \quad ; \quad \frac{P_{clad}}{P} = 1 - \frac{P_{core}}{P}$$

$$\alpha(v,m) = \alpha_1 \left(1 - \frac{P_{clad}}{P}\right) + \alpha_2 \frac{P_{clad}}{P}$$

$$\alpha(v,m) = \alpha_1 - \alpha_1 \frac{P_{clad}}{P} + \alpha_2 \frac{P_{clad}}{P}$$

$$\boxed{\alpha(v,m) = \alpha_1 + (\alpha_2 - \alpha_1) \frac{P_{clad}}{P}} \quad \text{(for step index)} \rightarrow (1)$$

$$\boxed{\alpha_{v,m}(r) = \alpha_1 + (\alpha_2 - \alpha_1) \frac{n^2(0) - n^2(r)}{n^2(0) - n_2^2}} \quad \text{(for graded index)} \rightarrow (2)$$

The total loss of the waveguide can be found by summing over all modes weighted by the fractional power in that mode.

For the case of a graded-index fiber the situation is much more complicated. In this case, both the attenuation co-efficients and modal power tend to be functions of the radial co-ordinate. At a distance r from the core axis the loss is in eq (2)

where α_1 & α_2 are the axial and cladding attenuation co-efficients, respectively and the loss encountered by a given mode is then

$$\boxed{\alpha_{gi} = \int_0^\infty \alpha(r) P(r)\, r\, dr \Big/ \int^\infty P(r)\, r\, dr}$$

$$\cancel{M = \frac{1}{x}\left[\frac{-x}{c}\lambda\frac{d^2n_1}{d\lambda}\right]}$$

$$\cancel{\boxed{M = -\frac{1}{c}\lambda\frac{d^2n_1}{d\lambda^2} \Rightarrow M = \frac{\lambda}{c}\left|\frac{d^2n_1}{d\lambda}\right|}}$$

Wave guide dispersion:

Information Capacity Determination:

A result of the dispession-induced signal distortion is that a light pulse will broaden as it travels along the fiber. As shown in fig below, this pulse broadening will eventually cause a pulse to overlap with neighbouring pulses. After a certain amount of overlap has occured, adjacent pulses can no longer be individually distinguished at the receiver and errors will occur. Thus the dispersive properties determine the limit of the information capacity of the fiber.

A measure of the information capacity of an optical wave-guide is usually specified by the band-width distance product in MHz. Km. For a step index fiber the various distortion effects tend to limit the bandwidth-distance product to about 20MHz.Km. In graded-index fibers the radial refractive index profile can be carefully selected so that pulse broadening is minimized at a specific operating wavelength.

This had led to bandwidth-distance products as high as 2.5 GHz. Km. Single-mode fibers can have capacities well in excess of this. A comparision of the information capacities of various optical fibers with the capacities of typical co-axial cables used for UHF and VHF transmission is shown in fig (b)

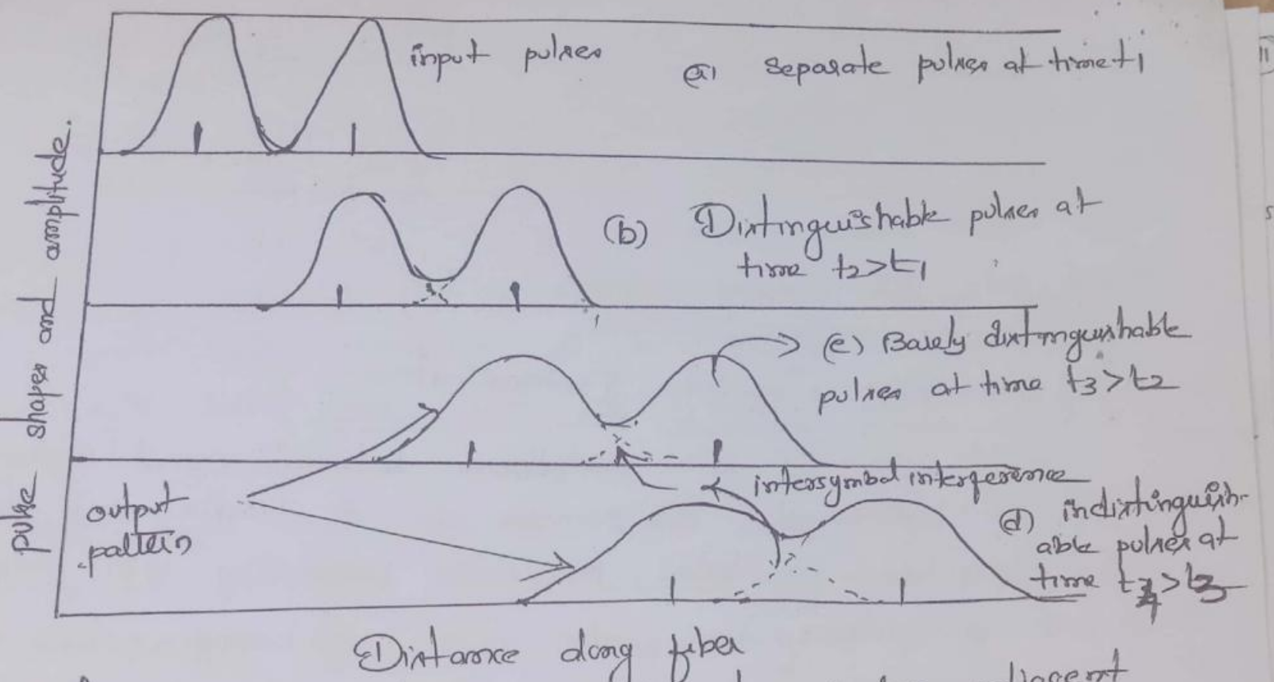
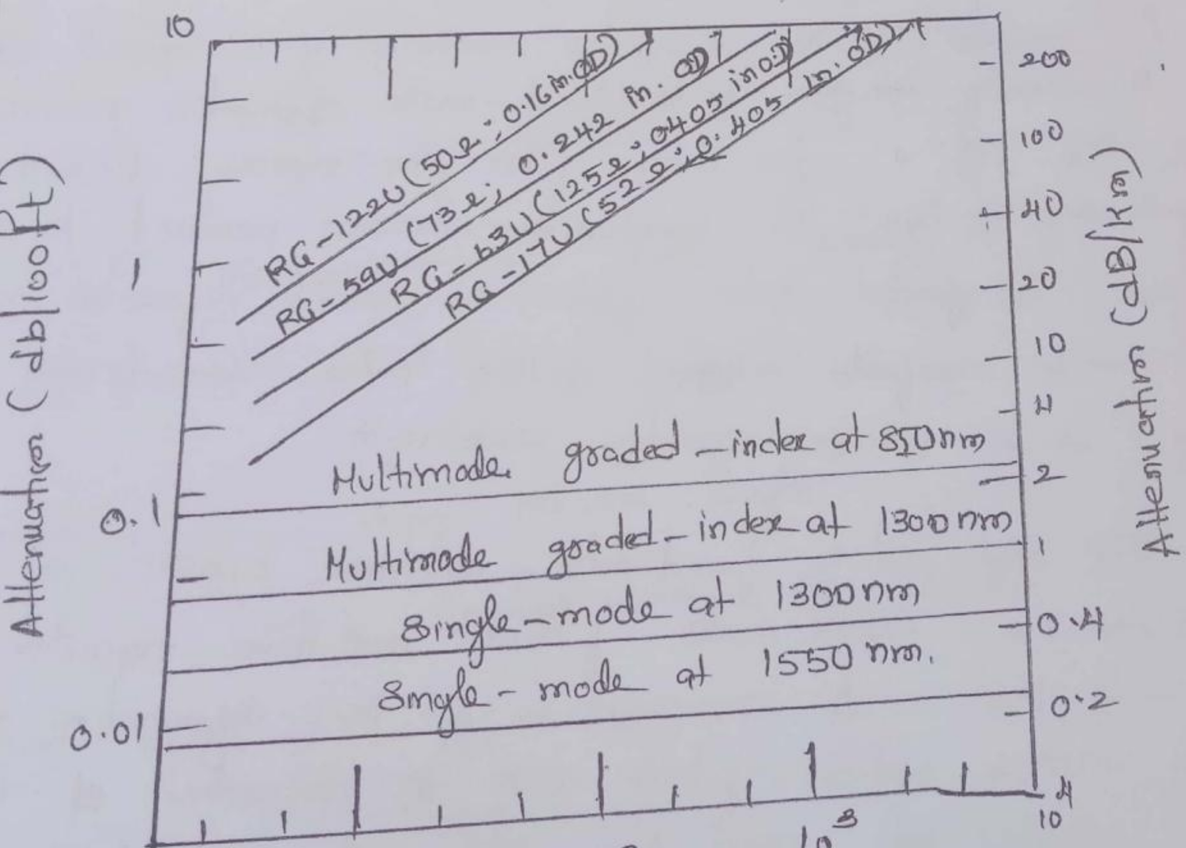


fig:- (a) Broadening and attenuation of two adjacent pulses as they travel along a fiber.



f(b):- A comparision of the attenuation as a function of frequency or data rate of various coaxial cables and several types of high B.W OF

Group Delay:-

Let us examine a signal that modulates an optical source. We shall assume that the modulated optical signal excites all modes equally at the input end of the fiber. Each mode thus carries an equal amount of energy through the fiber, Further more, each mode contains all the spectral components in the wavelength band over which the source emits. The signal may be considered as modulating each of these spectral component in the same way. As the signal propagates along the fiber, each spectral component can be assumed to travel independently, and to undergo a time delay (or) group delay per unit length in the direction of propagation given by

$$V_g \rightarrow \text{Group velocity}(V_g) = \frac{d\omega}{d\beta} = \frac{-2\pi c}{\lambda^2}\frac{d\lambda}{d\beta} \rightarrow (1)$$

Group velocity per unit length $\frac{\tau_g}{L}$

$\omega = 2\pi f$

$\omega = \frac{2\pi}{\lambda}$

$\frac{2\pi}{c/\lambda}$

$\omega = \frac{2\pi f}{c}$

$$\frac{\tau_g}{L} = \frac{1}{V_g} \Rightarrow \frac{d\beta}{d\omega} = \frac{-\lambda^2}{2\pi c}\frac{d\beta}{d\lambda} \rightarrow (2)$$

The amount of pulse spread is given by

$$\delta\tau = \frac{d\tau_g}{d\lambda}\sigma_\lambda \rightarrow (3)$$

where $\frac{d\tau_g}{d\lambda}$ = delay per unit length

σ_λ = Spectral width of the source

from eq (2) $\tau_g = \frac{-\lambda^2}{2\pi c} L \frac{d\beta}{d\lambda}$

from eq (3)

$$\delta\tau = \frac{d}{d\lambda}\left[\frac{-\lambda^2 L}{2\pi c}\frac{d\beta}{d\lambda}\right]\sigma_\lambda$$

$$\delta\tau = \frac{-L}{2\pi c}\left[2\lambda \frac{d\beta}{d\lambda} + \lambda^2 \frac{d^2\beta}{d\lambda^2}\right]\sigma\lambda$$

$\therefore$ Dispersion $\quad D = \frac{1}{L}\frac{d\tau_g}{d\lambda}$

$$D = \frac{d}{d\lambda}\left[\frac{1}{V_g}\right] \rightarrow$$ pulse broadening per unit distance per unit spectral width $\quad \because \frac{\tau_g}{L} = \frac{1}{V_g}$

$$\boxed{D = -\frac{2\pi c}{\lambda^2}\beta_2 \quad [ps/km/nm]}$$

It defines the pulse spread as a function of wave length and is measured in pico seconds per kilometer per nano meter [ps/(nm. km)]. It is a result of material and wave guide dispersion. In many theoretical treatments of intra modal dispersion it is assumed, for simplicity, that material dispersion and wave guide dispersion can be calculated seperately and added to give the total dispersion of the mode.

$$\boxed{D = D_{mat} + D_{wg}}$$

Basics of Dispersion losses:-

→ Dispersion is basically one of the limiting factors which decides, how much data can be transmitted through optical cable.

→ Due to dispersion broadening of the output pulse takes place as well as there can be intersymbol interference ISI.

→ All these factors, limit the information carrying capacity of optical cable.

→ The two major sources of dispersion are material dispersion and waveguide dispersion.

→ Material dispersion arises due to frequency dependent response of a material used to manufacture the cable.

→ Wave g when the speed of wave in a waveguide depends on its frequency then waveguide dispersion takes place.

Types of Dispersion Losses:-

There are two types of dispersion.

1. Intramodal dispersion (or) Chromatic Dispersion
2. Intermodal dispersion (or) modal delay. (multimode fibers)

1. Intramodal dispersion:- → The light source is used at input side. This converts an electrical signal into optical signal.

→ But this light source does not emits single wavelength.

→ In actual practice, this light sources emits band of wavelength. if the LED is used as lights source then this problem is more serious.

→ So the different spectral components will reach at the output at different times. This is called as intramod

→ This gives the spreading of output pulse.

dispersion.

→

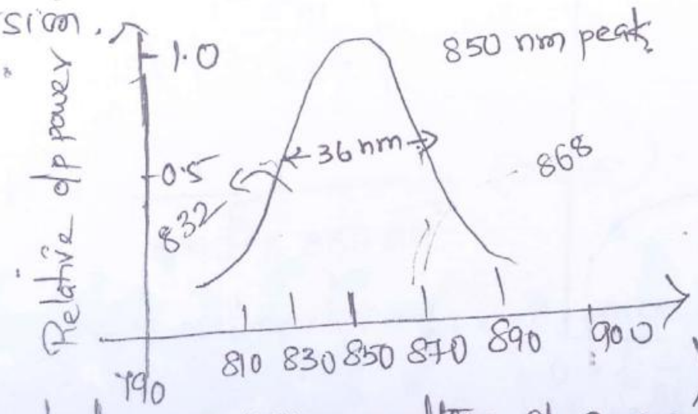


fig: Spectral emission pattern of a representative $Ga_{1-x}Al_xAs$ LED wi... ...of the spectral pattern at its half-po...

→ Because intramodal dispersion depends on the wavelength, its effect on signal distortion increases with the spectral width of the light source. The spectral width is the band of wavelengths over which the source emits light. This wavelength band normally is characterized by the root-mean-square (rms) spectral width σ_λ. Depending on the device structure of a light-emitting diode (LED), the spectral width is approximately 4 to 9 percent of a central wavelength. For example, as fig (a) illustrates, if the peak wavelength of an LED is 850 nm, a typical source spectral width would be 36 nm; that is such an LED emits most of its light in the 832-to-868 nm wavelength band. Laser diode optical sources exhibits much narrower spectral widths, with typical values being 1-2 nm for multimode lasers and 10^{-4} nm for single-mode lasers.

The two main causes of intramodal dispersion are as follows:

(a) Material dispersion :- Arises due to the variations of the refractive index of the core material as a function of wavelength. Material dispersion also is referred to as chromatic dispersion, since this is the same effect by which a prism spreads out a spectrum. This refractive index property causes a wavelength dependence of the group velocity of a given mode. That is pulse spreading occurs even when different wavelengths follow the same paths.

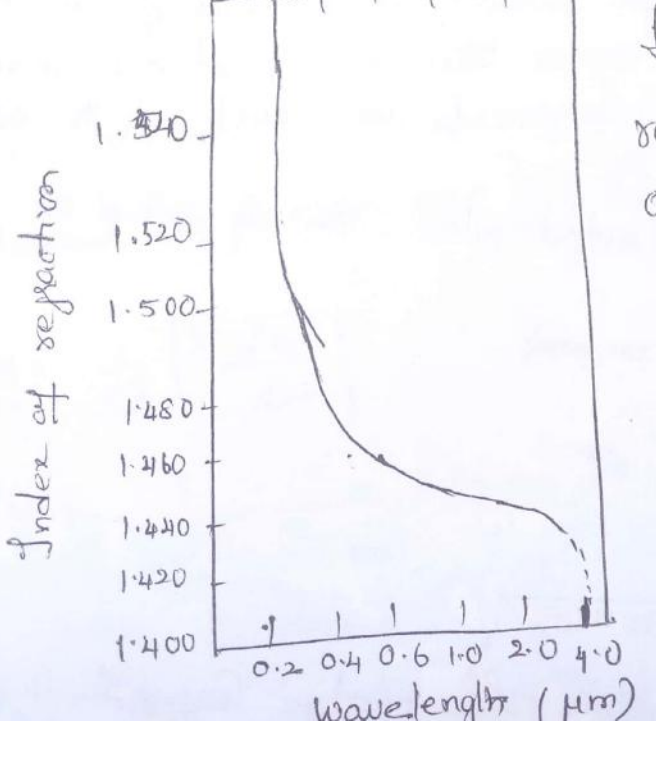
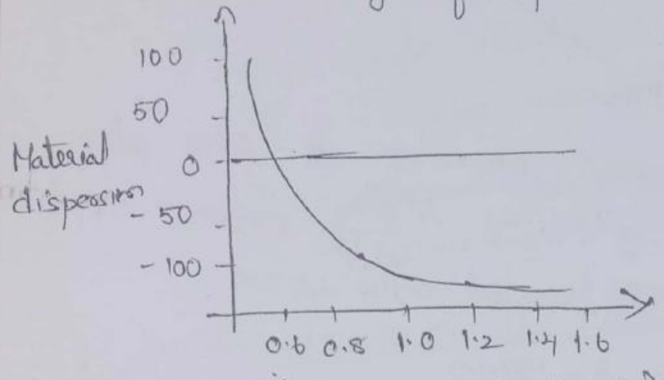


fig:- Variations in the index of refraction as a function of the optical wavelengths for silica.

→ Optical fiber operated at maximum wavelength,
The below fig shows the variation of material dispersion parameters 'M' with wave length for pure silica



→ It may be observed that the material dispersion tends to zero in larger wave length region.

→ Hence the material dispersion may be minimized at longer wavelengths.

Q. A glass fibre exhibits material dispersion given by modulus of $\left|\lambda^2 \cdot \frac{d^2 n_1}{d\lambda^2}\right|$ of 0.015. Determine the material dispersion parameter at a wavelength of 850 nm, estimate the RMS pulse broadening for km for a good LED source with an RMS spectral width of 20 nm.

Given that $\left|\lambda^2 \frac{d^2 n_1}{d\lambda^2}\right| = 0.015$

$\lambda = 850$ nm

Material dispersion $M = \frac{\lambda}{c}\left|\frac{d^2 n_1}{d\lambda^2}\right|$ ps/nm/km

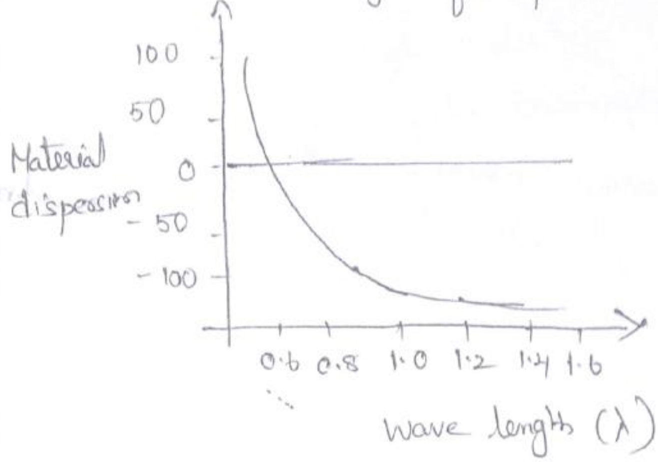
$$M = \frac{1}{c\lambda}\left|\lambda^2 \frac{d^2 n_1}{d\lambda^2}\right|$$

$$M = \frac{1}{3 \times 10^5 \times 850 \times 10^{-9}}(0.015) = 0.05882 \text{ ps/nm/km}$$

Rms pulse broadening is given by $\sigma\lambda = 20$ nm.

$$\sigma_m = \sigma\lambda \frac{L}{c}\left|\lambda \frac{d^2 n_1}{d\lambda^2}\right|$$

→ Optical fiber operated at maximum wavelength,
The below fig shows the variation of material dispersion parameters 'M' with wave length for pure silica



→ It may be observed that the material dispersion tends to zero in larger wave length region.

→ Hence the material dispersion may be minimized at longer wavelengths.

Q. A pure fibre exhibits material dispersion given by modulus of $\left|\lambda^2 \cdot \frac{d^2 n_1}{d\lambda^2}\right|$ of 0.015. Determine the material dispersion parameter at a wavelength of 850 nm, estimate the RMS pulse broadening for km for a good LED source with an RMS Spectral width of 20 nm.

Given that $\left|\lambda^2 \frac{d^2 n_1}{d\lambda^2}\right| = 0.015$

$\lambda = 850$ nm

Material dispersion $M = \frac{\lambda}{c}\left|\frac{d^2 n_1}{d\lambda^2}\right|$ ps/nm/km

$$M = \frac{1}{c\lambda}\left|\lambda^2 \frac{d^2 n_1}{d\lambda^2}\right|$$

$$M = \frac{1}{3\times10^5 \times 850\times10^{-9}}(0.015) = 0.05882 \text{ ps/nm/km}$$

Rms pulse broadening is given by $\sigma\lambda = 20$ nm.

$$\sigma_m = \sigma\lambda \frac{L}{c}\left|\lambda \frac{d^2 n_1}{d\lambda^2}\right|$$

$$= \sigma\lambda \frac{L}{c\lambda} \left| \lambda^2 \frac{d^2 n_1}{d\lambda^2} \right|$$

$$= \sigma\lambda \, L \cdot M$$

$$= 20 \times 10^{-9} \times 1 \times 0.05882.$$

$$\sigma M = 1.176 \times 10^{-9} \text{ (or) } 1.176 \text{ ns/km}$$

Q) Find the material dispersion parameter & Rms pulse broadening per km of the fiber has $\lambda^2 \frac{d^2 n_1}{d\lambda^2} = 0.055$, operating with wavelength 750nm & RMS spectral width 40 nm.

Given that $\lambda^2 \frac{d^2 n_1}{d\lambda^2} = 0.055$ $\lambda = 150\,nm$

Material dispersion $M = \frac{\lambda}{c} \left| \frac{d^2 n_1}{d\lambda^2} \right|$ ps/nm/km

$$= \frac{1}{c\lambda} \left| \frac{\lambda^2 d^2 n_1}{d\lambda^2} \right| \text{ ps/nm/km}$$

$$= \frac{1}{3 \times 10^5 \times 750 \times 10^{-9}} (0.055)$$

$$M = 0.244 \text{ ps/nm/km}$$

Rms pulse broadening is given by $\sigma\lambda = 40\,nm$

$$\sigma M = \sigma\lambda \frac{L}{c} \left| \frac{\lambda d^2 n_1}{d\lambda^2} \right|$$

$$= 40 \times 10^{-9} \times 1 \times 0.244$$

$$\sigma M = 9.76 \times 10^{-9} \text{ or } 9.76 \text{ ns/km}$$

Scanned with CamScanner

Material dispersion in Intramodal dispersion

(1) 'n' & 'λ' :- The refractive index of material (n) varies w.r.to operating wavelength (λ).

(2) The material is said to exhibit material dispersion w.r.to wavelength when the second differential of refractive index is not equal to zero.

$$\text{i.e } \frac{d^2n}{d\lambda^2} \neq 0$$

→ Material dispersion is also called as Chromatic dispersion (or) spectral dispersion.

V_g = group velocity

τ_g = Group delay

$$\tau_g \propto \frac{1}{v_g} \Rightarrow \frac{1}{d\omega/d\beta}$$

$$\tau_g = \frac{d\beta}{d\omega}$$

The pulse spread due to material dispersion may be obtained by considering group delay (τ_g) which is reciprocal of group velocity 'V_g'

The group delay τ_g is given by

$$\tau_g = \frac{d\beta}{d\omega} = \frac{1}{c}\left[n_1 - \lambda \frac{dn_1}{d\lambda}\right] \rightarrow (1)$$

where n_1 = Refractive index of core

The delay time τ_m due to material dispersion in a fiber of length 'L' is given by

$$\tau_m = \tau_g \times L$$

$$\tau_m = \frac{L}{c}\left[n_1 - \lambda \frac{dn_1}{d\lambda}\right] \rightarrow (2)$$

The r.m.s value of spreading out of a pulse using Taylor's series in equ (2)

$$\sigma_m = \sigma_\lambda \frac{d\tau_m}{d\lambda} + \sigma_\lambda^2 \frac{d^2\tau_m}{d\lambda^2} + \sigma_\lambda^3 \frac{d^3\tau_m}{d\lambda^3} + \text{-----} \rightarrow (3)$$

Differentiate τ_m w.r.to λ

$$\frac{d\tau_m}{d\lambda} = \frac{d}{d\lambda}\left[\frac{L}{c}\left(n_1 - \lambda \frac{dn}{d\lambda}\right)\right]$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$u = n_1, \quad v = \lambda \frac{dn}{d\lambda}$$

$$\frac{d\tau_m}{d\lambda} = \frac{L}{c}\left[\frac{dn_1}{d\lambda} - \left[\lambda \frac{d^2n_1}{d\lambda^2} + \frac{dn_1}{d\lambda}(1)\right]\right]$$

$$= \frac{L}{c}\left[\cancel{\frac{dn_1}{d\lambda}} - \lambda\frac{d^2n_1}{d\lambda^2} - \cancel{\frac{dn_1}{d\lambda}}\right]$$

$$\frac{d\tau_m}{d\lambda} = \frac{L}{c}\left[-\lambda \frac{d^2n_1}{d\lambda^2}\right] \rightarrow (4)$$

from eq (3) neglecting higher order mode because 'λ' is small

$$\lambda_m = \sigma_\lambda \frac{d\tau_m}{d\lambda} \rightarrow (5)$$

Sub eq (4) in eq (5)

$$\sigma_M = \sigma_\lambda\left[\frac{-L}{c}\lambda \frac{d^2n_1}{d\lambda^2}\right]$$

$$\boxed{\sigma_m = \sigma_\lambda \frac{L}{c}\left|\lambda \frac{d^2n_1}{d\lambda^2}\right|}$$

The material dispersion parameter m is defined by

$$M = \frac{1}{L}\frac{d\tau_m}{d\lambda} \Rightarrow M = \frac{1}{L}\left[\frac{-L}{c}\lambda\frac{d^2n_1}{d\lambda}\right] \Rightarrow \boxed{M = \frac{\lambda}{c}\left|\frac{d^2n_1}{d\lambda}\right|}$$

(b) Waveguide Dispersion:

Wave guide dispersion is chromatic dispersion which arises from waveguide effects. The dispersive effect of waveguide dispersion on pulse spreading can be approximated by assuming that the refractive index of the material is independent of wavelength. which occurs because a single mode fiber confines only about 80 percent of the optical power to the core. Dispersion thus arises, since the 20 percent of the light propagating in the cladding travels faster than the light confined to the core. The amount of waveguide dispersion depends on the fiber design, since the modal propagation constant β is a function of a/λ. The optical fiber dimension relative to the wavelength λ; there a is the core radius.

propagation constant → measure of the change undergone by the amplitude and phase of the wave as it propagates in a given direction

Let us consider the Group delay.

→ Now the dispersion is due to the guiding nature. The normalized propagation constant is denoted as 'b' and is defined as

$$b = \frac{\beta^2 - \beta_2^2}{\beta_1^2 - \beta_2^2} \rightarrow (1)$$

where β_1 = propagation constant in core

β_2 = propagation constant in cladding

β = propagation constant of a particular mode.

If β is bounded between β_1 & β_2 i.e $\beta_2 \leq \beta \leq \beta_1$ then

$$\therefore b = \frac{(\beta + \beta_2) - (\beta - \beta_2)}{(\beta_1 + \beta_2)(\beta_1 - \beta_2)} \rightarrow (2)$$

Since β_1 and β_2 is very close then

$$\therefore\ b = \frac{\beta - \beta_2}{\beta_1 - \beta_2}$$

$$\beta = b(\beta_1 - \beta_2) + \beta_2$$

$$\beta = \beta_2\left[1 + \frac{b(\beta_1 - \beta_2)}{\beta_2}\right] \rightarrow (3)$$

Let $\beta_1 = n_1 \beta_0$

$\beta_2 = n_2 \beta_0$

Substitute β_1 & β_2 values in eq (3)

$$\beta = n_2\beta_0\left[1 + \frac{b(n_1\beta_0 - n_2\beta_0)}{n_2\beta_0}\right]$$

$$\beta = n_2\beta_0\left[1 + \frac{b\cancel{\beta_0}(n_1 - n_2)}{\cancel{\beta_0}\, n_2}\right]$$

$$\beta = n_2\beta_0\left[1 + b\,\frac{n_1 - n_2}{n_2}\right] \qquad \therefore\ \frac{n_1 - n_2}{n_2} = \Delta$$

$$\beta = n_2\beta_0\left[1 + b\Delta\right] \rightarrow (4)$$

W.K.T

$\beta_2 = n_2\beta_0$

$\beta_0 = \frac{2\pi}{\lambda}$

& $\lambda = \frac{c}{f}$

$$\beta_0 = \frac{2\pi f}{c} \Rightarrow \frac{\omega}{c} \rightarrow (5)$$

Substitute β_0 value in eq (4)

$$\boxed{\beta = n_2\frac{\omega}{c}\left[1 + b\Delta\right]} \rightarrow (6)$$

$1 << \Delta \frac{db(V)}{dV}$ so

$$\boxed{\tau_g = \frac{L n_2}{c} \times \Delta \frac{db(V)}{dV}} \rightarrow (10) \text{ group delay}$$

Wave guide dispersion due to this group delay

$$D_{wg} = \frac{d\tau_g}{d\lambda} \rightarrow (11)$$

Substitute (10) in (11) eqn

$$D_{wg} = \frac{d}{d\lambda} \frac{L n_2}{c} \times \Delta \frac{db(V)}{dV}$$

$\because L, n_2, \Delta$ & c are constant

$$= \frac{L n_2 \Delta}{c} \times \frac{d}{d\lambda}\left[\frac{db(V)}{dV}\right] \times \frac{dV}{dV} \rightarrow (11)$$

multiply & divide by dV

W.K.T

$$V = \frac{2\pi a}{\lambda} NA$$

$$\frac{dV}{d\lambda} = \frac{d}{d\lambda}\left(\frac{1}{\lambda}\right) 2\pi a NA$$

$$\because \frac{d}{d\lambda}\left(\frac{1}{\lambda}\right) = -\frac{1}{\lambda^2}$$

$$\frac{dV}{d\lambda} = \frac{-2\pi a}{\lambda^2}(NA)$$

$$\frac{dV}{d\lambda} = \frac{-V}{\lambda} \rightarrow (12)$$

$$V = \frac{2\pi a}{\lambda}(NA)$$

$$V = \frac{2\pi f}{c} a(NA)$$

$$V = \frac{\omega}{c} a(NA) = V$$

Substitute (12) in (11) eq

$$D_{wg} = \frac{L n_2 \Delta}{c} \frac{d}{dV}\left[\frac{db(V)}{dV}\right] \times \frac{dV}{d\lambda}$$

$$D_{wg} = \frac{L n_2 \Delta}{c} \frac{d^2 b(V)}{dV^2} \times \frac{-V}{\lambda}$$

Let us first consider the group delay - that is the time required for a mode to travel along a fiber of length "L".

Group delay $\tau_g = L \dfrac{d\beta}{d\omega} \rightarrow (7)$ $\rightarrow$ $\left\{ \begin{array}{l} \therefore \dfrac{\tau_g}{L} = \dfrac{1}{V_g} \\ \tau_g = \dfrac{L}{V_g} = \dfrac{L}{d\omega/d\beta} = L\dfrac{d\beta}{d\omega} \\ V_g = \dfrac{d\omega}{d\beta} \end{array} \right.$

Substitute β value

$$\tau_g = L\frac{d}{d\omega}\left[\underbrace{\frac{n_2\omega}{c}}_{u}\underbrace{(1+b\Delta)}_{v}\right]$$

$$\tau_g = L\left[\frac{n_2\omega}{c}\frac{d}{d\omega}(1+b\Delta) + (1+b\Delta)\frac{d}{d\omega}\frac{n_2\omega}{c}\right] \quad \left(\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}\right)$$

$$\tau_g = L\left[\frac{n_2\omega}{c} \times \Delta\frac{db}{d\omega} + (1+b\Delta)\frac{n_2}{c}\right]$$

$$\tau_g = L\left[\frac{n_2}{c}(1+b\Delta) + \frac{n_2\omega}{c}\Delta\frac{db}{dV}\cdot\frac{dV}{d\omega}\right] \rightarrow (8)$$

w.k.T $\quad V = \dfrac{2\pi a}{\lambda}(NA)$

$$\lambda = \frac{c}{f} \rightarrow V = \frac{2\pi f}{c}a(NA)$$

$$V = \frac{\omega}{c}a(NA)$$

$$\frac{dV}{d\omega} = \frac{a(NA)}{c} \times \frac{\omega}{\omega} = \frac{V}{\omega} \rightarrow (9)$$

sub $\dfrac{dv}{d\omega}$ value in equ (8)

$$\tau_g = L\left[\frac{n_2}{c}(1+b\Delta) + \frac{n_2\cancel{\omega}}{c}\Delta\frac{db}{dV}\cdot\frac{V}{\cancel{\omega}}\right]$$

$$\tau_g = \frac{Ln_2}{c}\left[1+b\Delta + \Delta\frac{db(V)}{dV}\right]$$

$$\tau_g = \frac{Ln_2}{c}\left[1+\Delta\frac{db(V)}{dV}\right]$$

$$\boxed{D_{wg} = -\frac{L n_2 \Delta V}{c\lambda}\frac{d^2(bV)}{dV^2}}$$

polarisation Mode dispersion:

The effect of fiber birefringence on polarisation states of an optical signal causes pulse broadening. This is called polarisation mode dispersion

One inherent challenge to providing higher data rate communication is managing the dispersion effects on the system. Polarization mode dispersion, in high data rate systems, can significantl diminish the data-carrying capacity of a telecommunication.

" A fundamental property of single mode optical fiber and components, polarization Mode Dispersion (PMD) is a broadening of the input pulse due to a phase delay b/w input polarization states

Single mode optical fiber and components supports one fundamental mode, which consists of two orthogonal polarization modes Ideally, the core of an optical fiber is perfectly circular, and therefore has the same index of refraction for both polarization states. However, mechanical and thermal stresses introduced during manufacturing result in asymmetries in the fiber core geometry. This asymmetry introduces small index of refraction differences for the two polarization states, a property called birefringence. It also happens because of some external factors such as mechanical stress, bending, twisting & pinching of fiber

Birefringence causes one polarization mode to travel faster than the other, resulting in a difference in the propagation time called the differential group delay (DGD). DGD is the unit that is used to describe PMD.

DGD is typically measured in picoseconds.

Polarization refers to orientation of electric signal which refers along the length of the fiber.

It is given by $\Delta\sigma_{PMD} = \left| \frac{L}{V_{gx}} - \frac{L}{V_{gy}} \right|$.

where L - length of fiber.

V_{gx} & V_{gy} → Group velocities in x & y direction

$\Delta\sigma_{PMD}$ → Differential time delay between the two polarization components during propagation of the pulse over a distance L

A useful means of characterizing PMD for long fiber lengths is in terms of the mean value of the differential group delay

$$\boxed{\Delta\sigma_{PMD} = D_{PMD}\sqrt{L}}$$

where D_{PMD} is measured in $ps/\sqrt{Km}$, is the average PMD parameter

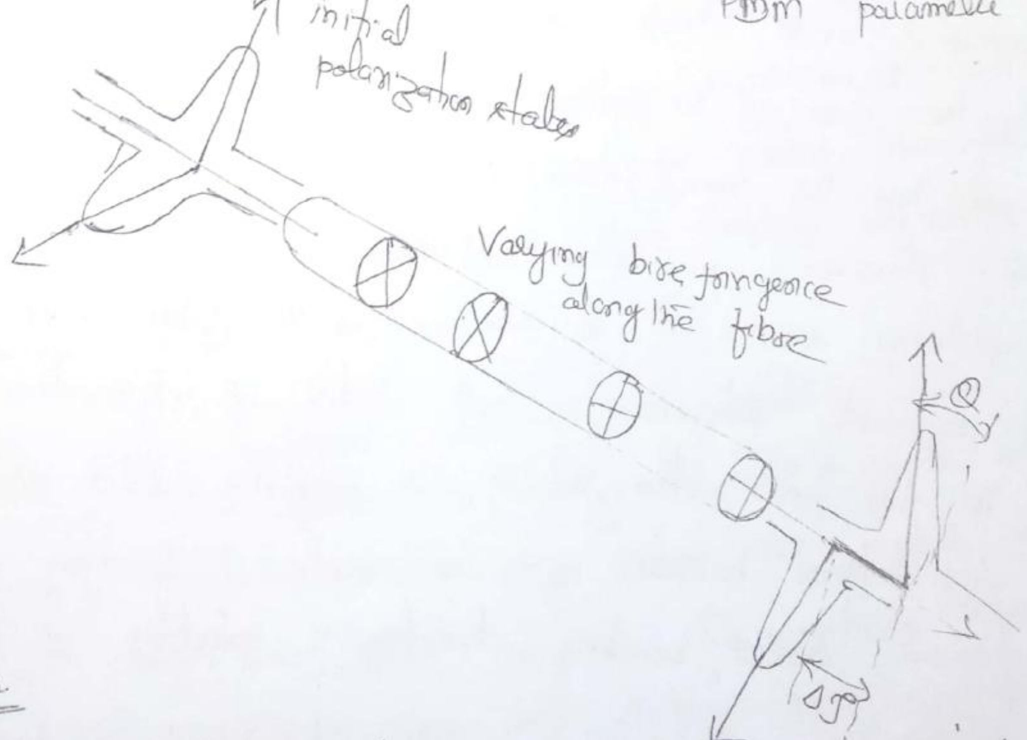


fig 1

Variation in the polarization states of an optical pulse as it passes through a fiber with varying birefringence along its length

Design optimization of Single mode fibers :-

Features of single mode fibers are :

1. Longer life
2. Low attenuation
3. Signal transfer quality is good
4. Modal noise is absent
5. Large bandwidth distance product

Basic design optimization includes the following:

1. Cut off wave length
2. Dispersion
3. Mode field diameter
4. Bending loss
5. Refractive index profile.

(a) Refractive - index profiles :

Dispersion of single mode silica fiber is lowest at 1300nm 1310 nm while its attenuation is minimum at 1550 nm. For achieving maximum transmission distance the dispersion zero should be at the wavelength of minimum attenuation. The wave guide dispersion is easier to control than the material dispersion. Therefore a variety of core-cladding refractive index configuration fibers.

The basic material dispersion is hard to alter significantly, but it is possible to modify the wave guide dispersion by changing from a simple step-index core profile design to more complicated index profiles. Researchers have thus examined a variety of core and cladding refractive index configuration for altering the behaviour of single mode fibers

Below figures shows representative refractive-index profiles of the four main categories:

1. 1300-nm-optimized fibers
2. dispersion-shifted fibers
3. dispersion-flattened fibers
4. Large-effive core area fibers.

The most popular single mode fibers used in telecommunication networks are near-step-index fibers, which are dispersion optimized for operation at 1300 nm.

1. 1300 nm - optimized fibers :- There are two types.

(a) Matched cladding
(b) depressed-cladding.

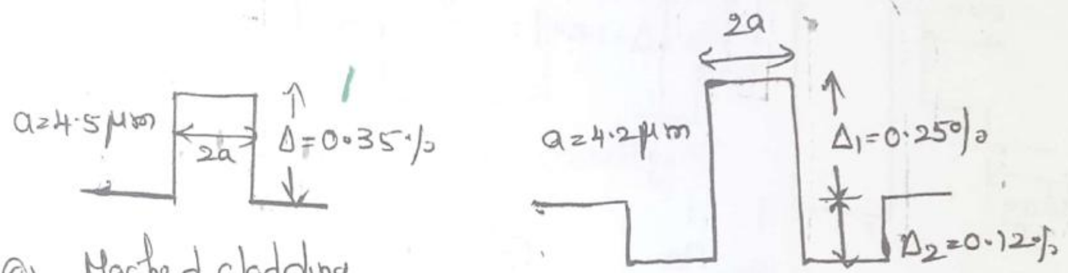
(a) Matched cladding fibers have a uniform refractive index cladding index differences are around 0.37 percent.

(b) In Depressed cladding fibers the cladding portion next to the core has a lower index than the outer cladding region. Mode field diameters are around 9 μm, and typical positive and negative index differences are 0.25 and 0.12 percent respectively.

2. Dispersion shifted fibers :- where as material dispersion depends only on the composition of the material, wave guide dispersion is a function of the profile. Thus, the wave guide dispersion can vary dramatically with the fiber design parameters. "The addition of waveguide and material dispersion can then shift the zero dispersion point to longer wave lengths. The resulting optical fibers are known as dispersion shifted fibers".

3. Dispersion flattened fibers

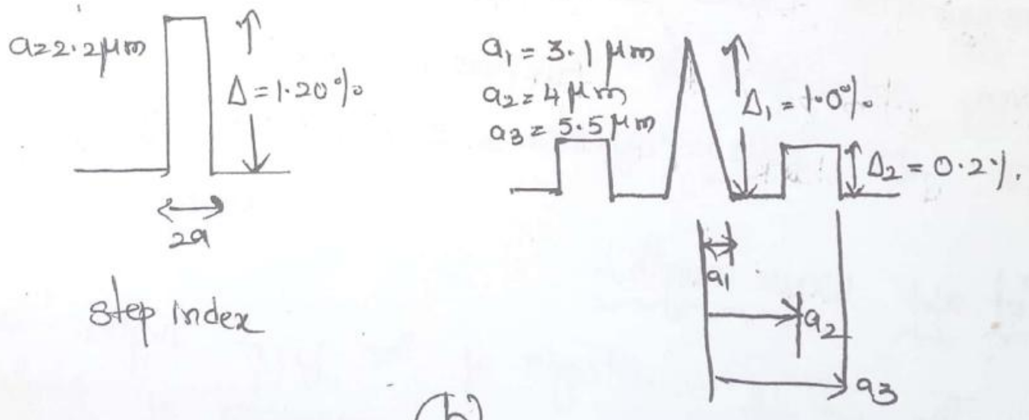
An alternative is to reduce fiber dispersion by spreading the dispersion minimum out over a wider range. This approach is known as dispersion flattening.
Dispersion-flattened fibers are more complex to design than the dispersion shifted fibers. Because dispersion must be considered over a much broader range of wavelengths. However they offer desirable characteristics over a wide span of wavelengths.



(a) Mached cladding

(b) Depressed cladding

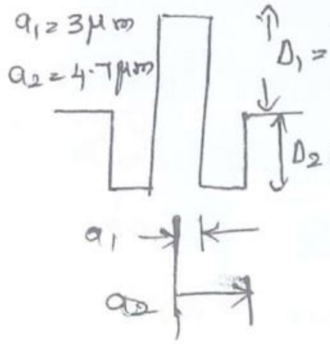
fig:- 1300-nm optimized refractive index profile

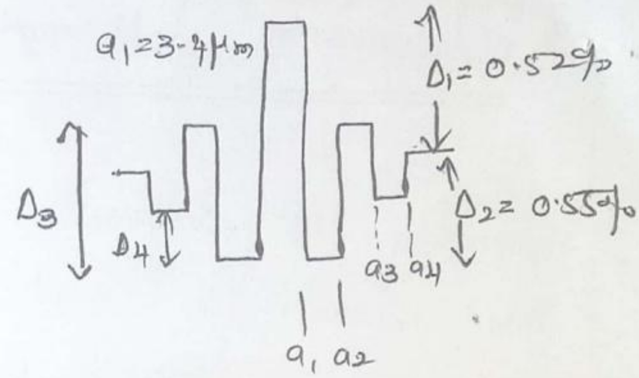


(b)

Triangular with annular ring

fig:- Dispersion shifted fibers.

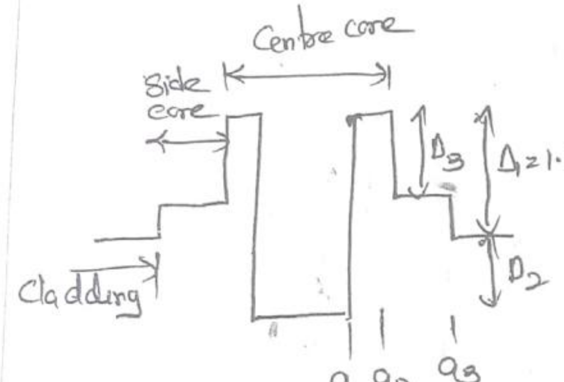


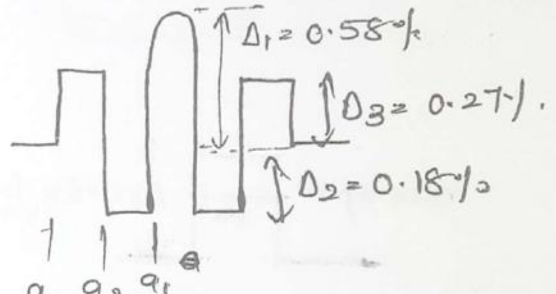


(c)

Double clad or W profile

~~Quadraple~~
Quadruple – clad profile.





(d)

Large – area dispersion shifted

Large area ~~dispersion~~ dispersion flattened

Representative cross sections of index profiles for (a) 1300-nm- optimized (b) dispersion – shifted (c) dispersion – flattened (d) large – effective – core – area fibers

(b) Cut off wavelength :-

The cut off wave length of the first higher – order mode (LP_{11}) is an important transmission parameter of single mode fibers. Since it seperates the single mode from the multi mode regions.

→ Cut off wave length is equal to the minimum wavelength at which an optical fiber will support single mode.

→ Above the cutoff wavelength, only the LP_{01} fundamental mode propagates. The fiber is a single mode fiber.

→ Below the cutoff wavelength, LP_{11} and other higher order modes propagate. The fiber becomes a multimode fiber.

→ Single mode operation occurs above the theoretical cutoff wavelength given by

$$\boxed{\lambda_c = \frac{2\pi a}{V_c}(n_1^2 - n_2^2)^{1/2}}$$

(or)

$$\boxed{\lambda_c = \frac{2\pi a n_1}{V_c}(2\Delta)^{1/2}}$$

(or)

$$\boxed{\lambda_c = \frac{2\pi a}{V_c}(NA)}$$

λ_c = theoritical cut off wavelength

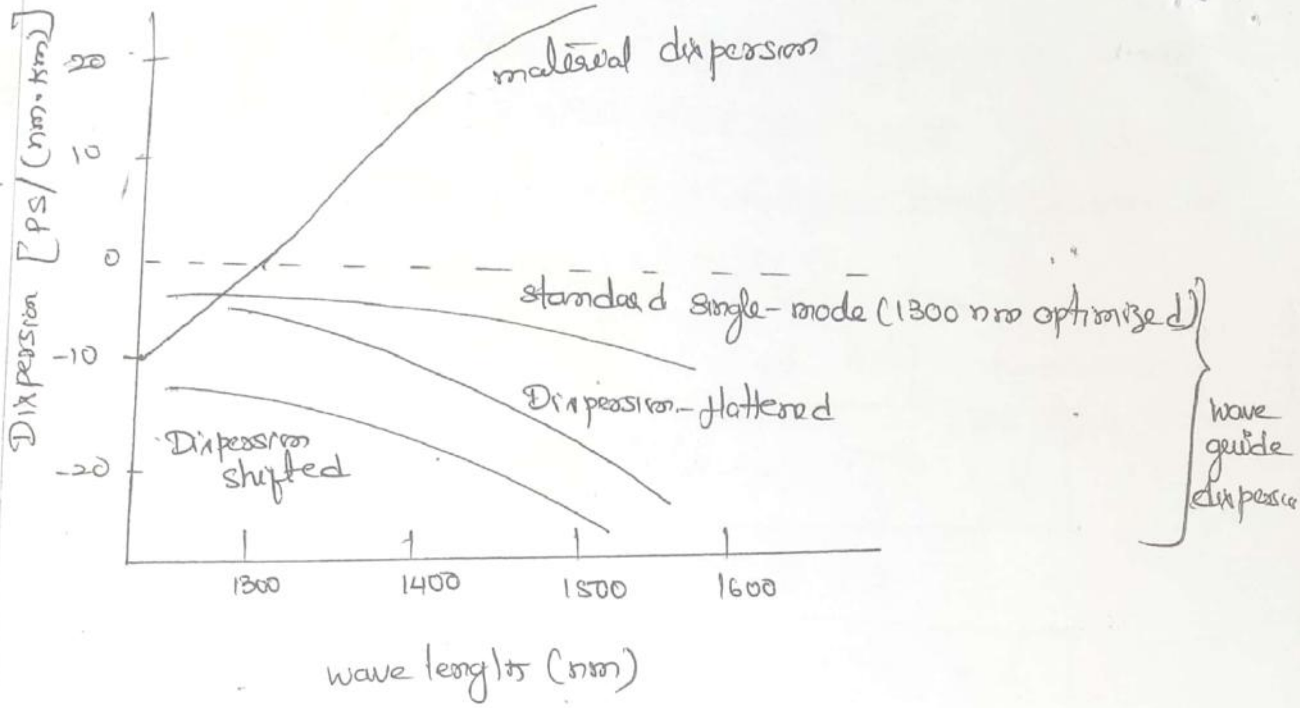
a = radius of the core

n_1 = refractive index of core

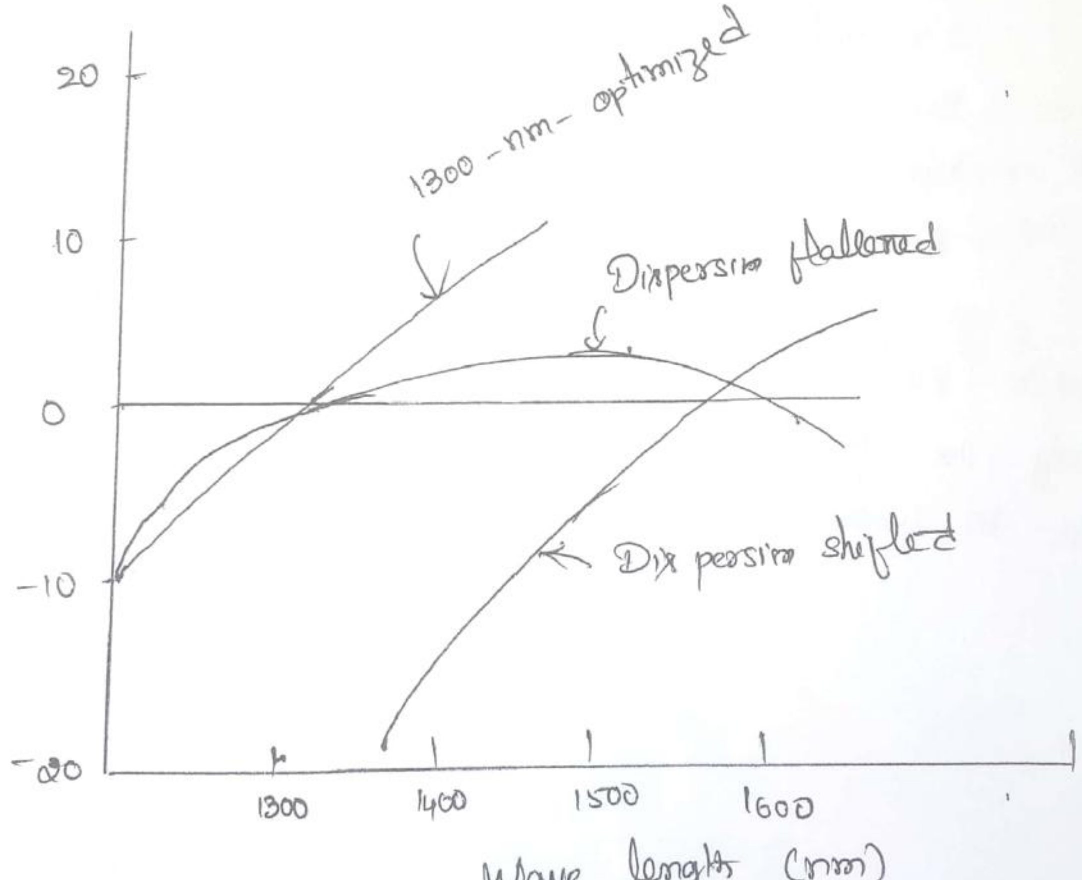
V_c = normalized frequency

Δ = Relative refractive index difference (b/w core and cladding)

With $V = 2.405$ for step index fibers. At this wavelength only the LP_{01} mode (i.e the HE_{11} mode) should propagate in the fiber.



(a)



(b)

* These two graphs are related to refractive index profile topic

(a) Typical wave guide and the common material dispersion of 3 different single mode fiber design

(b) Resultant total dispersions.

(b) Similarly at 62.5 μm we have $V = 26.5$, $M = 351$

(c) Finally at 100 μm, we have $V = 424$ and $M = 898$

Calculate the no. of modes of an optical fiber having diameter of 50 μm, $n_1 = 1.48$, $n_2 = 1.46$ and $\lambda = 0.82\,\mu m$

$d = 50\,\mu m$

$n_1 = 1.48$

$n_2 = 1.46$

$\lambda = 0.82\,\mu m$

$NA = (n_1^2 - n_2^2)^{1/2}$

$NA = (1.48^2 - 1.46^2)^{1/2}$

$NA = 0.243$

FOC sys & tech

No. of modes are given by

$$M = \frac{1}{2}\left[\frac{\pi d}{\lambda} . NA\right]^2$$

$$M = \frac{1}{2}\left[\frac{\pi\,(50\times\cancel{10^{-6}})}{0.82\times\cancel{10^{-6}}} . 0.243\right]^2$$

$$\boxed{M = 1083.}$$

Consider a multimode step-index optical fiber that has a core radius of 25 μm, a core index of 1.48, and an index difference $\Delta = 0.01$. Find the percentage of optical power that propagates in the cladding at 840 nm.

At an operating wavelength of 840 nm the value of V is

$$V = \frac{2\pi a}{\lambda} n_1 \sqrt{2\Delta} = \frac{2\pi \times 25\,\mu m \times 1.48}{0.84\,\mu m}\sqrt{2\times 0.01}$$

$$\boxed{V = 39}$$

The total no. of modes is

$$M = \frac{V^2}{2} = \frac{(39)^2}{2} = 760$$

$$\boxed{M = 760}$$

$$\text{Optical power } (P) = \frac{P_{clad}}{P} = \frac{4}{3\sqrt{M}} = 0.05$$

Approximately 5% of optical power propagates in the cladding