

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY ANANTAPUR

B.Tech IV-II Sem (E.C.E)

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(13A04804) RF INTEGRATED CIRCUITS

UNIT - I

Introduction RF systems - basic architectures, Transmission media and reflections, Maximum power transfer, Passive RLC Networks, Parallel RLC tank, Q, Series RLC networks, matching, Pi match, T match, Passive IC Components Interconnects and skin effect, Resistors, capacitors Inductors

UNIT - II

Review of MOS Device Physics - MOS device review, Distributed Systems, Transmission lines, reflection coefficient, the wave equation, examples, Lossy transmission lines, Smith charts - plotting Gamma, High Frequency Amplifier Design, Bandwidth estimation using open-circuit time constants, Bandwidth estimation, using short-circuit time constants, Rise time, delay and bandwidth, Zeros to enhance bandwidth, Shunt-series amplifiers, tuned amplifiers, Cascaded amplifiers

UNIT - III

Noise - Thermal noise, flicker noise review, Noise figure, LNA Design, Intrinsic MOS noise parameters, Power match versus, noise match, large signal performance, design examples & Multiplier based mixers, Mixer Design, Subsampling mixers.

UNIT - IV

RF Power Amplifiers, Class A, AB, B, C amplifiers, Class D, E, F amplifiers, RF Power amplifier design examples, Voltage controlled oscillators, Resonators, Negative resistance oscillators, Phase locked loops, Linearized PLL models, Phase detectors, charge pumps, Loop filters, and PLL design examples

UNIT - V

Frequency synthesis and oscillators, Frequency division, integer-N synthesis, Fractional frequency, synthesis, Phase noise, General considerations, and Circuit examples, Radio architectures, GSM radio architectures, CDMA, UMTS radio architectures

Textbooks:

1. The design of CMOS Radio frequency integrated circuits by Thomas H. Lee
Cambridge university press, 2004.
2. RF Micro Electronics by Behzad Razavi, Prentice Hall, 1997.

UNIT - 1

The Design of Radio Frequency Integrated circuits.

Introduction:-

3

* Electromagnetic spectrum.

↳ Defines the range of all types of electromagnetic radiations depending upon their wavelengths.

↳ Energy that travels (or) spreads out as it goes from one place to another.

Ex: Visible light that comes from the lamp in the house.

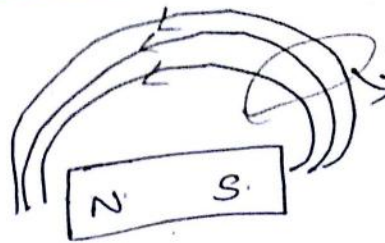
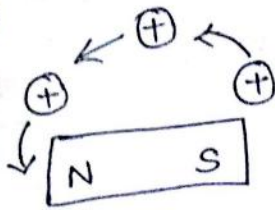
Radio waves that comes from a radio station are the two types of electromagnetic radiations.

Electromagnetic

↳ It gives the interrelationship b/w the electric current (or) electric fields and magnetic fields.

a) Electric field:-

↳ It is produced due to the motion of the charged particles.



b) Magnetic Field:-

↳ It depends on electrostatic forces.

like charges
 $\leftarrow (+) \text{ repel } (+) \rightarrow$

opp charges
 $(+) \rightarrow (-)$
 attract

Magnetic forces

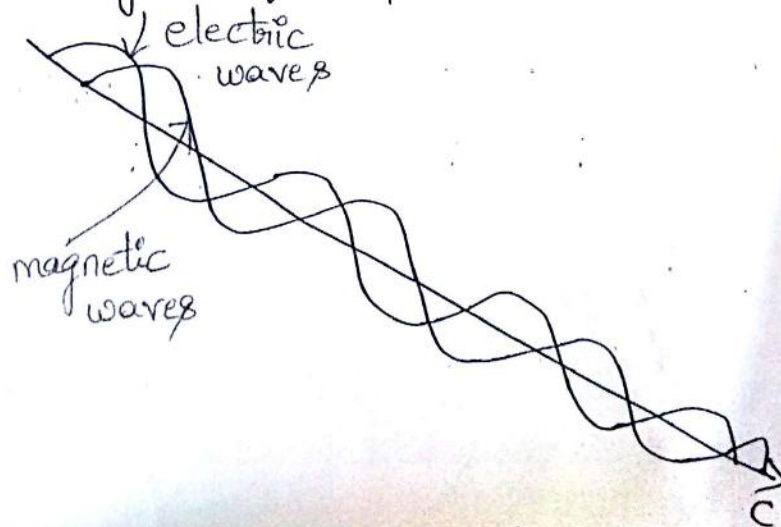
like poles so they repel.
 $\leftarrow [S | N] \quad [N | S] \rightarrow$

opp poles so they attract.
 $[N | S] \rightarrow \leftarrow [N | S]$

Electromagnetism (electricity & magnetism)

\Rightarrow Electric current produces a magnetic field.

\Rightarrow Faraday's law states that changes in electric field produces a magnetic field & vice-versa.

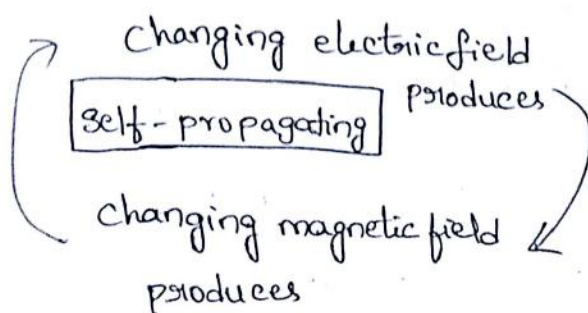




electric field.

Electromagnetic radiation is a self-propagating composite of an electric field & magnetic field".

- * Self-propagating (able to propagate by itself).
- * Composite (Combination).



Radio Frequency:-

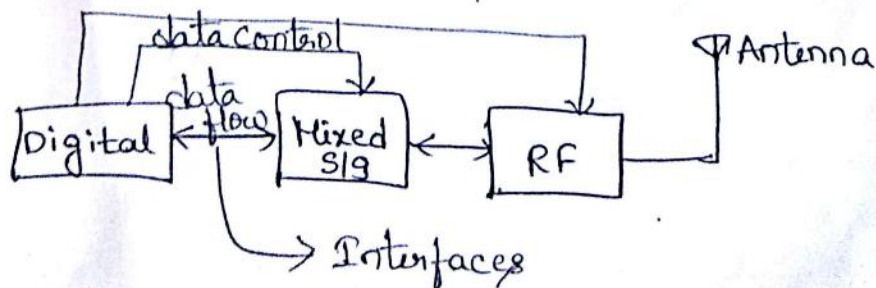
- * Radio freq (high freq, Tx, Rx)

↳ Any freq (or) freq of any sig that carries information is called Radio freq sig (wired or wireless).

↳ now-a-days

Ex: Cellular phones, WLAN, TV, AM-FM Radio etc.

Modern communication s/m's:-



* In most of the IC's the data control will be p:

→ Digital slg → Baseband processing
↓
(processor, Modems)

→ Mixed slg → ADC
DAC
Audio Amplifier.

→ RF slg → LNA's (Low Noise Amplifier)
Mixers (or) multiplier
power amplifiers.

Basically the RF slm consists of,

→ Transceiver architecture

↳ systems (interaction b/w the RF & Base station).

→ RF IC ckt → ckt design
ckt topologies.

CMOS

* The 3 m

CMOS RF Integrated Circuits

UNIT-1

Introduction to RF Systems

* The 3 sets of words used in this chapter are,

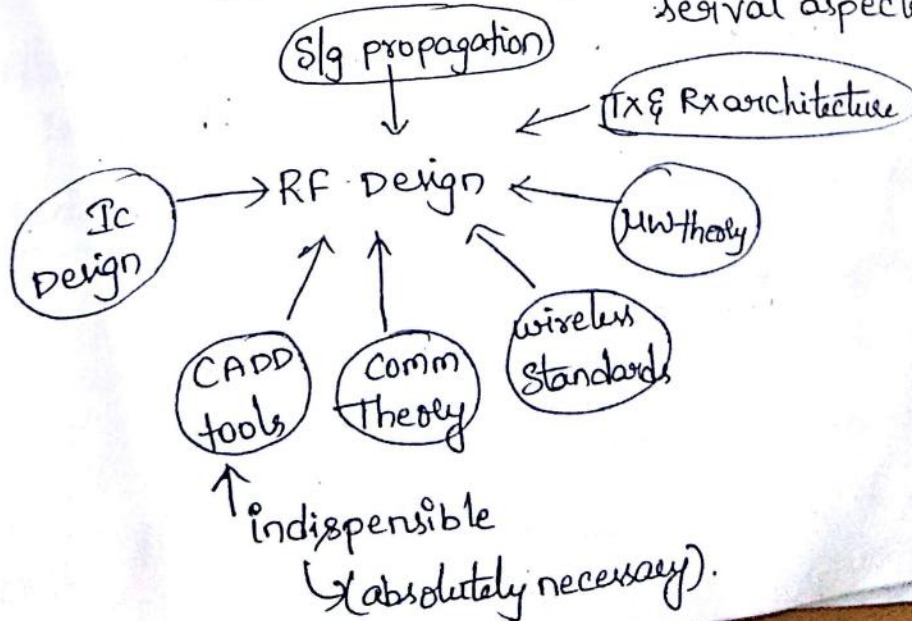
- Integrated circuits (IC's)
- Radio frequencies (RF)
- Complementary Metal oxide Semiconductor (CMOS)

(1) Integrated ckt's (IC's):-

The arrangement of passive & active components integrated on a single chip is called as Integrated circuits (IC's).

(2) Radio Frequency (RF):-

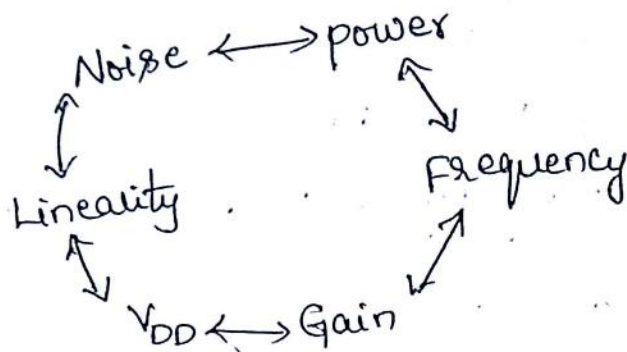
* RF is a "multi-disciplinary" (used with the help of several aspects).



RF Design Tradeoff:-

* Tradeoff

↳ situation that involves losing one quality in order to gain another quality.



3) CMOS:-

* The traditional reason for choosing the CMOS is,

CMOS digital ckt:-

- Required very few devices/gates.
- No static power dissipation (enhancement MOSFET)
↓ no channel.
- MOS devices can be easily scaled to achieve higher densities.
- Lower fabrication costs.

10S Analog ckt:-

⇒ possibility of soc (system-on-chip).



[single chip has Dig. sig, Mixed sig, Radio freq].

⇒ Reduces cost.

⇒ comparable to BJT, MOSFET has more speed and noise performance is better.

RF Band Designations

<u>Band.</u>	<u>Freq Ranges</u>	<u>Wavelength ranges.</u>
1. Extreme low freq (ELF)	$< 30\text{Hz}$	$> 10,000\text{Km}$.
2. Super low freq (SLF)	30Hz to 300Hz	$10,000\text{Km}$ to 1000Km .
3. ULF	300Hz to 3KHz	1000Km to 100Km .
4. VLF	3KHz to 30KHz	100Km to 10K
5. LF	30KHz to 300KHz	10K to 1K
6. MF	300KHz to 3MHz	1K to 100m
7. HF	3MHz to 30MHz	100m to 10m
8. VHF	30MHz to 300MHz	10m to 1m
9. UHF	300MHz to 3GHz	1m to 10cm .
10. SHF	3GHz to 30GHz	10cm to 1cm
11. EHF	$> 30\text{GHz}$	$< 1\text{cm}$.

* Another way of naming the course subject is,
"Circuits for cell phones."

↳ Because, a

→ CMOS implies that it is cheap and it is suitable for mass production.

→ IC → compact & tiny and also highly integrated.

→ RF → high freq. sigs. (3MHz to 30MHz)

* So, a device which provides cheap, mass production with a compact & highly integrated ckt is nothing but a mobile (or) cell phones.

* Apart from cell phones some more examples which provides cheap and high integration are,

→ WiFi

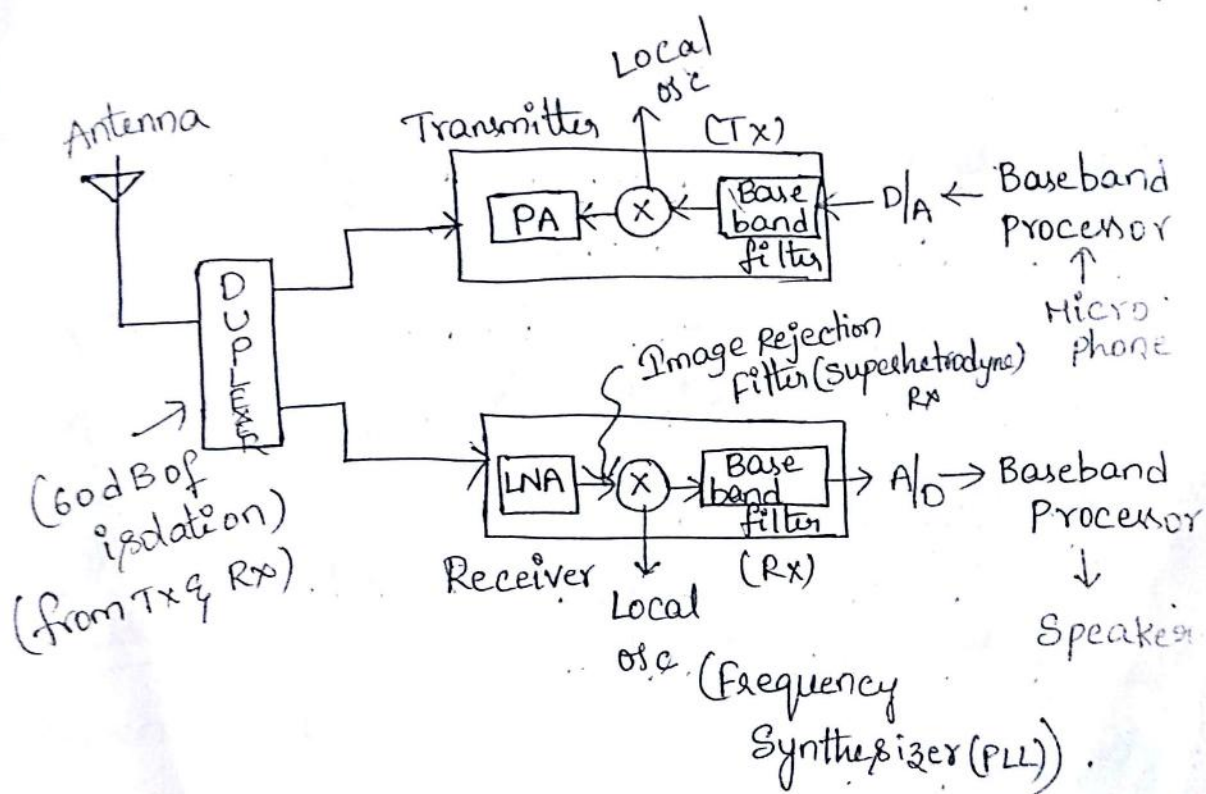
→ Bluetooth

→ GPS

→ FM Radios etc.

Basic Architectures:-

- * The cell phones & mobile phones has a Tx & Rx.
- * FM radio has only the Rx.
- * The handheld unit for the cell phone looks like,



(a) Mobile unit Architecture.

- Antenna
- Duplexer
- PA
- LNA
- Base band filters, A/D or D/A & Base band processors.

Explanation:-

- ⇒ Antenna is used for transmitting & receiving the s/g
- ⇒ The Tx & Rx uses the same antenna.
- ⇒ This is possible by a device which is called as "Duplexer".

Duplexer:-

- * It acts as a switch which separates the transmitter path and receiver path.
- * It can also be called as Mechanical switch as it isolates the Tx path & Rx path.
- * Typically it sometimes acts as a semimechanical switch since it provides low attenuation to the s/g and also filters the s/g.

Receiver Side:-

- ⇒ once the s/g is isolated by the duplexer from the Tx part.

(a) LNA:-

- ⇒ The Rx consists its first block as "Low Noise Amp".
- ⇒ During the receiving of the s/g there may be a chance

ing the s/g's
+ tiny s/g's from the atmosphere to be added to it.
These tiny s/g's has to be amplified so that we can hear the Tx s/g.

⇒ This is possible by the "LNA".

→ The LNA does not add much noise as its own.

[LNA → throws out the noise & keep the s/g (it will not be happened).
Every s/m adds noise if it burns power.]

⇒ The amplifier which adds low noise to the s/g has to be selected (i.e by the LNA is possible).

(b) Mixer (Multiplier):-

⇒ The second block of the Rx is Mixer with a local oscillator frequency.

⇒ When the cell phones receives the s/g its frequency will be 800MHz, 1600MHz etc which is extremely of high frequency and it is hard to work with them.

⇒ Hence this frequency has to be get down.

⇒ The frequency can be lowed down by using the Mixers (Multiplier) which down converts the high freq's to low freq's.

⇒ The local oscillator freqs for the Tx & Rx have to be different.

⇒ Since if they are of same freq then the Tx & Rx sigs will be the echo at the Rx.

⇒ The "frequency synthesizer" is used to match the base station frequency with the local oscillator freq.

⇒ In the frequency synthesizer the "phase locked loops" (PLL) are present.

⇒ For the superheterodyne Rx before the mixer it consists of an "Image rejection filter block". (heterodyne).

⇒ For homodyne Rx (or) direct down conversion filter it is not needed.

c) Base Band Filter:-

⇒ A baseband filter is a device that only passes frequencies inside the interval $(0, B)$, where B is the maximum freq of the sig.

Ex: Human voice occupies a spectrum from 0 Hz to 3400 Hz approximately.

⇒ The baseband filter only let those freqs pass.

Rx has to
Tx & Rx

A/D converter:-

⇒ It is used for converting the analog s/g into the digital signal.

(c) Baseband processor:-

⇒ The digital unit consists of the baseband processor.

⇒ At the Rx side the audio amplifier like (speaker) is selected as the baseband processor.

⇒ which is used for receiving the s/g whichever it is transmitted from the Tx side.

(2) Transmitter side:-

i. The power Amplifier (PA)

⇒ The last block of the Tx side is the power amplifier.

⇒ Basically the amplifiers are used for increasing the strength of the weak s/g's.

⇒ Strength in the sense power of the s/g gets increased.

⇒ The power amplifier strengthens very low-power, inaudible electronic audio s/g's to a level that is strong enough for driving the loudspeakers and being heard by the listeners.

ii. D/A converter:-

⇒ It is used for converting the digital s/g received from the Rx to analog s/g at the Tx.

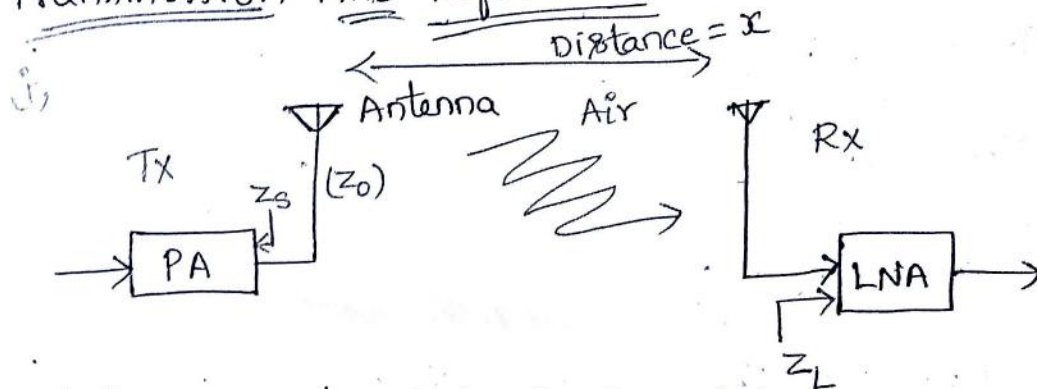
iii, Baseband processor:-

⇒ The baseband processor uses the microphone at the transmitter side.

iv, Local oscillator (Mixer):-

⇒ The mixer block functioning is same as the Rx block mixer but the frequency differs.

Transmission And Reflections:-



⇒ Let us consider Tx is the base station & Rx is the handset.

⇒ Between the two antennas of Tx & Rx the sig travels through the air.

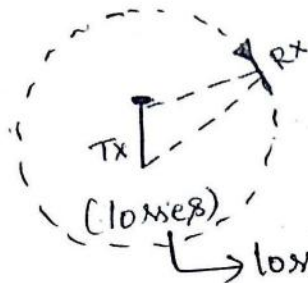
⇒ The sig travels through air in the form of waves called as Electromagnetic waves.

⇒ If the distance b/w these two antennas decreases then the power received by the Rx antenna decreases as a reciprocal of the square of the distance.

$$\text{Power Received by Rx} \propto 1/x^2$$

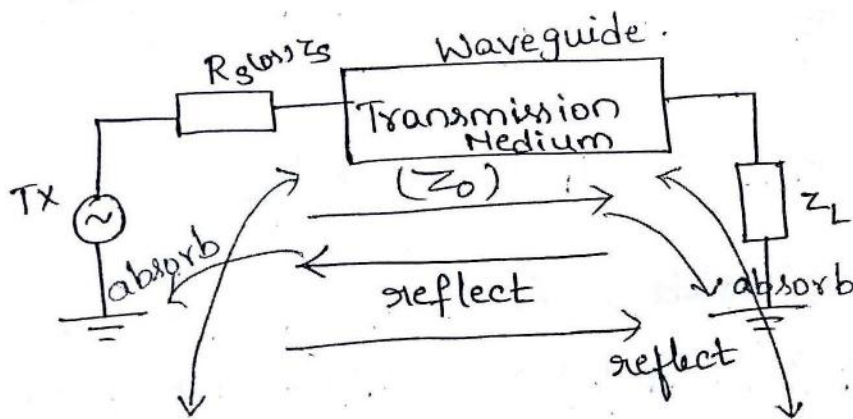
the antenna has some directivity (area).

As the area increases then more s/g can be received.



losses has to be minimized.

(ii) By using waveguides:-



* Every transmission medium has characteristic impedance

(as long as it transmits electromagnetic waves).

Ex: wired, atmosphere, optical fiber etc.

* When a wave is transmitted through the transmission medium the wave propagates over the transmission medium all the way to the receiver.

* When it hits the receiver, a portion of this wave gets absorbed by the Rx and a portion of the wave is reflected back.

- ⇒ The portion of this reflected s/g hits the source or again a portion of this s/g is also reflected back.
- ⇒ In this way the wave gets (or) moves back & forth.
- ⇒ The measurement of the wave which is absorbed by the source & reflected back to the Rx (vice versa) can be measured by using "Reflection coefficient".
- ⇒ Reflection coefficient is represented by Gamma (Γ).

$$\text{Reflection coefficient } (\Gamma) = \frac{\text{wave reflected}}{\text{wave incident}}$$

- ⇒ The amount of reflected wave at the load will be different from the amount of wave absorbed by the load.

$$\text{Ref Coefficient, } \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

where, Z_L = load impedance of Rx.

Z_0 = Characteristic impedance.

$$\text{III by } \Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}$$

where, Z_S = load impedance of Tx.

Z_0 = characteristic impedance.

For a non-conducting media, ^{Ex:-} [atmosphere doesn't conduct charges (or) current]

$$Z_0 = \sqrt{\mu/\epsilon}$$

where, μ = permeability of free space.

ϵ = dielectric constant.

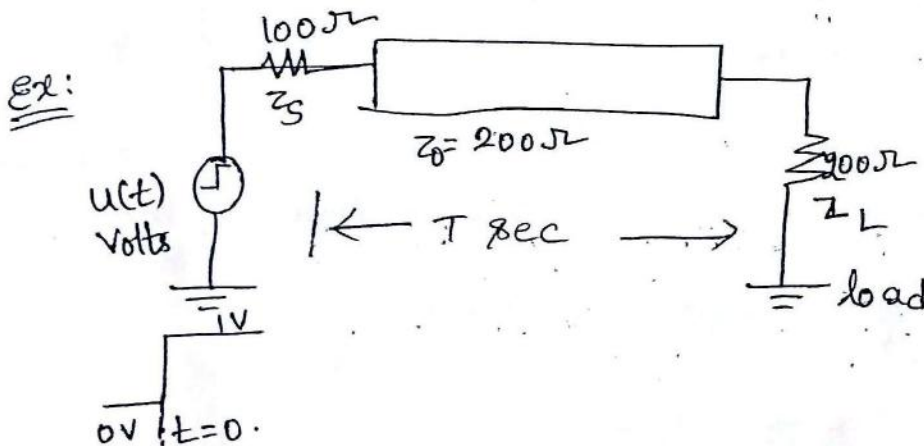
\Rightarrow For typical mediums, $Z_0 = \sqrt{\mu/\epsilon} \simeq \sqrt{\frac{\mu_0}{\epsilon_0}}$

\Rightarrow For a conducting medium,

$$Z_0 \simeq \sqrt{\frac{L'}{C'}}$$

where, $L' \rightarrow$ Inductance per meter.

$C' \rightarrow$ Capacitance per meter.



* Let $Z_s = 100 \Omega$

$Z_L = 900 \Omega$

$Z_0 = 200 \Omega$

Distance = T sec.

$$= 4/9 V.$$

Match.

To find out the voltage at a given time at a given location, the sum of all these voltage waves has to be considered.

⇒ Let us say at time T ,

$$2/3 + (-2/9) = \frac{6-2}{9} = 4/9 V.$$

⇒ At the source side, at time $2T$,

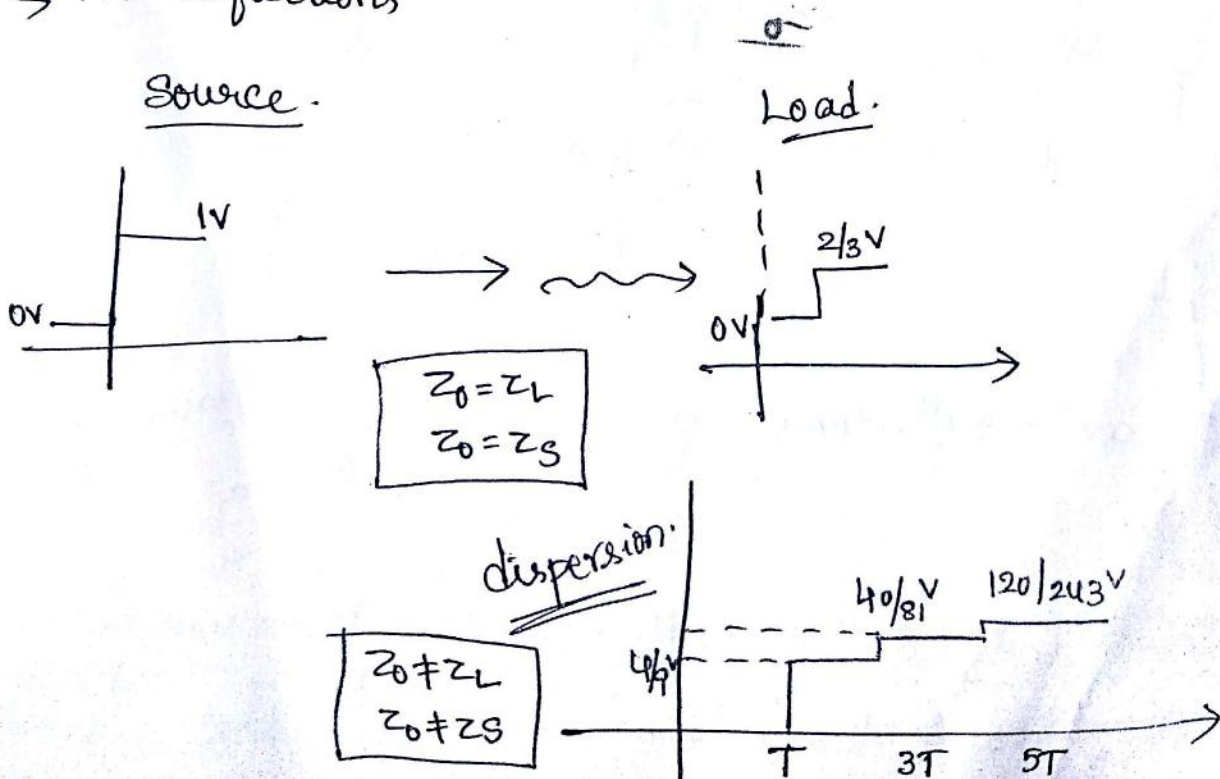
$$2/3 - 2/9 + 2/27 = 14/27 \text{ volts.}$$

⇒ At the load side at time $3T$,

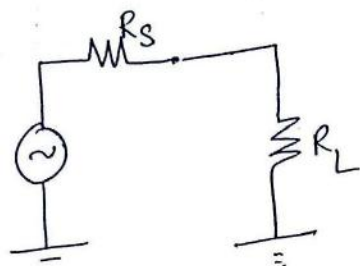
$$\frac{14}{27} - 2/81 = 40/81 V.$$

* Transmission media usually has losses.

→ No reflections

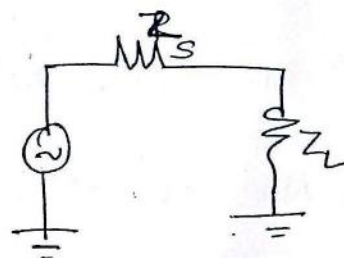
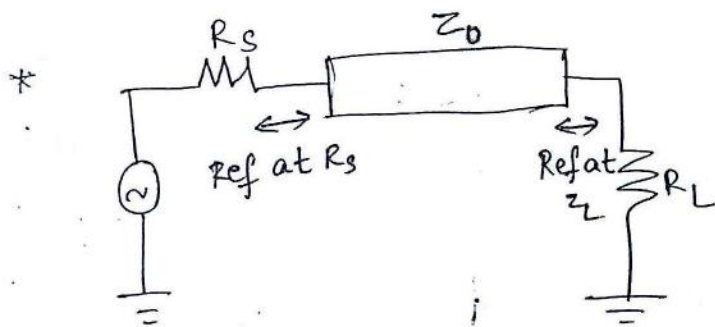


Maximum power Transfer Theorem:-



- * The source resistance is an integral part of the source.
- * The maximum power can be transferred if $R_L = R_s$.
- * The source resistance can't be separated from the voltage source.
- * If the impedances are considered then,

$$\underline{Z_L = Z_s^*}$$

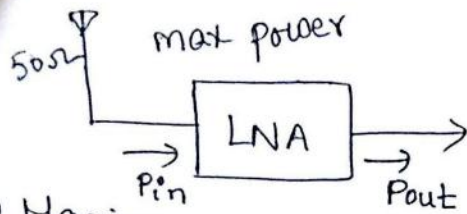


\Rightarrow To avoid reflections, $\left. \begin{matrix} R_L = Z_0 \\ R_s = Z_0 \end{matrix} \right\} R_s = R_L = Z_0$.

\Rightarrow Whenever there are no reflections then there will be maximum power transmission.

Matching.

for the RF s/m ,



⇒ Maximum power transferred to the LNA indicates that there are no reflections.

$$\Rightarrow P_{out} = P_{in} \times \text{Power gain}$$

↑ maximized ↑ maximized

⇒ The i/p power to the antenna will be very small.

0 dBm → 1 mW (power)

-100 dBm → 10^{-10} mW
(GSM phone) 0.1 pW.

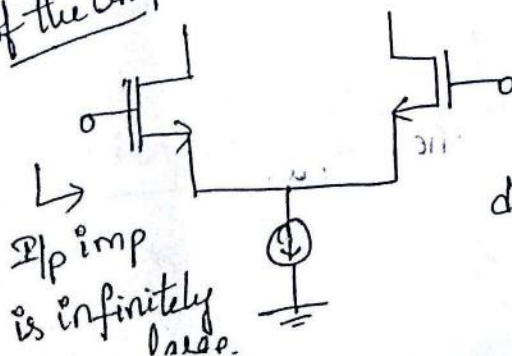
⇒ If the antenna is of 50Ω then,

$$10^{-13} \text{ W} = \frac{V_{50\Omega}^2}{50\Omega}$$

$$V^2 = 5 \times 10^{-12}$$

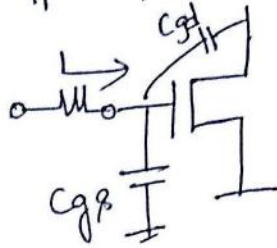
$$V \simeq 2.2 \times 10^{-6} \text{ V}$$

P_{ip} of the amp.

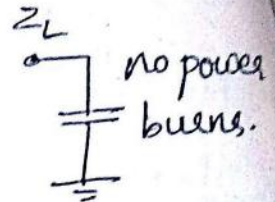


differential mode half ckt of an amp.

- * The i/p impedance is primarily capacitive,

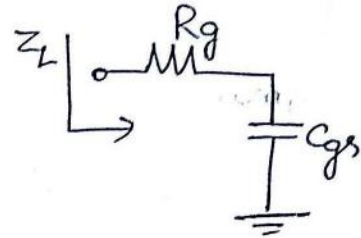


- * If the i/p is capacitive then the power can't burn.



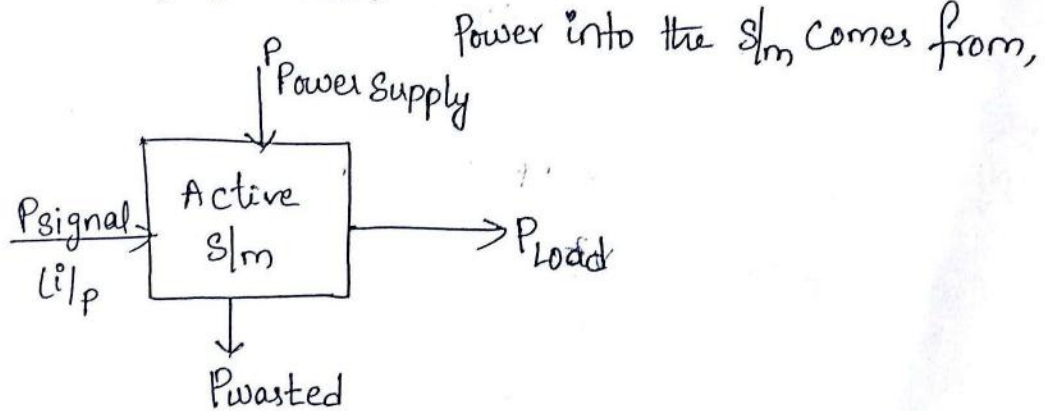
- * There will be a series resistance associated with the gate.

- * R_g burns the power. (i/p power).



- * power gain = $\frac{P_{out}}{P_{in}}$.

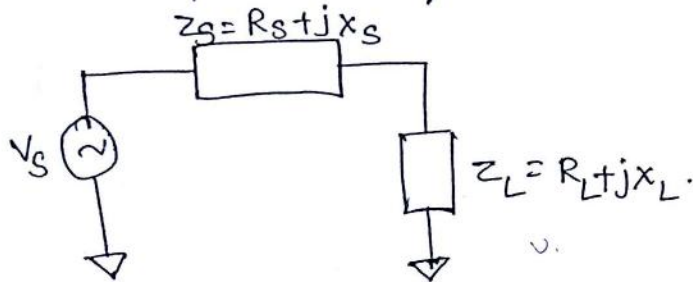
- * Let us consider, Active CKT, (MOSFET).



$$\text{Power Gain} = \frac{P_{load}}{P_{signal}}, \quad \underline{P_{load}} + P_{wasted} = \underline{P_{signal}} + P_{power\ supply}$$

for the P.G to be more than one, $\underline{P_{load}} > \underline{P_{signal}}$.

for Maximum power transfer theorem,



The power delivered to the load is,

$$\frac{|V_R|^2}{R_L} = \frac{R_L |V_s|^2}{(R_L + R_s)^2 + (X_L + X_s)^2}$$

where, V_R & V_s are the rms voltages across the load resistance & source.

Passive RLC circuits:-

* The RF circuits has relatively large ratio of passive components rather than the active components.

* Under passive RLC ckt's we come under,

→ Series RLC circuits

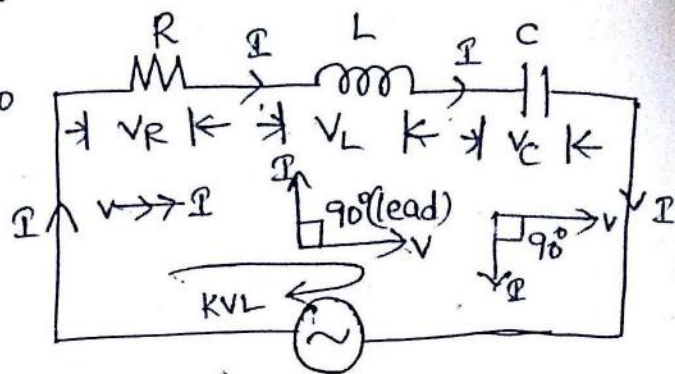
→ parallel RLC circuits

→ Quality factor (or) Q-factor.

(1) Series RLC circuit:-

* The applied voltage to the ckt is,

$$V = V_m \sin \omega t.$$



* As a result of this voltage $V = V_m \sin \omega t$ an alternating current ' i ' is generated.

* V = rms value of applied voltage.

* I = rms value of applied current.

* $V_R \rightarrow$ voltage across the resistor = IR .

* $V_L \rightarrow$ voltage across the inductor = IX_L .

* $V_C \rightarrow$ voltage across the Capacitor = IX_C .

\rightarrow rms quantities.

* Apply KVL to the ckt in vector (or) phasor form,

$$V = V_R + V_L + V_C \rightarrow \textcircled{1}$$

As V_L & V_C are driving in opposite directions,

$$V^2 = V_R^2 + (V_L - V_C)^2.$$

$$V^2 = (IR)^2 + (IX_L - IX_C)^2.$$

$$V^2 = I^2 [R^2 + (X_L - X_C)^2] \rightarrow \textcircled{2}$$

$$\therefore V = I \sqrt{R^2 + (X_L - X_C)^2}$$

Hence the current,

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} \rightarrow (3)$$

Since we know that, $I = V/Z$.

$$\text{Now, } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2} \rightarrow (4)$$

\therefore The resultant reactance is,

$$X = (X_L - X_C) = (\omega L - 1/\omega C).$$

Resonant Frequency

* The maximum peak frequency at which the inductance and Capacitance are equal.

\Rightarrow Hence the resonant frequency of the series RLC ckt is,

$$(\omega_0 L - 1/\omega_0 C) = 0.$$

$$\omega_0 L = 1/\omega_0 C.$$

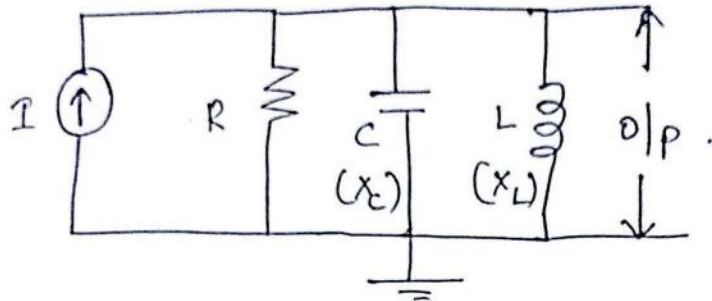
$$\omega_0^2 LC = 1$$

$$\omega_0^2 = 1/LC.$$

$$\therefore \boxed{\omega_0 = 1/\sqrt{LC}} \rightarrow (5)$$

(2) parallel RLC circuit:-

* The parallel RLC circuit is also called as "Tank circuit".



* The admittance of the tank ckt is,

$$Y = G + j\omega C + 1/j\omega L \rightarrow (1)$$

$$\therefore Y = G + j(\omega C - 1/\omega L) \rightarrow (2)$$

- * Capacitor Property \rightarrow it allows A.C & blocks D.C
- * Inductor Property \rightarrow as it allows D.C & blocks A.C.
- * At D.C (or) at low frequencies the Capacitor acts as open circuit.
- * Hence the admittance increasing levels depends on the inductor which plays an important role.
- * At high frequencies the inductor acts as open circuit.
- * Hence the admittance increasing levels depends on the Capacitor which plays an important role.

Matching:-

The resonant frequency for the parallel RLC circuit is,

$$\omega_0 C - 1/\omega_0 L = 0.$$

$$\omega_0 C = 1/\omega_0 L$$

$$\omega_0^2 LC = 1$$

$$\omega_0^2 = 1/LC.$$

$$\therefore \boxed{\omega_0 = 1/\sqrt{LC}} \rightarrow (3)$$

* Example:-

If $L = 1 \text{ nH}$ & $C = 1 \text{ pF}$,

$$\omega_0 = \frac{1}{\sqrt{1 \times 10^{-9} \times 1 \times 10^{-12}}}$$

$$\omega_0 = 5 \times 10^9$$

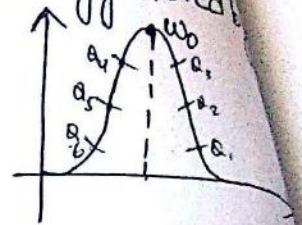
$$\therefore \omega_0 = 5 \text{ GHz}.$$

* It is an excellent transmission of frequency for the signal to be transmitted.

Note:- At resonance the frequency is purely real and is equal to G by the cancellation of reactive terms (inductors and capacitors).

(3) Quality Factor (Q):-

- * The quality factor is defined as at a given frequency the signal what will be the amount of energy stored it to its average power delivered.



$$Q = \omega \cdot \frac{\text{Peak Energy stored}}{\text{Avg power delivered}} \rightarrow \textcircled{i}$$

- * Quality factor is dimensionless. $\left[Q = \frac{\omega}{1/s} \cdot \frac{\text{Energy } J}{\frac{J}{s} \rightarrow \text{Power}} = \frac{J/s}{J/s} \right]$
- * Quality factor can be applied for both resonance & non-resonance sys.
- * A high order systems may exhibits multiple resonances, each with its own peak 'Q' value.

(a) parallel RLC ckt:-

- \Rightarrow Let the resonant frequency as ' ω_0 '.
- \Rightarrow The voltage across the network is $I_{in} \cdot R$.
- \Rightarrow The peak energy stored in either the Capacitance (or) inductor is equal to the energy stored in the network at any given time.
- \Rightarrow The peak Capacitance voltage at the resonance is $I_{pk} R$.

The ^{total} energy stored is,

$$E_{\text{tot}} = \frac{1}{2} C V^2$$

$$E_{\text{tot}} = \frac{1}{2} C (I_{\text{PK}} R)^2 \rightarrow (2)$$

\Rightarrow The average power dissipated by the resistor at resonance is,

$$P_{\text{avg}} = \frac{1}{2} I_{\text{PK}}^2 R \rightarrow (3)$$

$$\left[\begin{array}{l} P = VI \\ P = I \cdot (IR) \\ P = I^2 R \end{array} \right]$$

\therefore The quality factor (Q) of the n/w at resonance is,

Substitute eq (2) & (3) in eq (1),

$$Q = \omega_0 \cdot \frac{E_{\text{tot}}}{P_{\text{avg}}}$$

$$\therefore Q = \frac{1}{\sqrt{LC}} \cdot \frac{\frac{1}{2} C (I_{\text{PK}})^2 (R)^2}{\frac{1}{2} I_{\text{PK}}^2 \cdot R} \quad \text{or } Q = \omega_0 \cdot CR$$

$$Q = \frac{RC}{\sqrt{LC}}$$

$$Q = \frac{R \cdot \sqrt{C} \cdot \sqrt{C}}{\sqrt{L} \cdot \sqrt{C}}$$

$$\therefore \boxed{Q = \frac{R}{\sqrt{4/c}}} \rightarrow (4) \Rightarrow \boxed{Q = \frac{R}{Z_0}} \text{ Char imp}$$

* Hence as the resistance \uparrow quality factor \uparrow .

As the value of $\sqrt{4/c} \downarrow$ the quality factor \uparrow .

* The Quality factor is also equal to the magnitude of the Capacitive & inductive reactances at resonance.

$$\therefore |Z_C| = |Z_L| = \omega_0 L = \frac{L}{\sqrt{LC}} = \sqrt{L/C}$$

*
$$Q = \frac{R}{|Z_{LC}|} = \frac{R}{\omega_0 L} = \omega_0 R C$$

(b) Series RLC circuit:-

=> Let the resonant frequency is ω_0 .

=> The current across the network is $\frac{V_{in}}{R}$.

=> The peak current stored in capacitor (or) inductor is the energy stored in the n/w at any time.

=> The peak inductance at the resonance is; $\left(\frac{V_{PK}}{R}\right)$.

\therefore peak energy stored by the n/w is,

$$E_{tot} = \frac{1}{2} L(I)^2$$

$$E_{tot} = \frac{1}{2} L \left(\frac{V_{PK}}{R} \right)^2 \rightarrow \textcircled{5}$$

=> The avg power dissipated by the resistor at resonance is,

$$P_{avg} = \frac{1}{2} \frac{V_{PK}^2}{R} \rightarrow \textcircled{6}$$

$$\left[\begin{array}{l} P = VI \\ P = V \cdot V/R \\ P = V^2/R \end{array} \right]$$

The Q-factor of the n/w at resonance

$$Q = \omega_0 \cdot \frac{E_{\text{tot}}}{P_{\text{avg}}}$$

$$Q = \frac{1/\sqrt{LC}}{\frac{1/2 L (V_{PK})^2 (1/R)}{1/2 \cdot (V_{PK})^2 (1/R)}}$$

$$Q = \frac{1/\sqrt{LC}}{L/R}$$

$$Q = \frac{\sqrt{L} \cdot \sqrt{R}}{\sqrt{L} \sqrt{C} R}$$

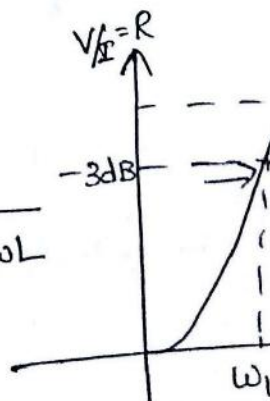
$$\therefore \boxed{Q = \frac{\sqrt{4C}}{R}} \rightarrow \textcircled{7}$$

Bandwidth of Q-factor:-

* For the ^{parallel RLC} n/w, the voltage developed is 'V' developed across $V = I \cdot \frac{1}{1/R + j\omega C + 1/j\omega L}$ RLC parallel

$$V = I \cdot \frac{j\omega LR}{R + j\omega L - \omega^2 LCR}$$

$$V = I \cdot \frac{j\omega LR}{R(1 - \omega^2 LC) + j\omega L}$$



* The frequency at which the response is 3dB below the maximum

f_{eq} is the bandwidth.

$$\therefore \omega_2 - \omega_1 = \frac{1}{Q} \cdot \omega_0$$

$$\therefore \boxed{B.W = \frac{1}{Q} \omega_0}$$

Branch currents at Resonance:-

- * At resonance the voltage across the r/w is $I_{in} R$.
- * Since the inductive & capacitive reactances are equal at resonance.
- * The inductance & capacitance branch currents will be equal in magnitude.

$$|I_L| = |I_C| = \frac{|V|}{Z} = \frac{|I_{in}| R}{\omega_0 L}$$

$$= \frac{|I_{in}| R \cdot \sqrt{LC}}{L} \quad [\because \omega_0 = 1/\sqrt{LC}]$$

$$= |I_{in}| \cdot \frac{R}{\sqrt{L/C}} \quad [\because R/\sqrt{L/C} = Q \text{ for Series RLCckt}]$$

$$\therefore \boxed{|I_L| = |I_C| = Q \cdot |I_{in}|}$$

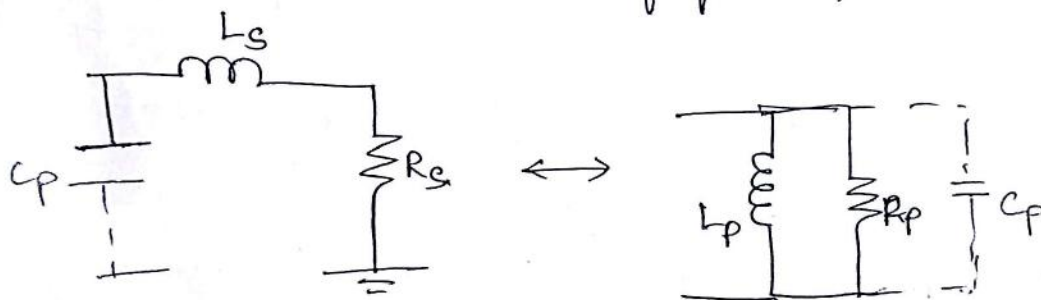
- * The current flowing in the inductive & capacitive branches is Q times as large as the net current flowing through the circuit.

Other Resonant RLC N/w's:-

Series to parallel conversions.

→ In the sense making the impedances of the series to be equal to the parallel.

→ This can't be possible at all the frequencies, it is only applicable for certain selected freq, and for resonant freq.



RL CKTs.

$$\Rightarrow \text{Resonant freq, } \omega_0 = \frac{1}{\sqrt{L_p C_p}} \rightarrow (1)$$

$$\text{For parallel ckt, } Q = \frac{R_p}{\sqrt{L_p / C_p}} \rightarrow (2)$$

$$Q = \frac{R_p}{\omega_0 L_p} = \frac{R_p}{\frac{1}{\sqrt{L_p C_p}} L_p} = \frac{R_p}{\frac{L_p}{\sqrt{L_p C_p}}} = \frac{R_p}{\sqrt{L_p / C_p}}$$

$$\left[\frac{R_p}{\frac{1}{\sqrt{L_p C_p}} \cdot L_p} = \frac{R_p}{\frac{L_p}{\sqrt{L_p C_p}}} \right]$$
$$\left[\frac{R_p}{\sqrt{L_p / C_p}} \right]$$

The i/p impedance,

$$Z_{imp}(P) = \frac{1}{\frac{1}{R_p} + \frac{1}{j\omega_0 L_p}} \rightarrow (4)$$

freq at which we want the
sig to be equal

$$= \frac{j\omega_0 L_p R_p}{j\omega_0 L_p + R_p}$$

$$= \frac{j\omega_0 L_p R_p (R_p - j\omega_0 L_p)}{(R_p + j\omega_0 L_p)(R_p - j\omega_0 L_p)} = \frac{j\omega_0 L_p R_p (R_p - j\omega_0 L_p)}{R_p^2 + \omega_0^2 L_p^2}$$

$$= \frac{j\omega_0^2 L_p^2 R_p + j\omega_0 L_p R_p^2}{R_p^2 + \omega_0^2 L_p^2} \checkmark \rightarrow (5)$$

We know that, $Q = \frac{R_p}{\omega_0 L_p} \Rightarrow Q^2 = \frac{R_p^2}{\omega_0^2 L_p^2}$

$$\Rightarrow 1 + Q^2 = 1 + \frac{R_p^2}{\omega_0^2 L_p^2}$$

$$\Rightarrow 1 + Q^2 = \frac{\omega_0^2 L_p^2 + R_p^2}{\omega_0^2 L_p^2}$$

$$\therefore \boxed{\frac{1}{1+Q^2} = \frac{\omega_0^2 L_p^2}{\omega_0^2 L_p^2 + R_p^2}} \rightarrow (6)$$

$$Z_{imp}(P) = \frac{R_p}{1+Q^2} + j \cdot \frac{\omega_0 L_p R_p^2}{\omega_0^2 L_p^2 + R_p^2} \cdot \frac{1}{1+Q^2}$$

$$\therefore Z_{imp}(P) = \frac{R_p}{1+Q^2} + j \frac{\omega_0 L_p Q^2}{1+Q^2} \rightarrow (7)$$

For Series ckt,

$$Z_{imp}(S) = R_s + j\omega_0 L_s \rightarrow (8)$$

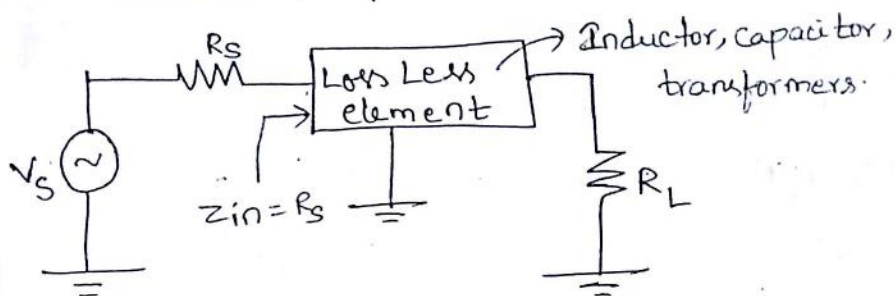
$$\boxed{R_s = R_p / (1+Q^2)} \quad \& \quad \boxed{L_s = L_p \left(\frac{Q^2}{1+Q^2} \right)}$$

} equating both the
eqs.

Matching:-

Unit - I

- * In order to get better performance of the s/g's, we need to make the load impedance to the characteristic imp.
- * The source resistance should be matched to the char imp
- * In order to get maximum power to be transferred the load resistance is equal to the source resistance.



⇒ In order to get the load resistance equal to the i/p impedance (or) source ($R_L = R_s$) the transmission media element should be a lossless element.

↓
[Any element that doesn't consume power]

* The designing of these lossless networks is called as "Matching n/w's".

* There are three types of matching n/w's. They are,

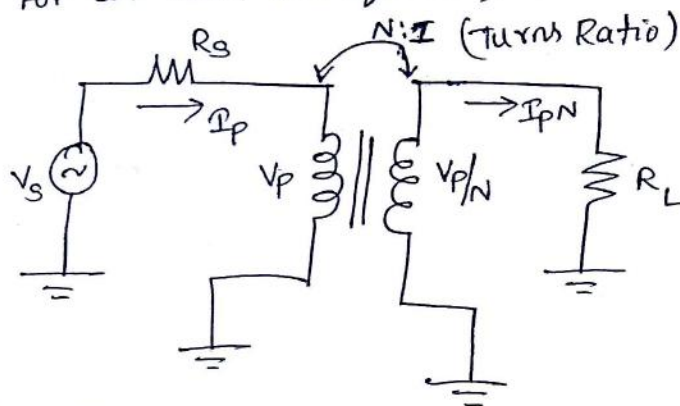
→ L-Match.

→ π -Match.

→ T-Match.

H) Match

For an ideal transformer,



* The i/p impedance across the primary is,

Primary vol = V_p , Sec vol = V_p/N

Current = I_p , Sec current = $I_p N$.

\therefore i/p impedance, $Z_{in} = \frac{V_p}{I_p}$ of primary

From Secondary i/p
turns, ~~turns~~,

$$\frac{V_p}{N} = I_p N R_L$$

$$\left\{ \frac{V_p}{I_p} \right\} = N^2 R_L$$

$\rightarrow Z_{in}$

$\therefore Z_{in} = N^2 R_L$

\therefore $N = \sqrt{\frac{Z_{in}}{R_L}}$

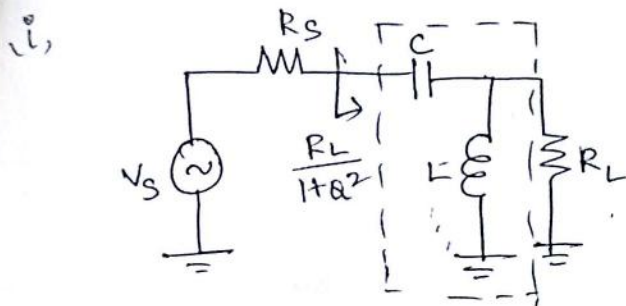
* $Z_{in} = R_s \Rightarrow N = \sqrt{R_s/R_L}$, but an ideal transformer can't be so easily obtained.

L-Match:-

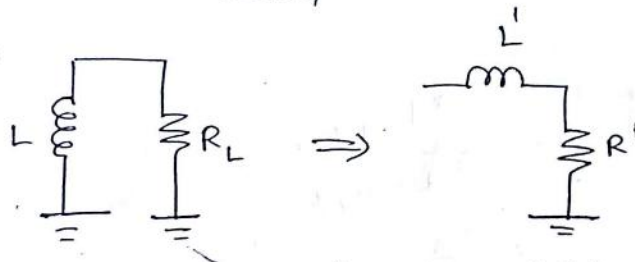
⇒ At all frequencies $R_L \neq R_S$.

⇒ At some discrete freq's say $f_1, f_2 \dots$ we can make $R_L = R_S$.

⇒ The easiest η/w that can be possible is the "Matching η/w ".



⇒ At ω_0 ,



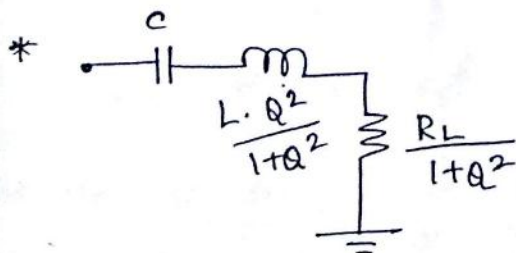
* The parallel inductor is transformed into the series inductor.

* These relationships are,

$$L' = L \frac{Q^2}{1+Q^2}$$

$$R' = \frac{R_L}{1+Q^2}$$

$$\left[\begin{array}{l} \therefore L_S = L_P \left(\frac{Q^2}{1+Q^2} \right) \\ R_S = \frac{R_P}{1+Q^2} \end{array} \right]$$



* At ω_0 if capacitor is placed then the capacitor resonance matches with the inductor resonance.

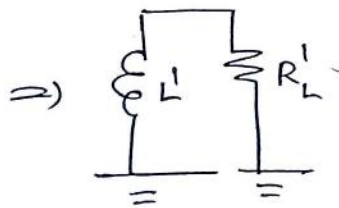
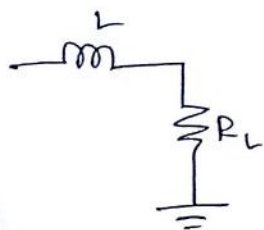
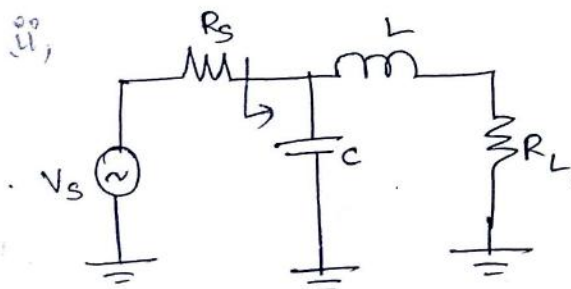
* pick the value of 'c' such that,

$$\omega_0^2 = \frac{1}{c \cdot L \frac{Q^2}{1+Q^2}}, \quad \omega_0^2 = \frac{1+Q^2}{CLQ^2}$$

$$\text{At } \omega_0, Z_{in} = \frac{R_L}{1+Q^2} = R_S.$$

$$\Rightarrow Q = \sqrt{R_L/R_S - 1}$$

\therefore When $R_L > R_S$ the Quality factor is a real part.



Series Imp converted parallel Imp.

$\left[\begin{array}{l} \therefore R_{eq} \rightarrow \text{large} \\ \quad \quad \quad \rightarrow \text{parallel} \\ R_{eq} \rightarrow \text{small} \\ \quad \quad \quad \rightarrow \text{series} \end{array} \right]$

* The relationships are,

$$R_L' = (1+Q^2) R_L$$

$$L' = \left(\frac{1+Q^2}{Q^2} \right) L.$$

At ω_0 , $R_L' = (1 + Q^2) R_L$

$$R_L' = (1 + Q^2) R_L = R_S.$$

$$\therefore (1 + Q^2) R_L = R_S.$$

$$\therefore R_L + R_L Q^2 = R_S$$

$$\Rightarrow R_L Q^2 = R_S - R_L.$$

$$Q^2 = \frac{R_S - R_L}{R_L}.$$

$$\therefore \boxed{Q = \sqrt{R_S/R_L - 1}}$$

* When $R_S > R_L$ the quality factor is a real part.

(2) π -Match:-

* For the L-Match it uses two components for transformation.

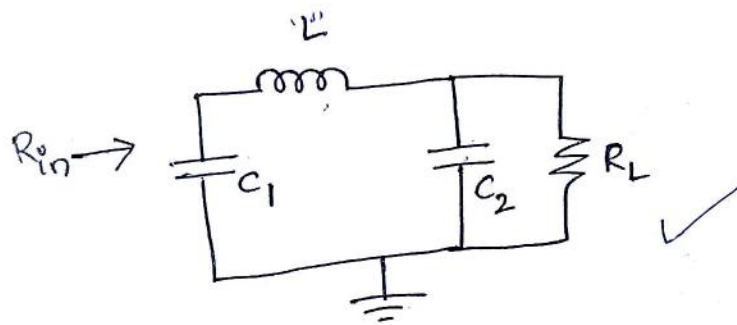
\rightarrow In the case of π -match instead of two components we use 3 component transformation.

* The L-Match is used for finding the quality factor individually.

\rightarrow In case of π -match along with the quality factor, the inductance & capacitance can be calculated.

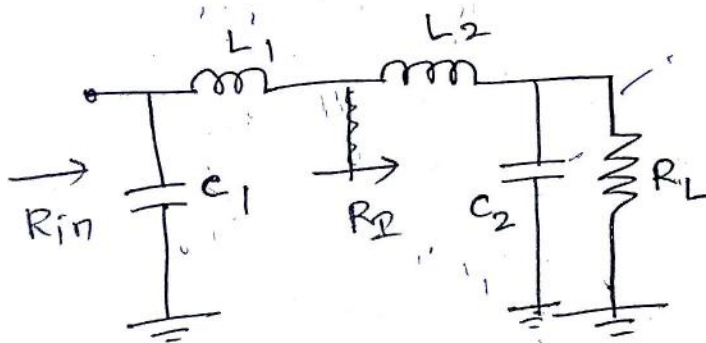
* The π -match is a Cascade of two L-match networks.

⇒ As it consists of two inductors & two capacitors, it can be easier to specify the center freq, quality factor & overall impedance transformation ratio.



T-Match N/w.

* Split the inductance into two parts.



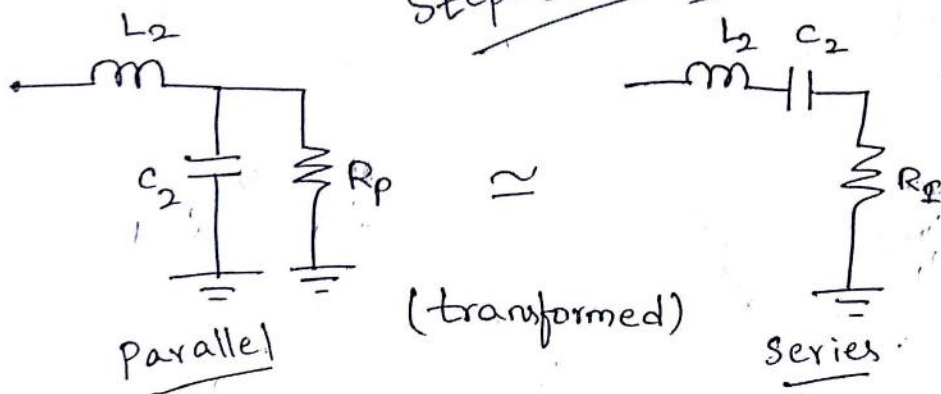
T-Match as Cascade of L-Matches.

* Two L-matches are connected in cascade so that one transforms the resistance down and the other transforms the resistance to be up.

* This is explained by the transformations of parallel to the series & series to the parallel.

a) parallel to series:-

Step-down transformation



* When parallel n/w is converted to series n/w then its resistance decreases.

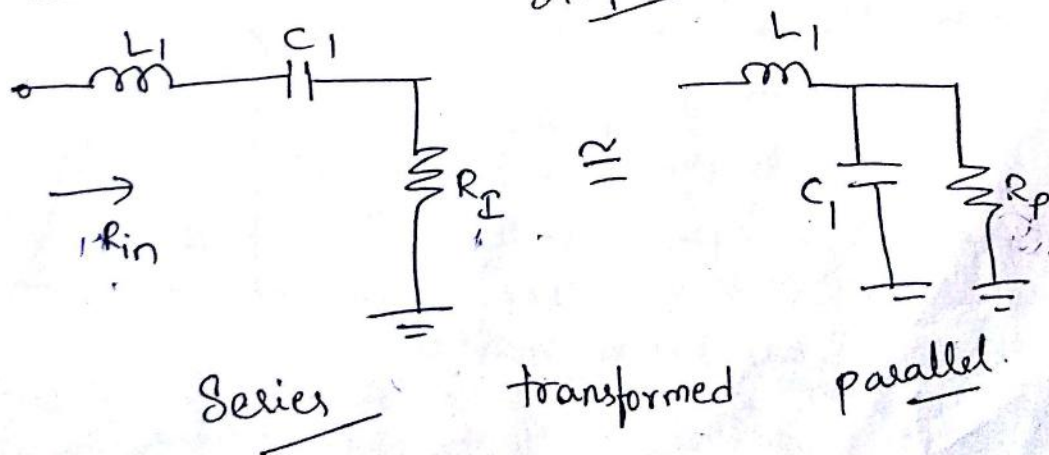
* When parallel n/w is converted to series n/w then its Capacitance increases.

\therefore Hence the Q-factor for the right hand side L-section is,

$$\frac{\omega_0 L_2}{R_P} = \sqrt{\frac{R_P}{R_I} - 1} = Q_{\text{right}}$$

(b) Series to parallel:-

Step-up transformation



- * When Series n/w is converted into parallel n/w its resistance \uparrow .
- * When Series n/w is converted into parallel n/w its inductance also increases.

\therefore Hence the Q-factor of the left hand side L-section (a) L-Match is,

$$\frac{\omega_0 L_1}{R_I} = \sqrt{\frac{R_{in}}{R_I} - 1} = Q_{Left}.$$

* The overall network Q-factor is,

$$Q = \frac{\omega_0 (L_1 + L_2)}{R_I} = \sqrt{\frac{R_{in}}{R_I} - 1} + \sqrt{\frac{R_p}{R_I} - 1}.$$

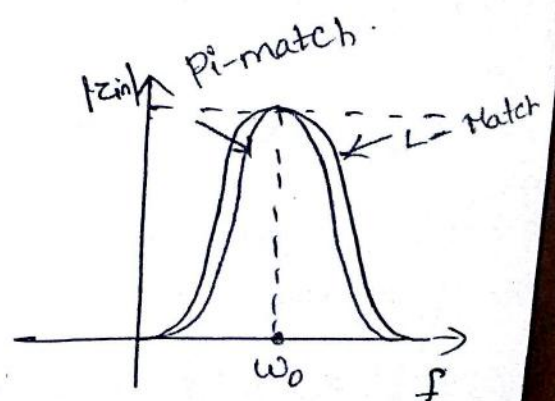
* The Inductance is,

$$L_1 + L_2 = \frac{Q R_I}{\omega_0}.$$

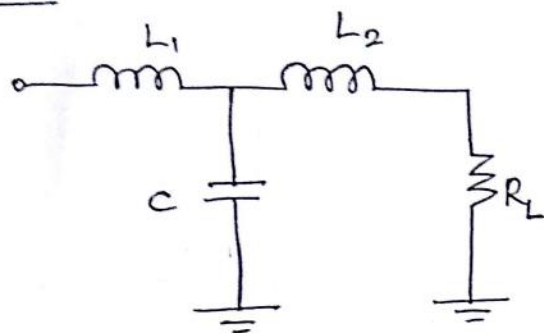
Note:-

* For L-Match $\rightarrow R_L / R_S = 1 + Q^2$
 $\Rightarrow Q = \sqrt{R_L / R_S - 1}.$

* For pi-Match $\rightarrow \frac{R_L}{R_S} = \frac{1 + Q_2^2}{1 + Q_1^2}$
 If $Q_2 \neq Q_1$ are larger than 1,
 $\Rightarrow \frac{Q_2^2}{Q_1^2} = R_L / R_S.$
 $\therefore Q_2^2 = Q_1^2 R_L / R_S$

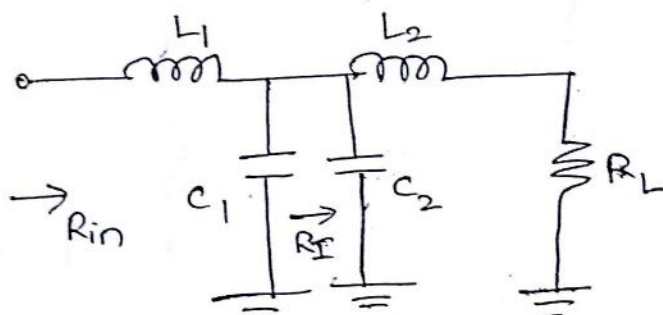


Match:-

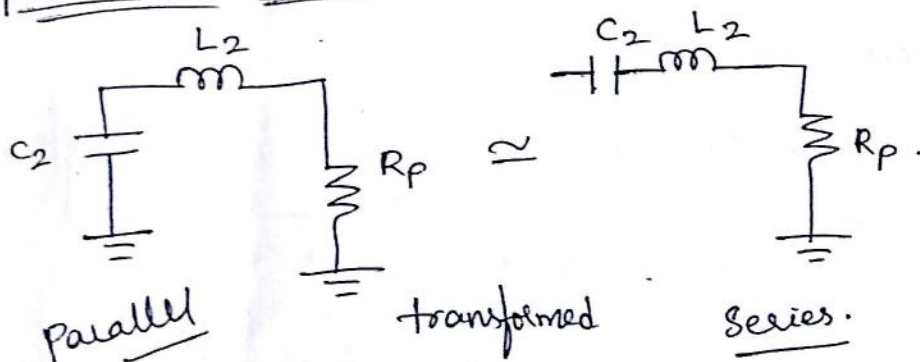


T-Match N/w.

* The Capacitor is splitted into two Capacitances.



(a) parallel to Series:-



∴ The value of the capacitance is,

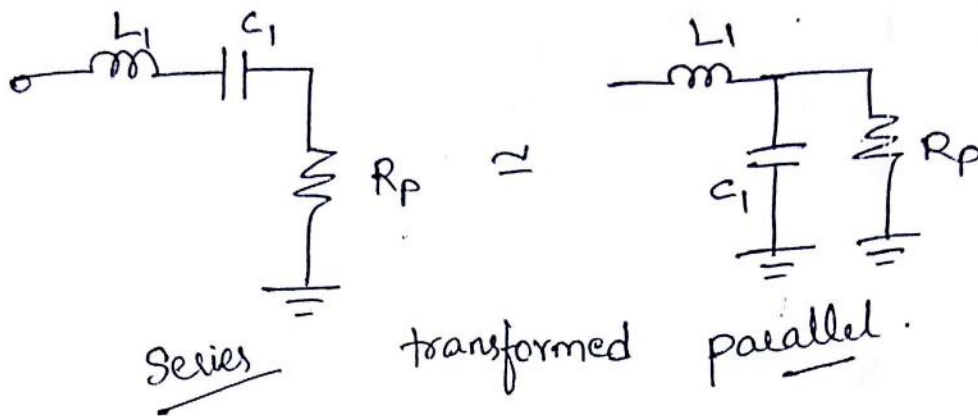
$$C_2 = \frac{Q_{\text{right}}}{\omega_0 R_p}$$

$$Q_{\text{left}} = \omega_0 C_2 R_{in}$$

$$C_2 = \frac{Q_{\text{right}}}{\omega_0 R_p}$$

$$Q_{\text{right}} = \omega_0 C_2 R_p$$

(b) Series to parallel:-



The Capacitance at right-hand side is,

$$C_2 = \frac{Q_{\text{right}}}{\omega_0 R_{\text{in}}}$$

$$C_1 = \frac{Q_{\text{left}}}{\omega_0 R_{\text{in}}}$$

$$Q_{\text{right}} = C_2$$

$$Q_{\text{left}} = \omega_0 C_1 R_{\text{in}}$$

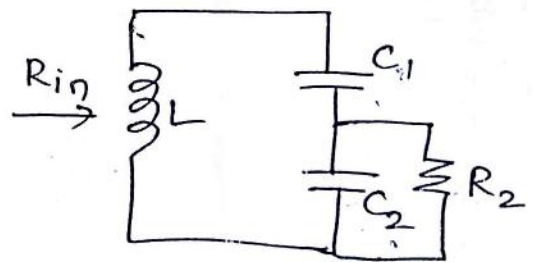
$$\therefore R_{\text{I}} \approx \frac{(\sqrt{R_{\text{in}}} + \sqrt{R_{\text{p}}})^2}{Q^2}$$

Tapped Capacitor:-

$$* Q = \omega_0 R_{\text{I}} (C_1 + C_2)$$

$$\Rightarrow \sqrt{\frac{R_{\text{I}}}{R_{\text{in}}} - 1} + \sqrt{\frac{R_{\text{I}}}{R_{\text{s}}} - 1}$$

$$C_1 + C_2 = \frac{Q}{\omega_0 R_{\text{I}}}, \quad L_1 = \frac{Q_{\text{left}} R_{\text{in}}}{\omega_0}, \quad L_2 = \frac{Q_{\text{right}} R_{\text{s}}}{\omega_0}$$

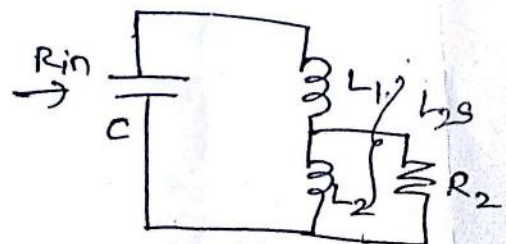


Tapped Inductor:-

$$Q = \omega_0 R_{\text{in}} C \Rightarrow C = \frac{Q}{\omega_0 R_{\text{in}}}$$

$$L_{2\text{S}} = L_2 \left[\frac{Q_2^2}{Q_2^2 + 1} \right]$$

$$R_{\text{S}} = \frac{R_2}{Q_2^2 + 1}, \quad R_{\text{S}} = R_{\text{in}} / Q_2^2 + 1 \Rightarrow Q_2 = \sqrt{\frac{R_2}{R_{\text{in}}} (Q_2^2 + 1) - 1}$$

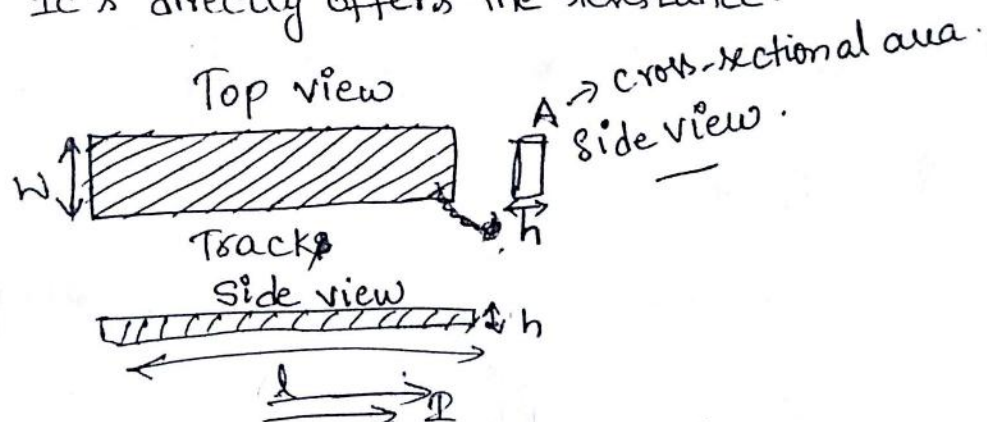


Passive IC components Interconnects:-

- * passive components are the components which doesn't need a power supply.
- * These are linear components.

Ex: Resistor, Capacitor, Inductor, Mutual Inductances, Transformers.

- * Modern resistors are made up of polysilicon material.
- * Metals can also be used for making the resistors.
- * Not all IC's directly offers the resistance.



- * The track has cross sectional area = A .

$$\text{Length} = l.$$

$$\text{height} = h.$$

$$\text{width} = w.$$

$$\therefore \text{Area of the track is } \underline{A = w \cdot h}.$$

- * Let current (I) is propagating through the resistor.

* The resistance $R = \frac{\rho l}{A}$

$\Rightarrow \rho \rightarrow$ resistivity.

$$\boxed{\rho = \frac{R \cdot A}{l}}$$

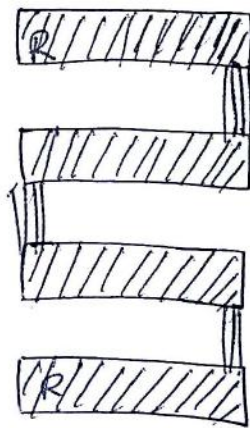
units $= \underline{\underline{\Omega \cdot m}}$

$$\left[\frac{R \cdot A}{l} = \frac{\Omega \cdot m^2}{m} \right]$$

* $\boxed{R = \left(\frac{\rho}{h} \right) \cdot \frac{l}{w}}$

* In this way the resistance can be calculated if the amount of resistance is small (say 100Ω , $200\Omega \dots$)

* If the value of the resistance is large enough then, lot of tracks has to be designed (say $1k\Omega$, $2k\Omega \dots$)

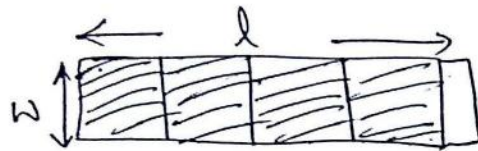


resistivity by making identical tracks n times the resistance of each track.

* Sometimes the value of the 'h' is defined by the designers.

* so, $\left(\frac{\rho}{h} \right)$ will be given in terms of Ω .

The resistance can also be calculated depending upon the no: of squares.



* No: of squares = l/w square.

* What ever may be the width of the track it doesn't matter.

$$\therefore R = \rho' l/w$$

$$\Rightarrow \text{Resistance} = \rho' \Omega/\text{square}$$

$$\therefore \text{Resistivity } (\rho') = R \cdot w/l$$

$$\Rightarrow \rho' = \frac{R \cdot w}{l}$$

$$\therefore \boxed{\text{Resistivity, } \rho' = \Omega/\text{square}}$$

Note:-

* compared to metals, poly silicon is more resistive.

* Poly silicon — Silicided \rightarrow specifically to reduce resistance.

— unilicided

(5 to 10 Ω/square):
 \rightarrow tolerance is poor (35%)

\rightarrow Temp Coeff,

$$TC = \frac{1}{R} \frac{dR}{dT}$$

→ unsilicided poly,

↳ higher resistivity.

→ Better tolerance (50%).

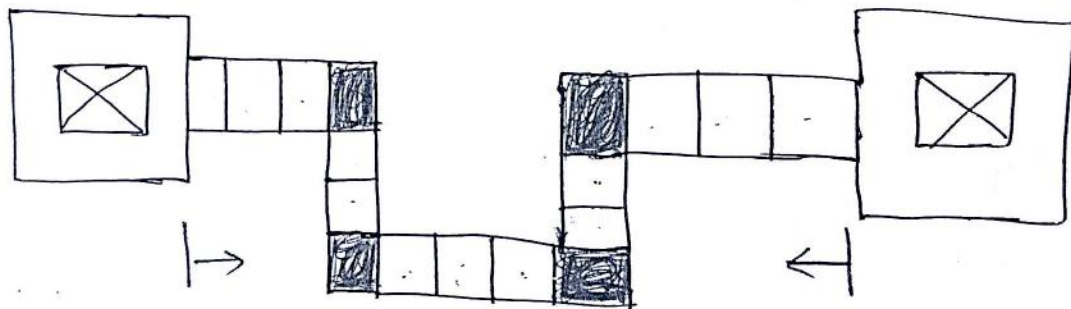
* A MOS transistor can also be used as a resistor.

* With a suitable gate-to-source voltage, a compact resistor can be formed.

* The resistance of a long-channel MOS transistor is,

$$r_{ds} \approx \left[\mu C_{ox} \frac{W}{L} (V_{GS} - V_T) - V_{DS} \right]^{-1}$$

Counting Squares:-



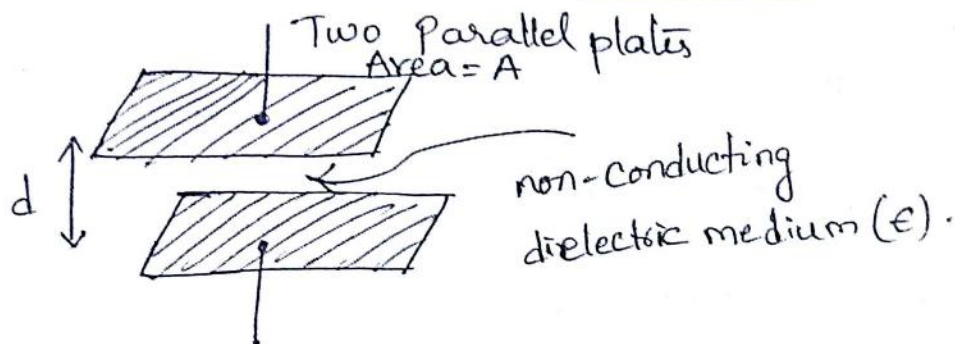
* The squares that are shaded has to be treated as 0.56 square.

* Hence the resistance b/w the two boundaries marked by the arrows is approximately 15.24 squares. $\left[13 \text{ full squares} + 4 \text{ half squares} \right]$
 $[13 + 4 \times 0.56 = 15.24 \text{ squares}]$

$$\begin{array}{r} 13 \\ 2 \cdot 24 \\ \hline 15.24 \end{array}$$

Capacitors

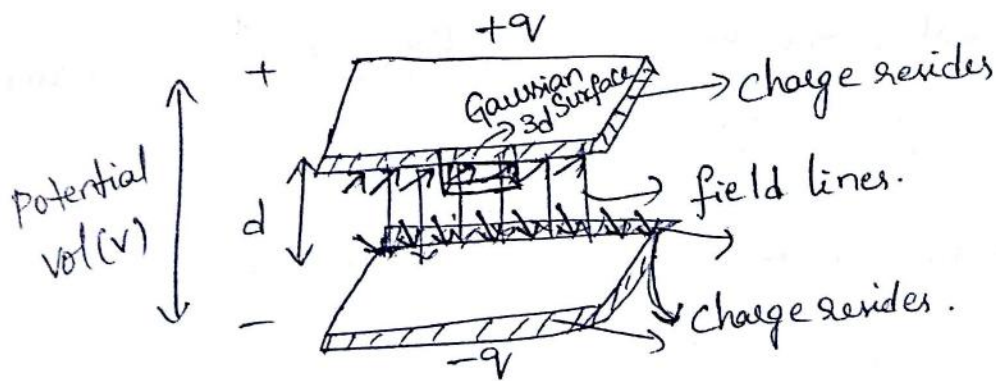
- * Two parallel plates separated by a non-conducting dielectric medium is called a Capacitor.



- * The Capacitance is,

$$C = \frac{\epsilon A}{d}$$

- i) * This is valid for two infinitely large parallel plates.



- * When two plates are kept in parallel with charges as $+q$ & $-q$.
- * Then the charge of $+q$ attracts $-q$.
 $-q$ attracts $+q$.
- * Hence the charge resides on, (Gauss law).

→ Bottom side of the top plate.

→ Top side of the bottom plate.

- * Due to this charge an electric field lines are generated.
- * Since the parallel plates are infinitely large enough the field lines go straight from one plate to the other.

* If area of the surface is A & the electric field is E ,

- * Now a 3d surface is placed in the electric field with an area ' a ' then,

$$\phi E = \frac{(q/A \cdot a)}{\epsilon} \rightarrow \text{charge enclosed.}$$

$$E = \frac{q}{A\epsilon} \rightarrow (1).$$

- * The potential across the plates is, $(q/A\epsilon)$ times the distance.

$$V = \frac{q}{A\epsilon} \cdot d \rightarrow (2).$$

- * The Capacitance is,

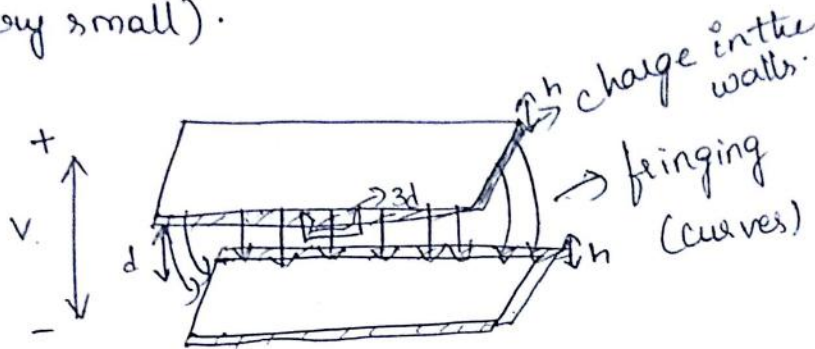
$$CV = q \rightarrow (3)$$

$$\therefore C \left(\frac{q}{A\epsilon} \right) \cdot d = q$$

$$\therefore C = \frac{q \cdot A\epsilon}{q \cdot d}$$

$$\therefore \boxed{C = \frac{\epsilon A}{d}}$$

If the area of the parallel plates is confined (made very small).



- * Whenever the area is confined only some of the field line goes straight.
- * The remaining field lines doesn't go straight.
- * These lines are called fringed lines or curved lines.
- * Now the Capacitance of this confined area parallel plates is,

$$C = \frac{\epsilon A}{d} + \text{fringe capacitance.}$$

fringe Capacitance \propto dielectric constant (ϵ).

inversely \propto distance ($1/d$).

\propto perimeter.

\propto height.

∴

inversely \propto (fuzz factor).

$$\therefore C = \frac{\epsilon A}{d} + \frac{\epsilon (\text{perimeter}) (h)}{d (\text{fuzz factor})}$$

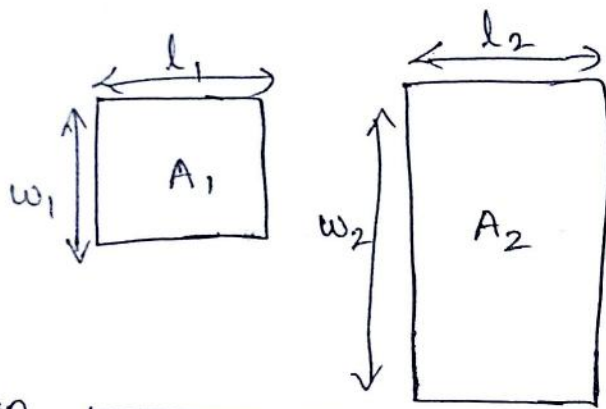
→ effective distance b/w the two plates > the distance b/w the plates.

* If two capacitors are to be matched (equal)

→ Their areas should be equal.

→ Their perimeters also should be equal.

* If one capacitor has to be double times the other capacitor then,



Then,

$$l_2 w_2 = 2 l_1 w_1$$

$$l_2 + w_2 = 2(l_1 + w_1)$$

If for N times is related then,

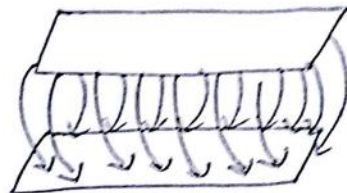
$$l_2 w_2 = N l_1 w_1$$

$$l_2 + w_2 = N(l_1 + w_1)$$

(equal) H_c

Capacitor on an IC:-

⇒ If two parallel plates are placed then the fringes will be more.

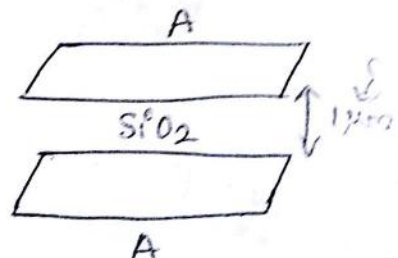


⇒ The distance b/w the two metal layers ^{on an IC} is $0.1 \mu\text{m}$ for 35nm and 28nm processor.

⇒ Its $1 \mu\text{m}$ for 0.13nm processor.

⇒ The material in between the two metal layers in the IC is silicon dioxide (SiO_2)

⇒ The SiO_2 is used to insulate all the diff metal layers.

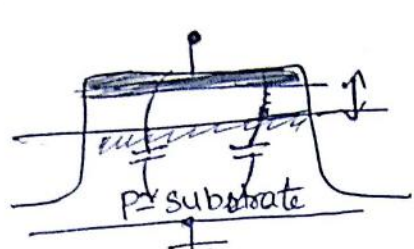


⇒ It has dielectric constant of 3.5 .

⇒ Some of the modern processors they offer some capacitors called MIM Capacitors (special capacitors which has distor b/w the two metals is less).

⇒ MIM — Metal Insulator Metal (Metal strip)

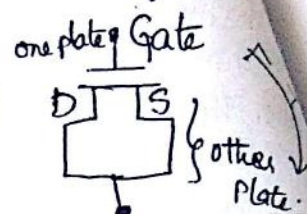
⇒ The other well known capacitor is the gate oxide of the MOSFET



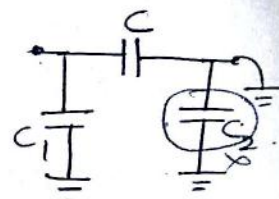
distance
greater density.

* Thickness of the gate oxide is well controlled & very small.

* The vol b/w the two nodes is more than the threshold vol. of the MOSFET. (MOSFET)

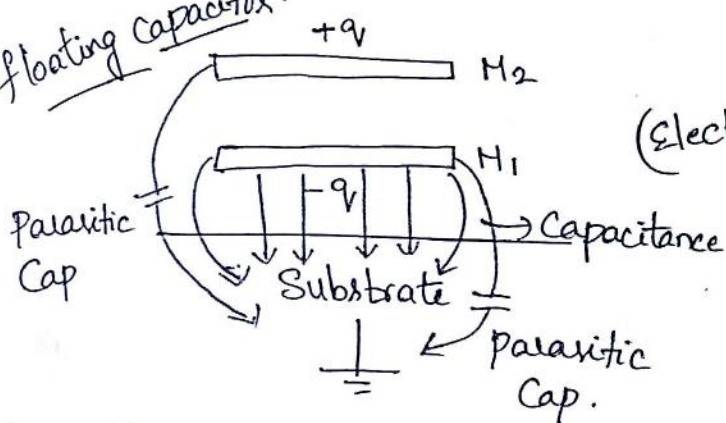


* The parasitic Capacitors are introduced in this MOSFET.

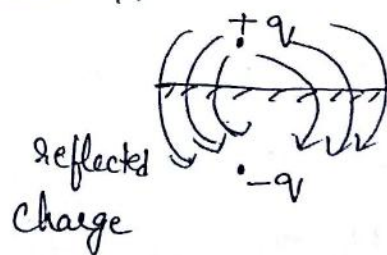


* Decoupling Capacitors \rightarrow large Capacitance can be obtained. One side of the capacitor is grounded.

* floating capacitor.

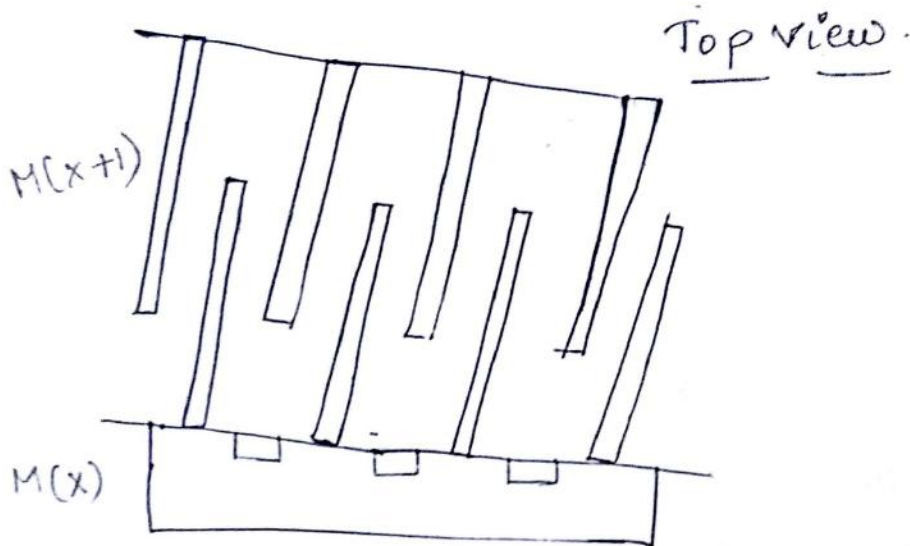


(Electrostatics)

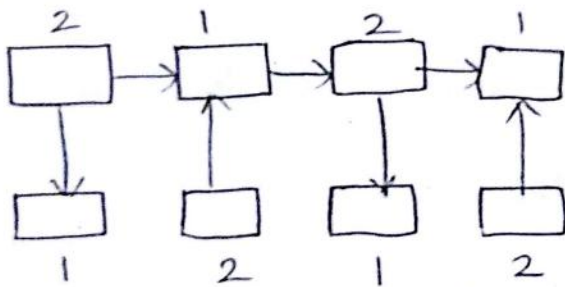


* Choose the metal layers away from the substrate of the gate, plane.

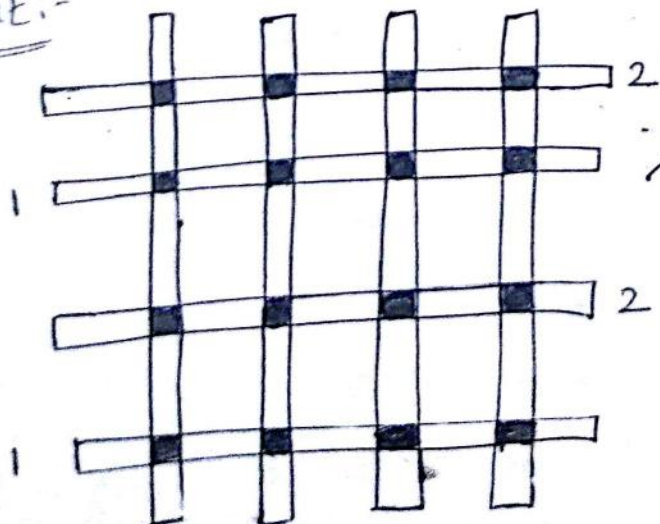
In order to reduce the parasitic capacitances by placing the parallel plates, the arrangement of metals is changed as,



Side view



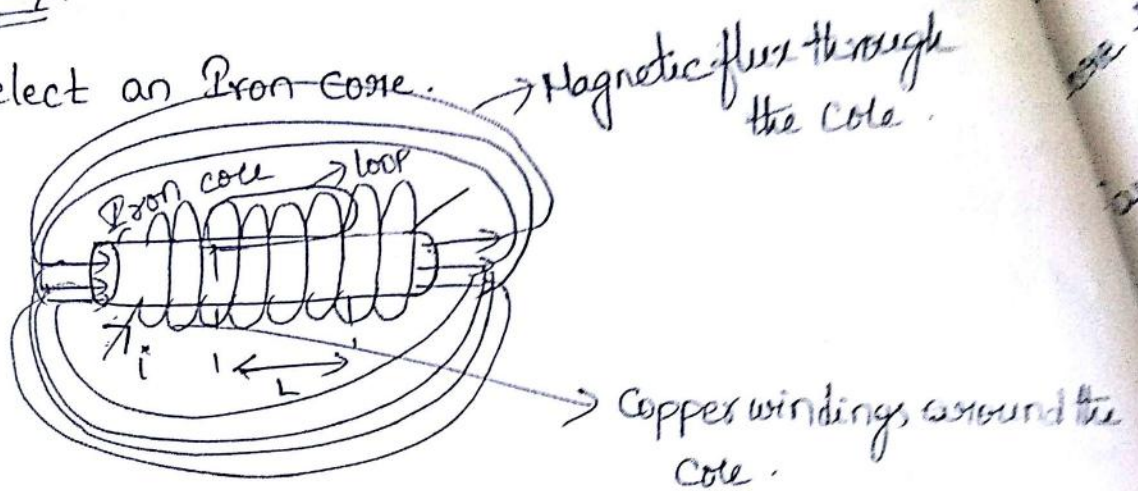
Layout:-



→ This arrangement gives
 → side-to-side capacitance
 → plate to plate capacitance

Inductors:-

- * Select an Iron core.



- * Iron core with a copper windings acts as a solenoid.

- * Let us consider the core has N turns/meter.

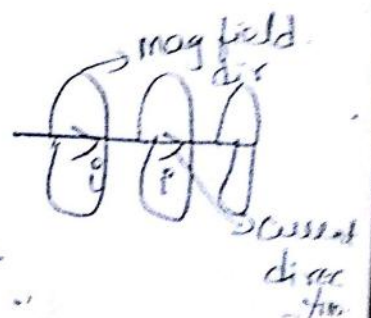
⇒ If we push a current (i) through a core then it creates a magnetic field around the core.

⇒ In order to specify the direction of the magnetic field.

⇒ According to Right hand thumb rule,

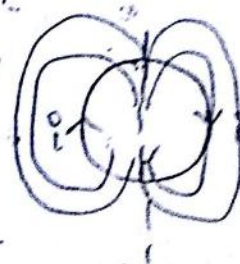
→ Thumb — direction of current.

→ Curl of four fingers — direction of magnetic field.



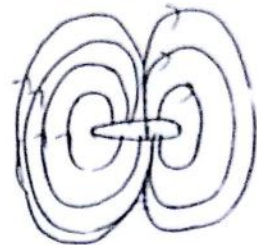
⇒ If current is passing in the circular core then,

→ Current generates individual magnetic flux on the right & left side of the core.



Hence by the side view of the core the dipole magnet is formed.

Side view.



dipole magnet

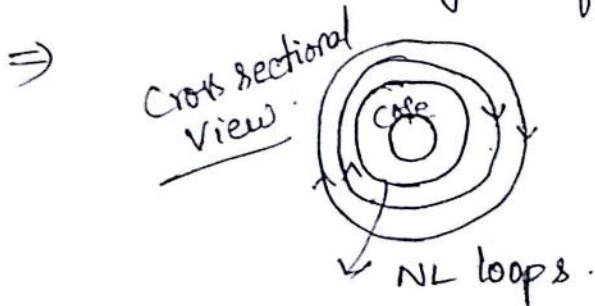
Faraday's Law:-

⇒ According to Faraday's law, change in the magnetic flux induces an potential voltage which opposes the change in this flux.

⇒ Hence the potential drop across the inductor is,

$$\text{Potential drop } \boxed{V(t) = \frac{d\phi}{dt}}$$

where, ϕ = magnetic flux through the solenoid.



⇒ If a core is circulated with the 'N' no: of loops with a current 'i' then each of these loops contributes flux around the loops.

⇒ Hence N L loops are formed.

* Ampere's law

→ In order to calculate exact amount of flux going through the solenoid.

→ The ampere's law with a part of Maxwell's Equation is used.

Maxwell's Equ:-

⇒ If the magnetic field along the loop is considered as l.
Then,

The Integral of the magnetic field around the loop.
= Amount of current enclosed within the loop.

$$\oint B \cdot dl = \frac{\mu_0}{4\pi} i N \oint dl \rightarrow (1)$$

$$\therefore B = \frac{\mu_0}{4\pi} i N \rightarrow (2)$$

If Area of the flux,

$$\phi = B \cdot \pi r^2 \rightarrow (3)$$

Substitute eq (2) in eq (3),

$$\phi = \frac{\mu_0}{4\pi} \cdot i \cdot N \cdot \pi r^2.$$

$$\therefore \boxed{\phi = \frac{\mu_0}{4\pi} i N r^2} \rightarrow (4)$$

This is the amount of flux passing through the solenoid.

The total voltage developed across each of these loops is i_p ,

$$V(t) = \frac{d\phi}{dt} \cdot NL \rightarrow (5)$$

$$\therefore V(t) = \frac{d}{dt} \left[\frac{\mu_0}{4} \cdot i N r^2 \right] \cdot NL$$

$$\therefore \boxed{V(t) = \left(\frac{\mu_0 r^2 N^2 L}{4} \right) \frac{di}{dt}}$$

\rightarrow Inductance.

* Hence the product of the inductance to the change in current gives the potential voltage across the solenoid.

Solenoid on an IC:-

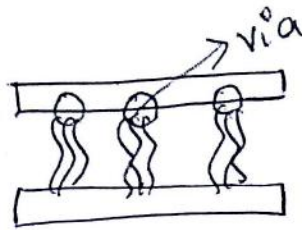
i, Solenoids can be placed on an IC in two ways.

\Rightarrow The metal cores can be placed one over the other with each core separated by a via.

* Via:- The punching of a hole with a specific depth & diameter for passing the core metal into it is called a via.

* Each via is highly resistive.

* Its resistance is $\approx 5\Omega$.



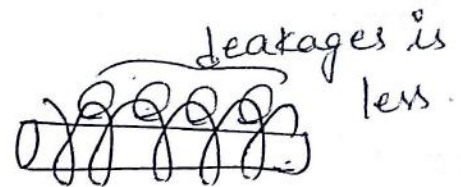
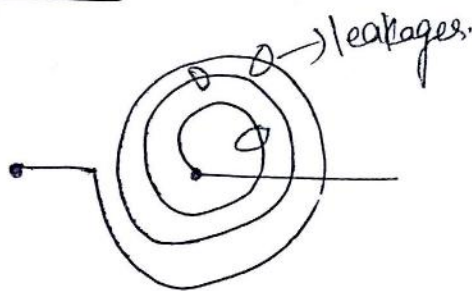
* Each via is poured with a metal over it and any one the via reaches the second core.

ii. The second way of placing the solenoids on an IC is,

→ Compress the spring into one single plane.

→ It has high conductivity.

Spiral Inductors.



⇒ If the core is placed in a straight line then the amount of leakages across the magnetic flux is less.

⇒ Indicates that the magnetic flux or field lines pass through the core completely.

⇒ But, if the core is selected in spiral then the amount of leakages are more.

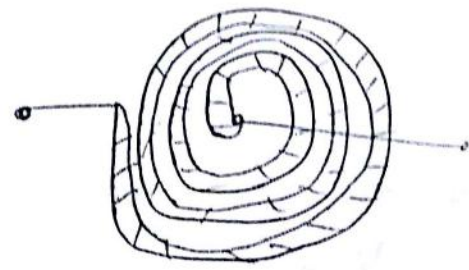
Non-Idealities:-

one of * The leakages in the inductor core are due to the non-idealities.

- copper losses
 - core losses
- > Non-idealities.

(a) copper Loss:-

- * When a wire is wound in a circular path.
- * Each wire has its resistance.
- * The resistance of the wire is calculated depending upon the no: of squares in the track.

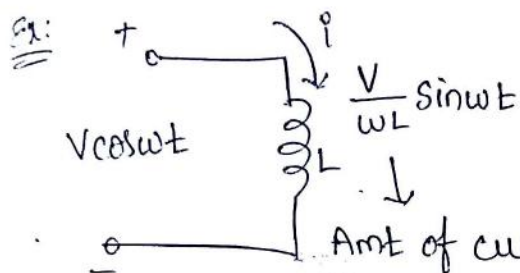


$$\therefore \boxed{R_{\text{copper loss}} = \frac{L}{w} \cdot \delta} \rightarrow \text{Surface resistivity.}$$

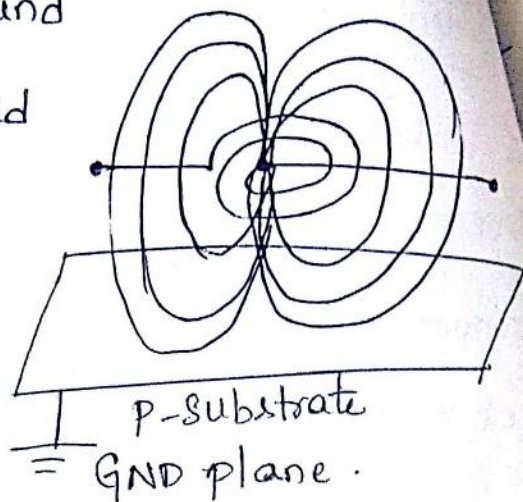
(b) core Losses:-

- * The Inductance doesn't accept any changes to be taken place in the magnetic flux.
- * Similarly the objects nearer to the inductors also doesn't accept any changes in the magnetic flux.
- * Hence an ϵ_{eff} developed by both these Inductance & the object.

* The current looping around this ground plane is called as "Eddy current".



Amt of current flowing through the inductor.



\therefore The magnetic flux of the coil is,

$$\phi \propto \frac{V}{\omega L} \sin \omega t.$$

$$\therefore \boxed{\frac{d\phi}{dt} \propto \frac{V}{L} \cos \omega t}$$

$$\boxed{\text{Emf} \propto \frac{V}{L} \cos \omega t}$$

$$\left[\because \frac{d\phi}{dt} = \text{Emf induced across the coil} \right]$$

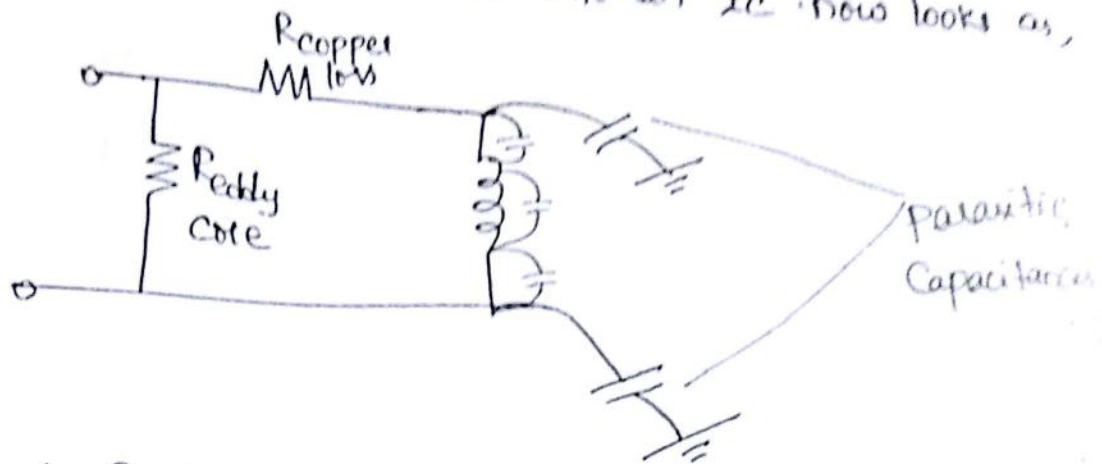
the amt of current $\boxed{i_{\text{eddy}} \propto \frac{V}{LR} \cos \omega t}$

* The amount of power loss is,

$$\text{Power loss} = \text{Emf} \times i_{\text{eddy}}.$$

$$\therefore \boxed{\text{power loss} = \frac{V^2}{L^2 R} \cos^2 \omega t}$$

the design of the inductor into an IC now looks as,



Metric of Inductor performance:-

* The Inductor performance can be improved by two factors

→ Quality factor

→ Resonant frequency.

i, High Resonant Frequency:- This can be obtained by,

→ Use topmost metal layer (parasitics to GND gets minimize)

→ Make gap b/w the turns to be large.

ii, Reduce copper losses:-

→ Using a thick wire (large Area).

iii, Reduce of eddy current losses:-

→ Use topmost metal layers.

→ Use highly resistive substrates.

$$\text{Power loss} = \frac{V^2}{R} \uparrow$$

Interconnections:-

⇒ An interconnection ^{is made with} is a thin film wire that electrically connects two (or) more components in an IC.

⇒ Due to these interconnections parasitics (unwanted components) of Capacitor, Resistor, Inductor are introduced.

⇒ These parasitics effects,

→ performance (propagation delay of the wires)

→ power consumption.

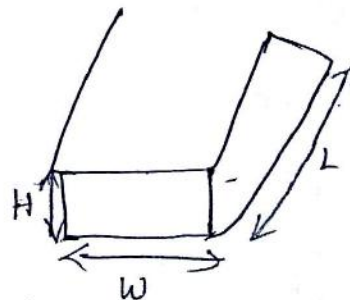
→ Reliability.

⇒ As the transistor is scaled down into a small size placed onto the IC, the no. of interconnections increases which causes an increase in parasitics.

Wire Resistance:-

⇒ * The Resistance of the wire,

$$R_{\text{wire}} = \frac{\rho L}{HW}$$



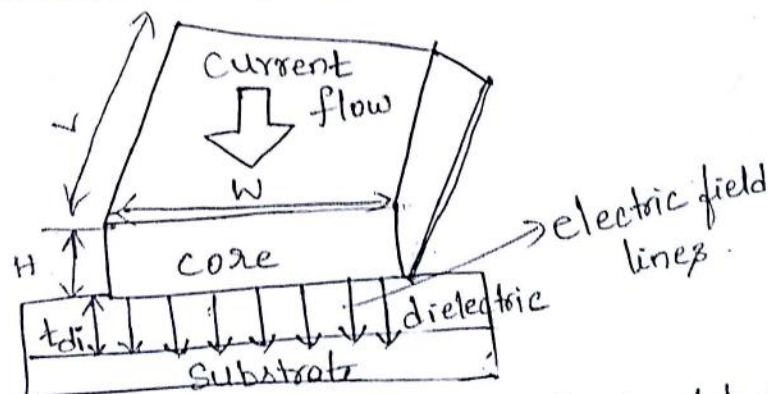
[$\because R/H = \text{Sheet Resistance } R_s$]

$$\therefore R_{\text{wire}} = R_s \cdot \frac{L}{W}$$

Material	$\rho (\Omega m)$
Silver (Ag)	1.6×10^{-8}
Copper (Cu)	1.7×10^{-8}
Gold (Au)	2.2×10^{-8}
Aluminium (Al)	2.7×10^{-8}
Tungsten (W)	5.5×10^{-8}

(2) wire capacitance:-

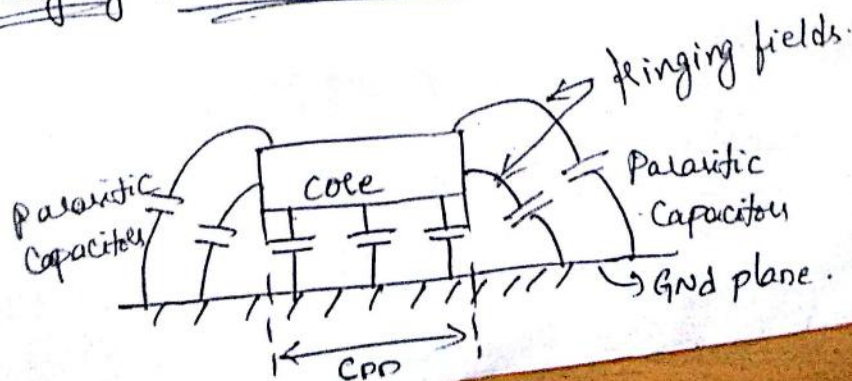
(a) plate-to-plate capacitance:-



From the fig, the amount of plate-to-plate capacitance is,

$$C_{pp} = \frac{\epsilon_{di}}{t_{di}} WL$$

(b) Fringing Field Capacitance:-



∴ The wire capacitance per unit length is,

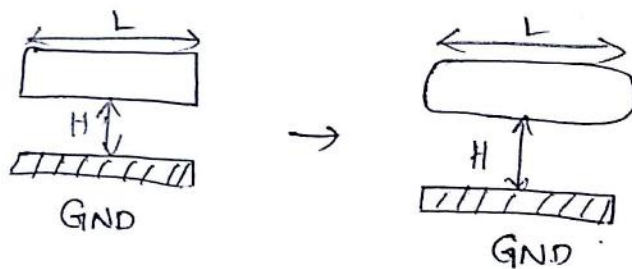
$$C_{\text{wire}} \cong C_{\text{pp}} + C_{\text{fringe}}$$

$$C_{\text{wire}} \cong \frac{w\epsilon_{di}}{t_{di}} + \frac{2\pi\epsilon_{di}}{\log(t_{di}/H)}$$

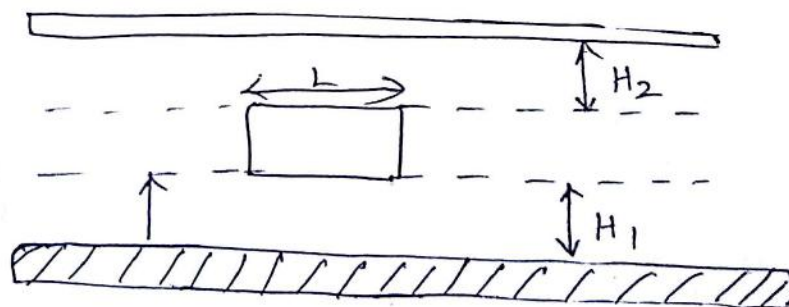
* For reducing the parasitics caused due to the interconnections.

* There are 3 methods (or) ways of arranging the wires.

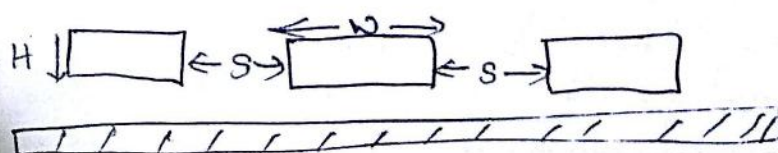
(i) Single conductor over GND.



(ii) wire sandwiched b/w two conductor planes.



(iii) Three adjacent wires over a single plane.



Skin Effect:-

⇒ At low frequencies the properties of interconnect deals with ,

→ Resistivity

→ Current-handling ability.

→ Capacitance.

⇒ At high frequencies the interconnections deals with inductance

⇒ As the frequency increases its inductance increases.

⇒ When the inductance increases its resistance also increases.

⇒ Due to this increase in the resistance the skin effect occurs.

Def

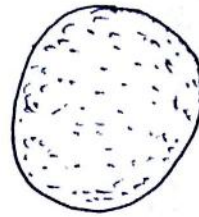
* The phenomenon arising due to the unequal distribution of current over the entire cross section of the conductor is called as "Skin Effect".

[OR]

* The tendency of current to flow primarily on the surface of the conductor as frequency increases.

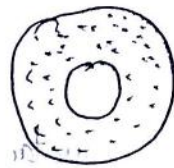
* As the length of the conductor increases the skin depth effect also increases.

* For the DC systems the distribution of current is throughout the system.

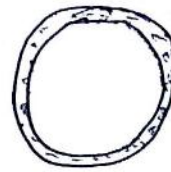


DC Resistance.

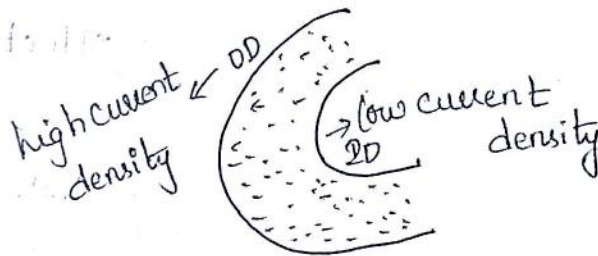
* For the AC systems, where the current tends to flow with higher density through the surface of the conductor (i.e. skin/surface).



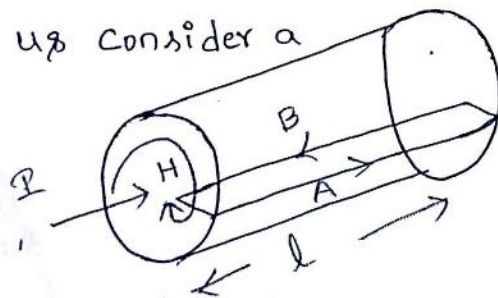
Low frequency A.C



High frequency A.C.



* Let us consider a solid cylinder conductor carrying a time-varying current.

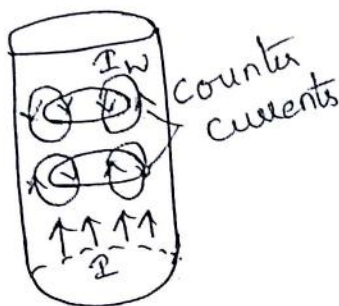


* A time varying current (I) generates a time-varying magnetic field (H).

* The time-varying field induces a voltage around the rectangular path according to Faraday's law.

Then according to ohm's law the induced voltage in turn produces a current flow along the same path.

- The current through the path will be opposite to each other.
- The current touching the surfaces gets its current to be added & the current below/above the surface gets diminished.



Formulae :-

- ⇒ The AC skin current density (J) in a conductor decreases exponentially from its value at the surface (J_s) according to the depth ' d ' from the surface.

$$J = J_s e^{-d/\delta}$$

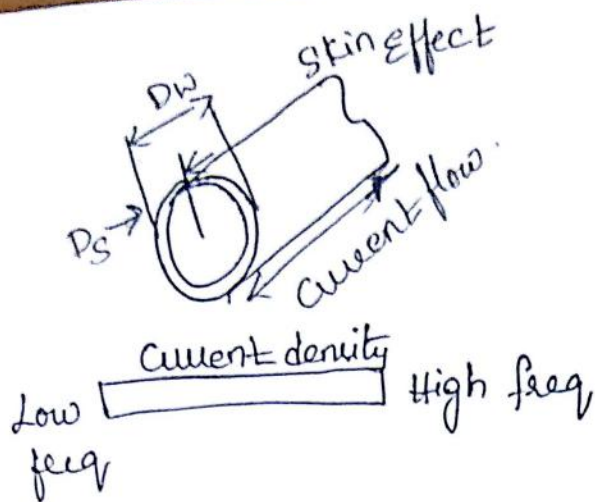
where $\delta \rightarrow$ skin depth.

- * The depth below the surface of the conductor at which the current density has fallen to $1/e$ (about 0.37) of $J_s \rightarrow$ skin depth.

- * In normal cases,

$$\delta = \sqrt{\frac{2\rho}{\omega\mu}}$$

where $\rho \rightarrow$ resistivity of conductor
 $\omega \rightarrow$ angular freq of current, $\omega = 2\pi f$
 $\mu \rightarrow$ absolute permeability.



Factors Affecting the skin depth:-

- Shape of the conductors.
- Type of the material.
- Diameter of the conductor.
- operational Frequency.

Material Effect on skin depth:-

- ⇒ Good conductors, the skin depth varies as the inverse square root of the conductivity.
- ⇒ Better conductivity implies that its skin depth reduces.
- ⇒ The overall resistance of the better conductor remains lower.
- ⇒ Skin Effect → inversely proportional to the square root of permeability of the conductor.
- ⇒ For Iron the conductivity is $\frac{1}{7}$ times that of Copper.
- ⇒ Iron wires is thus useful for A.C power lines.

⇒ The skin effect also reduces the effective thickness of laminations & power transformers.

⇒ Iron wires can also be used for D.C windings but it is impossible to use them at frequencies higher than 60Hz.

Mitigation of Skin Effect (Reduction).

⇒ Instead of these normal (wires/conductors) a type of cable called as "Litz wires" are used to reduce the skin effect for frequencies of few KHz to 1MHz.

⇒ The Litz wires consists of number of insulated wires woven together, so that over all magnetic field acts equally on the wires and the total amount is distributed equally among them.

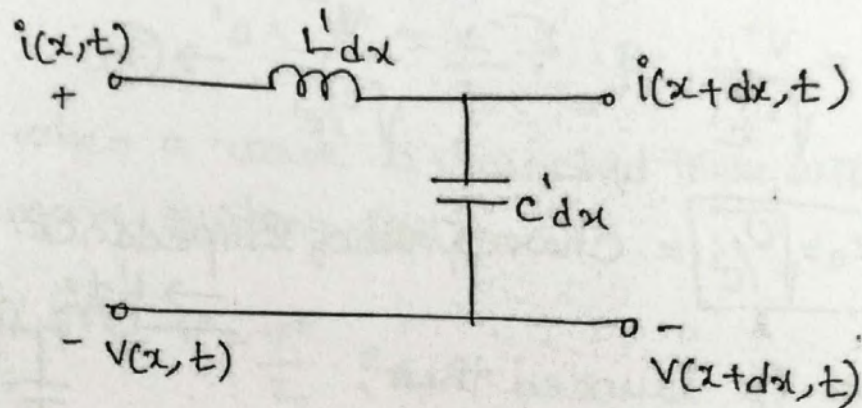
UNIT-II

Review of MOS Device physics

3

Transmission Lines:-

⇒ Transmission lines are the means of transmitting the signals from one point to the other point.



The differential equation for the CKT is,

$$\frac{d^2 V}{dx^2} = L' C' \frac{d^2 V}{dt^2} \rightarrow (1)$$

$$\frac{d^2 i}{dx^2} = L' C' \frac{d^2 i}{dt^2} \rightarrow (2)$$

The wave equation of the s/g is,

$$V(x,t) = \underbrace{V^+(x-ct)}_{\text{Forward moving wave}} + \underbrace{V^-(x+ct)}_{\text{Backward moving wave}} \rightarrow (3)$$

Forward moving wave

Backward moving wave.

$$i(x,t) = I^+(x-ct) + I^-(x+ct) \rightarrow (4)$$

⇒ The relationship b/w vol & current is,

$$\frac{dV}{dx} = -L' \frac{di}{dt} \rightarrow (5)$$

$$\frac{di}{dx} = -c' \frac{dv}{dt} \rightarrow (6)$$

The total amount of voltage to the wave is,

$$V^+ + V^- = cL' I^+ - cL' I^-$$

$$I^+ + I^- = cC' V^+ - cC' V^-$$

where, $I^+ = \frac{V^+}{\sqrt{L'/C'}} \quad \& \quad I^- = -\frac{V^-}{\sqrt{L'/C'}} \rightarrow (7)$

where, $Z_0 = \sqrt{L'/C'} = \text{Characteristic Impedance.}$

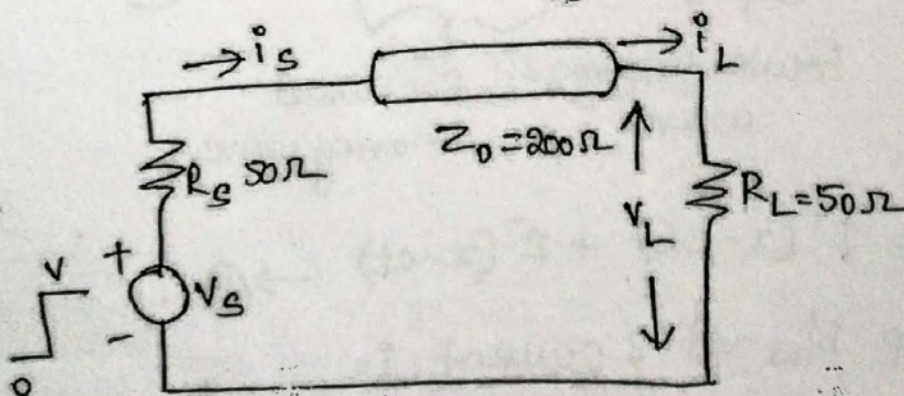
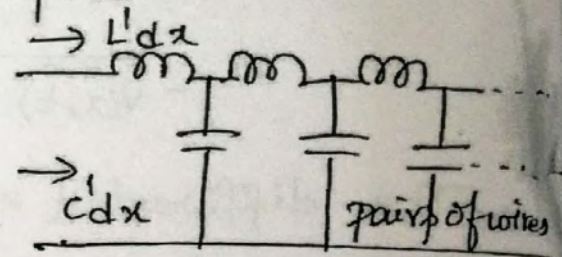
If a wave is launched then,

$$V(x,t) = V^+(x-ct) + V^-(x+ct)$$

$$i(x,t) = \frac{V^+}{Z_0} (x-ct) - \frac{V^-}{Z_0} (x+ct) \rightarrow (8)$$

Its characteristic Imp, $Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$

If $R' \& G' = 0$ then $Z_0 = \sqrt{L'/C'}$



If a wave is launched at the source end then,

$$V_S = V - i_S R_S \rightarrow (9)$$

At the load,

$$V_L = i_L R_L \rightarrow (10)$$

The source vol, $V_S = V^+ + V^-$.

$$i_S = \frac{V^+}{Z_0} - \frac{V^-}{Z_0}$$

* Initially when a wave is launched there will be no backward moving wave existing for it.

Hence, $V^+ = V - \frac{V^+}{Z_0} \cdot R_S$

$$V^+ + \frac{V^+}{Z_0} R_S = V$$

$$V^+ \left(\frac{Z_0 + R_S}{Z_0} \right) = V$$

$$\therefore V^+ = \frac{V Z_0}{Z_0 + R_S} \rightarrow (11)$$

$$V^+ = \frac{V Z_0}{Z_0 + R_S}$$

0V \swarrow wave travelling in the forward direction

* After the wave hits the load, $V_L = i_L R_L$.

$$(V^+ + V^-) = \left(\frac{V^+}{Z_0} - \frac{V^-}{Z_0} \right) \cdot R_L \leftrightarrow (12)$$

Substitute eq (11) in eq (12),

$$\left(\frac{V Z_0}{Z_0 + R_S} + V^- \right) = \left(\frac{V Z_0}{Z_0 + R_S} - \frac{V^-}{Z_0} \right) R_L$$

$$\left(\frac{V Z_0}{Z_0 + R_S} + V^- \right) = \frac{V R_L}{Z_0 + R_S} - \frac{V^- R_L}{Z_0}$$

$$\frac{V Z_0}{Z_0 + R_S} - \frac{V R_L}{Z_0 + R_S} = -\frac{V^- R_L}{Z_0} - V^-$$

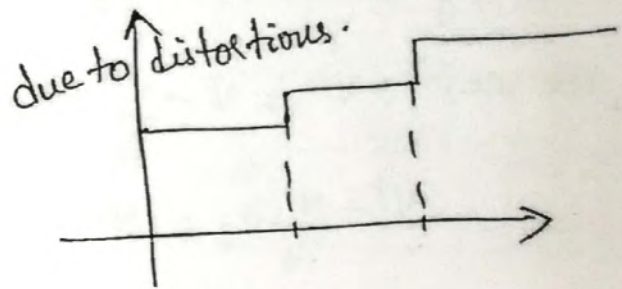
$$V \left[\frac{Z_0 - R_L}{Z_0 + R_S} \right] = -V^- \left[\frac{R_L - Z_0}{Z_0} \right]$$

$$\therefore V \left[\frac{Z_0 - R_L}{Z_0 + R_S} \right] = V^- \left[\frac{Z_0 - R_L}{Z_0} \right]$$

But.

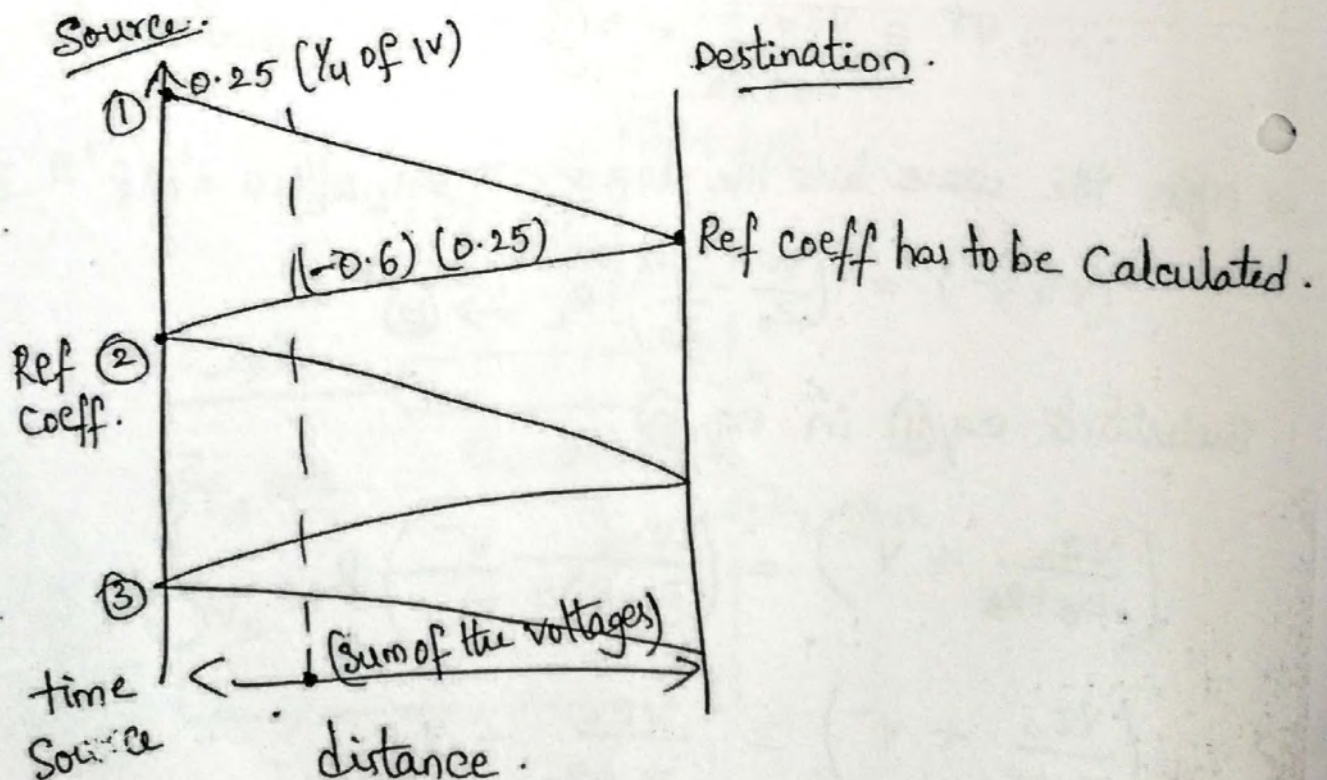
* If a wave of size V^+ hits the load then its reflection coefficient should be,

$$\boxed{\frac{V^-}{V^+} = \frac{R_L - Z_0}{R_L + Z_0}}$$



Ex: Let $R_S \& R_L = 50 \Omega$.

$$Z_0 = 200 \Omega$$



MOS Device Review:-

- * MOSFET is the most favoured component on an IC.
- * The specifications of an MOSFET for designing in an IC are,

- Its fabrication cost is less.
- Minimum no: of masks are used.
- widely available.
- cheapest.
- It has high i/p impedance compared to JFET.

Types:-

- Depletion mode MOSFET (D-MOSFET)

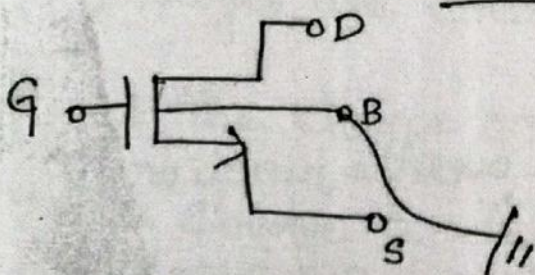
↳ The channel exists.

- Enhancement Mode MOSFET (E-MOSFET)

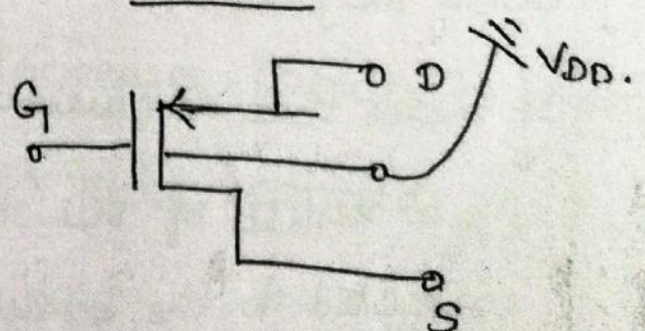
↳ The channel has to be created.

Symbols:-

n-channel.



P-channel.



Structure of MOSFET:-

If the body is connected to GND and the terminals of source & drain are connected to each other.

* Let $V_c = 0$, source & drain are at 0V.

* Then the gate terminal which looks like a (metal, dielectric, metal) capacitor.

* As the potential voltage across the gate terminal is increased slowly.

* A +ve charge is deposited on the +ve plate.

* But -ve charge is not deposited on the -ve plate since the bottom plate has no charged particles (electrons).

* The p-substrate is full of holes.

* This +ve charge goes on increases making this holes to be pushed out.

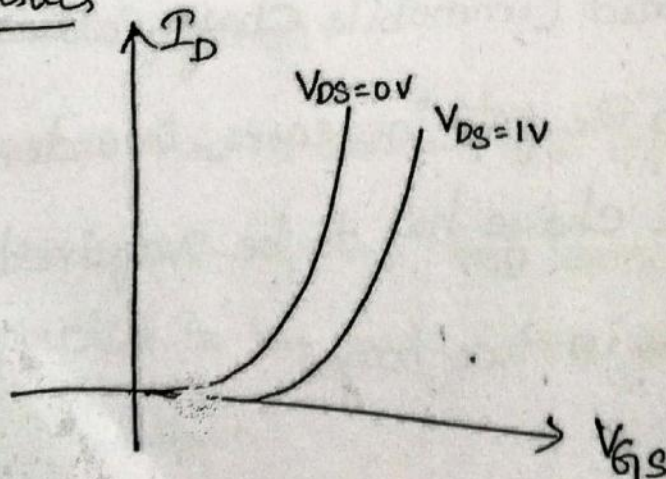
* As the holes are leaving out a fixed amt of negative charges are created (immobile charge carriers).

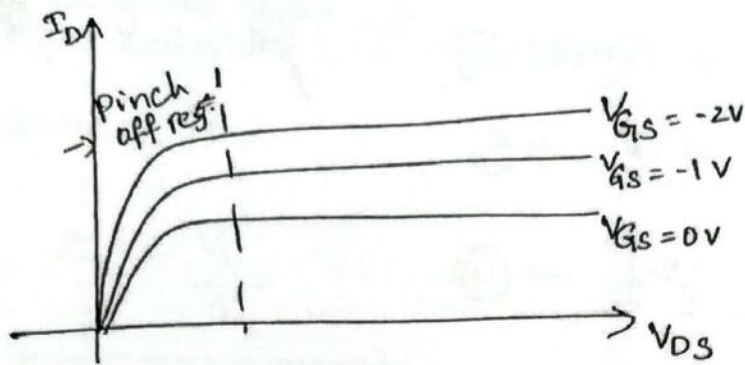
* As the charge on the gate increases, in order to balance this charge a -ve charge has to be required.

* This can be possible in two ways.

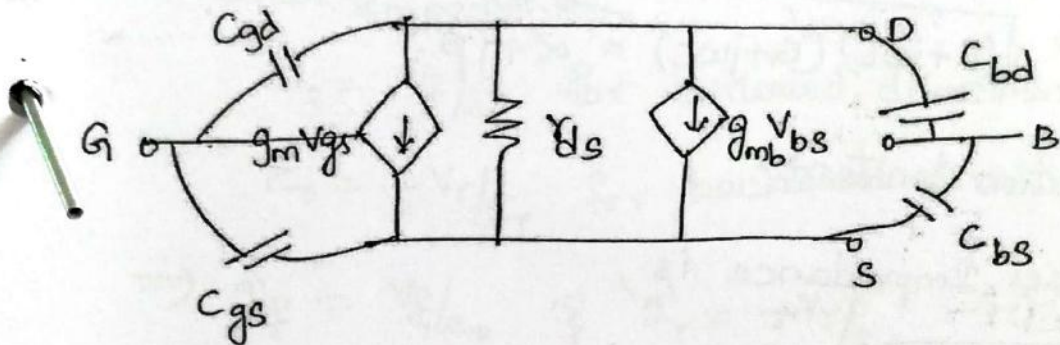
- * One possibility is some electrons move from all along path and injected into the body.
- * The other possibility is some electrons from source & drain of n-type jump into the p-substrate.
- * Hence they are arranged underneath the SiO_2 layer.
- * Thus channel is created (or) formed.
- * Channel electrically connects the source & drain.
- * If the drain has higher potential than the source then more no. of electrons are absorbed into the drain.
- * Shape of the channel gets changed.
- * Hence the density of the source is high & the density at the ~~load~~ drain is less.
- * As V_{DS} increases more & more then the channel gets pinched off.

Characteristics





Small S/g Model of MOSFET:-

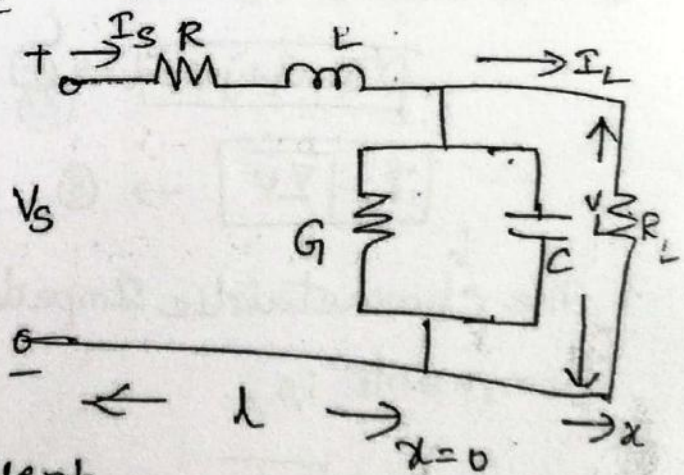


Transmission Line Equation:-

* V_S & I_S are the voltage & current at source side.

* V_L & I_L are the voltage & current at load side.

* Let V & I be the vol & current on the line at any arbitrary location.



Hence, $\frac{dV}{dx} = -(R + j\omega L)I \rightarrow (1)$

$\frac{dI}{dx} = -(G + j\omega C)V \rightarrow (2)$

Differentiating & combining the eqns we get,

$$\frac{d^2 V}{dz^2} = \gamma^2 V \rightarrow (3)$$

$$\frac{d^2 I}{dz^2} = \gamma^2 I \rightarrow (4)$$

where, $\gamma^2 = (R + j\omega L)(G + j\omega C)$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \rightarrow (5)$$



Propagation Constant.

* The Series Impedance is,

$$Z = R + j\omega L \rightarrow (6)$$

* The Shunt admittance is,

$$Y = G + j\omega C \rightarrow (7)$$

$$\therefore \gamma = \sqrt{ZY} \rightarrow (8)$$

* The Characteristic Impedance $= Z_0$ related to the R, L, C components is,

$$Z_0 = \sqrt{Z/Y}$$

$$\therefore Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \rightarrow (9)$$

$$\text{If } R \text{ \& } G = 0, \quad Z_0 = \sqrt{L/C} \rightarrow (10)$$

The transmission line reaches the load and the load impedance is,

$$Z_L = \frac{V_L}{I_L}$$

$$Z_L = \frac{(V_f + V_r)}{(I_f + I_r)} \rightarrow (11)$$

The Char Impedance,

$$Z_0 = V_f / I_f \text{ for forward direction.}$$

$$Z_0 = -V_r / I_r \text{ for backward direction.}$$

$$\Rightarrow I_f = V_f / Z_0 \quad \& \quad I_r = -V_r / Z_0 \rightarrow (12)$$

By Substituting the values of I_f & I_r in eq (11) we get,

$$Z_L = \frac{V_f + V_r}{\frac{V_f}{Z_0} - \frac{V_r}{Z_0}} \rightarrow (13)$$

$$\Rightarrow Z_L = \frac{(V_f + V_r) Z_0}{(V_f - V_r)}$$

$$\Rightarrow V_f Z_L - V_r Z_L = V_f Z_0 + V_r Z_0$$

$$\Rightarrow V_f (Z_L - Z_0) = V_r (Z_L + Z_0)$$

$$\therefore \frac{V_r}{V_f} = \frac{Z_L - Z_0}{Z_L + Z_0} \rightarrow (14)$$

$$\text{Ref coefficient } \rho = \frac{V_r}{V_f} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Lossless Transmission Lines:-

- * A transmission line is said to be lossless if the conductivities of the lines are perfect ($\sigma_c = 0$).
- * If the dielectric medium b/w the lines is lossless (i.e.) ($\sigma_d = 0$).
- * For a lossless T.L,

$$\gamma = \alpha + j\beta$$

$$\text{If } \alpha = 0,$$

$$\gamma = j\beta \rightarrow (1).$$

According to the T.L Equ,

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}.$$

$$\text{If } R = 0 \text{ \& } G = 0,$$

$$\gamma = j\omega\sqrt{LC} \rightarrow (2).$$

Equate (1) \& (2),

$$\gamma = j\omega\sqrt{LC} = j\beta.$$

$$\therefore \boxed{\beta = \omega\sqrt{LC}}, \alpha = 0. \rightarrow (3).$$

* The velocity of propagation of a lossless T.L is,

$$v_p = \frac{\omega}{\beta}$$

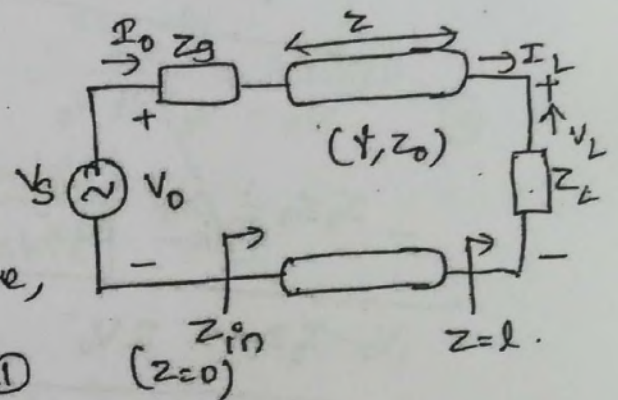
$$\therefore v_p = \frac{\omega}{\omega\sqrt{LC}}$$

$$\boxed{v_p = 1/\sqrt{LC}} \rightarrow (4).$$

By Transmission Lines:-

Consider a transmission line of length 'l', characterized by γ & z_0 (γ, z_0) and connect it to the load z_L .

* Let the T.L. extends from, $z=0$ at the generator to $z=l$ at the load.



* The voltage & current eqns are,

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \rightarrow (1)$$

$$I_s(z) = \frac{V_0^+}{z_0} e^{-\gamma z} - \frac{V_0^-}{z_0} e^{+\gamma z} \rightarrow (2)$$

* To find V_0^+ & V_0^- ,

$$\left. \begin{aligned} V_0 &= V(z=0) \\ I_0 &= I(z=0) \end{aligned} \right\} \rightarrow (3)$$

Substituting these in eqns,

$$V_s(0) = V_0^+ e^0 + V_0^- e^0$$

$$I_s(0) = \frac{V_0^+}{z_0} e^0 - \frac{V_0^-}{z_0} e^0$$

$$\therefore V_0 = V_0^+ + V_0^- \rightarrow (4)$$

$$I_0 = \frac{V_0^+}{z_0} - \frac{V_0^-}{z_0}$$

$$I_0 z_0 = V_0^+ - V_0^- \rightarrow (5)$$

$$\begin{aligned} \textcircled{1} \quad V_0 &= V_0^+ + V_0^- \\ I_0 Z_0 &= V_0^+ - V_0^- \quad (\text{Add}) \\ \hline V_0 + I_0 Z_0 &= 2V_0^+ \end{aligned}$$

$$\therefore \boxed{V_0^+ = \frac{1}{2}(V_0 + I_0 Z_0)} \rightarrow \textcircled{6}$$

$$\begin{aligned} \textcircled{2} \quad V_0 &= V_0^+ + V_0^- \\ -I_0 Z_0 &= V_0^+ - V_0^- \quad (\text{sub}) \\ \hline V_0 - I_0 Z_0 &= 2V_0^- \end{aligned}$$

$$\therefore \boxed{V_0^- = \frac{1}{2}(V_0 - I_0 Z_0)} \rightarrow \textcircled{7}$$

* Similarly at the load,

If $V_L = V(z=l)$, $I_L = I(z=l)$ we get,

$$V_0^+ = \frac{1}{2}(V_L + Z_0 I_L) e^{+\gamma l} \rightarrow \textcircled{8}$$

$$V_0^- = \frac{1}{2}(V_L - Z_0 I_L) e^{-\gamma l} \rightarrow \textcircled{9}$$

\therefore The input impedance,

$$Z_{in} = \frac{V_g(z)}{I_g(z)}$$

$$Z_{in} = \frac{V_g(z)}{I_g(z)} = \frac{V_0^+ + V_0^-}{\frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}}$$

$$Z_{in} = Z_0 \frac{(V_0^+ + V_0^-)}{(V_0^+ - V_0^-)} \rightarrow \textcircled{10}$$

Substitute eqs ⑧ & ⑨ in eq ⑩,

$$Z_{in} = z_0 \left[\frac{V_L \left(\frac{e^{\gamma l} + e^{-\gamma l}}{2} \right) + z_0 I_L \left(\frac{e^{\gamma l} - e^{-\gamma l}}{2} \right)}{V_L \left(\frac{e^{\gamma l} - e^{-\gamma l}}{2} \right) + z_0 I_L \left(\frac{e^{\gamma l} + e^{-\gamma l}}{2} \right)} \right] \Rightarrow \textcircled{11}$$

$$\therefore \frac{e^{\gamma l} + e^{-\gamma l}}{2} = \cosh \gamma l \quad \& \quad \frac{e^{\gamma l} - e^{-\gamma l}}{2} = \sinh \gamma l.$$

$$Z_{in} = z_0 \left[\frac{V_L \cosh \gamma l + z_0 I_L \sinh \gamma l}{V_L \sinh \gamma l + z_0 I_L \cosh \gamma l} \right] \rightarrow \textcircled{12}$$

Take $\cosh \gamma l$ common,

$$Z_{in} = z_0 \left[\frac{V_L + z_0 I_L \tanh \gamma l}{V_L \tanh \gamma l + z_0 I_L} \right] \quad \left[\because \frac{\sinh \gamma l}{\cosh \gamma l} = \tanh \gamma l \right] \rightarrow \textcircled{13}$$

Take I_L common,

$$Z_{in} = z_0 \left[\frac{V_L/I_L + z_0 \tanh \gamma l}{z_0 + V_L/I_L \tanh \gamma l} \right] \rightarrow \textcircled{14}$$

$$\because z_L = V_L/I_L.$$

$$Z_{in} = z_0 \left[\frac{z_L + z_0 \tanh \gamma l}{z_0 + z_L \tanh \gamma l} \right] \rightarrow \textcircled{15} \quad \underline{\underline{[Lossy P.L.]}}$$

For a lossless P.L.,

$$\gamma = j\beta \quad \& \quad \tanh \gamma l = j \tan \beta l.$$

$$Z_{in} = z_0 \left[\frac{z_L + j z_0 \tan \beta l}{z_0 + j z_L \tan \beta l} \right] \rightarrow \textcircled{16} \quad \underline{\underline{[Lossless P.L.]}}$$

Distributed systems:-

- * The systems are distinguished depending upon their operating frequency.
- * The operating frequency consists of

Low frequency

High frequency.
- * If we are discussing about low frequency then the "Circuit theory" come into existence.
- * Those systems which deals with low frequency circuit theory comes under "Lumped ckt systems".
- * If we are discussing about high frequency then the "field theory" come into existence.
- * Those systems, which deals with high frequency circuit (or) field theory comes under "Distributed systems".
- * At low frequencies the identification of components is clear (R, C, L).
- * At high frequencies the identification of components is difficult since they are glaucy.
- * The Lumped ckt's work on the Kirchoff's Laws (KCL & KVL).
- * The distributed ckt's work on the Maxwell's Equations.

The Maxwell's Equations (for free space) in differential form are,

$$\nabla \cdot \mu_0 H = 0 \rightarrow (1)$$

$$\nabla \cdot \epsilon_0 E = \rho \rightarrow (2)$$

$$\nabla \times H = J + \epsilon_0 \frac{dE}{dt} \rightarrow (3)$$

$$\nabla \times E = -\mu_0 \frac{dH}{dt} \rightarrow (4)$$

Eq (1): says that there is no net magnetic charge.
(if magnetic charge exists it causes divergence in the magnetic field).
↓ (how much flow is expanding).

→ Eq (2): Gauss law.

→ There is net electric charge and is equal to the amount of resistivity.

→ But it is of very small value.

→ Eq (3): Ampere's law

→ Both the ordinary current & rate of change of electric field produce the same effect on the magnetic field.

→ Eq (4): Faraday's law

→ Changes in the magnetic field causes a change in the electric field with a opposing potential voltage.

* The wave behaviour can be estimated depending on two equations.

As a specific example setting $\mu_0 = 0$ makes the electric field to change allowing the gradient of potential.

* The line integral of the E-field around any closed path is zero

$$V = \oint E \cdot dl = \oint (\nabla \phi) \cdot dl = 0.$$

* This is the field theoretical expression for KVL.

* The curl of H depends only on the current density 'J'.

Hence,

$$\nabla \times H = J$$

* This is the field theoretical expression for KCL.

* The speed of the light is,

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

* Setting μ_0 & ϵ_0 to zero makes the speed of light to be infinite.

* The condition for the high freq distributed sys is $l \ll \lambda$.

Conclusion:-

\Rightarrow The boundary between the lumped ckt theory and distributed sys depends on the ckt-elements and the relative wavelengths of the waves.

Smith chart & Applications:-

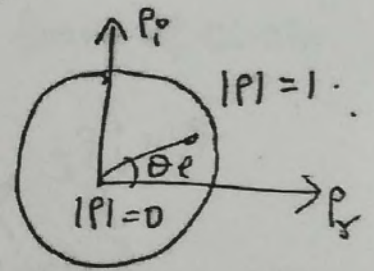
Def:- Smith chart is a polar plot of the reflection coefficient in terms of normalized impedance ($r+jx$).

(or)

Smith chart is a graphical plot of normalized resistance and reactance in the reflection coefficient plane.

construction of Smith chart:-

- * It is a construction within a circle with unit radius for $|P| \leq 1$.
- * Smith chart provides the relationship between,



→ Reflection coefficient (P)

→ Load impedance (Z_L)

→ characteristic imp (Z_0).

The reflection coefficient,

$$P = \frac{Z_L - Z_0}{Z_L + Z_0} \rightarrow \textcircled{1}$$

$$P = |P| \angle \theta_P = P_r + jP_i \rightarrow \textcircled{2}$$

* Smith charts are constructed in terms of normalised as (z_L/z_0) .

$$z_n = \frac{z_L}{z_0} = r + jx \rightarrow (3)$$

Equating eqs ① & ② we get,

$$p_r + j p_i = \frac{z_L - z_0}{z_L + z_0}$$

Take z_0 common,

$$p_r + j p_i = \left[\frac{\frac{z_L/z_0 - 1}{z_0}}{\frac{z_L/z_0 + 1}{z_0}} \right]$$

$$p_r + j p_i = \frac{z_n - 1}{z_n + 1} \rightarrow (4) \quad \left[\because z_L/z_0 = z_n \right]$$

$$(p_r + j p_i)(z_n + 1) = z_n - 1$$

$$p_r z_n + p_r + j p_i z_n + j p_i = z_n - 1$$

$$p_r z_n + j p_i z_n - z_n = -1 - p_r - j p_i$$

$$z_n (p_r + j p_i - 1) = -(1 + p_r + j p_i)$$

$$\therefore z_n = \frac{-(1 + p_r + j p_i)}{[(p_r - 1) + j p_i]}$$

$$\left[\because \frac{-A}{B} = A/-B \right]$$

$$z_n = \frac{[(1 + p_r) + j p_i]}{[(1 - p_r) - j p_i]} \rightarrow (5)$$

for obtaining the real part & imaginary parts, perform complex conjugate of eq (5).

$$z_n = \frac{[(1+p_r) + j p_i]}{[(1-p_r) - j p_i]} \times \frac{[(1-p_r) + j p_i]}{[(1-p_r) + j p_i]} = r + jx.$$

Solving this we get,

$$r = \frac{1 - p_r^2 - p_i^2}{(1-p_r)^2 + p_i^2}, \quad x = \frac{2p_i}{(1-p_r)^2 + p_i^2} \rightarrow (6).$$

* Now the equation (6) is reduced into the form of circle equation.

$$\left[p_i - \frac{r}{r+1} \right]^2 + p_i^2 = \left(\frac{1}{1+r} \right)^2 \rightarrow (7) \text{ (r-circle)}$$

$$[p_i - 1]^2 + \left[p_i - \frac{1}{x} \right]^2 = \left(\frac{1}{x} \right)^2 \rightarrow (8) \text{ (x-circle)}$$

* The equations (7) & (8) are compared with the general circle equation,

$$(x-k)^2 + (y-h)^2 = a^2 \rightarrow (9)$$

Comparing Eq (7) & (9) we get,

→ The resistance of the circle has center at (p_r, p_i) .

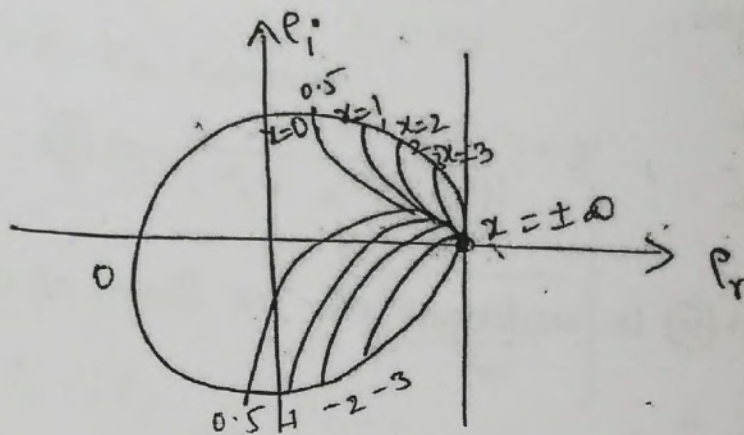
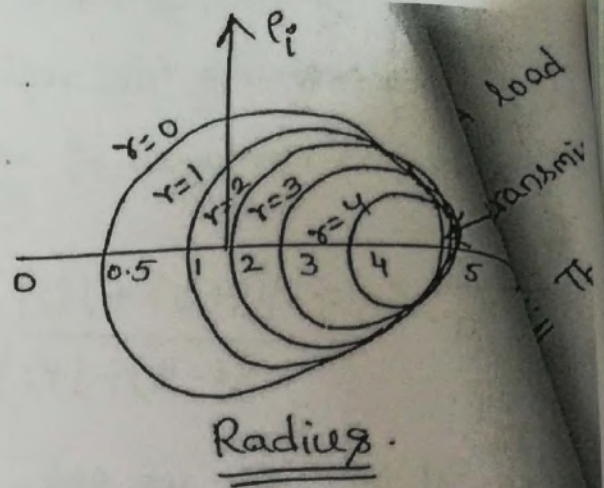
$$(P_r, P_i) = \left(\frac{r}{1+r}, 0 \right)$$

and radius = $\left(\frac{1}{1+r} \right)$.

* The reactance of the circle

$$(P_r, P_i) = \left(1, \frac{1}{x} \right)$$

with radius $\left(\frac{1}{x} \right)$.



Applications:-

- ⇒ It is used for finding the parameters of mismatched transmission lines.
- ⇒ Used to calculate the normalised admittance from the normalised impedance.
- ⇒ Find the VSWR for a given load.
- ⇒ If impedance of a transmission lines.

(12)
A load of $(100 + j150)\Omega$ is connected to a 75Ω lossless transmission line. Find the values of Ref coeff (Γ) & VSWR.

Sol: The normalized impedance,

$$Z_n = \frac{Z_L}{Z_0}$$

$$\therefore Z_n = \frac{100 + j150}{75} = 1.33 + j2$$

$(r + jx)$.

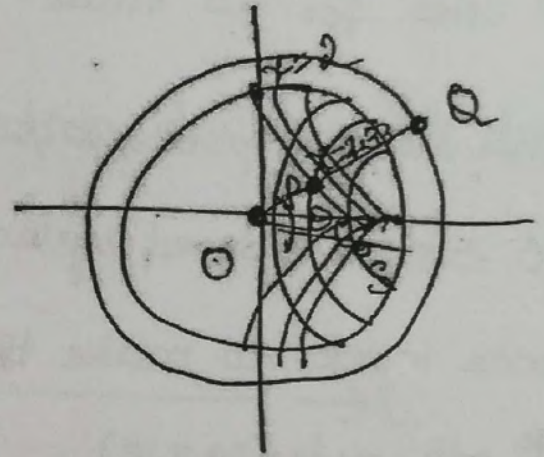
$$\therefore r = 1.33.$$

$$x = 2.$$

\therefore Locate this point as 'p'.

$$(a) |\Gamma| = \frac{OP}{OQ}$$

$$|\Gamma| = \frac{6\text{cm}}{9.1\text{cm}} = 0.659.$$



$$(b) \text{ VSWR, } S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$S = \frac{1 + 0.659}{1 - 0.659}$$

$$S = 4.865.$$

$$\therefore \angle \theta_r = \text{Angle pos}$$

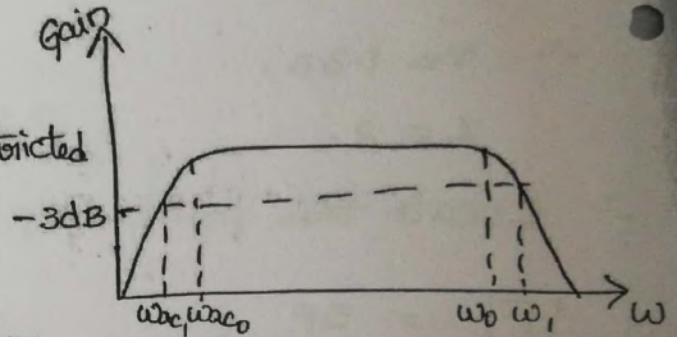
Bandwidth Estimations:-

* The standard steps for calculating the bandwidth of a network are,

- Deriving the i/p - o/p transfer function.
- Set $s = j\omega$.
- Find the magnitude of the resultant expression.
- Set the magnitude $= \frac{1}{\sqrt{2}}$ of the mid value.
- Solve for ' ω ' value.

* This standard process is restricted to certain ordered systems.

* Hence in order to make the s/m for n th order,



→ The simulators are used for finding the quantitative verification methods.

→ Two such approximation methods are,

→ Bandwidth estimation using open ckt time Response.

→ Bandwidth estimation using short ckt time Response.

* The o.c time constant provides an estimation for high freq, roll off pts.

* The s.c time constant provides an estimation for low freq, roll off points.

Q5. Bandwidth Estimation Using openckt time response:

* It is also called as "zero value time constant".

Limitations:

→ It works when the s/m has only poles.

→ The poles should not have complex conjugate poles.

* The octr identify the elements which are responsible for bandwidth limitations.

* Let us consider a transfer function which consists of all poles only,

$$\frac{V_o(s)}{V_i(s)} = \frac{a_0}{(\tau_1 s + 1)(\tau_2 s + 1) \dots (\tau_n s + 1)} \rightarrow \textcircled{1}$$

* Multiplying the terms in the denominator leads to a polynomial that shall be expressed as,

$$b_n s^n + b_{n-1} s^{n-1} + \dots + b_1 s + 1 \rightarrow \textcircled{2}$$

$$(\tau_1 s + 1)(\tau_2 s + 1)$$

$$\tau_1 \tau_2 s^2 + \tau_1 s + \tau_2 s + 1$$

$$\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2) s + 1 \rightarrow \textcircled{3}$$

comparing $\textcircled{2}$ & $\textcircled{3}$ eqns,

* b_0 The coefficient b_0 is the product of all the constants.

* b_1 is the sum of all the time constants.

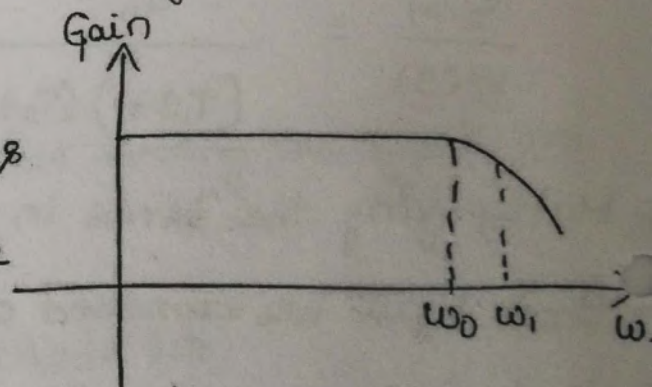
* The -3dB frequency the first order term typically dominates the higher order terms.

$$\frac{V_o(s)}{V_i(s)} \simeq \frac{a_0}{b_1 s + 1}$$

$$\therefore \frac{V_o(s)}{V_i(s)} \simeq \frac{a_0}{\left(\sum_{i=1}^n \tau_i\right) s + 1} \rightarrow (3)$$

* τ_i is the time constant corresponding to the poles in the network.

* The estimated B.W for the s/m is the reciprocal of the sum of the effective time constant.



$$B.W = \omega_h \simeq 1/b_1 = \frac{1}{\sum_{i=1}^n \tau_i} \rightarrow (4)$$

$$\omega_{h, est} \simeq \frac{1}{\sum_{i=1}^n \tau_i} \rightarrow (5)$$

If we consider a linear n/w comprising of only the resistors and capacitors then,

$$T_{j0} = R_{j0} \cdot C_{j0}$$

\therefore The estimated B.W of the linear n/w is,

$$B.W = \frac{1}{\sum_{j=1}^m R_{j0} C_{j0}} \rightarrow \textcircled{6}$$

(0) \rightarrow open ckt time constant

2) Bandwidth Estimation for Short ckt Time constant:-

* For s.c time constant low freq is estimated.

* For s.c time constant both zeros and poles are to be considered.

* Consider a transfer function as,

$$\frac{V_o(s)}{V_i(s)} = \frac{K \cdot s^n}{(s+s_1)(s+s_2) + \dots + (s+s_n)} \rightarrow \textcircled{1}$$

* $(s+s_1)(s+s_2)$

$$s^2 + s_1s + ss_2 + s_1s_2$$

$$s^2 + s(s_1+s_2) + s_1s_2 \rightarrow \textcircled{2}$$

* Multiplying the terms in the denominator leads to the polynomial

as, $s^n + b_1s^{n-1} + \dots + b_{n-1}s + b_n \rightarrow \textcircled{3}$

Equating eqns (2) & (3) we get,

b_1 - Sum of pole frequency.

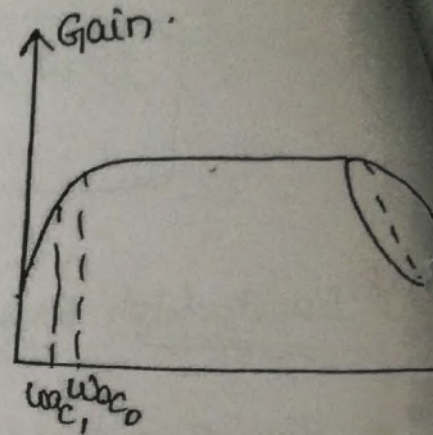
b_n - product of pole frequency.

* At -3dB freq, the higher order term dominates,

$$\frac{V_o(s)}{V_i(s)} \approx \frac{K s^n}{s^n + b_1 s^{n-1}} \quad (s^n/s)$$

$$\approx \frac{K \cdot \cancel{s^n}}{\cancel{s^n} [1 + b_1/s]}$$

$$\frac{V_o(s)}{V_i(s)} \approx \frac{K \cdot s}{s + b_1} \rightarrow (4)$$



where, $b_1 \rightarrow$ Sum of all the pole freqs.

* The B.W estimated frequency is,

$$(B.W)_{esh} \approx b_1 \approx \sum_{i=1}^n s_i \rightarrow (5)$$

* For a linear n/w consisting of resistor & Capacitor then the estimated B.W is,

$$(B.W)_{esh} \approx \sum_{j=1}^m \frac{1}{R_j s C_j} \rightarrow (6)$$

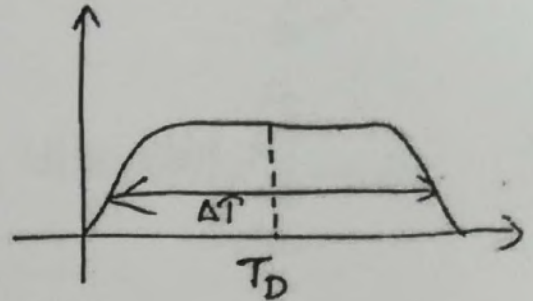
(s) \rightarrow short ckt time constant.

Delay Time:-

* Delay time is the measure of the time it takes for the impulse response to reach its "center of mass."

* Integration in time domain

\iff Differentiation in freq domain



$$\rightarrow x(t) \rightarrow [h(t)] \rightarrow x(t)h(t)$$

$$\rightarrow \text{step } x(t) \rightarrow [h(t)] \rightarrow h(t)$$

$$t_1 \rightarrow [h(t)] \rightarrow t_1 h(t)$$

$$t_2 \rightarrow [h(t)] \rightarrow t_2 h(t)$$

\vdots

$$t_n \rightarrow [h(t)] \rightarrow t_n h(t)$$

* The transfer function of the impulse response,

$$P.F = \frac{t_1 h(t)}{h(t)} + \frac{t_2 h(t)}{h(t)} + \dots + \frac{t_n h(t)}{h(t)}$$

* The delay time,

$$T_D = \frac{\int_{-\infty}^{\infty} t h(t) \cdot dt}{\int_{-\infty}^{\infty} h(t) \cdot dt} \rightarrow \text{①}$$

* This equation is also called as "Elmore delay".

* Now the time domain in terms of frequency domain

$$\int_{-\infty}^{\infty} h(t) \cdot dt \xleftrightarrow{FT} d/d_f H(f).$$

$$\int_{-\infty}^{\infty} t h(t) \cdot dt \xleftrightarrow{} \frac{j}{2\pi} d/d_f H(f).$$

$$\xleftrightarrow{} \frac{j}{2\pi} \times \frac{j}{j} d/d_f H(f).$$

$$\therefore \int_{-\infty}^{\infty} t h(t) \cdot dt \xleftrightarrow{} \frac{-1}{2\pi j} d/d_f H(f) \Big|_{f=0} \rightarrow (2)$$

* The denominator of the eq(1) represents the zero-state response of the gain (i.e) D.C gain.

$$\text{Hence, } \int_{-\infty}^{\infty} h(t) \cdot dt = H(0) \rightarrow (3)$$

Substitute eq (2) & (3) in eq(1),

$$T_D = \frac{\int_{-\infty}^{\infty} t h(t) \cdot dt}{\int_{-\infty}^{\infty} h(t) \cdot dt}$$

$$T_D = \frac{-1}{2\pi j H(0)} \cdot d/d_f H(f) \Big|_{f=0} \rightarrow (4)$$

consider two systems with impulse responses $h_1(t)$ & $h_2(t)$ with corresponding fourier transforms $H_1(f)$ & $H_2(f)$.

The overall time delay is ,

$$T_{D, tot} = \frac{-1}{j2\pi H_1(0)H_2(0)} \cdot \frac{d}{df} H_1 H_2(f) \Big|_{f=0} \rightarrow \textcircled{5}$$

* By expanding the equation we get,

$$T_{D, tot} = \frac{-1}{j2\pi H_1(0)H_2(0)} \left[H_2(0) \frac{dH_1}{df} \Big|_{f=0} + H_1(0) \frac{dH_2}{df} \Big|_{f=0} \right] \rightarrow \textcircled{6}$$

* The amount of the total delay for the impulse response is thus obtained as,

$$\boxed{T_{D, tot} = T_{D_1} + T_{D_2}} \rightarrow \textcircled{7}$$

Rise Time (ΔT):-

* The quantity ΔT is a measure of the duration of the impulse response.

[OR]

* It is measure of 10-90% of rise & fall times of the impulse response (or) step response.

* The rise time is twice the center of the mass (T_D) of $h(t)$.

$$\therefore \underline{\Delta T = 2(\text{center of mass})}.$$

$$\Delta T = 2 \left[\frac{\int_{-\infty}^{\infty} t h(t) \cdot dt}{\int_{-\infty}^{\infty} h(t) \cdot dt} \right] \rightarrow (1)$$

$$\frac{\Delta T}{2} = \frac{\int_{-\infty}^{\infty} t h(t) \cdot dt}{\int_{-\infty}^{\infty} h(t) \cdot dt}$$

Squaring on both sides,

$$\left(\frac{\Delta T}{2} \right)^2 = \frac{\int_{-\infty}^{\infty} t^2 h(t) \cdot dt}{\int_{-\infty}^{\infty} h^2(t) \cdot dt}$$

$$\left[\because \int_{-\infty}^{\infty} t \cdot h(t) \cdot dt \Leftrightarrow j/2\pi \frac{d}{df} H(f) \right].$$

$$\int_{-\infty}^{\infty} t^2 h(t) \cdot dt = \left(\frac{j}{2\pi}\right)^2 \frac{d^2}{df^2} H^2(f)$$

$$\int_{-\infty}^{\infty} t^2 h(t) \cdot dt = \frac{-1}{(2\pi)^2} \frac{d^2}{df^2} H^2(f) \Big|_{f=0} \rightarrow (3)$$

$$\int_{-\infty}^{\infty} h^2(t) \cdot dt \Leftrightarrow H(0) \rightarrow (4)$$

Substituting eq (3) & (4) in eq (2),

$$\left(\frac{\Delta T}{2}\right)^2 = \frac{\frac{-1}{(2\pi)^2} \cdot \frac{d^2}{df^2} H^2(f)}{H(0)}$$

$$(\Delta T)^2 = 4 \left[\frac{-1}{4\pi^2 H(0)} \cdot \frac{d^2}{df^2} H^2(f) \Big|_{f=0} \right]$$

Since $(\Delta T)^2 = t_{rise}^2$.

$$t_{rise}^2 = \frac{4}{4} \left[\frac{-1}{4\pi^2 H(0)} \cdot \frac{d^2}{df^2} H^2(f) \Big|_{f=0} \right]$$

* Consider a cascaded s/m's then,

$$t_{rise, tot}^2 = t_{rise 1}^2 + t_{rise 2}^2 \rightarrow (5)$$

$$t_{rise, tot} = \sqrt{t_{rise 1}^2 + t_{rise 2}^2} \rightarrow (6)$$

Bandwidth:-

* The bandwidth of the impulse response is measured at the normalized frequency of the closed loop transfer function.

* Consider the closed loop of Transfer function with second order S/m,

$$\frac{C(s)}{R(s)} = M(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \rightarrow (1)$$

* Set $s = j\omega$,

$$\begin{aligned} M(j\omega) &= \frac{\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n(j\omega) + \omega_n^2} \\ &= \frac{\omega_n^2}{-\omega^2 + j2\zeta\omega_n\omega + \omega_n^2} \\ &= \frac{\cancel{\omega_n^2}}{\cancel{\omega_n^2} \left[\frac{-\omega^2}{\omega_n^2} + j2\zeta\frac{\omega}{\omega_n} + 1 \right]} \end{aligned}$$

$$M(j\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\zeta\omega/\omega_n} \rightarrow (2)$$

* The normalized freq, $u = (\omega/\omega_n)$.

$$M(j\omega) = \frac{1}{(1-u^2) + j2\zeta u} \rightarrow (3)$$

$$\text{Magnitude } |M(j\omega)| = \left[\frac{1}{1 - u^2 + (2\zeta u)^2} \right]^{1/2} \rightarrow (4)$$

$$\angle M(j\omega) = -\tan^{-1} \left(\frac{2\zeta u}{1-u^2} \right)$$

If the slm of the linear η/ω is estimated for finding the bandwidth then,

$$\underline{U_b = \omega_b \overset{B.W}{\bigg|} \omega_n}$$

$$M(j\omega) = \frac{1}{[(1-U_b^2)^2 + 4\zeta^2 U_b^2]^{1/2}} = 1/\sqrt{2} \rightarrow (5)$$

Squaring on both sides & cross multiplication,

$$(1-U_b^2)^2 + 4\zeta^2 U_b^2 = 2.$$

$$1 + U_b^4 - 2U_b^2 + 4\zeta^2 U_b^2 = 2.$$

$$U_b^4 - 2U_b^2(1-2\zeta^2) - 1 = 0.$$

$$\text{Let, } U_b^2 = x.$$

$$x^2 - 2x(1-2\zeta^2) - 1 = 0.$$

$$x = \frac{2(1-2\zeta^2) \pm \sqrt{4(1-2\zeta^2)^2 + 4}}{2}$$

$$x = \frac{2(1-2\zeta^2) \pm 2\sqrt{(1-2\zeta^2)^2 + 1}}{2}$$

$$x = \cancel{2} \left[(1-2\zeta^2) \pm \sqrt{2+4\zeta^4-4\zeta^2} \right] \cancel{2}$$

$$\therefore \underline{U_b = \sqrt{x}}.$$

$$U_b = \left[(1 - 2\epsilon^2) + \sqrt{2 - 4\epsilon^2 + 4\epsilon^4} \right]^{1/2}.$$

\therefore Normalized B.W,

$$B.W = U_b = \frac{\omega_b}{\omega_n}.$$

$$\therefore \omega_b = U_b \cdot \omega_n.$$

$$\therefore B.W = \omega_n \left[(1 - 2\epsilon^2) + \sqrt{2 - 4\epsilon^2 + 4\epsilon^4} \right]^{1/2} \rightarrow (6)$$

High Frequency Amplifiers:-

* High frequency amplifiers are designed for negative feedback systems.

* Such high frequency amplifiers are,

→ Shunt-Series Amplifier.

→ Cascaded Amplifier

→ Tuned Amplifier.

* These amplifiers provide,

→ Reduction in the dependency on device parameters.

→ Broader Bandwidth.

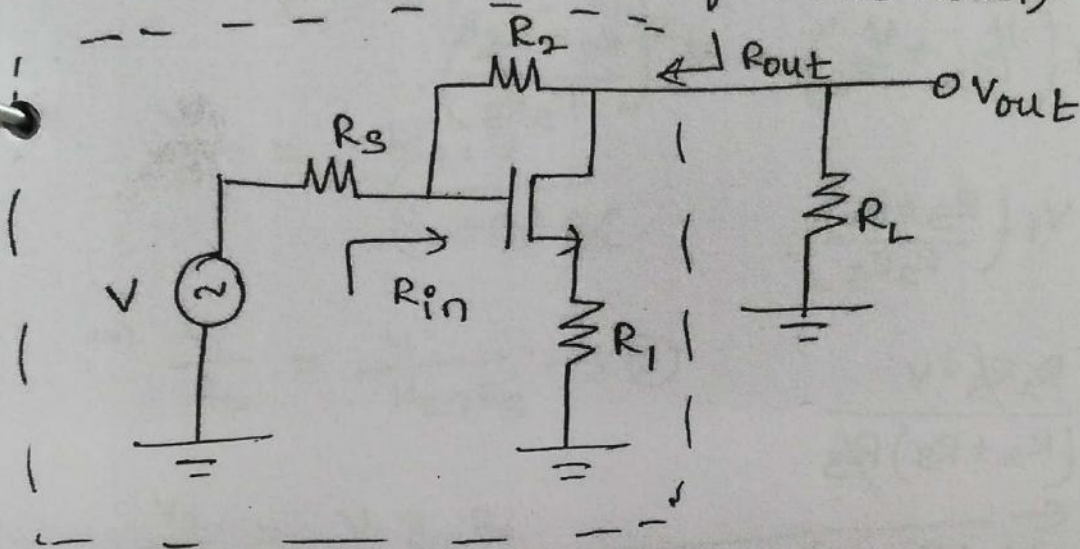
Shunt-Series Amplifier:-

In these amplifier there are two experimental tests,

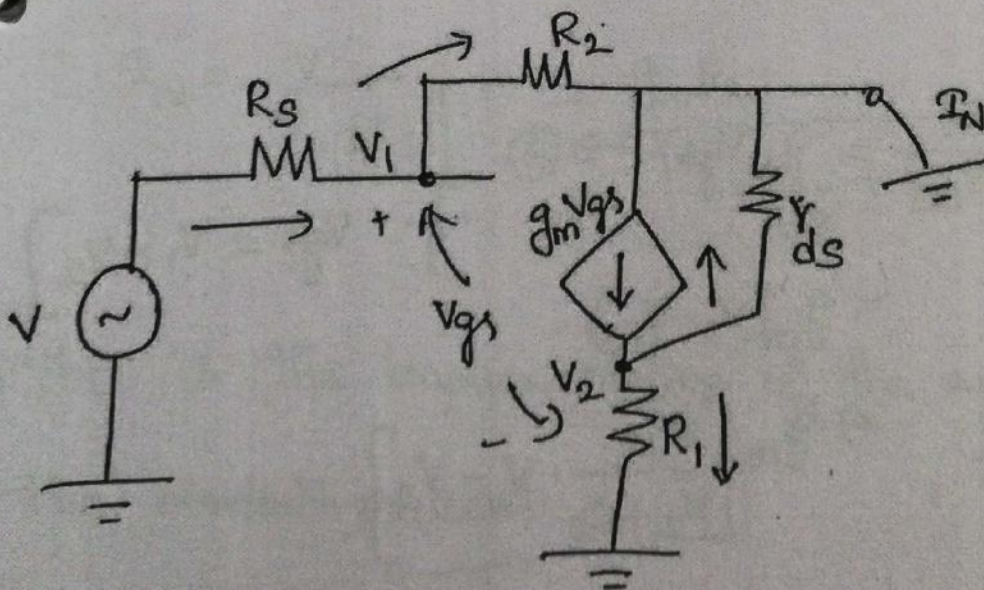
→ LF Gain.

→ Estimate the B.W.

Low Freq Gain:- (Norton Equivalent Model).



* Hence the current source is shunt with an o/p resistance.



From the fig,

Across V_1 ,
$$\frac{V - V_1}{R_S} = \frac{V_1}{R_2} \rightarrow (1)$$

$$\frac{V}{R_S} - \frac{V_1}{R_S} = \frac{V_1}{R_2}$$

$$\frac{V}{R_S} = \frac{V_1}{R_2} + \frac{V_1}{R_S}$$

$$\frac{V}{R_S} = V_1 \left(\frac{1}{R_2} + \frac{1}{R_S} \right) = V_1 \left(\frac{R_S + R_2}{R_2 R_S} \right)$$

$$\therefore \frac{V}{R_S} = V_1 \left(\frac{R_S + R_2}{R_2 R_S} \right)$$

$$\therefore V_1 = \frac{R_2 R_S \cdot V}{(R_2 + R_S) R_S}$$

$$\therefore V_1 = V \cdot \left(\frac{R_2}{R_2 + R_S} \right) \rightarrow (2)$$

Across V_2 ,

$$\frac{V_2}{R_1} + \frac{V_2}{r_{ds}} = g_m V_{gs} \rightarrow (3)$$

$$\left[\because V_{gs} = V_1 - V_2 \right]$$

$$= g_m (V_1 - V_2)$$

$$\Rightarrow g_m \left[\frac{R_2}{R_2 + R_S} \cdot V - V_2 \right]$$

$$\therefore \frac{V_2}{R_1} + \frac{V_2}{r_{ds}} + V_2 g_m = V \cdot \frac{g_m R_2}{R_2 + R_S}$$

$$V_2 \left(\frac{1}{R_1} + \frac{1}{r_{ds}} + g_m \right) = V \cdot \frac{g_m R_2}{R_2 + R_S}$$

$$\therefore V_2 = \frac{V \cdot g_m R_2}{(R_2 + R_S) \left(\frac{1}{R_1} + \frac{1}{r_{ds}} + g_m \right)} \rightarrow (4)$$

Hence the amount of the current at the nodes is,

$$I_N = V_1/R_2 - V_2/R_1$$

$$\frac{V_1}{R_2} = \frac{\cancel{R_2} \cdot V}{(R_2 + R_S) \cancel{R_2}}$$

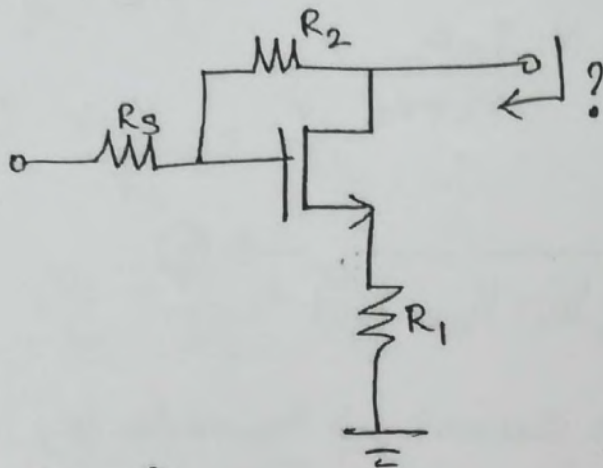
$$\Rightarrow \frac{V_1}{R_2} = \frac{V}{R_2 + R_S} \rightarrow (5)$$

$$\frac{V_2}{R_1} = \frac{V \cdot g_m R_2}{\left[(R_2 + R_S) \left(\frac{1}{R_1} + \frac{1}{r_{ds}} + g_m \right) \right] R_1} \rightarrow (6)$$

$$\therefore I_N = \frac{V}{R_2 + R_S} \left[1 - \frac{g_m R_2}{1 + \frac{R_1}{r_{ds}} + g_m R_1} \right] \rightarrow (7)$$

* This is the implementation of the amplifier for calculating the Norton's current.

Exp 2. o/p Imp:-



From the fig,

$$\frac{V_x}{R_1} = \frac{V - V_x}{r_{ds}} + g_m V_{gs}.$$

$$\Rightarrow \frac{V - V_x}{r_{ds}} + g_m \left(V \cdot \frac{R_s}{R_s + R_2} - V_x \right).$$

$$V_x \left(\frac{1}{R_1} + \frac{1}{r_{ds}} + g_m \right) = V \left(\frac{1}{r_{ds}} + \frac{g_m R_s}{R_s + R_2} \right).$$

$$V_x = \frac{V \left(\frac{1}{r_{ds}} + \frac{g_m R_s}{R_s + R_2} \right)}{\frac{1}{R_1} + \frac{1}{r_{ds}} + g_m} \rightarrow (8)$$

$$I_N = \frac{V}{R_2 + R_s} + \frac{V_x}{R_1} \rightarrow (9)$$

$$I_N = \frac{V \left(\frac{1}{r_{ds}} + \frac{g_m R_s}{R_s + R_2} \right)}{\left(\frac{1}{R_1} + \frac{1}{r_{ds}} + g_m \right) R_1} + \frac{V}{R_2 + R_s}.$$

$$I = \frac{V \left(\frac{1}{r_{ds}} + \frac{g_m R_s}{R_s + R_2} \right)}{1 + R_1 / r_{ds} + g_m R_1} + \frac{V}{R_2 + R_s}$$

$$I = \frac{V \cdot \left(1 + R_1 / r_{ds} + g_m R_1 + g_m R_s + \frac{R_2 + R_s}{r_{ds}} \right)}{\left(1 + R_1 / r_{ds} + g_m R_1 \right) (R_2 + R_s)} \rightarrow (10)$$

$$R_{out} = \frac{V}{I}$$

$$R_{out} = \frac{\left(1 + R_1 / r_{ds} + g_m R_1 \right) (R_2 + R_s)}{1 + R_1 / r_{ds} + g_m R_1 + g_m R_s + \frac{R_2 + R_s}{r_{ds}}} \rightarrow (11)$$

If $R_1 = 0$, $R_2 = \text{infinitely large}$,

$$R_{out} = \frac{R_2}{R_2 / r_{ds}} = r_{ds} \rightarrow (12)$$

$$\therefore \text{Gain} = \frac{I_N \cdot R_{out}}{V} = \frac{1 + R_1 / r_{ds} + g_m R_1 + g_m R_2}{1 + R_1 / r_{ds} + g_m R_1 + g_m R_s + \frac{R_2 + R_s}{r_{ds}}}$$

[r_{ds} is very large].

$$\text{Gain} \simeq \frac{1 + g_m (R_1 - R_2)}{1 + g_m (R_1 + R_s)}$$

$$\boxed{\text{Gain} \simeq \frac{R_1 - R_2}{R_1 + R_s}} \rightarrow (13), \quad g_m \text{ is very large.}$$

Estimated B.W.:-

- * If the system consists of all the poles and the feedback γ_n and on A_m CE
then the bandwidth of the s/m depends on the Gain.

Gain \downarrow B.W \uparrow

- * If the system consists of zeros & complex conjugate poles with increasing order of s/m's then the bandwidth of the s/m depends on the delay.

Delay \uparrow B.W \uparrow .

Cascaded Amplifier:-

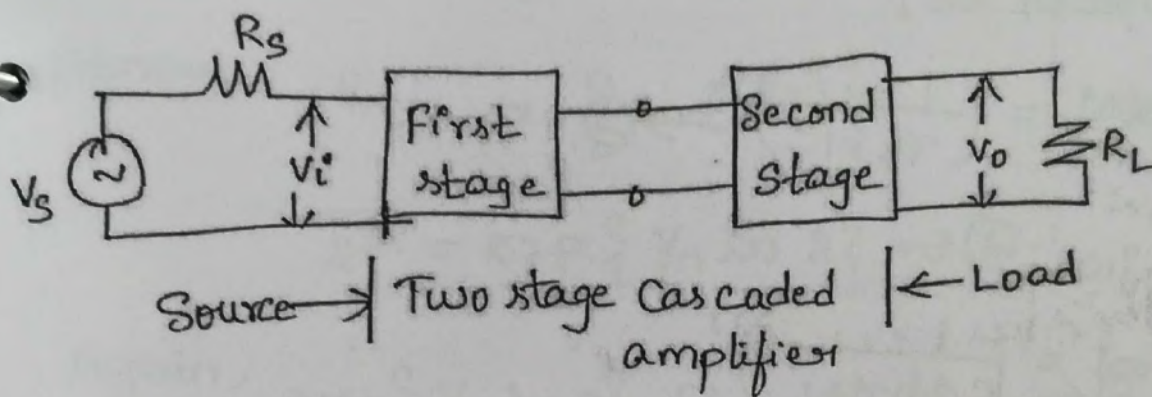
Introduction:-

- * If the voltage (or) power gain obtained from a single stage small s/g amplifier is not sufficient for practical applications, then one have to use more than one stage of amplification to achieve necessary voltage & power gain.
- * Such an amplifier is called as "Multistage Amplifier".
- * The o/p of one stage is fed as an i/p to the next stage.
- * Such a connection is commonly called as "Cascading".

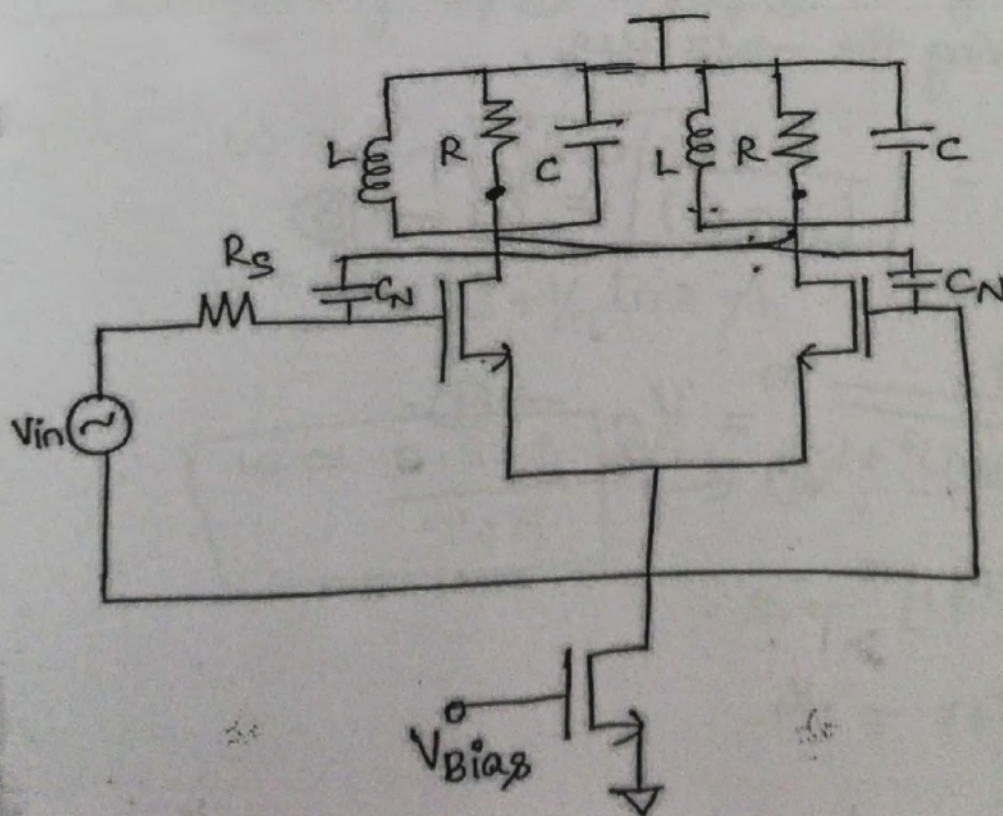
In amplifiers Cascading is done to achieve correct input and output impedances for specific applications.

Defn.: A multistage amplifier using two (or) more single stage CE amplifiers is called as "Cascaded Amplifier".

* A multistage amplifier with CE as first stage and CB as second stage is called as "Cascoded Amplifier".



Cascaded Amplifier ckt for MOSFET:-



* Let us consider each amplifier stage has a unit dc and a single pole.

* The amplifier transfer function is,

$$H(s) = \frac{1}{\tau s + 1} \rightarrow \textcircled{1}$$

* A cascaded 'n' such amplifiers will therefore have an overall transfer function as,

$$A(s) = \left(\frac{1}{\tau s + 1} \right)^n \rightarrow \textcircled{2}$$

* Set $s = j\omega$,

$$A(j\omega) = \left(\frac{1}{j\omega\tau + 1} \right)^n.$$

* The B.W can be computed by its magnitude of the transfer function & solving the -3dB freq,

$$|A(j\omega)| = \left| \frac{1}{(j\omega\tau + 1)} \right|^n = 1/\sqrt{2} \rightarrow \textcircled{3}$$

$$\therefore \left(\frac{1}{\sqrt{(\omega\tau)^2 + 1}} \right)^n = 1/\sqrt{2} \rightarrow \textcircled{4}$$

$$[(\omega\tau)^2 + 1]^n = 2.$$

$$(\omega\tau)^2 + 1 = 2^{1/n}.$$

$$(wT)^2 = 2^{1/n} - 1.$$

$$w^2 = 1/T^2 \cdot [2^{1/n} - 1].$$

$$\therefore \boxed{w = 1/T \sqrt{2^{1/n} - 1}} \rightarrow (5)$$

* For larger 'n' values,

$$e^x \equiv \exp \{ \ln(x) \}.$$

Hence, $2^{1/n} = \exp \{ \ln(2^{1/n}) \}.$

$$2^{1/n} = \exp \{ 1/n \ln 2 \} \rightarrow (6).$$

Again, $\exp \{ 1/n \ln 2 \} \cong 1 + 1/n \ln 2 \rightarrow (7)$

\therefore substituting eq (7) in eq (5) we get,

$$w = 1/T \sqrt{2^{1/n} - 1}$$

$$w = 1/T \sqrt{1 + 1/n \ln 2 - 1}$$

$$\therefore \boxed{w \cong \frac{0.833}{T\sqrt{n}}} \rightarrow (8)$$

Overall Gain^{BW} of the Cascaded Amp:-

- * If the overall gain of the amplifier is G .
- * Then each amplifier stage has the gain as $G^{1/n}$.
- * The Gain-bandwidth product of the amplifier is ω_T .
- * Then the Bandwidth of the amplifier is,

$$BW = \frac{\omega_T}{G^{1/n}} \rightarrow (9)$$

- * Hence the total bandwidth of the Cascaded amplifier is,

$$(BW)_{tot} \approx \frac{\omega_T}{G^{1/n}} \cdot \frac{\sqrt{\ln 2}}{\sqrt{n}} \rightarrow (10)$$

- * The reciprocal of the B.W is,

$$\frac{1}{(BW)_{tot}} \approx \left(\frac{1}{\omega_T \sqrt{\ln 2}} \cdot \sqrt{n} \right) G^{1/n}$$

- * To max the total B.W, minimize its reciprocal.

$$\frac{d}{dn} (\sqrt{n} G^{1/n}) = 0.$$

$$UV' + U'V = 0.$$

$$V' = \frac{d}{dn} [G^{1/n}] = G^{1/n} \cdot \log G \cdot -1/n^2$$

$$U' = \frac{d}{dn} (\sqrt{n}) = 1/2\sqrt{n}$$

$$-\sqrt{n} \cdot \frac{G^{1/n}}{n^2} \cdot \ln G + 1/2\sqrt{n} G^{1/n} = 0.$$

$$\frac{d}{dx} [a^{f(x)}] = a^{f(x)} \cdot \log a \cdot f'(x)$$

$$-\sqrt{n} \cdot \frac{G^{1/n}}{n \cdot \sqrt{n} \cdot n} \cdot \ln G + \frac{1}{2\sqrt{n}} G^{1/n} = 0$$

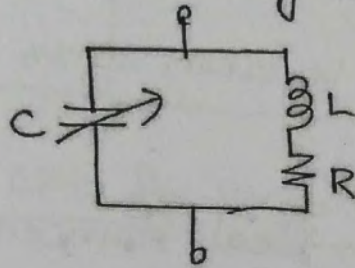
$$\frac{G^{1/n}}{\sqrt{n}} \left[\frac{-\ln G}{n} + 1/2 \right] = 0$$

$$\log_e G = 1/2 \Rightarrow G = e^{1/2}$$

new
To amplifier load

Tuned Amplifier:-

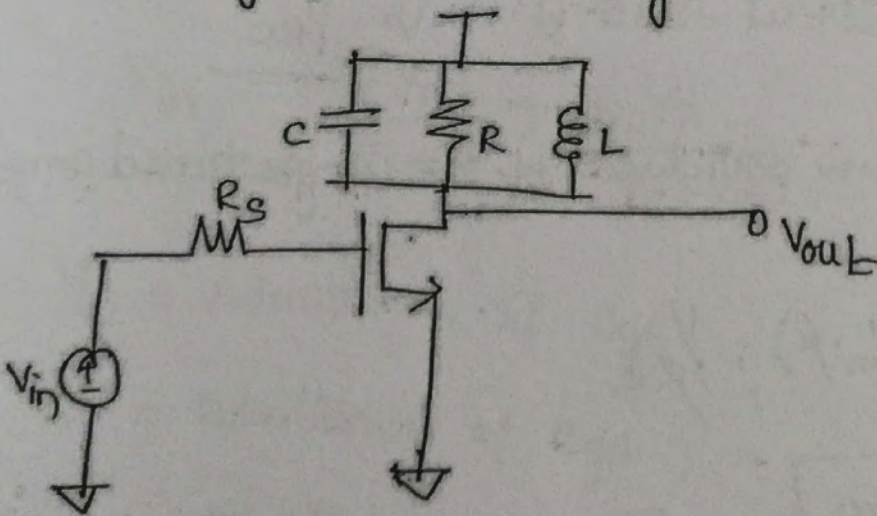
To amplify the selective range of frequencies, the resistive load R_c is replaced by a tuned ckt.



Tuned ckt

- * The tuned ckt is capable of amplifying a signal over a narrow band of freq's centered at f_r .
- * The amplifier with such a tuned ckt as a load are called as Tuned Amplifier.
- * Tuned amplifiers are used extensively in communication system to provide selective amplification of wanted sig's.

→ filtering of unwanted signals.



(a) Amplifier with single tuned load.

Case ii, If $V_s = 0$.

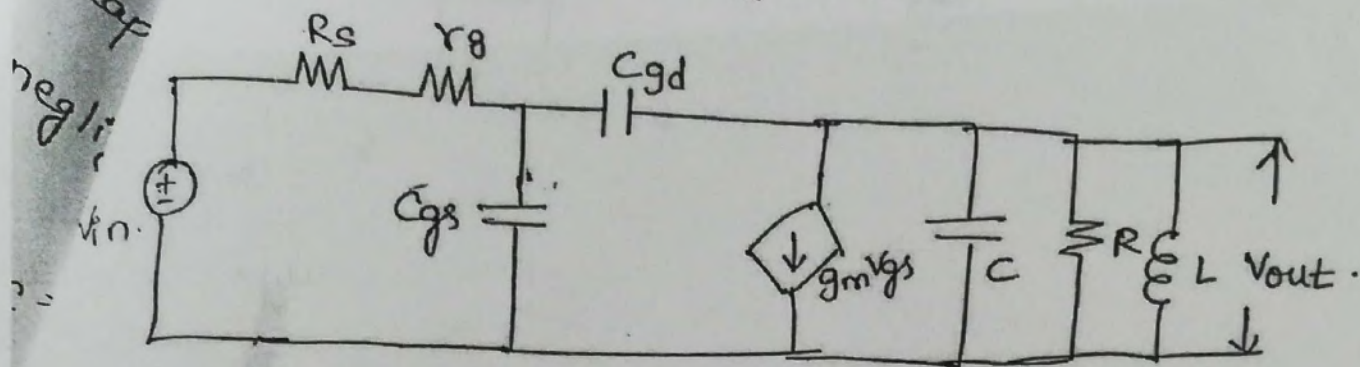
- * If we drive from a zero-impedance source then the caps at drain, source & gate (i.e) (C_{gd} & C_{gs}) becomes negligible small.
- * Hence the o/p of the ckt is taken across the Capacitor 'c' of the tuned ckt.
- * Hence the ckt acts as an ideal transconductor driving a Parallel RLC ckt.
- * At low freq's the inductor acts as \rightarrow short ckt.
- * At high freq's the Capacitor acts as \rightarrow short ckt.
- * At both these freq's the amount of Gain = 0.
- * At resonant freq. of the ckt, the gain becomes $g_m R$.
(since inductance & capacitance cancels each other).
- * For the ckt the total -3dB B.W is $1/R_c$.
- * Hence the Gain B.W product of the single tuned amplifier

is,

$$G \cdot BW = (g_m \cdot R) \cdot \frac{1}{R_c}$$

$$\therefore \boxed{G \cdot BW = \frac{g_m}{c}} \rightarrow \textcircled{1}$$

If $V_S \neq 0$, $R_S = \text{non-zero value}$.



(b) Eqn ckt for single tuned amp ckt.

* If the non-zero source resistance & non-zero series gate resistance are considered then its equivalent capacitance is,

$$C_{eq} = C_{gd} [1 + g_m R_{eq}] \rightarrow (2)$$

$$[\because R_{eq} = R_S + r_g]$$

$$C_{eq} = C_{gd} [1 + g_m (R_S + r_g)] \rightarrow (3)$$

The overall admittance of the amplifier is,

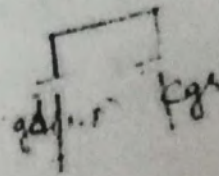
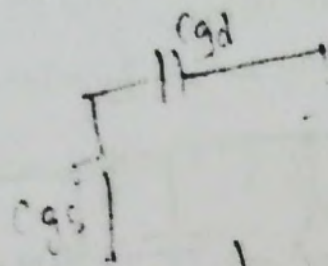
$$Y_{in} = \frac{Y_L Y_F}{Y_L + Y_F} + \frac{g_m Y_F}{Y_L + Y_F} \rightarrow (4)$$

Y_{in} = Admittance of C_{gs} .

Y_F = Admittance of C_{gd} .

Y_L = Admittance of RLC ckt.

Explanation



Miller Theorem

$$\left(\frac{Z}{1-K}, \frac{Z \cdot K}{K-1} \right)$$

$$C_{gd}(1-A) + C = C_{eq}$$

↑
(capacitor)

$$C_{eq} = C_{gd} [1 - A]$$

$$\text{gain } A = \frac{g_m \cdot Z}{1 + g_m \cdot R_{eq}}$$

$$A = -g_m \cdot R_{eq}$$

$$A = -g_m (R_s + R_{eq})$$

$$C_{eq} = C_{gd} [1 + g_m (R_s + R_{eq})]$$

Noise

The sensitivity of comm. syms is limited by noise. The broadest definition of noise as "everything except the desired sigl". In other words we can say "unwanted sigl" as noise.

In audio systems, noise is recognizable as a continuous hiss. In video the noise manifests itself as "snow" of analog TV syms.

Thermal Noise

Thermally agitated charge carriers in a conductor tends to a randomly varying current that gives rise to a random vtz.

Because the noise process is random, we can not identify a specific value, we can only characterize the noise as mean square or rms value.

The thermal noise power is proportional to T . Specifically,

ly,

$$\text{Available noise Power } P_{NA} = k \cdot T \cdot \Delta f$$

Hence, k - Boltzmann's constant
 $= 1.38 \times 10^{-23} \text{ J/K}$

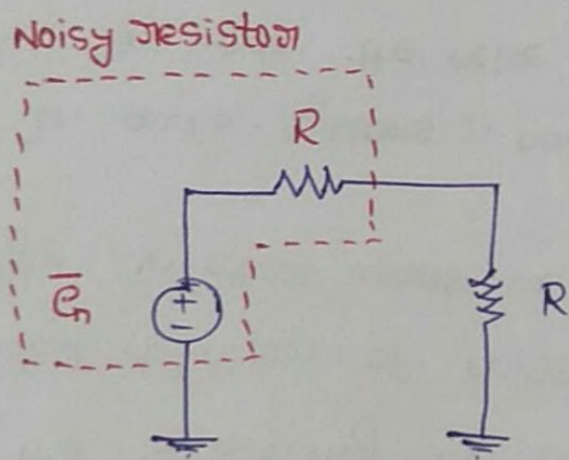
T - Absolute Temperature in kelvins.

Δf - Noise B.W. in Hz.

The Thermal noise power is constant at all freq, hence it is often called as white noise, by analogy with white light.

The available noise power P_{NA} is the maximum power that can be delivered to the load.

consider the n/w given below.



The available Noise power $P_{NA} = k \cdot T \cdot \Delta f = \frac{\bar{e}_n^2}{4R}$

where

\bar{e}_n - open ckt rms noise vtz generated by the resistor 'R' over the B.W Δf at given temperature.

The mean-square open ckt noise vtz

$$\bar{e}_n^2 = 4kTR\Delta f.$$

Generally noise is specified in terms of spectral density

E_n is

$$\frac{\bar{e}_n^2}{\Delta f} \quad (\text{or}) \quad \frac{\bar{e}_n}{\sqrt{\Delta f}} = 4kTR.$$

To reduce the noise of given resistance,

* Keep the temperature as low as possible.

* Limit the B.W. to the minimum useful value.

The noise B.W. $\Delta f = \frac{1}{|H_{pk}|^2} \int_0^\infty |H(f)|^2 \cdot df$

here, H_{pk} - Peak value of filter vtz transfer function.

Let us consider, a single pole RC LPF, w.k.T. its 3dB B.W. is $\frac{1}{2\pi RC}$. but the equivalent noise B.W Δf is

$$\Delta f = \frac{1}{11^2} \int_0^{\infty} \frac{1}{1 + (2\pi f RC)^2} \cdot df$$

From the basics of Filters

$$\text{LPF Gain} = \frac{1}{\sqrt{1 + (2\pi f RC)^2}}$$

& here treated as $H(f)$.

$$= \frac{1}{2\pi RC} \cdot \arctan(2\pi f RC) \Big|_0^{\infty}$$

$$\int_0^{\infty} \frac{1}{1+x^2} \cdot dx = \arctan x$$

$$= \frac{1}{2\pi RC} \cdot \left(\frac{\pi}{2} - 0\right)$$

$$= \frac{1}{4RC} \quad (\text{or}) \cdot \frac{\pi}{2} \cdot f_{3dB}$$

\therefore single pole LPF has a noise B.W about 1.57 times 3dB band width. Similarly, a critically damped second order LPF has a noise B.W about 1.22 times 3dB B.W.

\therefore As the rolloff becomes steeper, 3dB & noise B.W's tend to converge.

However, by taking several considerations into account, the noise v_{tz} is

$$\overline{e_n^2} = \frac{h \cdot \omega \cdot R \Delta f}{\pi} \cdot \coth \left[\frac{h\omega}{4\pi kT} \right]$$

where h - plank const.

$$= 6.62 \times 10^{-34} \text{ J}\cdot\text{s}$$

In fact there is no difference b/w this equation & earlier equ. for freq's below 80 THz at room temperature.

Note that, purely reactive elements generate no thermal noise. But, the noisy currents flowing through any impedance, reactance or resistance will give rise to a noisy v_{tz} .

Thermal Noise in MOSFETs

Drain current Noise :

Since FETs are v_{tz} controlled resistors, they exhibit thermal noise. Particularly, it can be experienced in triode region.

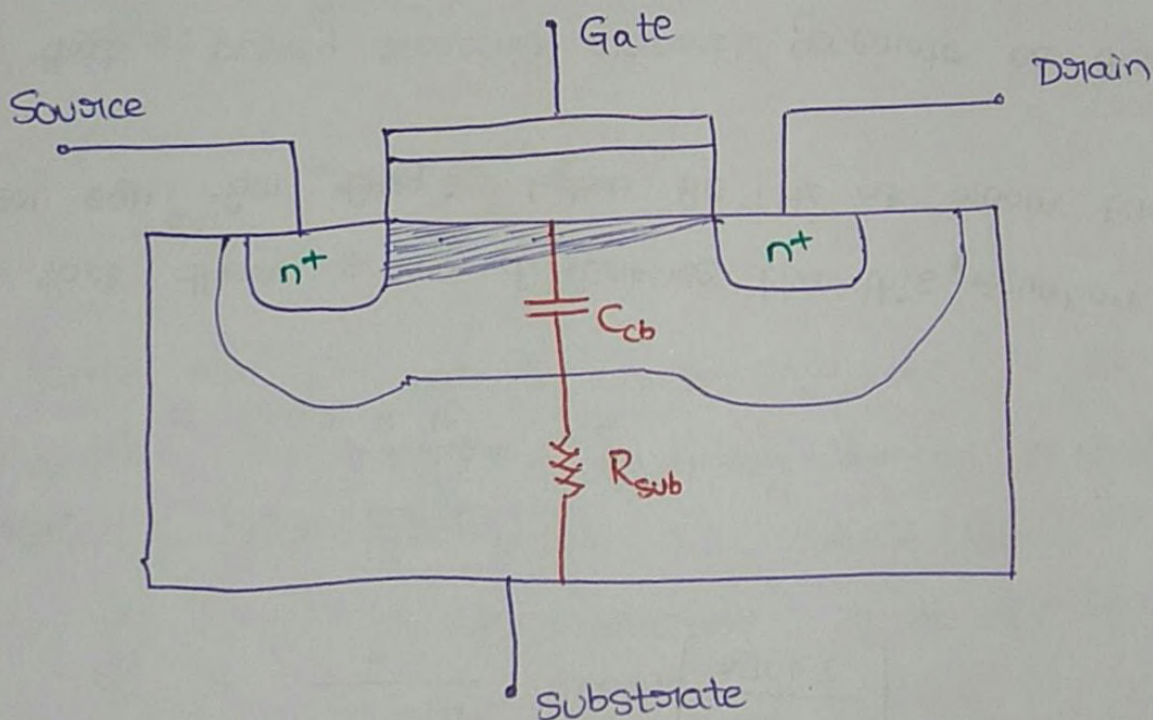
The detailed theoretical considerations leads to following expression

$$\overline{i_{nd}^2} = 4kT \alpha g_{do} \Delta f$$

where - g_{do} - drain to source conductance at zero V_{DS} .

$\alpha = 1$ at $V_{DS} = 0$. ϵ_1

α in log devices, decreases toward a value $\frac{2}{3}$ in saturation



The above picture depicts, how the thermal noise caused by substrate resistance can produce considerable effects

at main terminals.

At low freq's, C_{cb} is negligible

$\therefore R_{sub}$ contributes some noisy drain current

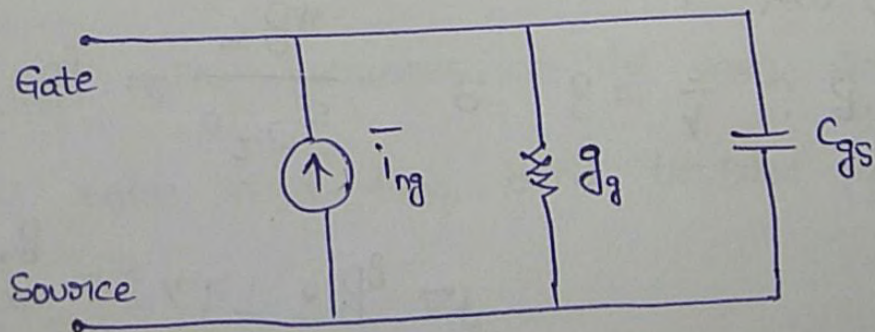
$$\overline{i_{nd-sub}^2} = 4kT R_{sub} g_{mb}^2 \Delta f$$

At high freq's

$$\overline{i_{nd-sub}^2} = \frac{4kT R_{sub} g_{mb}^2 \Delta f}{1 + (\omega R_{sub} C_{cb})^2}$$

Gate Noise :

The fluctuating channel potential couples capacitively into the gate terminal, leading to a noisy gate current.

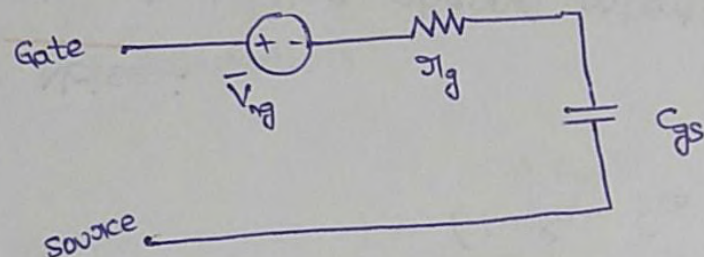


The gate noise may be expressed as

$$\overline{i_{ng}^2} = 4kT \delta g_g \Delta f$$

where $g_g = \frac{\omega^2 C_{gs}^2}{5 \cdot g_{do}}$; $\delta = \frac{4}{3}$; gate noise co-efficient in long channel devices.

Its alternate gate noise ckt model is



To derive this alternate model, the $\parallel RC$ n/w is transformed into an equivalent series RC n/w.

$$\therefore \text{series resistance } r_g = \frac{1}{g_g} \cdot \frac{1}{(Q^2+1)}$$

$$= \frac{1}{g_g \cdot Q^2} ; Q^2 \gg 1$$

$$r_g = \frac{1}{5 \cdot g_{do}}$$

The equivalent series noise vtz source is found to be

$$\overline{V_{ng}^2} = \overline{i_{ng}^2} \cdot g_g^2$$

$$= 4kT \delta \frac{g_g}{\omega_g} \Delta f \cdot \frac{1}{g_g} \cdot g_g$$

$$\overline{V_{ng}^2} = 4kT \delta g_g \cdot \Delta f$$

which gives a constant spectral density.

However, it is reasonable to assume that δ continues to be about twice as large as γ . Hence, γ is typically 1-2 for short channel NMOS devices, & $\therefore \delta$ is 2-4.

Flicker Noise ($\frac{1}{f}$ noise or pink noise)

The flicker noise increases as the freq. decreases.

The total noise in a freq. band bounded by lower freq f_L & higher freq f_H .

$$\overline{N^2} = \int_{f_L}^{f_H} \frac{k}{f} \cdot df = k \cdot \ln \frac{f_H}{f_L}$$

\overline{N} - rms noise (voltage or current)

K - empirical parameter (i.e., device specific).

Flicker Noise in Resistors :

The resistor exhibits $\frac{1}{f}$ noise only when there is DC current flowing through it. ξ is dependent of various parameters as given in below equ.

$$\overline{e_n^2} = \frac{K}{f} \cdot \frac{R_D^2}{A} \cdot V^2 \cdot \Delta f$$

where, A - area of resistor

R_D - sheet resistance

V - vtz across resistor.

$K \approx 5 \times 10^{-28} \text{ s}^2 \cdot \text{m}^2$ for diffused & ion implanted resistor.

Flicker Noise in MOSFETs :

Flicker noise is most prominent in devices that are sensitive to surface phenomena. Hence MOSFETs exhibit more pink noise than bipolar devices.

charge trapping phenomena is generally used to explain $\frac{1}{f}$ noise in transistors. Some types of defects & certain impurities can randomly trap & release charge.

Larger MOSFETs exhibit less $\frac{1}{f}$ noise b'cos their larger gate capacitance smooths the fluctuations in channel charge.

The mean square $\frac{1}{f}$ drain noise current

$$\overline{i_n^2} = \frac{k}{f} \frac{g_m^2}{W L C_{ox}^2} \cdot \Delta f$$

$$= \frac{k}{f} \cdot \omega_T^2 \cdot A \cdot \Delta f$$

where A - area of gate
 $= WL$

ω_T - Gain-B.W product

Thus for a fixed transconductance, a larger gate area & a thinner dielectric reduce the noise.

Noise Figure

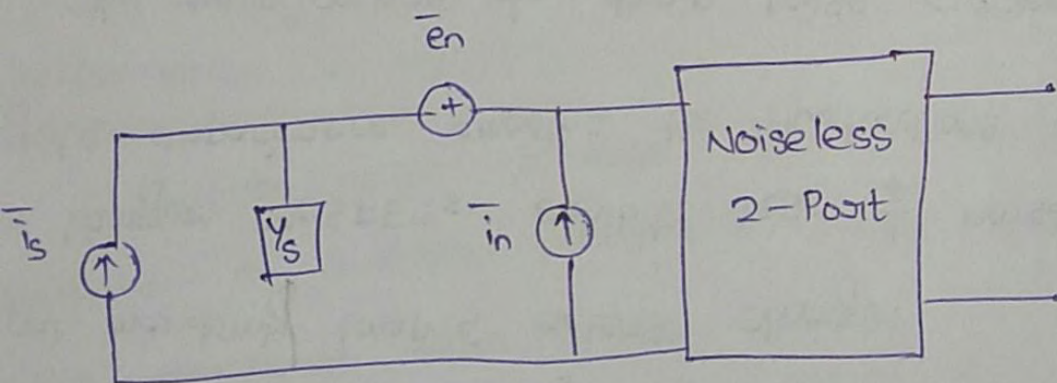
Noise Factor is an useful measure of noise performance,

The Noise Figure [NF or F_{dB}] is simply the noise Factor (F) expressed in dB.

To define & understand the noise factor, consider a noisy (but linear) 2-port n/w. driven by a source that has an admittance Y_s & an equivalent shunt noise current \bar{I}_s . Let the net effect of all noise sources can be represented by just one pair of ext sources: a noise vtz & a noise current.

Conventionally, the source is at a temperature of 290 K,

the noise factor $F = \frac{\text{Total o/p noise power}}{\text{o/p noise due to i/p source.}}$



The noise factor is a measure of degradation in SNR.
Larger the degradation, larger the noise factor.

Assume that, noise powers of the source E_s of the 2-port are uncorrelated, then the expression for noise factor

$$F = \frac{\overline{i_s^2} + \overline{|i_n + Y_s \cdot e_n|^2}}{\overline{i_s^2}} \quad \text{--- (1)}$$

Now, let to accommodate the possibility of correlation b/w e_n & i_n , express i_n as sum of two components i_c & i_u , where i_c is correlated with e_n & i_u is not.

$$\therefore i_n = i_c + i_u$$

$\therefore i_c$ is correlated with e_n ,

$$i_c = Y_c \cdot e_n$$

where Y_c - correlation admittance.

Substitute in eqn (1).

$$F = 1 + \frac{\overline{|(i_c + i_u) + Y_s \cdot e_n|^2}}{\overline{i_s^2}}$$

$$= 1 + \frac{\overline{i_u^2} + |Y_c + Y_s|^2 \cdot \overline{e_n^2}}{\overline{i_s^2}}$$

The above eqn has 3 independent noise sources & their resistances or conductances are

$$R_n = \frac{\overline{e_n^2}}{4kT \Delta f}$$

$$G_u = \frac{\overline{i_u^2}}{4kT \Delta f}$$

$$G_s = \frac{\overline{i_s^2}}{4kT \cdot \Delta f}$$

$$\therefore F = 1 + \frac{G_u + |Y_c + Y_s|^2 \cdot R_n}{G_s}$$

$$= 1 + \frac{G_u + [(G_c + G_s)^2 + (B_c + B_s)^2] \cdot R_n}{G_s}$$

$$\because Y = G + jB$$

conductance
+ susceptance

LNA Design

The first stage of R_x is typically a Low Noise Amp [LNA], & its main function is to provide enough gain to overcome the noise of subsequent stages. Along with this an LNA should accommodate large sigs without distortion while adding noise as low as possible. Also it has to present specific impedance such as 50Ω to the i/p source.

To develop a design strategy that balances gain, i/p impedance, noise figure & power consumption, it is required to know Four noise parameters G_c , B_c , R_n & G_u .

Intrinsic MOSFET Two port Noise Parameters

W.K.T.

$$\overline{e_n^2} = 4kTR_n\Delta f$$

$$\therefore \text{But also } \overline{e_n^2} = \frac{\overline{i_{nd}^2}}{g_m^2} = \frac{4kT \gamma g_{do} \Delta f}{g_m^2}$$

$$\therefore R_n = \frac{\overline{e_n^2}}{4kT \Delta f}$$

$$= \frac{\frac{4kT \cdot g_{do} \cdot \Delta f}{g_m^2}}{4kT \cdot \Delta f}$$

$$R_n = \frac{g_{do}}{g_m^2} \quad (\text{cor}) \quad R_n = \frac{1}{\omega} \cdot \frac{1}{g_m} \quad ; \quad \text{where} \quad \omega = \frac{g_m}{g_{do}}$$

W.K.T.

$$G_u = \frac{\overline{i_u^2}}{4kT \cdot \Delta f}$$

$$\text{Correlation admittance } Y_c = \frac{i_c}{e_n} = G_c + jB_c \quad \text{--- (1)}$$

After a great analysis [Refer page No : 366-367]

$$Y_c = j\omega C_{gs} - j\omega C_{gs} \frac{g_m}{g_{do}} |c| \sqrt{\frac{\delta}{51}}$$

$$\text{where } c = \frac{\overline{i_{ng} \cdot i_{nd}^*}}{\sqrt{\overline{i_{ng}^2}} \cdot \sqrt{\overline{i_{nd}^2}}} \quad ; \quad \text{correlation coefficient}$$

$$\therefore Y_c = j\omega C_{gs} \left[1 - \infty |c| \sqrt{\frac{\delta}{5n}} \right] \quad \text{--- (2)}$$

\therefore From the above eqn, correlation admittance is purely imaginary.

comparing the real & imaginary parts of eqn (1) & (2) gives

$$G_c = 0$$

$$B_c = \omega C_{gs} \left[1 - \infty |c| \sqrt{\frac{\delta}{5n}} \right]$$

The induced gate noise is sum of correlated & uncorrelated gate noises.

$$\therefore \overline{i_{ng}^2} = \overline{(i_{ngc} + i_{ngu})^2}$$

$$= 4k \cdot T \cdot \Delta f \cdot \delta \cdot g_g |c|^2 +$$

$$4kT \cdot \Delta f \cdot \delta \cdot g_g (1 - |c|^2).$$

In the above eqn, second term is uncorrelated position of gate noise current.

$$\begin{aligned}\therefore G_u &= \frac{i_u^2}{4kT \cdot \Delta f} \\ &= \frac{4 \cdot kT \cdot \Delta f \cdot \delta \cdot g_g (1 - |c|^2)}{4 \cdot kT \cdot \Delta f}\end{aligned}$$

$$= \delta \cdot g_g (1 - |c|^2)$$

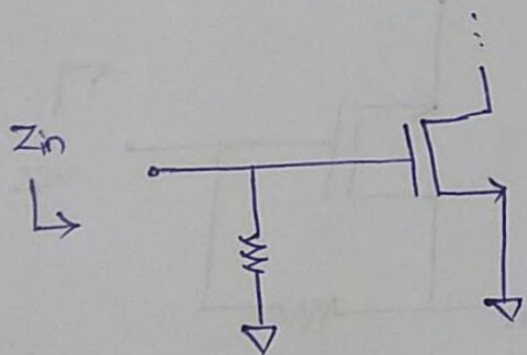
$$G_u = \delta \cdot (1 - |c|^2) \cdot \frac{\omega^2 \cdot C_{gs}^2}{5g_{do}}$$

Power Match Vs Noise Match

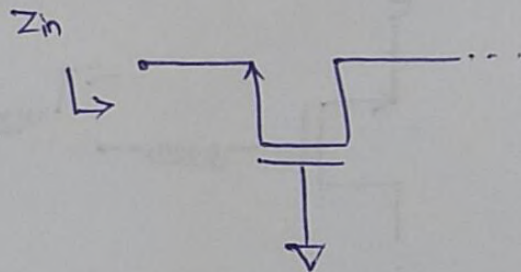
Using optimum noise matching, minimum noise fig of LNA is obtained. On the other hand, Power gain [conjugate impedance matching (from max. power transfer theorem)] yields the max. available power gain for a ckt. These two matchings are contradictory. Fortunately, it is possible, simultaneously noise & power matching in CMOS Technology.

For max. power gain matching, the i/p impedance of LNA must have a resistive term. Then matching n/w transforms this resistance to the real part of source impedance.

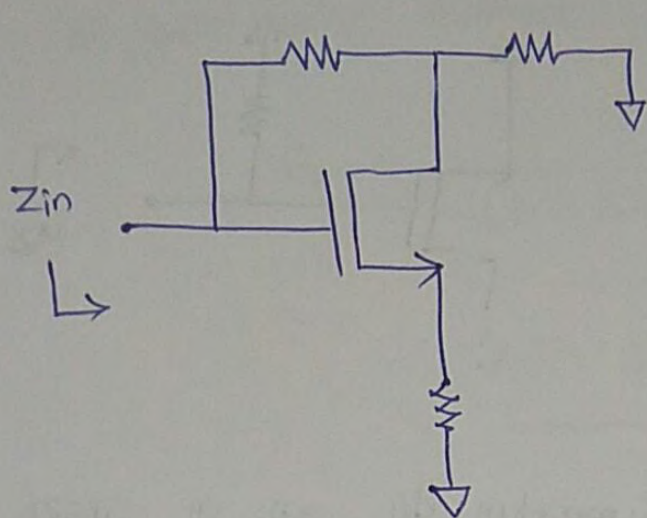
Different classic techniques to produce required resistive term in the i/p impedance of LNA are shown below.



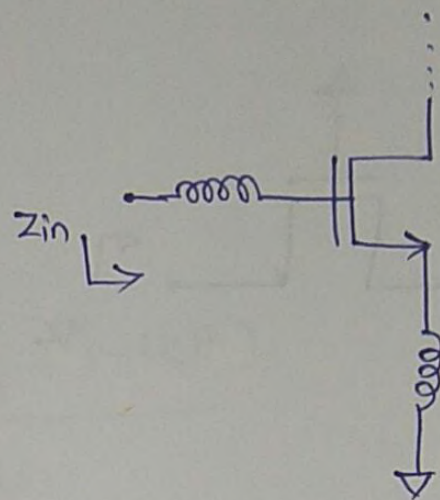
(a)



(b)



(c)



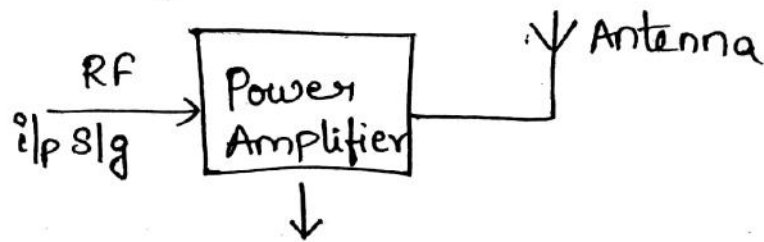
(d)

In case of common gate stage [fig (b)], the resistive term is part of i/p impedance to the source of the CG transistor.

In case of common source [Fig (a) & (d)] or cascode stage i.e., shunt-series amp [Fig (c) ; biasing & V_{in} & V_{out} are not shown for convenience], the i/p impedance is capacitive & hence a resistive part should be added to the i/p impedance. This is done by a \parallel resistance in the gate [Fig (a)] or a resistive feedback [fig (c)]

RF Power Amplifier.Introduction:-

⇒ The power amplifier is the last block of the transmitter.



↓
To pump out as much energy as possible to the antenna.

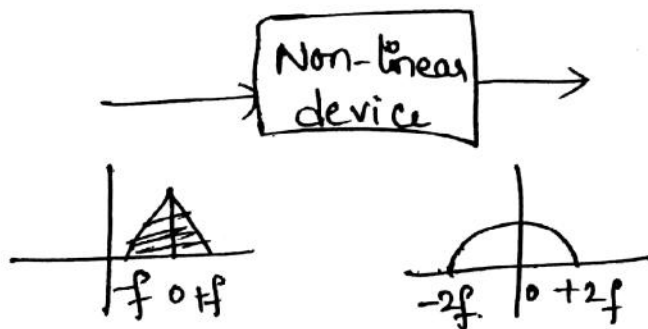
⇒ The PA provides broadcast of the messages.

⇒ The main considerations of the power amplifier is,

→ Efficiency

→ Linearity.

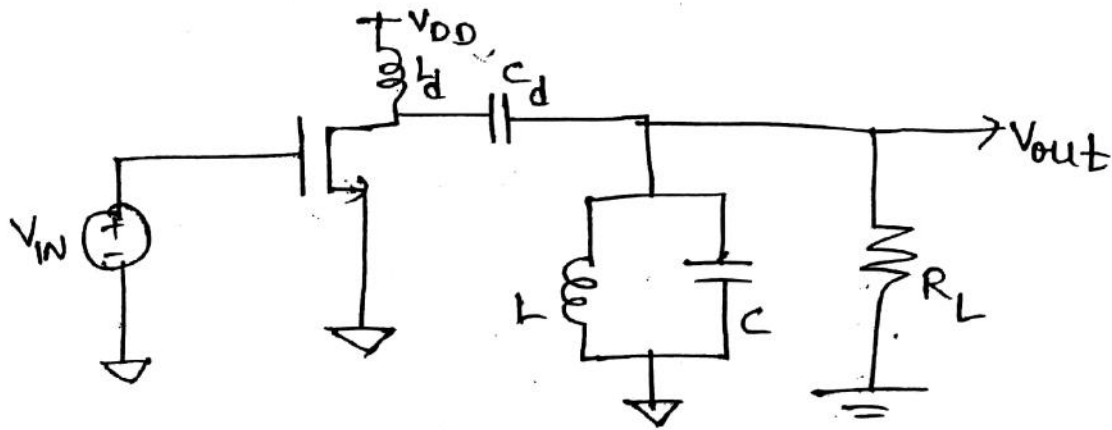
⇒ If a s/m is non-linear then,



⇒ Power amplifier is used to increase the amount of the power at the o/p.

Generalized power amplifier model:-

class



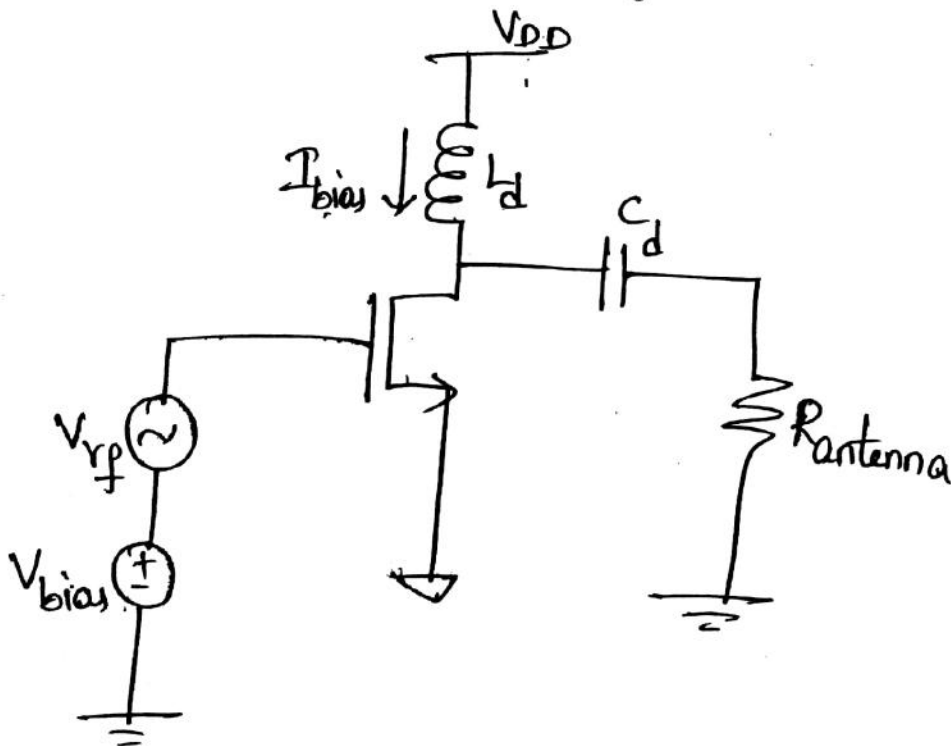
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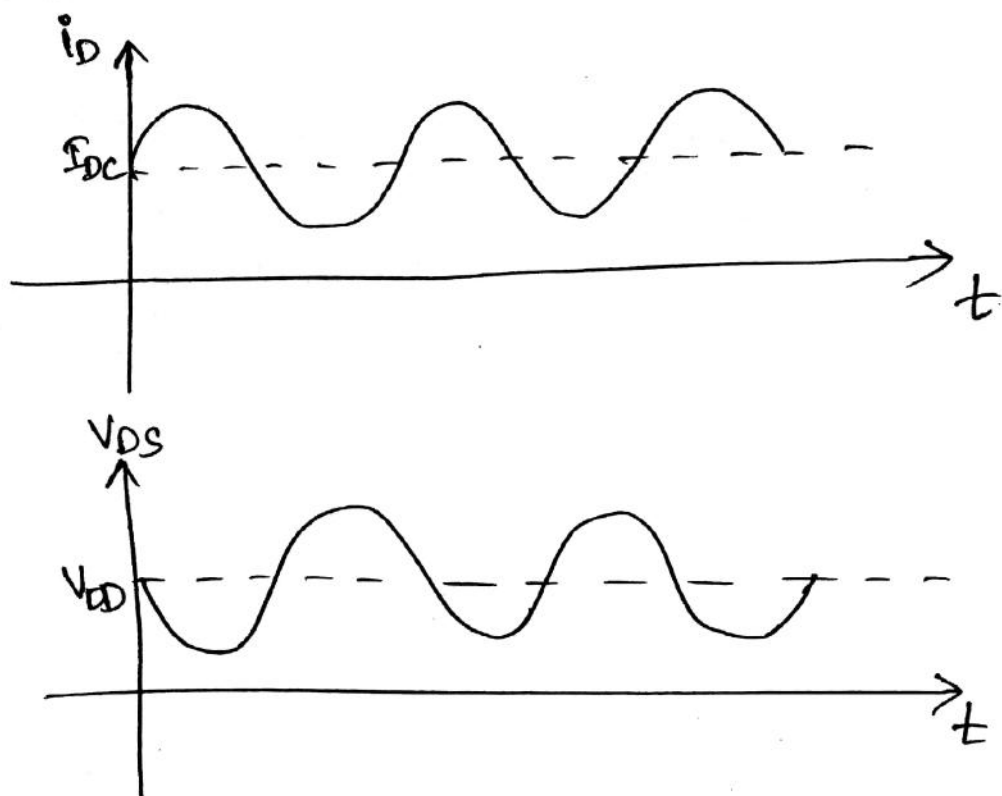
- \Rightarrow ' R_L ' is the load resistance through which the o/p power is delivered
- \Rightarrow The inductor ' L_d ' feeds DC power to the drain and is assumed to be large enough so that the current through it is substantially constant.
- \Rightarrow The advantage of this configuration is that the o/p capacitance is absorbed by the tank-ckt first.
- \Rightarrow Filtering provided by this tank ckt reduces the emissions caused by the non-linearities.
- \Rightarrow Depending upon the conduction angle and the biasing of the amplifiers, the power amplifiers are classified as,

- \rightarrow class A
- \rightarrow class B
- \rightarrow class C
- \rightarrow class AB
- \rightarrow class D
- \rightarrow class E
- \rightarrow class F

Class A power Amplifier:-

- ⇒ For the class A power amplifier the tank ckt reduces the distortions and provides linearity.
- ⇒ The conduction angle for the class A power amp is 360° .
- ⇒ The drain current for the amplifier is equal to the full cycle of time period.
- ⇒ MOSFET in the class A PA is in the saturation region.
- ⇒ That means the current through the MOSFET will be always > 0 to make MOSFET ON.
- ⇒ MOSFET has the voltage < 0 to make MOSFET OFF.





Drain vol & current of an ideal class A PA.

⇒ Assume that the drain current is,

$$i_D = I_{DC} + i_{rf} \sin \omega_0 t \rightarrow \textcircled{1}$$

\downarrow bias current \downarrow amp of the s/g component of the drain current.

' ω_0 ' - s/g freq (resonant freq).

⇒ The o/p voltage is product of the s/g current & the load resistance,

$$V_o = -i_{rf} R_L \sin \omega_0 t \Rightarrow \textcircled{2}$$

⇒ The avg s/g power delivered to the resistor R_L is,

$$P_{rf} = \frac{i_{rf}^2 R_L}{2} \rightarrow \textcircled{3}$$

⇒ The bias current should be equal to the offset current to get the maximum amount of the power.

$$I_{DC} = i_{rf} \rightarrow (4).$$

⇒ The DC i/p power,

$$P_{DC} = I_{DC} V_{DD}.$$

$$P_{DC} = i_{rf} V_{DD} \rightarrow (5).$$

⇒ The efficiency of the power amp is,

$$\eta = \frac{\text{RF o/p power}}{\text{DC i/p power}}.$$

$$\eta = \frac{P_{rf}}{P_{DC}} = \frac{(i_{rf}^2)(R/2)}{i_{rf} V_{DD}}.$$

$$\eta = i_{rf}' \cdot \frac{R}{2V_{DD}}.$$

$$\therefore \% \eta = \frac{1}{2} \left(\frac{i_{rf} R}{V_{DD}} \right) \rightarrow (6)$$

* Hence the drain current for the class A PA is just only 50%.

* For a class A power amplifier maximum drain-to-source voltage is $2V_{DD}$.

* The peak drain current of the amp is $2V_{DD}/R$.

* Stress on the devices also specifies the efficiency.



Normalized power o/p capability.

$$P_N = \frac{\text{Actual o/p power}}{\text{Product of max device vol \& current}}$$

$$P_N = \frac{P_{rf}}{V_{DS,PK} \times I_{D,max}}$$

$$P_N = \frac{\frac{V_{DD}^2}{2R}}{(2V_{DD}) \left(\frac{2V_{DD}}{R} \right)}$$

$$P_N = \frac{\cancel{V_{DD}^2}}{2\cancel{R}} \times \frac{\cancel{R}}{4\cancel{V_{DD}^2}}$$

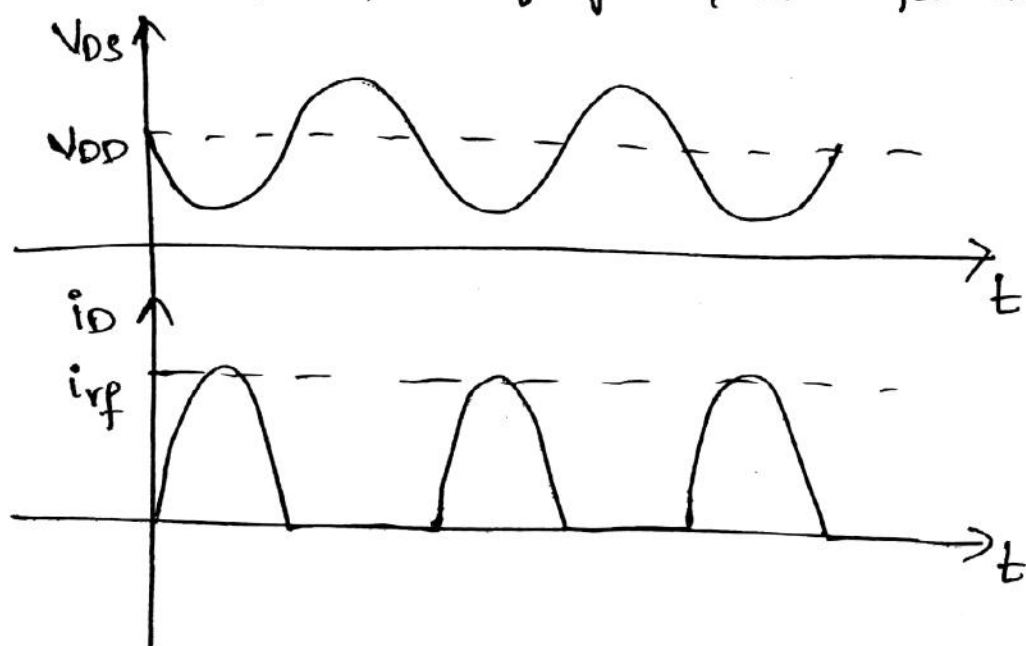
$$\boxed{P_N = 1/8} \rightarrow \textcircled{7}$$

* Hence the class A PA provides linearity at a cost of low efficiency & relatively large device stresses.

* Class A PA are rarely used in RF PA.

to sou, 2. Class B power Amplifier:-

- ⇒ For class B power amplifier the conduction angle is 180° .
- ⇒ The biasing is arranged to shunt off the d/p device for every half cycle.
- ⇒ For this type of amp, we assume the drain current to be maximum for one half cycle & zero for another half cycle.



Drain voltage & current for class B PA.

- ⇒ The drain current of the amp is,

$$i_D = i_{rf} \sin \omega_0 t \quad \text{for } i_D > 0 \rightarrow \textcircled{1}$$

- ⇒ The fundamental component of the PA is,

$$\hat{i}_{\text{fun}} = \frac{2}{\pi} \int_0^{\pi/2} (i_{rf} \sin \omega_0 t) (\sin \omega_0 t) \cdot dt$$

$$= \frac{2}{\pi} \int_0^{\pi/2} i_{rf} \sin^2 \omega_0 t \cdot dt$$

$$= \frac{2}{T} i_{rf} \int_0^{T/2} \frac{1 - \cos 2\omega_0 t}{2\omega_0} \cdot dt$$

$$= \frac{1}{T} i_{rf} \left[\int_0^{T/2} 1 \cdot dt - \int_0^{T/2} \cos 2\omega_0 t \cdot dt \right]$$

$$= \frac{1}{T} i_{rf} \left[(t)_0^{T/2} - \left(\frac{\sin 2\omega_0 t}{2\omega_0} \right)_0^{T/2} \right]$$

$$i_{fun} = \frac{1}{T} i_{rf} (T/2) - \frac{1}{T\omega_0} \cdot \frac{\sin 2\omega_0 (T/2)}{2}$$

* At max resonant freq, the value of $\omega_0 = 1$ and the higher order harmonic terms are eliminated.

$$i_{fun} = i_{rf}/2 \rightarrow (2)$$

* The o/p voltage,

$$V_{out} \simeq i_D \cdot R_L$$

$$V_{out} \simeq \frac{i_{rf}}{2} R_L \sin \omega_0 t \rightarrow (3)$$

* The max value of V_{out} is V_{DD} . Hence the current,

$$i_{rf, max} = \frac{2V_{DD}}{R_L} \rightarrow (4)$$

* The o/p power,

$$P_o = \frac{V_o^2}{2R_L}$$

$$P_{o, max} = V_{DD}^2 / R_L \rightarrow (5)$$

⇒ The D.C. drain

⇒ The D.C i/p power can be obtained by the avg of the drain current,

$$\bar{i}_D = \frac{1}{T} \int_0^{T/2} \frac{2V_{DD}}{R_L} \sin \omega_0 t \cdot dt.$$

$$= \frac{1}{2\pi} \int_0^{\pi} \frac{2V_{DD}}{R_L} \sin \phi \cdot d\phi.$$

$$= \frac{1}{2\pi} \cdot \frac{2V_{DD}}{R_L} \left[-\cos \phi \right]_0^{\pi}$$

$$= \frac{V_{DD}}{\pi R_L} \left[-\cos \pi + \cos(0) \right].$$

$$= \frac{V_{DD}}{\pi R_L} \times (2).$$

$$\bar{i}_D = \frac{2V_{DD}}{\pi R_L} \rightarrow (6)$$



∴ The D.C i/p power of the amp,

$$P_{DC} = \frac{2V_{DD}}{\pi R_L} \cdot V_{DD} = \frac{2V_{DD}^2}{\pi R_L} \rightarrow (7).$$

∴ The Efficiency of the amp is,

$$\eta_0 = \frac{P_{o,max}}{P_{DC}}$$

$$= \frac{\frac{V_{DD}^2}{2R}}{\frac{2V_{DD}^2}{\pi R}} = \frac{\cancel{V_{DD}^2}}{2R} \times \frac{\pi R}{\cancel{2V_{DD}^2}}$$

$$\eta_0 = \frac{\pi}{4} = 0.785$$

P.L. $\eta_a = 22.5\%$

3. Class C power Amplifier:-

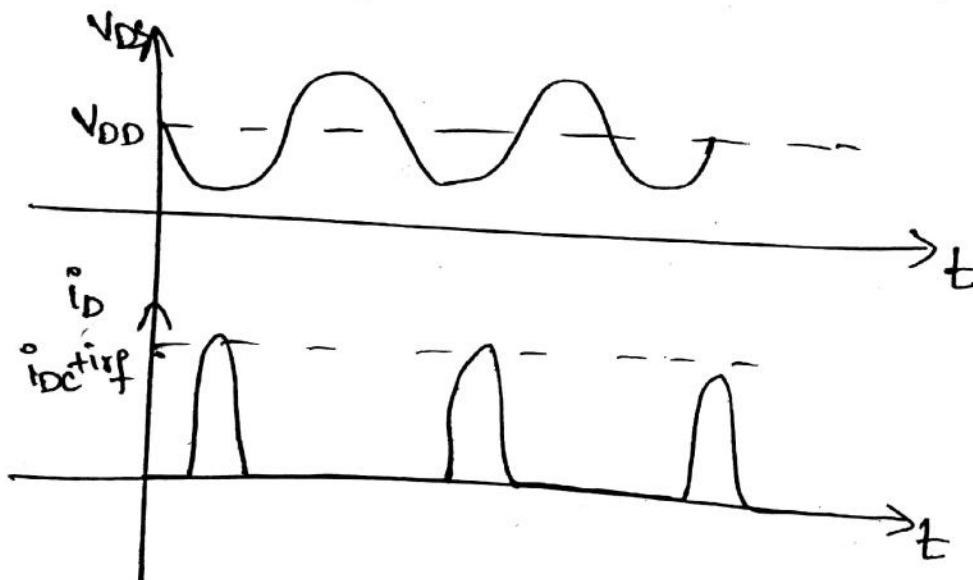
⇒ The conduction angle for the class C PA is < 180°.

⇒ The MOSFET has the gate biasing to conduct for less than the half cycle.

⇒ The drain current,

$$i_D = I_{DC} + i_{rf} \sin \omega_0 t, \quad i_D > 0 \rightarrow (1)$$

⇒ To simplify the equation we rewrite the term sine by cos.



Drain vol & current for class C PA.

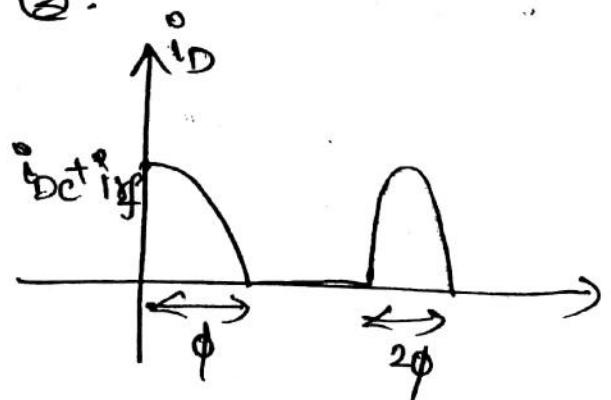
⇒ The drain current for the PA is,

$$i_D = I_{DC} + i_{rf} \cos \omega_0 t \rightarrow (2)$$

⇒ The conduction angle,

$$\phi = \cos^{-1} \left(-\frac{I_{DC}}{i_{rf}} \right)$$

$$2\phi = 2\cos^{-1} \left(-\frac{I_{DC}}{i_{rf}} \right) \rightarrow (3)$$



The bias current, $I_{DC} = -i_{rf} \cos \phi \rightarrow (4)$.

* The -ve sign is due to the conduction angle $< 180^\circ$, the offset current will be negative.

\Rightarrow The avg drain current,

$$\overline{I_D} = \frac{1}{2\pi} \int_{-\phi}^{\phi} (I_{DC} + i_{rf} \cos \phi) \cdot d\phi.$$

$$= \frac{I_{DC}}{2\pi} [\phi]_{-\phi}^{\phi} + \frac{i_{rf}}{2\pi} \left[\int_{-\phi}^{\phi} \cos \phi \cdot d\phi \right].$$

$$= \frac{I_{DC}}{2\pi} [\cancel{\phi}] + \frac{i_{rf}}{2\pi} [\sin \phi]_{-\phi}^{\phi}$$

$$= \frac{I_{DC}\phi}{\pi} + \frac{i_{rf}}{2\pi} [\sin \phi - \sin(-\phi)].$$

$$= \frac{I_{DC}\phi}{\pi} + \frac{i_{rf}}{2\pi} [2 \sin \phi].$$

$$\overline{I_D} = \frac{I_{DC}}{\pi} \phi + \frac{i_{rf}}{\pi} (\sin \phi) \rightarrow (5).$$

Substitute eq (4) in eq (5),

$$\overline{I_D} = \frac{(-i_{rf} \cos \phi) \cdot \phi}{\pi} + \frac{i_{rf}}{\pi} \sin \phi.$$

$$\overline{I_D} = \frac{i_{rf}}{\pi} [\sin \phi - \phi \cos \phi] \rightarrow (6).$$

The fundamental component of the drain current,

i_{fun}

$$i_{fun} = \frac{2}{T} \int_0^T i_D \cos \omega_0 t \cdot dt \rightarrow (7)$$

Sub the value of i_D in eq (7),

$$i_{fun} = \frac{2}{T} \int_0^T (I_{DC} + i_{rf} \cos \omega_0 t) \cdot \cos \omega_0 t \cdot dt$$

$$= \frac{2}{T} \left[\int_0^T I_{DC} \cos \omega_0 t \cdot dt + \int_0^T i_{rf} \cos^2 \omega_0 t \cdot dt \right]$$

$$= \frac{2}{T} \left[I_{DC} \int_0^T \cos \omega_0 t \cdot dt + i_{rf} \int_0^T \cos^2 \omega_0 t \cdot dt \right]$$

$$= \frac{2}{T} \left[\frac{I_{DC}}{\omega_0} (\sin \omega_0 t)_0^T + i_{rf} \int_0^T \left(\frac{1 + \cos 2\omega_0 t}{2} \right)_0^T \right]$$

$$= \frac{2}{T} \left[\frac{I_{DC}}{\omega_0} (\sin \omega_0 t)_0^T + \frac{i_{rf}}{2} (t)_0^T + \frac{i_{rf}}{2} \left(\frac{\sin 2\omega_0 t}{2\omega_0} \right)_0^T \right]$$

$$= \frac{2}{T} \left[\frac{I_{DC}}{\omega_0} (\sin \omega_0 t)_0^T + \frac{i_{rf}}{2} (t)_0^T + \frac{i_{rf}}{4\omega_0} (\sin 2\omega_0 t)_0^T \right] \rightarrow (8)$$

Multiply the eq by ω_0 ,

$$i_{fun} = \frac{2}{T} \left[\frac{I_{DC}}{\omega_0} \cdot \omega_0 (\sin \omega_0 t)_0^T + \frac{i_{rf}}{2} \omega_0 (t)_0^T + \frac{i_{rf}}{4\omega_0} \cdot \omega_0 (\sin 2\omega_0 t)_0^T \right]$$

$$i_{fun} = \frac{2}{T} \left[I_{DC} (\sin \omega_0 t)_0^T + \frac{ir_f}{2} \omega_0 (t)_0^T + \frac{ir_f}{4} (\sin 2\omega_0 t)_0^T \right]$$

$$= \frac{2}{T} \left[I_{DC} \sin \omega_0 T + \frac{ir_f}{2} \omega_0 T + \frac{ir_f}{4} \sin 2\omega_0 T \right]$$

$$= \frac{1}{T} \left[2 I_{DC} \sin \omega_0 T + ir_f \omega_0 T + \frac{ir_f}{2} \sin 2\omega_0 T \right]$$

Multiply & divide the eqn by 2.

$$= \frac{1}{2T} \left[4 I_{DC} \sin \omega_0 T + 2ir_f \omega_0 T + ir_f \sin 2\omega_0 T \right] \rightarrow (9)$$

* Assume $T = \pi$ & $\omega_0 T = \phi$,

$$\therefore i_{fun} = \frac{1}{2\pi} \left[4 I_{DC} \sin \phi + 2ir_f \phi + ir_f \sin 2\phi \right] \rightarrow (10)$$

Substitute the value of I_{DC} in eq (10),

$$i_{fun} = \frac{1}{2\pi} \left[\underset{(2 \times 2)}{-4ir_f \cos \phi \sin \phi} + 2ir_f \phi + ir_f \sin 2\phi \right]$$

$$\left[2 \cos A \sin B = \sin(A+B) - \sin(A-B) \right]$$

$$= \frac{1}{2\pi} \left[-2ir_f \sin 2\phi + 2ir_f \phi + ir_f \sin 2\phi \right]$$

$$i_{fun} = \frac{ir_f}{\pi} \left[2\phi - \sin 2\phi \right] \rightarrow (11)$$

* The max o/p vol,

$$V_{DD} = \frac{i_{rf}}{2\pi} (2\phi - \sin 2\phi) \cdot R \rightarrow (12).$$

i_{rf} in terms of V_{DD} ,

$$i_{rf} = \frac{2\pi V_{DD}}{R(2\phi - \sin 2\phi)} \rightarrow (13).$$

* The peak drain current is equal to the sum of the fund
drain current & bias current.

$$i_{D,PK} = i_{rf} + i_{bias} \text{ (or) } I_{DC}.$$

$$\therefore i_{D,PK} = \frac{2\pi V_{DD}}{R(2\phi - \sin 2\phi)} + \overset{\text{same}}{\frac{i_{rf}}{\pi} (\sin \phi - \phi \cos \phi)}.$$

$$i_{D,PK} = \frac{2\pi V_{DD}}{R(2\phi - \sin 2\phi)} \left[1 + \frac{(\sin \phi - \phi \cos \phi)}{\pi} \right].$$

→ The Efficiency of the PA,

$$\eta_o = \frac{i_{fun}}{I_{DC}} \rightarrow (14).$$

$$\eta_o = \frac{\frac{i_{rf}}{2\pi} (2\phi - \sin 2\phi)}{\frac{i_{rf}}{\pi} (\sin \phi - \phi \cos \phi)}$$

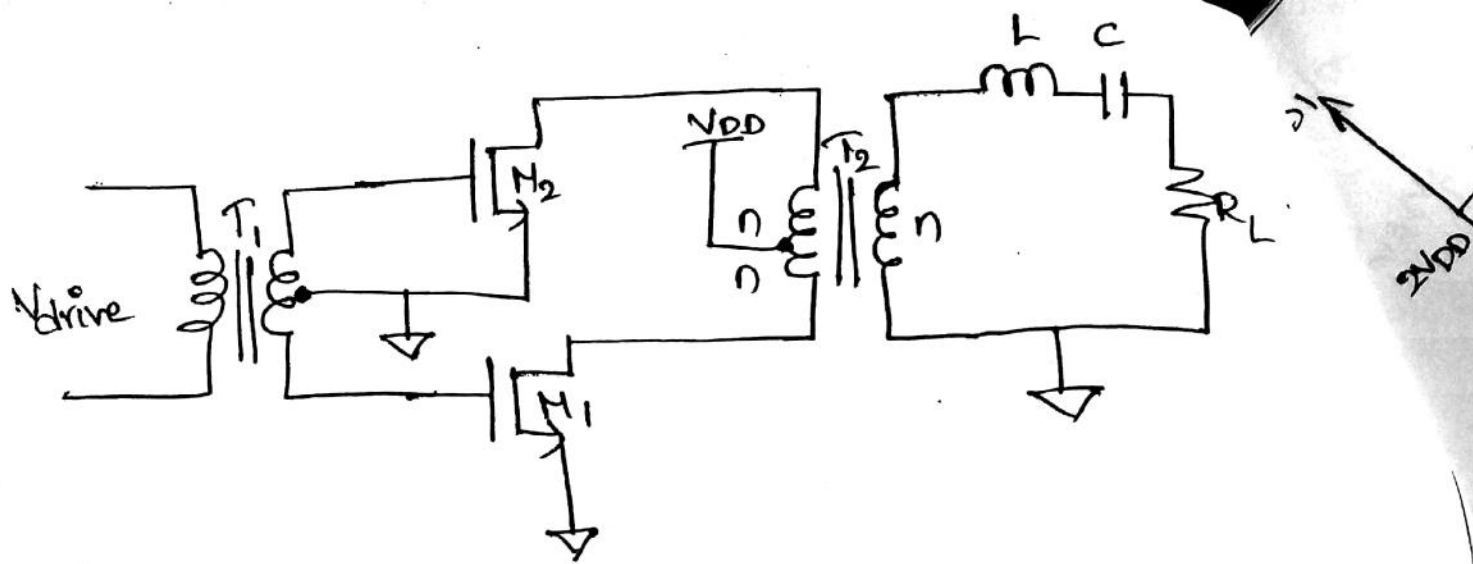
$$\therefore \boxed{\eta_o = \frac{(2\phi - \sin 2\phi)}{2(\sin \phi - \phi \cos \phi)}} \rightarrow (15).$$

Class AB power Amplifier:-

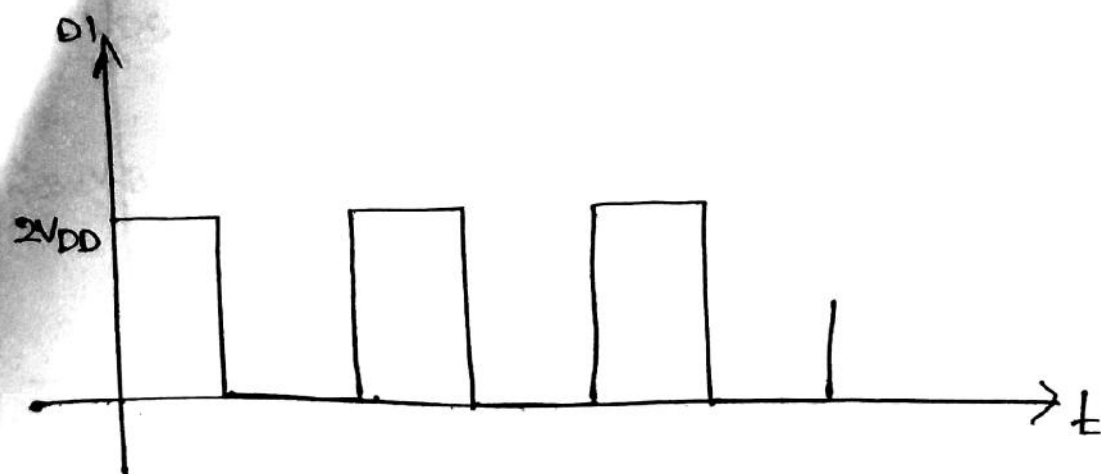
- ⇒ For class A PA the MOSFET conducts for 100% of the time period.
- ⇒ For class B amplifier the MOSFET conducts for 50% of the time period.
- ⇒ For class C PA the MOSFET conducts between 0 to 50% of the time period.
- ⇒ As the name itself suggests the class AB power amp conducts between 50% to 100% of the time period.
- ⇒ As a result its efficiency is in b/w 50% to 78.5% (or) exactly equal to 78.5%.

5. class D power Amplifier:-

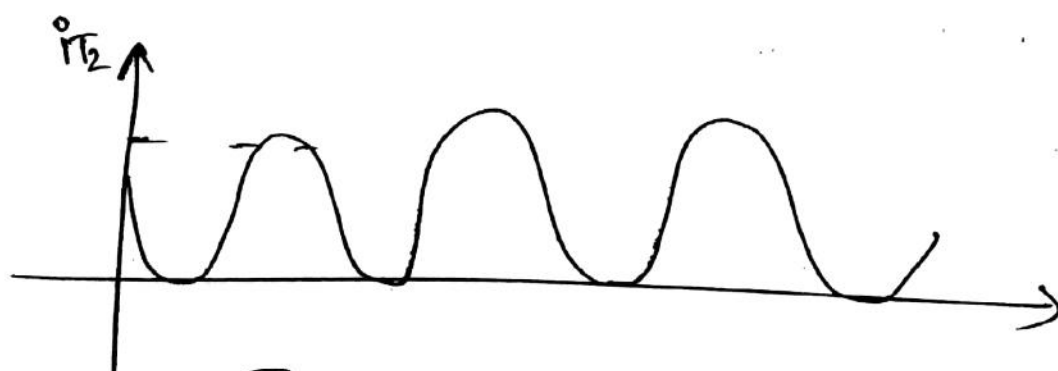
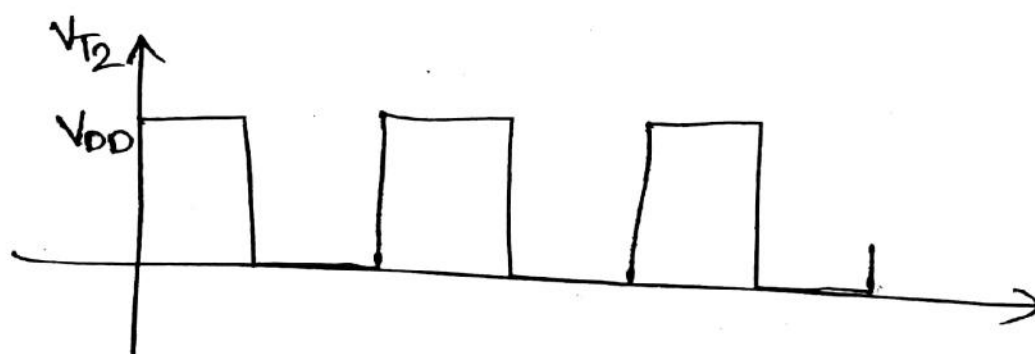
- * The power amplifiers so far discussed use the active devices as controlled current source.
- Another application is to use the device as a switch.
- ⇒ Hence the class D - amplifier is also called as "switching amplifier".



- ⇒ Since the power handling devices (MOSFET) work as a perfect binary switch, no time is wasted in b/w the transition stages.
- ⇒ No power is also wasted in the zero i/p conditions.
- ⇒ An ideal binary switch will pass all current through it with no voltage across it when it is in ON state.
- ⇒ When it is in OFF state the entire voltage remains across it and no current flows through it.
- ⇒ This means that no power is wasted across the switching element during amplification.
- ⇒ Hence it provides maximum amount of efficiency.
- ⇒ The higher the efficiency means low thermal dissipation which means the amplifier dissipates less amount of power at the o/p.



M_1 drain voltage & current

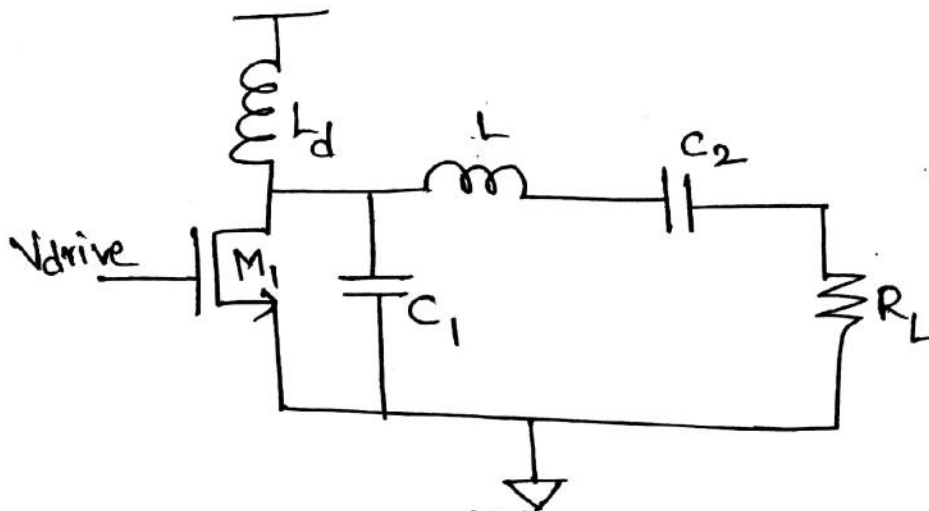


T_2 secondary vol & current for ideal class D PA.

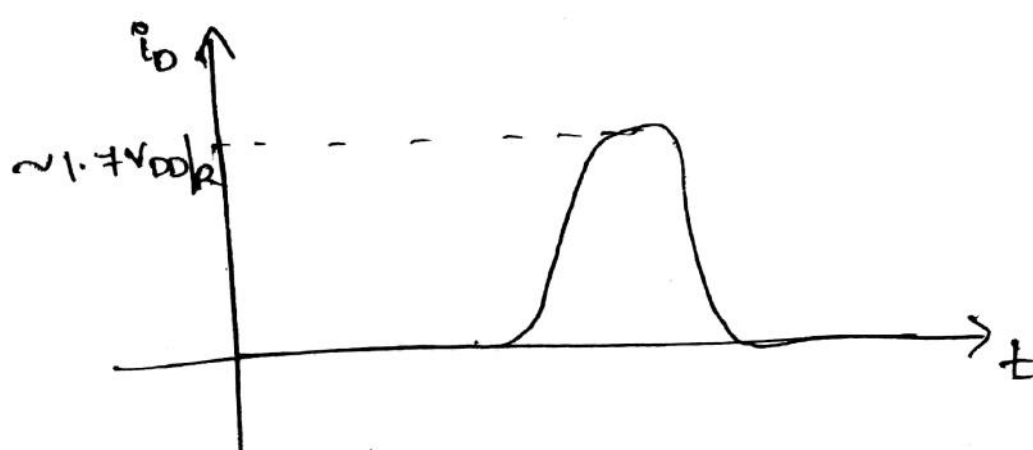
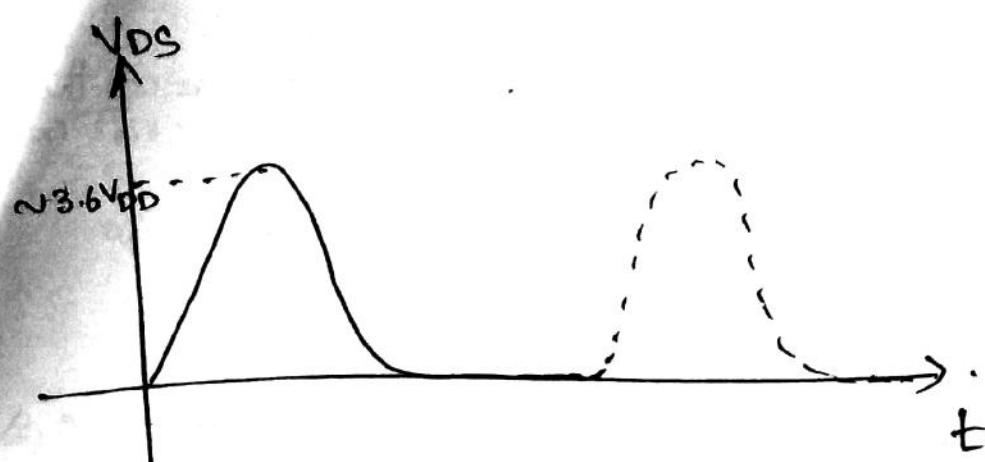
* The normalized power for class D power amp is,

$$P_N \approx 0.32$$

6. Class E power Amp:-



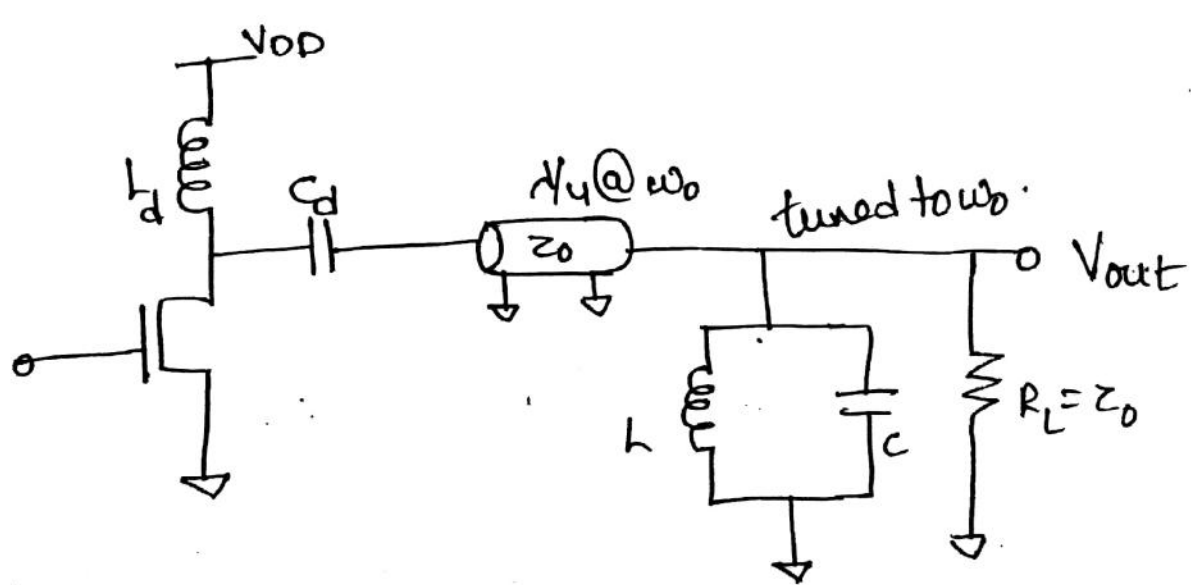
- * Using transistors (or) FET as switches provides improved efficiency, but due to imperfections in the real switches there may be some problem.
- * The associated dissipation degrades the efficiency.
- * To prevent this gross losses, the switches must be quite fast relative to the freq. of operation.
- * Hence the Class E amplifier is used by which it modifies the ckt to force a zero switch vol for a non-zero interval of time about the instant of switch which reduces the power dissipation.



* Normalized power o/p Capability,

$$P_N \approx 0.098$$

Class F power Amplifier:-



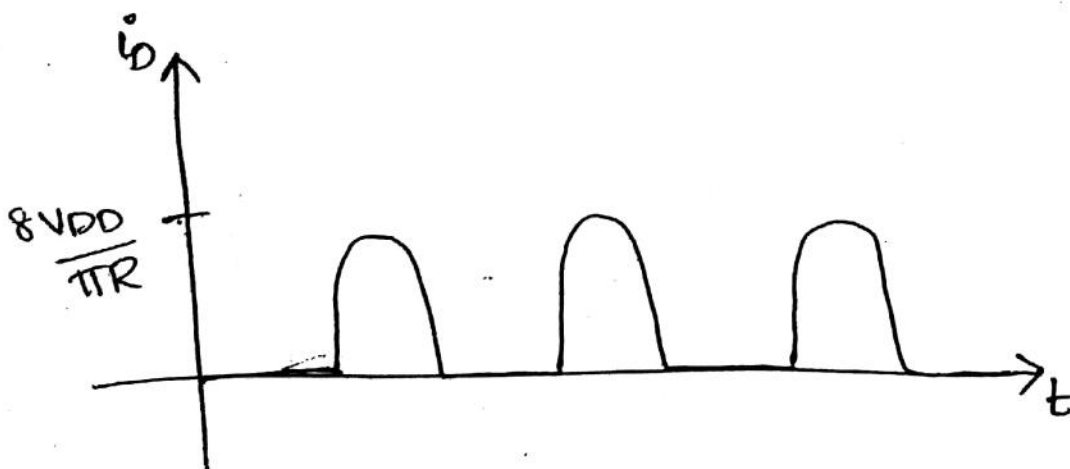
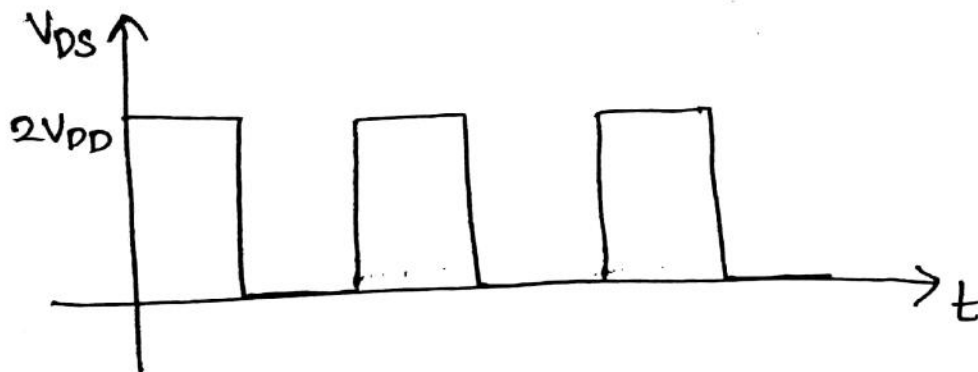
⇒ The o/p tank ckt is used to tune the resonance at the carrier frequency.

⇒ The Q-factor is high enough.

⇒ The length of the transmission line is chosen to be a quarter-wavelength at the carrier frequency.

⇒ The quarter-wavelength of line has an "impedance reciprocation" property.

⇒ The i/p impedance of such a line is proportional to the reciprocal of the termination impedance.



Drain Voltage & current for ideal class F Amp.

⇒ The peak-to-peak voltage of the fundamental component of V_{DS} is,

$$(4/\pi) 2V_{DD} \rightarrow (1).$$

⇒ The ^{avg} o/p power delivered to the load by the fundamental component of the peak voltage is,

$$P_o = \frac{[(4/\pi) V_{DD}]^2}{2R} \rightarrow (2).$$

$$P = VI$$

$$P = V \cdot \frac{V}{R}$$

$$P = \frac{V^2}{2R}$$

$$P = \frac{(4/\pi)^2}{2}$$

⇒ The peak drain current,

$$i_{D,PK} = \frac{2V_{DD}}{R} \cdot \frac{4}{\pi}$$

$$i_{D,PK} = \frac{8}{\pi} \cdot \frac{V_{DD}}{R} \rightarrow (3).$$

∴ The normalized power handling capability is,

$$P_N = \frac{P_o}{V_{DS,max} \cdot i_{D,PK}}$$

$$= \frac{\frac{[(4/\pi) V_{DD}]^2}{2R}}{2V_{DD} \cdot \left(\frac{8}{\pi} \cdot \frac{V_{DD}}{R}\right)}$$

$$P_N = \frac{1}{2\pi} \approx 0.16.$$

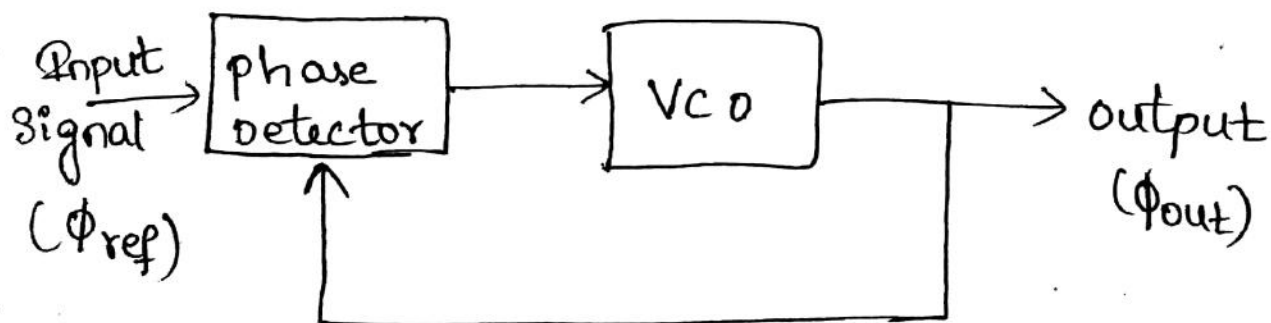
$$\frac{\frac{16}{\pi^2} V_{DD}^2}{2R} \cdot \frac{1}{2V_{DD} \cdot \left(\frac{8}{\pi} \cdot \frac{V_{DD}}{R}\right)}$$

$$\frac{\frac{16}{\pi^2} V_{DD}^2}{2R} \cdot \frac{1}{2V_{DD} \cdot \left(\frac{8}{\pi} \cdot \frac{V_{DD}}{R}\right)}$$

$$\frac{16 V_{DD}^2}{\pi^2 R^2} \cdot \frac{1}{16 V_{DD}^2 R^2}$$

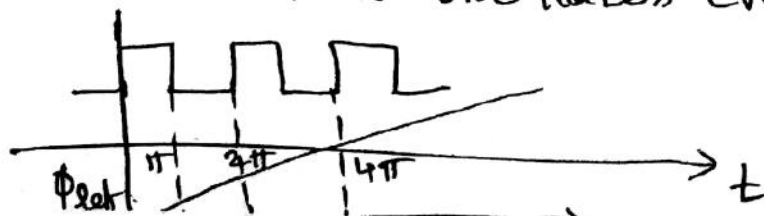
Phase Locked Loops:-

- ⇒ A PLL may be used to generate an o/p signal whose frequency is a programmable, rational multiple of a fixed frequency.
- ⇒ The o/p of frequency synthesizers may be used as the local oscillator signal in superhetrodyne transceivers.
- ⇒ PLL's may also be used to perform frequency modulation and demodulation.
- ⇒ It can also be used to generate the carrier from an i/p sig in which the carrier has been suppressed.

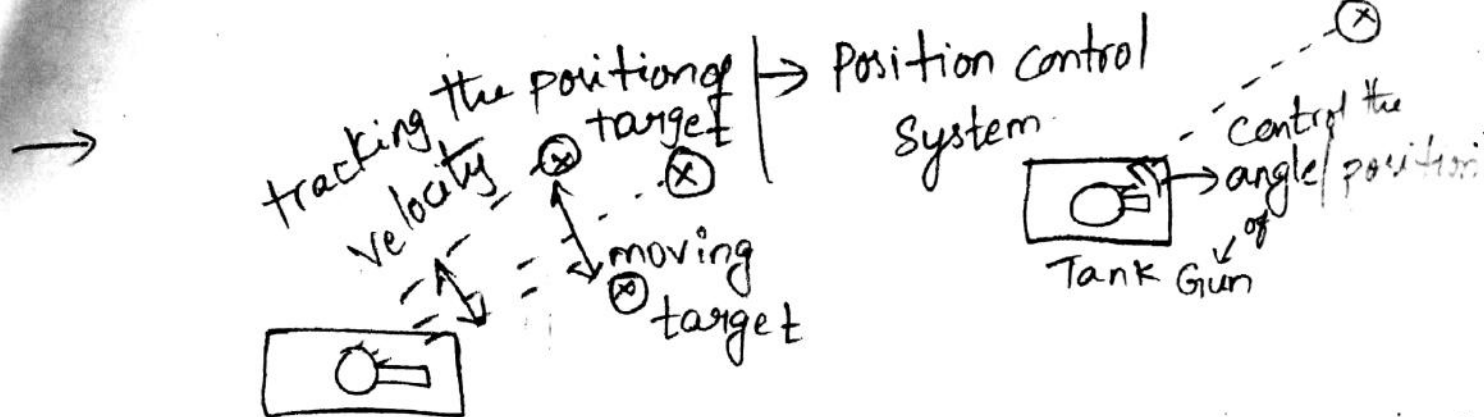


Phase - Locked Loop Architecture .

- ⇒ The reference oscillator oscillates every time.

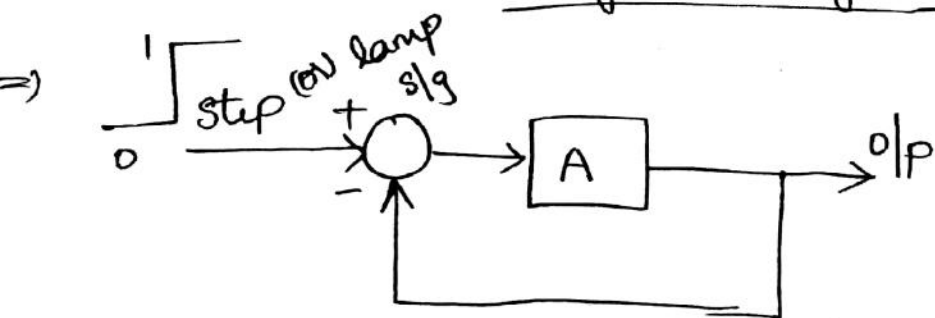


⇒ The ϕ_{ref} looks like a ramp signal.



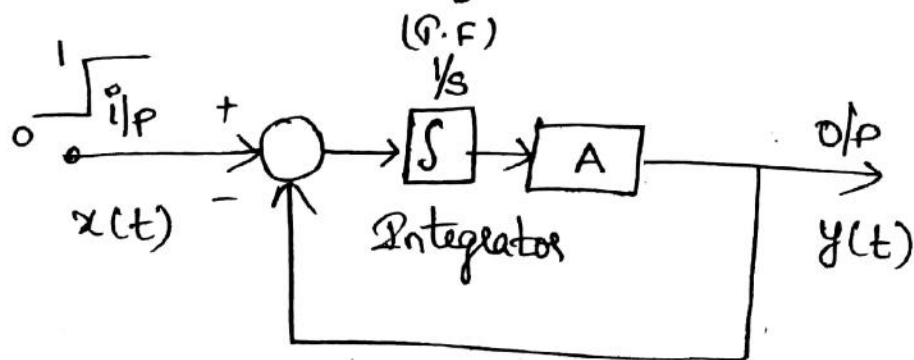
velocity control slm

⇒ The i/p signal is a continuously changing ramp signal hence the PLL uses velocity control system.



$$\text{Gain} = \frac{o/p}{i/p} = \frac{A}{1+A}$$

If $A = 5$ then $G = 5/6$



⇒ This integral eliminates the noise errors.

$$\begin{aligned}\frac{O/P}{I/P} &= \frac{A/s}{1+A/s} \\ &= \frac{\frac{A/s}{s+A}}{s} = \frac{A}{s} \cdot \frac{s}{s+A} \\ &= \frac{A}{A[1+s/A]} = \frac{1}{1+s/A}\end{aligned}$$

⇒ $I/P \quad U(t) = 1/s$

$$Y(s) = \frac{1}{s} \cdot \frac{1}{1+s/A}$$

⇒ Partial fractions, $Y(s) = \frac{K_1}{s} + \frac{K_2}{1+s/A}$

⇒ If mul both sides by s .

$$\frac{1}{1+s/A} = K_1 + \frac{K_2 s}{1+s/A}$$

If $s=0$, $\boxed{K_1 = 1}$

⇒ If multiply both sides by $(1+s/A)$

$$\frac{1}{s} = K_2 + \frac{K_1(1+s/A)}{s}$$

If $s=-A$,

$$\boxed{K_2 = -1/A}$$

$$\therefore K_1 + \frac{K_2}{1+s/A} = \frac{1}{s} - \frac{1}{s+A}$$

$\downarrow U(t) \quad \quad \downarrow U(t) \cdot e^{tA}$

$A \rightarrow \infty$,

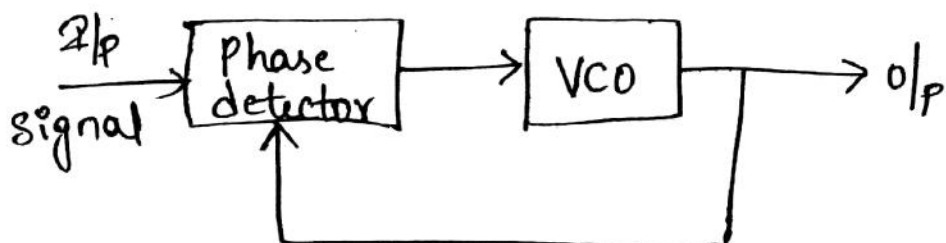
$U(t) = 1$

$U(t) \cdot e^{tA} = 0$

Hence the o/p is equal to i/p.

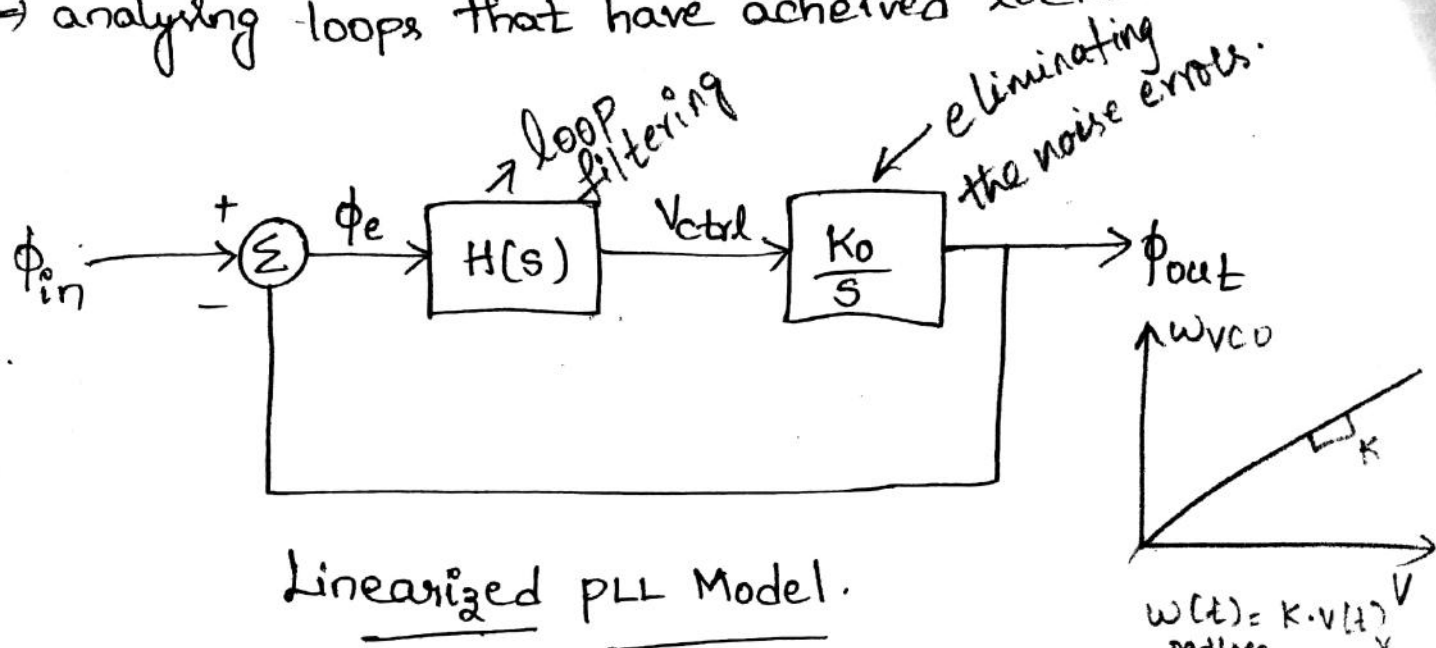
Linearized PLL Models:-

PLL Architecture:-



- * The PLL architecture consists of a phase detector and voltage controlled oscillator (VCO).
- * The phase detector compares the phase of an incoming reference sig with the VCO and produces an o/p that is a function of ^{some} phase difference.
- * The VCO simply generates a signal whose frequency is function of some control voltage.
- * The phase detector drives the VCO frequency in a direction that reduces the phase difference.
- * The feedback provided by the s/m is Negative Feedback
- * Once the loop achieves lock, the phase of the i/p reference and VCO o/p sigs have a fixed phase relationship (mostly 0° or 90°).

- ⇒ Although both the phase detector and VCO are ^{high first order} non-linear. It is assumed that they are linear when ^{the} analysing loops that have achieved locks.



- * The consequence of choosing phase as the i/p-o/p Variable is that the VCO, whose o/p freq, depends on a control vol is modeled as an integrator, since phase is the integral of freq.
- ⇒ The VCO gain constant K_0 has units of radians/sec-vol.
- ⇒ It describes the change in o/p freq, resulting from a specified change in control voltage.
- ⇒ The phase detector is modeled as a simple subtractor that generates a phase error o/p ϕ_e (i.e) the diff b/w the i/p & o/p phases.
- ⇒ To accomodate gain scaling factors & the option of additional filtering in the loop, a block with T.F. $H(s)$ is included.

le high
ciber.
first-order PLL:-

- ⇒ The PLL in which the function $H(s)$ is simply a scalar gain (let it be K_D with units vol/radian).
- ⇒ The loop transmission possesses just a single pole, hence this type of loop is known as first-order PLL.
- ⇒ The bandwidth and steady-state phase errors are strongly coupled in this type of loop.
- ⇒ The i/p - o/p transfer function is derived as,

$$\frac{\phi_{\text{out}}(s)}{\phi_{\text{in}}(s)} = \frac{K_0 K_D}{s + K_0 K_D} \rightarrow \textcircled{1}. \quad (K_0 \text{ is the VCO gain constant})$$

The closed-loop B.W is,

$$\omega_h = K_0 K_D \rightarrow \textcircled{2}.$$

- ⇒ The B.W & phase error are linked to each other. To derive the i/p-to-error transfer fun,

$$\frac{\phi_e(s)}{\phi_{\text{in}}(s)} = \frac{s}{s + K_0 K_D} \rightarrow \textcircled{3}.$$

⇒ If we assume that the i/p sig is a constant freq of second-order,
then,

$$\phi_{in}(s) = \frac{\omega_i}{s^2} \rightarrow (4).$$

$$\phi_e(s) = \frac{\phi_i(s) \cdot s}{s + K_0 K_D} = \frac{\omega_i \cdot s}{s^2 + K_0 K_D}$$

The int

$$\therefore \phi_e(s) = \frac{\omega_i}{s(s + K_0 K_D)} \rightarrow (5).$$

The steady-state error with a constant freq input is,

$$\lim_{s \rightarrow 0} s \phi_e(s) = \frac{\omega_i}{K_0 K_D}.$$

$$[\because K_0 K_D = \omega_h].$$

$$\lim_{s \rightarrow 0} s \phi_e(s) = \frac{\omega_i}{\omega_h} \rightarrow (6).$$

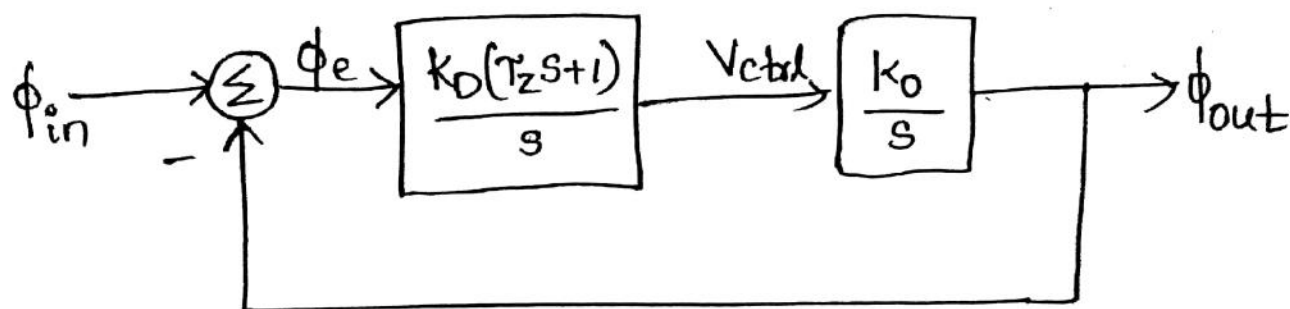
⇒ Hence the steady-state phase error is simply the ratio of the input frequency to the loop bandwidth.

⇒ A large loop bandwidth requires a small steady-state phase error.

⇒ An increase in gain raises the loop transmission uniformly at all freqs, a bandwidth increase necessarily reduces the phase error.

Second-order PLL:-

* The 90° negative phase shift contributed by the added integrator has to be offset by the positive phase shift of the loop.



Model of Second-order PLL.

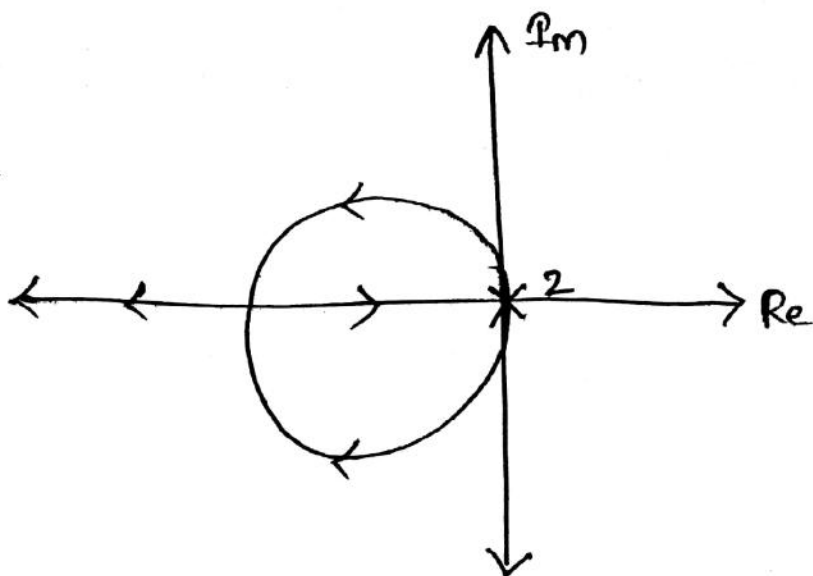
\Rightarrow The constant K_D has the units of Vol/sec.

\Rightarrow The stability of this loop can be explained with the root-locus diagram.

\Rightarrow As the loop transmission magnitude increases (by $\uparrow K_D K_O$), the increase in cross over frequency allows more positive phase shift to offset the negative phase shift of the poles.

\Rightarrow The phase Transfer function is,

$$\frac{\phi_{out}}{\phi_{in}} = \frac{\tau_z s + 1}{(s^2 / K_D K_O) + \tau_z s + 1} \rightarrow \textcircled{1}$$



Root locus of second-order PID.

$$\omega_n = \sqrt{K_D K_0} \rightarrow (2).$$

$$\xi = \frac{\omega_n \tau_z}{2} \left[\frac{\omega_n}{\omega_z} \right] \omega_n = \sqrt{K_D K_0}.$$

$$\xi = \frac{\tau_z \sqrt{K_D K_0}}{2} \rightarrow (3).$$

\therefore The crossover freq for the loop may be expressed as,

$$\omega_c = \left[\frac{\omega_n^4}{2\omega_z^2} + \omega_n^2 \sqrt{\frac{1}{4} \left(\frac{\omega_n}{\omega_z} \right)^4 + 1} \right]^{1/2} \rightarrow (4).$$

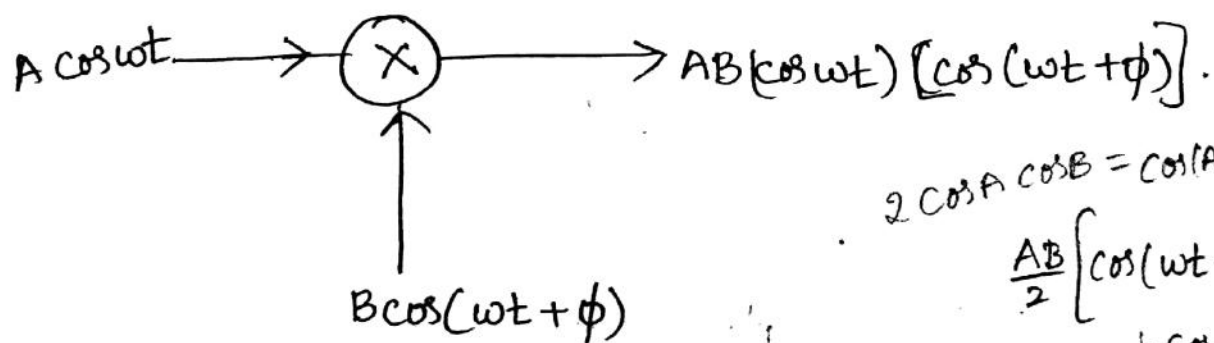
\Rightarrow The cross over frequency above the zero frequency is,

$$\boxed{\omega_c \approx \frac{\omega_n}{\omega_z}} \rightarrow (5).$$

Phase Detectors:-

a) The Analog Multiplier As A phase Detector:-

⇒ The PLL's which have sine-wave inputs and sine-wave V_{cc} the most common phase detector is the multiplier, after implemented with a Gilbert-type topology.



$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$\frac{AB}{2} \left[\cos(\omega t + \omega t + \phi) + \cos(\omega t - \omega t - \phi) \right]$$

Multiplier as phase detector.

$$\frac{AB}{2} \left[\cos(-\phi) + \cos(2\omega t + \phi) \right]$$

⇒ The o/p of the multiplier is expressed as, $\cos(-\phi) = \cos \phi$

$$AB \cos \omega t \cos(\omega t + \phi) = \frac{AB}{2} \left[\cos \phi + \cos(2\omega t + \phi) \right] \rightarrow \textcircled{1}$$

⇒ The o/p of the multiplier consists of a DC term and a double-freq term.

⇒ The phase detector requires only the DC term.

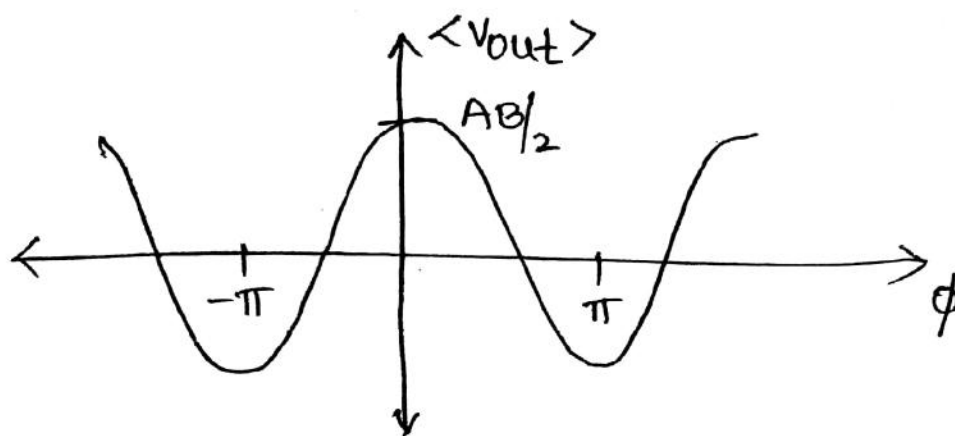
⇒ Hence the avg o/p of the phase detector is,

$$(AB \cos \omega t \cos(\omega t + \phi)) = \frac{AB}{2} (\cos \phi) \rightarrow \textcircled{2}$$

⇒ The phase detector gain constant is a function of phase angle and is,

$$K_D = \frac{d}{d\phi} (V_{out}) = -\frac{AB}{2} [\sin \phi] \rightarrow (3).$$

* The average o/p as a function of phase angle is,



Multiplier phase detector o/p vs phase difference

⇒ The phase detector gain constant is zero when the phase difference is zero and is greatest when the i/p phase difference is 90° .

⇒ Hence the loop should be arranged to lock to a phase difference of 90° .

⇒ Thus a multiplier is often called as "Quadrature phase detector".

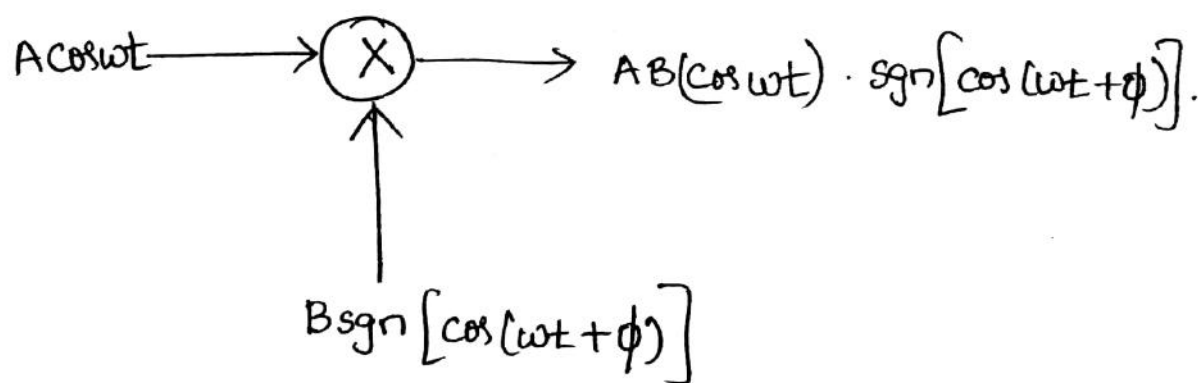
ction (12)
 (a) When the loop is locked in quadrature, the phase detector has an incremental gain constant as,

$$k_D|_{\phi=\pi/2} = \left. \frac{d}{d\phi}(V_{out}) \right|_{\phi=\pi/2} = -\frac{AB}{2} \rightarrow (4).$$

$\sin \pi/2 = 1$

(b) Commutating Multiplier As a phase Detector:-

* In this method consider one as the sine-wave and the other to be the square-wave i/p.



Multiplier with one Square-wave i/p.

* The signum fun is defined as,

$$\operatorname{sgn}(x) = 1 \text{ if } x > 0 \rightarrow (1).$$

$$\operatorname{sgn}(x) = -1 \text{ if } x < 0 \rightarrow (2).$$

* A square wave of amplitude AB has a fundamental component whose amplitude is ~~$4AB/\pi$~~ $4AB/\pi$.

⇒ The avg o/p of the multiplier is, ^{with the fundamental component}

$$(V_{out}) = \frac{4}{\pi} \cdot \frac{AB}{2} [\cos \phi]$$

$$(V_{out}) = \frac{2}{\pi} AB [\cos \phi] \rightarrow (3).$$

⇒ The corresponding phase detector gain is,

$$K_D|_{\phi=\pi/2} = \left. \frac{d}{d\phi} (V_{out}) \right|_{\phi=\pi/2}$$

$$\Rightarrow -\frac{2AB}{\pi} \rightarrow (4).$$

⇒ Multiplication of a signal by a periodic signum function is equivalent to inverting the phase of the signal periodically.

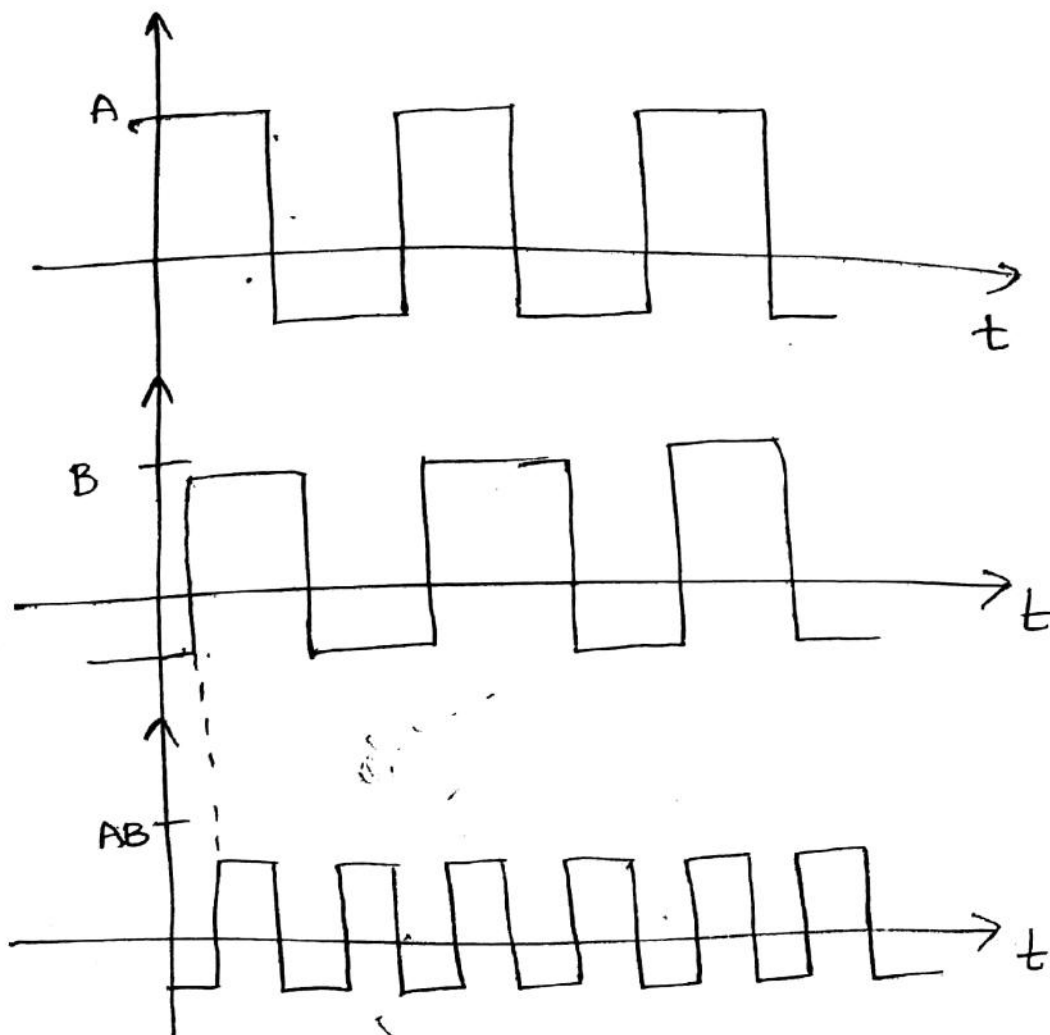
⇒ Hence the multiplier in this method is used as a switch (known as commutators).

⇒ Thus implementing multiplier as switches rather than the previous one is easier.

⇒ They are implemented by some technologies (such as CMOS).

(C) The Exclusive-OR Gate as phase detector:-

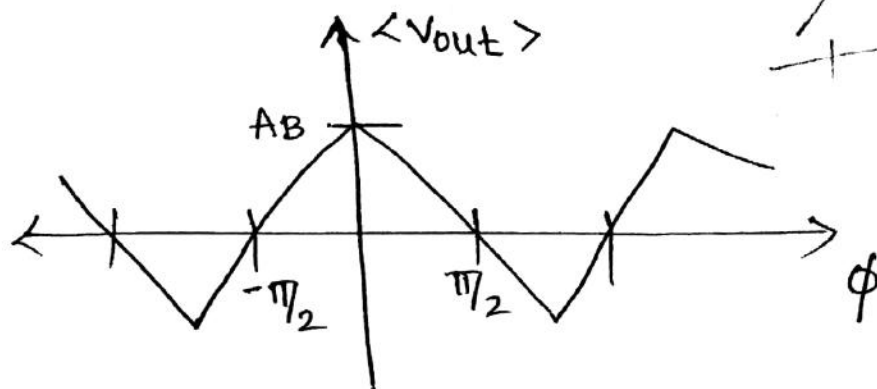
- ⇒ In this method the analog multiplier is driven with square waves on both inputs.
- ⇒ Here we analyse the situation by using the fourier series for each of the i/p's, multiplying them and so on.
- ⇒ In this case as the i/p phase difference changes, the o/p takes the form of a square wave Varying duty cycle with 50% duty cycle of quadrature loop.



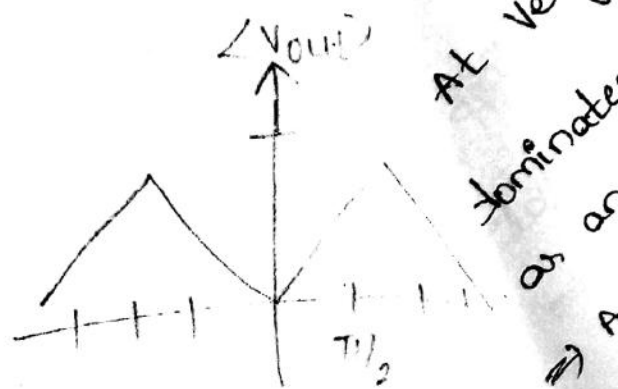
Multiplier i/p's & o/p.

* The phase detector constant is,

$$K_D = \frac{2}{\pi} AB \rightarrow \textcircled{1}$$



Multiplier characteristics with
two square-wave i/p's.



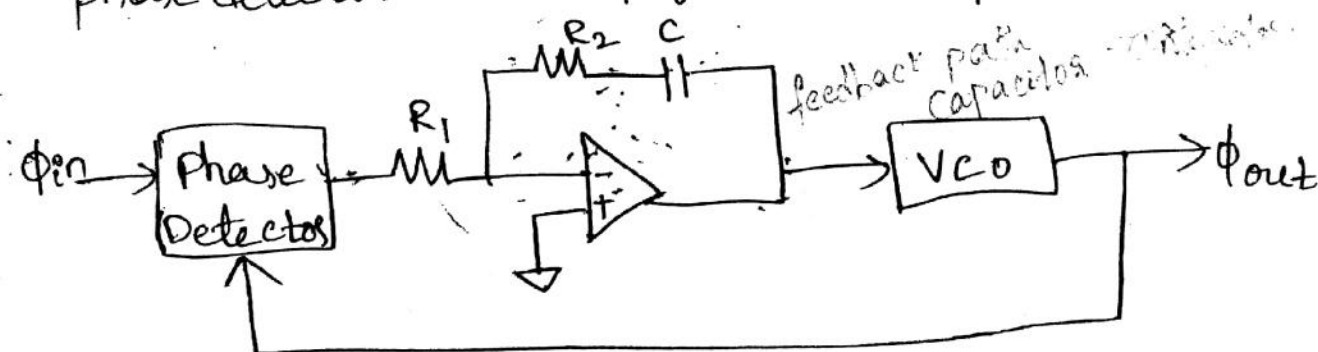
Char of the
as a function of
phase φ

Charge pump and Loop Filters:-

⇒ Generally zero phase error has to be locked.

⇒ The VCO requires some control voltage to produce an o/p of the desired freq.

⇒ To provide this control voltage with a zero o/p from the phase detector the loop filter must provide an integration.



PLL with loop filter

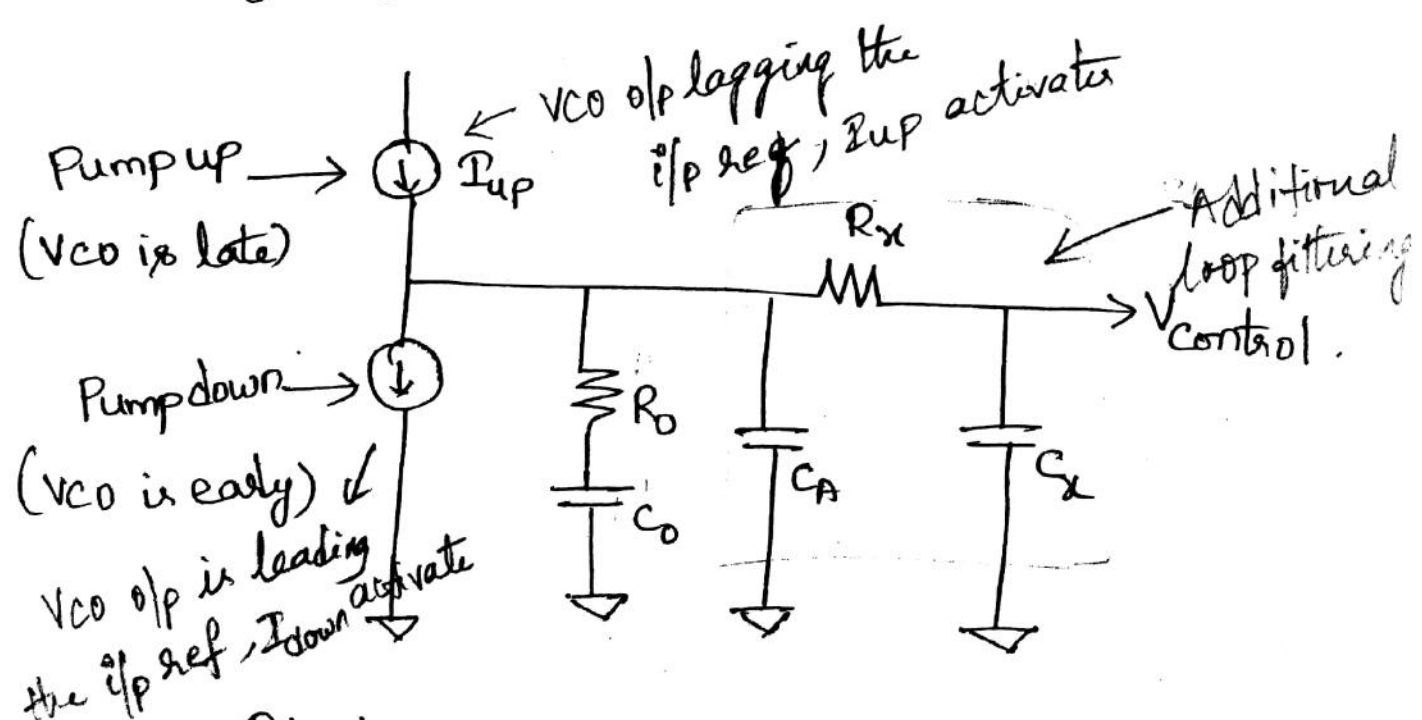
At very low frequencies the Capacitor's impedance dominates the op-amp's feedback hence the loop filter behaves as an integrator.

⇒ As the freq increases, the Capacitive reactance decreases and eventually equals the series resistance ' R_2 '.

⇒ At high frequencies, the Capacitive reactance becomes increasingly negligible compared to R_2 and hence the gain ultimately be equal to $-R_2/R_1$.

⇒ The alternative to the op-amp loop filter is charge pump.

⇒ In this charge pump the phase detector controls one (or) more current sources, and the RC n/w provides the necessary loop.



Idealized PLL charge pump loop filter.

⇒ Here the charge pump provides the necessary loop ~~negative~~ feedback.

⇒ The phase detector is assumed to provide a digital "pump up" or "pump down" signal.

⇒ If the phase detector determines that the VCO o/p is lagging the i/p reference, it activates the top current source, depositing charge onto the capacitor (pumping up).

⇒ If the VCO o/p is ahead of the i/p reference, the bottom current source is activated by withdrawing the charge from the capacitor (pumping down).

⇒ The elements C_A , R_x and C_x provides additional filtering to the ckt.

⇒ When the detector is used with charge pumps then their net pump current is,

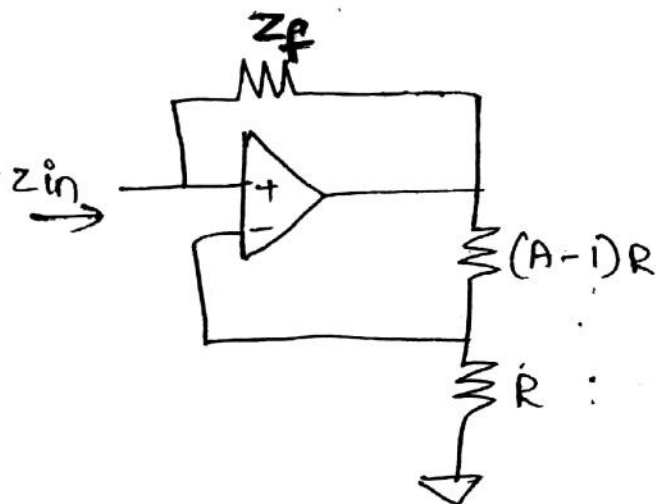
$$I = I_{\text{pump}} \frac{\Delta\phi}{2\pi}$$

where, $I_{\text{pump}} = I_{\text{up}} = I_{\text{down}}$.

⇒ This current multiplied by the impedance of the filter n/w connected to the current sources, gives the o/p voltage.

loop negative Resistance oscillators:-

- ⇒ A perfectly lossless resonant ckt acts as an oscillator.
- ⇒ The negative impedance converter (NIC) with a simple op-amp ckt provides both positive & negative feedback



Generalized Impedance Converter

⇒ The i/p impedance related to the feedback impedance is ,

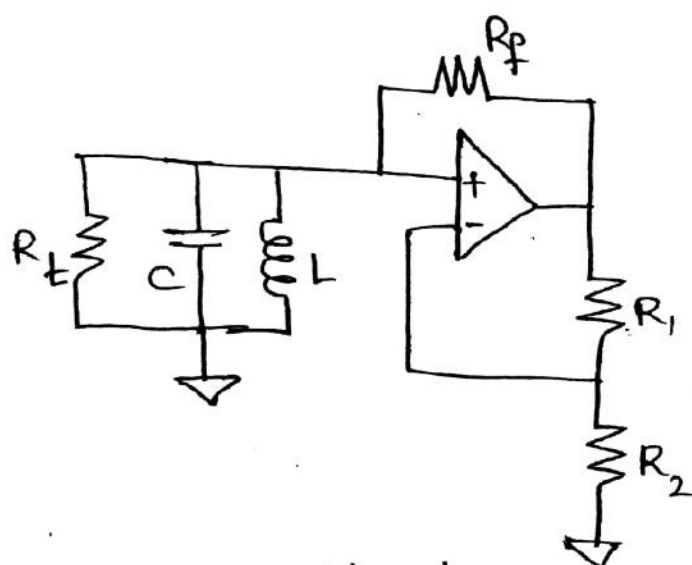
$$Z_{in} = \frac{Z_f}{1-A}$$

* If the closed-loop gain 'A' is set equal to '2' then the i/p impedance is algebraic inverse of the feedback imp

$$\left[\begin{array}{l} Z_{in} = \frac{Z_f}{1-2} \\ Z_{in} = \frac{Z_f}{-1} \end{array} \right] \left| \begin{array}{l} Z_{in} = 1/Z_f \\ Z_{in} \propto 1/Z_f \end{array} \right.$$

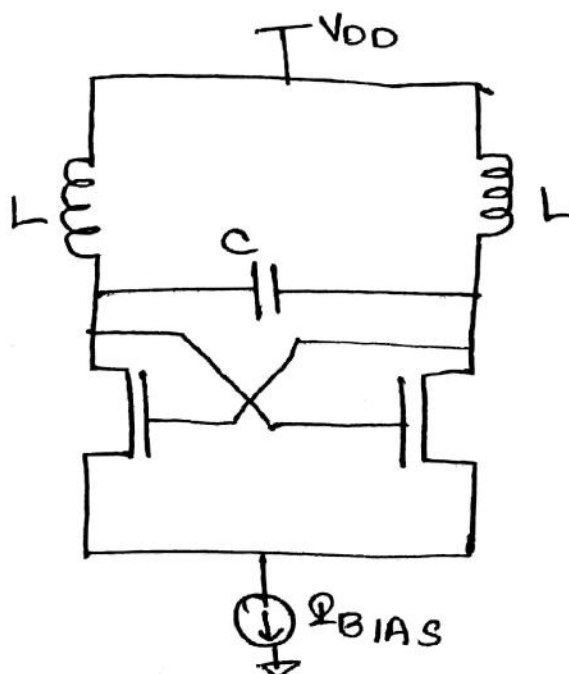
⇒ If the feedback impedance is a pure positive resistance then the i/p impedance will be a purely negative resistance.

⇒ This negative resistance may be used to offset the positive resistance of the resonators to produce an oscillator.

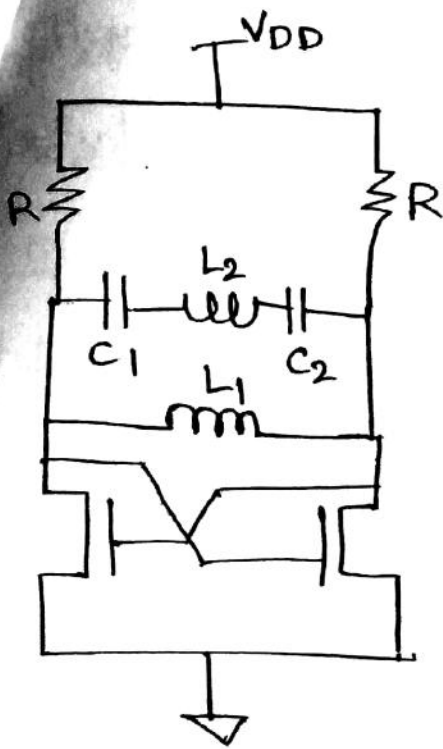


Negative Resistance oscillator.

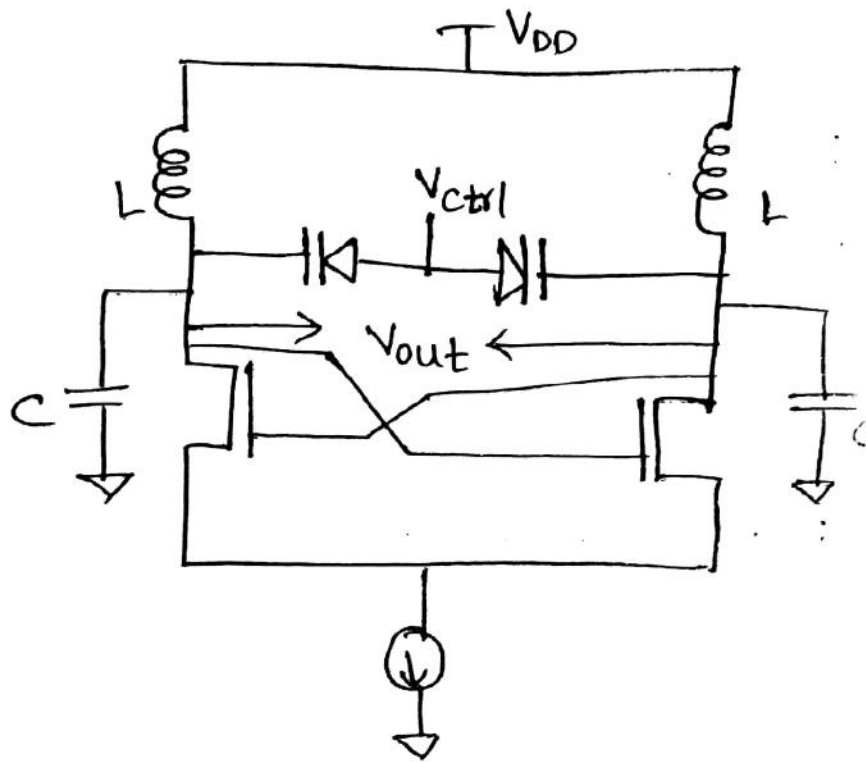
* The condition $R_t > R_f$ should be assumed.



Simple Differential negative resistance oscillator.



Neg Resistance oscillator
with modified tank ckt.



Voltage controlled negative
resistance oscillator.

Resonators:-

(a) Quarter-wave Resonators:-

⇒ At high freqs, it becomes difficult to obtain adequate 'Q' from lumped resonators because the required components values are impractical.

⇒ The distributed resonator such as a quarter-wave piece of transmission line is used.

⇒ Q is proportional to Ratio of Energy stored
Energy dissipated

⇒ Some distributed structures store energy in the volume of volume and dissipation is due to surface effects in the volume.

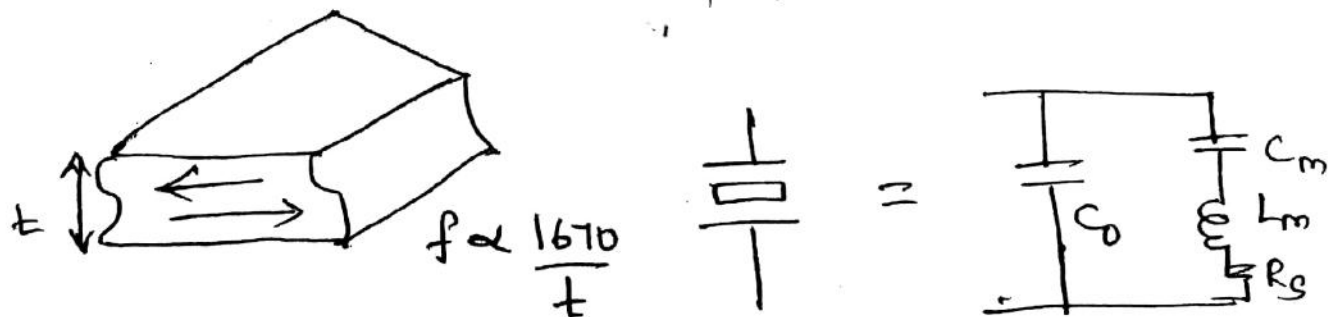
⇒ Hence Volume/Surface area ratio is important in determining the Q-factor.

b) Quartz Crystals:-

⇒ The most commonly made non-RLC resonators is made up of quartz.

⇒ Quartz is a piezoelectric material, and it exhibits a reciprocal transduction (shuffles, converts) between mechanical strain and electric charge.

⇒ When a mechanical strain is applied, charges appear across the crystal.



Symbol & model for crystal.

the
pts
x The quartz Crystals used at radio freqs employ a bulk shear mode.

- ⇒ In this mode, the resonant freq is inversely proportion to the thickness of the slab.
- ⇒ The quartz Crystal has the quality factor Q from the range b/w 10^4 to 10^6 .
- ⇒ The Capacitance ' C_0 ' represents the parallel plate Capacitance associated with the contacts & the lead wires
- ⇒ The C_m & L_m represents the mechanical energy storage
- ⇒ Resistance ' R_g ' accounts for the nonzero lossless real time systems.

⇒ The resistance ' R_g ' can be expressed as,

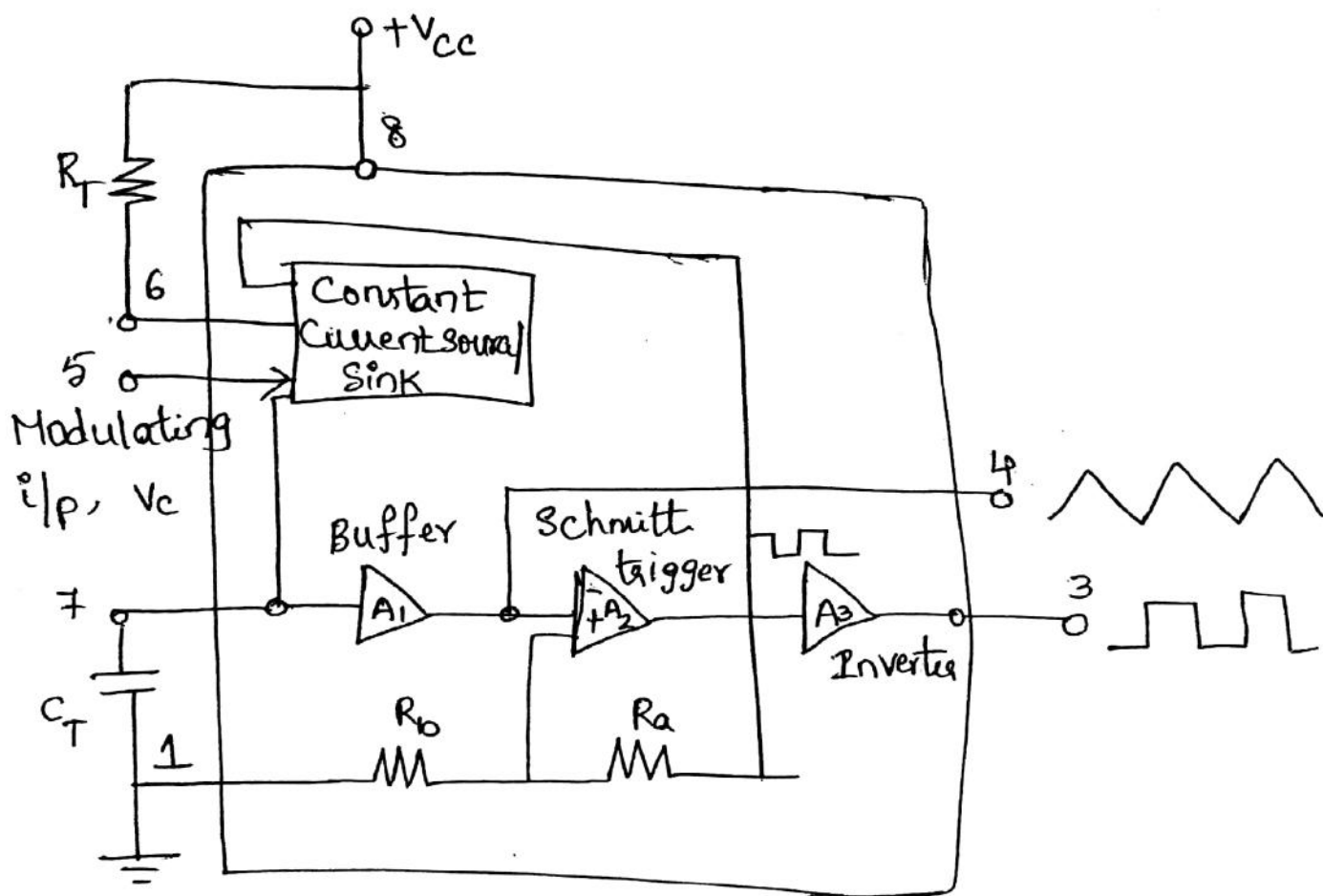
$$R_g \approx \frac{5 \times 10^8}{f_0}$$

⇒ For effective series resistance the square of the overtone mode is,

$$R_g \approx \frac{5 \times 10^8}{f_0} N^2.$$

Voltage controlled oscillator (VCO):-

⇒ The voltage controlled oscillator is used to produce
converting low freq signals such as EEG's, EKG into an
audio freq range.



Block Diagram of VCO.

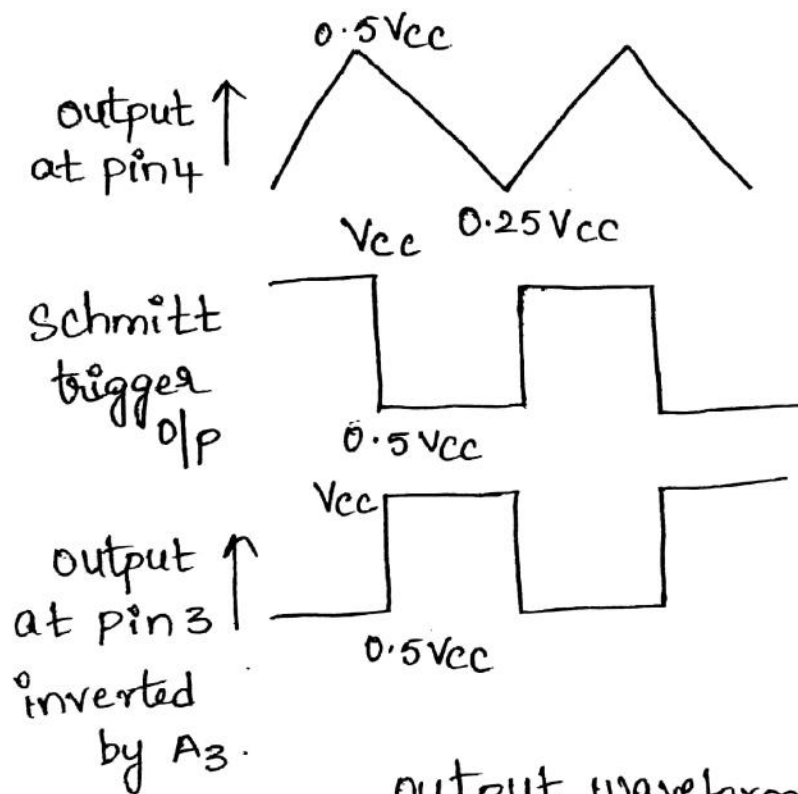
- (Ct)
- ⇒ The timing capacitor is linearly charged (or) discharged by a constant current source/sink.
 - ⇒ The amount of current can be controlled by ^{changing} the voltage ' V_c ' applied at the modulating i/p (pin 5) (or) by changing the timing resistor ' R_T ' external to IC chip.

duce
5

- ⇒ The voltage at pin 6 is held at the same voltage as pin 5.
- ⇒ If the modulating voltage at pin 5 is increased, the voltage at pin 6 also increases,
- ⇒ The VCO produced audio signals can be transmitted over telephone lines (or) a two way radio communication systems for diagnostic purposes (or) can be recorded on a magnetic tape for further reference.
- ⇒ The voltage across the capacitor ' C_T ' is applied to the invert i/p terminal of schmitt trigger ' A_2 ' via buffer amplifier ' A_1 '.
- ⇒ The o/p voltage swing of the schmitt trigger is designed to be V_{CC} to $0.5 V_{CC}$.
- ⇒ If $R_a = R_b$ in the +ve feedback loop, the voltage at the non-inverting i/p terminal of A_2 swings from $0.5 V_{CC}$ to $0.25 V_{CC}$.
- ⇒ When the voltage on the capacitor ' C_T ' exceeds $0.5 V_{CC}$ during charging, the o/p of the schmitt trigger will be Low ($0.5 V_{CC}$).
- ⇒ The Capacitor discharges when it is at $0.25 V_{CC}$, the o/p of the schmitt trigger will be HIGH (V_{CC}).
- ⇒ The Capacitor charges and discharges leads to a triangular voltage waveform across ' C_T ' which is produced at the pin 4.

⇒ The square^{wave} o/p of the schmitt trigger is inverted inverter A_3 and is produced at the pin 3.

⇒ The inverter A_3 is basically a current amplifier used to drive the load.



output waveforms of VCO.

⇒ The total voltage on the Capacitor changes from $0.25V_{cc}$ to $0.5V_{cc}$.

Thus $\Delta V = 0.25V_{cc}$.

⇒ The Capacitor charges with a constant current source as,

$$\frac{\Delta V}{\Delta t} = \frac{i}{C_T} \rightarrow (1)$$

$$\frac{0.25V_{cc}}{\Delta t} = \frac{i}{C_T}$$

$$\therefore \Delta t = \frac{0.25V_{cc}C_T}{i} \rightarrow (2).$$

∴ The time period 'T' of the triangular waveform = $2\Delta t$.

∴ The freq of oscillator f_0 ,

$$f_0 = \frac{1}{T} = \frac{1}{2\Delta t}$$

$$\Rightarrow \frac{1}{2 \times \frac{0.25 V_{CC} C_T}{i}}$$

$$f_0 = \frac{i}{0.5 V_{CC} C_T} \rightarrow (3)$$

$$\text{But, } i = \frac{V_{CC} - V_c}{R_T} \rightarrow (4)$$

$$\therefore \boxed{f_0 = \frac{2(V_{CC} - V_c)}{C_T R_T V_{CC}}} \rightarrow (5)$$

* Hence the o/p frequency of the VCO can be changed either by changing,

→ R_T

→ C_T (or)

→ the voltage V_c at the modulating i/p terminal pin 5.

* The modulating i/p voltage is usually varied from $0.75 V_{CC}$ to V_{CC} .

If $V_c = (7/8) V_{CC}$ then VCO o/p freq is,

$$f_0 = \frac{2[V_{CC} - (7/8)V_{CC}]}{C_T R_T V_{CC}} = \frac{1}{4 R_T C_T}$$

$$\boxed{f_0 = \frac{0.25}{C_T R_T}} \rightarrow (6)$$

Voltage to Frequency Conversion Factor:-

⇒ The vol to freq conversion factor K_V is,

$$K_V = \frac{\Delta f_o}{\Delta V_c} \rightarrow (7).$$

⇒ ΔV_c is the modulation vol required to produce the freq shift Δf_o for a VCO.

⇒ If we assume that the original freq is f_o and the new freq is f_1 then,

$$\Delta f_o = f_1 - f_o$$

$$\begin{aligned} &= \frac{2(V_{cc} - V_c + \Delta V_c)}{C_T R_T V_{cc}} - \frac{2(V_{cc} - V_c)}{C_T R_T V_{cc}} \\ &= \frac{2V_{cc} - 2V_c + 2\Delta V_c - 2V_{cc} + 2V_c}{C_T R_T V_{cc}} \end{aligned}$$

$$\Delta f_o = \frac{2\Delta V_c}{C_T R_T V_{cc}} \rightarrow (8).$$

$$\therefore \Delta V_c = \frac{\Delta f_o C_T R_T V_{cc}}{2} \rightarrow (9).$$

From eq (6),

$$C_T R_T = \frac{0.25}{f_o}$$

$$\Delta V_c = \Delta f_o \cdot \frac{0.25}{f_o} \cdot V_{cc} = \frac{\Delta f_o V_{cc}}{8 f_o}$$

$$\therefore K_V = \frac{\Delta f_o}{\Delta V_c} = \frac{2}{8 f_o / V_{cc}} = \frac{V_{cc}}{4 f_o}$$

$$\Delta V_c = \frac{\Delta f_o C_T R_T V_{cc}}{2}$$

$$f_o = \frac{0.25}{R_T C_T}$$

$$R_T C_T = \frac{0.25}{f_o}$$

$$\frac{\Delta f_o \times 0.25 \times V_{cc}}{f_o \times 2}$$

$$\frac{\Delta f_o \times V_{cc}}{\frac{0.25}{2} \times \frac{1}{8} f_o}$$

$$\frac{\Delta f_o \cdot 8 f_o}{\Delta f_o \cdot V_{cc}}$$

$$\left[f_o = \frac{0.25}{R_T C_T} \right].$$

UNIT-V

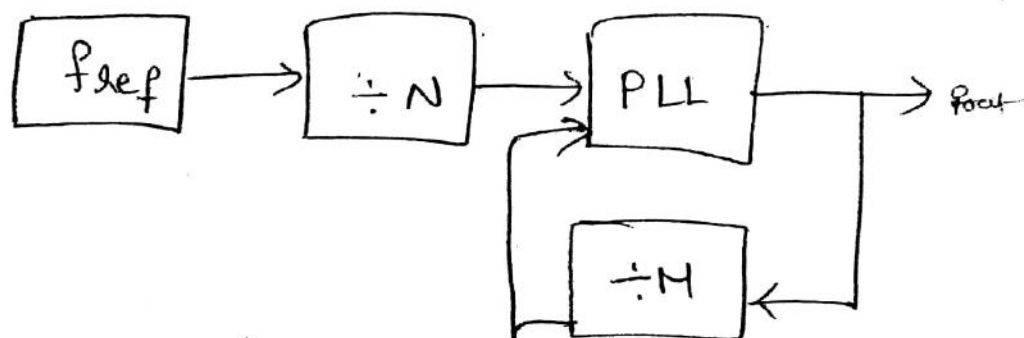
FREQUENCY SYNTHESIS AND OSCILLATORS.

Introduction:-

- ⇒ Oscillators with high quality factor (Q) exhibits best spectral density.
- ⇒ Most of the transceivers operate at different frequencies which causes lack of tuning capability.
- ⇒ In order to overcome the problem of tuning capability the resonators are used for each frequency.
- ⇒ A frequency divider is used for all synthesizers to properly model the effect on loop stability.

Integer-N Synthesis:-

- * The simplest PLL frequency synthesizer uses one reference oscillator and two frequency dividers.



Classic PLL Frequency Synthesizer.

⇒ The loop forces the VCO to a frequency that makes the i/p's to the PLL equal in freq.

$$\frac{f_{ref}}{N} = \frac{f_{out}}{N} \rightarrow (1).$$

$$\therefore f_{out} = \frac{M}{N} \cdot f_{ref} \rightarrow (2).$$

⇒ Thus by varying the divide moduli M & N , any rational multiple of the i/p reference frequency can be generated.

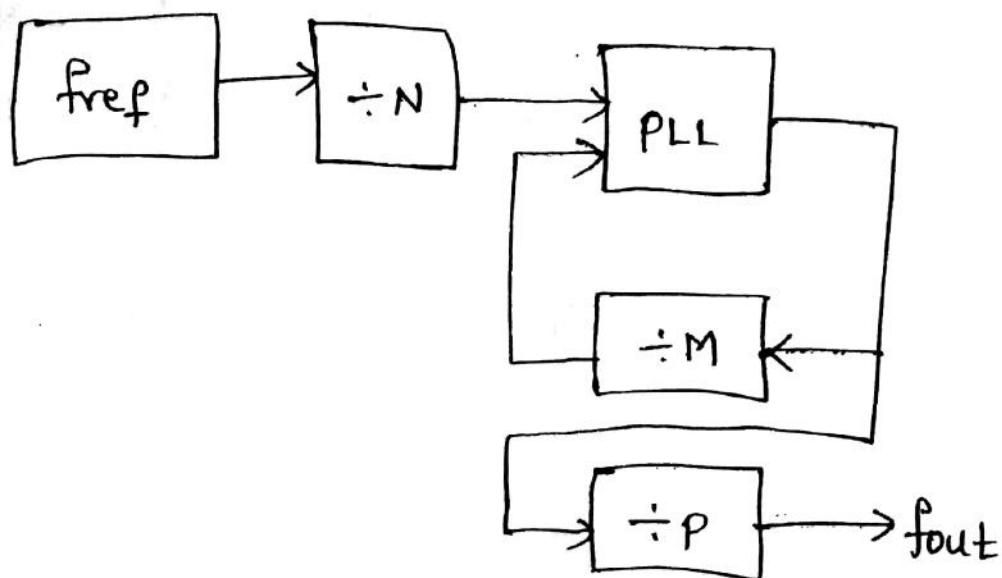
⇒ The o/p frequency can be incremented in steps of f_{ref}/N and this freq. represents the rate at which phase detection is performed in the PLL.

⇒ For this synthesizer,

$$f_{out} = \frac{M}{NP} \cdot f_{ref} \rightarrow (3).$$

⇒ This modification therefore improves the loop bandwidth constraint by a factor of P .

⇒ The PLL oscillates p times faster and the $\div M$ counter runs as much faster as the previous synthesizer.



Modified PLL frequency synthesizer.

⇒ The another modification of the integer-N synthesizer is the divider logic block.

⇒ This logic consists of two counters and a dual-modulus prescaler (divider).

⇒ one counter called the channel-spacing (or "swallow") counter is made programmable to enable channel selection.

⇒ The other counter is called as frame counter (also known as program counter) which determines the total no: of prescaler cycles that performs certain operations.

⇒ The prescaler initially divides by (N+1) until the channel-spacing counter overflows, then it divides by (N') until the frame counter overflows and the cycle repeats.

⇒ If 's' is the maximum value of the channel-spacing counter and 'F' is the maximum value of the frame count then the prescaler divides the VCO o/p by $(N+1)$ for s cycles.

⇒ Then by $F-s$ for N cycles.

⇒ The effective overall divide modulus 'M' is,

$$M = (N+1)s + (F-s)N$$

$$M = Ns + s + NF - Ns$$

$$\therefore M = NF + s \rightarrow \textcircled{4}$$

⇒ The o/p frequency increment is thus equal to the reference frequency.

Fractional Frequency Synthesis:- [Synthesizers with Dithering Module]

⇒ In the integer module the desired channel spacing directly constrains the loop bandwidth.

⇒ In this freq synthesizer there will be a two module to generate channel spacings that are smaller than the reference frequency.

⇒ Changing the percentage of time spent on any one module will change the effective modulus (avg modulus) of the signal, so that the o/p can be incremented by frequency steps smaller than the i/p reference frequency.

⇒ There are many strategies for switching b/w two modules which yields the same avg modulus.

⇒ The most common strategy used by the fractional-N synthesizer is, the one which divides VCO o/p by a one modulus $(N+1)$ for every 'K' VCO cycles.

⇒ The other by (N) for the next cycles.

⇒ Hence the average divide factor is,

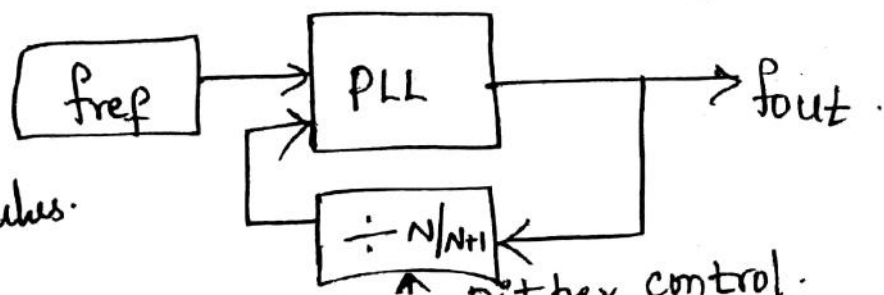
$$N_{eff} = (N+1)\left(\frac{1}{K}\right) + N\left(1 - \frac{1}{K}\right)$$

$$= N \cdot \left(\frac{1}{K}\right) + \left(\frac{1}{K}\right) + N - N\left(\frac{1}{K}\right)$$

$$N_{eff} = N + \frac{1}{K}$$

$$f_{out} = N_{eff} f_{ref} = \left(N + \frac{1}{K}\right) f_{ref}$$

Block D/g for
freq synthesizer
with digital modulus.



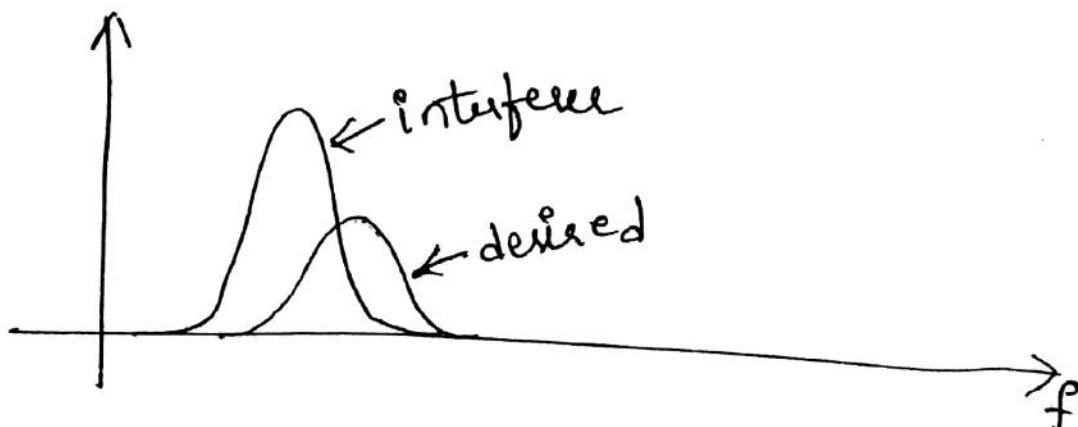
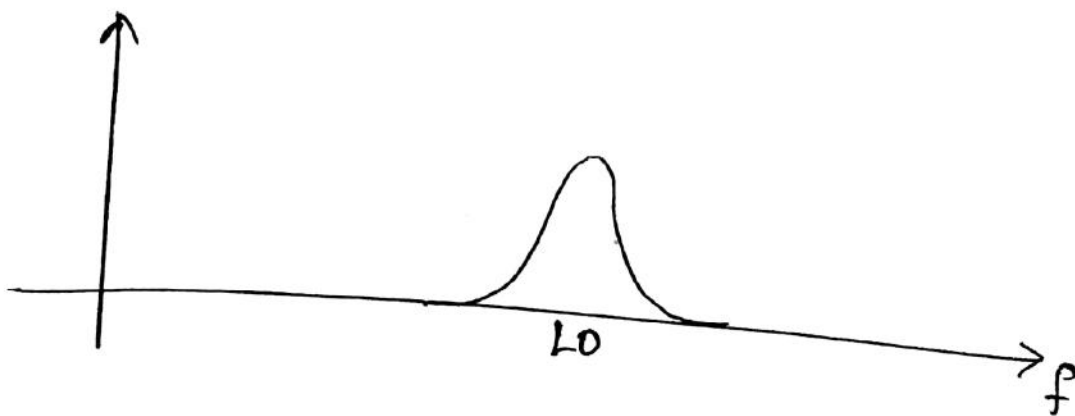
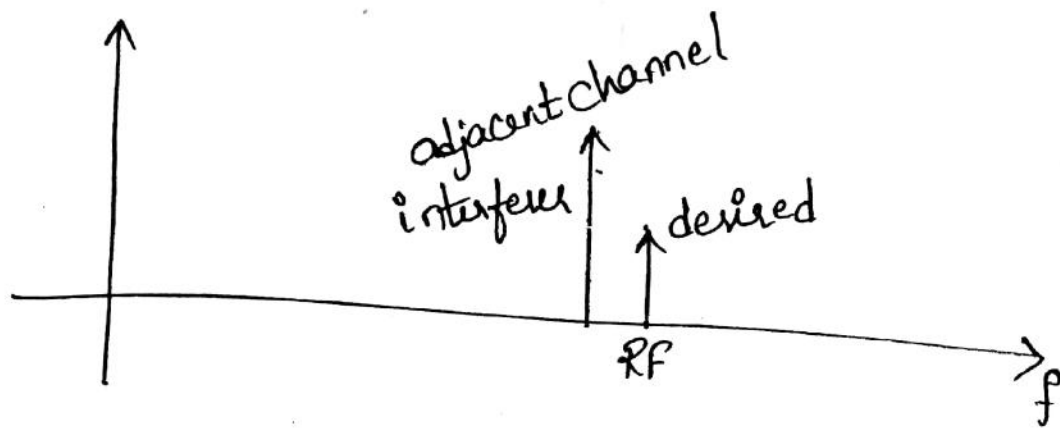
Phase Noise:-

- ⇒ The tuned oscillators produce o/p's with higher spectral density than relaxation oscillators.
- ⇒ High-Q resonator attenuates spectral components from the center frequency.
- ⇒ This reduces the distortions (distortions are suppressed).
- ⇒ In addition to suppressing distortions, a resonator also attenuates spectral components contributed by the source such as thermal noise etc.
- ⇒ phase noise is to minimize the problem of "reciprocal mixing".
- ⇒ In a superhetrodyne receiver the local oscillator is completely noise-free.
- ⇒ Hence the two RF signals simply translate the frequency b/w each other.
- ⇒ If the local oscillator is impure (includes some noise) then the two RF sigs heterodyne with the 'LO' to produce a pair of IF sigs.

⇒ Since the 'LO' spectrum sig is of non-zero, the downconverted RF sig cant be translated efficiently.

⇒ Hence reciprocal mixing takes place.

↳ heterodyning of RF sigs with those unwanted components.

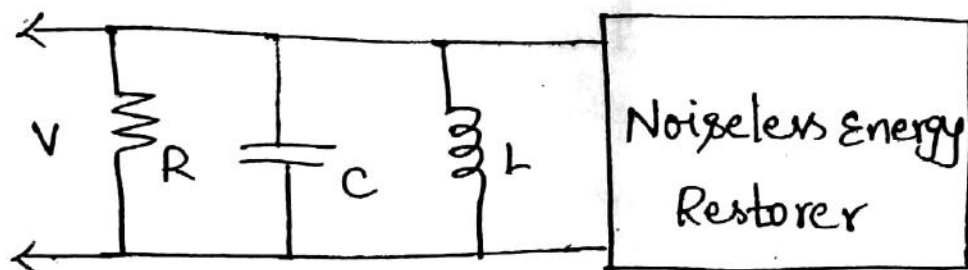


Reciprocal Mixing due to LO phase noise.

General considerations:-

⇒ Assume that the energy restorer is noiseless.

⇒ The tank resistance is therefore the only noisy element in the model.



"Perfectly Efficient" RLC oscillator.

⇒ The signal energy stored by the tank is,

$$E_{sig} = \frac{1}{2} C V_{pk}^2 \rightarrow (1)$$

⇒ The mean-square sig (carrier) voltage is,

$$\overline{V_{sig}^2} = \frac{E_{sig}}{C} \rightarrow (2)$$

⇒ The total mean-square noise voltage can be obtained by integrating the resistor's thermal noise density over the noise B.W of the RLC resonator,

$$\overline{V_n^2} = 4KTR \int_0^\infty \left| \frac{Z(f)}{R} \right|^2 df$$

$$\overline{V_n^2} = 4KTR \cdot \frac{1}{4R_c} = \frac{KT}{C} \rightarrow (3).$$

\Rightarrow combining the eqs (2) & (3), we obtain a noise-to-carrier ratio as,

$$\begin{aligned} \frac{N}{C} &= \frac{\overline{V_n^2}}{\overline{V_{sig}^2}} \\ &= \frac{\frac{KT}{C}}{\frac{E_{sig}}{C}} = \frac{KT}{C} \times \frac{C}{E_{sig}} \end{aligned}$$

$$\therefore \frac{N}{C} = \frac{KT}{E_{sig}} \rightarrow (4).$$

\therefore The signal levels has to be maximized to minimize the noise-to-carrier ratio.

\Rightarrow The resonator Q can be defined as Energy stored by the signal divided by the energy dissipated.

$$Q = \frac{\omega_0 E_{sig}}{P_{diss}} \rightarrow (5).$$

$$\Rightarrow E_{sig} = \frac{Q \cdot P_{diss}}{\omega_0} \rightarrow (6)$$

From eq (4), $\frac{N}{C} = \frac{KT}{E_{sig}}$

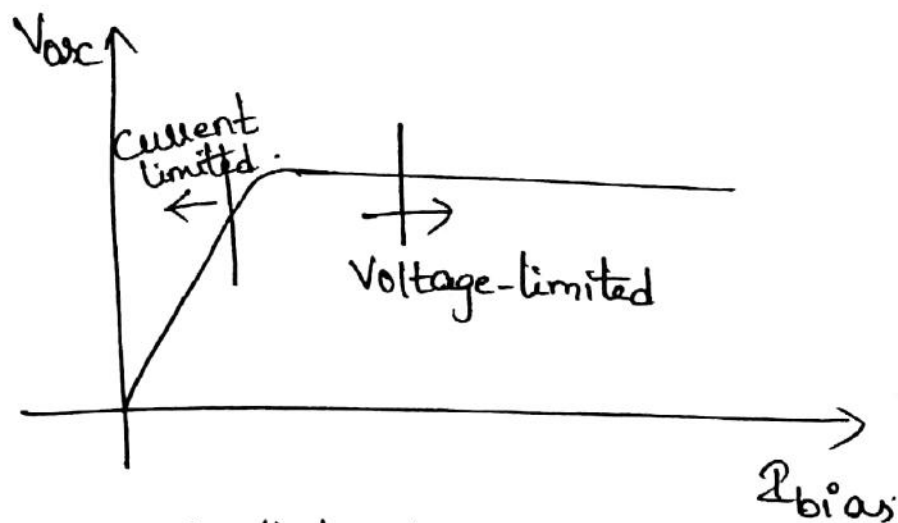
Substitute eq (6) in eq (4),

$$\frac{N}{C} = \frac{KT}{E_{sig}}$$

$$\frac{N}{C} = \frac{KT}{\frac{Q P_{dis}}{\omega_0}}$$

$$\frac{N}{C} = \frac{KT \omega_0}{Q P_{dis}} \rightarrow (7)$$

Hence the noise-to-carrier ratio is inversely proportional to the product of the resonator Q & the power consumed.
 \Rightarrow It is directly proportional to the oscillation frequency.



Oscillator operating regimes.

⇒ The o/p amplitude on bias current is,

$$V_{sig} = I_{BIAS} \cdot R \rightarrow (8)$$

where 'R' is a constant of proportionality with the dimension of resistance.

⇒ This constant is in turn proportional to the equivalent parallel tank resistance.

$$V_{sig} \propto I_{BIAS} R_{tank} \rightarrow (9)$$

⇒ The carrier power,

$$P_{sig} \propto I_{BIAS}^2 R_{tank} \rightarrow (10)$$

⇒ The mean-square noise voltage in terms of the tank capacitor is,

$$\overline{V_n^2} = \frac{KT}{C} \rightarrow (11)$$

But it can also be expressed in terms of tank inductance,

$$\overline{V_n^2} = \frac{KT}{C} = \frac{KT}{1/\omega_0^2 L} = KT\omega_0^2 L \rightarrow (12)$$

∴ The noise-to-carrier ratio in the current-limited region is,

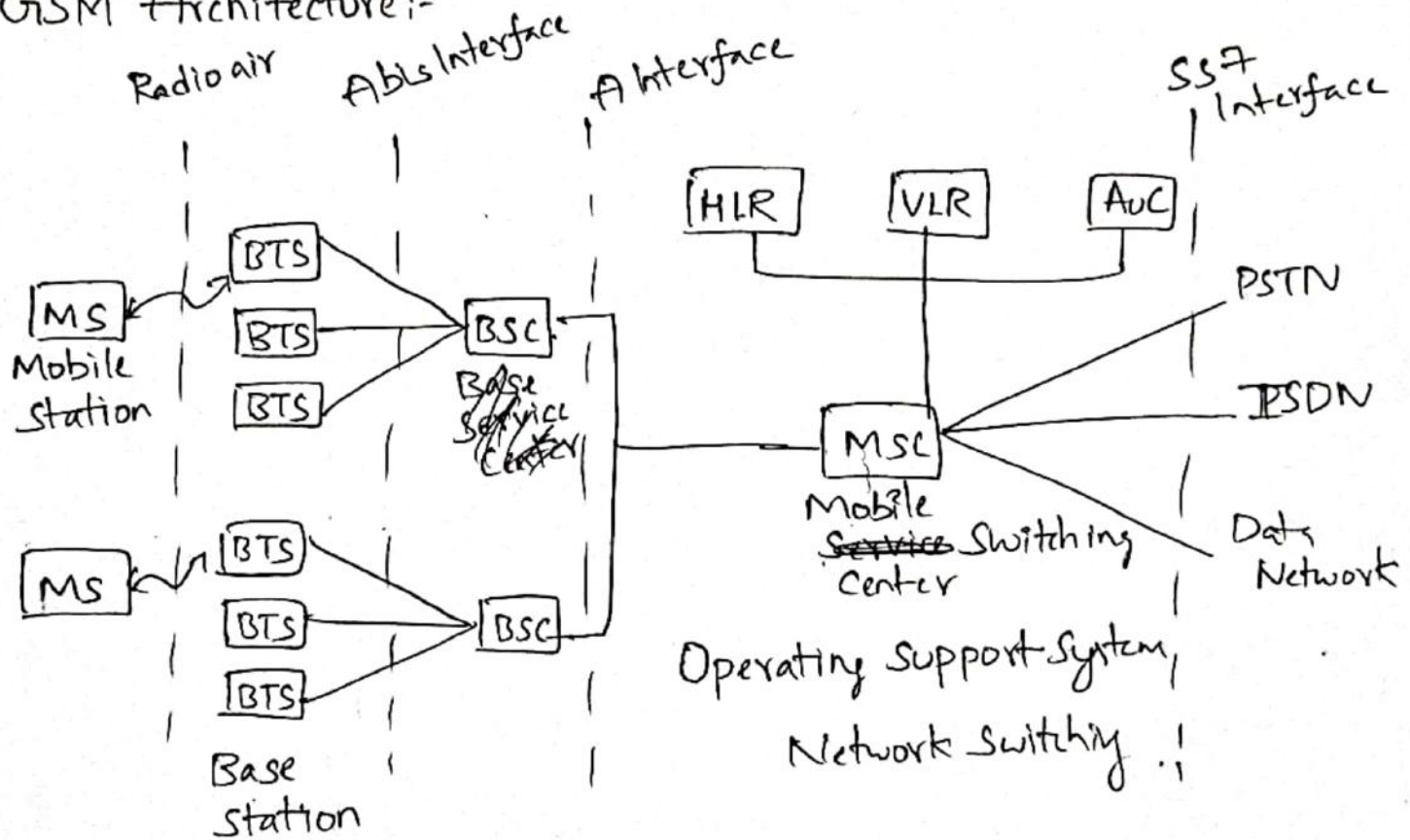
$$\boxed{\frac{N}{C} \propto \frac{KT\omega_0^2 L}{I_{BIAS}^2 R_{tank}}} \rightarrow (13)$$

GSM

- * GSM network comprises of many functional units.
- * GSM N/w can be broadly classified into 4 types

1. MS - Mobile station.
2. BSS - Base station Subsystem
3. NSS - Network switching Subsystem
4. OSS - Operating support Subsystem

GSM Architecture:-



HLR - Home Location Register

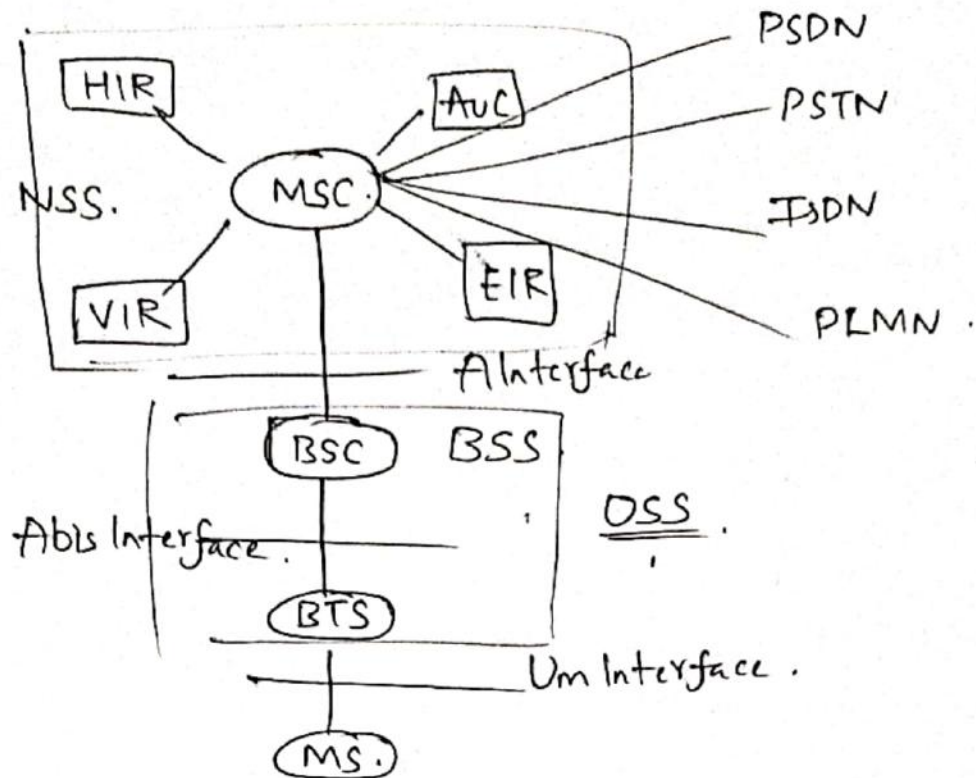
VLR - Visitor Location Register

AuC - Authentication Center

BSC - Base Station Controller

BTS - Base Transceiver station.

Simple pictorial View of GSM Network.



EIR- Equipment Identity Register,

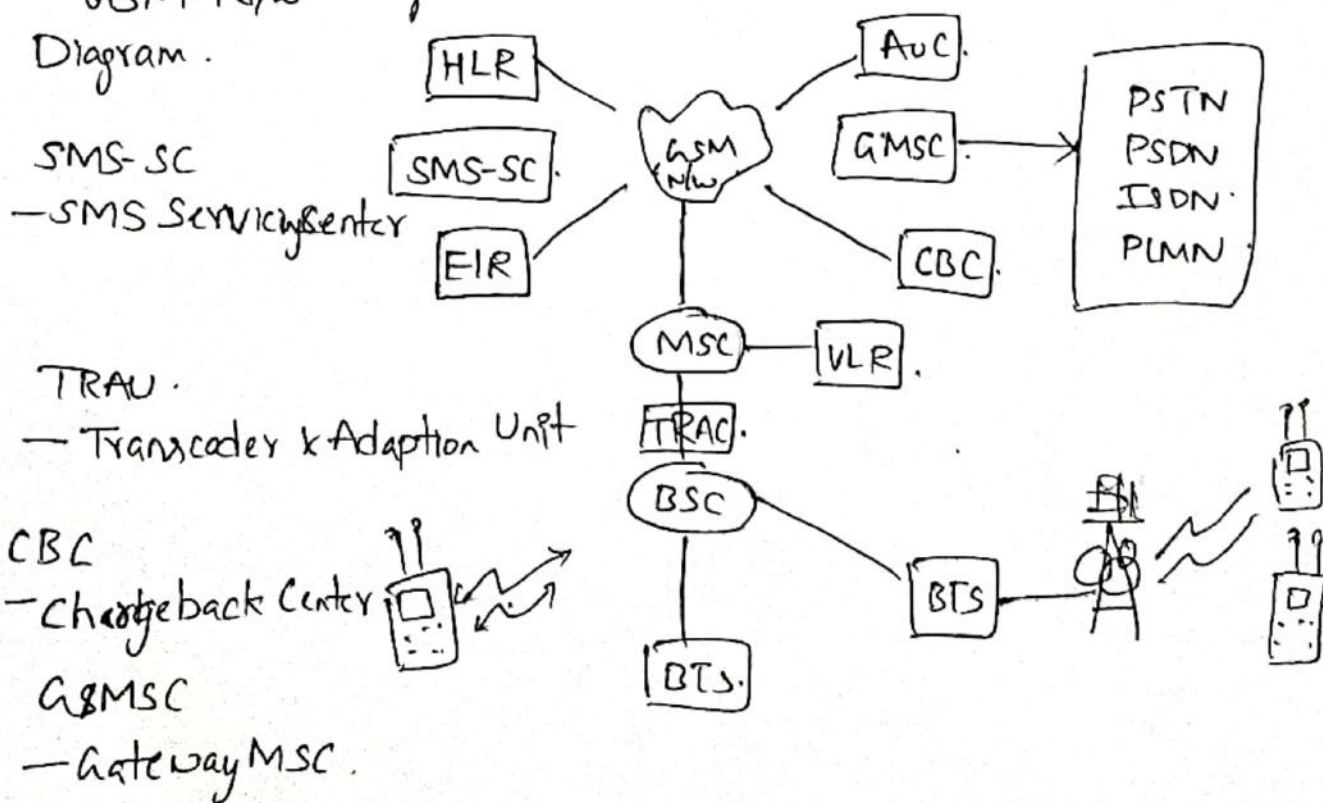
* MS and BSS communicate across the Um Interface. It is also known as air Interface/ Radio link.

* BSS and NSS communicate across the A Interface,

* BTS and BSC communicate across the Abis Interface.

GSM N/w along with added elements:-

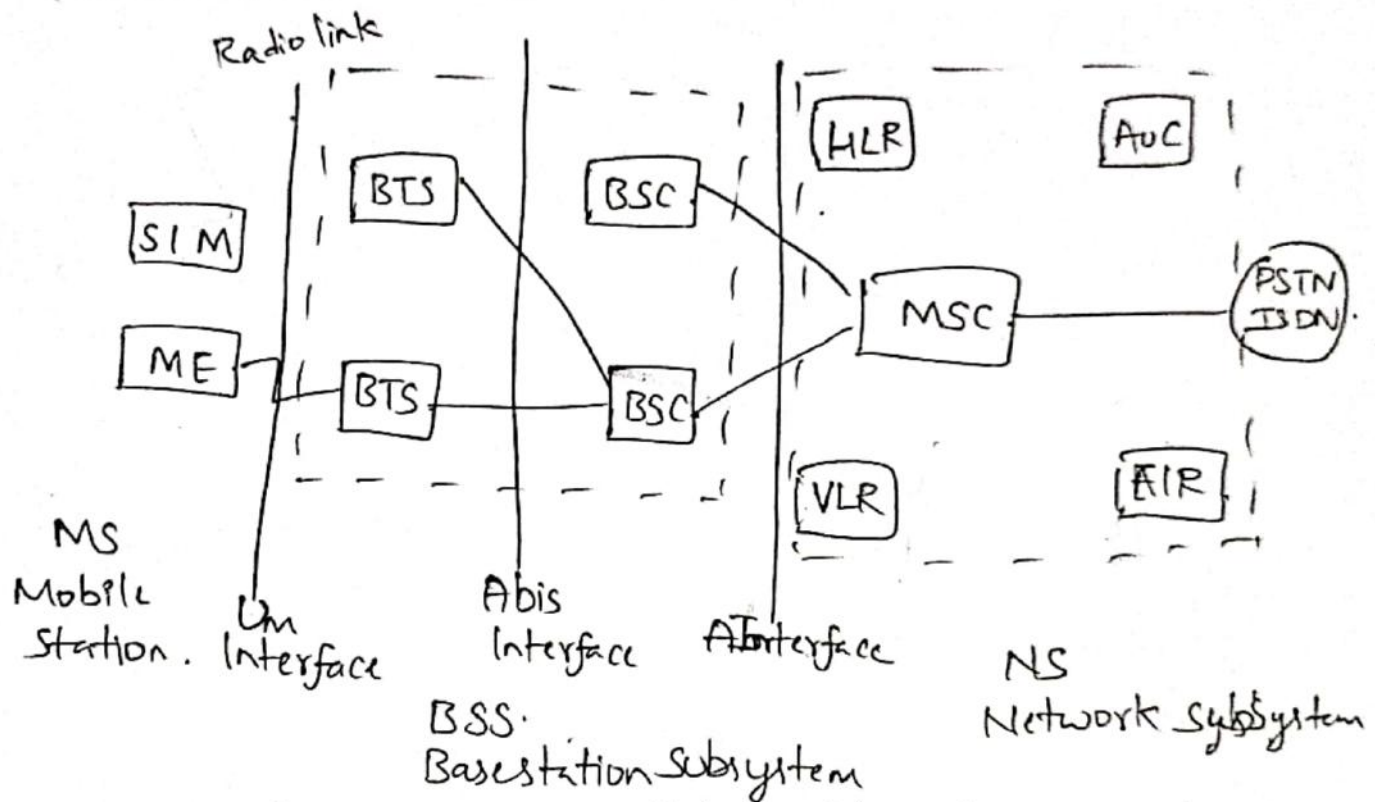
Diagram.



CDMA

- * A digital multiple access technique specified by the Telecommunication Industry Association (TIA) as "IS-95"
- * One of the unique aspects of CDMA is, while there are certainly limits to the number of phone calls that can be handled by a carrier, this not a fixed number.
- * CDMA is a digital air interface standard, which is eight to fifteen times the capacity of analog.

General Architecture of CDMA.



MS - Mobile station SIM - Subscriber Identity Module

ME - Mobile Equipment

→ Functions of MS:-

* Personal Mobility.

IMEI & IMSI

- International Mobile Equipment Identity

- International Mobile Subscriber Identity

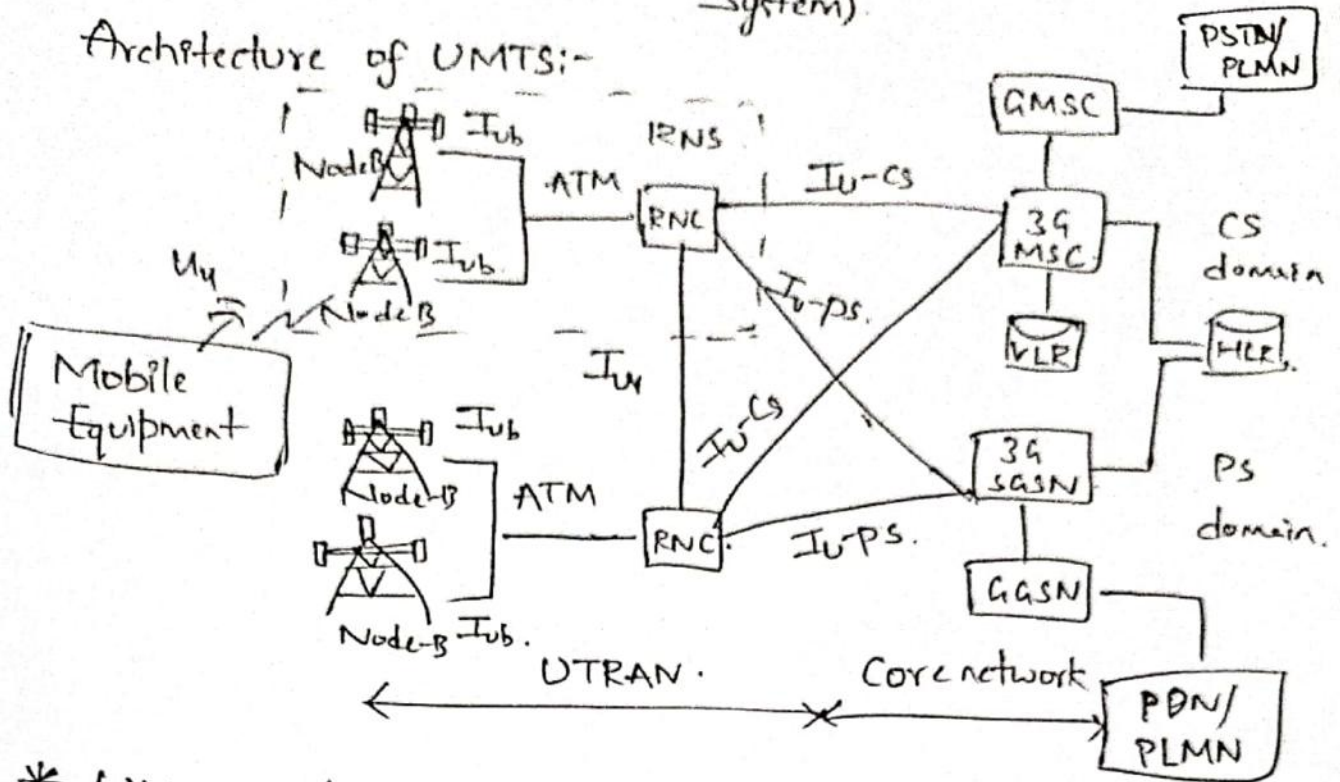
BTS - Base Transceiver Station

BSC - Base Station Controller

MSC - Mobile Switching Center

UMTS (Universal Mobile Telecommunication System).

Architecture of UMTS:-



- * UMTS system uses the same core N/w as the GPRS and uses entirely new radio interface.
- * The new Radio N/w in UMTS is called - "UTRAN" (Universal Terrestrial Radio Access Network).
- * I_u is the UTRAN interface b/w the Radio Network Controller (RNC) and Core Network (CN).
- * Mobile terminal in UMTS is called "User Equipment".
- * User Equipment is connected to Node-B over high speed U_u (upto 2Mbps) interface.
- * Several Node-B's are controlled by a single RNC's over I_{ub} interfaces.
- * The packet switched data is transmitted through I_{u-PS} interface.
- * The circuit switched data is transmitted through I_{u-CS} interface.