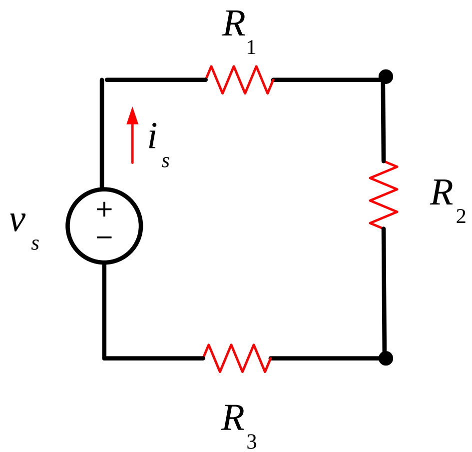
Electrical Circuits-1

UNIT-I

INTRODUCTION:

we have analyzed relatively simple resistive circuits by applying KVL and KCL in combination with Ohm's law.



Applying KVL and Ohm's law, we have,

$$v_s - iR_1 + iR_2 + iR_3 = 0$$

Solving for i , we have,

$$i = \frac{v_s}{R_1 + R_2 + R_3}$$

However as circuit become more complicated and involve more elements, this direct method become cumbersome

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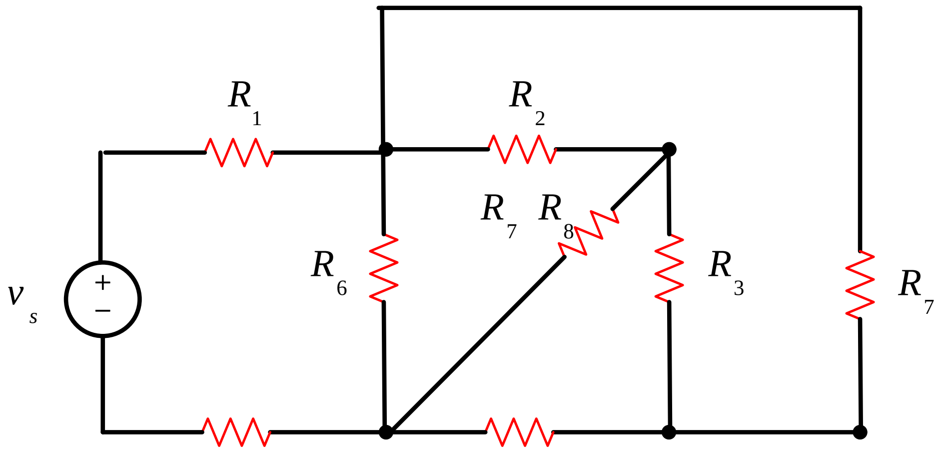
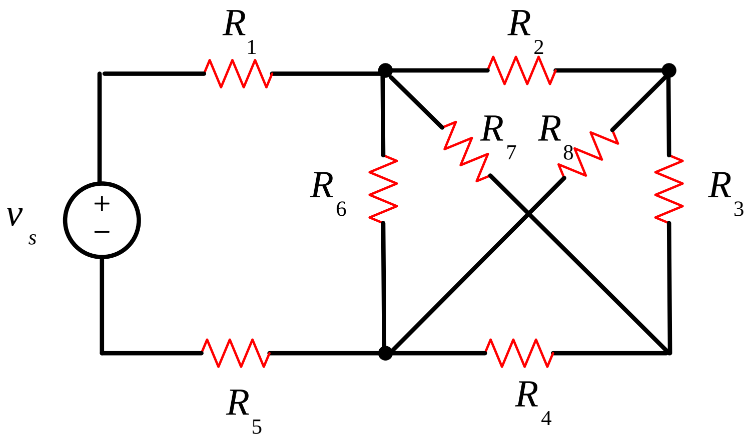


In this chapter we introduce two powerful techniques of circuit analysis that aid in the analysis of complex circuit which they are known as

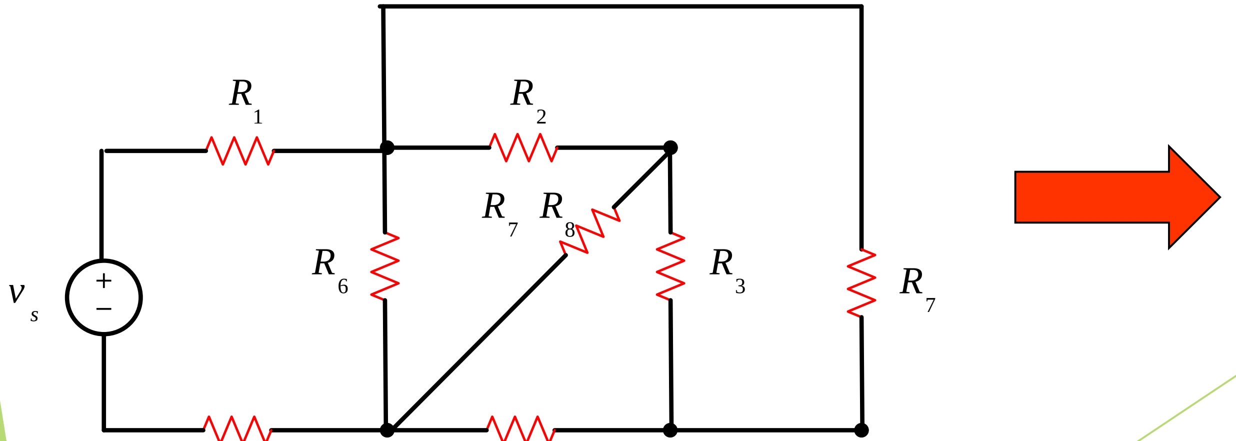
Node-Voltage Method or Nodal Analysis **Mesh-Current Method or Mesh Analysis**

In addition to these method we will develop other method such as source transformations, Thevenin and Norton equivalent circuit.

Planar Circuit : a circuit that can be drawn on a plane with no crossing branches as shown

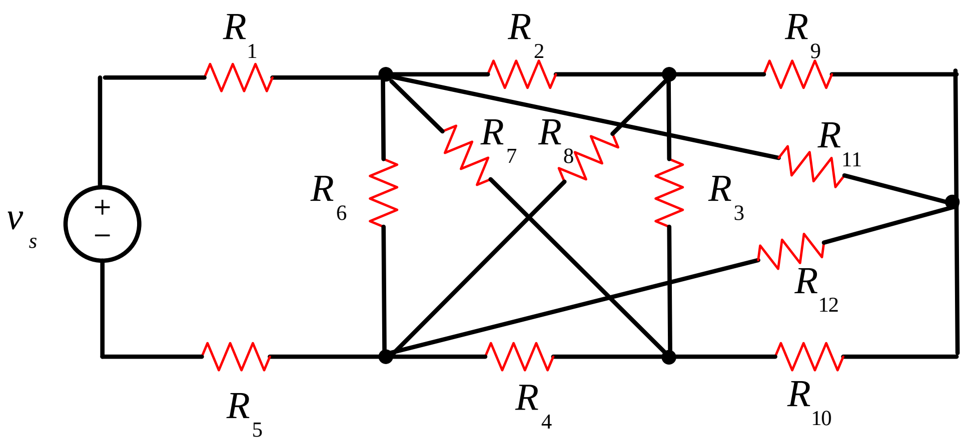


The circuit shown can be redrawn which is equivalent to the above circuit



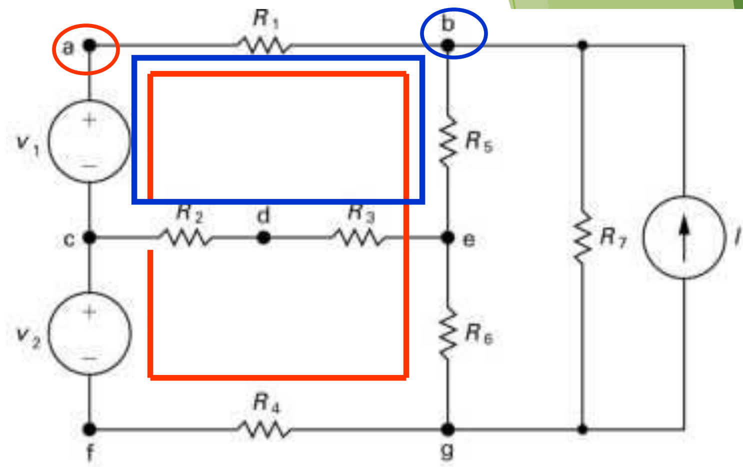
Planar Circuit

The circuit shown is non planar circuit because it can not be redrawn to make it planar the circuit



Describing a Circuit – The vocabulary

Consider the following circuit



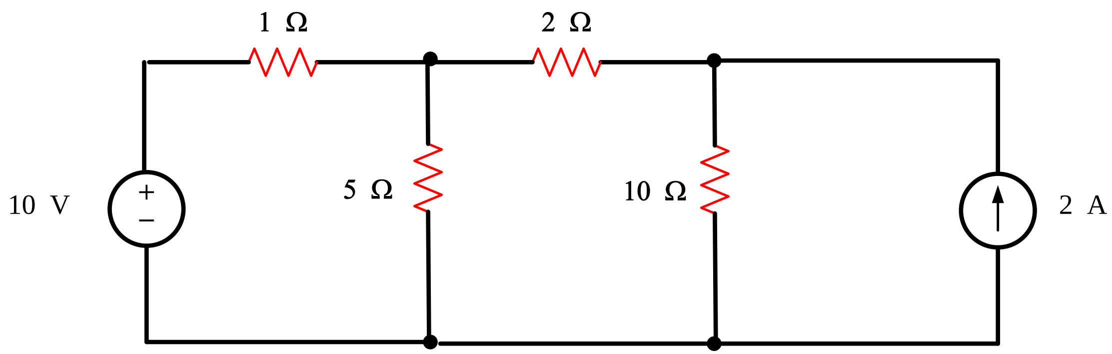
Node	A point where two or more circuit elements join	a
Essential node	A node where three or more circuit elements join	b
path	A trace of adjoining basic elements with no elements included	$v_1 - R_1 - R_5 - R_6$
branch	A path that connects two nodes	R_1
Essential branch	A path that connects two essential nodes without passing through an essential node	$v_1 - R_1$
loop	A path whose last node is the same as the starting node	$v_1 - R_1 - R_5 - R_6 - R_4 - v_2$
mesh	A loop that does not enclose any other loops	$v_1 - R_1 - R_5 - R_3 - R_2$
Planar Circuit	A circuit that can be drawn on a plane with no crossing branches	

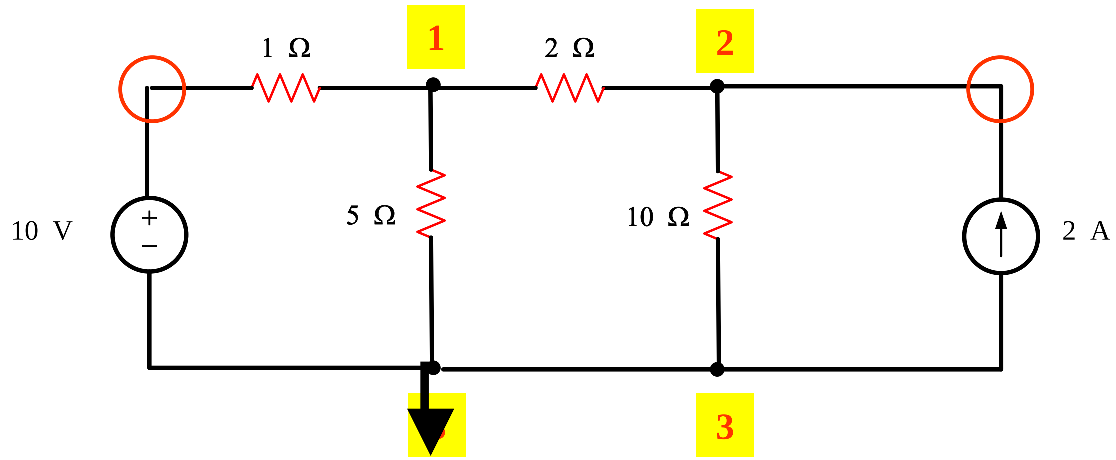
NODE-VOLTAGE METHOD

Nodal analysis provide a general procedure for analyzing circuits using node voltages as the circuit variables

Node-Voltage Method is applicable to both planar and nonplanar circuits

Using the circuit shown, we can summarize the node-voltage methods as shown next :



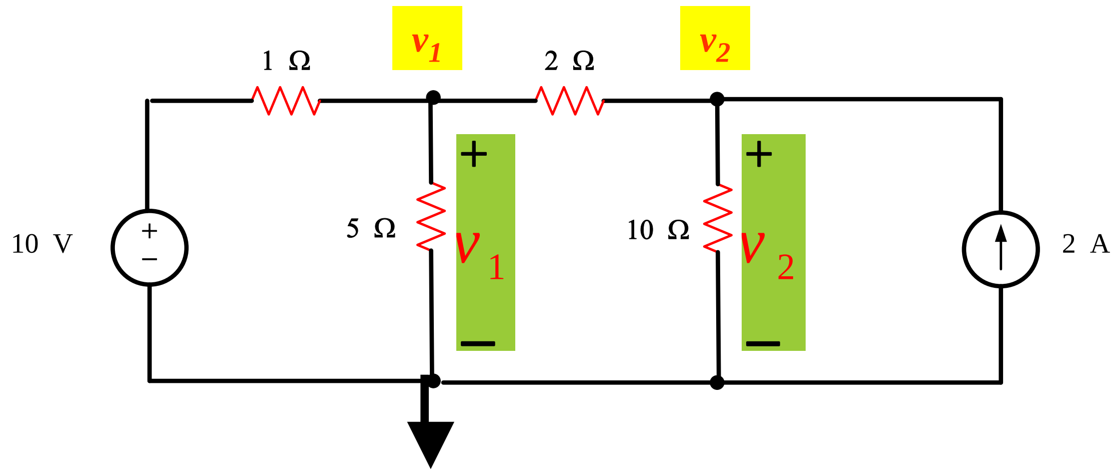


Step 1 identify all essentials nodes *Do not select the non essentials nodes*

Step 2 select one of the essentials nodes (**1**, **2**, or **3**) as a reference node

Although the choice is arbitrary the choice for the reference node is were most of branches, example node **3**

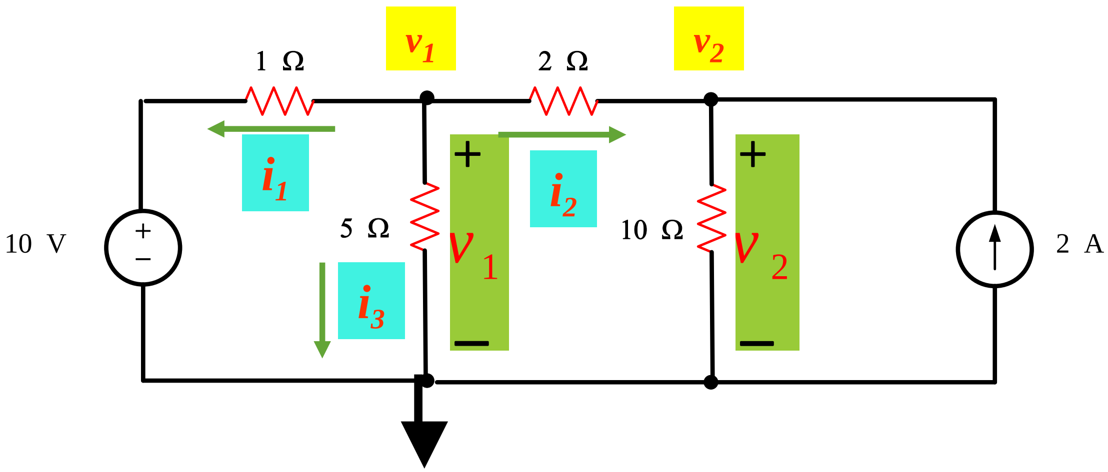
Selecting the reference node will become apparent as you gain experience using this method (i,.e, solving problems)



A node voltage is defined as the voltage rise from the reference node to a nonreference node

<u>Step 3</u> label all nonrefrence essentials nodes with alphabetical label as v_1, v_2...

<u>Step 4</u> write KCL equation on all labels nonrefrence nodes as shown next

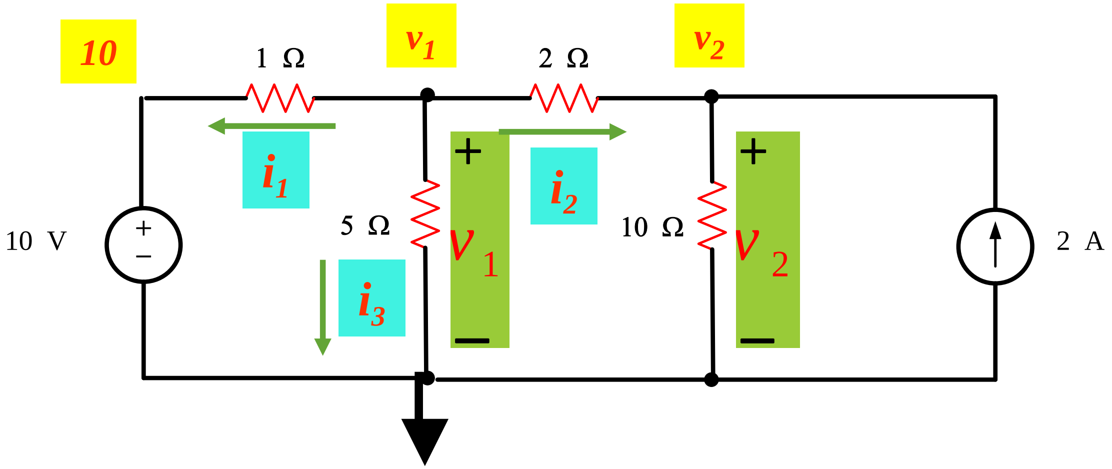


KCL at node 1 $i_1 + i_2 + i_3 = \mathbf{0}$ Let us find i_1 , i_2 , i_3

By applying KVL $(1)i_1 + 10 - v_1 = 0 \Rightarrow i_1 = \dfrac{v_1 - 10}{(1)}$

KVL on the middle mesh, we have $-v_1 + (2)i_2 + v_2 = 0 \Rightarrow i_2 = \dfrac{v_1 - v_2}{2}$

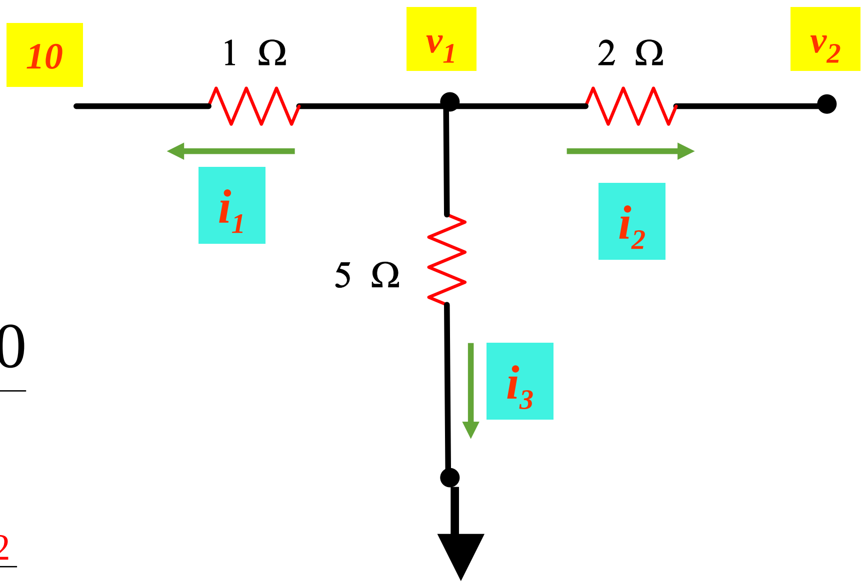
similarly $v_1 - 0 = (5)i_3 \Rightarrow i_3 = \dfrac{v_1}{(5)}$



Therefore $i_1 + i_2 + i_3 = \mathbf{0}$ We have

$$\frac{v_1 - 10}{1} + \frac{v_1 - v_2}{2} + \frac{v_1}{5} = \mathbf{0}$$

If we look at this KCL equation, we see that the current we notice that the potential at the left side of the **1 Ω** resistor which is tied to the + of the 10 V source is 10 V because the – is tied to the reference



$$i_1 = \frac{v_1 - 10}{1}$$

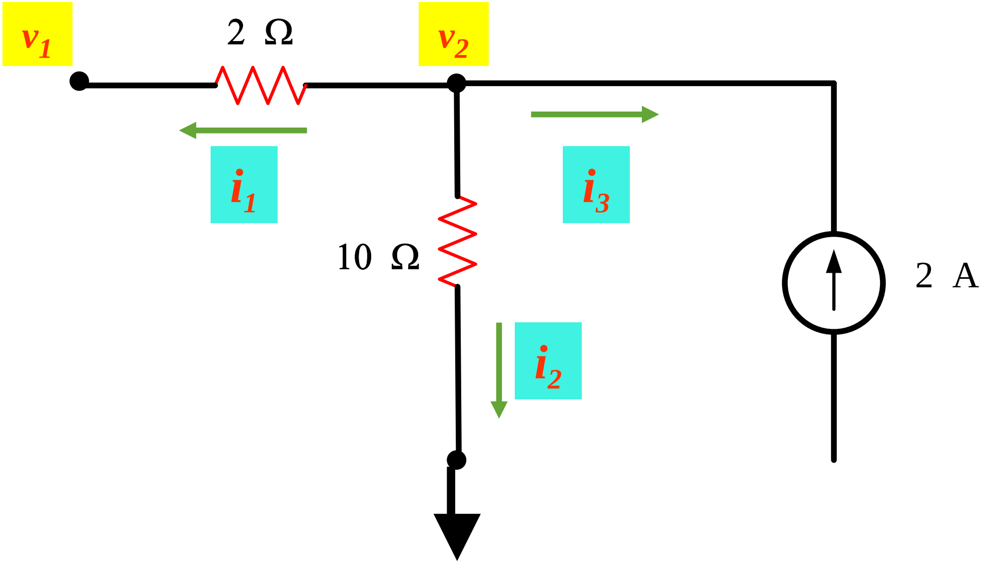
$$i_2 = \frac{v_1 - v_2}{2}$$

$$i_3 = \frac{v_1}{5}$$

Therefore we can write **KCL** at node 1 without doing **KVL**'s as we did previously

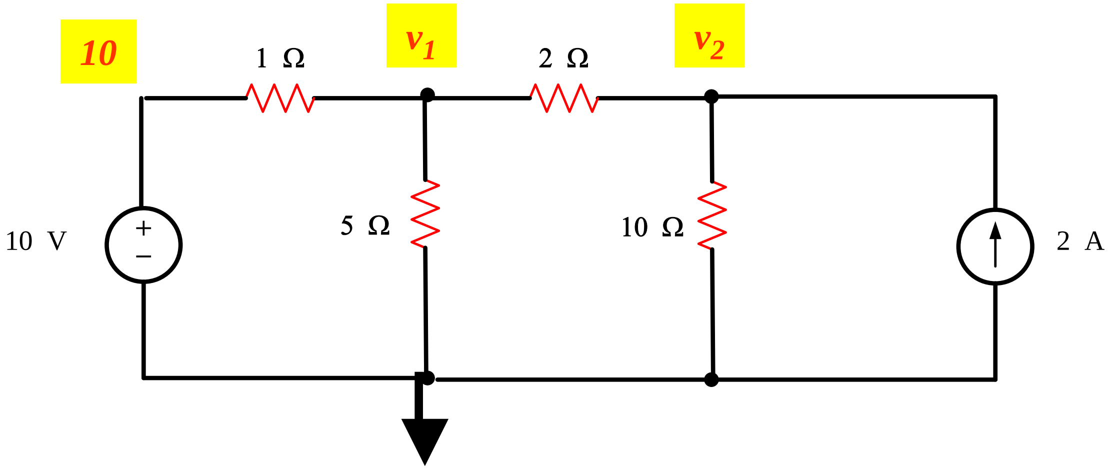
$$\frac{v_1 - 10}{1} + \frac{v_1 - v_2}{2} + \frac{v_1}{5} = \mathbf{0}$$

Similarly



KCL at node 2

$$\frac{v_2 - v_1}{2} + \frac{v_2}{10} - 2 = \mathbf{0}$$



$$\frac{v_1 - 10}{1} + \frac{v_1 - v_2}{2} + \frac{v_1}{5} = \mathbf{0}$$

$$\frac{v_2 - v_1}{2} + \frac{v_2}{10} - 2 = \mathbf{0}$$

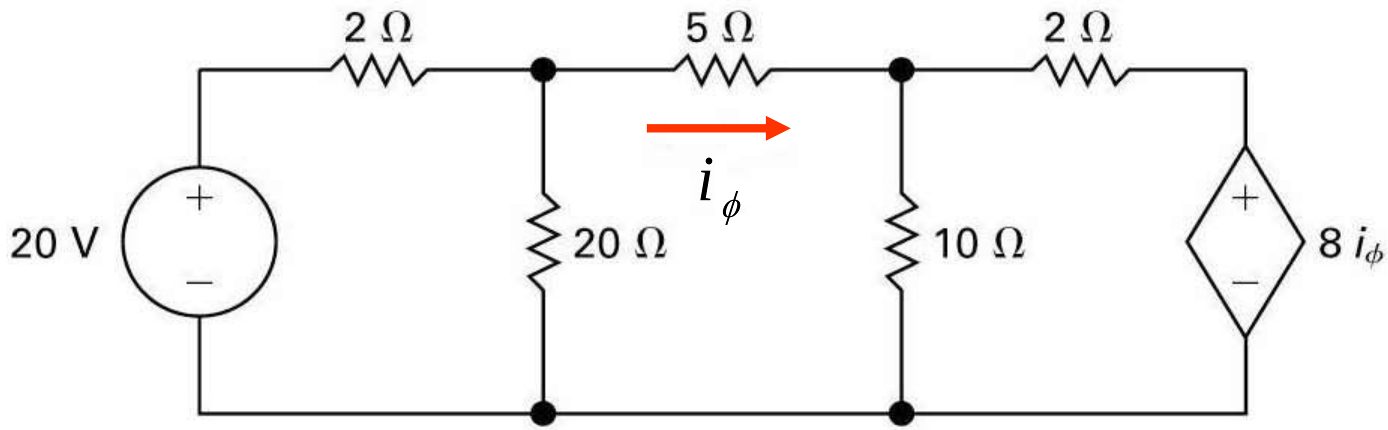
Two equations and two unknowns namely v_1 , v_2 we can solve and have

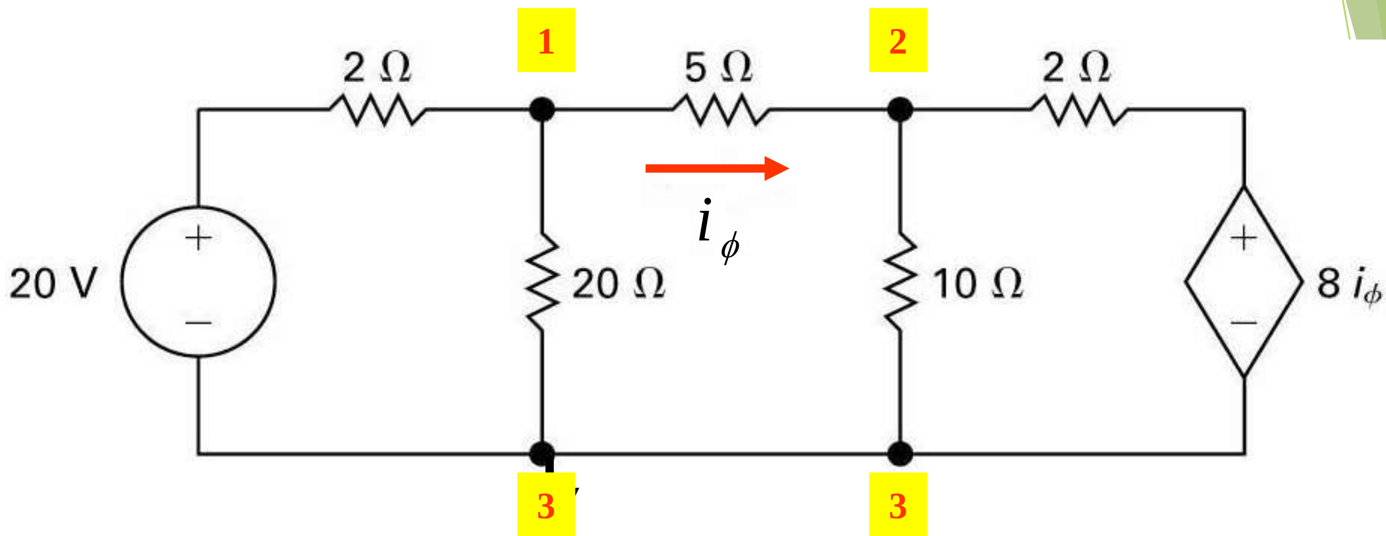
$$v_1 = \frac{100}{11} = 9.09 \ \ V \qquad\qquad v_2 = \frac{120}{11} = 10.91 \ \ V$$

The Node-Voltage Method and Dependent Sources

If the circuit contains dependent source, the node-voltage equations must be supplemented with the constraint equations imposed by the dependent source as will be shown in the example next

Example 1. Use the node voltage method to find the power dissipated in the **5 Ω** resistor

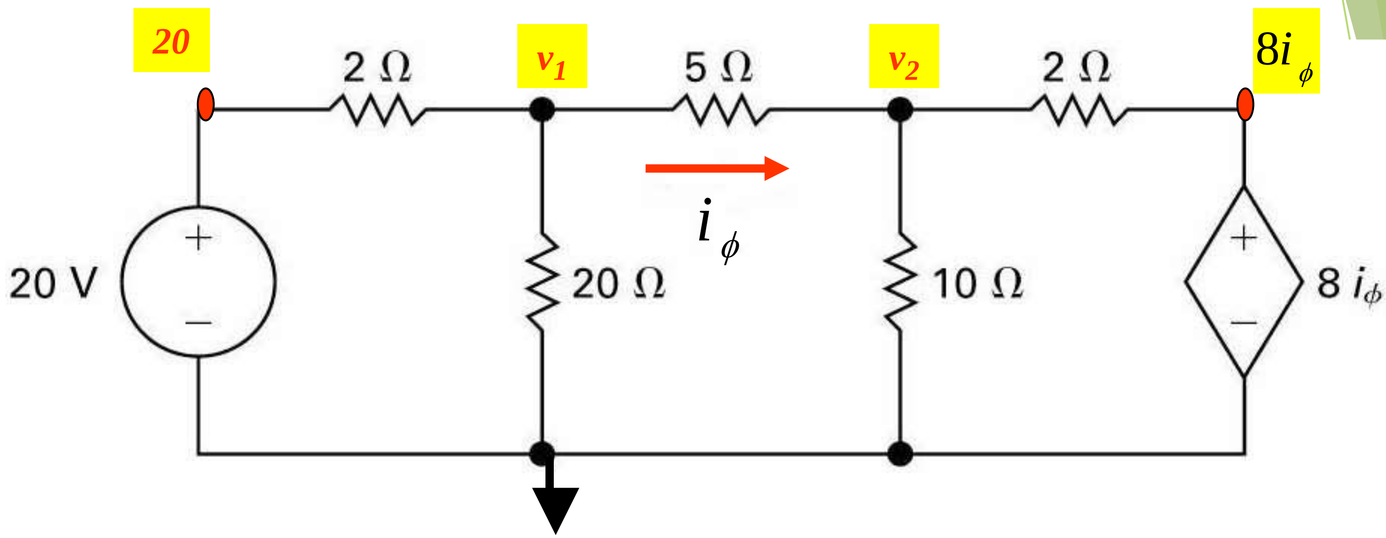




The circuit has three essentials nodes **1**, **2**, and **3** and one of them will be the reference

We select node **3** as the reference node since it has the most branches (i.e, **4** branches)

We need two node-voltage equations to describe the circuit



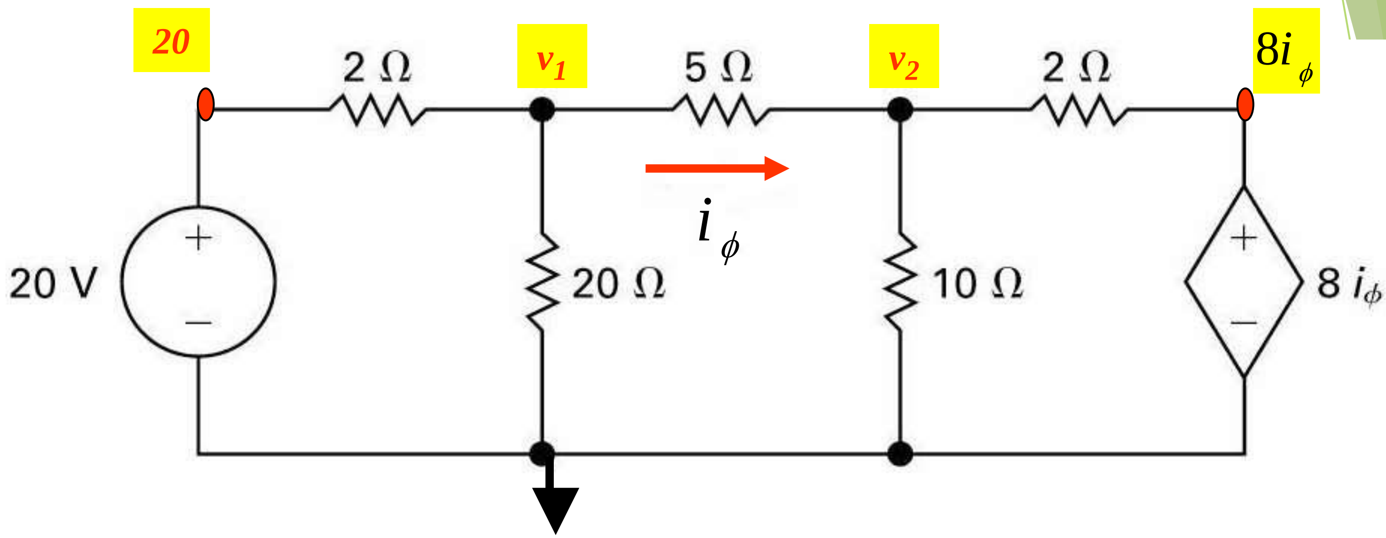
The circuit has two non essentials nodes which are connected to voltage sources and will impose the constrain imposed by the value of the voltage sources on the non essential nodes voltage

Applying KCL at node 1

$$\frac{v_1 - 20}{2} + \frac{v_1}{20} + \frac{v_1 - v_2}{5} = 0$$

Applying KCL at node 2

$$\frac{v_2 - v_1}{5} + \frac{v_2}{10} + \frac{v_2 - 8i_\phi}{2} = 0$$



$$\frac{v_1 - 20}{2} + \frac{v_1}{20} + \frac{v_1 - v_2}{5} = 0 \qquad \frac{v_2 - v_1}{5} + \frac{v_2}{10} + \frac{v_2 - 8i_\phi}{2} = 0$$

The second node equation contain the current i_ϕ which is related to the nodes voltages as

$$i_\phi = \frac{v_1 - v_2}{5}$$

Substituting in the second node equation, we have the following nodes equations

$$0.75 v_1 - 0.2 v_2 = 10$$

$$-v_1 + 1.6 v_2 = 0$$

Solving we have

$$v_1 = 16 \text{ V} \qquad v_2 = 10 \text{ V}$$

$$i_\phi = \frac{16-10}{5} = \mathbf{1.2} \text{ A} \qquad p_{5\Omega} = (\mathbf{1.2})^2(5) = \mathbf{7.2} \text{ W}$$

Nodal Analysis With Voltage Sources

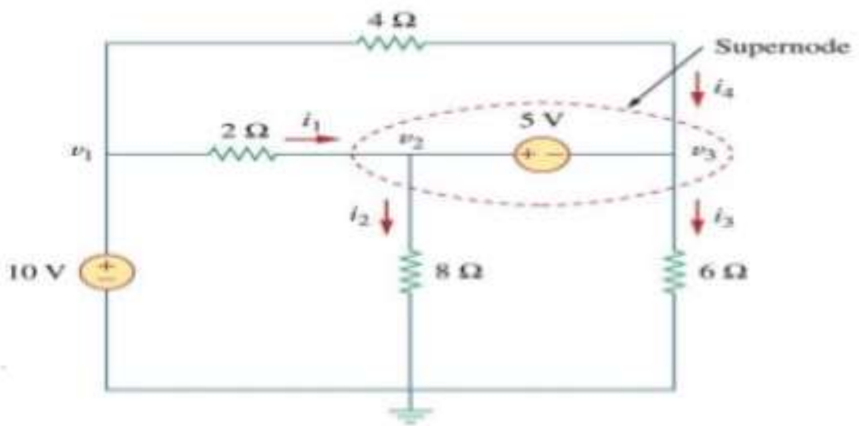
We now consider how voltage sources affect nodal analysis:

CASE 1: If a voltage source is connected between the reference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source

E.g.: v_1 = 10V

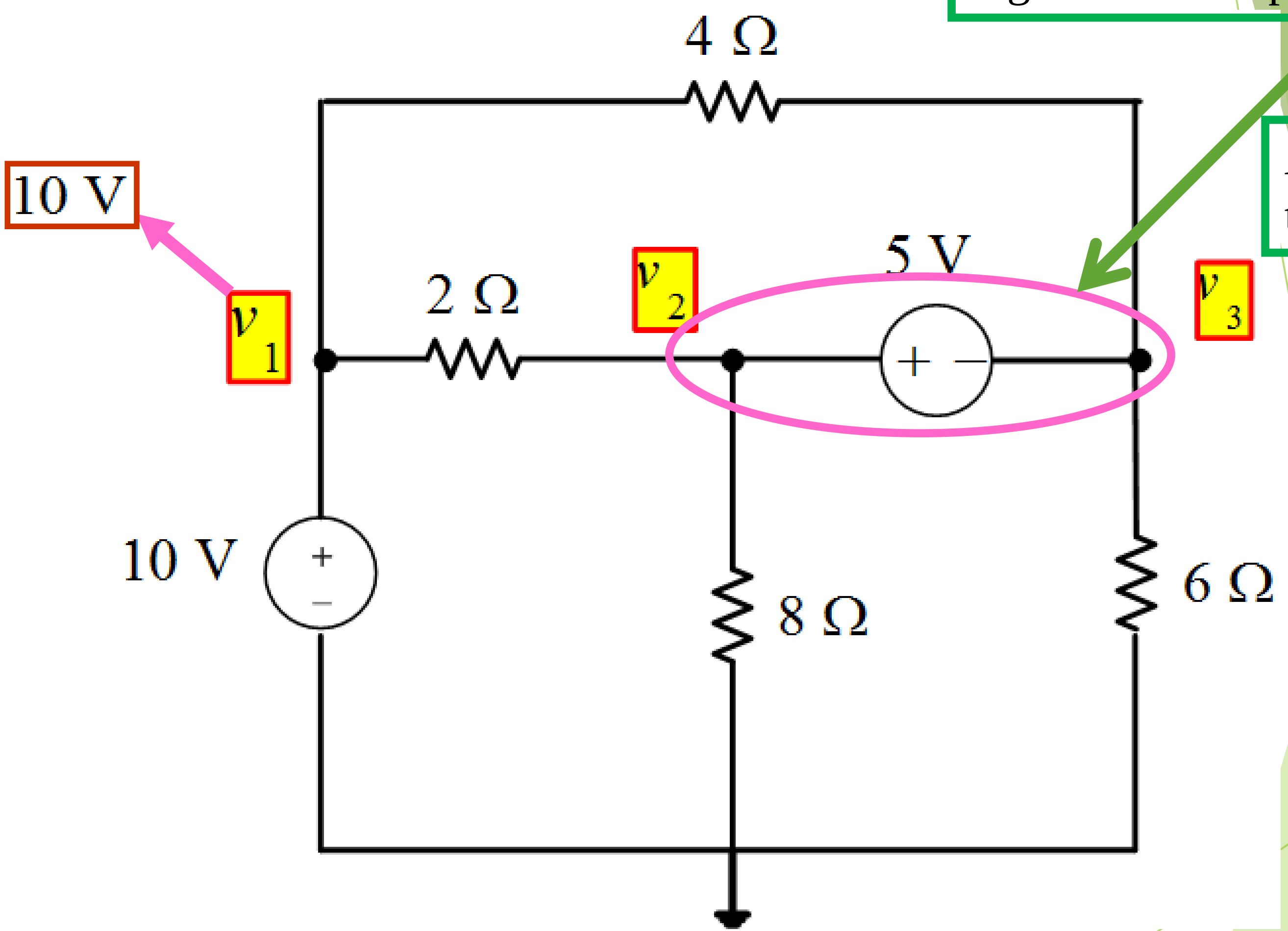
CASE 2: If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a *generalized node* or *supernode*; apply both KCL and KVL to determine the node voltages

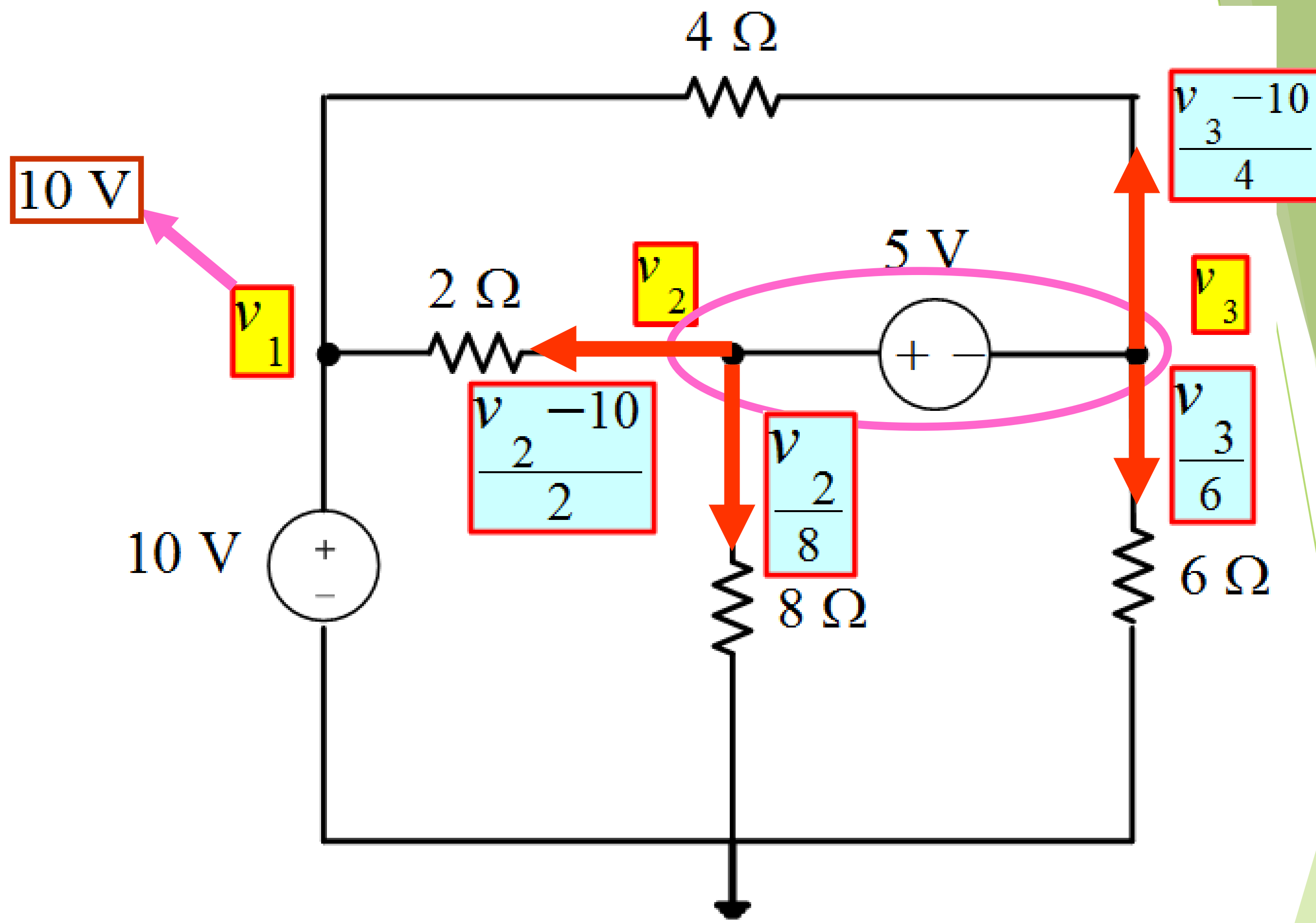
E.g.: nodes 2 and 3 form a supernode



Enclose the voltage source in a region called supper node

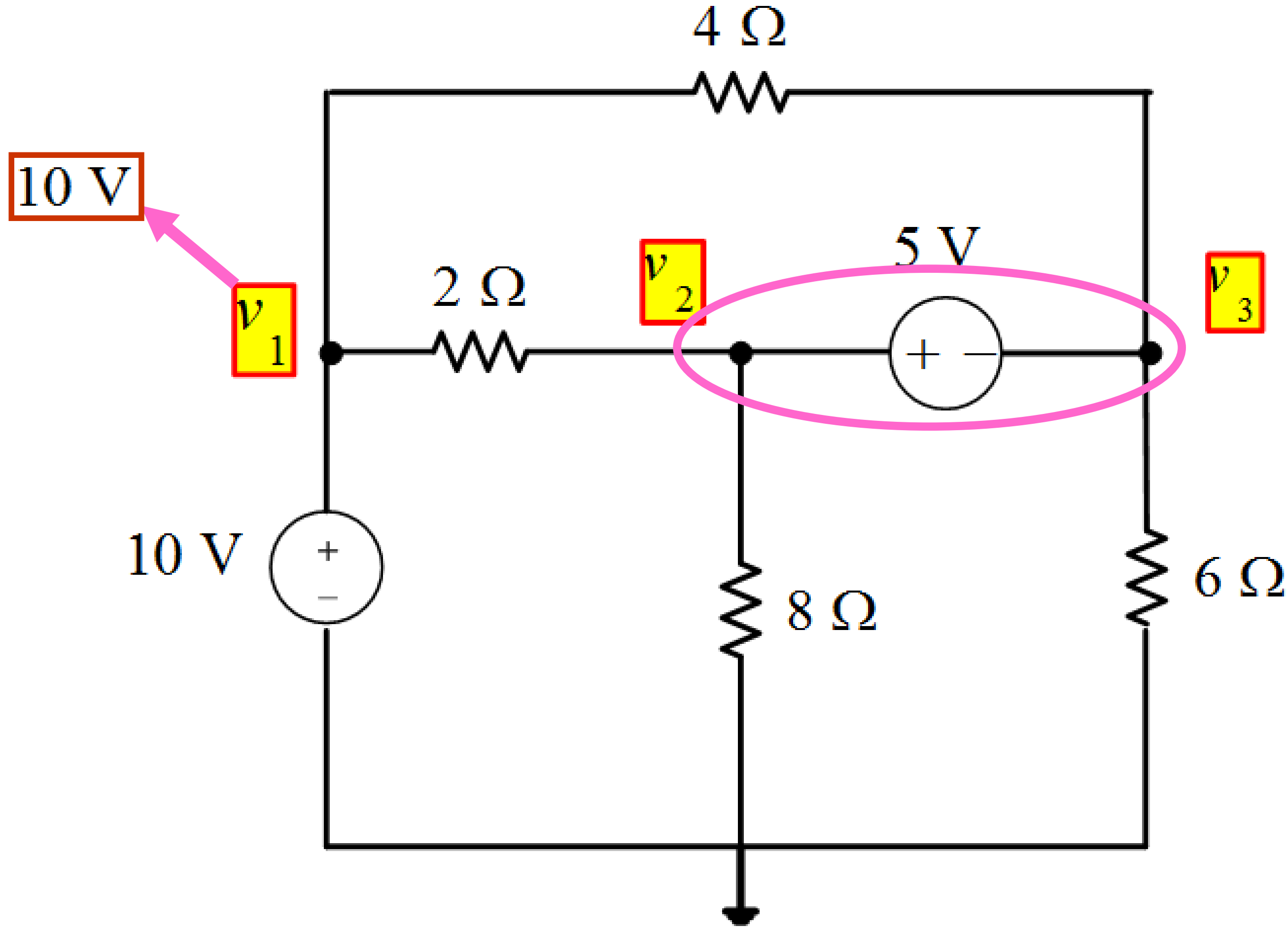
Apply KCL to that region





KCL on the Supper node

$$\frac{v_3}{6} + \frac{v_3-10}{4} + \frac{v_2}{8} + \frac{v_2-10}{2} = 0 \Rightarrow 3v_2 + 3v_2 = 36$$



KCL on the Supper node $$3v_2 + 3v_2 = 36 \quad (1)$$

The second equation come from the constraint on the voltage source

$$v_2 - v_3 = 5 \quad (2)$$

Two equations and two unknowns , we can solve

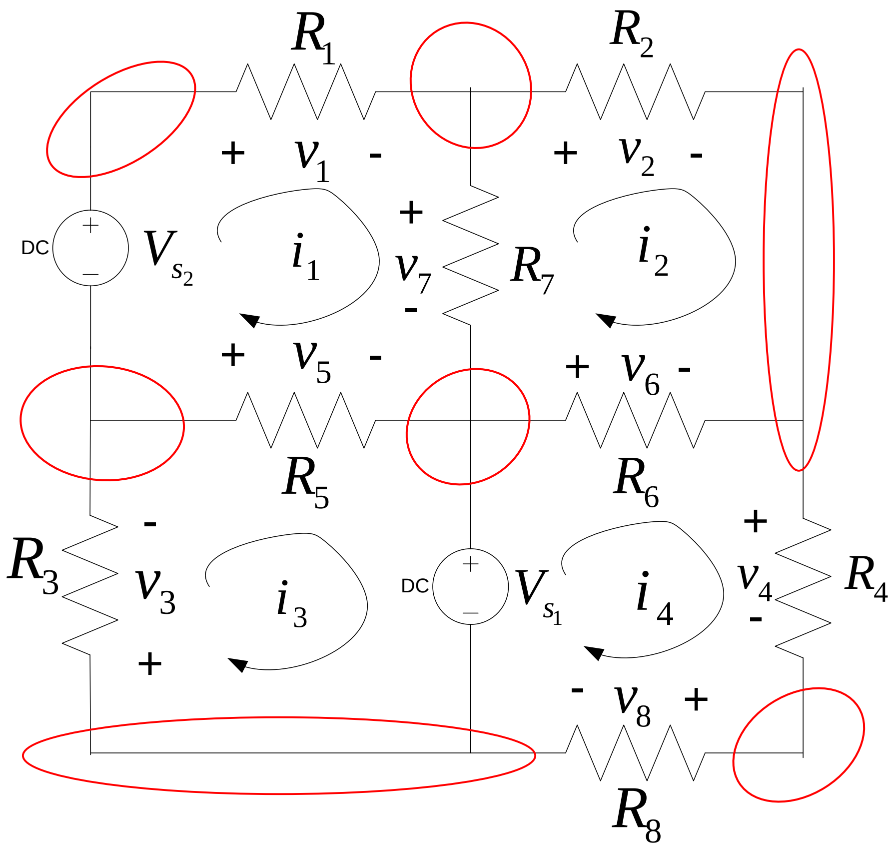
Mesh Analysis

- Mesh analysis applies KVL to find unknown currents.
- It is only applicable to **planar** circuits (a circuit that can be drawn on a plane with no branches crossing each other).
- A mesh is a loop that does not contain any other loops.
- The current through a mesh is known as the mesh current.
- Assume for simplicity that the circuit contains only voltage sources.

Mesh Analysis Steps

1. Assign mesh currents i_1, i_2, i_3, … i_l, to the l meshes,
2. Apply KVL to each of the l meshes and use Ohm's law to express the voltages in terms of the mesh currents,
3. Solve the l resulting simultaneous equations to find the mesh currents.

Example



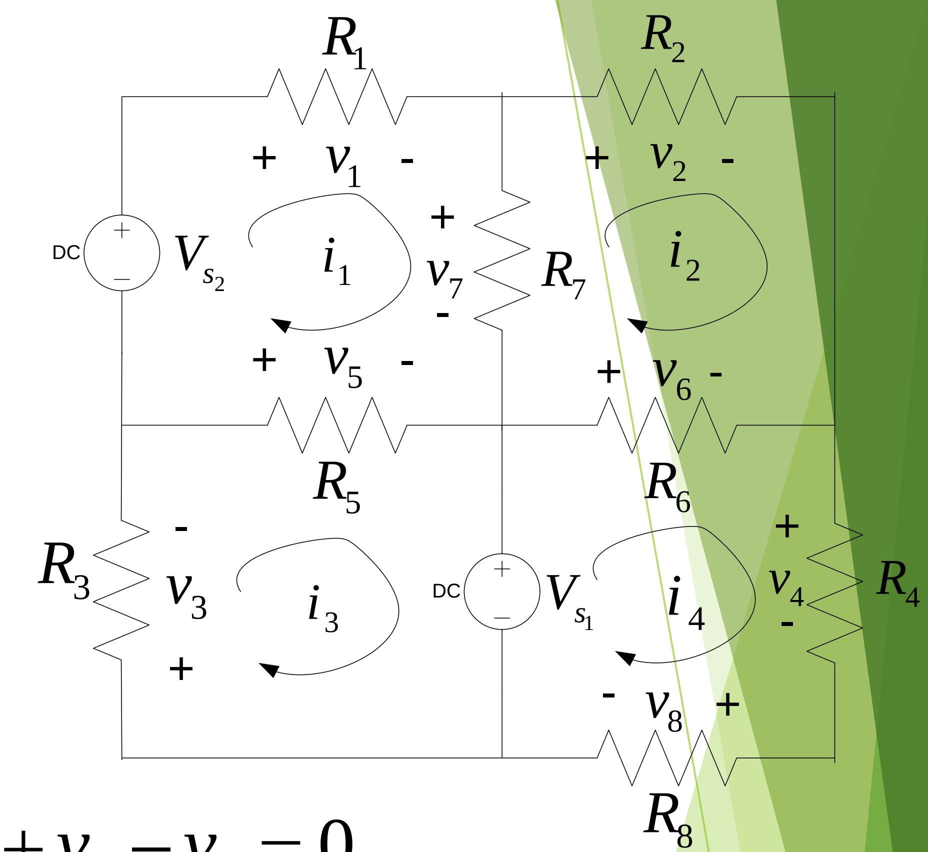
Number of nodes, n = **7**

Number of loops, l = **4**

Number of branches, b = **10**

$$l = b - n + 1$$

Example



Apply KVL to each mesh

Mesh 1: $$-V_{s_2} + v_1 + v_7 - v_5 = 0$$

Mesh 2: $$v_2 - v_6 - v_7 = 0$$

Mesh 3: $$v_5 + v_{s_1} + v_3 = 0$$

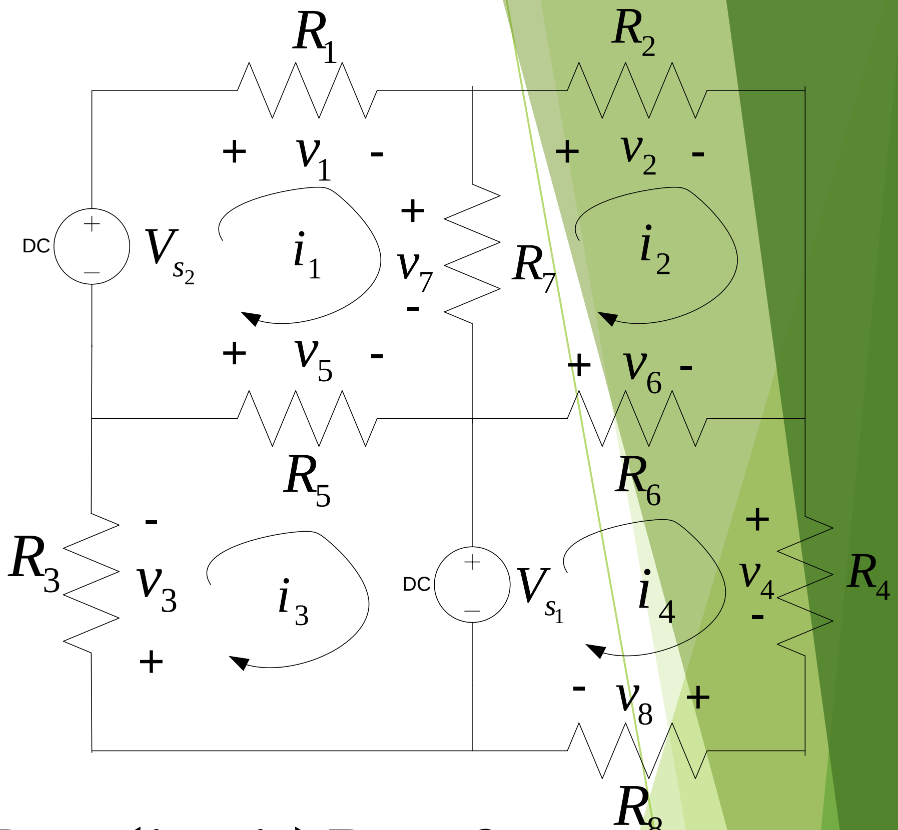
Mesh 4: $$v_4 + v_8 - V_{s_1} + v_6 = 0$$

Mesh 1: $$-V_{s_2} + v_1 + v_7 - v_5 = 0$$

Mesh 2: $$v_2 - v_6 - v_7 = 0$$

Mesh 3: $$v_5 + v_{s_1} + v_3 = 0$$

Mesh 4: $$v_4 + v_8 - V_{s_1} + v_6 = 0$$



Express the voltage in terms of the mesh currents:

Mesh 1: $$-V_{s_2} + i_1R_1 + (i_1 - i_2)R_7 + (i_1 - i_3)R_5 = 0$$

Mesh 2: $$i_2R_2 + (i_2 - i_4)R_6 + (i_2 - i_1)R_7 = 0$$

Mesh 3: $$(i_3 - i_1)R_5 + V_{s_1} + i_3R_3 = 0$$

Mesh 4: $$i_4R_4 + i_4R_8 - V_{s_1} + (i_4 - i_2)R_6 = 0$$

Mesh 1: $$-V_{s_2} + i_1R_1 + (i_1 - i_2)R_7 + (i_1 - i_3)R_5 = 0$$

Mesh 2: $$i_2R_2 + (i_2 - i_4)R_6 + (i_2 - i_1)R_7 = 0$$

Mesh 3: $$(i_3 - i_1)R_5 + V_{s_1} + i_3R_3 = 0$$

Mesh 4: $$i_4R_4 + i_4R_8 - V_{s_1} + (i_4 - i_2)R_6 = 0$$

Mesh 1: $$(R_1 + R_5 + R_7)i_1 - R_7i_2 - R_5i_3 = V_{s_2}$$

Mesh 2: $$-R_7i_1 + (R_2 + R_6 + R_7)i_2 - R_6i_4 = 0$$

Mesh 3: $$-R_5i_1 + (R_3 + R_5)i_3 = -V_{s_1}$$

Mesh 4: $$-R_6i_2 + (R_4 + R_6 + R_8)i_4 = V_{s_1}$$

Mesh 1: $$(R_1+R_5+R_7)i_1-R_7 i_2-R_5 i_3=V_{s_2}$$

Mesh 2: $$-R_7 i_1+(R_2+R_6+R_7)i_2-R_6 i_4=0$$

Mesh 3: $$-R_5 i_1+(R_3+R_5)i_3=-V_{s_1}$$

Mesh 4: $$-R_6 i_2+(R_4+R_6+R_8)i_4=V_{s_1}$$

$$\begin{pmatrix} R_1+R_5+R_7 & -R_7 & -R_5 & 0 \\ -R_7 & R_2+R_6+R_7 & 0 & -R_6 \\ -R_5 & 0 & R_3+R_5 & 0 \\ 0 & -R_6 & 0 & R_4+R_6+R_8 \end{pmatrix}\begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix}=\begin{pmatrix} V_{s_2} \\ 0 \\ -V_{s_1} \\ V_{s_1} \end{pmatrix}$$

$$\begin{pmatrix} R_1+R_5+R_7 & -R_7 & -R_5 & 0 \\ -R_7 & R_2+R_6+R_7 & 0 & -R_6 \\ -R_5 & 0 & R_3+R_5 & 0 \\ 0 & -R_6 & 0 & R_4+R_6+R_8 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{pmatrix} = \begin{pmatrix} V_{s_2} \\ 0 \\ -V_{s_1} \\ V_{s_1} \end{pmatrix}$$

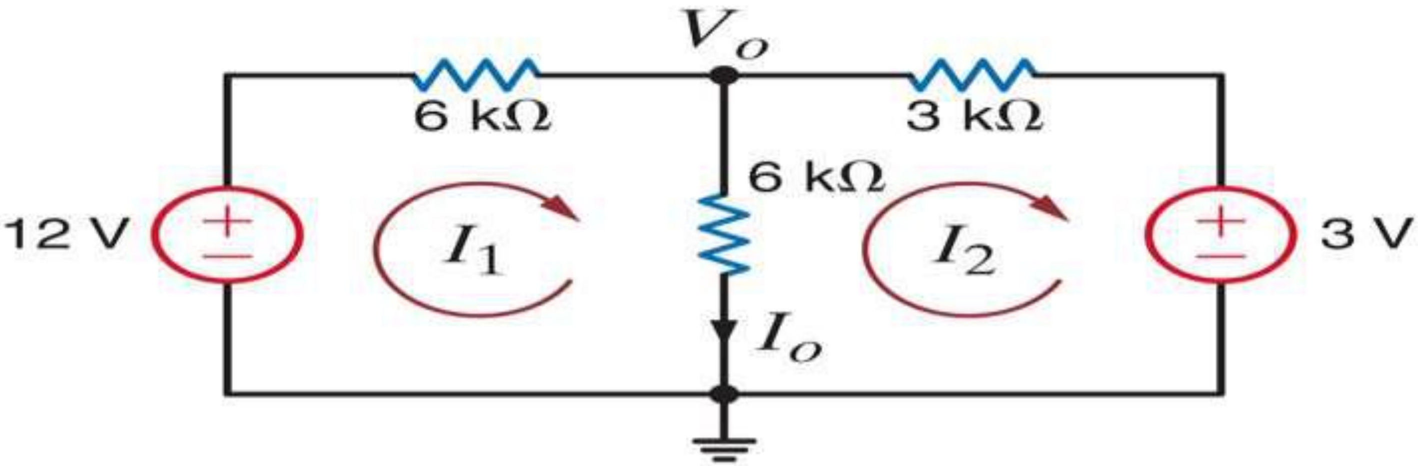
$$\mathbf{Ri} = \mathbf{v}$$

$\mathbf{R}$ is an *l* x *l* *symmetric* resistance matrix

$\mathbf{i}$ is a 1 x *l* vector of mesh currents

$\mathbf{v}$ is a vector of voltages representing "known" voltages

EXAMPLE: FIND Io



<u>SHORTCUT:</u> POLARITIES ARE NOT NEEDED.
APPLY OHM'S LAW TO EACH ELEMENT AS KVL IS BEING WRITTEN

KVL @ I_1 $$-12 + 6kI_1 + 6k(I_1 - I_2) = 0$$

KVL @ I_2 $$6k(I_2 - I_1) + 3kI_2 + 3 = 0$$

REARRANGE

$$12kI_1 - 6kI_2 = 12$$

$$-6kI_1 + 9kI_2 = -3 \quad */2 \text{ and add}$$

$$12kI_2 = 6 \Rightarrow I_2 = 0.5mA$$

$$12kI_1 = 12 + 6kI_2 \Rightarrow I_1 = \frac{5}{4}mA$$

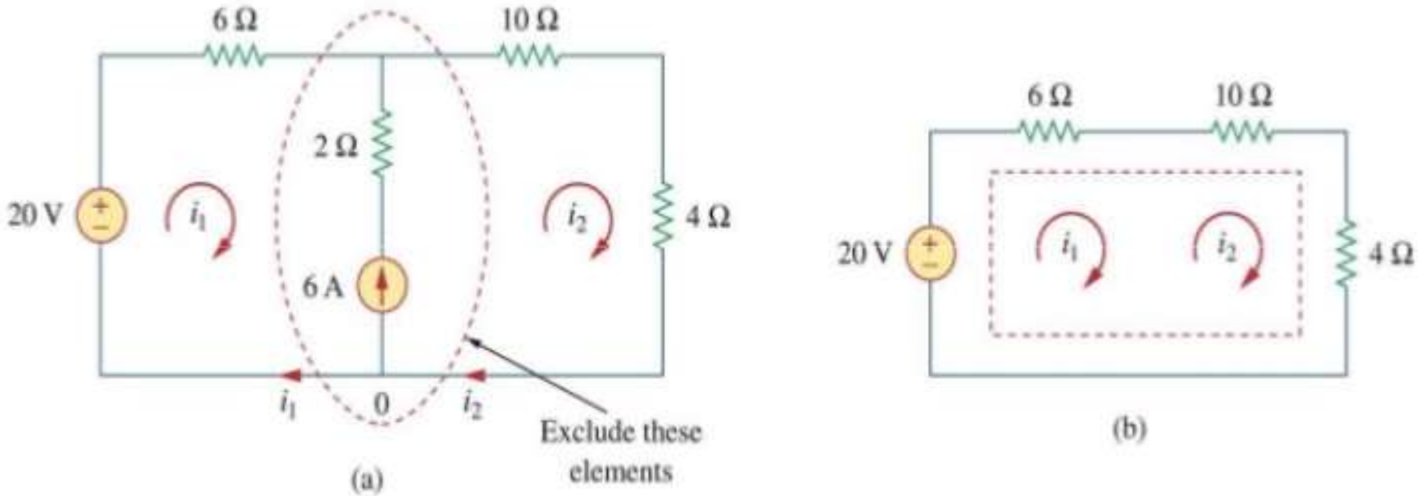
EXPRESS VARIABLE OF INTEREST AS FUNCTION OF LOOP CURRENTS

$$I_O = I_1 - I_2$$

MESH ANALYSIS WITH CURRENT SOURCES

CASE 2

- When a current source exists between two meshes, we create a supermesh by excluding the current source and any elements connected in series with it



- Applying KVL in (b),
- $-20 + 6i_1 + 10i_2 + 4i_2 = 0$ → $6i_1 + 14i_2 = 20$ ……….(i)
- Applying KCL to a node where the two meshes intersect,
- $i_2 = i_1 + 6$ ………… (ii)
- Solve (i) and (ii), → $i_1 = -3.2$ A, $i_2 = 2.8$A

DUALITY

‘The two networks are said to be dual networks, if the mesh equations of one network is equal to the node equation of the other network’.

- The principle of duality means that the solution to the characterizing equations for one circuit can be applied to its dual circuit.
  - The equations that describe the currents and voltages in one circuit are the same set of equations for the its dual, where the variables have been interchanged.

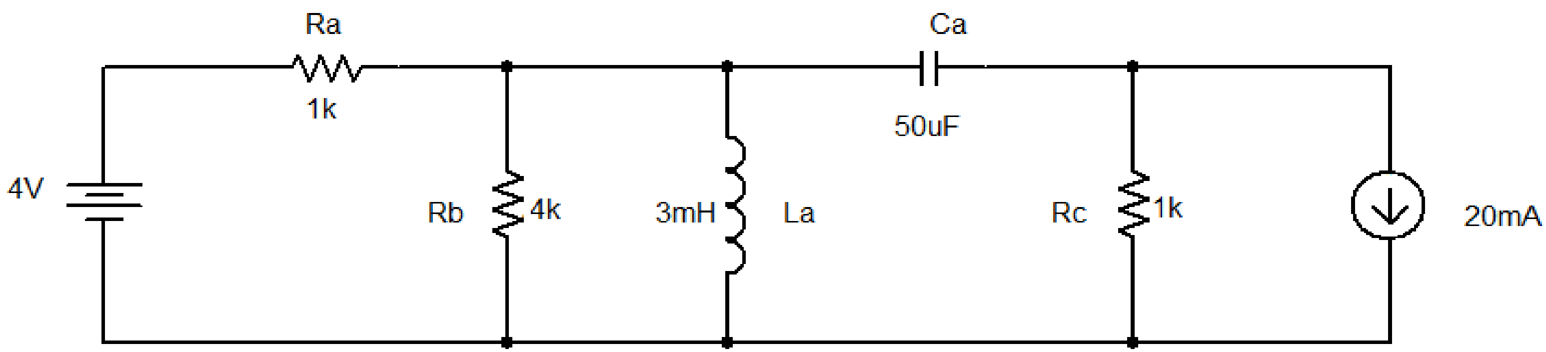
Dual Pairs		
Resistance (R) ------	--	Conductance (G)
Inductance (L) ------	--	Capacitance (C)
Voltage (v) ------	--	Current (i)
Voltage Source -----	---	Current Source
Node -------	-	Mesh/Loop
Series Path	--	Parallel Path

To Construct Dual Circuits

1. Place a node at the center of each mesh of the circuit.
2. Place a reference node (ground) outside of the circuit.
3. Draw lines between nodes such that each line crosses an element.
4. Replace the element by its dual pair.
5. Determine the polarity of the voltage source and direction of the current source.

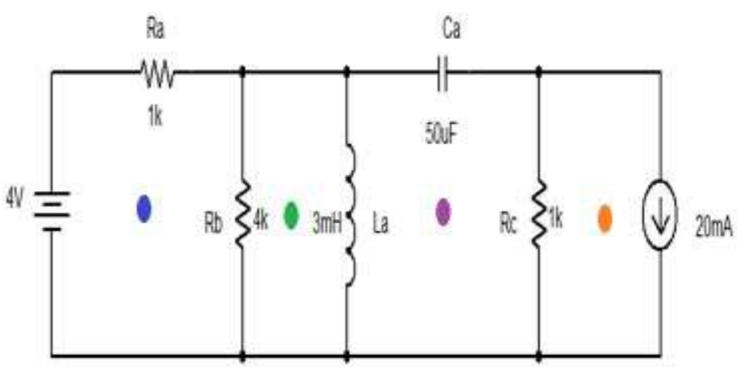
   A voltage source that produces a positive mesh current has as its dual a current source that forces current to flow from the reference ground to the node associated with that mesh.

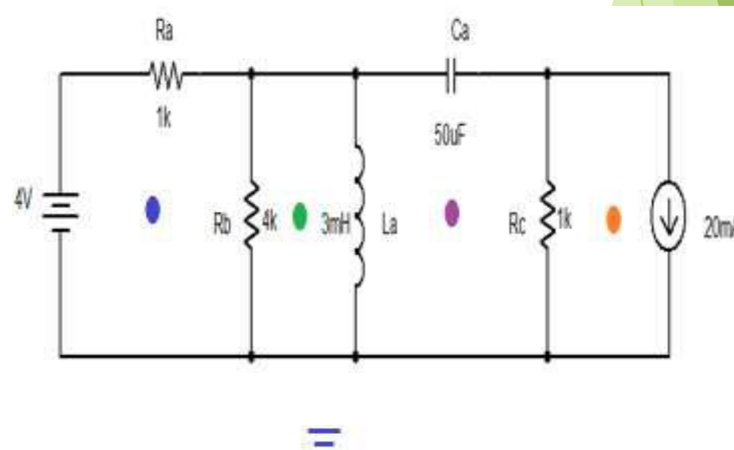
Circuit 1



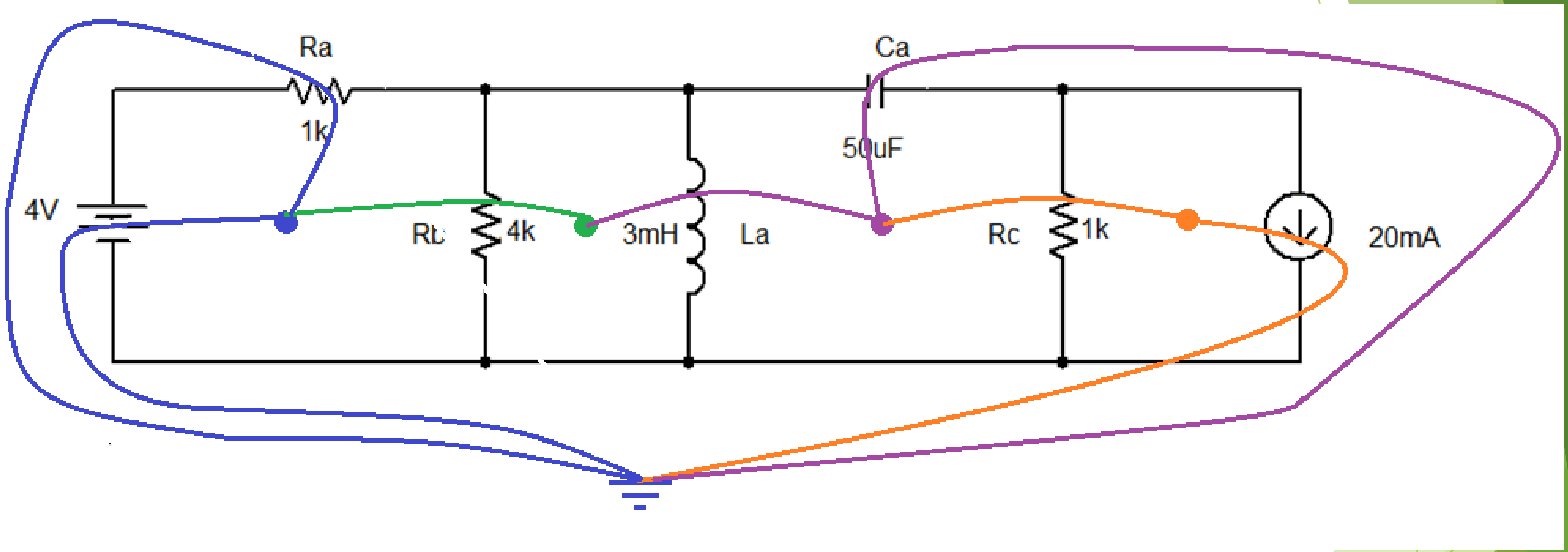
Step 1

Step 2





Step 3



If the line from the new node leaves the circuit after it is drawn through the component, the line must go directly to the reference ground that was drawn in Step 2.

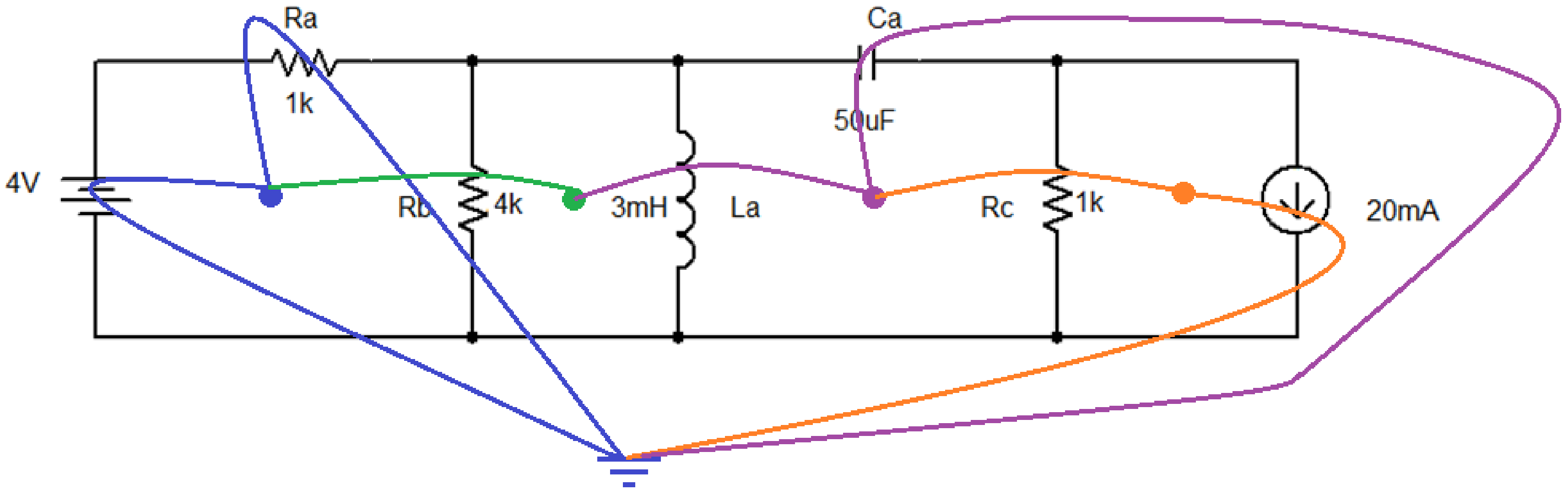
Step 4

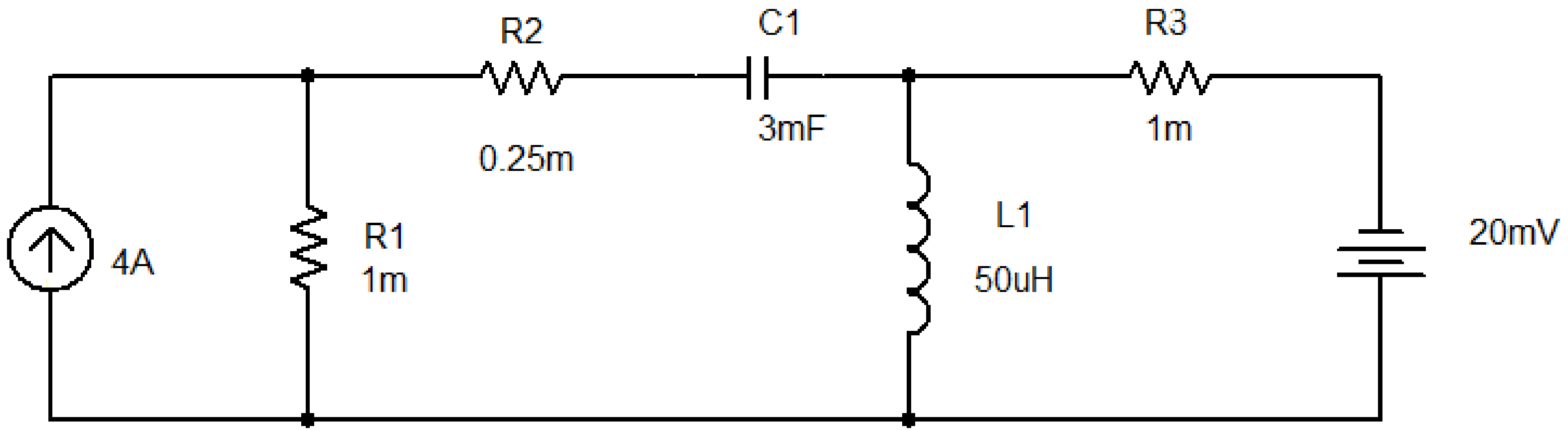
Component in Original circuit	Its Dual
Voltage source (4 V)	Current source (4 A)
Resistor Ra (1 kΩ)	Conductor R1 (1/1kΩ = 1 mΩ)
Resistor Rb (4 kΩ)	Conductor R2 (1/4kΩ = 0.25 mΩ)
Resistor Rc (4 kΩ)	Conductor R3 (1/1kΩ = 1 mΩ)
Inductor La (3 mH)	Capacitor C1 (3 mF)
Capacitor Ca (50 μF)	Inductor L1 (50 μH)
Current Source (20 mA)	Voltage source (20 mV)

Step 5

- In the original circuit:
  - The voltage source forces current to flow towards Ra.
    - Its dual should force current to flow from the reference ground to the node that is shared by the current source and R1, the dual of Ra.
  - The current source causes current to flow from the node where Rc is connected towards the other meshes.
    - Its dual should cause current to flow from the node between it and R3 to distributed node (reference) of the rest of the circuit.

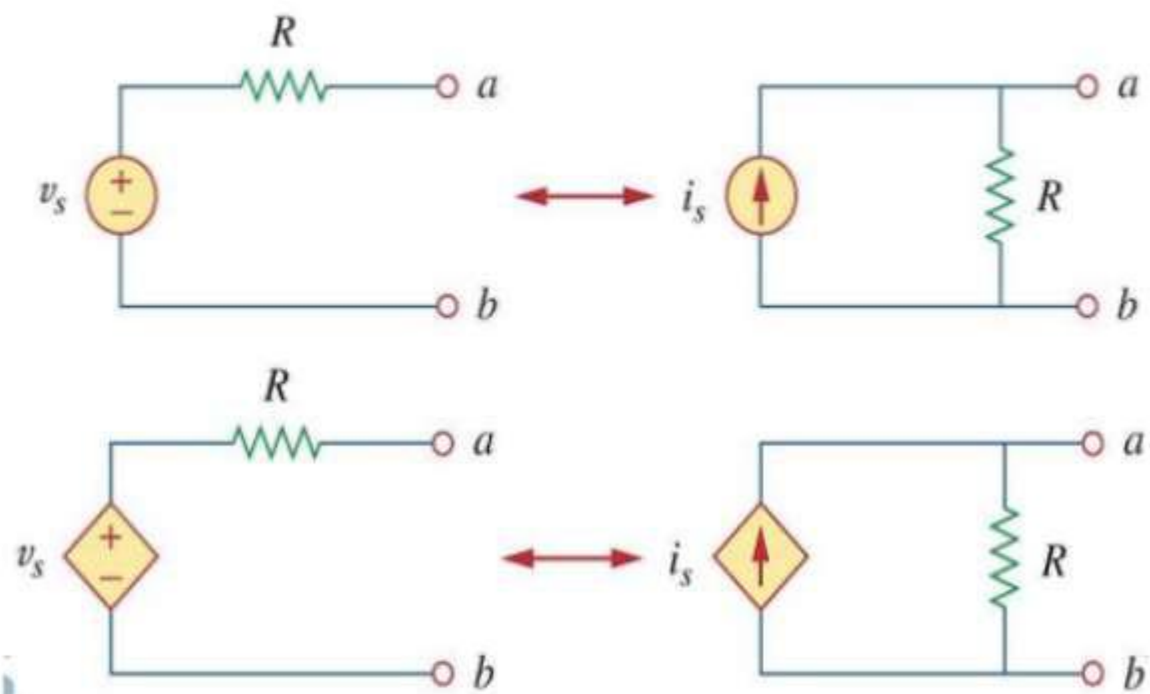
Draw Dual Circuit





SOURCE TRANSFORMATION

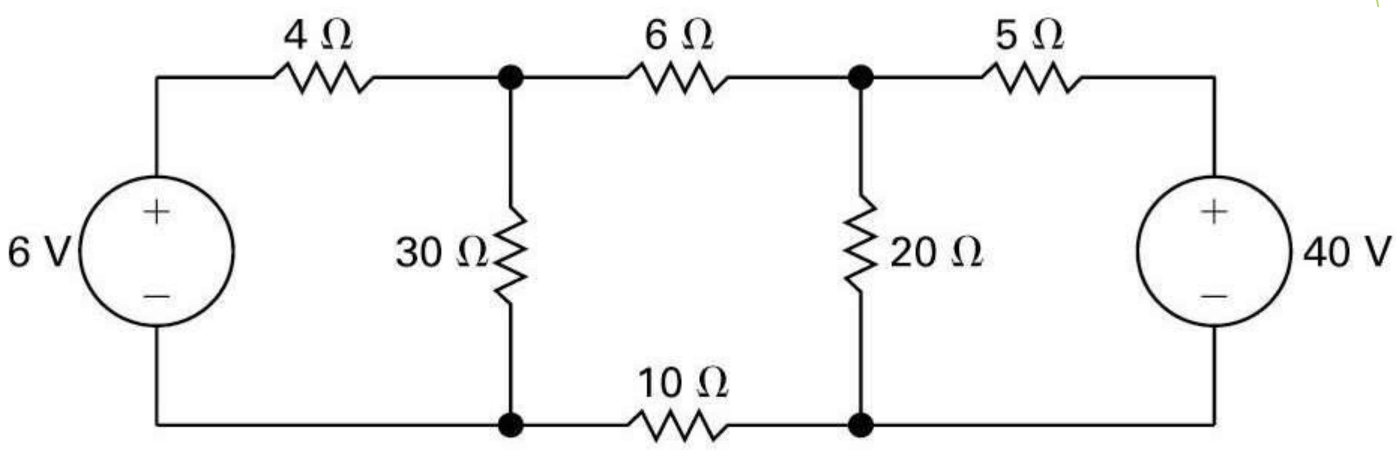
- series parallel combination and wye-delta transformation help simplify circuits
- source transformation is another tool for simplifying circuits
- is the process of replacing a voltage source, V_s in series with a resistor by a current source in parallel with a resistor, or vice versa
- basic to these tools is the concept of equivalence



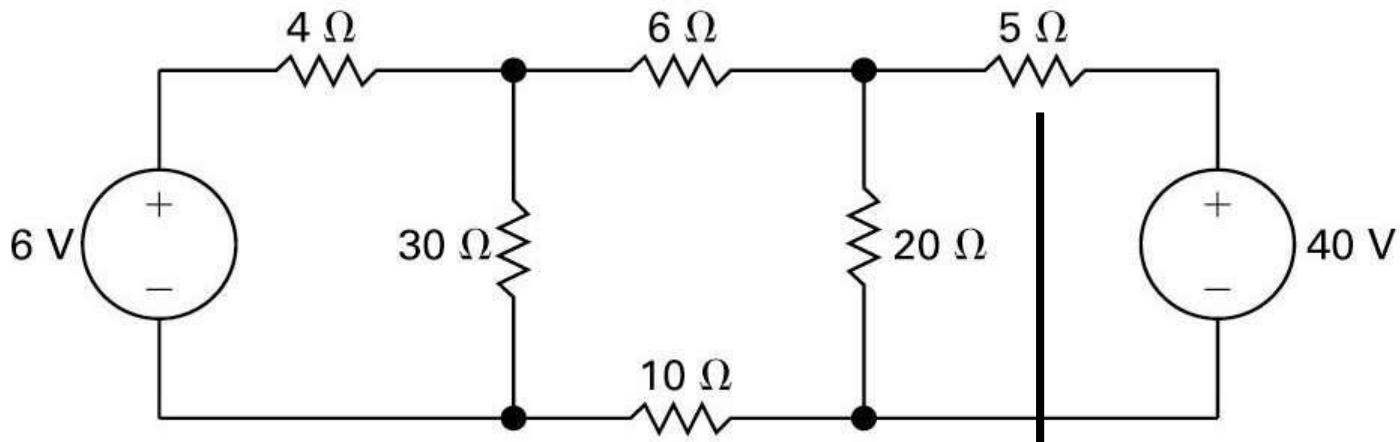
$$v_s = i_s R$$

$$i_s = \frac{v_s}{R}$$

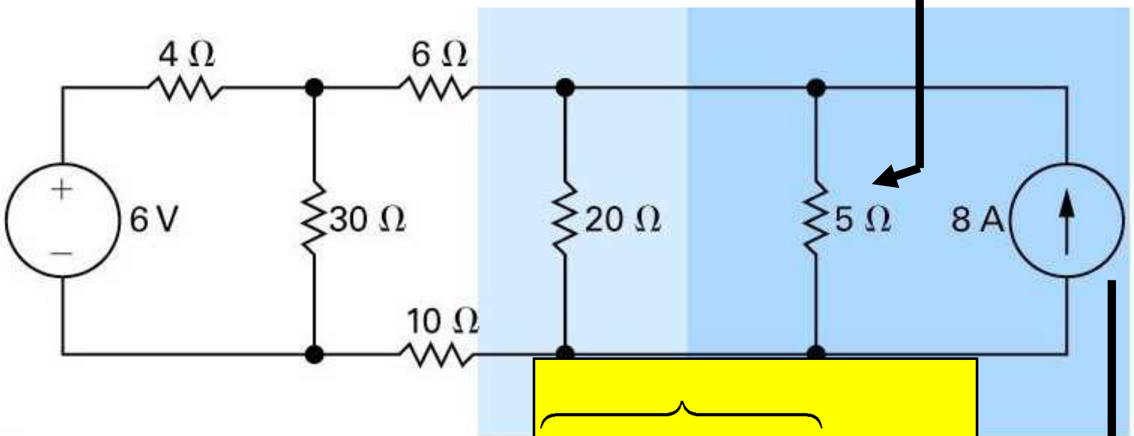
Example : (a) find the power associated with the 6 V source
(b) State whether the 6 V source is absorbing or delivering power



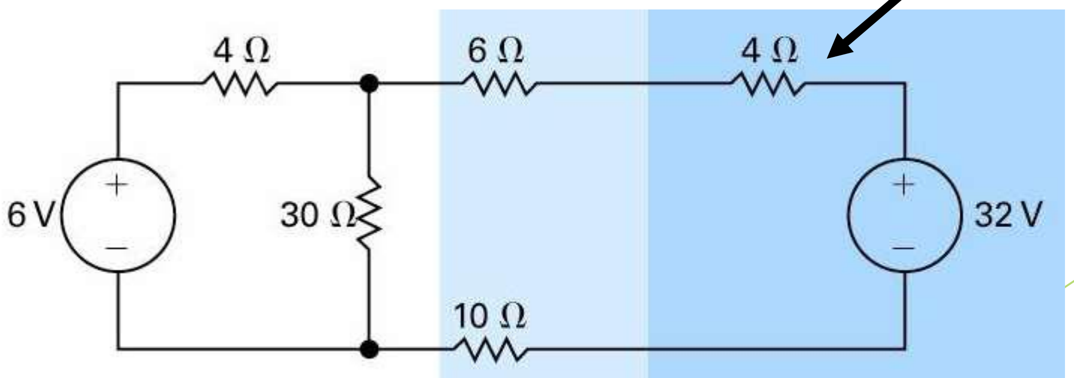
We are going to use source transformation to reduce the circuit, however note that we will not alter or transfer the **6 V** source because it is the objective.



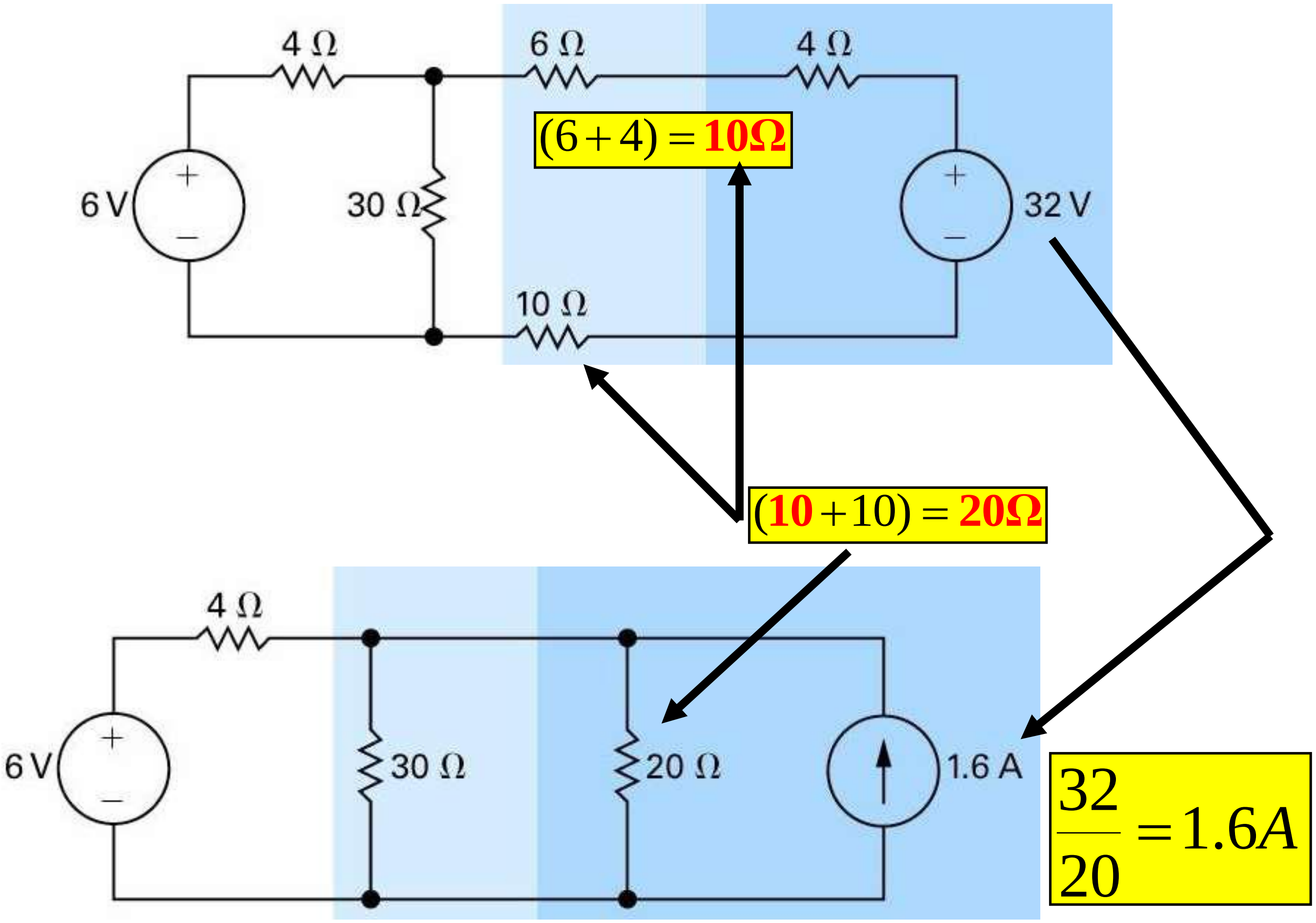
$$\frac{40}{5} = 8A$$

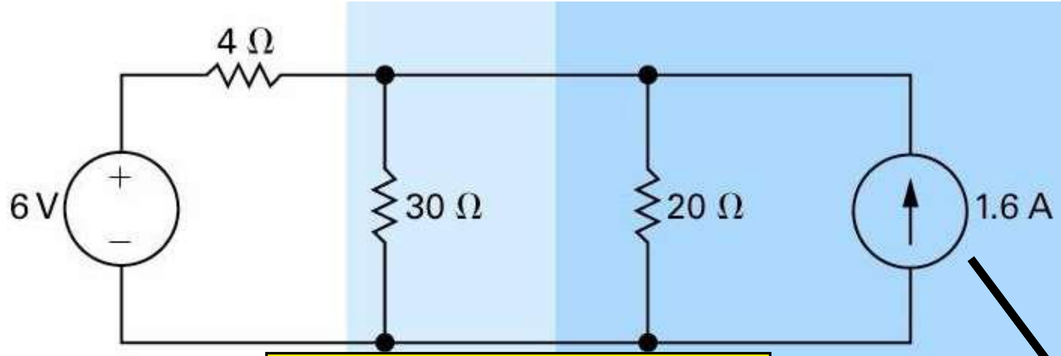


$$\overbrace{(20\Omega \,\|\, 5\Omega)} = \mathbf{4\Omega}$$

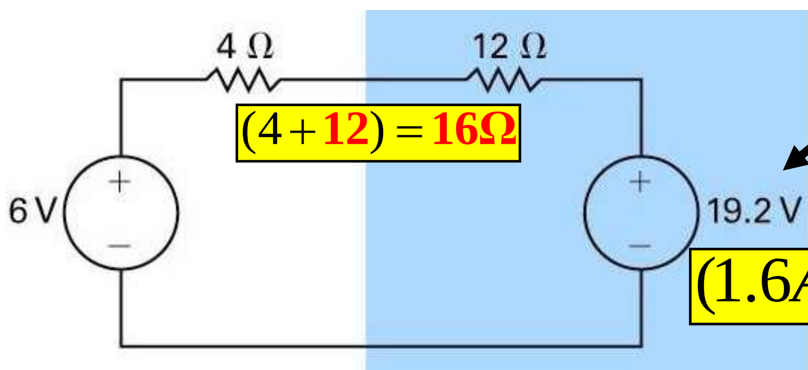


$$(8A)(\mathbf{4\Omega}) = 32\ \text{V}$$


4 Ω
6 Ω
4 Ω
$(6+4)=10\Omega$
6 V
30 Ω
32 V
10 Ω
$(10+10)=20\Omega$
4 Ω
6 V
30 Ω
20 Ω
1.6 A
$\frac{32}{20}=1.6A$

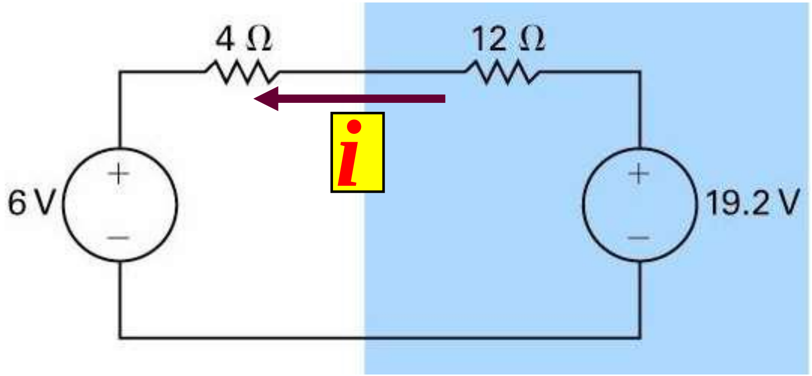


$$\overbrace{(30\Omega \,\|\, 20\Omega)} = \mathbf{12\Omega}$$



$$(4 + \mathbf{12}) = \mathbf{16\Omega}$$

$$(1.6A)(\mathbf{12\Omega}) = 19.2\ \text{V}$$



$$i = \frac{19.2-6}{(4+12)} = 0.825 \text{ A}$$

$$\Rightarrow P_{6V} = (0.825)(6) = 4.95 \text{ W}$$

It should be clear if we transfer the 6V during these steps you will not be able to find the power associated with it

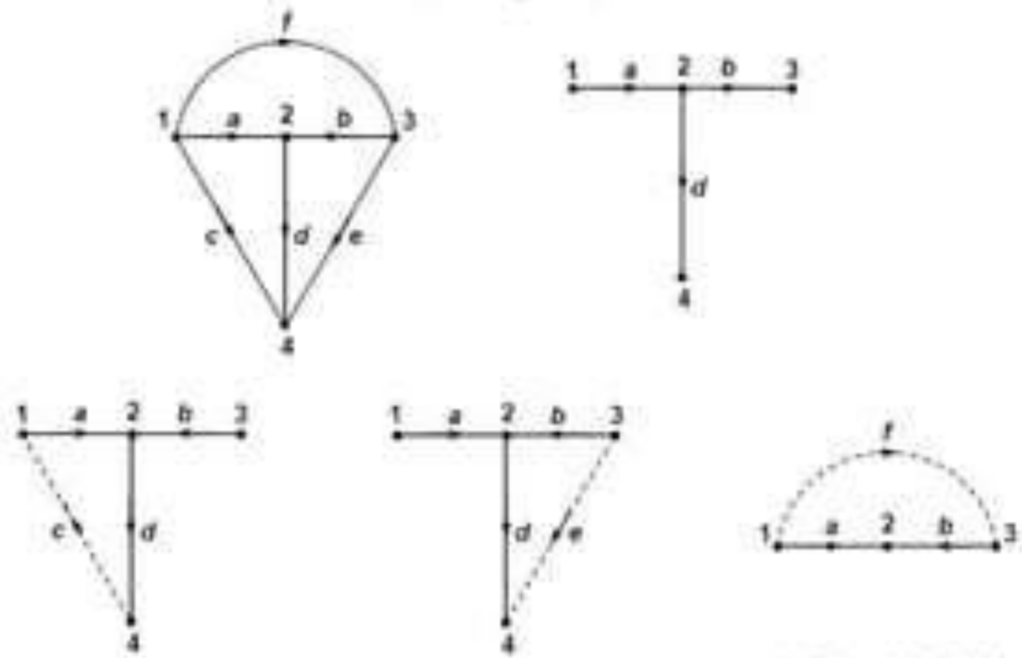
TIE-SET

- A *tie-set* is a set of branches that form a closed path in a graph such that the closed path contains one link and remainders are twigs.
- The closed path is also known as loop.
- A tree of a graph does not have any closed path.
- Upon adding a link to the tree a closed path or loop is created.
- Therefore on adding each link creates one loop and
- So the number of loops in a graph will be equal to number of links.

TIE-SET

- In other words, the number of tie-sets will be equal to number of links.
- For a graph with b branches and n nodes, the possible tie-sets are given by $(b - n + 1)$.
- The tie-set is also called as *fundamental circuit* or *f-circuit*.
- Therefore, a fundamental tie-set is a tie-set formed by one and only one link and a set of twigs.

TIE-SET



(c) Tie-set 1: [a, c, d]

(d) Tie-set 2: [b, d, e]

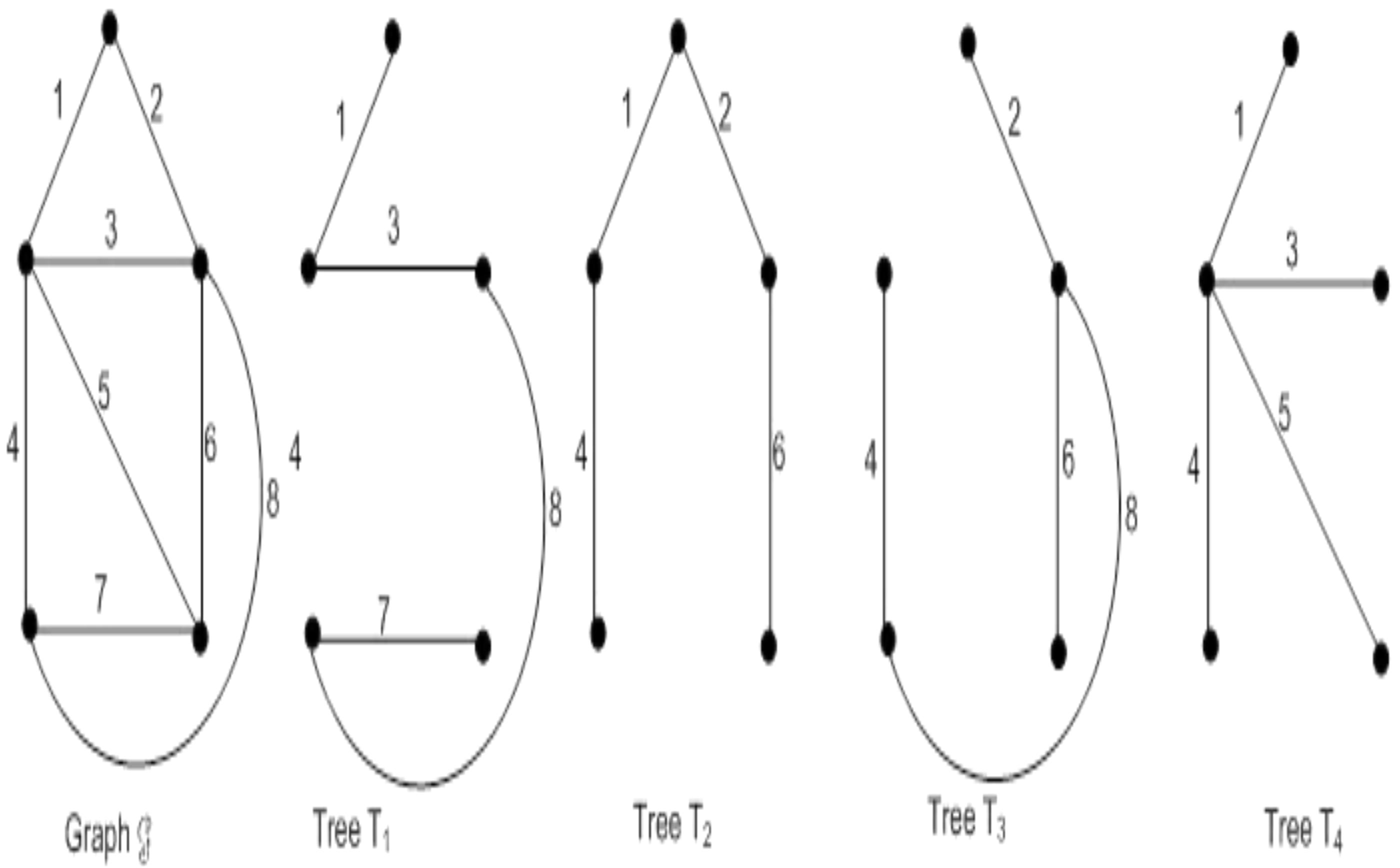
(e) Tie-set 3: [a, b, f]

Loop and cut set Analysis

- Loop and cut set are more flexible than node and mesh analyses and are useful for writing the state equations of the circuit commonly used for circuit analysis with computers.
- The loop matrix **B** and the cutset matrix **Q** will be introduced.

Fundamental Theorem of Graph Theory

A tree of a graph is a connected subgraph that contains all nodes of the graph and it has no loop. Tree is very important for loop and curset analyses. A Tree of a graph is generally not unqiue. Branches that are not in the tree are called links.


1
2
3
4
5
6
7
8
Graph $\mathcal{G}$
Tree T_1
Tree T_2
Tree T_3
Tree T_4

Electrical Circuits-1

UNIT-II

a) Self Inductance

- It called *self inductance* because it relates the voltage induced in a coil by a time varying current in the same coil.
- Consider a single inductor with N number of turns when current, *i* flows through the coil, a magnetic flux, Φ is produces around it.

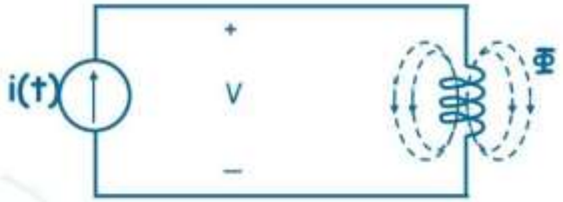


Fig. 1

- According to Faraday's Law, the voltage, *v* induced in the coil is proportional to N number of turns and rate of change of the magnetic flux, Φ;

$$v = N\frac{d\phi}{dt} \text{.......}(1)$$

- But a change in the flux Φ is caused by a change in current, *i*.

Hence; $\frac{d\phi}{dt} = \frac{d\phi}{di}\frac{di}{dt} \text{.......}(2)$

- Thus, (2) into (1) yields; $v = N\frac{d\phi}{di}\frac{di}{dt} \text{.......}(3)$ or $v = L\frac{di}{dt} \text{.......}(4)$

- From equation (3) and (4) the self inductance L is define as;

$L = N\frac{d\phi}{di}$ $[\mathrm{H}] \text{.......}(5)$ The unit is in Henrys (H)

b) Mutual Inductance

- When two inductors or coils are in close proximity to each other, magnetic flux caused by current in one coil links with the other coil, therefore producing the induced voltage.
- Mutual inductance is the ability of one inductor to induce a voltage across a neighboring inductor.

Consider the following two cases:

- Case 1:

  two coil with self - inductance L_1 and L_2 which are in close proximity which each other (Fig. 2). Coil 1 has N_1 turns, while coil 2 has N_2 turns.

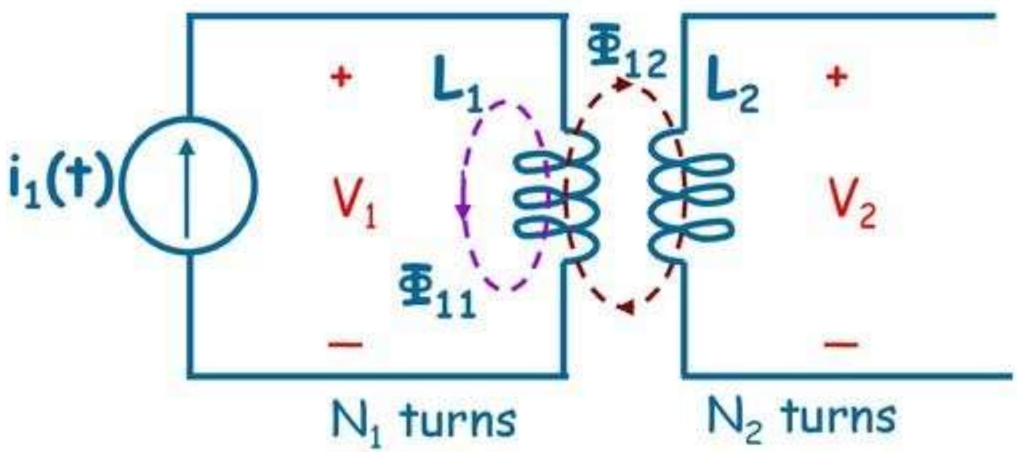


Fig. 2

- Magnetic flux Φ_1 from coil 1 has two components;

  * Φ_{11} links only coil 1.

  * Φ_{12} links both coils.

Hence; $\Phi_1 = \Phi_{11} + \Phi_{12}$ ……. (6)

Thus, voltage induces in coil 1:

$$v_1 = N_1 \frac{d\phi_{11}}{di_1} \frac{di_1}{dt} = L_1 \frac{di_1}{dt} \text{.......(7)}$$

Voltage induces in coil 2

$$v_2 = N_2 \frac{d\phi_{12}}{di_1} \frac{di_1}{dt} = M_{21} \frac{di_1}{dt} \text{.......(8)}$$

Subscript 21 in M_{21} means the mutual inductance on coil 2 due to coil 1

- *Case 2:*

Same circuit but let current i_2 flow in coil 2.(Fig. 3)

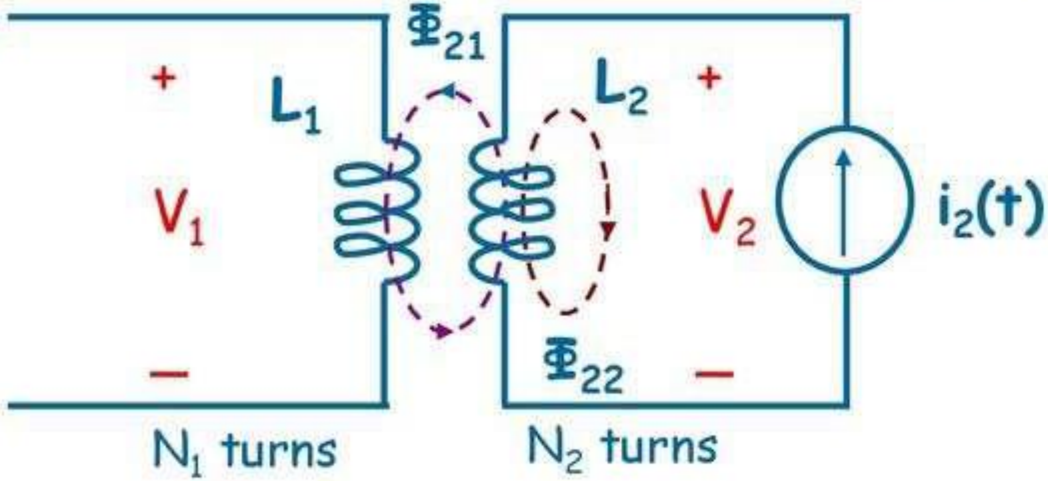


Fig. 3

- The magnetic flux Φ_2 from coil 2 has two components:
  - * Φ_{22} links only coil 2.
  - * Φ_{21} links both coils.

Hence; $\Phi_2 = \Phi_{21} + \Phi_{22}$ .... (9)

Thus; voltage induced in coil 2

$$v_2 = N_2 \frac{d\phi_{22}}{di_2}\frac{di_2}{dt} = L_2 \frac{di_2}{dt} \text{.......(10)}$$

Subscript 12 in M_{12} means the Mutual Inductance on coil 1 due to coil 2

Voltage induced in coil 1

$$v_1 = N_1 \frac{d\phi_{21}}{di_2}\frac{di_2}{dt} = M_{12} \frac{di_2}{dt} \text{.......(11)}$$

Since the two circuits and two current are the same:

$$\boxed{M_{21} = M_{12} = M}$$

Mutual inductance M is measured in Henrys (H)

7-2 COUPLING COEFFICIENT, (k)

- It is measure of the magnetic coupling between two coils.
- Range of k : $0 \leq k \leq 1$
  - $k = 0$ means the two coils are NOT COUPLED.
  - $k = 1$ means the two coils are PERFECTLY COUPLED.
  - $k < 0.5$ means the two coils are LOOSELY COUPLED.
  - $k > 0.5$ means the two coils are TIGHTLY COUPLED.
- k depends on the closeness of two coils, their core, their orientation and their winding.
- The coefficient of coupling, k is given by;

$$\boxed{k = \frac{M}{\sqrt{L_1 L_2}}}$$

or

$$\boxed{M = k\sqrt{L_1 L_2}}$$

7-3 DOT DETERMINATION

- Required to determine polarity of "mutual" induced voltage.
- A dot is placed in the circuit at one end of each of the two magnetically coupled coils to indicate the direction of the magnetic flux if current enters that dotted terminal of the coil.

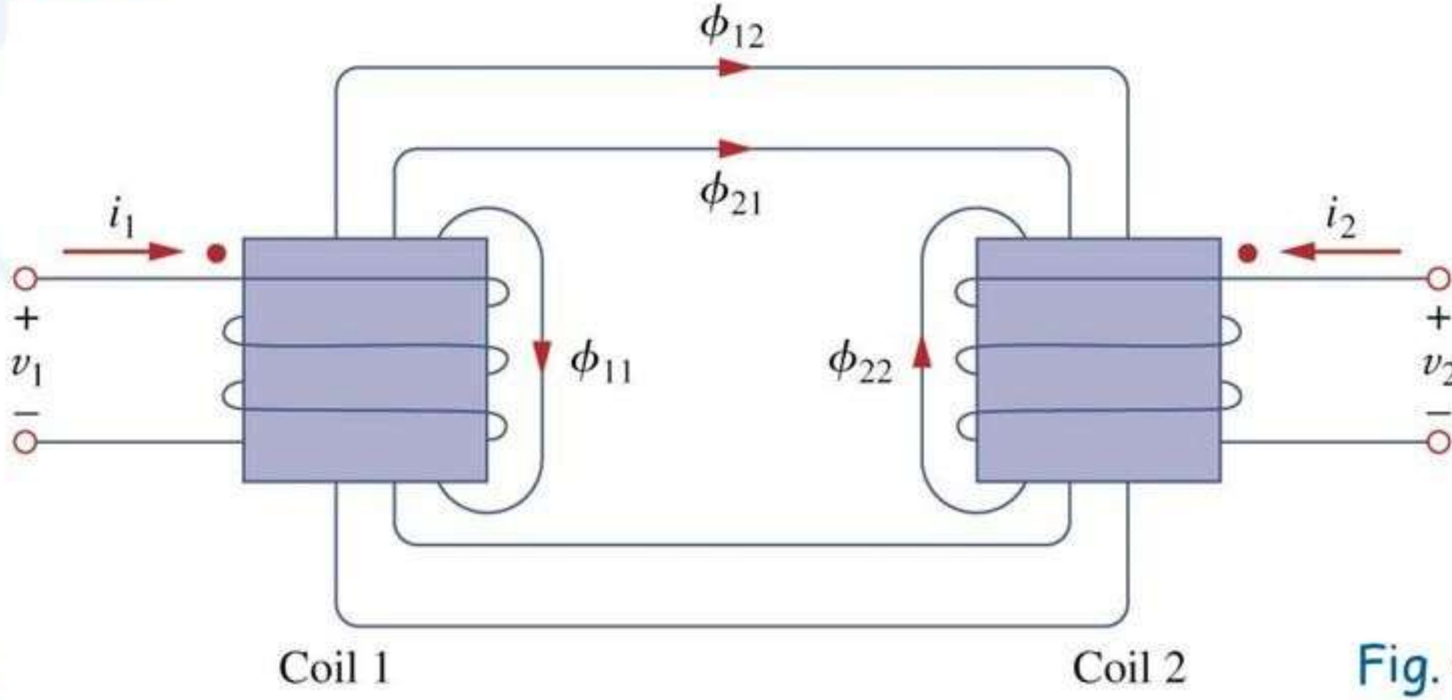


Fig. 4

- **Dot convention is stated as follows:**

  if a current ENTERS the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is POSITIVE at the dotted terminal of the second coil.

- **Alternatively;**

  if a current LEAVES the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is NEGATIVE at the dotted terminal of the second coil.

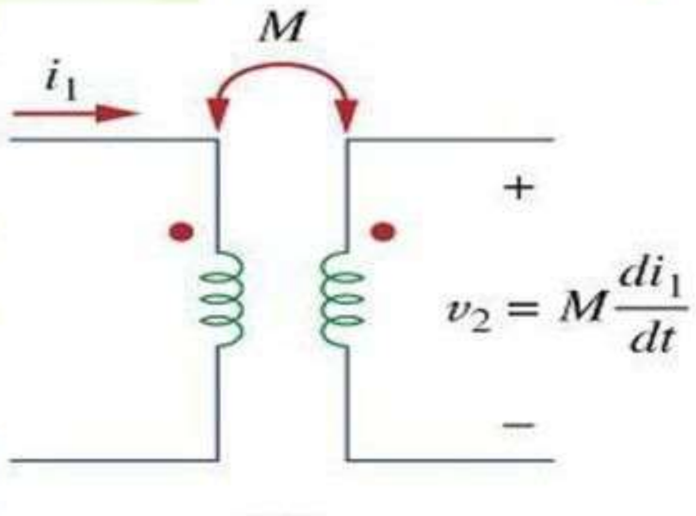
- The following dot rule may be used:

  i. when the assumed currents both entered or both leaves a pair of couple coils by the dotted terminals, the signs on the L - terms.

  ii. if one current enters by a dotted terminals while the other leaves by a dotted terminal, the sign on the M - terms will be opposite to the signs on the L - terms.

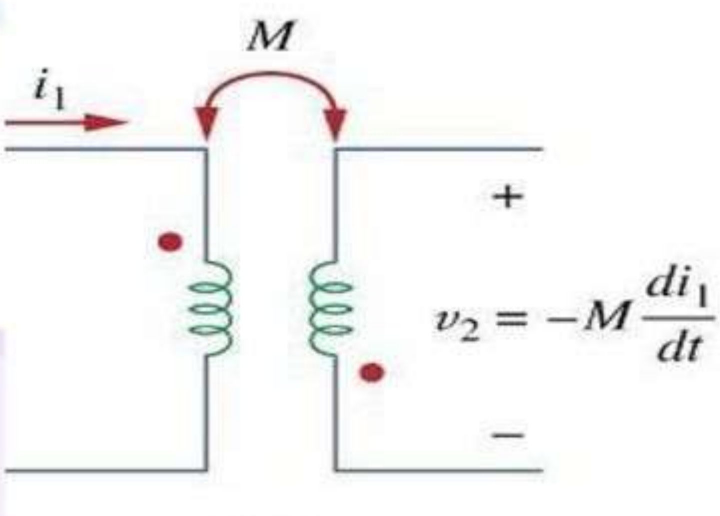
- Once the polarity of the mutual voltage is already known, the circuit can be analyzed using *mesh method*.

Application of the dot convention is illustrated in the four pairs of mutual coupled coils. (Fig. a,b,c,d)



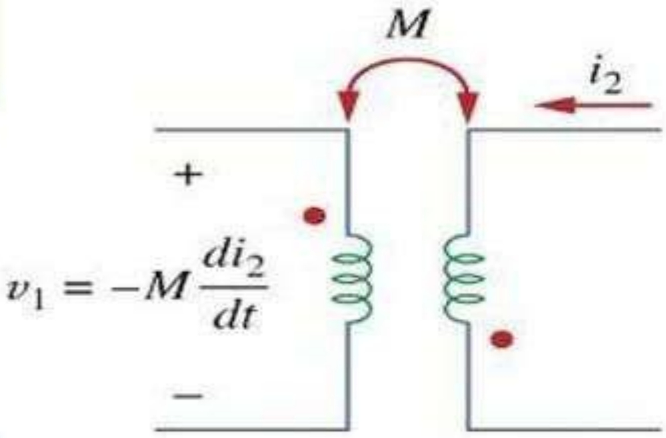
(a)

The sign of the mutual voltage v_2 is determined by the reference polarity for v_2 and the direction of i_1. Since i_1 enters the dotted terminal of coil 1 and v_2 is positive at the dotted terminal of coil 2, the mutual voltage is $+M\, di_1/dt$. (Fig. a)

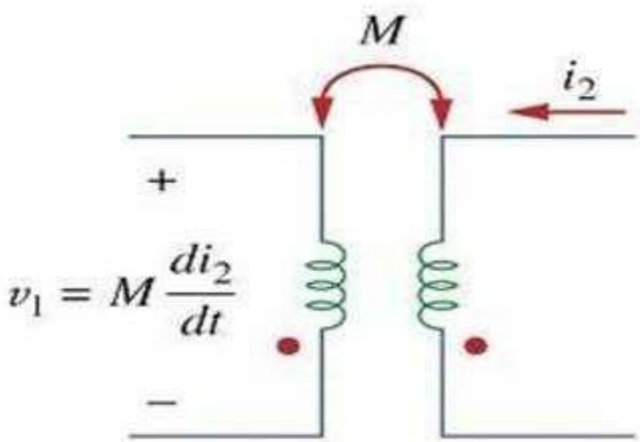


(b)

Current i_1 enters the dotted terminal of coil 1 and v_2 is negative at the dotted terminal of coil 2. The mutual voltage is $-M\, di_1/dt$. (Fig. b)



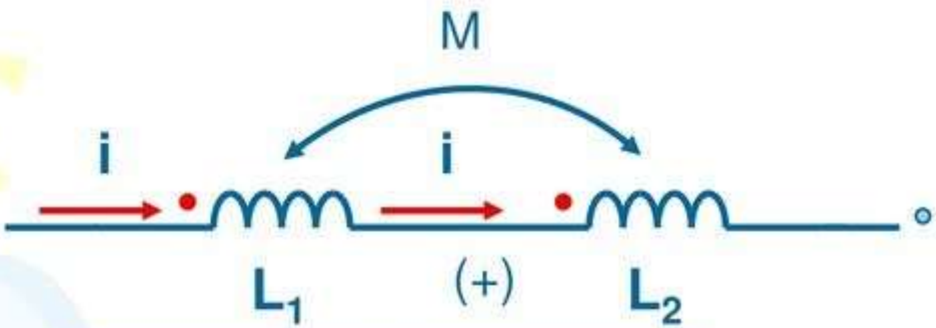
(c)



(d)

Same reasoning with Fig. a and fig. b are applies to the coil in Fig. c and Fig. d.

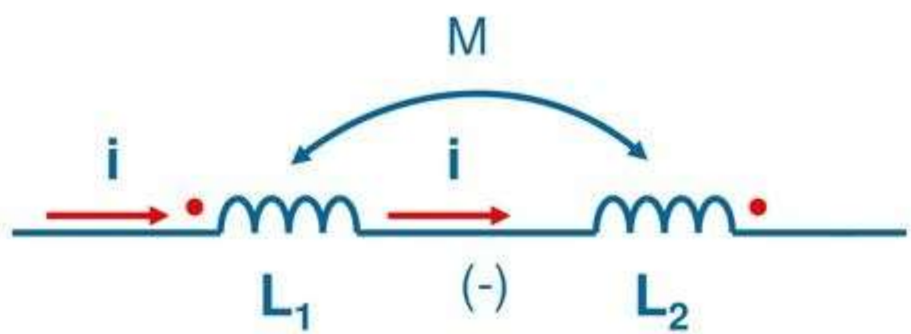
Dot convention for coils in series



Series - aiding connection

$$L = L_1 + L_2 + 2M$$

Series - opposing connection



$$L = L_1 + L_2 - 2M$$

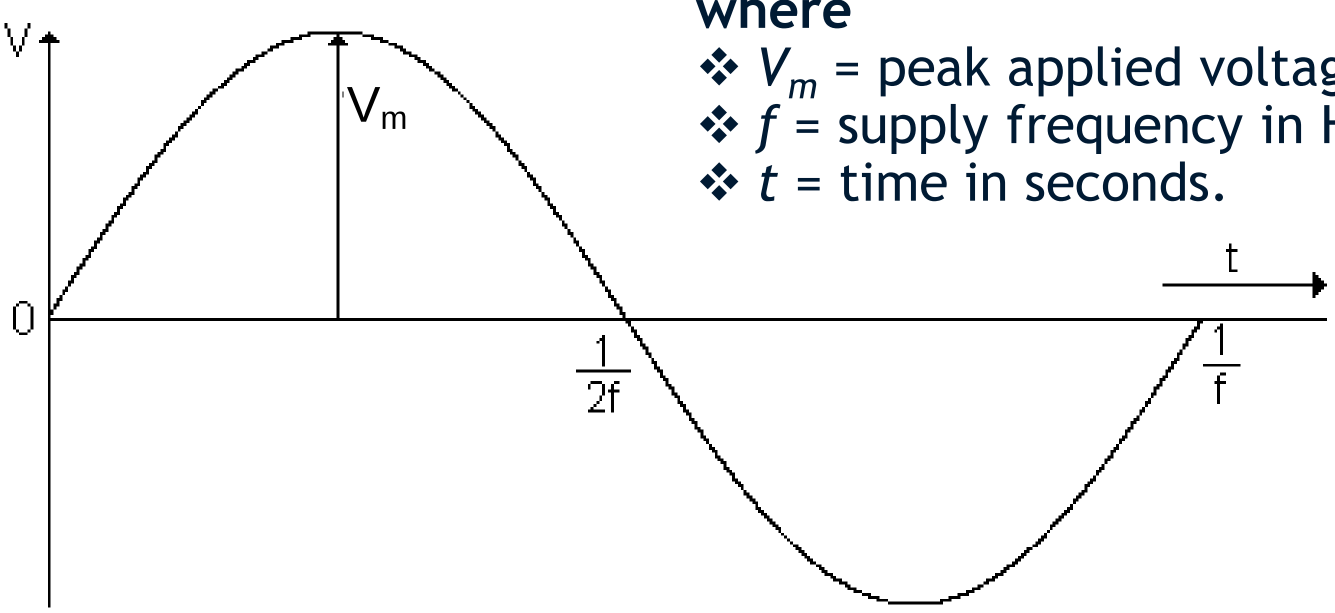
Unit 3
Single phase AC Circuits

Introduction

- Electricity supply systems are normally **ac** (alternating current).
- The supply voltage varies sinusoidal $v = V_m \sin(2\pi f t)$
- instantaneous applied voltage, **OR** $v = V_m \sin(\omega t)$

where

- V_m = peak applied voltage in volts
- f = supply frequency in Hz
- t = time in seconds.



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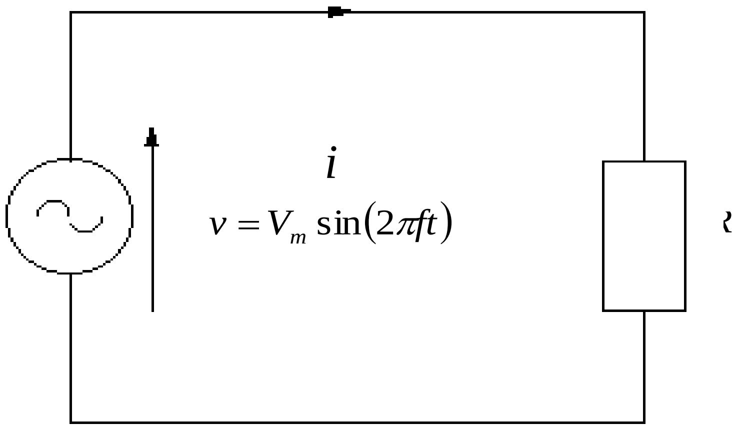
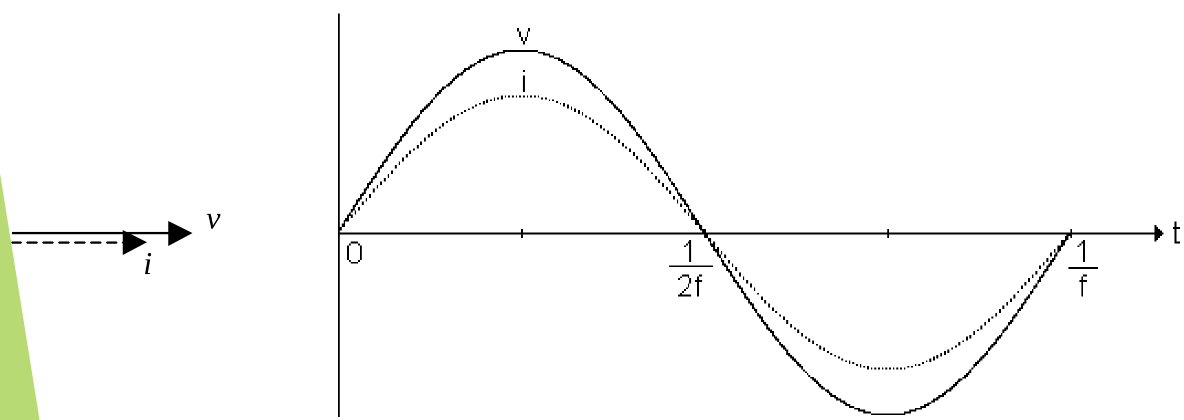
Resistance connected to an AC supply



Instantaneous current, $i = \dfrac{v}{R}$

$$i = \frac{V_m}{R}\sin(2\pi ft)$$

$$i = I_m \sin(2\pi ft)$$



Current and Voltage are in phase

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Root Mean Square (rms) Voltage and Current

- The “effective” values of voltage and current over the whole cycle

- rms voltage is $$V = \frac{V_m}{\sqrt{2}}$$

- rms current is $$I = \frac{I_m}{\sqrt{2}}$$

“RMS value of an alternating current is that steady state current (dc) which when flowing through the given resistor for a given amount of time produces the same amount of heat as produced by the alternative current when flowing through the same resistance for the same time”

Meters normally indicate rms quantities and this value is equal to the DC value

Other representations of Voltage or Current are

- maximum or peak value
- average value

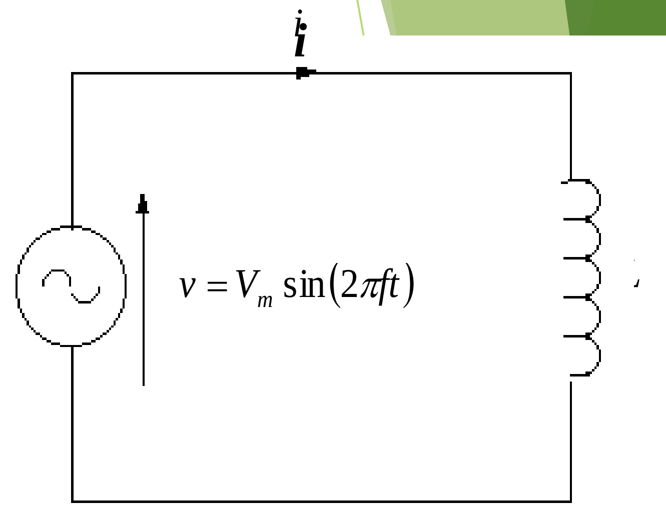
Inductance connected to an AC supply

$$v = L\frac{di}{dt}$$ *i – instantaneous current*

$$v = V_m \sin(2\pi f t) \Rightarrow i = \frac{-V_m}{2\pi f L}\cos(2\pi f t) \qquad \omega = 2\pi f$$

$$i = \frac{-V_m}{\omega L}\cos(\omega t) \Rightarrow I_m = \frac{-V_m}{\omega L}\Big|_{t=0}$$

$$i = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) \Rightarrow$$ **Current lags Voltage by 90 degree**

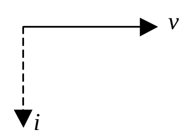


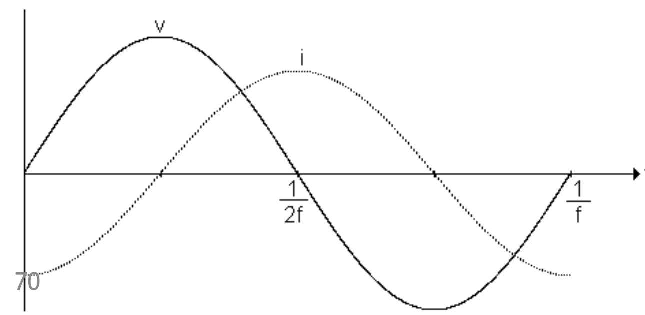
rms current $$I = \frac{V}{\omega L} = \frac{V}{2\pi f L}$$

Using complex numbers and the *j* operator $$I = \frac{-j}{\omega L}V$$

Inductive Reactance $$X_L = 2\pi f L = \omega L$$

$$I = -j\frac{V}{X_L} = \frac{V}{jX_L}$$



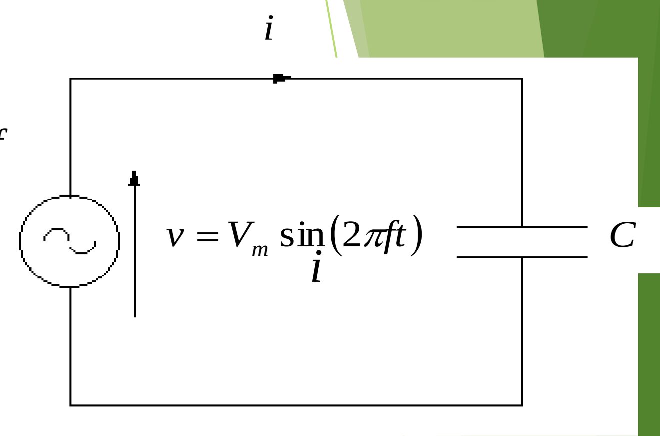


Phasor diagram and wave form

Capacitance connected to an AC supply

$$i = C\frac{dv}{dt}$$

$$v = V_m \sin(2\pi f t) \Rightarrow i = 2\pi f C V_m \cos(2\pi f t) \qquad \omega = 2\pi f$$



$$i = \omega C V_m \cos(\omega t) \Rightarrow I_m = \omega C V_m$$

$$i = \omega C V_m \sin\left(\omega t + \frac{\pi}{2}\right) \Rightarrow$$

Current leads Voltage by 90 degrees

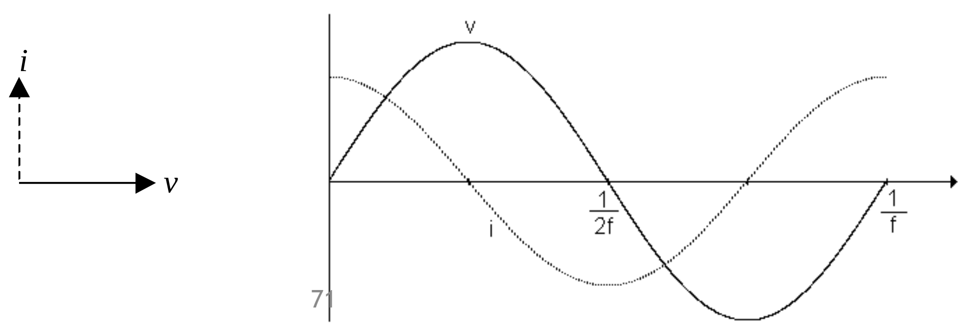
rms current $\quad \mathrm{I} = \omega \mathrm{C} \mathrm{V} = 2\pi \mathrm{f} \mathrm{C} \mathrm{V}$

Using complex numbers and the *j* operator $\quad \mathrm{I} = +\mathrm{j}\omega \mathrm{C} \mathrm{V}$

Capacitance Reactance

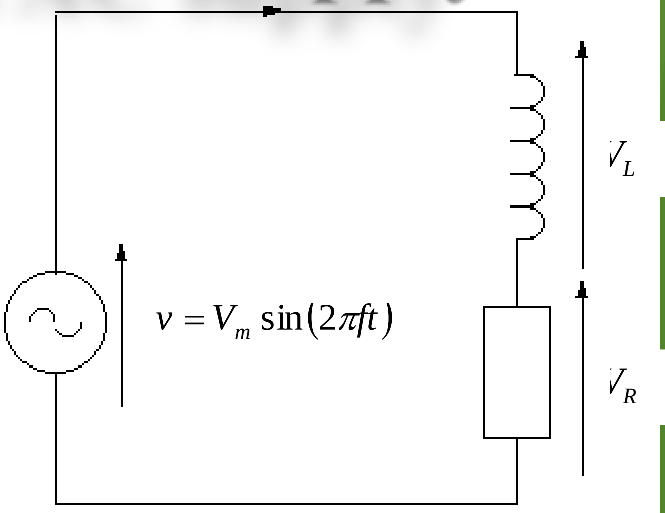
$$X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C}$$

$$I = +j\frac{V}{X_C} = -\frac{V}{jX_C} = \frac{V}{(-jX_C)}$$



Phasor diagram and wave form

R and L in series with an AC supply



$$V = V_R + V_L$$

But $V_R = IR$ **and** $V_L = I \cdot jX_L$

$$\therefore \quad V = I(R + jX_L)$$

And $I = \dfrac{V}{R + jX_L}$ Where, $X_L = \omega L = 2\pi f L$ $\quad \therefore \quad I = \dfrac{V}{R + j\omega L}$

Complex Impedance $Z = R + j\omega L$

Cartesian Form $I = \dfrac{V}{R + j\omega L} \cdot \dfrac{R - j\omega L}{R - j\omega L}$ ➡ $I = \left[\dfrac{VR}{R^2 + \omega^2 L^2}\right] - j\left[\dfrac{V\omega L}{R^2 + \omega^2 L^2}\right]$

-j **indicates that the current <u>lags</u> the voltage**

Complex Impedance: $Z = R + j\omega L$

Cartesian Form: $I = \left[\dfrac{VR}{R^2 + \omega^2 L^2}\right] - j\left[\dfrac{V\omega L}{R^2 + \omega^2 L^2}\right]$

In Polar Form

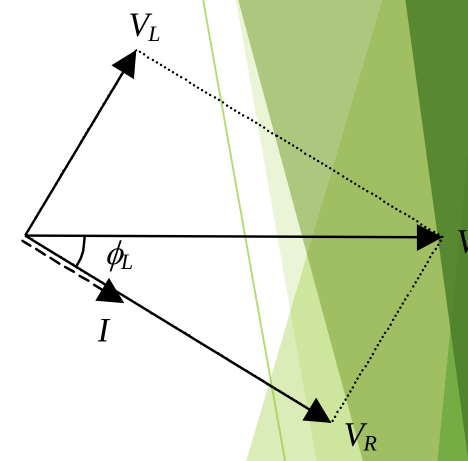
$$I = \frac{V}{\sqrt{R^2 + \omega^2 L^2}} \angle(-\phi_L)$$

$$\phi_L = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

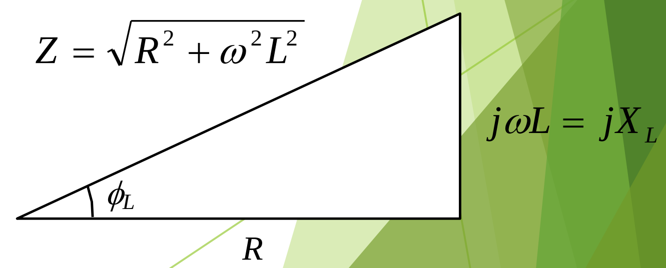
$-\phi_L$ indicates lagging current.

$$|I| = \frac{V}{\sqrt{R^2 + \omega^2 L^2}}$$

phasor diagram constructed with RMS quantities



Power factor, p.f. $= \cos(\phi_L) = \cos\left(\tan^{-1}\dfrac{\omega L}{R}\right)$

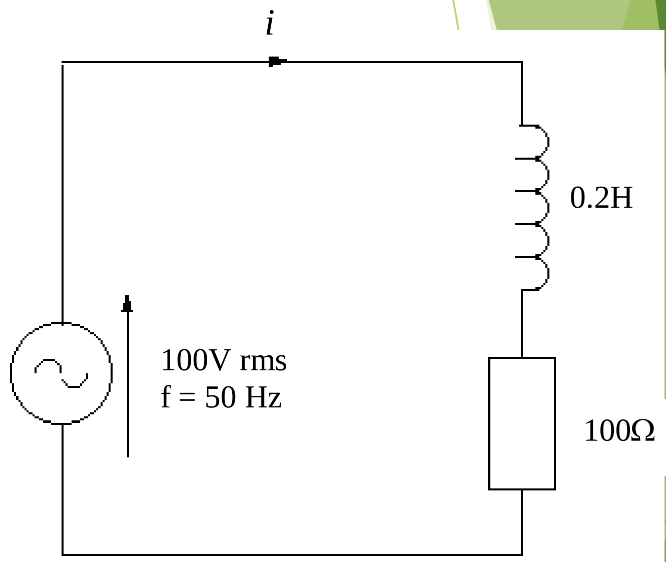


Complex impedance: $Z = R + j\omega L$

$$|Z| = \sqrt{R^2 + \omega^2 L^2}$$

Exercise:

For the circuit shown below, calculate the rms current I & phase angle ϕ_L



Use:

$$I = \frac{V}{\sqrt{R^2 + \omega^2 L^2}} \angle(-\phi_L)$$

$$I = \left[\frac{VR}{R^2 + \omega^2 L^2}\right] - j\left[\frac{V\omega L}{R^2 + \omega^2 L^2}\right]$$

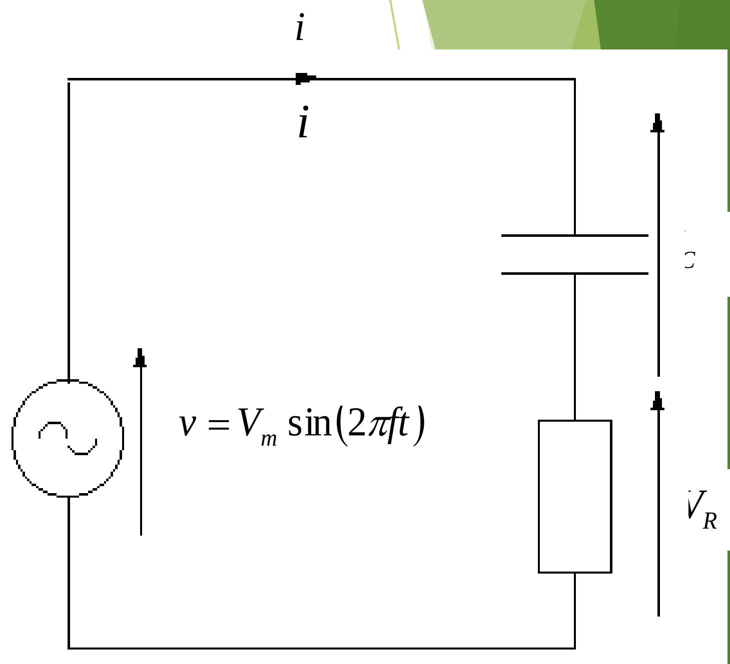
$$\phi_L = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Answer: I = 0.85A ∠-32.10

R and C in series with an AC supply

$V = V_C + V_R$ **But** $V_R = IR$ **and** $V_C = I(-jX_C)$

$\therefore \quad \mathrm{V} = \mathrm{I}(\mathrm{R} - \mathrm{jX_C}) \Rightarrow \mathrm{I} = \dfrac{\mathrm{V}}{\mathrm{R} - \mathrm{jX_C}}$



but $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC}$ $\quad \therefore \mathrm{I} = \dfrac{\mathrm{V}}{\mathrm{R} - (\mathrm{j}/\omega\mathrm{C})}$

Complex Impedance $Z = (R - j/\omega C)$

The current, I **in Cartesian form is given by**

$$I = \left[\frac{VR}{R^2 + \dfrac{1}{\omega^2 C^2}}\right] + j\left[\frac{V/\omega C}{R^2 + \dfrac{1}{\omega^2 C^2}}\right]$$

+j signifies that the current **leads** the voltage.

Complex Impedance: $Z = \left(R - j/\omega C\right)$

***I* Cartesian form:** $I = \left[\dfrac{VR}{R^2 + \dfrac{1}{\omega^2 C^2}}\right] + j\left[\dfrac{V/\omega C}{R^2 + \dfrac{1}{\omega^2 C^2}}\right]$

In Polar Form

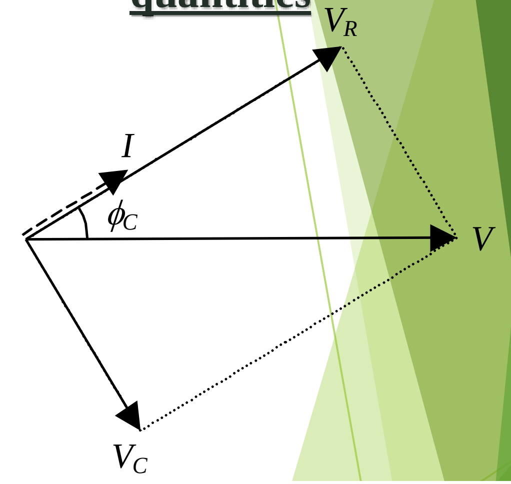
$$I = \frac{V}{\sqrt{R^2 + \dfrac{1}{\omega^2 C^2}}} \angle + \phi_C$$

$+\phi_C$ identifies current **leading** voltage

$$\phi_C = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

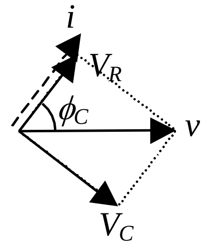
$$|I| = \frac{V}{\sqrt{R^2 + \dfrac{1}{\omega^2 C^2}}}$$

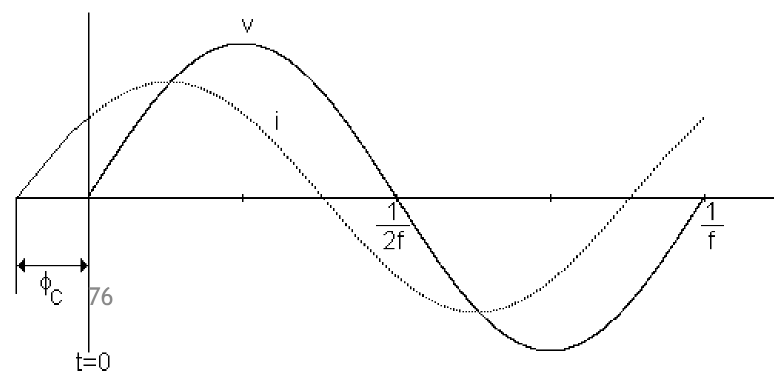
phasor diagram drawn with RMS quantities



Power Factor $= \cos\left(\phi_C\right)$

$$= \cos\left(\tan^{-1}\left(\frac{1}{\omega CR}\right)\right)$$

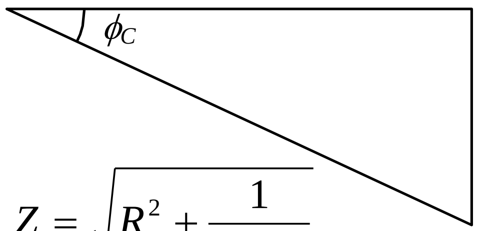




sinusoidal current leading the voltage

$$\therefore \quad Z = R - \frac{j}{\omega C}$$

$$|Z| = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$



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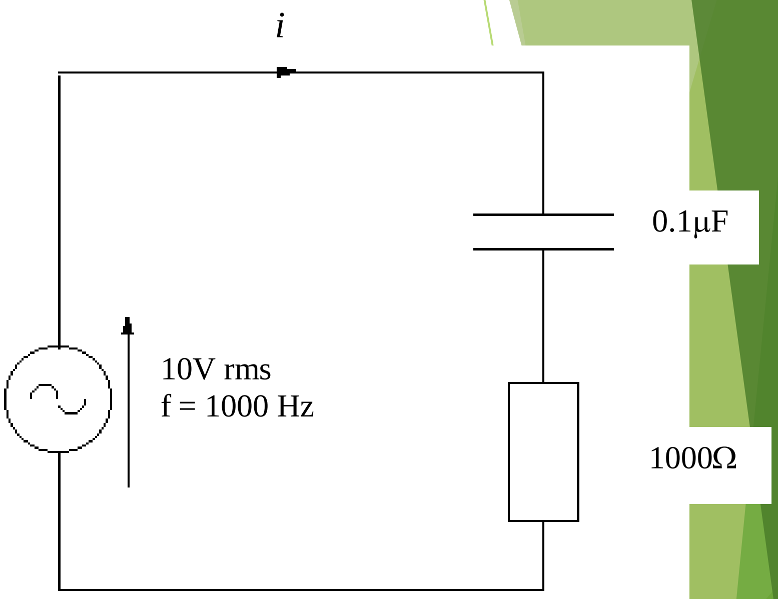
Exercise:

For the circuit shown, calculate the rms current I & phase angle ϕ_L

Use: $$I = \frac{V}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \angle + \phi_C$$

$$I = \left[\frac{VR}{R^2 + \frac{1}{\omega^2 C^2}} \right] + j \left[\frac{V / \omega C}{R^2 + \frac{1}{\omega^2 C^2}} \right]$$



Answer: $I = 5.32\text{mA} \angle 57.90$

RLC in series with an AC supply

$$V = V_R + V_L + V_C$$

We know that: $V_R = IR$ $\quad V_L = I(jX_L)$ $\quad V_C = I(-jX_C)$

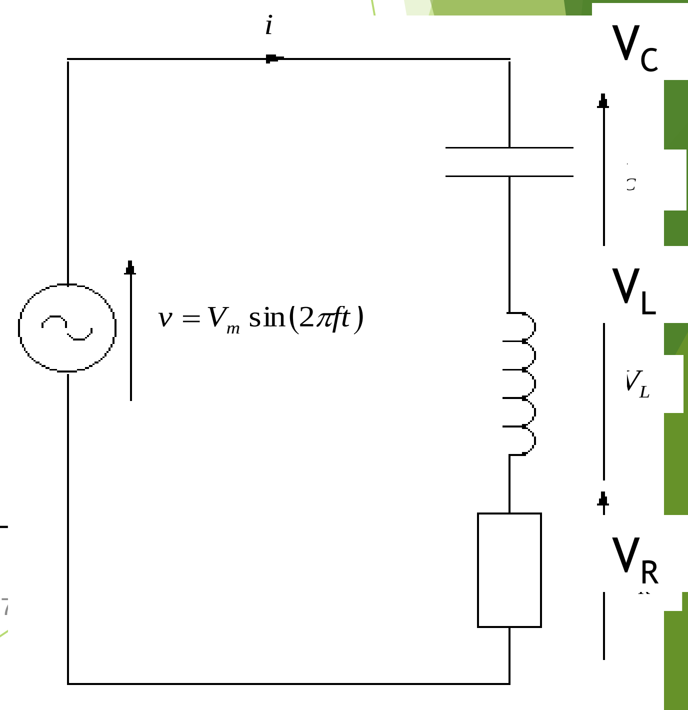
$$\therefore \mathrm{V} = \mathrm{I}[\mathrm{R} + \mathrm{j}\mathrm{X}_\mathrm{L} - \mathrm{j}\mathrm{X}_\mathrm{C}] = \mathrm{I}[\mathrm{R} + \mathrm{j}(\mathrm{X}_\mathrm{L} - \mathrm{X}_\mathrm{C})]$$

But $X_L = \omega L$ **&** $X_C = 1/\omega C$

$$\therefore \mathrm{I} = \frac{\mathrm{V}}{\mathrm{R} + \mathrm{j}\,(\omega\,\mathrm{L} - 1/\omega\mathrm{C})}$$

Complex Impedance

$$Z = R + j\left(\omega\,L - \frac{1}{\omega\,C}\right) \quad |Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$



From previous page ➡ $$I=\frac{V}{R+j\left(\omega L-1/\omega C\right)}$$

$$I=\frac{VR}{R^2+\left(\omega L-1/\omega C\right)^2}-j\frac{V\left(\omega L-1/\omega C\right)}{R^2+\left(\omega L-1/\omega C\right)^2}$$

$$I=\frac{V}{R^2+\left(\omega L-1/\omega C\right)^2}\left[R-j\left(\omega L-1/\omega C\right)\right]$$

$$I=\frac{V}{\sqrt{R^2+\left(\omega L-1/\omega C\right)^2}}\angle-\phi_s$$

The phasor diagram (and hence the waveforms) depend on the relative values of ωL and $1/\omega C$. Three cases must be considered

$$\phi_s=\tan^{-1}\left(\frac{\omega L-1/\omega C}{R}\right)$$

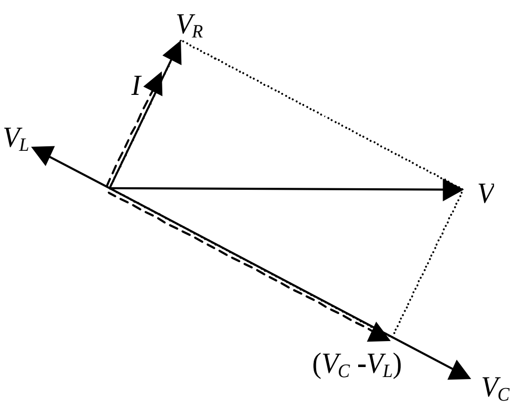
or

$$\phi_s=\tan^{-1}\left(\frac{X_L-X_C}{R}\right)$$

$$|I|=\frac{V}{\sqrt{R^2+\left(\omega L-1/\omega C\right)^2}}$$

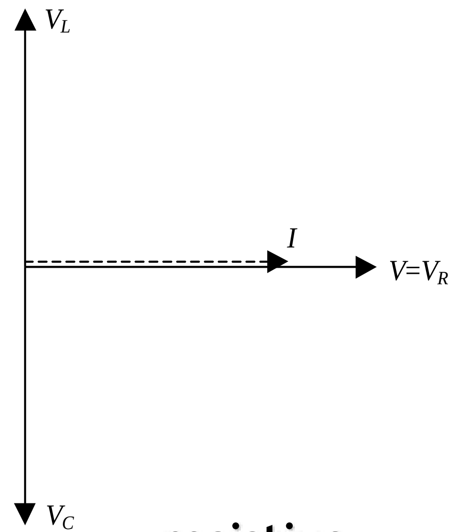
From previous page ➡ $$|I| = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

(i) $\omega L < 1/\omega C \quad V_L < V_C$



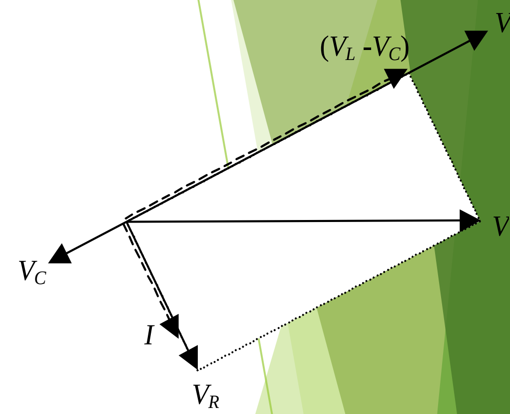
capacitive

(ii) $\omega L = 1/\omega C \quad V_L = V_C$



resistive

(iii) $\omega L > 1/\omega C \quad V_L > V_C$



inductive

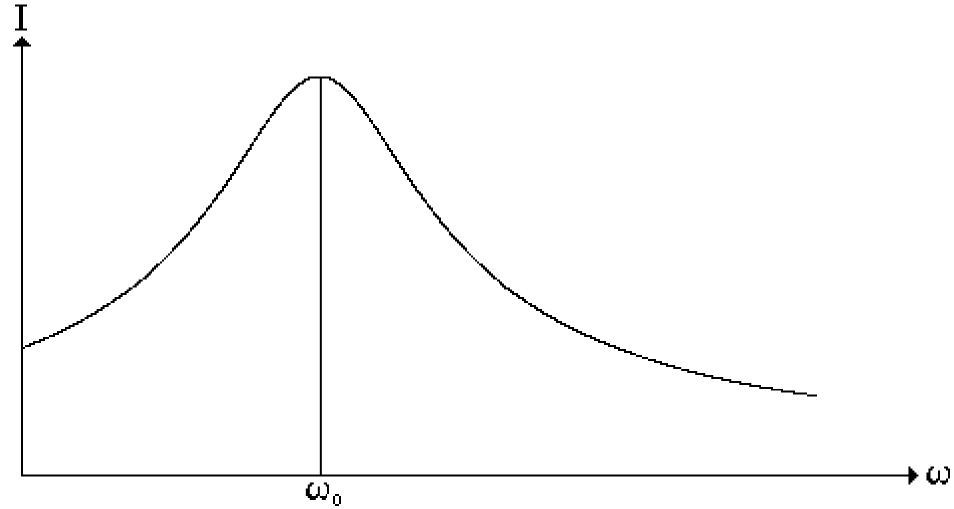
Resonant frequency $$f_o = \frac{1}{2\pi\sqrt{LC}}$$

From previous page ➡ $$|I| = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

From the above equation for the current it is clear that the magnitude of the current varies with ω (and hence frequency, f). This variation is shown in the graph

at ω_o, $$\omega L = 1/\omega C \qquad I = \frac{V}{R}\angle 0°$$

$|V_L| = |V_C|$ and they may be greater than V



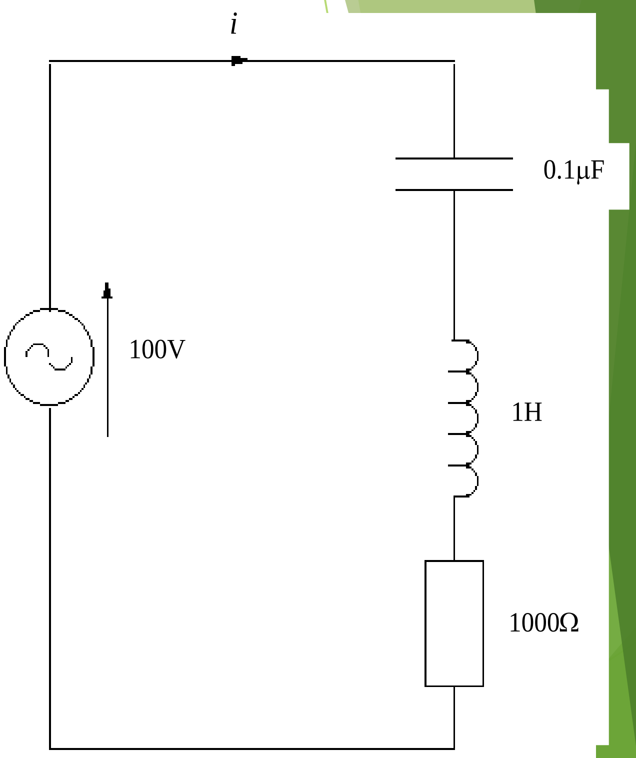
$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \& \quad f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

- ❑ **f_o is called the series resonant frequency.**
- ❑ **This phenomenon of series resonance is utilised in radio tuners.**

Exercise:

For circuit shown in figure, calculate the current and phase angle and power factor when frequency is

(i) 159.2Hz, (ii) 1592.Hz and (iii) 503.3Hz



How about you try this ?

Answer:

(i) 11.04 mA + 83.6°, 0.111 leading
(ii) 11.04mA, -83.6°, 0.111 lagging
(iii) 100mA, 00, 1.0 (in phase)

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AC Supply in Parallel with C, and in Series R &L

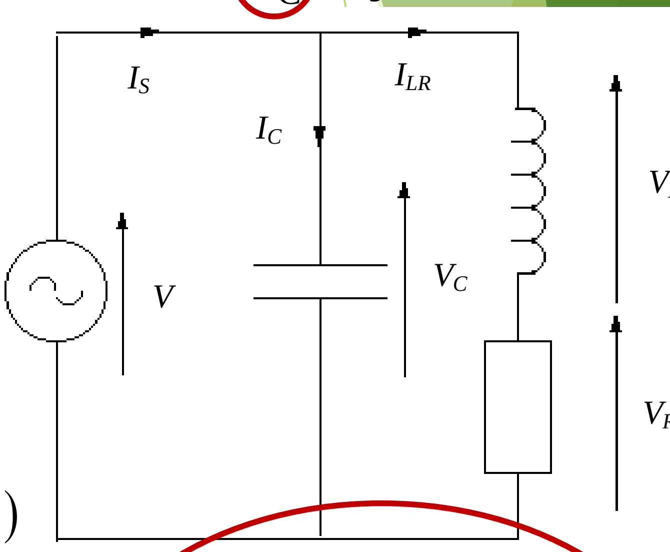
$$I_S = I_C + I_{LR} \qquad V = V_C = V_R + V_L$$

Can U name the Laws?

We know that: $V_C = I_C(-jX_C) = I_C(-j/\omega C) = V \Rightarrow I_C = j\omega CV$

$$V_R = \underline{I_{LR}R}$$

$$V_L = I_{LR}(jX_L) = \underline{I_{LR}(j\omega L)}$$

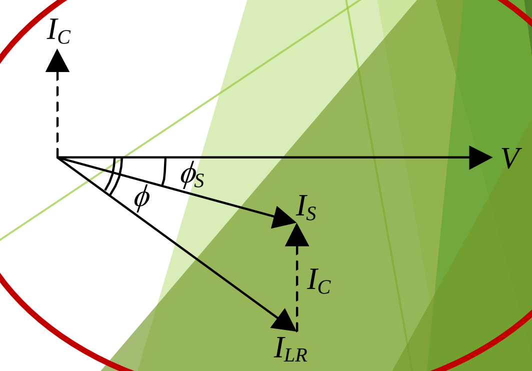


Substituting for the different Voltage components gives:

$$V = I_{LR}(R + j\omega L) \Rightarrow I_{LR} = \frac{V}{R + j\omega L} = \frac{V}{R^2 + \omega^2 L^2}(R - j\omega L)$$

and $I_S = I_C + I_{LR} \Rightarrow I_S = j\omega CV + \frac{V}{R^2 + \omega^2 L^2}(R - j\omega L)$

Hence, $$I_S = \frac{V}{R^2 + \omega^2 L^2}\left[R + j\omega\left(CR^2 + \omega^2 CL^2 - L\right)\right]$$



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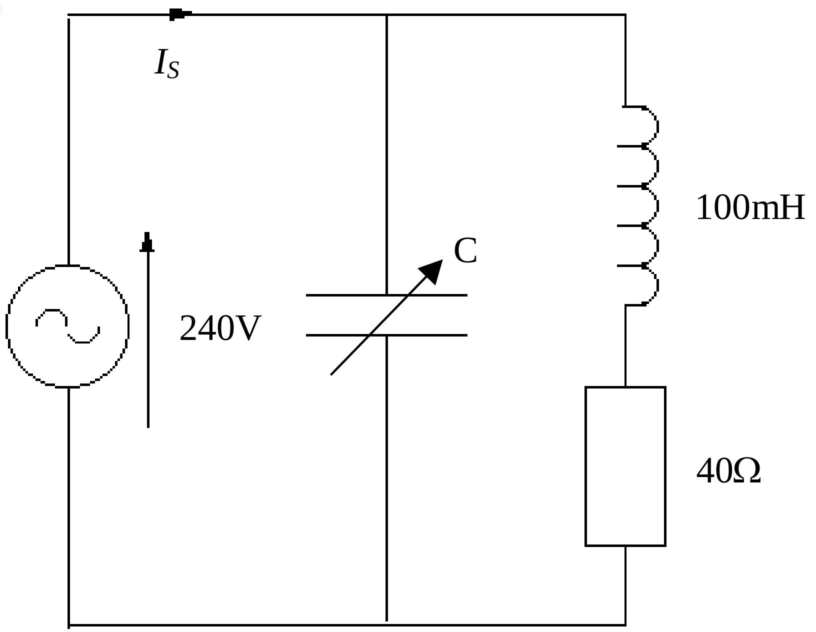
Exercise:

For the circuit shown calculate the minimum supply current, I_s and the corresponding capacitance C. Frequency is 50 Hz.

How about you try this one too?

☺



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Answer: I_{Smin} = 3.71A C = 38.6μF

Power Dissipation

We know that: *power dissipation* $|_{instantaneous}$ = *voltage* $|_{instantaneous}$ × *current* $|_{instantaneous}$

$$\therefore p = v \times i$$

Hence, instantaneous voltage, ➡ $v = V_m \sin(\omega t)$

instantaneous current, ➡ $i = I_m \sin(\omega t \pm \phi)$

$$p = vi = V_m \sin(\omega t) I_m \sin(\omega t \pm \phi)$$

$$p = \frac{V_m I_m}{2}\left[\cos(2\omega t \pm \phi) - \cos(\pm\phi)\right]$$

$$P = \frac{V_m I_m}{2}\cos\phi \quad \textbf{but} \quad V = \frac{V_m}{\sqrt{2}} \quad \& \quad I = \frac{I_m}{\sqrt{2}}$$

Therefore, **net power transfer** ➡ $P = VI\cos(\phi)$

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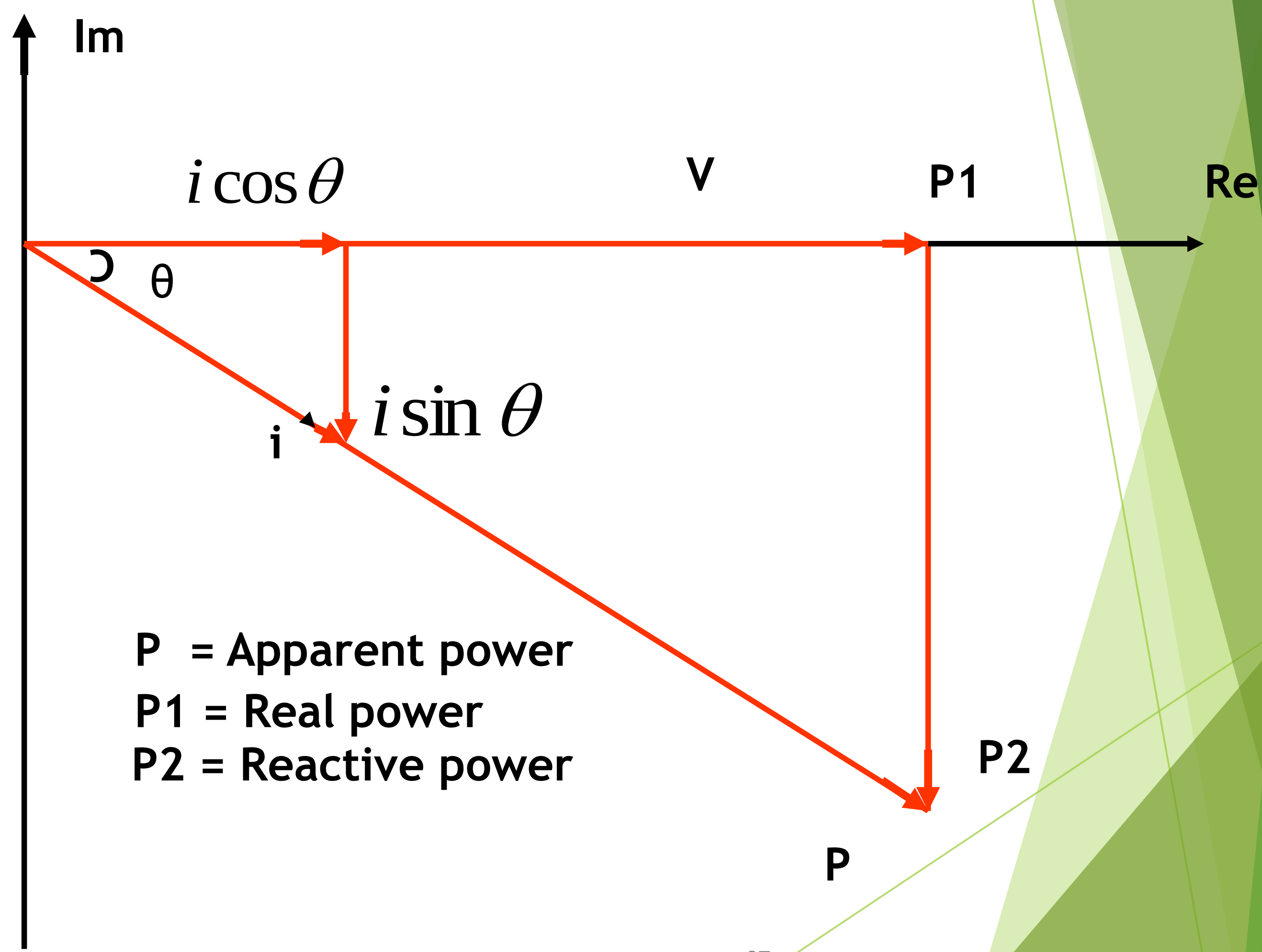
Real, Apparent and Reactive Power



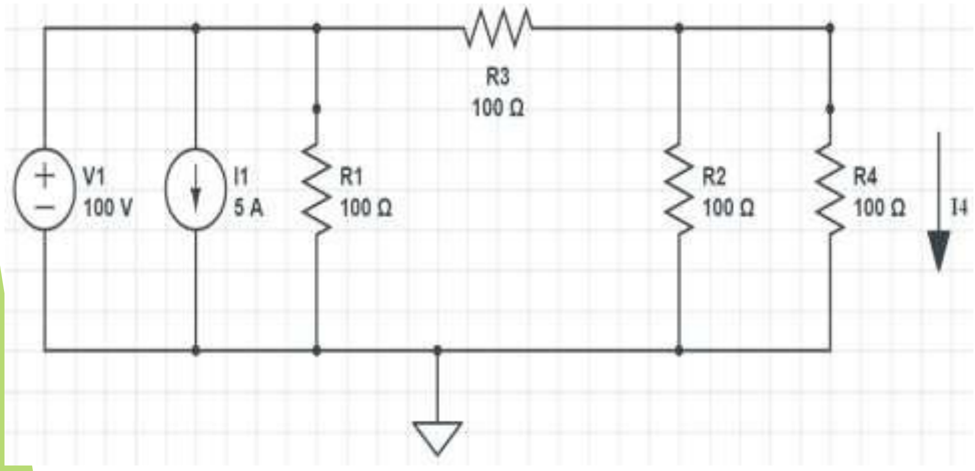
P = Apparent power
P1 = Real power
P2 = Reactive power

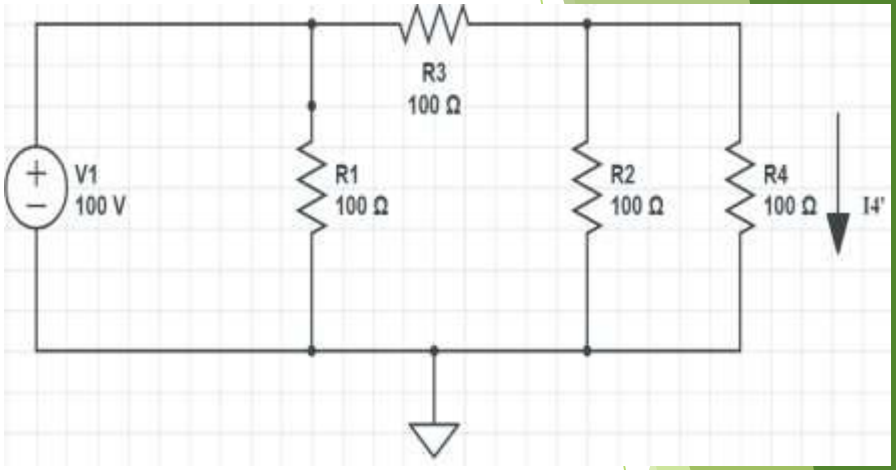
Electrical Circuits-1

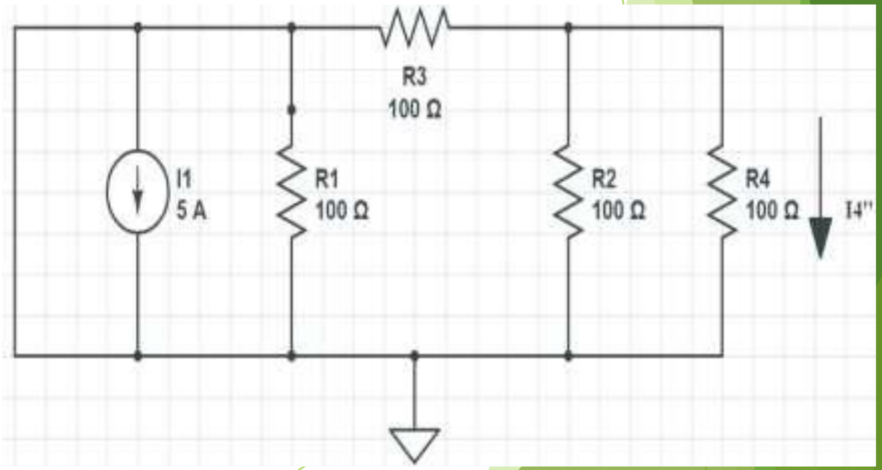
UNIT-IV

Superpostion - Introduction

What’s the current on R4?







$$I4 = I4' + I4''$$

Superpostion

The **superposition theorem** can be used to find solution to networks with many sources.

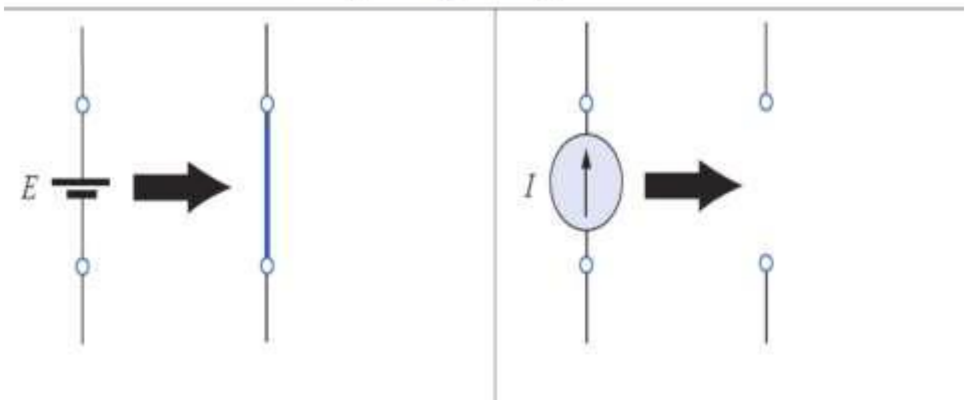
The current through, or voltage across, any element of a network is equal to the algebraic sum of the currents or voltages produced independently by each source.

Since the effect of each source will be determined independently, the number of networks to be analyzed will equal the number of sources.

$$\begin{matrix}\text{Number of networks} \\ \text{to be analyzed}\end{matrix} = \begin{matrix}\text{Number of} \\ \text{independent sources}\end{matrix}$$

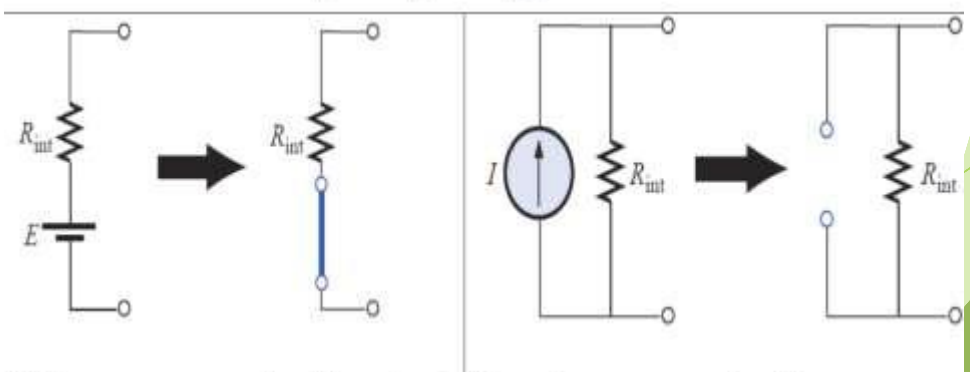
Superpostion

Removing the effects of ideal sources.



Voltage source are replaced by a short | Current source are replaced by an open

Removing the effects of practical sources.



Voltage source are replaced by a short | Current source are replaced by an open

Superpostion

I_1 and I_2 in the same direction: $I_T = I_1 + I_2$

Power from Source 1 is: $P_1 = I_1^2 \cdot R$

Power from Source 2 is: $P_2 = I_2^2 \cdot R$

If we assume total power is just the sum of powers delivered by each source then:

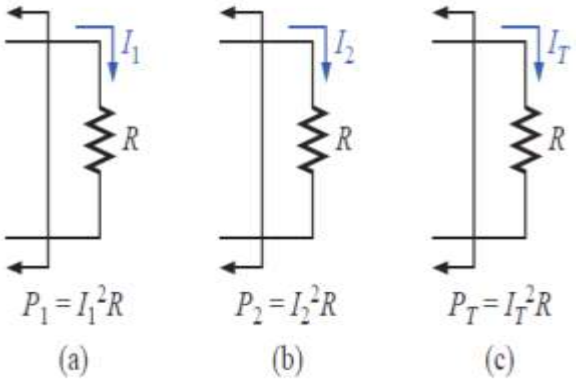
$$P_T = P_1 + P_2 = R \cdot \left(I_1^2 + I_2^2\right)$$

$$\Rightarrow I_T^2 = I_1^2 + I_2^2$$

This is not equal to the value calculated from the theorem that is:

$$I_T^2 = (I_1 + I_2)^2 = I_1^2 + 2 \cdot I_1 \cdot I_2 + I_2^2$$

Which is different.

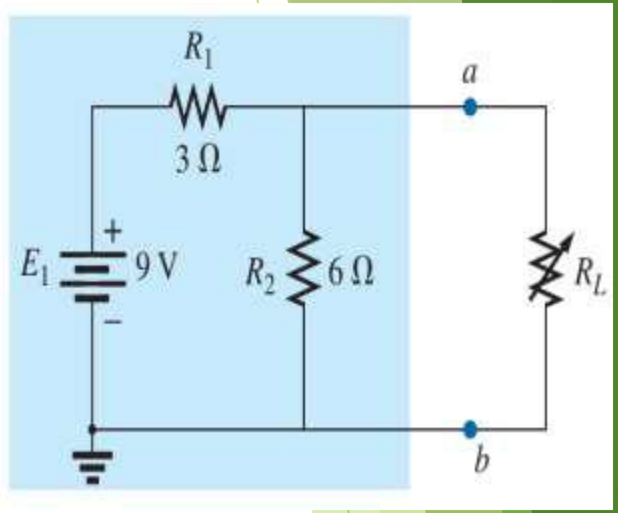


$P_T \neq P_1 + P_2$

the total power delivered to a resistive element must be determined using the total current through or the total voltage across the element and cannot be determined by a simple sum of the power levels established by each source.

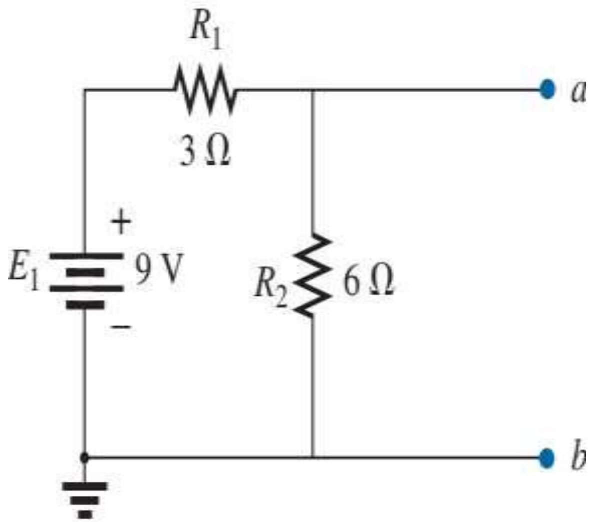
Thévenin’s theorem - Example

Find the Thevenin equivalent circuit for the shaded area.



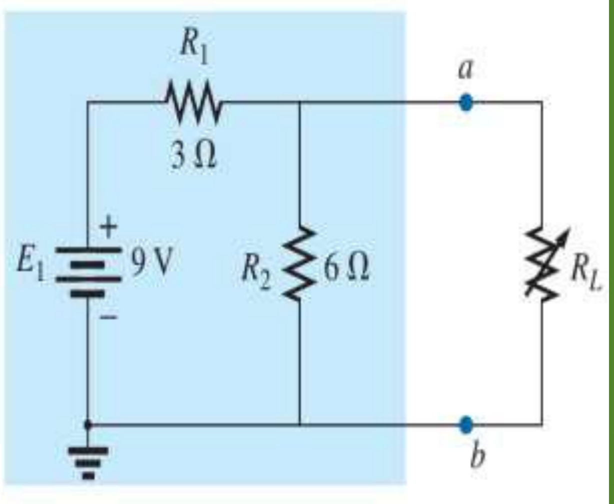
Step 1: Remove that portion of the network where the Thévenin equivalent circuit is found.

Step 2: Mark the terminals of the remaining

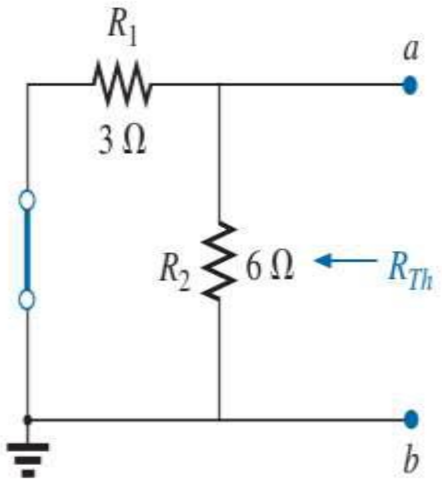


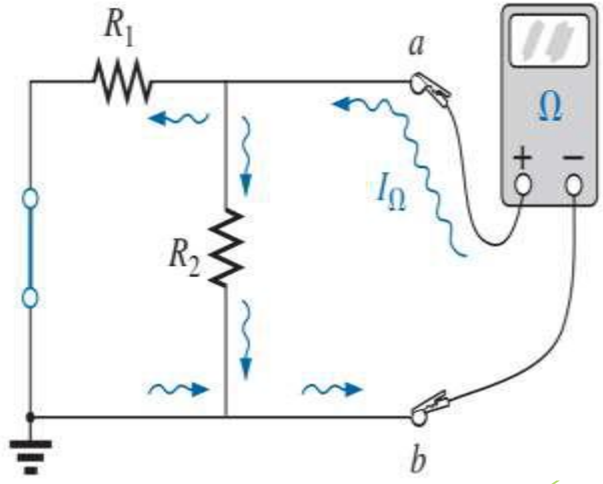
Thévenin's theorem - Example

Find the Thevenin equivalent circuit for the shaded area.



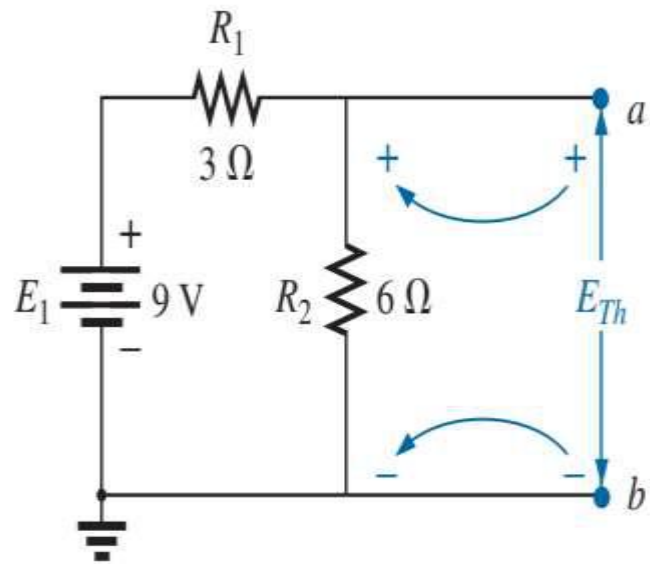
Step 3: Calculate R_{TH} by first setting all sources to zero and then finding the resultant resistance between the two marked terminals

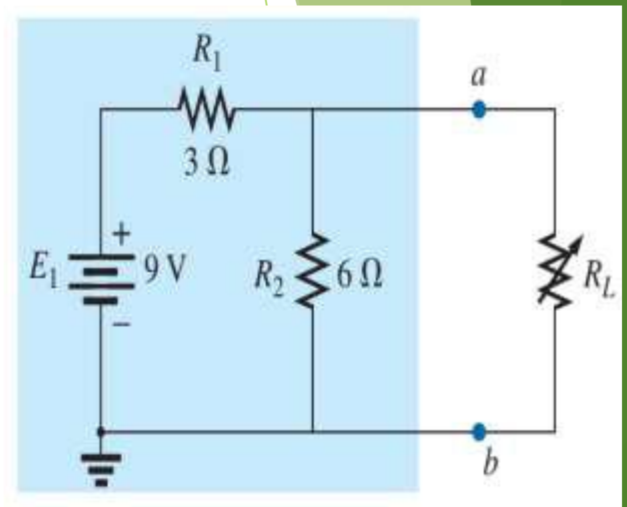




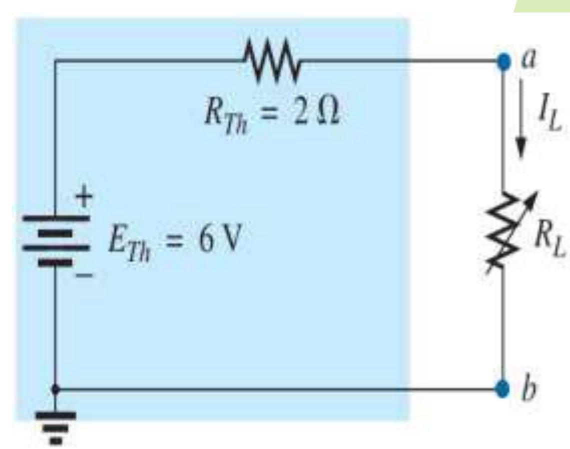
Thévenin's theorem - Example

Step 4: Find E_{TH} (open-circuit voltage) by measuring the voltage between the two ma





$$E_{Th} = \frac{R_2 E_1}{R_2 + R_1} = \frac{(6\,\Omega)(9\,\text{V})}{6\,\Omega + 3\,\Omega} = \frac{54\,\text{V}}{9} = \mathbf{6\,V}$$

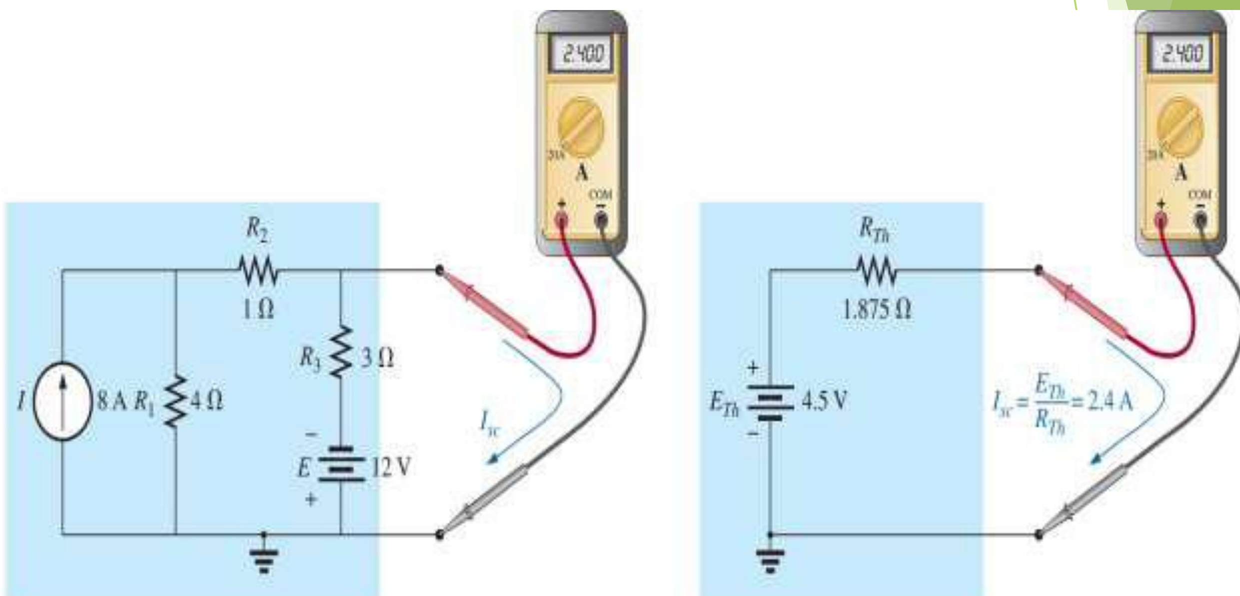


Thévenin Resistance – Alternative Method

The Thévenin resistance can also be determined by placing a **short circuit** across the output terminals and finding the current through the short circuit.

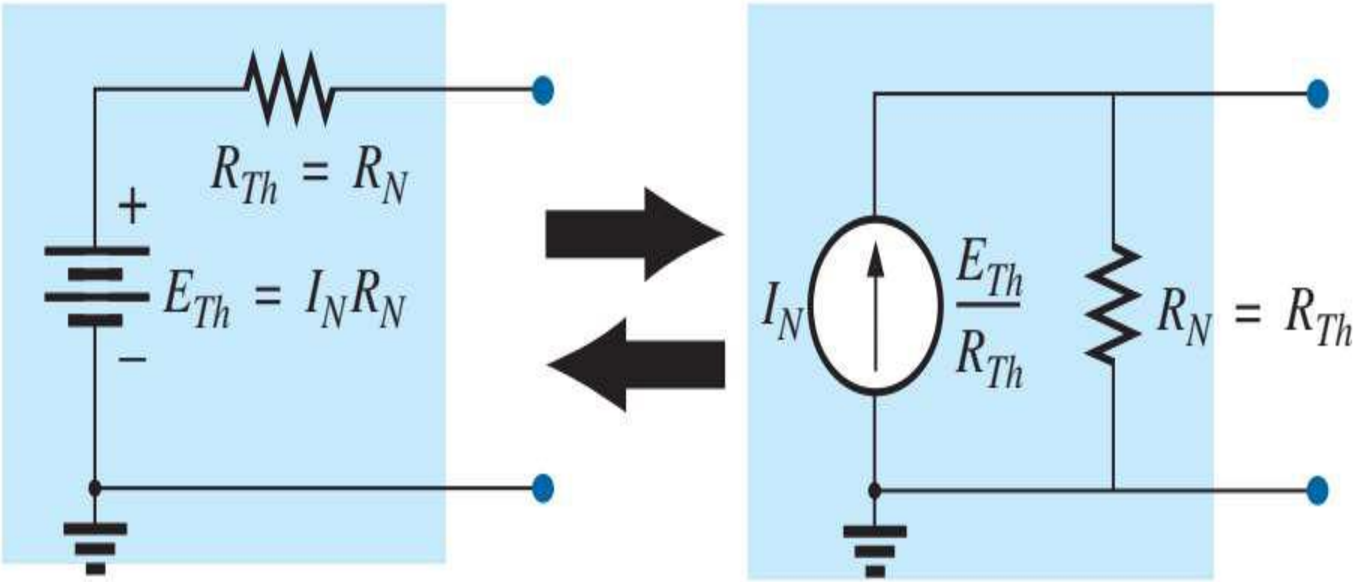
$$R_{Th} = \frac{E_{Th}}{I_{sc}}$$

$$\boxed{R_{Th} = \frac{V_{oc}}{I_{sc}}}$$

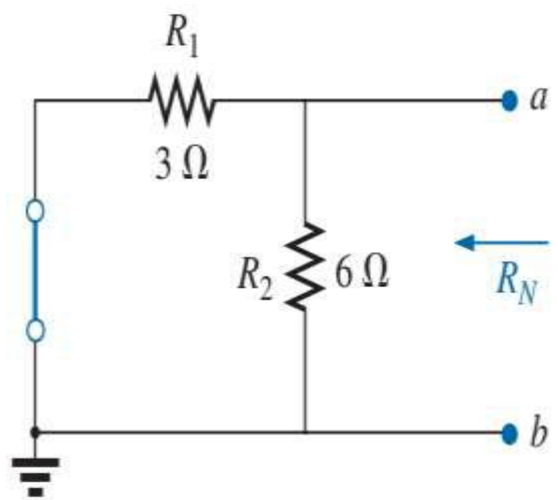


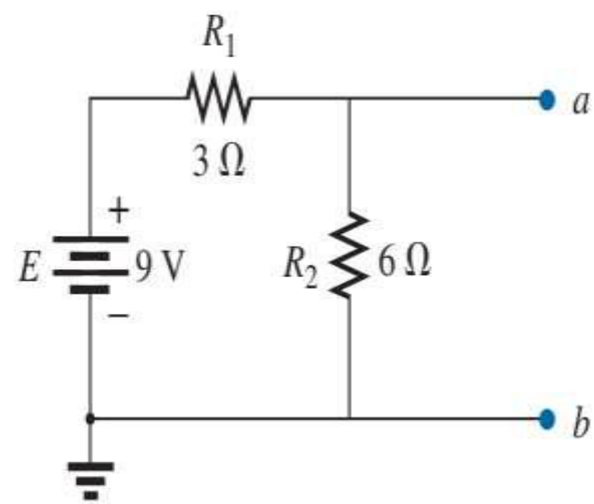
Norton’s Theorem

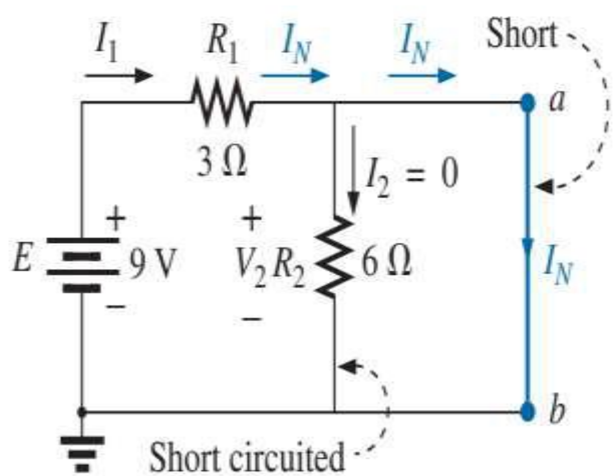
$$R_N = R_{TH}$$
$$I_N = I_{sc}$$



Norton’s Theorem – Example 1







$$R_N = R_1 \| R_2 = 3\,\Omega \| 6\,\Omega = \frac{(3\,\Omega)(6\,\Omega)}{3\,\Omega + 6\,\Omega} = \frac{18\,\Omega}{9} = \mathbf{2\,\Omega}$$

$$I_N = \frac{E}{R_1} = \frac{9\text{ V}}{3\,\Omega} = \mathbf{3\text{ A}}$$

RECIPROCITY THEOREM

- The reciprocity theorem is applicable only to single-source networks.
- It is, therefore, not a theorem employed in the analysis of multisource networks described thus far.

The theorem states the following:

“The current I in any branch of a network, due to a single voltage source E anywhere else in the network, will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current I was originally measured.”

- In other words, the location of the voltage source and the resulting current may be interchanged without a change in current.

- In the representative network of Fig. 1(a), the current I due to the voltage source E was determined. If the position of each is interchanged as shown in Fig. 1(b), the current I will be the same value as indica

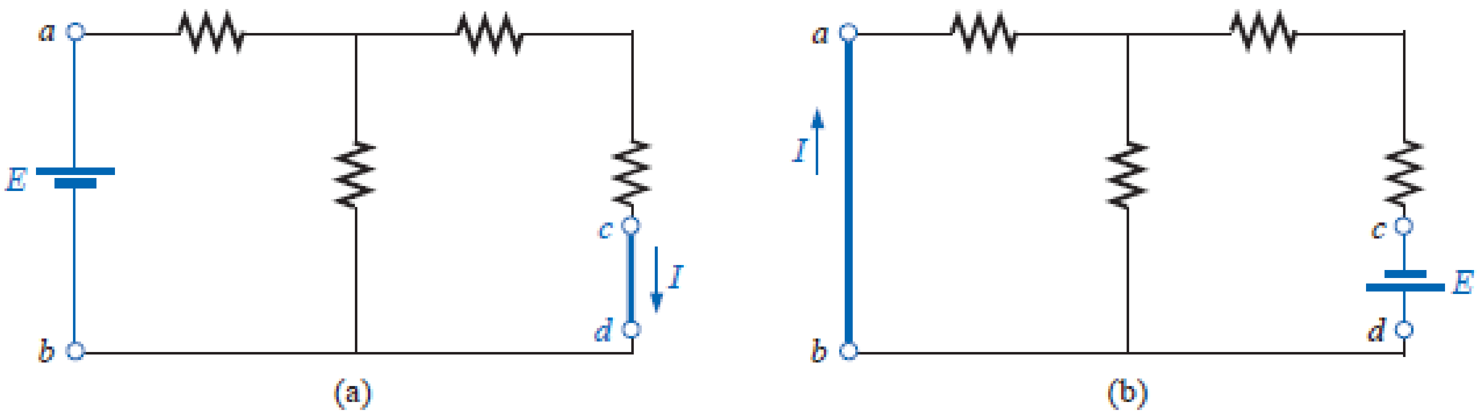


FIGURE-1

To demonstrate the validity of this statement and the theorem, consider the network of Fig-2, in which values for the elements have been assigned.

The total resistance

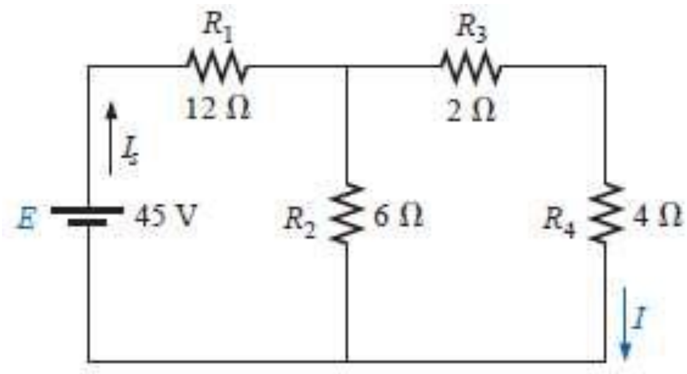


Fig-2

$$R_T = R_1 + R_2 \parallel (R_3 + R_4) = 12\,\Omega + 6\,\Omega \parallel (2\,\Omega + 4\,\Omega)$$
$$= 12\,\Omega + 6\,\Omega \parallel 6\,\Omega = 12\,\Omega + 3\,\Omega = 15\,\Omega$$

And

$$I_s = \frac{E}{R_T} = \frac{45\text{ V}}{15\,\Omega} = 3\text{ A}$$

$$I = \frac{3\text{ A}}{2} = 1.5\text{ A}$$

For the network of Fig-3,

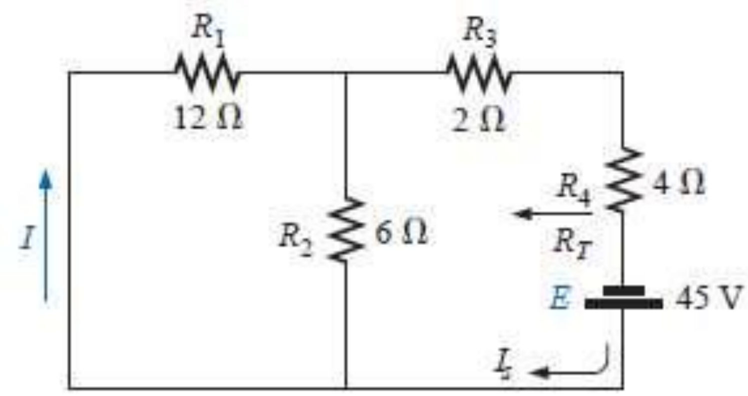


Fig-3

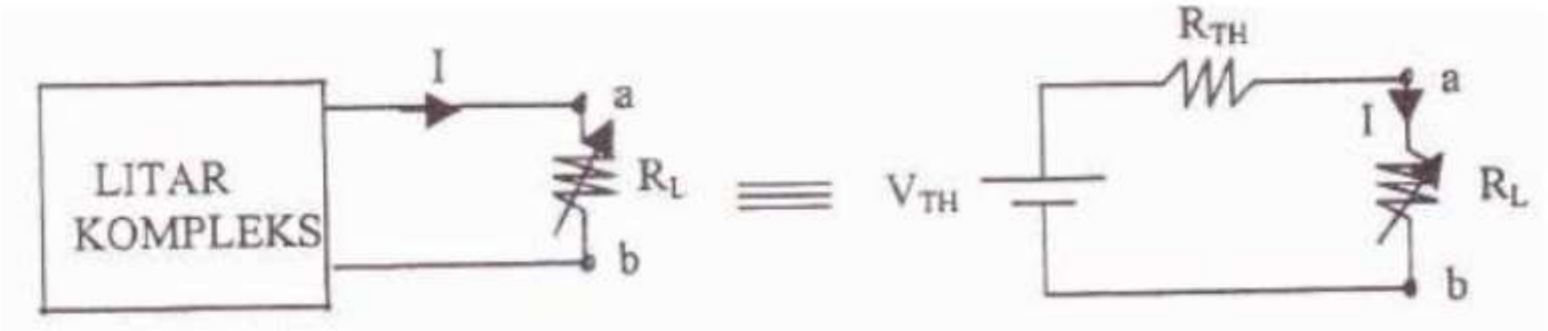
$$R_T = R_4 + R_3 + R_1 \| R_2$$
$$= 4\ \Omega + 2\ \Omega + 12\ \Omega \| 6\ \Omega = 10\ \Omega$$

$$I_s = \frac{E}{R_T} = \frac{45\ \text{V}}{10\ \Omega} = 4.5\ \text{A}$$

$$I = \frac{(6\ \Omega)(4.5\ \text{A})}{12\ \Omega + 6\ \Omega} = \frac{4.5\ \text{A}}{3} = \mathbf{1.5\ A}$$

Maximum power transfer

- a circuit will supply maximum power to the load if the **load resistance R_L is equal to the equivalent resistance** seen by the load
- Maximum power transfer can be obtained by replace a complex circuit with the **Thevenin equivalent circuit** or Norton equivalent circuit

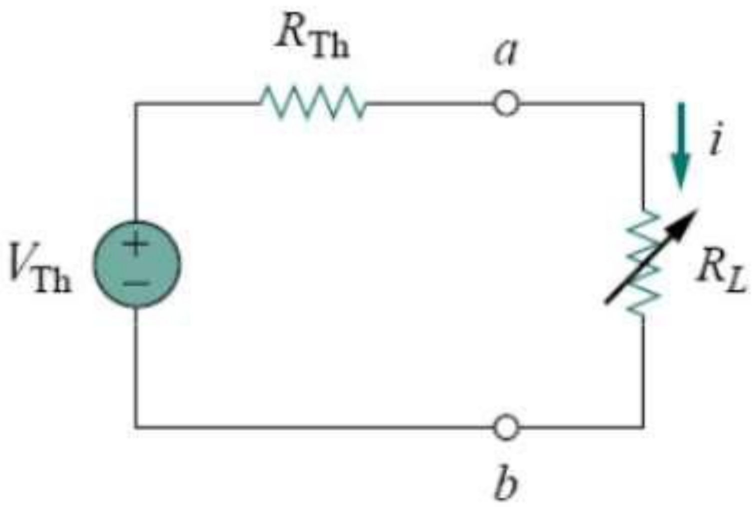


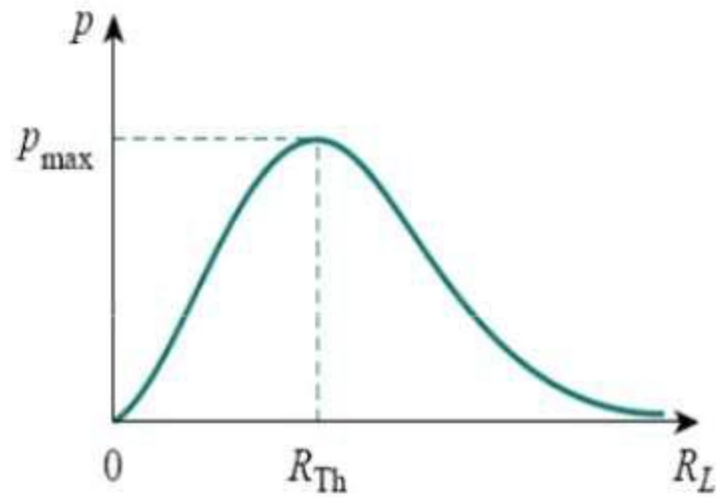
Maximum power transfer cont.

- Power to the load R_L, $P_{RL} = I^2 R_L = \left(\dfrac{V_{TH}}{R_{TH} + R_L} \right)^2 R_L$
- Condition maximum power transfer $R_L = R_{TH}$
- Therefore, maximum power supplied to the load is:

$$P_{RLmak} = \frac{V_{TH}^{\;2}}{4R_L} = \frac{V_{TH}^{\;2}}{4R_{TH}}$$

Maximum power transfer cont.





$$R_L = R_{Th}$$

$$p_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

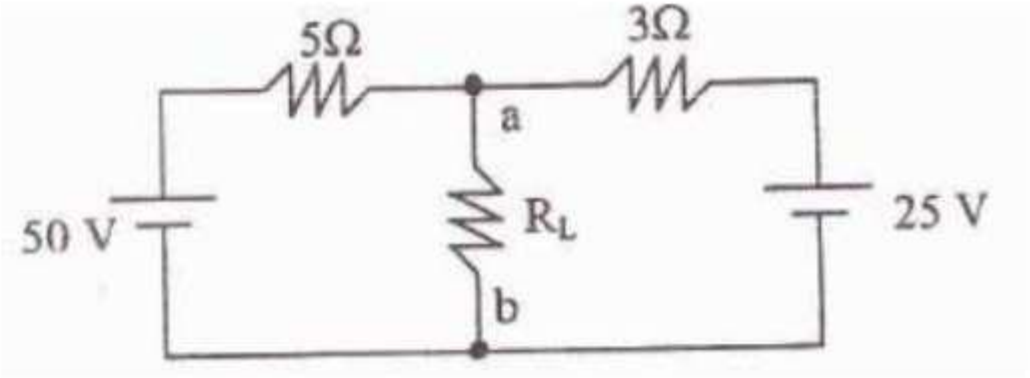
Maximum power transfer cont.

- Examples

From circuit below, calculate:-

a) Value RL when maximum output power

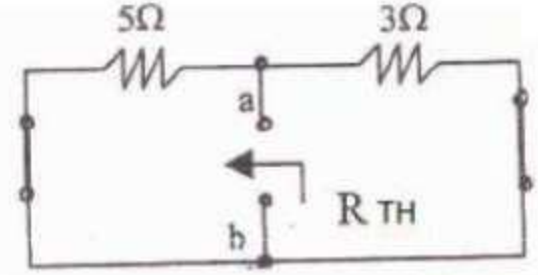
b) Maximum power absorb by load RL



Maximum power transfer cont.

- Solution.

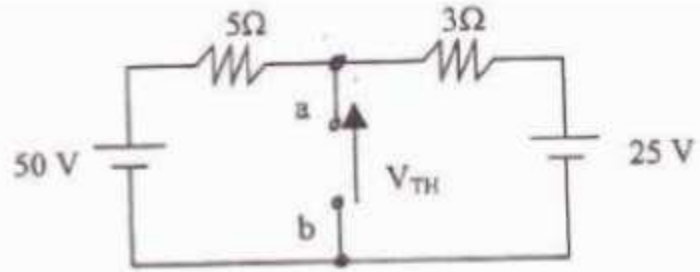
a) Remove RL from circuit and off all source from circuit to find RTH



$$R_{TH} = (5 // 3) = \frac{5 \times 3}{5 + 3} = 1.875\Omega$$

So, for get maximum power, value $R_L = R_{TH} = 1.875\ \Omega$

b) Get the thevenin voltage at terminal a-b when RL removed from circuit

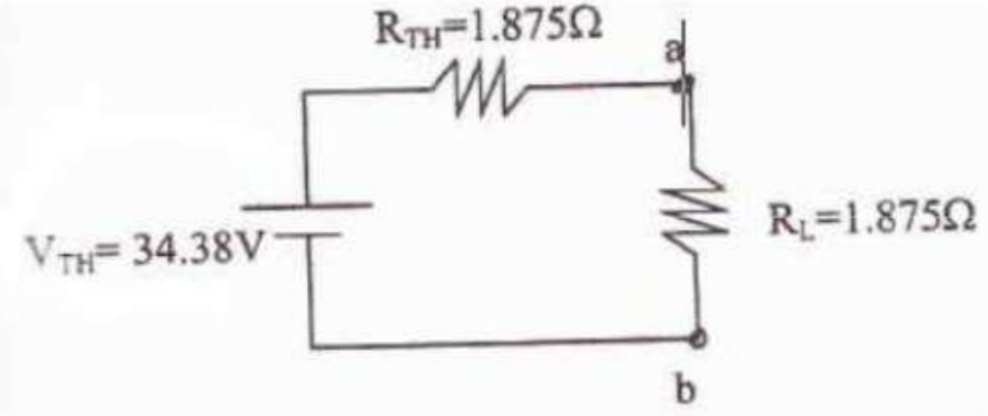


$$\frac{V_{TH} - 50}{5} + \frac{V_{TH} - 25}{3} = 0$$

$$\therefore V_{TH} = 34.38\ V$$

Maximum power transfer cont.

So, draw Thevenin equivalent circuit when maximum power transfer happen.



$$P_{mak} = \frac{V_{TH}^2}{4R_L} = 157.6\,W$$

Compensation Theorem

- It is one of the important theorems in Network Analysis , which finds it’s application mostly in calculating the sensitivity of electrical networks & bridges and solving electrical networks.

- The Compensation Theorem states that :-

For the sake of branch responses calculations ; Any resistance in a branch of an linear bilateral electrical network can be replaced by a voltage source which provides the same voltage as the voltage dropped in the resistance.

- In any linear bilateral Electrical Network If in any Branch have it’s initial resistance (or impedance in case of AC) “R” conducting a current of “I” through it, And if the resistance of the branch is changed by a factor of R , with it’s final resistance R+ R , the final effect in various branches due to the change in the resistance of the branch can be calculated by injecting an extra voltage source along with the resistance in modified branch.

- The above statement can be clarified with the following illustration.

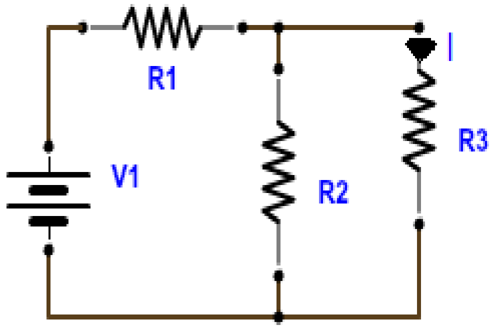


fig: 2.a Electrical Network

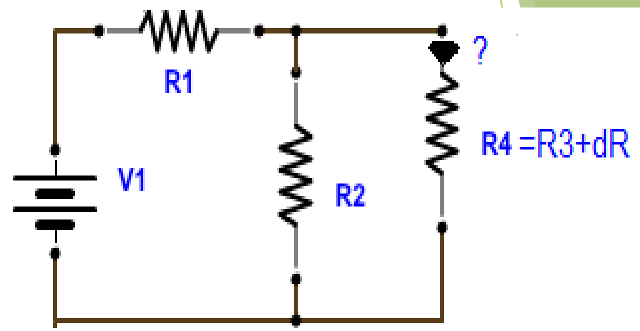


fig: 2.b Addition of dR to R3

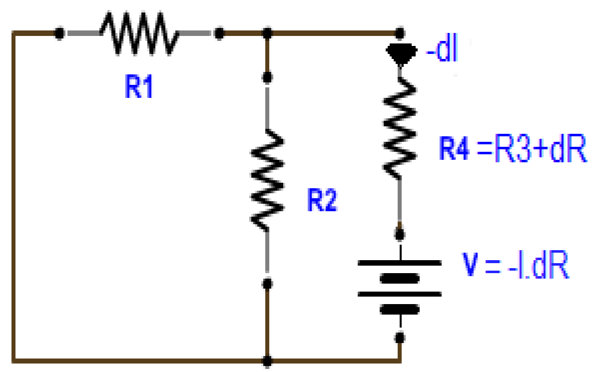


fig: 2.c Calculation of branch response due to change in R3

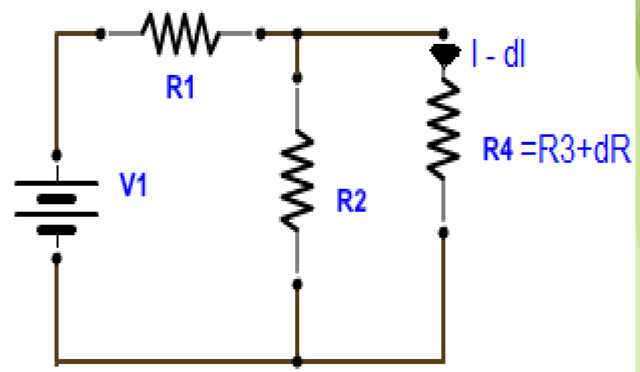


fig: 2.d New branch response due to change in R3

- In the figure above,
- In fig: 2.a , The current “I” flows through R3 when V1 acts upon it. In fig: 2.b , the R3 is changed to R4 where R4=R3+dR , or R3 is increased by dR. This can also be thought of as an extra dR added in series with R3. Now , we don’t know how much current flows through the branch when R3 is increased by dR , so to calculate the current flowing through the branch due to the effect of dR , as per Compensation theorem in fig: 2.c we add an extra V=-I.dR along with R4 and calculate the current flowing through the branch due to the V or dR to be -dI. Now in fig: 2.d we add the currents in fig: 2.a and 2.c using superposition theorem to find the new current to be I-dI.

Example 2.95. *Calculate the values of new currents in the network illustrated in Fig. 2.200 when the resistor R_3 is increased (in place of s) by 30 %.*

Solution. In the given circuit, the values of various branch currents are

$$I_1 = 75/(5+10) = 5 \text{ A}$$
$$I_2 = I_3 = 2.5 \text{ A}$$

Now, value of

$$R_3 = 20 + (0.3 \times 20) = 26\ \Omega$$
$$\therefore \quad \Delta R = 6\ \Omega$$
$$V = -I_3\,\Delta R$$
$$= -2.5 \times 6 = -15 \text{ V}$$

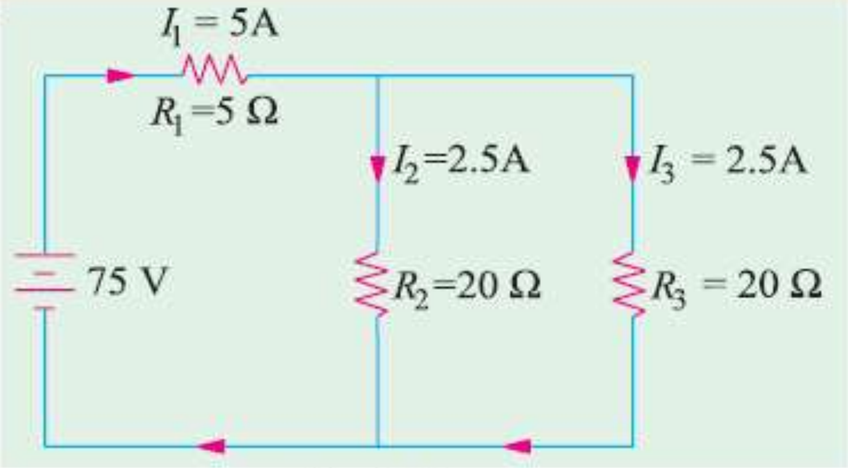
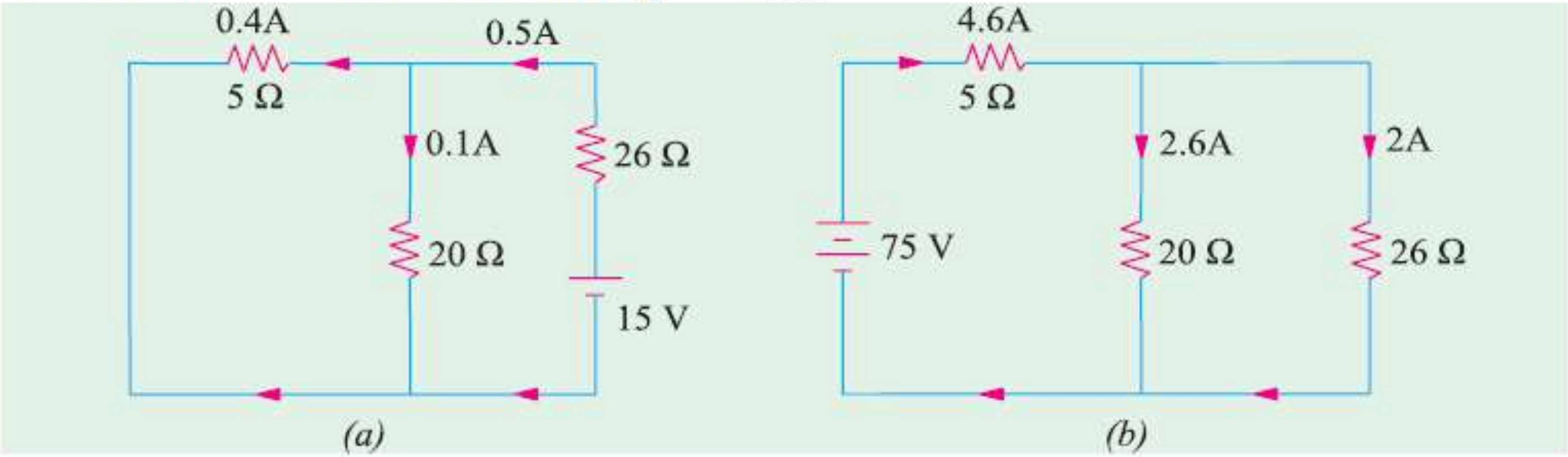


Fig. 2.200

The compensating currents produced by this voltage are as shown in Fig. 2.201 (*a*).

When these currents are added to the original currents in their respective branches the new current distribution becomes as shown in Fig. 2.201 (*b*)



Tellegen's Theorem

- Tellegen's Theorem is a general network theorem
- It is valid for any lump network

For a lumped network whose element assigned by associate reference direction for branch voltage v_k and branch current j_k

The product $v_k j_k$ is the power delivered at time t by the network to the element k

If all branch voltages and branch currents satisfy KVL and KCL then

$$\sum_{k=1}^{b} v_k j_k = 0$$

b = number of branch

Tellegen's Theorem

Suppose that $\hat{v}_1, \hat{v}_2, \ldots\ldots \hat{v}_b$ and $\hat{j}_1, \hat{j}_2, \ldots\ldots \hat{j}_b$ is another sets of branch voltages and branch currents and if $\hat{v}_k$ and $\hat{j}_k$ satisfy KVL and KCL

Then

$$\sum_{k=1}^{b} \hat{v}_k \hat{j}_k = 0 \qquad \sum_{k=1}^{b} v_k j_k = 0$$

$$\sum_{k=1}^{b} v_k \hat{j}_k = 0 \quad \text{and} \quad \sum_{k=1}^{b} \hat{v}_k j_k = 0$$

Tellegen's Theorem

Applications

Tellegen's Theorem implies the law of energy conservation.

Since

$$\sum_{k=1}^{b} v_k \, j_k = 0$$

"The sum of power delivered by the independent sources to the network is equal to the sum of the power absorbed by all branches of the network".

MILLIMAN'S THEOREM

- **Millman's Theorem** is nothing more than a long equation, applied to any circuit drawn as a set of parallel-connected branches, each branch with its own voltage source and series resistance: ... The polarity of all voltages in **Millman's Theorem** is referenced to the same point.

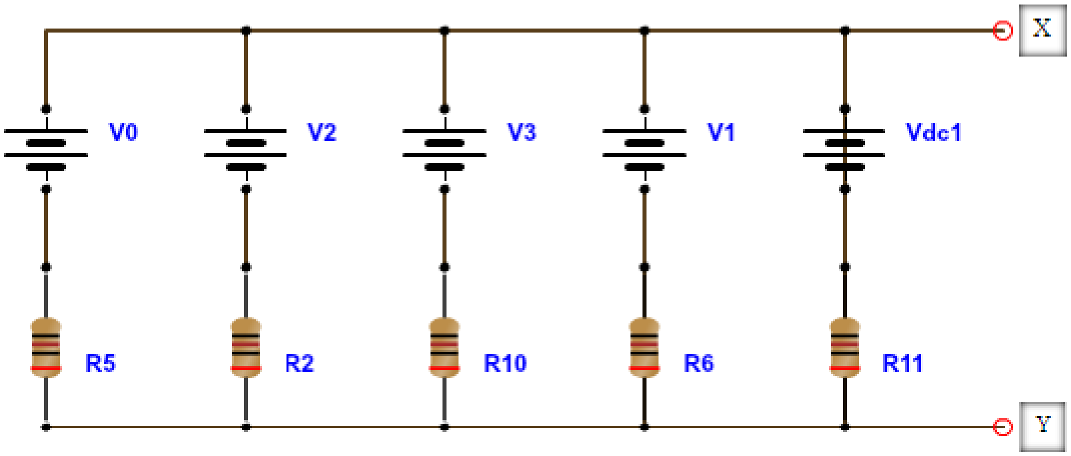
- **Applying Millman’s Theorem To circuits!**
- We can apply Millman’s Theorem to circuits as following:

Circuits with Voltage sources

- According to Millman’s theorem , in circuits with voltage sources:
- The total voltage or potential difference build up between any two points in a circuit is equal to:

$$V = \frac{\sum V_x \times G_x}{G_x}$$

- For example in the following circuit:



- Here, The potential difference between X and Y is:

$$V_{xy} = \frac{\frac{V_0}{R_5} + \frac{V_2}{R_2} + \frac{V_3}{R_{10}} + \frac{V_1}{R_6} + \frac{V_{dc1}}{R_{11}}}{\frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{R_{10}} + \frac{1}{R_6} + \frac{1}{R_{11}}}$$

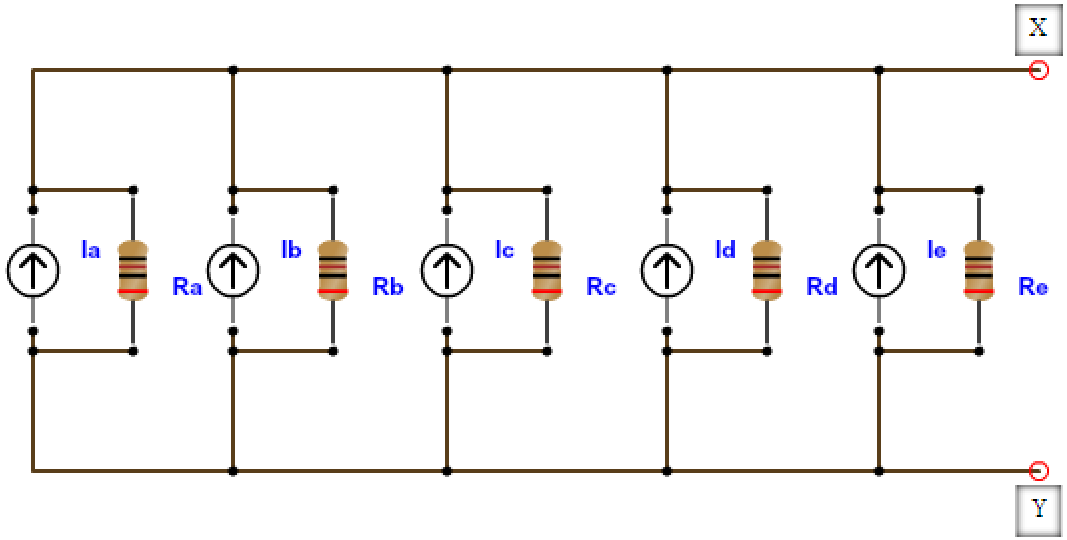
Circuits with Current Sources:

- According to Millman’s theorem , in circuits with voltage sources:
- The total voltage or potential difference between any two terminals in a circuit is equal to:

$$V = \frac{\sum i}{\sum G}$$

- Where,
- i = the current flowing through each branch.

- For example in the following circuit:



- The potential difference between X and Y is:

$$V_{XY} = \frac{I_a + I_b + I_c + I_d + I_e}{\frac{1}{R_a} + \frac{1}{R_b} + \frac{1}{R_c} + \frac{1}{R_d} + \frac{1}{R_e}}$$

Example 2.110. *Use Millman's theorem, to find the common voltage across terminals A and B and the load current in the circuit of Fig. 2.222.*

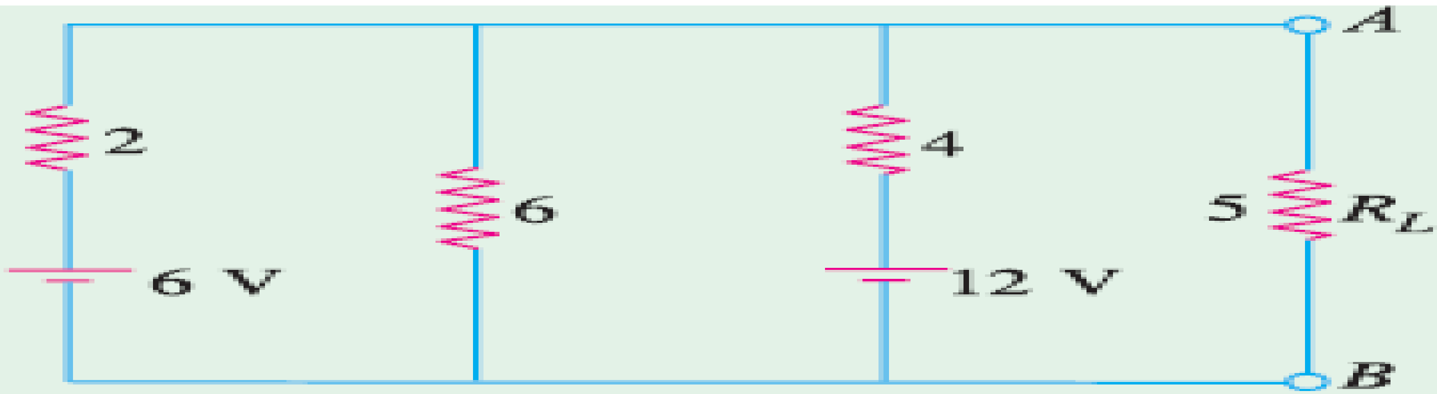


Fig. 2.222

Solution. As per Millman's Theorem,

$$V_{AB} = \frac{6/2 + 0/6 + 12/4}{1/2 + 1/6 + 1/4} = \frac{6}{11/12} = 6.55 \text{ V}$$

$$\therefore \quad V_{th} = 6.55 \text{ V}$$

$$R_{th} = 2 \,||\, 6 \,||\, 4 = 12/11 \; \Omega$$

$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{6.55}{(12/11) + 5} = 1.05 \text{ A}$$

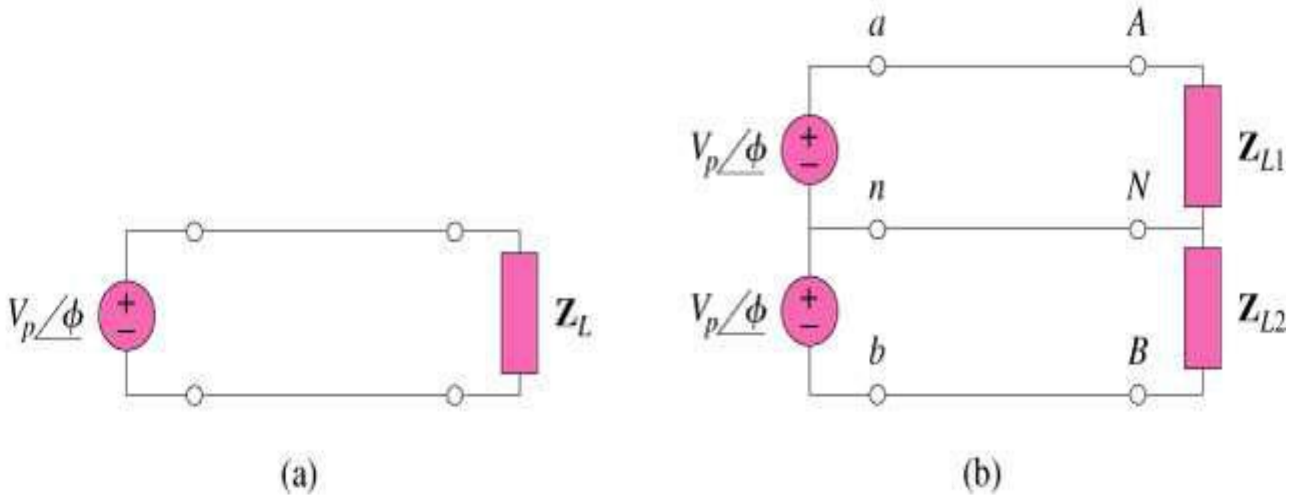
ELECTRICAL CIRCUITS UNIT-V

Three phase Circuits

Three phase Circuits

- An AC generator designed to develop a single sinusoidal voltage for each rotation of the shaft (rotor) is referred to as a **single-phase AC generator.**
- If the number of coils on the rotor is increased in a specified manner, the result is a **Polyphase AC generator**, which develops more than one AC phase voltage per rotation of the rotor
- In general, three-phase systems are preferred over single-phase systems for the transmission of power for many reasons.
  - 1. Thinner conductors can be used to transmit the same kVA at the same voltage, which reduces the amount of copper required (typically about 25% less).
  - 2. The lighter lines are easier to install, and the supporting structures can be less massive and farther apart.
  - 3. Three-phase equipment and motors have preferred running and starting characteristics compared to single-phase systems because of a more even flow of power to the transducer than can be delivered with a single-phase supply.
  - 4. In general, most larger motors are three phase because they are essentially self-starting and do not require a special design or additional starting circuitry

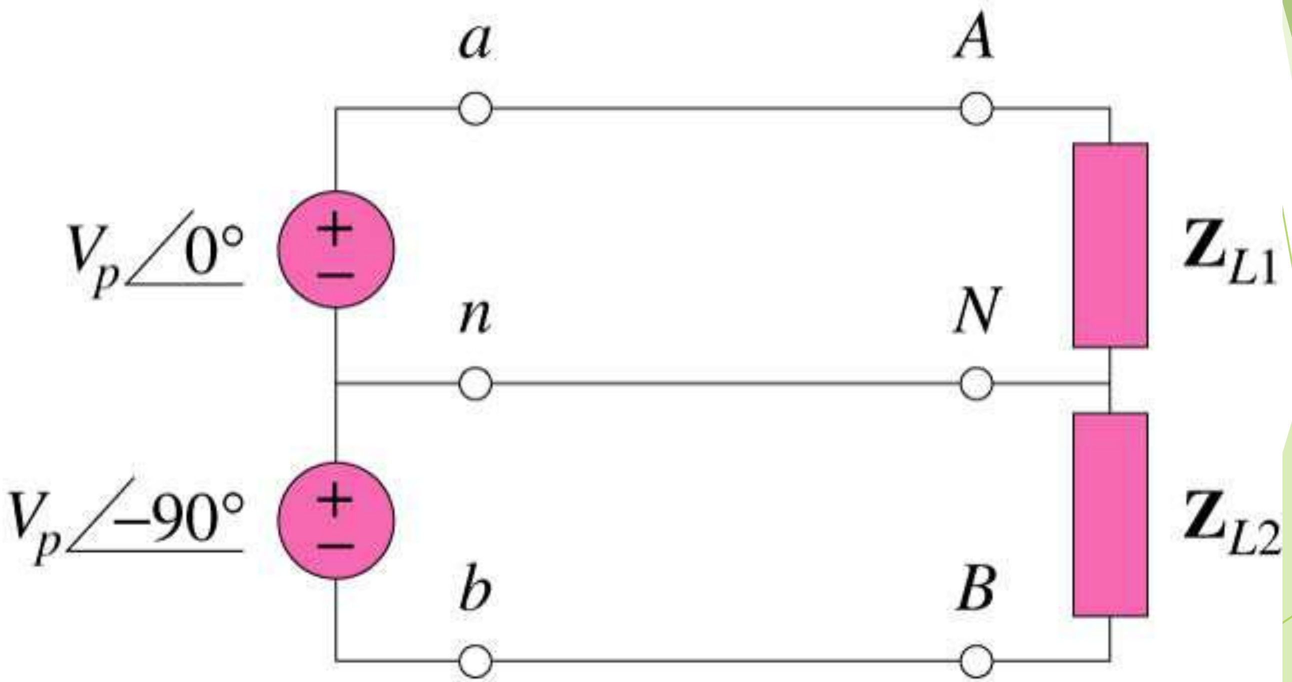
Single Phase, Three phase Circuits



a) Single phase systems two-wire type

b) Single phase systems three-wire type.
Allows connection to both 120 V and 240 V.

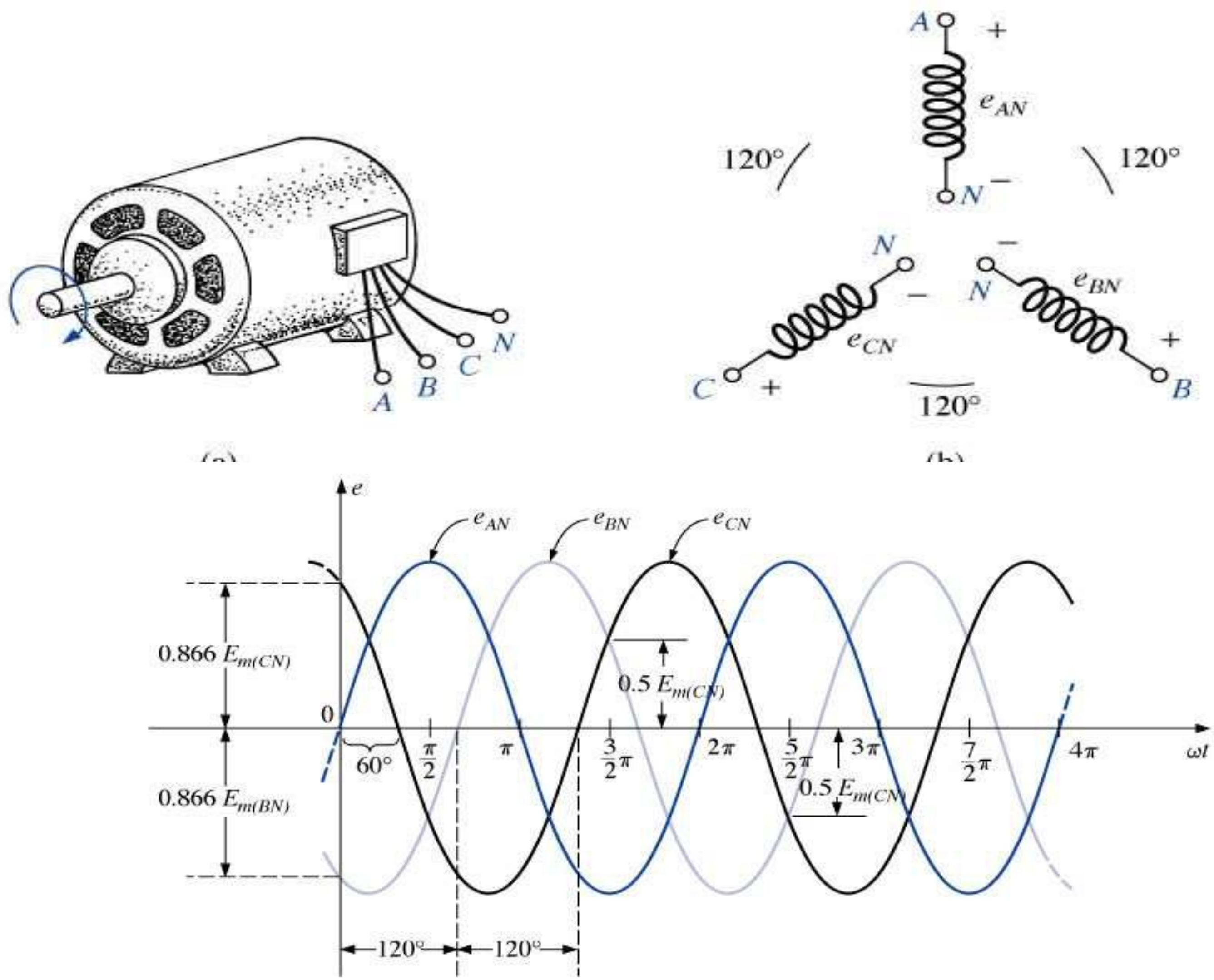
Two-phase three-wire system. The AC sources operate at different phases.



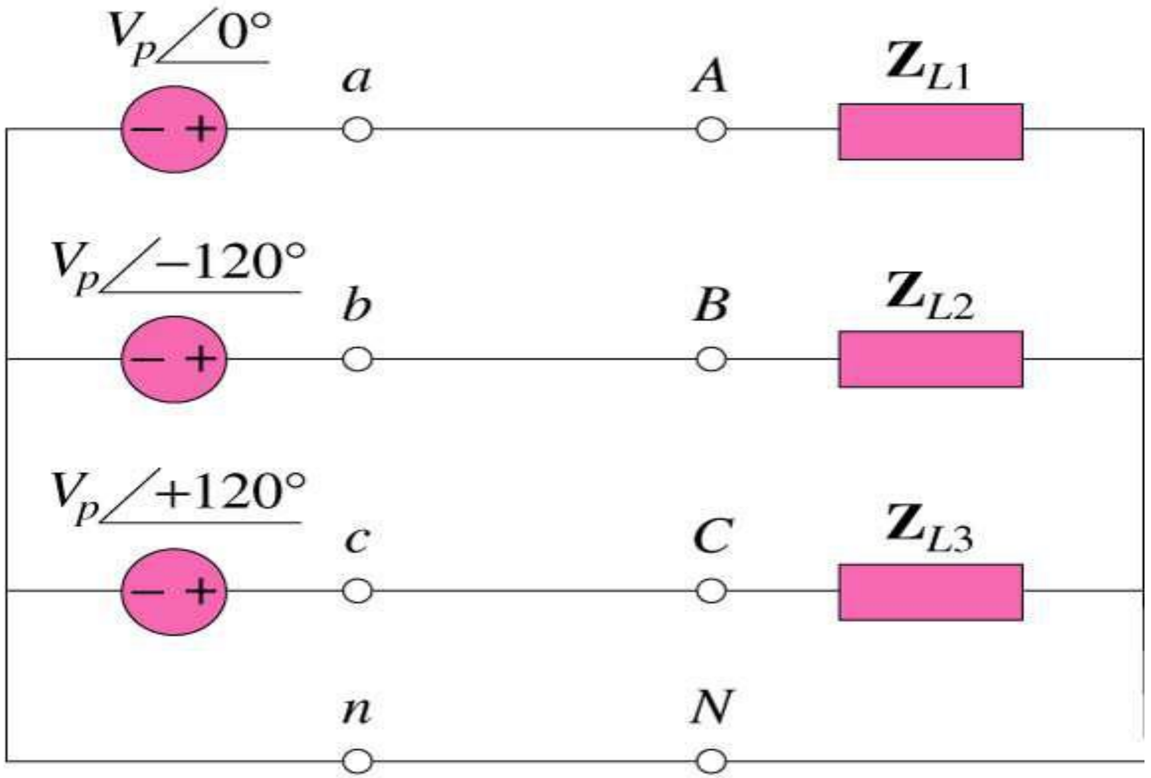
Three-phase Generator

- The three-phase generator has three induction coils placed 120° apart on the stator.
- The three coils have an equal number of turns, the voltage induced across each coil will have the same peak value, shape and frequency.

Three-phase Generator



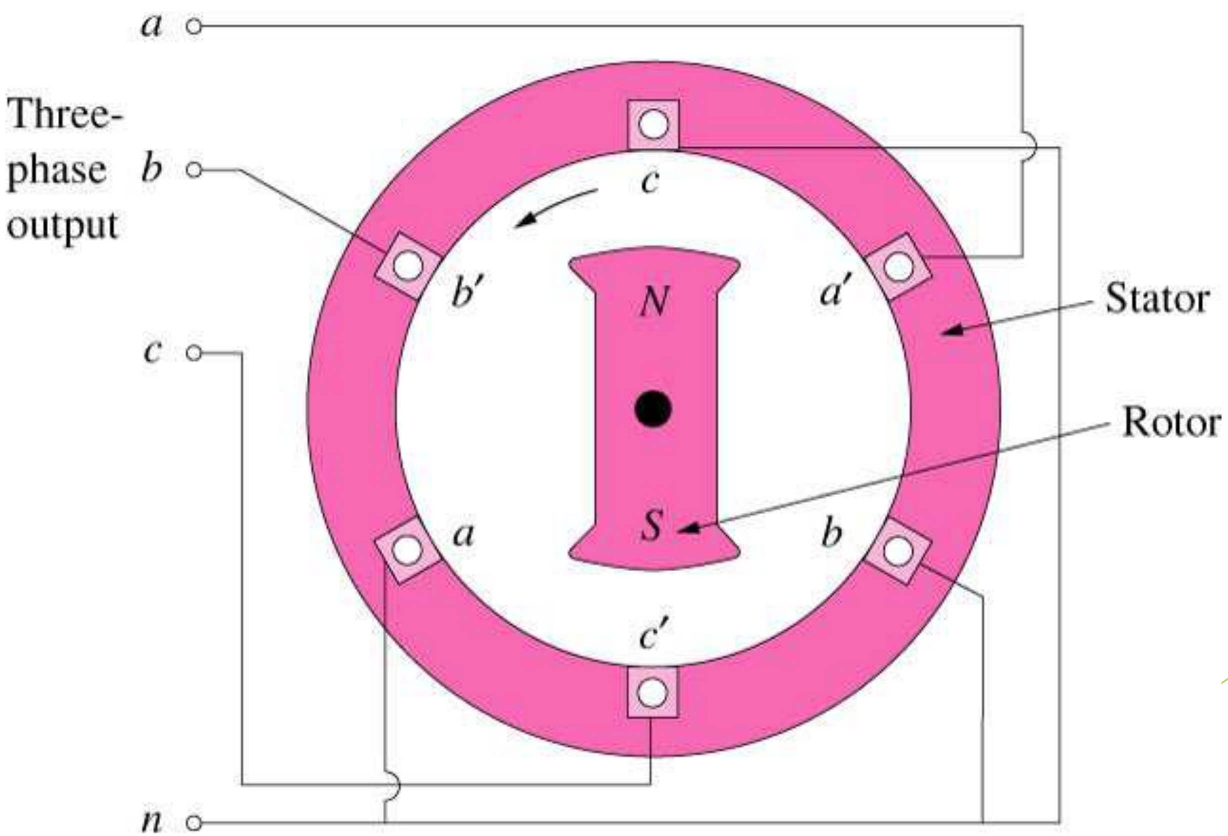
Balanced Three-phase Voltages



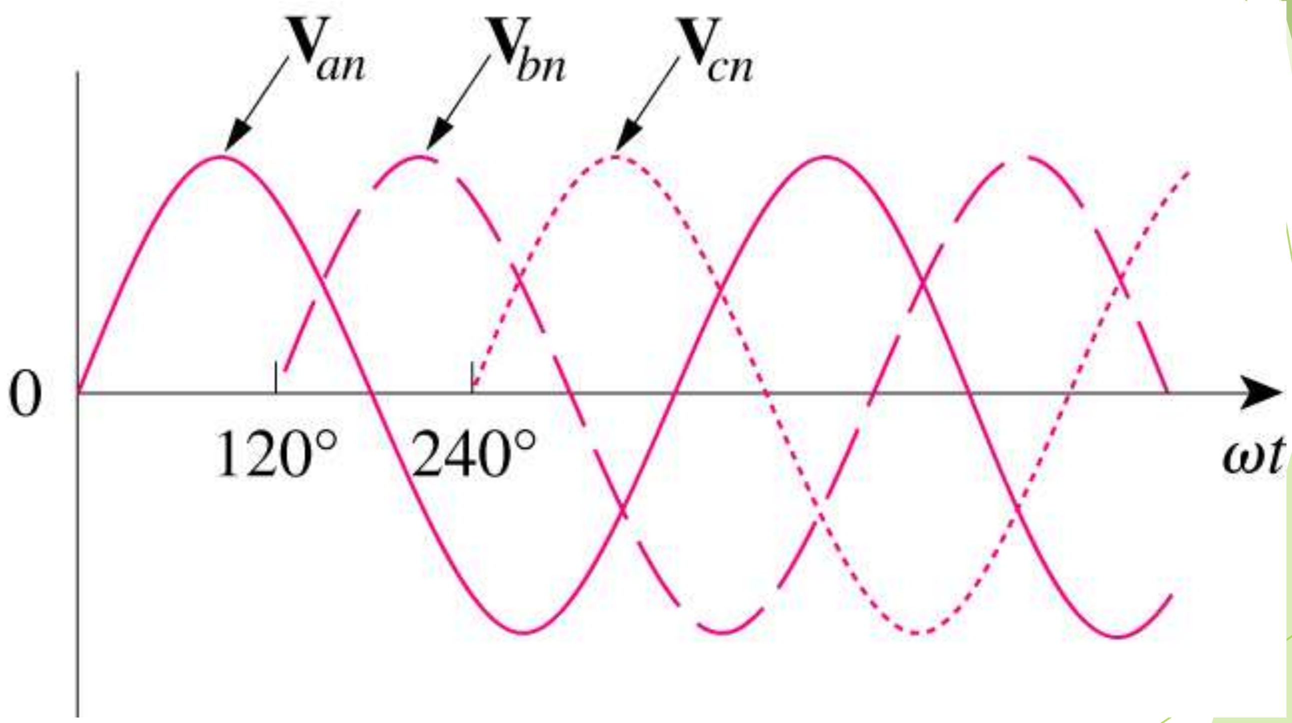
Three-phase four-wire system

Balanced Three-phase Voltages

A Three-phase Generator



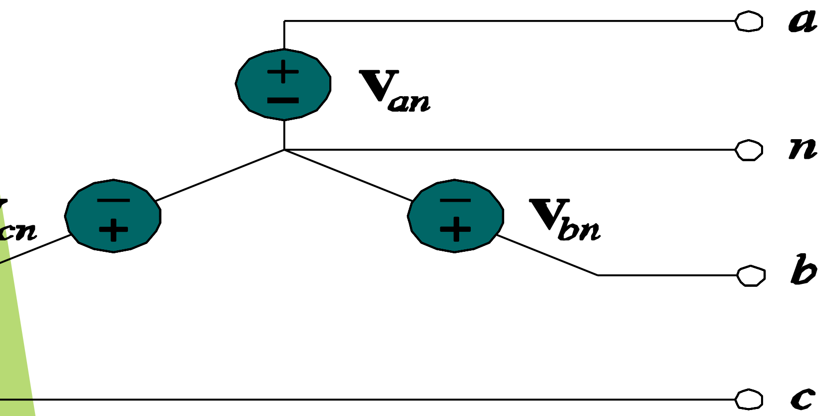
Voltages having 120° phase difference



Balanced Three phase Voltages

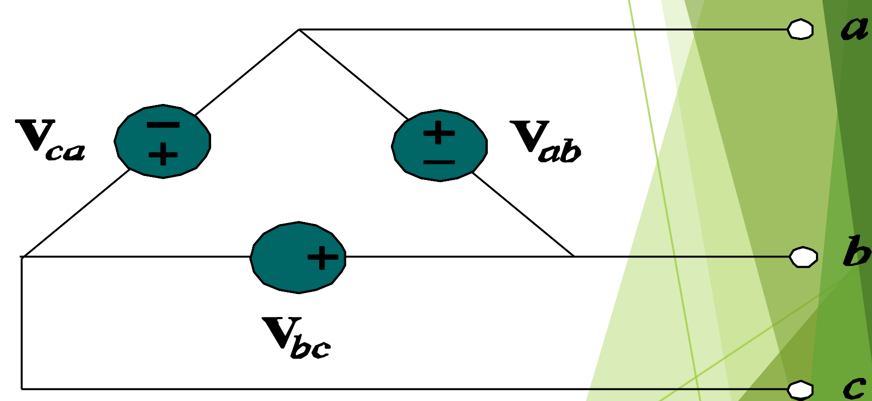
a) Wye Connected Source

b) Delta Connected Source



(a)

$$V_{an} = V_p \angle 0°$$
$$V_{bn} = V_p \angle -120°$$
$$V_{cn} = V_p \angle -240°$$

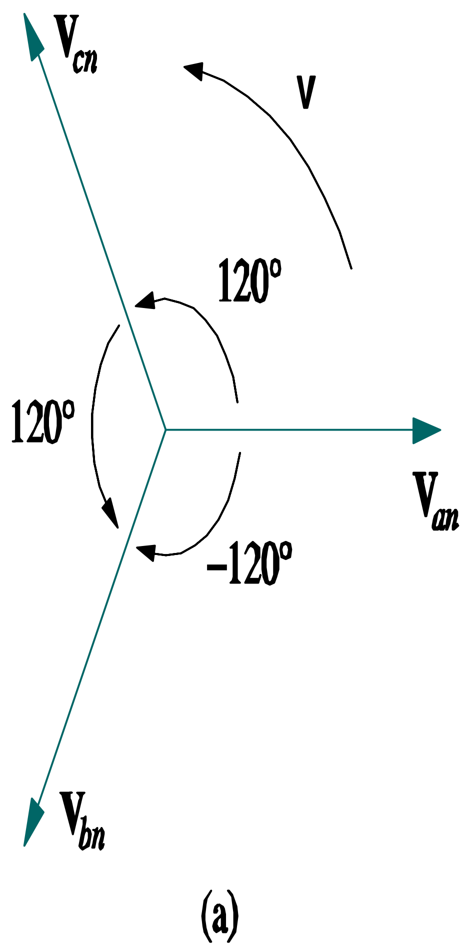


(b)

$$V_{an} = V_p \angle 0°$$
$$V_{bn} = V_p \angle +120°$$
$$V_{cn} = V_p \angle +240°$$

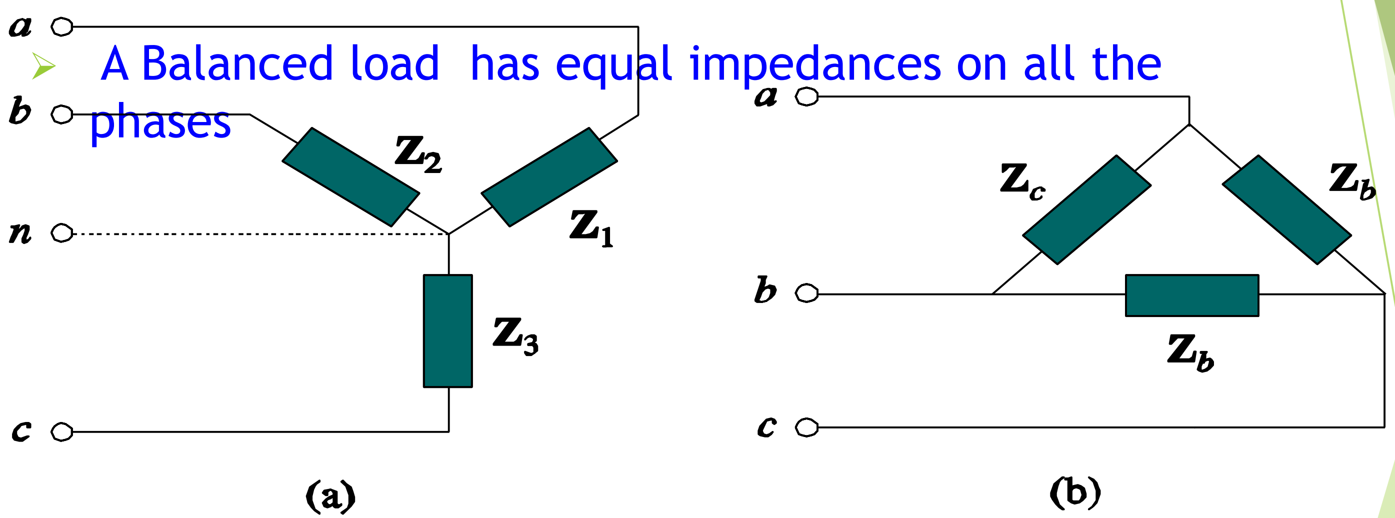
Phase Sequence

- a) abc or positive sequence b) acb or negative sequence



Balanced Three phase Loads

- A Balanced load has equal impedances on all the phases



a) Wye-connected load

b) Delta-connected load

cond

Balanced Impedance Conversion:

Conversion of Delta circuit to Wye or Wye to Delta.

$$Z_Y = Z_1 = Z_2 = Z_3$$

$$Z_\Delta = Z_a = Z_b = Z_c$$

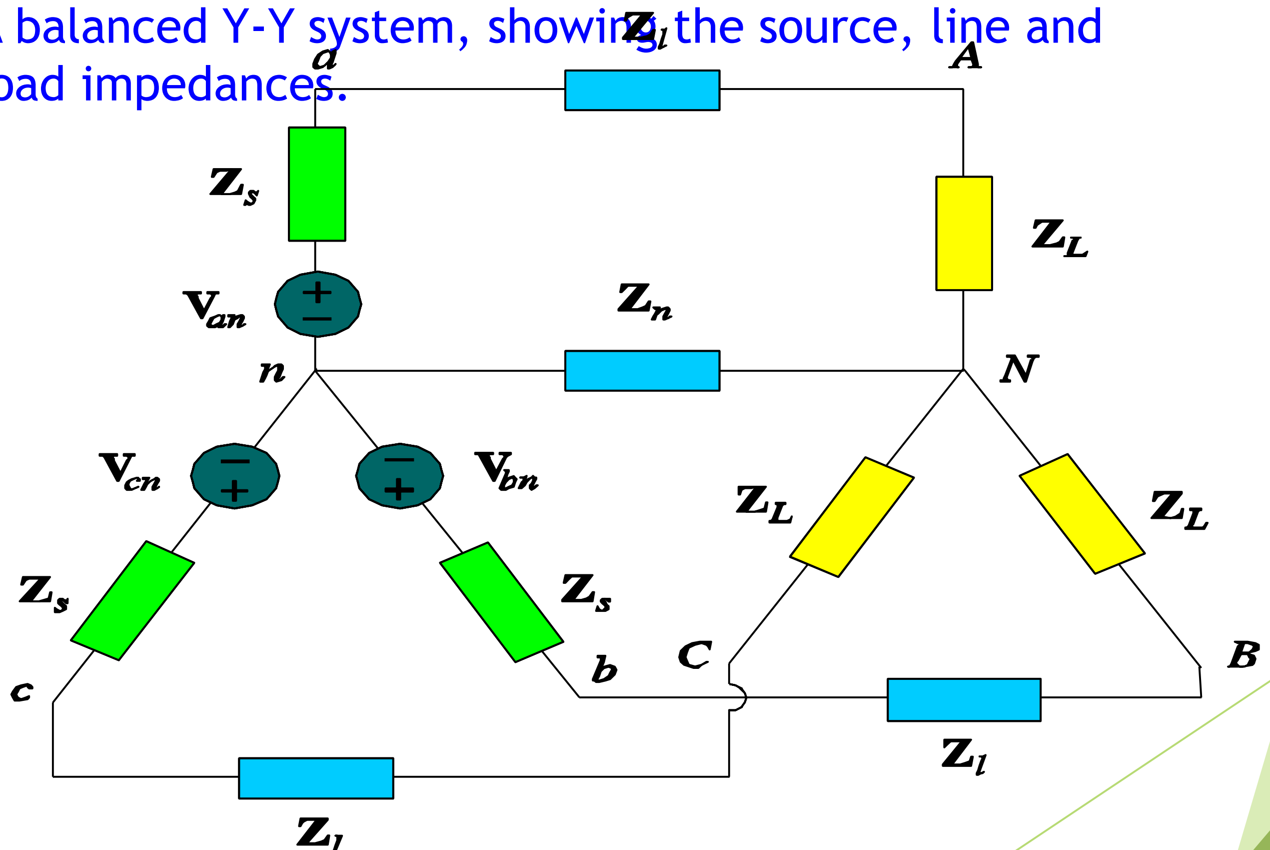
$$\boxed{Z_\Delta = 3Z_Y \qquad Z_Y = \frac{1}{3}Z_\Delta}$$

Three phase Connections

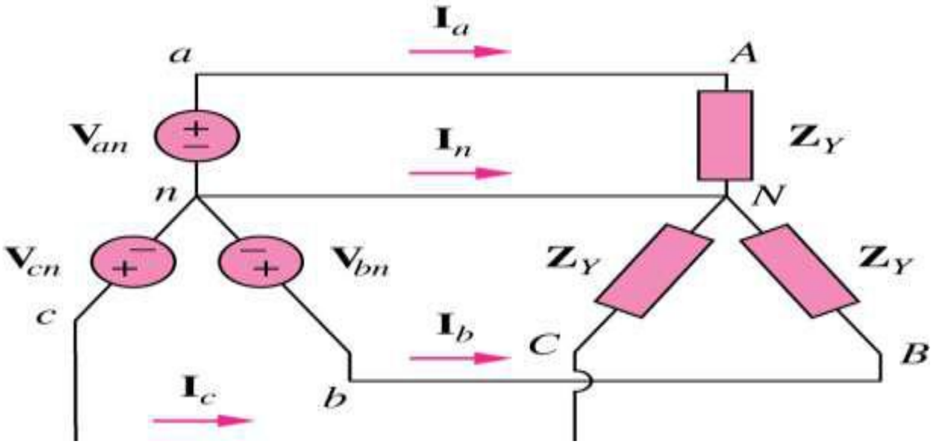
- Both the three phase source and the three phase load can be connected either Wye or DELTA.
- We have 4 possible connection types.
  - Y-Y connection
  - Y-Δ connection
  - Δ-Δ connection
  - Δ-Y connection
- Balanced Δ connected load is more common.
- Y connected sources are more common.

Balanced Wye-wye Connection

- A balanced Y-Y system, showing the source, line and load impedances.



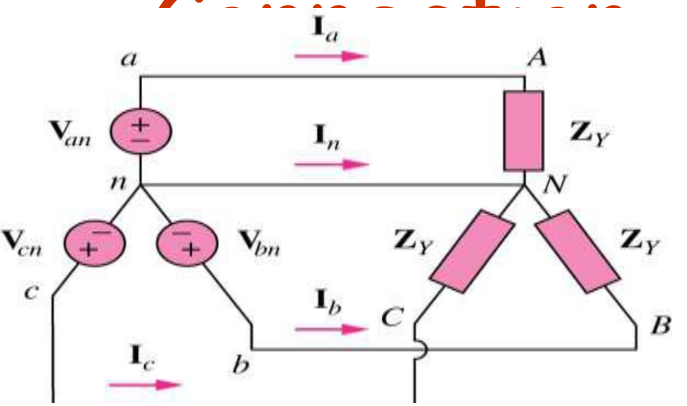
Balanced Wye-wye Connection



Line current In add up to zero.
Neutral current is zero:
$I_n = -(I_a + I_b + I_c) = 0$

Phase voltages are: Van, Vbn and Vcn.
The three conductors connected from a to A, b to B and c to C are called LINES.
The voltage from one line to another is called a **LINE voltage**
Line voltages are: Vab, Vbc and Vca
Magnitude of line voltages is √3 times the magnitude of phase voltages.
$V_L = \sqrt{3}\, V_p$

Balanced Wye-wye Connection



Line current *I*n add up to zero. **Neutral current is zero:**

$I_n = -(I_a + I_b + I_c) = 0$

Magnitude of line voltages is √3 times the magnitude of phase voltages.

$V_L = \sqrt{3}\, V_p$

$$V_{an} = V_p\angle 0°, \quad V_{bn} = V_p\angle -120°, \quad V_{cn} = V_p\angle +120°$$

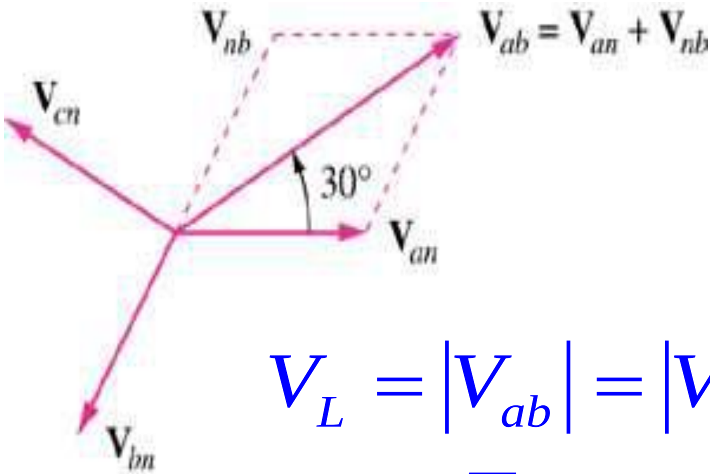
$$V_{ab} = V_{an} + V_{nb} = V_{an} - V_{bn} = \sqrt{3}V_p\angle 30°$$

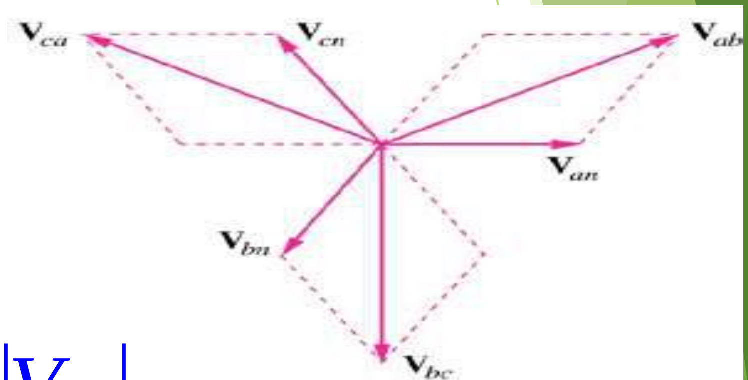
$$V_{bc} = V_{bn} - V_{cn} = \sqrt{3}V_p\angle -90°$$

$$V_{ca} = V_{cn} - V_{an} = V_{an} + V_{bn} = \sqrt{3}V_p\angle -210°$$

Balanced Wye-wye Connection

- Phasor diagram of phase and line voltages





$$
\begin{aligned}
V_L &= |V_{ab}| = |V_{bc}| = |V_{ca}| \\
&= \sqrt{3}\,|V_{an}| = \sqrt{3}\,|V_{bn}| = \sqrt{3}\,|V_{cn}| \\
&= \sqrt{3}V_p
\end{aligned}
$$

- Cond

P.P.12.2 Calculate the line voltages and line currents of a Y-Y connection.

$$Z_{Source} = (0.4 + j0.3), \quad Z_{Line} = (0.6 + j0.7), \quad Z_{Load} = (24 + j19)$$

(a) $\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = 120\angle 30° - 120\angle -90°$

$\mathbf{V}_{ab} = (103.92 + j60) + j120 \qquad \mathbf{V}_{ab} = \underline{\mathbf{207.85\angle 60°\ V}}$

Alternatively, using the fact that $\mathbf{V}_{ab}$ leads $\mathbf{V}_{an}$ by 30° and has a magnitude of $\sqrt{3}$ times that of $\mathbf{V}_{an}$,

$$\mathbf{V}_{ab} = \sqrt{3}(120)\angle(30° + 30°) = 207.85\angle 60°$$

Following the abc sequence,

$\mathbf{V}_{bc} = \underline{\mathbf{207.85\angle -60°\ V}} \qquad \mathbf{V}_{ca} = \underline{\mathbf{207.85\angle -180°\ V}}$

(b) $\mathbf{I}_a = \dfrac{\mathbf{V}_{an}}{\mathbf{Z}}$

$\mathbf{Z} = Z_{Source} + Z_{Line} + Z_{Load} = (0.4 + j0.3) + (24 + j19) + (0.6 + j0.7)$

$\mathbf{Z} = 25 + j20 = 32\angle 38.66°$

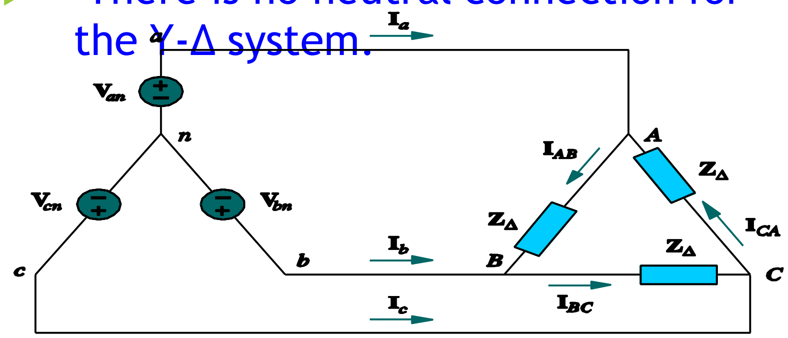
$$\mathbf{I}_a = \frac{120\angle 30°}{32\angle 38.66°} = \underline{\mathbf{3.75\angle -8.66°\ A}}$$

Following the abc sequence, $\mathbf{I}_b = \mathbf{I}_a\angle -120° = \underline{\mathbf{3.75\angle -128.66°\ A}}$

$\mathbf{I}_c = \mathbf{I}_a\angle -240° = \underline{\mathbf{3.75\angle -248.66°\ A}}$

Balanced Wye-delta Connection

- Three phase sources are usually Wye connected and three phase loads are Delta connected.
- There is no neutral connection for the Y-Δ system.



$$I_{AB} = \frac{V_{AB}}{Z_\Delta}$$

$$I_{BC} = \frac{V_{BC}}{Z_\Delta}$$

$$I_{CA} = \frac{V_{CA}}{Z_\Delta}$$

$$I_a = I_{AB} - I_{CA} = I_{AB}\sqrt{3}\angle -30^\circ$$

$$I_b = I_{BC} - I_{AB} = I_{BC}\sqrt{3}\angle -30^\circ$$

$$I_c = I_{CA} - I_{BC} = I_{CA}\sqrt{3}\angle -30^\circ$$

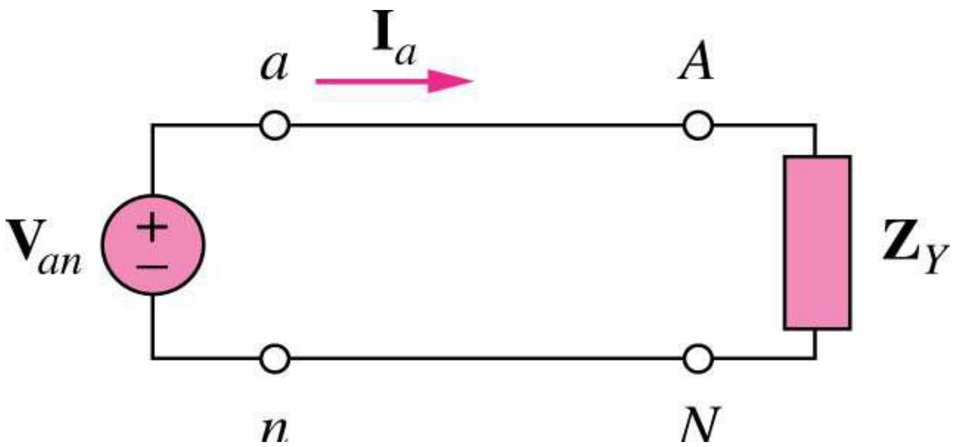
$$I_L = |I_a| = |I_b| = |I_c|$$

$$I_p = |I_{AB}| = |I_{BC}| = |I_{CA}|$$

$$I_L = \sqrt{3} I_p$$

Single Phase Equivalent of Balanced Y-Y Connection

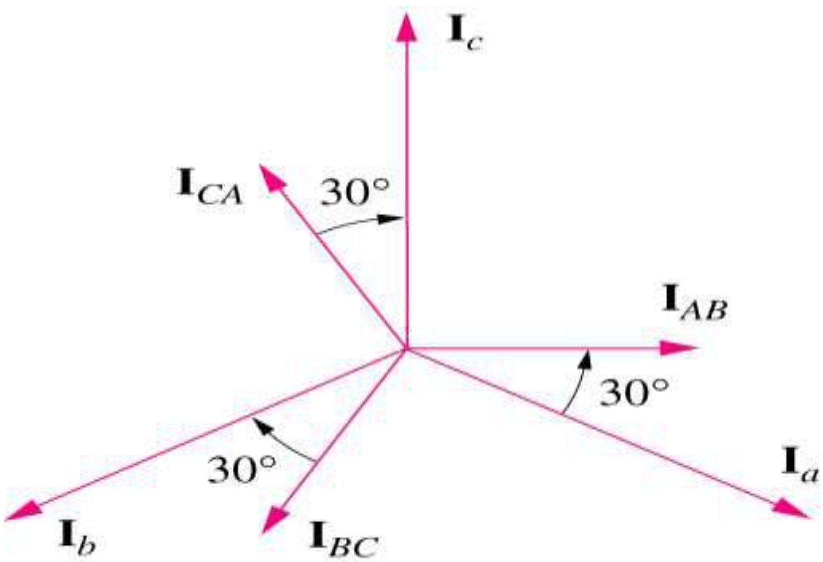
- We look Balanced three phase circuits can be analyzed on “per phase “ basis..
- at one phase, say phase *a* and analyze the single phase equivalent circuit.
- Because the circuiit is balanced, we can easily obtain other phase values using their phase relationships.



$$I_a = \frac{V_{an}}{Z_Y}$$

Balanced Wye-delta Connection

- Phasor diagram of phase and line currents

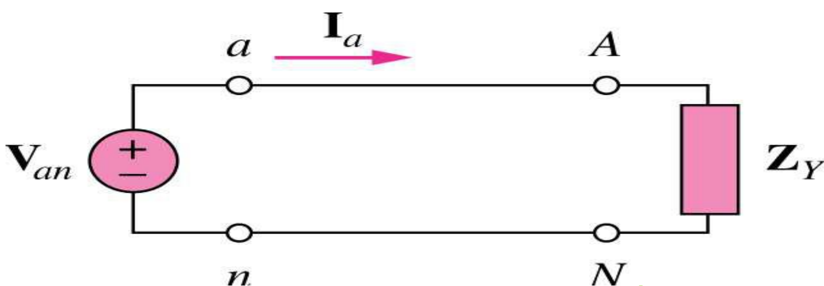


$$I_L = |I_a| = |I_b| = |I_c|$$

$$I_p = |I_{AB}| = |I_{BC}| = |I_{CA}|$$

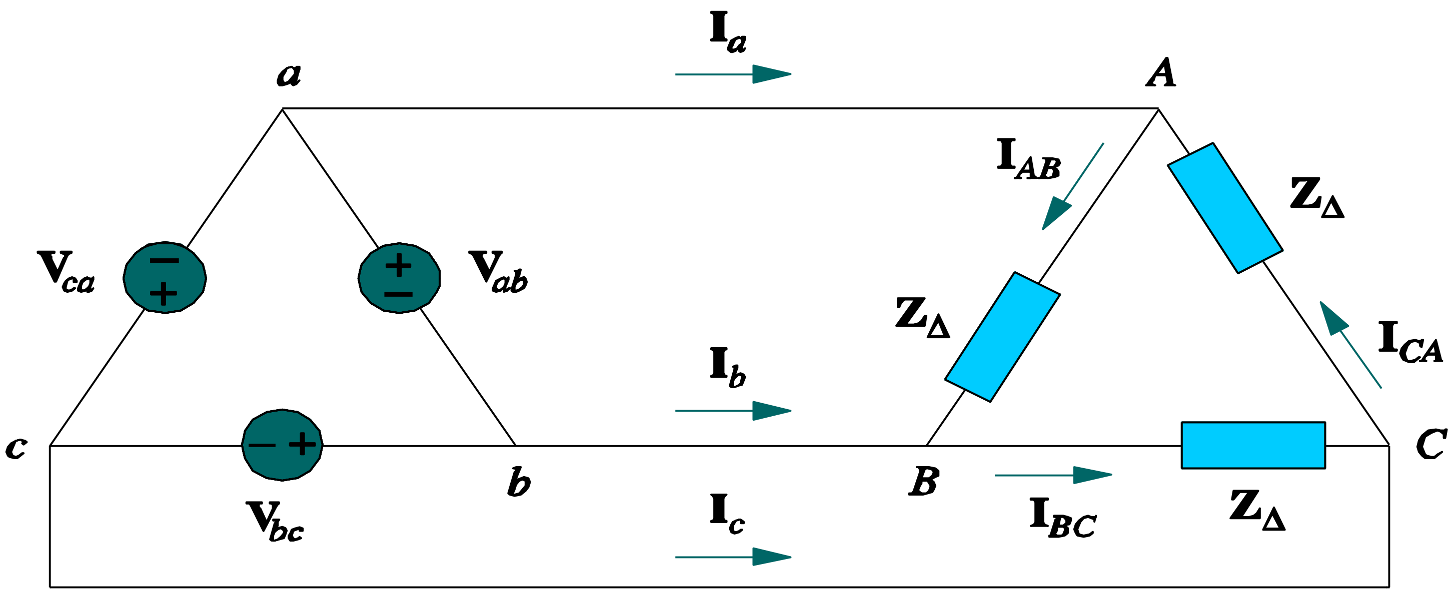
$$I_L = \sqrt{3} I_p$$

Single phase equivalent circuit of the balanced Wye-delta connection



Balanced Delta-delta Connection

- Both the source and load are Delta connected and balanced.



$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}}, \quad I_{BC} = \frac{V_{BC}}{Z_{\Delta}}, \quad I_{CA} = \frac{V_{CA}}{Z_{\Delta}}$$

$$I_a = I_{AB} - I_{CA}, \quad I_b = I_{BC} - I_{AB}, \quad I_c = I_{CA} - I_{BC}$$

Balanced Delta-wye Connection

- Transforming a Delta connected source to an equivalent Wye connection