UNIT I

MECHANISMS, MACHINE AND STRUCTURE

Mechanism: If a number of bodies are assembled in such a way that the motion of one causes constrained and predictable motion to the others, it is known as a mechanism.

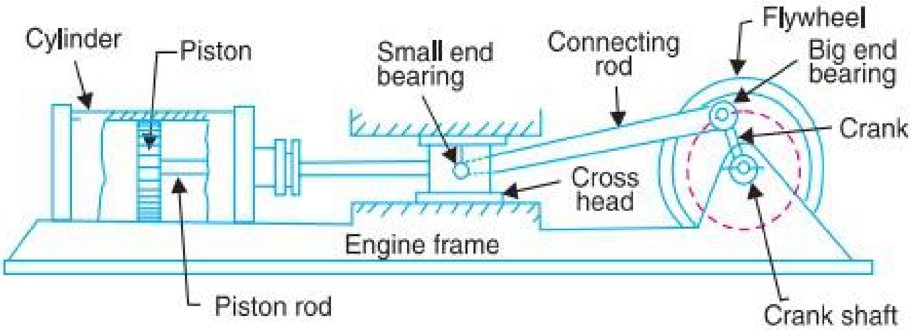
Machine:A machine is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of desired work.

Analysis: Analysis is the study of motions and forces concerning different parts of an existing mechanism.

Synthesis: Synthesis involves the design of its different parts.

Kinematic Link or Element:

Each part of a machine, which moves relative to some other part, is known as a kinematic link (or simply link) or element. A link may consist of several parts, which are rigidly fastened together, so that they do not move relative to one another. For example, in a reciprocating steam engine, as shown in below figure, piston, piston rod and crosshead constitute one link ; connecting rod with big and small end bearings constitute a second link ; crank, crank shaft and flywheel a third link and the cylinder, engine frame and main bearings a fourth link.



Reciprocating steam engine.

A link or element need not to be a rigid body, but it must be a resistant body. A body is said to be a resistant body if it is capable of transmitting the required forces with negligible deformation. Thus a link should have the following two characteristics:

1. It should have relative motion, and
2. It must be a resistant body.

Types of Links:

In order to transmit motion, the driver and the follower may be connected by the following three types of links:

1. Rigid link. A rigid link is one which does not undergo any deformation while transmitting motion. Strictly speaking, rigid links do not exist. However, as the deformation of a connecting rod, crank etc. of a reciprocating steam engine is not appreciable, they can be considered as rigid links.

2. Flexible link. A flexible link is one which is partly deformed in a manner not to affect the transmission of motion. For example, belts, ropes, chains and wires are flexible links and transmit tensile forces only.

3. Fluid link. A fluid link is one which is formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only, as in the case of hydraulic presses, jacks and brakes.

Structure:

It is an assemblage of a number of resistant bodies (known as members) having no relative motion between them and meant for carrying loads having straining action. A railway bridge, a roof truss, machine frames etc., are the examples of a structure.

Difference between a Machine and a Structure:

The following differences between a machine and a structure are important from the subject point of view:

1. The parts of a machine move relative to one another, whereas the members of a structure do not move relative to one another.
2. A machine transforms the available energy into some useful work, whereas in a structure no energy is transformed into useful work.
3. The links of a machine may transmit both power and motion, while the members of a structure transmit forces only.

Kinematic Pair: The two links or elements of a machine, when in contact with each other, are said to form a pair. If the relative motion between them is completely or successfully constrained (i.e. in a definite direction), the pair is known as kinematic pair.

Classification of Kinematic Pairs:

1. **According to the type of relative motion between the elements.** The kinematic pairs according to type of relative motion between the elements may be classified as discussed below:

(a) Sliding pair. When the two elements of a pair are connected in such a way that one can only slide relative to the other, the pair is known as a sliding pair. The piston and cylinder, cross-head and guides of a reciprocating steam engine, ram and its guides in shaper, tail stock on the lathe bed etc. are the examples of a sliding pair. A little consideration will show, that a sliding pair has a completely constrained motion.

(b) Turning pair. When the two elements of a pair are connected in such a way that one can only turn or revolve about a fixed axis of another link, the pair is known as turning pair. A shaft with collars at both ends fitted into a circular hole, the crankshaft in a journal bearing in an engine, lathe spindle supported in head stock, cycle wheels turning over their axles etc. are the examples of a turning pair. A turning pair also has a completely constrained motion.

(c) Rolling pair. When the two elements of a pair are connected in such a way that one rolls over another fixed link, the pair is known as rolling pair. Ball and roller bearings are examples of rolling pair.

(d) Screw pair. When the two elements of a pair are connected in such a way that one element can turn about the other by screw threads, the pair is known as screw pair. The lead screw of a lathe with nut, and bolt with a nut are examples of a screw pair.

(e) Spherical pair. When the two elements of a pair are connected in such a way that one element (with spherical shape) turns or swivels about the other fixed element, the pair formed is called a spherical pair. The ball and socket joint, attachment of a car mirror, pen stand etc., are the examples of a spherical pair.

2. According to the type of contact between the elements. The kinematic pairs according to the type of contact between the elements may be classified as discussed below:

(a) Lower pair. When the two elements of a pair have a surface contact when relative motion takes place and the surface of one element slides over the surface of the other, the pair formed is known as lower pair. It will be seen that sliding pairs, turning pairs and screw pairs form lower pairs.

(b) Higher pair. When the two elements of a pair have a line or point contact when relative motion takes place and the motion between the two elements is partly turning and partly sliding,then the pair is known as higher pair. A pair of friction discs, toothed gearing, belt and rope drives, ball and roller bearings and cam and follower are the examples of higher pairs.

3. According to the type of closure. The kinematic pairs according to the type of closure between the elements may be classified as discussed below:

(a) Self closed pair. When the two elements of a pair are connected together mechanically in such a way that only required kind of relative motion occurs, it is then known as self closed pair. The lower pairs are self closed pair.

(b) Force - closed pair. When the two elements of a pair are not connected mechanically but are kept in contact by the action of external forces, the pair is said to be a force-closed pair. The cam and follower is an example of force closed pair, as it is kept in contact by the forces exerted by spring and gravity.

Types of Constrained Motions:

1. Completely constrained motion. When the motion between a pair is limited to a definite direction irrespective of the direction of force applied, then the motion is said to be a completely constrained motion. For example, the piston and cylinder (in a steam engine) form a pair and the motion of the piston is limited to a definite direction (i.e. it will only reciprocate) relative to the cylinder irrespective of the direction of motion of the crank, as shown in above reciprocating steam engine figure.

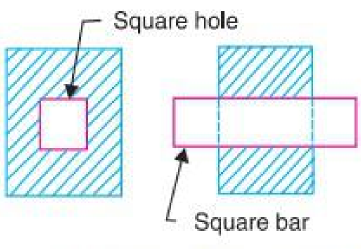


Fig 2.Square bar in a square hole.

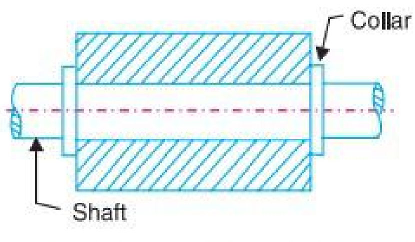


Fig 3.Shaft with collars in a circular hole.

The motion of a square bar in a square hole, as shown in Fig. 2, and the motion of a shaft with collars at each end in a circular hole, as shown in Fig. 3, are also examples of completely constrained motion.

2. Incompletely constrained motion. When the motion between a pair can take place in more than one direction, then the motion is called an incompletely constrained motion. The change in the direction of impressed force may alter the direction of relative motion between the pair. A circular bar or shaft in a circular hole, as shown in Fig. 4, is an example of an incompletely constrained motion as it may either rotate or slide in a hole. These both motions have no relationship with the other.

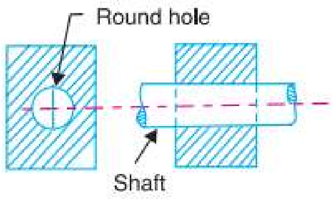


Fig 4.Shaft in a circular hole.

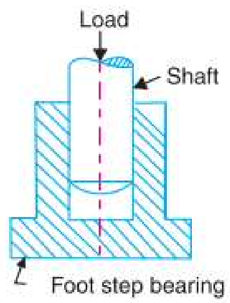


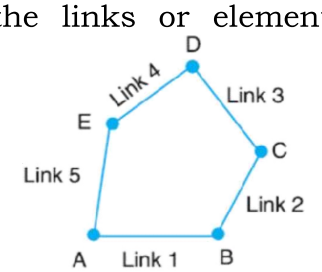
Fig 5. Shaft in a foot step bearing.

3. Successfully constrained motion. When the motion between the elements, forming a pair,is such that the constrained motion is not completed by itself, but by some other means, then the motion is said to be successfully constrained motion. Consider a shaft in a foot-step bearing as shown in Fig. 5. The shaft may rotate in a bearing or it may move upwards. This is a case of incompletely constrained motion. But if the load is placed on the shaft to prevent axial upward movement of the shaft, then the motion of the pair is said to be successfully constrained motion. The motion of an I.C. engine valve (these are kept on their seat by a spring) and the piston reciprocating inside an engine cylinder are also the examples of successfully constrained motion.

Kinematic Chain:

When the kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion (i.e. completely or successfully constrained motion), it is called a kinematic chain.

In other words, a kinematic chain may be defined as a combination of kinematic pairs, joined in such a way that each link forms a part of two pairs and the relative motion between the links or elements is completely or successfully constrained.



If each link is assumed to form two pairs with two adjacent links, then the relation between the number of pairs (p) forming a kinematic chain and the number of links (l) may be expressed in the form of an equation:

$$l = 2p - 4 \qquad \ldots (i)$$

Since in a kinematic chain each link forms a part of two pairs, therefore there will be as many links as the number of pairs.

Another relation between the number of links (l) and the number of joints (j) which constitute a kinematic chain is given by the expression:

$$j = \frac{3}{2} l - 2 \qquad \ldots (ii)$$

The equations (i) and (ii) are applicable only to kinematic chains, in which lower pairs are used.

Let us apply the above equations to the following cases to determine whether each of them is a kinematic chain or not.

1. Consider the arrangement of three links A B, BC and CA with pin joints at A, B and C as shown in below Fig. In this case,

1. Consider the arrangement of three links AB, BC and CA with pin joints at A, B and C as shown in Fig. 5.6. In this case,

Number of links, $l = 3$

Number of pairs, $p = 3$

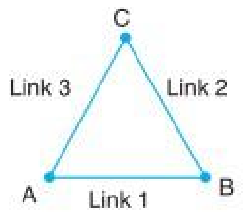
and number of joints, $j = 3$

From equation (*i*), $l = 2p - 4$

or $3 = 2 \times 3 - 4 = 2$

i.e. L.H.S. > R.H.S.

Now from equation (*ii*),



Arrangement of three links.

$$j = \frac{3}{2} l - 2 \quad \text{or} \quad 3 = \frac{3}{2} \times 3 - 2 = 2.5$$

i.e. L.H.S. > R.H.S.

Since the arrangement of three links, as shown in Fig. 5.6, does not satisfy the equations (i) and (ii) and the left hand side is greater than the right hand side, therefore it is not a kinematic chain and hence no relative motion is possible. Such type of chain is called locked chain and forms a rigid frame or structure which is used in bridges and trusses.

2. Consider the arrangement of four links AB, BC, CD and DA as shown in below Fig. In this case,

$$l = 4,\ p = 4, \text{ and } j = 4$$

From equation (*i*), $l = 2p - 4$

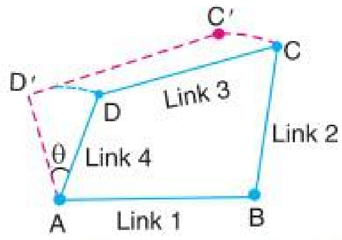
$$4 = 2 \times 4 - 4 = 4$$

i.e. L.H.S. = R.H.S.

From equation (*ii*), $j = \frac{3}{2}l - 2$

$$4 = \frac{3}{2} \times 4 - 2 = 4$$

i.e. L.H.S. = R.H.S.



Arrangement of four links.

Since the arrangement of four links, as shown in above Fig., satisfy the equations (i) and (ii), therefore it is a **kinematic chain of one degree of freedom.**

A chain in which a single link such as AD in above Fig. is sufficient to define the position of all other links, it is then called a kinematic chain of one degree of freedom.

A little consideration will show that in above Fig., if a definite displacement (say 6) is given to the link AD, keeping the link AB fixed, then the resulting displacements of the remaining two links BC and CD are also perfectly definite. Thus we see that in a four bar chain, the relative motion is completely constrained. Hence it may be called as a **constrained kinematic chain**, and it is the basis of all machines.

3. Consider an arrangement of five links, as shown in below Fig. In this case,

$$l = 5,\ p = 5, \text{ and } j = 5$$

From equation (*i*),

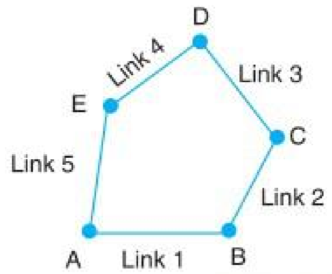
$$l = 2p - 4 \quad \text{or} \quad 5 = 2 \times 5 - 4 = 6$$

i.e. L.H.S. < R.H.S.

From equation (*ii*),

$$j = \frac{3}{2}l - 2 \quad \text{or} \quad 5 = \frac{3}{2} \times 5 - 2 = 5.5$$

i.e. L.H.S. < R.H.S.



Arrangement of five links.

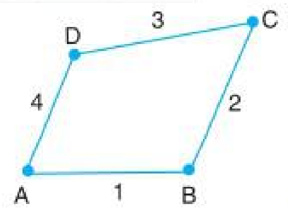
Since the arrangement of five links, as shown in above Fig. does not satisfy the equations and left hand side is less than right hand side, therefore it is not a kinematic chain. Such a type of chain is called **unconstrained chain** i.e. the relative motion is not completely constrained. This type of chain is of little practical importance.

Types of Joints in a Chain:

The following types of joints are usually found in a chain:

Binary joint:

When two links are joined at the same connection, the joint is known as binary joint. For example, a chain as shown below figure, has four links and four binary joins at A, B, C and D. In order to determine the nature of chain, i.e. whether the chain is a locked chain (or structure) or kinematic chain or unconstrained chain, the following relation between the number of links and the number of binary joints, as given by A.W. Klein, may be used:



$$j + \frac{h}{2} = \frac{3}{2} l - 2$$

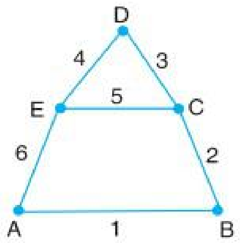
Where,

j = Number of binary joints,

h = Number of higher pairs, and

l = Number of links.

When h = 0, the above equation, may be written as j= (3/2) l – 2

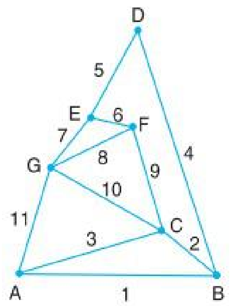


Ternary joint:

When three links are joined at the same connection, the joint is known as ternary joint. It is equivalent to two binary joints as one of the three links joined carry the pin for the other two links.

Quaternary joint:

When four links are joined at the same connection, the joint is called a quaternary joint. It is equivalent to three binary joints. In general, when l number of links are joined at the same connection, the joint is equivalent to (l – 1) binary joints.



Mechanism:

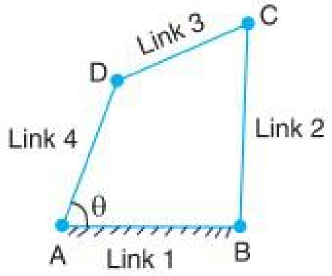
When one of the links of a kinematic chain is fixed, the chain is known as mechanism. It may be used for transmitting or transforming motion

A mechanism with four links is known as simple mechanism, and the mechanism with more than four links is known as compound mechanism. When a mechanism is required to transmit power or to do some particular type of work, it then becomes a machine. In such cases, the various links or elements have to be designed to withstand the forces (both static and kinetic) safely.

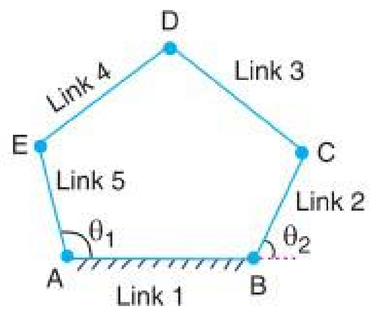
A little consideration will show that a mechanism may be regarded as a machine in which each part is reduced to the simplest form to transmit the required motion.

Number of Degrees of Freedom for Plane Mechanisms:

In the design or analysis of a mechanism, one of the most important concern is the number of degrees of freedom (also called movability) of the mechanism. It is defined as the number of input parameters (usually pair variables) which must be independently controlled in order to bring the mechanism into a useful engineering purpose. It is possible to determine the number of degrees of freedom of a mechanism directly from the number of links and the number and types of joints which it includes.

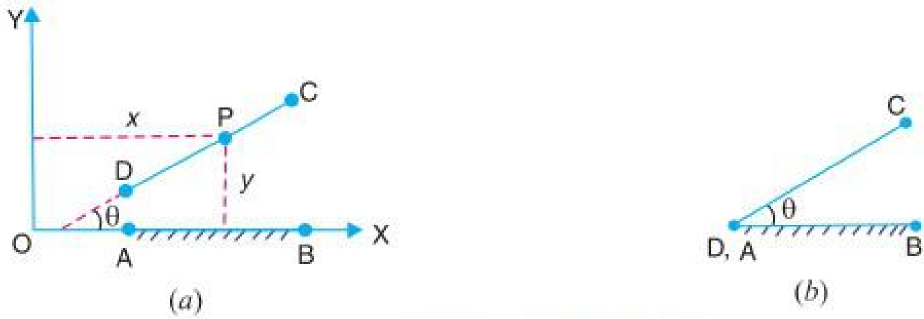


(*a*) Four bar chain.



(*b*) Five bar chain.

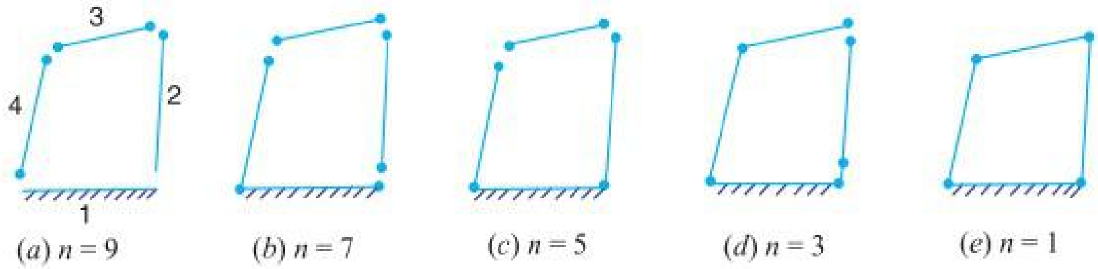
Consider a four bar chain, as shown in Fig.(a). A little consideration will show that only one variable such as θ is needed to define the relative positions of all the links. In other words, we say that the number of degrees of freedom of a four bar chain is one. Now, let us consider a five bar chain, as shown in Fig.(b). In this case two variables such as θ_1 and θ_2 are needed to define completely the relative positions of all the links. Thus, we say that the number of degrees of freedom is * two. In order to develop the relationship in general, consider two links AB and CD in a plane motion as shown in below Fig. (a).



Links in a plane motion.

The link AB with co-ordinate system OXY is taken as the reference link (or fixed link). The position of point P on the moving link CD can be completely specified by the three variables, i.e. the co-ordinates of the point P denoted by x and y and the inclination θ of the link CD with X-axis or link A B. In other words, we can say that each link of a mechanism has three degrees of freedom before it is connected to any other link. But when the link CD is connected to the link A B by a turning pair at A, as shown in above Fig.(b), the position of link CD is now determined by a single variable θ and thus has one degree of freedom.

From above, we see that when a link is connected to a fixed link by a turning pair (i.e. lower pair), two degrees of freedom are destroyed. This may be clearly understood from below Fig., in which the resulting four bar mechanism has one degree of freedom (i.e. n = 1).



Four bar mechanism.

Now let us consider a plane mechanism with l number of links. Since in a mechanism, one of the links is to be fixed, therefore the number of movable links will be (l – 1) and thus the total number of degrees of freedom will be 3 (l – 1) before they are connected to any other link. In general, a mechanism with l number of links connected by j number of binary joints or lower pairs (i.e. single degree of freedom pairs) and h number of higher pairs (i.e. two degree of freedom pairs), then the number of degrees of freedom of a mechanism is given by

$$n = 3\,(l - 1) - 2\,j - h \qquad \ldots (i)$$

This equation is called Kutzbach criterion for the movability of a mechanism having plane motion.

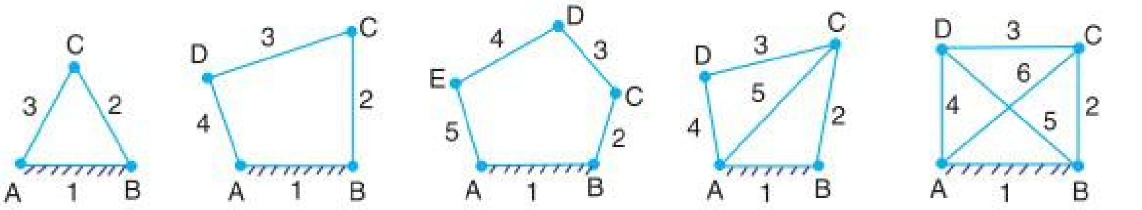
If there are no two degree of freedom pairs (i.e. higher pairs), then h = 0. Substituting h = 0 in equation (i), we have

$$n = 3\,(l - 1) - 2\,j \qquad \text{... (ii)}$$

Application of Kutzbach Criterion to Plane Mechanisms:

We have discussed in the previous article that Kutzbach criterion for determining the number of degrees of freedom or movability (n) of a plane mechanism is

$$n = 3\,(l - 1) - 2\,j - h$$



(*a*) Three-bar mechanism. (*b*) Four bar mechanism. (*c*) Five bar mechanism. (*d*) Five bar mechanism. (*e*) Six bar mechanism.

Plane mechanisms.

The number of degrees of freedom or movability (n) for some simple mechanisms having no higher pair (i.e. h = 0), as shown in above Fig., are determined as follows:

1. The mechanism, as shown in Fig. (*a*), has three links and three binary joints, *i.e.* $l = 3$ and $j = 3$.

$$\therefore \qquad n = 3\,(3 - 1) - 2 \times 3 = 0$$

2. The mechanism, as shown in Fig. (*b*), has four links and four binary joints, *i.e.* $l = 4$ and $j = 4$.

$$\therefore \qquad n = 3\,(4 - 1) - 2 \times 4 = 1$$

3. The mechanism, as shown in Fig. (*c*), has five links and five binary joints, *i.e.* $l = 5$, and $j = 5$.

$$\therefore \qquad n = 3\,(5 - 1) - 2 \times 5 = 2$$

4. The mechanism, as shown in Fig. (*d*), has five links and six equivalent binary joints (because there are two binary joints at *B* and *D*, and two ternary joints at *A* and *C*), *i.e.* $l = 5$ and $j = 6$.

$$\therefore \qquad n = 3\,(5 - 1) - 2 \times 6 = 0$$

5. The mechanism, as shown in Fig. (*e*), has six links and eight equivalent binary joints (because there are four ternary joints at *A*, *B*, *C* and *D*), *i.e.* $l = 6$ and $j = 8$.

$$\therefore \qquad n = 3\,(6 - 1) - 2 \times 8 = -1$$

It may be noted that

(a) When n = 0, then the mechanism forms a structure and no relative motion between the links is possible, as shown in Fig. (a) and (d).

(b) When n = 1, then the mechanism can be driven by a single input motion, as shown in Fig. (b).

(c) When n = 2, then two separate input motions are necessary to produce constrained motion for the mechanism, as shown in Fig. (c).

(d) When n = – 1 or less, then there are redundant constraints in the chain and it forms a statically indeterminate structure, as shown in Fig. (e).

The application of Kutzbach's criterion applied to mechanisms with a higher pair or two degree of freedom joints is shown in below Fig.

(a) (b)

Mechanism with a higher pair.

In Fig. (*a*), there are three links, two binary joints and one higher pair, *i.e.* $l = 3, j = 2$ and $h = 1$.

$$\therefore \qquad n = 3\,(3 - 1) - 2 \times 2 - 1 = 1$$

In Fig. (*b*), there are four links, three binary joints and one higher pair, *i.e.* $l = 4$, $j = 3$ and $h = 1$

$$\therefore \qquad n = 3\,(4 - 1) - 2 \times 3 - 1 = 2$$

Here it has been assumed that the slipping is possible between the links (*i.e.* between the wheel and the fixed link). However if the friction at the contact is high enough to prevent slipping, the joint will be counted as one degree of freedom pair, because only one relative motion will be possible between the links.

Grubler's Criterion for Plane Mechanisms:

The Grubler's criterion applies to mechanisms with only single degree of freedom joints where the overall movability of the mechanism is unity. Substituting n = 1 and h = 0 in Kutzbach equation, we have

$$1 = 3\,(l - 1) - 2\,j \qquad \text{or} \qquad 3l - 2j - 4 = 0$$

This equation is known as the Grubler's criterion for plane mechanisms with constrained motion.

A little consideration will show that a plane mechanism with a movability of 1 and only single degree of freedom joints cannot have odd number of links. The simplest possible mechanisms of this type are a four bar mechanism and a slider-crank mechanism in which l = 4 and j = 4.

Inversion of Mechanism:

When one of links is fixed in a kinematic chain, it is called a mechanism. So we can obtain as many mechanisms as the number of links in a kinematic chain by fixing, in turn, different links in a kinematic chain. This method of obtaining different mechanisms by fixing different links in a kinematic chain, is known as **inversion of the mechanism.**

It may be noted that the relative motions between the various links is not changed in any manner through the process of inversion, but their absolute motions (those measured with respect to the fixed link) may be changed drastically.

Types of Kinematic Chains:

The most important kinematic chains are those which consist of four lower pairs, each pair being a sliding pair or a turning pair. The following three types of kinematic chains with four lower pairs are important from the subject point of view:

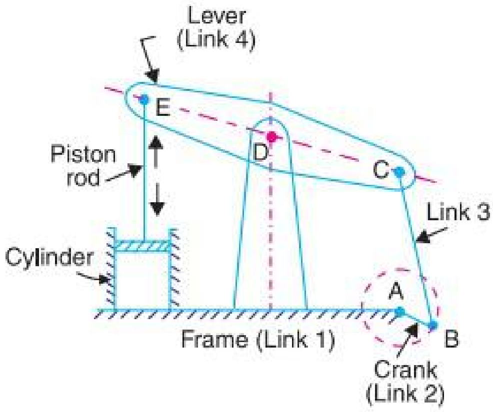
1. Four bar chain or quadric cyclic chain,
2. Single slider crank chain, and
3. Double slider crank chain

Inversions of Four Bar Chain:

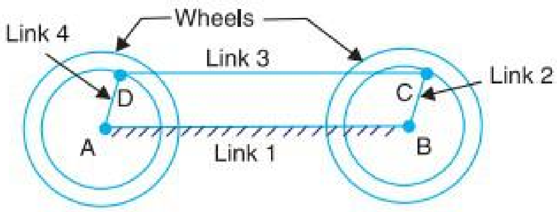
Though there are many inversions of the four bar chain, yet the following are important from the subject point of view:

1. Beam engine (crank and lever mechanism).

A part of the mechanism of a beam engine (also known as crank and lever mechanism) which consists of four links, is shown in below Fig. In this mechanism, when the crank rotates about the fixed centre A, the lever oscillates about a fixed centre D. The end E of the lever CDE is connected to a piston rod which reciprocates due to the rotation of the crank. In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion.



Beam engine.



Coupling rod of a locomotive.

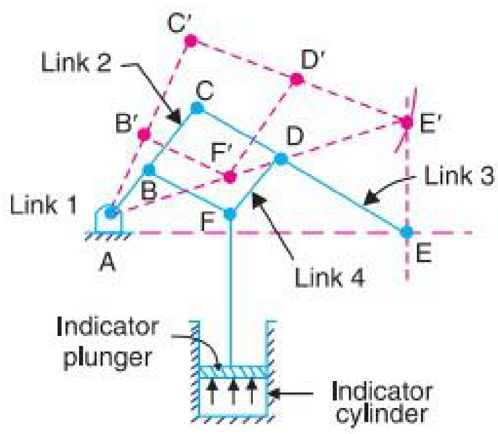
2. Coupling rod of a locomotive (Double crank mechanism).

The mechanism of a coupling rod of a locomotive (also known as double crank mechanism) which consists of four links, is shown in above Fig.

In this mechanism, the links AD and BC (having equal length) act as cranks and are connected to the respective wheels. The link CD acts as a coupling rod and the link A B is fixed in order to maintain a constant centre to centre distance between them. This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.

3. Watt's indicator mechanism (Double lever mechanism). A *Watt's indicator mechanism (also known as Watt's straight line mechanism or double lever mechanism) which consists of four links, is shown in below Fig. The four links are : fixed link at A, link AC, link CE and link BFD. It may be noted that BF and FD form one link because these two parts have no relative motion between them. The links CE and BFD act as levers. The displacement of the link BFD is directly proportional to the pressure of gas or steam which acts on the indicator plunger. On any small displacement of the mechanism, the tracing point E at the end of the link CE traces out approximately a straight line.

The initial position of the mechanism is shown in Fig. by full lines whereas the dotted lines show the position of the mechanism when the gas or steam pressure acts on the indicator plunger.

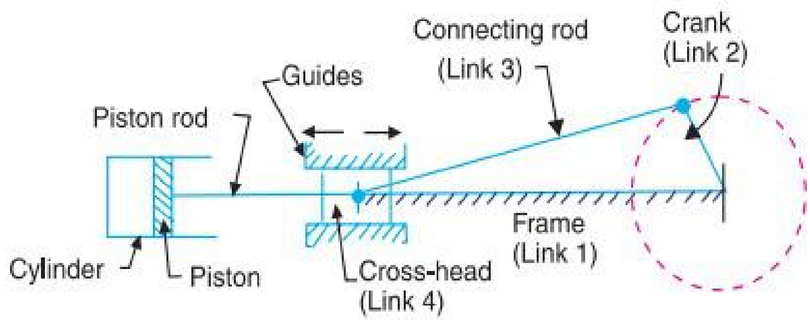


Watt's indicator mechanism.

Single Slider Crank Chain:

A single slider crank chain is a modification of the basic four bar chain. It consists of one sliding pair and three turning pairs. It is usually found in reciprocating steam engine mechanism. This type of mechanism converts rotary motion into reciprocating motion and vice versa.

In a single slider crank chain, as shown in below Fig., the links 1 and 2, links 2 and 3, and links 3 and 4 form three turning pairs while the links 4 and 1 form a sliding pair.

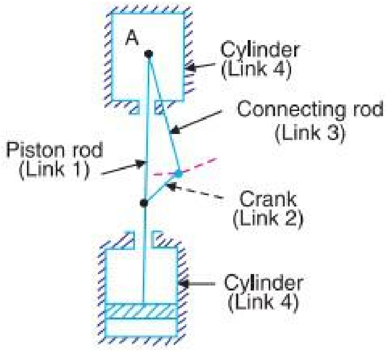


Single slider crank chain.

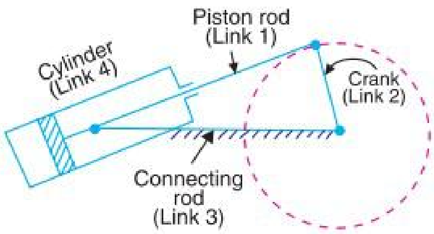
The link 1 corresponds to the frame of the engine, which is fixed. The link 2 corresponds to the crank; link 3 corresponds to the connecting rod and link 4 corresponds to cross-head. As the crank rotates, the cross-head reciprocates in the guides and thus the piston reciprocates in the cylinder.

Inversions of Single Slider Crank Chain: We have seen in the previous article that a single slider crank chain is a four-link mechanism. We know that by fixing, in turn, different links in a kinematic chain, an inversion is obtained and we can obtain as many mechanisms as the links in a kinematic chain. It is thus obvious, that four inversions of a single slider crank chain are possible. These inversions are found in the following mechanisms.

1. Pendulum pump or Bull engine. In this mechanism, the inversion is obtained by fixing the cylinder or link 4 (i.e. sliding pair), as shown in below Fig. In this case, when the crank (link 2) rotates, the connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at A and the piston attached to the piston rod (link 1) reciprocates. The duplex pump which is used to supply feed water to boilers have two pistons attached to link 1, as shown in Fig.



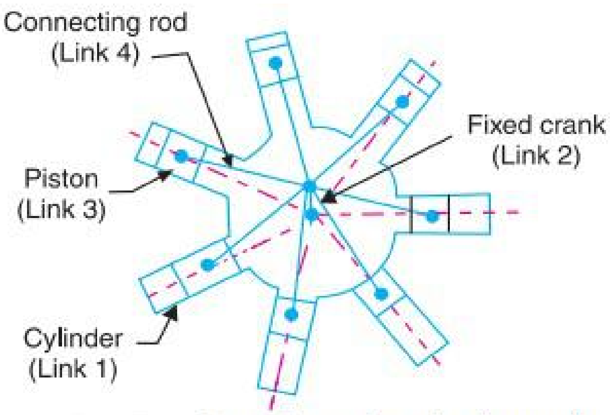
Pendulum pump.



Oscillating cylinder engine.

2. Oscillating cylinder engine. The arrangement of oscillating cylinder engine mechanism, as shown in above Fig., is used to convert reciprocating motion into rotary motion. In this mechanism, the link 3 forming the turning pair is fixed. The link 3 corresponds to the connecting rod of a reciprocating steam engine mechanism. When the crank (link 2) rotates, the piston attached to piston rod (link 1) reciprocates and the cylinder (link 4) oscillates about a pin pivoted to the fixed link at A.

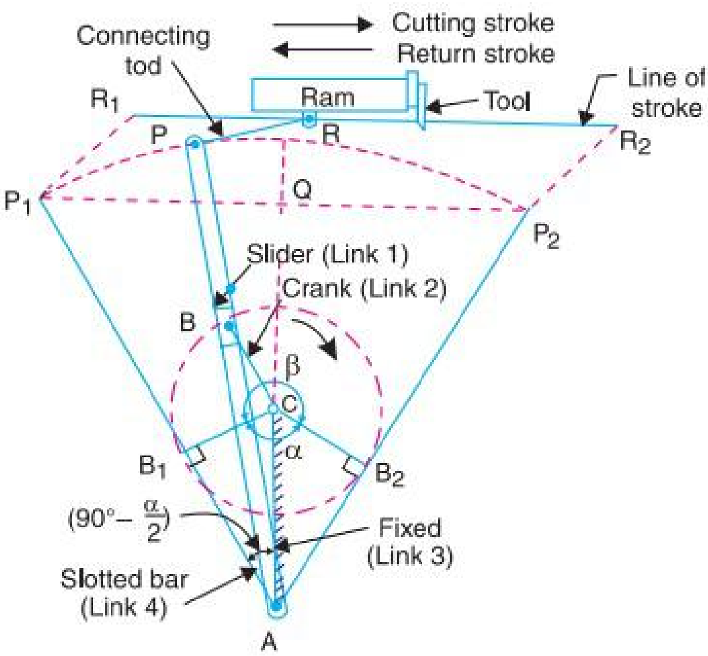
3. Rotary internal combustion engine or Gnome engine. Sometimes back, rotary internal combustion engines were used in aviation. But now-a-days gas turbines are used in its place. It consists of seven cylinders in one plane and all revolves about fixed centre D, as shown in below Fig., while the crank (link 2) is fixed. In this mechanism, when the connecting rod (link 4) rotates, the piston (link 3) reciprocates inside the cylinders forming link 1.



Rotary internal combustion engine.

4. Crank and slotted lever quick return motion mechanism. This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines.

In this mechanism, the link AC (i.e. link 3) forming the turning pair is fixed, as shown in below Fig. The link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank CB revolves with uniform angular speed about the fixed centre C. A sliding block attached to the crank pin at B slides along the slotted bar AP and thus causes AP to oscillate about the pivoted point A. A short link PR transmits the motion from AP to the ram which carries the tool and reciprocates along the line of stroke $R_1 R_2$. The line of stroke of the ram (i.e. $R_1 R_2$) is perpendicular to AC produced.



Crank and slotted lever quick return motion mechanism

In the extreme positions, AP_1 and AP_2 are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position CB_1 to CB_2 (or through an angle β) in the clockwise direction. The return stroke occurs when the crank rotates from the position CB_2 to CB_1 (or through angle α) in the clockwise direction. Since the crank has uniform angular speed, therefore,

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360° - \beta} \quad \text{or} \quad \frac{360° - \alpha}{\alpha}$$

Since the tool travels a distance of $R_1 R_2$ during cutting and return stroke, therefore travel of the tool or length of stroke

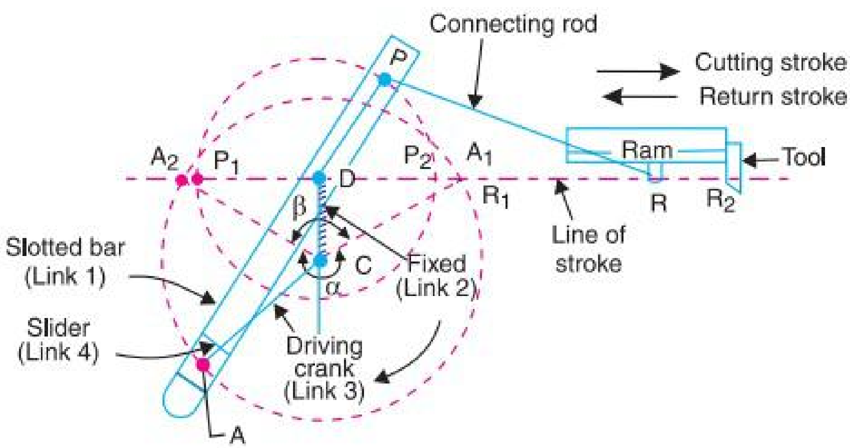
$$= R_1R_2 = P_1P_2 = 2P_1Q = 2AP_1 \sin \angle P_1AQ$$

$$= 2AP_1 \sin\left(90° - \frac{\alpha}{2}\right) = 2AP \cos\frac{\alpha}{2} \qquad \text{..... } (\because AP_1 = AP)$$

$$= 2AP \times \frac{CB_1}{AC} \qquad \text{... } \left(\because \cos\frac{\alpha}{2} = \frac{CB_1}{AC}\right)$$

$$= 2AP \times \frac{CB}{AC} \qquad \text{... } (\because CB_1 = CB)$$

5. Whitworth quick return motion mechanism. This mechanism is mostly used in shaping and slotting machines. In this mechanism, the link CD (link 2) forming the turning pair is fixed, as shown in below Fig. The link 2 corresponds to a crank in a reciprocating steam engine. The driving crank CA (link 3) rotates at a uniform angular speed. The slider (link 4) attached to the crank pin at A slides along the slotted bar PA (link 1) which oscillates at a pivoted point D. The connecting rod PR carries the ram at R to which a cutting tool is fixed. The motion of the tool is constrained along the line RD produced, i.e. along a line passing through D and perpendicular to CD.



Whitworth quick return motion mechanism

When the driving crank CA moves from the position CA1 to CA2 (or the link DP from the position DP_1 to DP_2) through an angle α in the clockwise direction, the tool moves from the left hand end of its stroke to the right hand end through a distance 2PD.

Now when the driving crank moves from the position CA_2 to CA_1 (or the link DP from DP_2 to DP_1) through an angle β in the clockwise direction, the tool moves back from right hand end of its stroke to the left hand end.

A little consideration will show that the time taken during the left to right movement of the ram (i.e. during forward or cutting stroke) will be equal to the time taken by the driving crank to move from CA_1 to CA_2. Similarly, the time taken during the right to left movement of the ram (or during the idle or return stroke) will be equal to the time taken by the driving crank to move from CA_2 to CA_1.

Since the crank link CA rotates at uniform angular velocity therefore time taken during the cutting stroke (or forward stroke) is more than the time taken during the return stroke. In other words, the mean speed of the ram during cutting stroke is less than the mean speed during the return stroke. The ratio between the time taken during the cutting and return strokes is given by

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\alpha}{\beta} = \frac{\alpha}{360^\circ - \alpha} \quad \text{or} \quad \frac{360^\circ - \beta}{\beta}$$

Note. In order to find the length of effective stroke $R_1 R_2$, mark $P_1 R_1 = P_2 R_2 = PR$. The length of effective stroke is also equal to 2 PD.

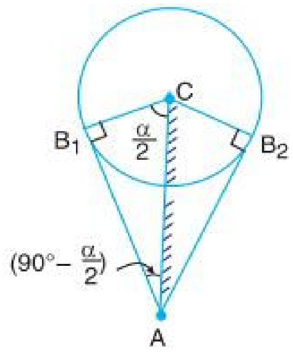
P) A crank and slotted lever mechanism used in a shaper has a centre distance of 300 mm between the centre of oscillation of the slotted lever and the centre of rotation of the crank. The radius of the crank is 120 mm. Find the ratio of the time of cutting to the time of return stroke.

Solution. Given: AC = 300 mm; CB_1 = 120 mm

The extreme positions of the crank are shown in below Fig. We know that

$$\sin \angle CAB_1 = \sin(90° - \alpha/2)$$

$$= \frac{CB_1}{AC} = \frac{120}{300} = 0.4$$



$\therefore$ $$\angle CAB_1 = 90° - \alpha/2$$

$$= \sin^{-1} 0.4 = 23.6°$$

or $$\alpha/2 = 90° - 23.6° = 66.4°$$

and $$\alpha = 2 \times 66.4 = 132.8°$$

We know that

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{360° - \alpha}{\alpha} = \frac{360° - 132.8°}{132.8°} = 1.72 \text{ Ans.}$$

P) In a crank and slotted lever quick return motion mechanism, the distance between the fixed centres is 240 mm and the length of the driving crank is 120 mm. Find the inclination of the slotted bar with the vertical in the extreme position and the time ratio of cutting stroke to the return stroke.

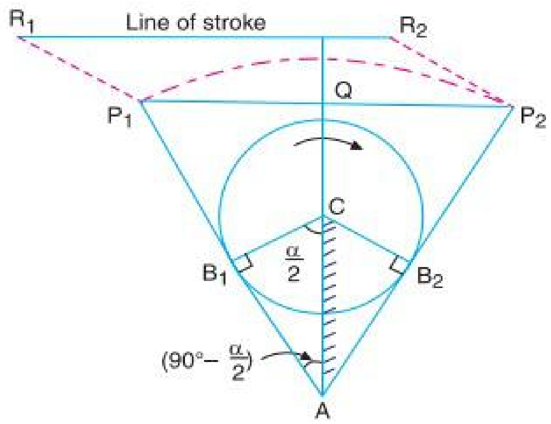
If the length of the slotted bar is 450 mm, find the length of the stroke if the line of stroke passes through the extreme positions of the free end of the lever.

Solution. Given : $AC = 240$ mm ; $CB_1 = 120$ mm ; $AP_1 = 450$ mm

Inclination of the slotted bar with the vertical

Let $\angle CAB_1$ = Inclination of the slotted bar with the vertical.

The extreme positions of the crank are shown in Fig. 5.29. We know that



$$\sin \angle CAB_1 = \sin\left(90° - \frac{\alpha}{2}\right)$$

$$= \frac{B_1C}{AC} = \frac{120}{240} = 0.5$$

$$\therefore \angle CAB_1 = 90° - \frac{\alpha}{2}$$

$$= \sin^{-1} 0.5 = 30° \text{ Ans.}$$

Time ratio of cutting stroke to the return stroke

We know that

$$90° - \alpha/2 = 30°$$

$\therefore$ $$\alpha/2 = 90° - 30° = 60°$$

or $$\alpha = 2 \times 60° = 120°$$

$$\therefore \frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{360° - \alpha}{\alpha} = \frac{360° - 120°}{120°} = 2 \text{ Ans.}$$

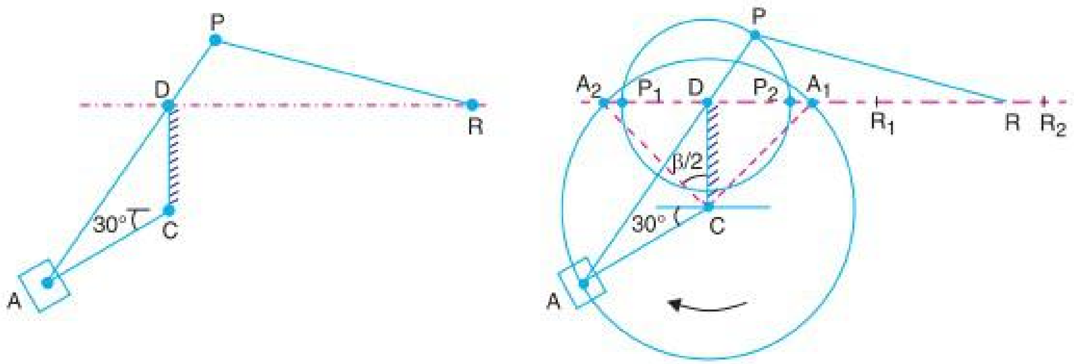
Length of the stroke

We know that length of the stroke,

$$R_1R_2 = P_1P_2 = 2\,P_1Q = 2\,AP_1 \sin(90° - \alpha/2)$$
$$= 2 \times 450 \sin(90° - 60°) = 900 \times 0.5 = 450 \text{ mm } \textbf{Ans.}$$

P) In a Whitworth quick return motion mechanism, as shown in Fig. 5.32, the distance between the fixed centers is 50 mm and the length of the driving crank is 75 mm. The length of the slotted lever is 150 mm and the length of the connecting rod is 135 mm. Find the ratio of the time of cutting stroke to the time of return stroke and also the effective stroke.

Solution. Given : $CD = 50$ mm ; $CA = 75$ mm ; $PA = 150$ mm ; $PR = 135$ mm



The extreme positions of the driving crank are shown in Fig. From the geometry of the figure,

$$\cos\beta/2 = \frac{CD}{CA_2} = \frac{50}{75} = 0.667 \qquad \ldots (\because CA_2 = CA)$$

$\therefore \qquad \beta/2 = 48.2°$ or $\beta = 96.4°$

Ratio of the time of cutting stroke to the time of return stroke

We know that

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{360 - \beta}{\beta} = \frac{360 - 96.4}{96.4} = 2.735 \ \textbf{Ans.}$$

Length of effective stroke:

In order to find the length of effective stroke (i.e. R_1 R_2), draw the space diagram of the mechanism to some suitable scale, as shown in above Fig. Mark $P_1 R_2 = P_2R_2 = PR$. Therefore by measurement we find that, Length of effective stroke $= R_1R_2 = 87.5$ mm Ans.

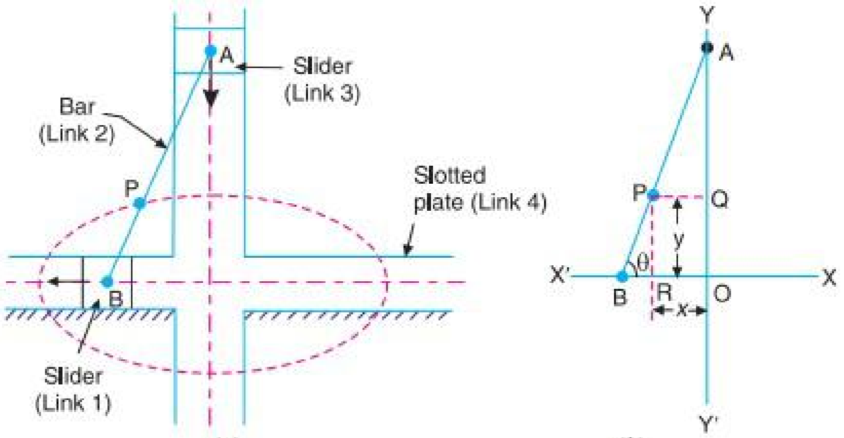
Double Slider Crank Chain:

A kinematic chain which consists of two turning pairs and two sliding pairs is known as double slider crank chain, as shown in below Fig. We see that the link 2 and link 1 form one turning pair and link 2 and link 3 form the second turning pair. The link 3 and link 4 form one sliding pair and link 1 and link 4 form the second sliding pair.

Inversions of Double Slider Crank Chain:

The following three inversions of a double slider crank chain are important from the subject point of view: 1. Elliptical trammels. It is an instrument used for drawing ellipses. This inversion is obtained by fixing the slotted plate (link 4), as shown in Fig. The fixed plate or link 4 has two straight grooves cut in it, at right angles to each other. The link 1 and link 3, are known as sliders and form sliding pairs with link 4. The link A B (link 2) is a bar which forms turning pair with links 1 and 3.

When the links 1 and 3 slide along their respective grooves, any point on the link 2 such as P traces out an ellipse on the surface of link 4, as shown in Fig. (a). A little consideration will show that AP and BP are the semi-major axis and semi-minor axis of the ellipse respectively. This can be proved as follows:



(a) (b)

Let us take OX and O Y as horizontal and vertical axes and let the link BA is inclined at an angle θ with the horizontal, as shown in Fig. (b). Now the co-ordinates of the point P on the link B A will be

$$x = PQ = AP\cos\theta;\ \text{and}\ y = PR =$$

or

$$\frac{x}{AP} = \cos\theta;\ \text{and}\ \frac{y}{BP} = \sin\theta$$

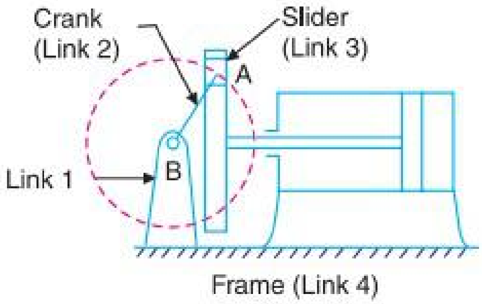
Squaring and adding,

$$\frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = \cos^2\theta + \sin^2\theta = 1$$

This is the equation of an ellipse. Hence the path traced by point P is an ellipse whose semimajor axis is AP and semi-minor axis is BP.

Scotch yoke mechanism:

This mechanism is used for converting rotary motion into a reciprocating motion. The inversion is obtained by fixing either the link 1 or link 3. In below Fig. link 1 is fixed. In this mechanism, when the link 2 (which corresponds to crank) rotates about B as centre, the link 4 (which corresponds to a frame) reciprocates. The fixed link 1 guides the frame.



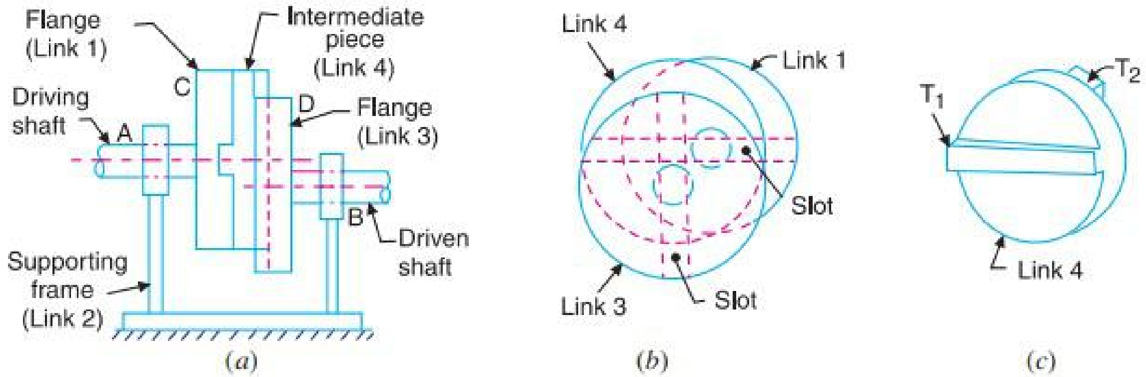
Scotch yoke mechanism.

Oldham's coupling:

An oldham's coupling is used for connecting two parallel shafts whose axes are at a small distance apart. The shafts are coupled in such a way that if one shaft rotates, the other shaft also rotates at the same speed. This inversion is obtained by fixing the link 2, as shown in Fig. (a). The shafts to be connected have two flanges (link 1 and link 3) rigidly fastened at their ends by forging.

The link 1 and link 3 form turning pairs with link 2. These flanges have diametrical slots cut in their inner faces, as shown in Fig. (b). The intermediate piece (link 4) which is a circular disc, have two tongues (i.e. diametrical projections) T_1 and T_2 on each face at right angles to each other, as shown in Fig. (c). The tongues on the link 4 closely fit into the slots in the two flanges (link 1 and link 3). The link 4 can slide or reciprocate in the slots in the flanges.

When the driving shaft A is rotated, the flange C (link 1) causes the intermediate piece (link 4) to rotate at the same angle through which the flange has rotated, and it further rotates the flange D (link 3) at the same angle and thus the shaft B rotates. Hence links 1, 3 and 4 have the same angular velocity at every instant. A little consideration will show, that there is a sliding motion between the link 4 and each of the other links 1 and 3.



Oldham's coupling.

If the distance between the axes of the shafts is constant, the centre of intermediate piece will describe a circle of radius equal to the distance between the axes of the two shafts. Therefore, the maximum sliding speed of each tongue along its slot is equal to the peripheral velocity of the centre of the disc along its circular path.

Let ω = Angular velocity of each shaft in rad/s, and

r = Distance between the axes of the shafts in metres.

$\therefore$ Maximum sliding speed of each tongue (in m/s),

$$v = \omega . r$$

UNIT II

VELOCITY AND ACCELERATION ANALYSIS OF MECHANISMS

Displacement: The displacement of a body is its change of position with reference to a certain fixed point.

Velocity: It is the rate of change of displacement of a body with respect to time. It is vector quantity.

Linear velocity: It is the rate of change of displacement of a body along a straightline with respect to time.

Angular velocity: It is the rate of change of angular position of a body with respect to time.

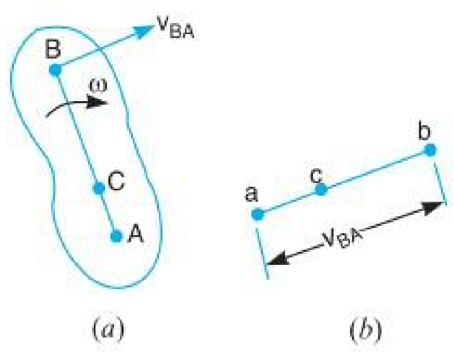
Acceleration: It is the rate of change of velocity of a body with respect to time. It is vector quantity.

Angular Acceleration: It is the rate of change of angular velocity of a body with respect to time.

Motion of a Link:

Consider two points A and B on a rigid link AB, as shown in below figure (a). Let one of the extremities (B) of the link move relative to A, in a clockwise direction. Since the distance from A to B remains the same, therefore there can be no relative motion between A and B, along the line AB. It is thus obvious, that the relative motion of B with respect to A must be perpendicular to AB.

Hence velocity of any point on a link with respect to another point on the same link is always perpendicular to the line joining these points on the configuration (or space) diagram.



Motion of a Link.

The relative velocity of B with respect to A (i.e. v_{BA}) is represented by the vector ab and is perpendicular to the line AB as shown in above fig. (b).

Let ω = Angular velocity of the link AB about A.

We know that the velocity of the point B with respect to A,

$$v_{BA} = \overline{ab} = \omega . AB \qquad ...(i)$$

Similarly, the velocity of any point C on AB with respect to A,

$$v_{CA} = \overline{ac} = \omega . AC \qquad ...(ii)$$

From equations *(i)* and *(ii)*,

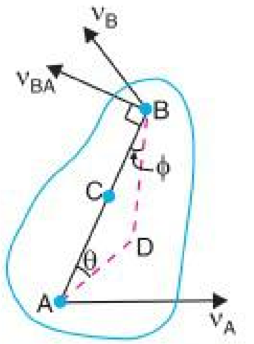
$$\frac{v_{CA}}{v_{BA}} = \frac{\overline{ac}}{\overline{ab}} = \frac{\omega . AC}{\omega . AB} = \frac{AC}{AB} \qquad ...(iii)$$

Thus, we see from equation (iii), that the point c on the vector ab divides it in the same ratio as C divides the link AB.

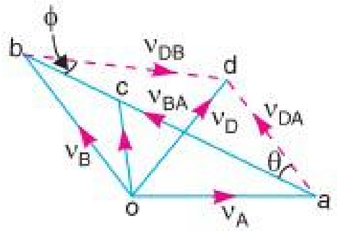
Velocity of a Point on a Link by Relative Velocity Method:

Consider two points A and B on a link as shown in below figure (a). Let the absolute velocity of the point A i.e. v_A is known in magnitude and direction and the absolute velocity of the point B i.e. v_B is known in direction only. Then the velocity of B may be determined by drawing the velocity diagram as shown in below fig. (b). The velocity diagram is drawn as follows :

1. Take some convenient point o, known as the pole.
2. Through o, draw oa parallel and equal to v_A, to some suitable scale.
3. Through a, draw a line perpendicular to AB of fig.(a). This line will represent the velocity of B with respect to A, i.e. v_{BA}.
4. Through o, draw a line parallel to v_B intersecting the line of v_{BA} at b.
5. Measure ob, which gives the required velocity of point B (v_B), to the scale.



(a) Motion of points on a link.



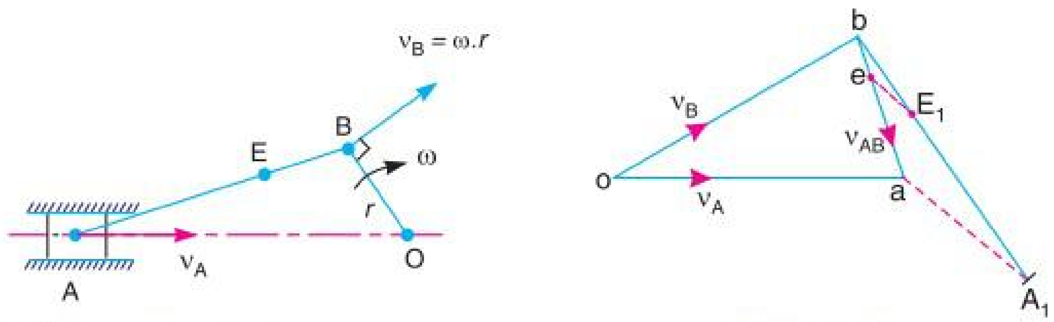
(b) Velocity diagram.

Velocities in Slider Crank Mechanism:

A slider crank mechanism is shown in below fig. (a). The slider A is attached to the connecting rod AB. Let the radius of crank OB be r and let it rotates in a clockwise direction, about the point O with uniform angular velocity ω rad/s. Therefore, the velocity of B i.e. v_B is known in magnitude and direction. The slider reciprocates along the line of stroke AO.

The velocity of the slider A (i.e. v_A) may be determined by relative velocity method as discussed below:

1. From any point o, draw vector ob parallel to the direction of v_B (or perpendicular to OB) such that ob = v_B = ω.r, to some suitable scale, as shown in fig. (b).



(a) Slider crank mechanism. (b) Velocity diagram.

2. Since AB is a rigid link, therefore the velocity of A relative to B is perpendicular to AB. Now draw vector ba perpendicular to AB to represent the velocity of A with respect to B i.e. v_{AB}.

3. From point o, draw vector oa parallel to the path of motion of the slider A (which is along AO only). The vectors ba and oa intersect at a. Now oa represents the velocity of the slider A i.e. v_A, to the scale.

The angular velocity of the connecting rod AB (ω_{AB}) may be determined as follows:

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{ab}{AB} \qquad \text{(Anticlockwise about A)}$$

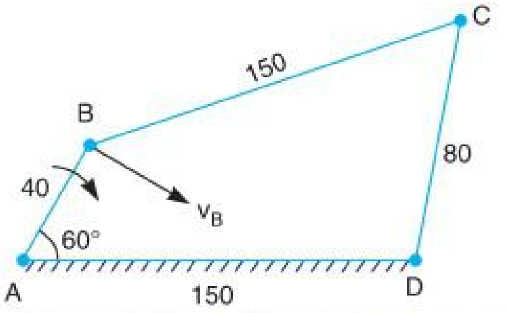
The direction of vector ab (or ba) determines the sense of ω_{AB} which shows that it is anticlockwise.

P) In a four bar chain ABCD, AD is fixed and is 150 mm long. The crank AB is 40 mm long and rotates at 120 r.p.m. clockwise, while the link CD = 80 mm oscillates about D. BC and AD are of equal length. Find the angular velocity of link CD when angle BAD = 60°.

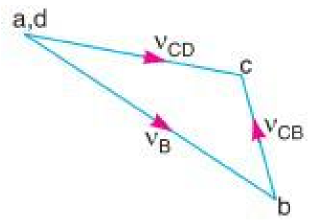
Solution. Given : N_{BA} = 120 r.p.m. or $\omega_{BA} = 2\pi \times 120/60 = 12.568$ rad/s

Since the length of crank AB = 40 mm = 0.04 m, therefore velocity of B with respect to A or velocity of B, (because A is a fixed point),

$$v_{BA} = v_B = \omega_{BA} \times AB = 12.568 \times 0.04 = 0.503 \text{ m/s}$$



(a) Space diagram (All dimensions in mm).



(b) Velocity diagram.

First of all, draw the space diagram to some suitable scale, as shown in above fig. (a). Now the velocity diagram, as shown in above fig. (b), is drawn as discussed below:

1. Since the link AD is fixed, therefore points a and d are taken as one point in the velocity diagram. Draw vector ab perpendicular to BA, to some suitable scale, to represent the velocity of B with respect to A or simply velocity of B (i.e. v_{BA} or v_B) such that

vector ab = v_{BA} = v_B = 0.503 m/s

2. Now from point b, draw vector bc perpendicular to CB to represent the velocity of C with respect to B (i.e. v_{CB}) and from point d, draw vector dc perpendicular to CD to represent the velocity of C with respect to D or simply velocity of C (i.e. v_{CD} or v_C). The vectors bc and dc intersect at c.

By measurement, we find that

v_{CD} = v_C = vector dc = 0.385 m/s

We know that CD = 80 mm = 0.08 m

∴ Angular velocity of link CD,

$$\omega_{CD} = \frac{v_{CD}}{CD} = \frac{0.385}{0.08} = 4.8 \text{ rad/s (clockwise about } D\text{)} \textbf{ Ans.}$$

P) The crank and connecting rod of a theoretical steam engine are 0.5 m and 2 m long respectively. The crank makes 180 r.p.m. in the clockwise direction. When it has turned 45° from the inner dead centre position, determine: 1. velocity of piston, 2. angular velocity of connecting rod, 3. velocity of point E on the connecting rod 1.5 m from the gudgeon pin.

Solution. Given : N_{BO} = 180 r.p.m. or $\omega_{BO} = 2\pi \times 180/60 = 18.852$ rad/s

Since the crank length $OB = 0.5$ m, therefore linear velocity of B with respect to O or velocity of B (because O is a fixed point),

$$v_{BO} = v_B = \omega_{BO} \times OB = 18.852 \times 0.5 = 9.426 \text{ m/s}$$

... (Perpendicular to BO)

1. Velocity of piston

First of all draw the space diagram, to some suitable scale, as shown in below fig. (a). Now the velocity diagram, as shown in fig. (b), is drawn as discussed below :

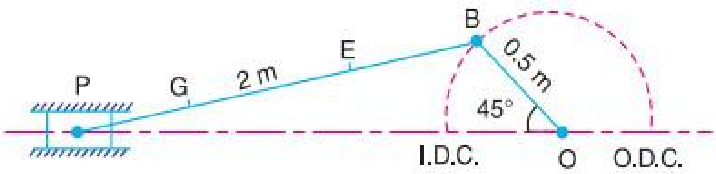
1. Draw vector ob perpendicular to BO, to some suitable scale, to represent the velocity of B with respect to O or velocity of B such that

$$\text{vector } ob = v_{BO} = v_B = 9.426 \text{ m/s}$$

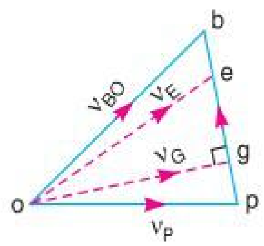
2. From point b, draw vector bp perpendicular to BP to represent velocity of P with respect to B (*i.e.* v_{PB}) and from point o, draw vector op parallel to PO to represent velocity of P with respect to O (*i.e.* v_{PO} or simply v_P). The vectors bp and op intersect at point p.

By measurement, we find that velocity of piston P,

$$v_P = \text{vector } op = 8.15 \text{ m/s} \quad \textbf{Ans.}$$



(*a*) Space diagram.



(*b*) Velocity diagram.

2. Angular velocity of connecting rod

From the velocity diagram, we find that the velocity of P with respect to B,

$$v_{PB} = \text{vector } bp = 6.8 \text{ m/s}$$

Since the length of connecting rod PB is 2 m, therefore angular velocity of the connecting rod,

$$\omega_{PB} = \frac{v_{PB}}{PB} = \frac{6.8}{2} = 3.4 \text{ rad/s (Anticlockwise)} \quad \textbf{Ans.}$$

3. Velocity of point E on the connecting rod

The velocity of point E on the connecting rod 1.5 m from the gudgeon pin (i.e. PE = 1.5 m) is determined by dividing the vector bp at e in the same ratio as E divides PB in above fig. (a). Join oe. The vector oe represents the velocity of E. By measurement, we find that velocity of point E,

$$v_E = \text{vector oe} = 8.5 \text{ m/s}$$

Instantaneous center of rotation:

A link as a whole may be considered to be rotating about an imaginary centre or a given centre at a given instant. This is known as the instantaneous centre or centre of rotation. Such a centre has zero velocity or the link is at rest at this point.

Instantaneous centre varies from instant to instant for different positions of the link. The locus of these centres is termed the centrode.

Properties of the Instantaneous Centre:

The following properties of the instantaneous centre are important from the subject point of view:

1. A rigid link rotates instantaneously relative to another link at the instantaneous centre for the configuration of the mechanism considered.

2. The two rigid links have no linear velocity relative to each other at the instantaneous centre. At this point (i.e. instantaneous centre), the two rigid links have the same linear velocity relative to the third rigid link.

Number of Instantaneous Centres in a Mechanism:

The number of instantaneous centres in a constrained kinematic chain is equal to the number of possible combinations of two links. The number of pairs of links or the number of instantaneous centres is the number of combinations of n links taken two at a time. Mathematically, number of instantaneous centres,

$$N = \frac{n(n-1)}{2}, \text{ where } n = \text{Number of links}$$

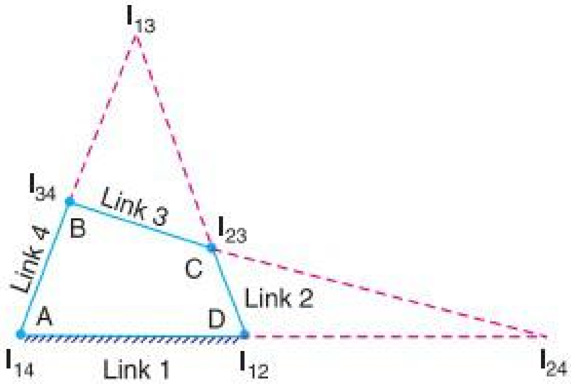
Types of Instantaneous Centres:

1. Fixed instantaneous centres, **2.** Permanent instantaneous centres, and **3.** Neither fixed nor permanent instantaneous centres.

The first two types i.e. fixed and permanent instantaneous centres are together known as **primary instantaneous centres** and the third type is known as **secondary instantaneous centres**.

Consider a four bar mechanism ABCD as shown in below fig. The number of instantaneous centres (N) in a four bar mechanism is given by

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$



The instantaneous centres I_{12} and I_{14} are called the **fixed instantaneous centres** as they remain in the same place for all configurations of the mechanism. The instantaneous centres I_{23} and I_{34} are the **permanent instantaneous centres** as they move when the mechanism moves, but the joints are of permanent nature. The instantaneous centres I_{13} and I_{24} are **neither fixed nor permanent instantaneous centres** as they vary with the configuration of the mechanism.

Location of Instantaneous Centres:

The following rules may be used in locating the instantaneous centres in a mechanism:

1. When the two links are connected by a pin joint (or pivot joint), the instantaneous centre lies on the centre of the pin as shown in below fig. (a). Such a instantaneous centre is of permanent nature, but if one of the links is fixed, the instantaneous centre will be of fixed type.
2. When the two links have a pure rolling contact (i.e. link 2 rolls without slipping upon the fixed link 1 which may be straight or curved), the instantaneous centre lies on their point of contact, as shown in below fig. (b). The velocity of any point A on the link 2 relative to fixed link 1 will be perpendicular to I_{12} A and is proportional to I_{12} A . In other words

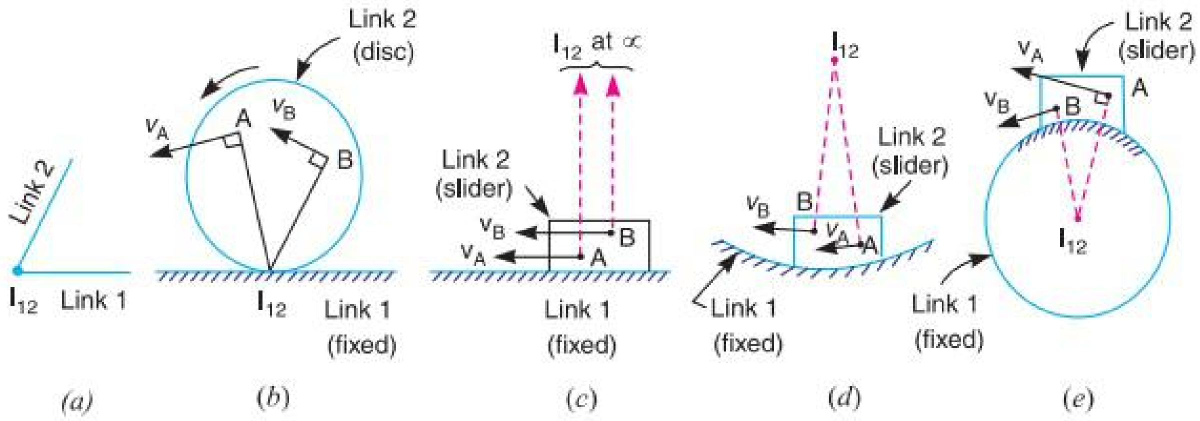
$$\frac{v_A}{v_B} = \frac{I_{12}A}{I_{12}B}$$

3. When the two links have a sliding contact, the instantaneous centre lies on the common normal at the point of contact. We shall consider the following three cases :

(a) When the link 2 (slider) moves on fixed link 1 having straight surface as shown in Fig. (c), the instantaneous centre lies at infinity and each point on the slider have the same velocity.

(b) When the link 2 (slider) moves on fixed link 1 having curved surface as shown in Fig. (d), the instantaneous centre lies on the centre of curvature of the curvilinear path in the configuration at that instant.

(c) When the link 2 (slider) moves on fixed link 1 having constant radius of curvature as shown in Fig. (e), the instantaneous centre lies at the centre of curvature i.e. the centre of the circle, for all configuration of the links.



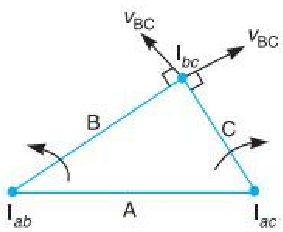
Location of instantaneous centres.

Aronhold Kennedy (or Three Centres in Line) Theorem:

The Aronhold Kennedy's theorem states **that if three bodies move relatively to each other, they have three instantaneous centres and lie on a straight line**. Consider three kinematic links A, B and C having relative plane motion. The number of instantaneous centres (N) is given by

$$N = \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$$

n = Number of links = 3



Aronhold Kennedy's theorem.

The two instantaneous centres at the pin joints of B with A, and C with A (i.e. I_{ab} and I_{ac}) are the permanent instantaneous centres. According to Aronhold Kennedy's theorem, the third instantaneous centre I_{bc} must lie on the line joining I_{ab} and I_{ac}.

In order to prove this, let us consider that the instantaneous centre I_{bc} lies outside the line joining I_{ab} and I_{ac} as shown in above fig. The point I_{bc} belongs to both the links B and C. Let us consider the point I_{bc} on the link B. Its velocity v_{BC} must be perpendicular to the line joining I_{ab} and I_{bc}. Now consider the point I_{bc} on the link C. Its velocity v_{BC} must be perpendicular to the line joining I_{ac} and I_{bc}.

Method of Locating Instantaneous Centres in a Mechanism:

Consider a pin jointed four bar mechanism as shown in below fig. (a). The following procedure is adopted for locating instantaneous centres.

1. First of all, determine the number of instantaneous centres (N) by using the relation

$$N = \frac{n(n-1)}{2}, \text{ where } n = \text{Number of links.}$$

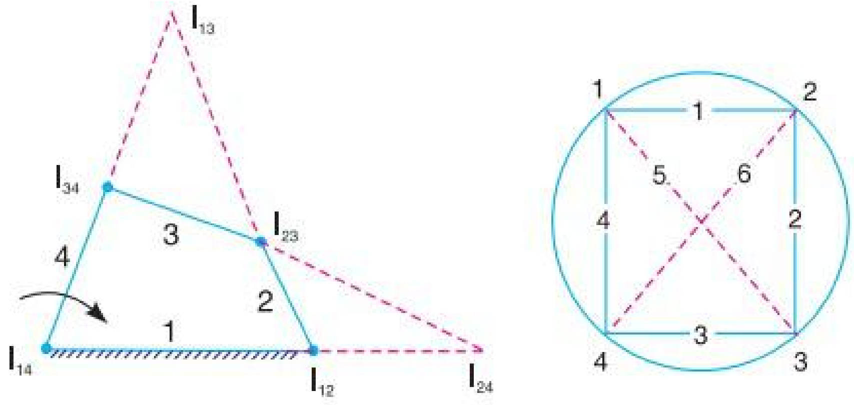
In the present case, $$N = \frac{4(4-1)}{2} = 6 \qquad ...(\because n = 4)$$

2. Make a list of all the instantaneous centres in a mechanism. Since for a four bar mechanism, there are six instantaneous centres, therefore these centres are listed as shown in the following table (known as book-keeping table).

Links	1	2	3	4
Instantaneous centres (6 in number)	12 13 14	23 24	34	--

3. Locate the fixed and permanent instantaneous centres by inspection. In below fig.(a), I_{12} and I_{14} are fixed instantaneous centres and I_{23} and I_{34} are permanent instantaneous centres.

4. Locate the remaining neither fixed nor permanent instantaneous centres (or secondary centres) by Kennedy's theorem. This is done by circle diagram as shown in above fig. (b). Mark points on a circle equal to the number of links in a mechanism. In the present case, mark 1, 2, 3, and 4 on the circle.
5. Join the points by solid lines to show that these centres are already found. In the circle diagram [Fig. (b)] these lines are 12, 23, 34 and 14 to indicate the centres I_{12}, I_{23}, I_{34} and I_{14}.

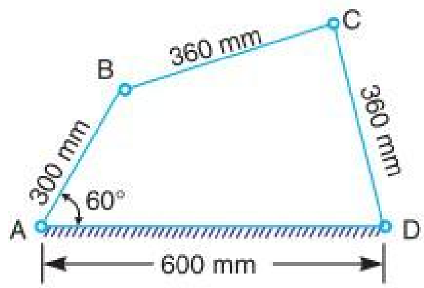


(*a*) Four bar mechanism. (*b*) Circle diagram.

Method of locating instantaneous centres.

6. In order to find the other two instantaneous centres, join two such points that the line joining them forms two adjacent triangles in the circle diagram. The line which is responsible for completing two triangles, should be a common side to the two triangles. In Fig. b), join 1 and 3 to form the triangles 123 and 341 and the instantaneous centre I_{13} will lie on the intersection of I_{12} I_{23} and I_{14} I_{34}, produced if necessary, on the mechanism. Thus the instantaneous centre I_{13} is located. Join 1 and 3 by a dotted line on the circle diagram and mark number 5 on it. Similarly the instantaneous centre I_{24} will lie on the intersection of I_{12} I_{14} and I_{23} I_{34}, produced if necessary, on the mechanism. Thus I_{24} is located. Join 2 and 4 by a dotted line on the circle diagram and mark 6 on it. Hence all the six instantaneous centres are located.

P) In a pin jointed four bar mechanism, as shown in below fig, AB = 300 mm, BC = CD = 360 mm, and AD = 600 mm. The angle BAD = 60°. The crank AB rotates uniformly at 100 r.p.m. Locate all the instantaneous centres and find the angular velocity of the link BC.



Given : N_{AB} = 100 r.p.m or $\omega_{AB} = 2\,\pi \times 100/60 = 10.47$ rad/s

Since the length of crank AB = 300 mm = 0.3 m, therefore velocity of point B on link AB,

$v_B = \omega_{AB} \times AB = 10.47 \times 0.3 = 3.141$ m/s

Location of instantaneous centres

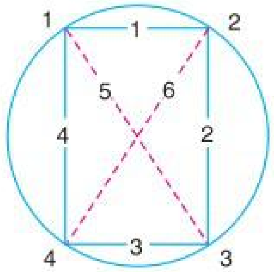
The instantaneous centres are located as discussed below:

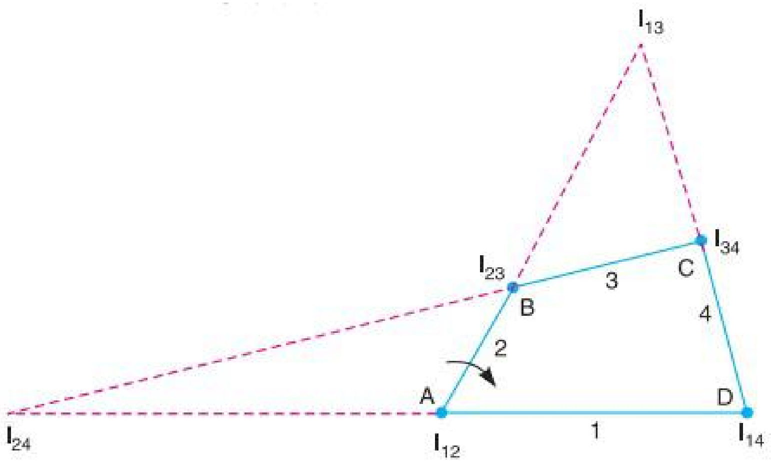
1. Since the mechanism consists of four links (i.e. n = 4), therefore number of instantaneous centres,

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

2. Locate the fixed and permanent instantaneous centres by inspection. These centres are I_{12}, I_{23}, I_{34} and I_{14}, as shown in below fig.

3. Locate the remaining neither fixed nor permanent instantaneous centres by Aronhold Kennedy's theorem. This is done by circle diagram as shown in below fig. Mark four points (equal to the number of links in a mechanism) 1, 2, 3, and 4 on the circle.





4. Join the points by solid lines to show that these centres are already found. In the circle diagram these lines are 12, 23, 34 and 14 to indicate the centres I_{12}, I_{23}, I_{34} and I_{14}.
5. join 1 and 3 to form the triangles 123 and 341 and the instantaneous centre I_{13} will lie on the intersection of I_{12} I_{23} and I_{14} I_{34}, produced if necessary, on the mechanism. Thus the instantaneous centre I_{13} is located. Join 1 and 3 by a dotted line on the circle diagram and mark number 5 on it.
6. Similarly the instantaneous centre I_{24} will lie on the intersection of I_{12} I_{14} and I_{23} I_{34}, produced if necessary, on the mechanism. Thus I_{24} is located. Join 2

and 4 by a dotted line on the circle diagram and mark 6 on it. Hence all the six instantaneous centres are located.

Angular velocity of the link BC:

Let ω_{BC} = Angular velocity of the link BC.

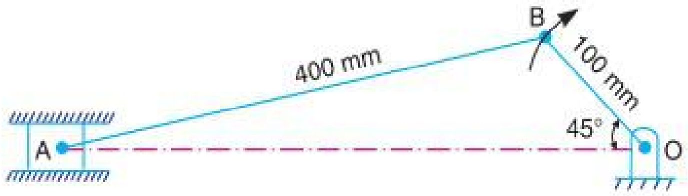
Since B is also a point on link BC, therefore velocity of point B on link BC,

$$v_B = \omega_{BC} \times I_{13} B$$

By measurement, we find that $I_{13} B = 500 \text{ mm} = 0.5 \text{ m}$

$$\therefore \quad \omega_{BC} = \frac{v_B}{I_{13} B} = \frac{3.141}{0.5} = 6.282 \text{ rad/s} \quad \textbf{Ans.}$$

P) Locate all the instantaneous centres of the slider crank mechanism as shown in below fig. The lengths of crank OB and connecting rod AB are 100 mm and 400 mm respectively. If the crank rotates clockwise with an angular velocity of 10 rad/s, find: 1. Velocity of the slider A, and 2. Angular velocity of the connecting rod AB.



Solution. Given : $\omega_{OB} = 10$ rad/ s; $OB = 100$ mm $= 0.1$ m

We know that linear velocity of the crank OB,

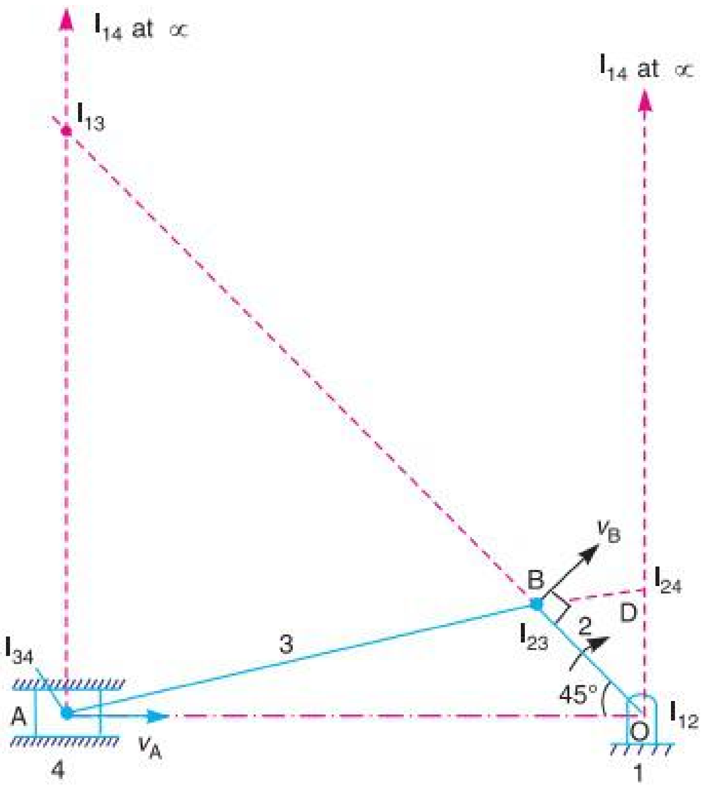
$$v_{OB} = v_B = \omega_{OB} \times OB = 10 \times 0.1 = 1 \text{ m/s}$$

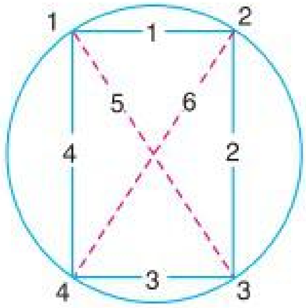
Location of instantaneous centres

The instantaneous centres in a slider crank mechanism are located as discussed below:

1. Since there are four links (*i.e.* $n = 4$), therefore the number of instantaneous centres,

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$





2. Locate the fixed and permanent instantaneous centres by inspection. These centres are I_{12}, I_{23} and I_{34} as shown in above fig. Since the slider (link 4) moves on a straight surface (link 1), therefore the instantaneous centre I_{14} will be at infinity.

3. Locate the other two remaining neither fixed nor permanent instantaneous centres, by Aronhold Kennedy's theorem. This is done by circle diagram as shown in above fig. Mark four points 1, 2, 3 and 4 (equal to the number of links in a mechanism) on the circle to indicate I_{12}, I_{23}, I_{34} and I_{14}.

4. join 1 and 3 to form the triangles 123 and 341 and the instantaneous centre I_{13} will lie on the intersection of I_{12} I_{23} and I_{14} I_{34}, produced if necessary, on the mechanism. Thus the instantaneous centre I_{13} is located. Join 1 and 3 by a dotted line on the circle diagram and mark number 5 on it.

5. Similarly the instantaneous centre I_{24} will lie on the intersection of I_{12} I_{14} and I_{23} I_{34}, produced if necessary, on the mechanism. Thus I_{24} is located. Join 2 and 4 by a dotted line on the circle diagram and mark 6 on it. Hence all the six instantaneous centres are located.

By measurement, we find that

$$I_{13}A = 460 \text{ mm} = 0.46 \text{ m} \text{ ; and } I_{13}B = 560 \text{ mm} = 0.56 \text{ m}$$

1. *Velocity of the slider A*

Let v_A = Velocity of the slider A.

We know that $\dfrac{v_A}{I_{13}A} = \dfrac{v_B}{I_{13}B}$

or $$v_A = v_B \times \frac{I_{13}A}{I_{13}B} = 1 \times \frac{0.46}{0.56} = 0.82 \text{ m/s} \quad \textbf{Ans.}$$

2. *Angular velocity of the connecting rod AB*

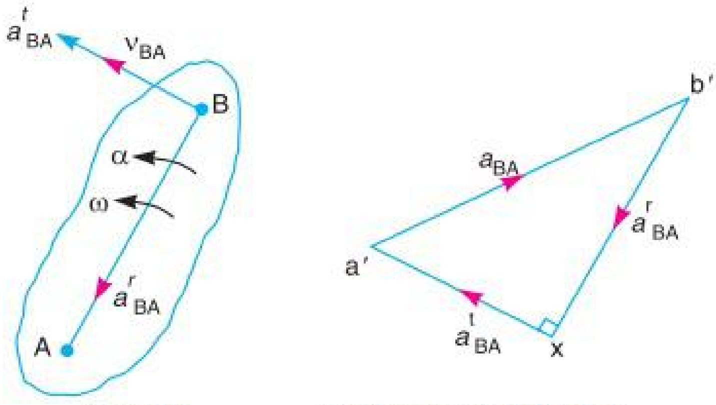
Let ω_{AB} = Angular velocity of the connecting rod AB.

We know that $\dfrac{v_A}{I_{13}A} = \dfrac{v_B}{I_{13}B} = \omega_{AB}$

$$\omega_{AB} = \frac{v_B}{I_{13}B} = \frac{1}{0.56} = 1.78 \text{ rad/s} \quad \textbf{Ans.}$$

Acceleration Diagram for a Link:

Consider two points A and B on a rigid link as shown in below fig. (a). Let the point B moves with respect to A, with an angular velocity of ω rad/s and let α rad/s^2 be the angular acceleration of the link AB.



(*a*) Link. (*b*) Acceleration diagram.

Acceleration for a link.

Acceleration of a particle (whose velocity changes both in magnitude and direction) at any instant have the following two components:

1.Centripetal or Radial component, 2.Tangential component

1. The **centripetal or radial component**, which is perpendicular to the velocity of the particle at the given instant.

2. The **tangential component**, which is parallel to the velocity of the particle at the given instant.

Thus for a link AB, the velocity of point B with respect to A (i.e. v_{BA}) is perpendicular to the link AB as shown in above fig. (a). Since the point B moves with respect to A with an angular velocity of ω rad/s, therefore centripetal or radial component of the acceleration of B with respect to A,

$$a_{BA}^r = \omega^2 \times \text{Length of link } AB = \omega^2 \times AB = v_{BA}^2 / AB \qquad \ldots\left(\because \omega = \frac{v_{BA}}{AB}\right)$$

This radial component of acceleration acts perpendicular to the velocity v_{BA}, In other words, it acts parallel to the link AB.

We know that tangential component of the acceleration of B with respect to A,

$$a_{BA}^t = \alpha \times \text{Length of the link } AB = \alpha \times AB$$

This tangential component of acceleration acts parallel to the velocity v_{BA}. In other words, it acts perpendicular to the link AB.

In order to draw the acceleration diagram for a link AB, as shown in above fig. (b), from any point b', draw vector b'x parallel to BA to represent the radial component of acceleration of B with respect to A i.e. a^r_{BA} and from point x draw vector xa' perpendicular to BA to represent the tangential component of acceleration of B with respect to A i.e. a^t_{BA}. Join b' a'. The vector b'a' (known as acceleration image of the link AB) represents the total acceleration of B with respect to A (i.e. a_{BA}) and it is the vector sum of radial component (a^r_{BA}) and tangential component (a^t_{BA}) of acceleration.

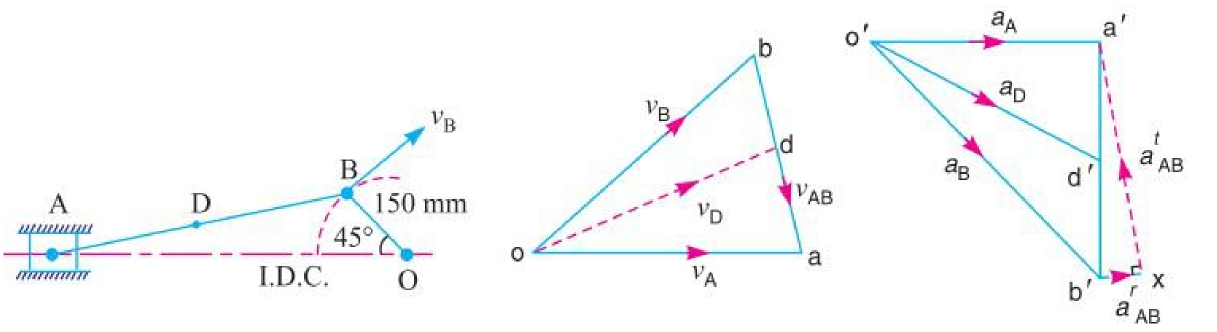
P) The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine : 1. linear velocity and acceleration of the midpoint of the connecting rod, and 2. angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead centre position.

Solution. Given : N_{BO} = 300 r.p.m. or $\omega_{BO} = 2\pi \times 300/60 = 31.42$ rad/s; OB = 150 mm = 0.15 m ; BA = 600 mm = 0.6 m

We know that linear velocity of B with respect to O or velocity of B,

$$v_{BO} = v_B = \omega_{BO} \times OB = 31.42 \times 0.15 = 4.713 \text{ m/s}$$

...(Perpendicular to BO)



(*a*) Space diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

1. Linear velocity of the midpoint of the connecting rod:

First of all draw the space diagram, to some suitable scale; as shown in above fig. (a). Now the velocity diagram, as shown in Fig. (b), is drawn as discussed below:

1. Draw vector *ob* perpendicular to *BO*, to some suitable scale, to represent the velocity of *B* with respect to *O* or simply velocity of *B i.e.* v_{BO} or v_B, such that

$$\text{vector } ob = v_{BO} = v_B = 4.713 \text{ m/s}$$

2. From point *b*, draw vector *ba* perpendicular to *BA* to represent the velocity of *A* with respect to *B i.e.* v_{AB}, and from point *o* draw vector *oa* parallel to the motion of *A* (which is along *AO*) to represent the velocity of *A i.e.* v_A. The vectors *ba* and *oa* intersect at *a*.

By measurement, we find that velocity of *A* with respect to *B*,

$$v_{AB} = \text{vector } ba = 3.4 \text{ m/s}$$

and

$$\text{Velocity of } A,\ v_A = \text{vector } oa = 4 \text{ m/s}$$

3. In order to find the velocity of the midpoint *D* of the connecting rod *AB*, divide the vector *ba* at *d* in the same ratio as *D* divides *AB*, in the space diagram. In other words,

$$bd / ba = BD/BA$$

Note: Since *D* is the midpoint of *AB*, therefore *d* is also midpoint of vector *ba*.

4. Join *od*. Now the vector *od* represents the velocity of the midpoint *D* of the connecting rod *i.e.* v_D.

By measurement, we find that

$$v_D = \text{vector } od = 4.1 \text{ m/s} \quad \textbf{Ans.}$$

Acceleration of the midpoint of the connecting rod

We know that the radial component of the acceleration of *B* with respect to *O* or the acceleration of *B*,

$$a^r_{BO} = a_B = \frac{v^2_{BO}}{OB} = \frac{(4.713)^2}{0.15} = 148.1 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. c) is drawn as discussed below:

1. Draw vector *o′b′* parallel to *BO*, to some suitable scale, to represent the radial component of the acceleration of *B* with respect to *O* or simply acceleration of *B i.e.* a^r_{BO} or a_B, such that

$$\text{vector } o'b' = a^r_{BO} = a_B = 148.1 \text{ m/s}^2$$

2. The acceleration of A with respect to B has the following two components:

(*a*) The radial component of the acceleration of A with respect to B *i.e.* a^r_{AB}, and

(*b*) The tangential component of the acceleration of A with respect to B *i.e.* a^t_{AB}. These two components are mutually perpendicular.

Therefore from point b', draw vector $b'x$ parallel to AB to represent $a^r_{AB} = 19.3 \text{ m/s}^2$ and from point x draw vector xa' perpendicular to vector $b'x$ whose magnitude is yet unknown.

3. Now from o', draw vector $o'a'$ parallel to the path of motion of A (which is along AO) to represent the acceleration of A *i.e.* a_A. The vectors xa' and $o'a'$ intersect at a'. Join $a'b'$.

4. In order to find the acceleration of the midpoint D of the connecting rod AB, divide the vector $a'b'$ at d' in the same ratio as D divides AB. In other words

$$b'd' / b'a' = BD / BA$$

Note: Since D is the midpoint of AB, therefore d' is also midpoint of vector $b'a'$.

5. Join $o'd'$. The vector $o'd'$ represents the acceleration of midpoint D of the connecting rod *i.e.* a_D.

By measurement, we find that

$$a_D = \text{vector } o'd' = 117 \text{ m/s}^2 \textbf{ Ans.}$$

2. *Angular velocity of the connecting rod*

We know that angular velocity of the connecting rod AB,

$$\omega_{AB} = \frac{v_{AB}}{BA} = \frac{3.4}{0.6} = 5.67 \text{ rad/s}^2 \text{ (Anticlockwise about } B\text{)} \textbf{ Ans.}$$

Angular acceleration of the connecting rod

From the acceleration diagram, we find that

$$a^t_{AB} = 103 \text{ m/s}^2 \qquad \text{...(By measurement)}$$

We know that angular acceleration of the connecting rod AB,

$$\alpha_{AB} = \frac{a^t_{AB}}{BA} = \frac{103}{0.6} = 171.67 \text{ rad/s}^2 \text{ (Clockwise about } B\text{)} \textbf{ Ans.}$$

Coriolis Component of Acceleration:

When a point on one link is sliding along another rotating link, such as in quick return motion mechanism, then the coriolis component of the acceleration must be calculated.

Consider a link OA and a slider B as shown in below fig. (a). The slider B moves along the link OA. The point C is the coincident point on the link OA.

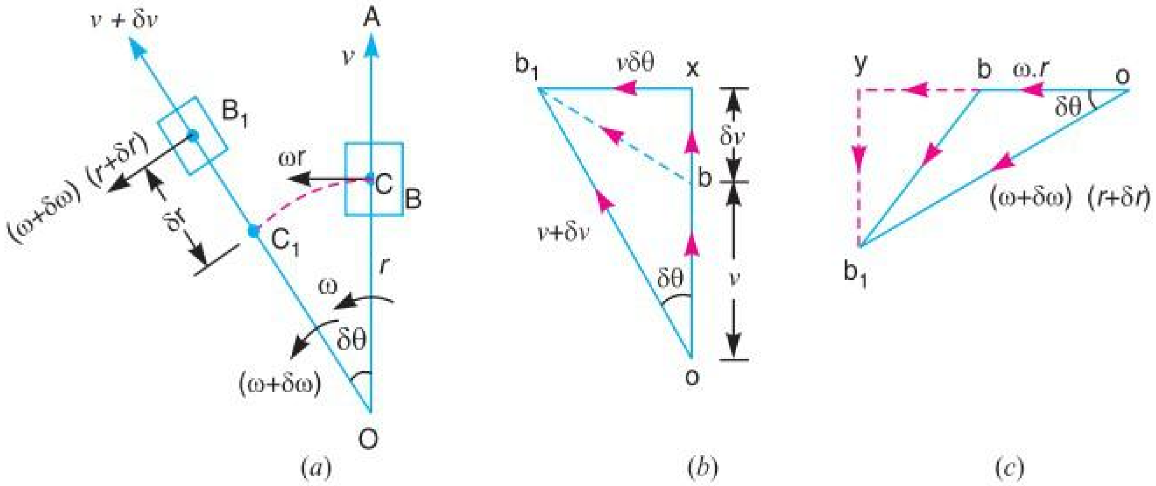
Let ω = Angular velocity of the link OA at time t seconds.

v = Velocity of the slider B along the link OA at time t seconds.

$\omega.r$ = Velocity of the slider B with respect to O (perpendicular to the link OA) at time t seconds, and

$(\omega + \delta\omega)$, $(v + \delta v)$ and $(\omega + \delta\omega)\,(r + \delta r)$

= Corresponding values at time $(t + \delta t)$ seconds.



Coriolis component of acceleration.

Let us now find out the acceleration of the slider B with respect to O and with respect to its coincident point C lying on the link OA.

Fig. (b) shows the velocity diagram when their velocities v and (v + δv) are considered. In this diagram, the vector bb_1 represents the change in velocity in time δt sec ; the vector bx represents the component of change of velocity bb_1 along OA (i.e. along radial direction) and vector xb_1 represents the component of change of velocity bb_1 in a direction perpendicular to OA (i.e. in tangential direction). Therefore

$$bx = ox - ob = (v + \delta v)\cos\delta\theta - v \uparrow$$

Since δθ is very small, therefore substituting $\cos\delta\theta = 1$, we have

$$bx = (v + \delta v - v) \uparrow = \delta v \uparrow$$

...(Acting radially outwards)

and $$xb_1 = (v + \delta v)\sin\delta\theta$$

Since δθ is very small, therefore substituting $\sin\delta\theta = \delta\theta$, we have

$$xb_1 = (v + \delta v)\,\delta\theta = v.\delta\theta + \delta v.\delta\theta$$

Neglecting $\delta v.\delta\theta$ being very small, therefore

$$xb_1 = v.\overset{\leftarrow}{\delta\theta} \quad \ldots$$

Fig. (c) shows the velocity diagram when the velocities ω.r and (ω + δω) (r + δr) are considered. In this diagram, vector bb_1 represents the change in velocity; vector yb_1 represents the component of change of velocity bb_1 along OA (i.e. along radial direction) and vector by represents the component of change of velocity bb_1 in a direction perpendicular to OA (i.e. in a tangential direction). Therefore

$$yb_1 = (\omega + \delta\omega)\,(r + \delta r)\sin\delta\theta \downarrow$$
$$= (\omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r)\sin\delta\theta$$

Since δθ is very small, therefore substituting sin δθ = δθ in the above expression, we have

$$yb_1 = \omega.r.\delta\theta + \omega.\delta r.\delta\theta + \delta\omega.r.\delta\theta + \delta\omega.\delta r.\delta\theta$$

$$= \omega.r.\delta\theta \downarrow, \text{ acting radially inwards} \quad \text{...(Neglecting all other quantities)}$$

and $$by = oy - ob = (\omega + \delta\omega)\,(r + \delta r)\cos\delta\theta - \omega.r$$
$$= (\omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r)\cos\delta\theta - \omega.r$$

Since δθ is small, therefore substituting cos δθ = 1, we have

$$by = \omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r - \omega.r = \omega.\delta r + r.\delta\omega \quad \text{...(Neglecting } \delta\omega.\delta r)$$

...(Perpendicular to OA and towards left)

Therefore, total component of change of velocity along radial direction

$$= bx - yb_1 = (\delta v - \omega.r.\delta\theta) \uparrow \quad \text{...(Acting radially outwards from } O \text{ to } A)$$

∴ Radial component of the acceleration of the slider B with respect to O on the link OA, acting radially outwards from O to A,

$$a^r_{BO} = \text{Lt}\,\frac{\delta v - \omega.r.\delta\theta}{\delta t} = \frac{dv}{dt} - \omega.r \times \frac{d\theta}{dt} = \frac{dv}{dt} - \omega^2.r \uparrow \quad \text{...}(i)$$

...($\because d\theta/dt = \omega$)

Also, the total component of change of velocity along tangential direction,

$$= xb_1 + by = v.\overleftarrow{\delta\theta} + (\omega.\overleftarrow{\delta r} + r.\delta\omega)$$

...(Perpendicular to OA and towards left)

∴ Tangential component of acceleration of the slider B with respect to O on the link OA, acting perpendicular to OA and towards left,

$$a^t_{BO} = \text{Lt}\,\frac{v.\delta\theta + (\omega.\delta r + r.\delta\omega)}{\delta t} = v\frac{d\theta}{dt} + \omega\frac{dr}{dt} + r\frac{d\omega}{dt}$$
$$= v.\omega + \omega.v + r.\alpha = (2\,v.\overleftarrow{\omega} + r.\alpha) \quad \text{...}(ii)$$

...($\because dr/dt = v$, and $d\omega/dt = \alpha$)

Now radial component of acceleration of the coincident point C with respect to O, acting in a direction from C to O,

$$a^r_{CO} = \omega^2.r \uparrow \quad \text{...}(iii)$$

and tangential component of acceleraiton of the coincident point C with respect to O, acting in a direction perpendicular to CO and towards left,

$$a^t_{\text{CO}} = \overleftarrow{\alpha.r} \uparrow \quad \text{...}(iv)$$

Radial component of the slider B with respect to the coincident point C on the link OA, acting radially outwards,

$$a^r_{\text{BC}} = a^r_{\text{BO}} - a^r_{\text{CO}} = \left(\frac{dv}{dt} - \omega^2.r\right) - \left(-\omega^2.r\right) = \frac{dv}{dt} \uparrow$$

and tangential component of the slider B with respect to the coincident point C on the link OA acting in a direction perpendicular to OA and towards left,

$$a^t_{\text{BC}} = a^t_{\text{BO}} - a^t_{\text{CO}} = (2\omega.v + \alpha.r) - \alpha.r = 2\overleftarrow{\omega.v}$$

This tangential component of acceleration of the slider B with respect to the coincident point C on the link is known as ***coriolis component of acceleration*** and is always perpendicualr to the link.

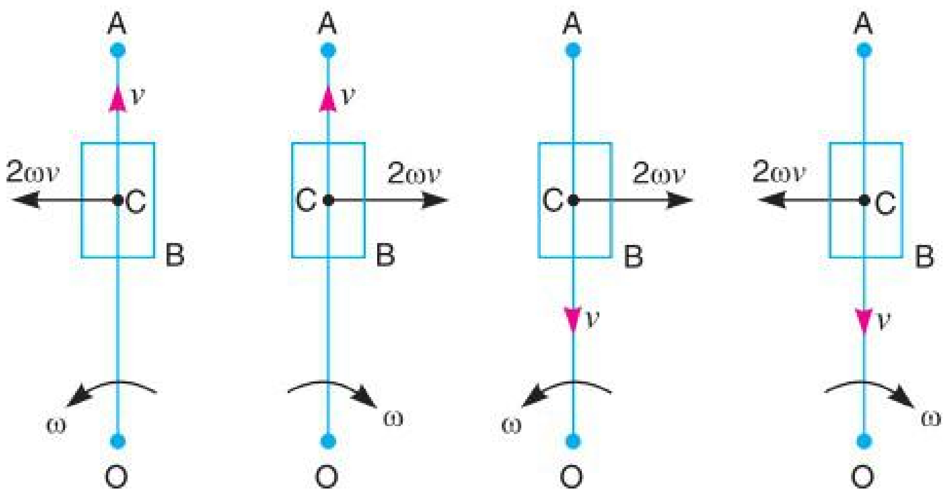
∴ Coriolis component of the acceleration of B with respect of C,

$$a^c_{\text{BC}} = a^t_{\text{BC}} = 2\,\omega.v$$

where ω = Angular velocity of the link OA, and

v = Velocity of slider B with respect to coincident point C.

In the above discussion, the anticlockwise direction for ω and the radially outward direction for v are taken as ***positive.*** It may be noted that the direction of coriolis component of acceleration changes sign, if either ω or v is reversed in direction. But the direction of coriolis component of acceleration will not be changed in sign if both ω and v are reversed in direction. It is concluded that the direction of coriolis component of acceleration is obtained by rotating v, at 90°, about its origin in the same direction as that of ω.

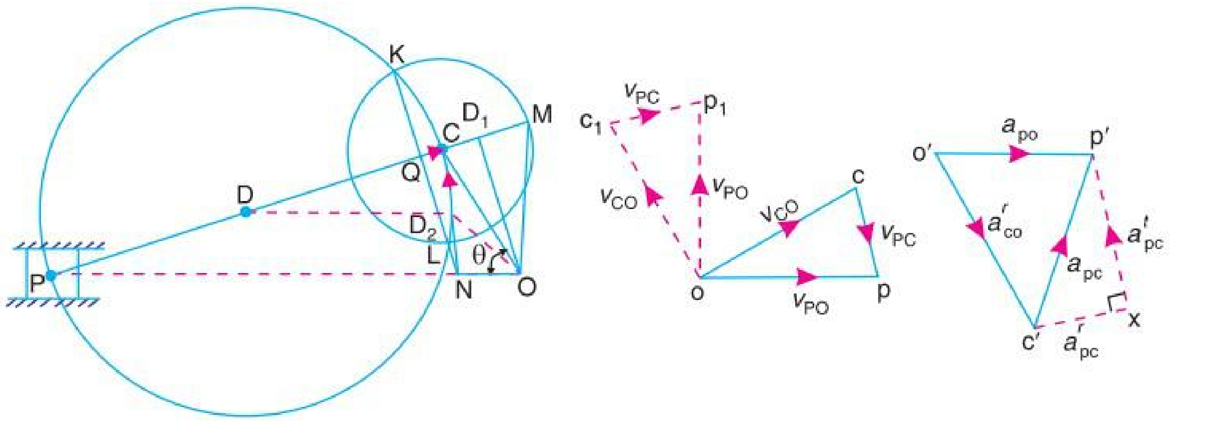


Direction of coriolis component of acceleration.

The direction of coriolis component of acceleration (2 ω.v) for all four possible cases, is shown in above fig. The directions of ω and v are given.

Klien’s Construction:

Let OC be the crank and PC the connecting rod of a reciprocating steam engine, as shown in below fig. (a). Let the crank makes an angle θ with the line of stroke PO and rotates with uniform angular velocity ω rad/s in a clockwise direction. The Klien’s velocity and acceleration diagrams are drawn as discussed below:



(*a*) Klien’s acceleration diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Klien’s construction.

Klien’s velocity diagram:

First of all, draw OM perpendicular to OP; such that it intersects the line PC produced at M. The triangle OCM is known as **Klien’s velocity diagram**. In this triangle OCM,

OM may be regarded as a line perpendicular to PO,

CM may be regarded as a line parallel to PC, and ...(since It is the same line.)

CO may be regarded as a line parallel to CO.

We have already discussed that the velocity diagram for given configuration is a triangle ocp as shown in Fig. (b). If this triangle is revolved through 90°, it will be a triangle oc_1p_1 , in which oc_1 represents v_{CO} (i.e. velocity of C with respect to O or velocity of crank pin C) and is parallel to OC,

op_1 represents v_{PO} (*i.e.* velocity of *P* with respect to *O* or velocity of cross-head or piston *P*) and is perpendicular to *OP*, and

c_1p_1 represents v_{PC} (*i.e.* velocity of *P* with respect to *C*) and is parallel to *CP*.

A little consideration will show, that the triangles oc_1p_1 and *OCM* are similar. Therefore,

$$\frac{oc_1}{OC} = \frac{op_1}{OM} = \frac{c_1 p_1}{CM} = \omega \text{ (a constant)}$$

$$\frac{v_{CO}}{OC} = \frac{v_{PO}}{OM} = \frac{v_{PC}}{CM} = \omega$$

$$v_{CO} = \omega \times OC\,;\ v_{PO} = \omega \times OM, \text{ and } v_{PC} = \omega \times CM$$

Thus, we see that by drawing the Klien's velocity diagram, the velocities of various points may be obtained without drawing a separate velocity diagram.

Klien's acceleration diagram:

The Klien's acceleration dia gram is drawn as discussed below:

1. First of all, draw a circle with C as centre and CM as radius.

2. Draw another circle with PC as diameter. Let this circle inter sect the previous circle at K and L.

3. Join KL and produce it to intersect PO at N. Let KL intersect PC at Q. This forms the quadrilateral CQNO, which is known as **Klien's acceleration diagram**.

(i) $o'c'$ represents a^r_{CO} (*i.e.* radial component of the acceleration of crank pin *C* with respect to *O*) and is parallel to *CO*;

(ii) $c'x$ represents a^r_{PC} (*i.e.* radial component of the acceleration of crosshead or piston *P* with respect to crank pin *C*) and is parallel to *CP* or *CQ*;

(iii) xp' represents a^t_{PC} (*i.e.* tangential component of the acceleration of *P* with respect to *C*) and is parallel to *QN* (because *QN* is perpendicular to *CQ*); and

(iv) $o'p'$ represents a_{PO} (*i.e.* acceleration of *P* with respect to *O* or the acceleration of piston *P*) and is parallel to *PO* or *NO*.

A little consideration will show that the quadrilateral $o'c'x\,p'$ [Fig. 15.2 (*c*)] is similar to quadrilateral *CQNO* [Fig. 15.2 (*a*)]. Therefore,

$$\frac{o'c'}{OC} = \frac{c'x}{CQ} = \frac{xp'}{QN} = \frac{o'p'}{NO} = \omega^2 \text{ (a constant)}$$

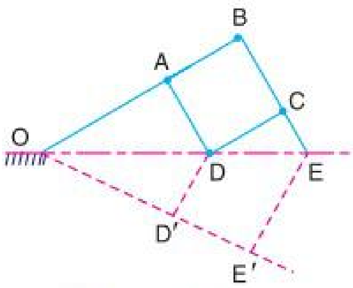
Thus we see that by drawing the Klien's acceleration diagram, the acceleration of various points may be obtained without drawing the separate acceleration diagram.

UNIT – III

STRAIGHT LINE MOTION MECHANISMS, STEERING MECHANISMS, AND HOOKE'S JOINT

Pantograph:

A pantograph is an instrument used to reproduce to an enlarged or a reduced scale and as exactly as possible the path described by a given point.



Pantograph

It consists of a jointed parallelogram ABCD as shown in below fig. It is made up of bars connected by turning pairs. The bars BA and BC are extended to O and E respectively, such that

$$OA/OB = AD/BE$$

Thus, for all relative positions of the bars, the triangles OAD and OBE are similar and the points O, D and E are in one straight line. It may be proved that point E traces out the same path as described by point D.

From similar triangles OAD and OBE, we find that

$$OD/OE = AD/BE$$

Let point O be fixed and the points D and E move to some new positions D′ and E′. Then

$$OD/OE = OD'/OE'$$

A little consideration will show that the straight line DD′ is parallel to the straight line EE′. Hence, if O is fixed to the frame of a machine by means of a turning pair and D is attached to a point in the machine which has rectilinear motion relative to the frame, then E will also trace out a straight line path. Similarly, if E is constrained to move in a straight line, then D will trace out a straight line parallel to the former.

A pantograph is mostly used for the reproduction of plane areas and figures such as maps, plans etc., on enlarged or reduced scales. A modified form of pantograph is used to collect electricity/power at the top of an electric locomotive.

Straight Line Mechanisms:

One of the most common forms of the constraint mechanisms is that it permits only relative motion of an oscillatory nature along a straight line. The mechanisms used for this purpose are called **straight line mechanisms**. These mechanisms are of the following two types:

1. in which only turning pairs are used, and
2. in which one sliding pair is used. These two types of mechanisms may produce exact straight line motion or approximate straight line motion.

Exact Straight Line Motion Mechanisms Made up of Turning Pairs:

The principle adopted for a mathematically correct or exact straight line motion is described in Fig.9.2. Let O be a point on the circumference of a circle of diameter OP. Let OA be any chord and B is a point on OA produced, such that

$$OA \times OB = \text{constant}$$

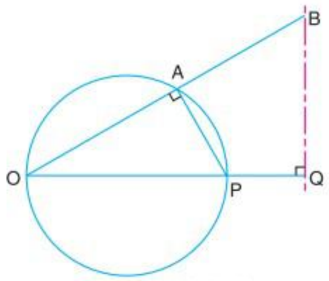
Then the locus of a point B will be a straight line perpendicular to the diameter OP. This may be proved as follows:

Draw BQ perpendicular to OP produced. Join AP. The triangles OAP and OBQ are similar.

$$\therefore \quad \frac{OA}{OP} = \frac{OQ}{OB}$$

or $$OP \times OQ = OA \times OB$$

or $$OQ = \frac{OA \times OB}{OP}$$



Exact straight line motion mechanism.

But OP is constant as it is the diameter of a circle, therefore, if OA × OB is constant, then OQ will be constant. Hence the point B moves along the straight path BQ which is perpendicular to OP.

Following are the two well known types of exact straight line motion mechanisms made up of turning pairs.

Peaucellier mechanism. It consists of a fixed link OO_1 and the other straight links O_1A, OC, OD, AD, DB, BC and CA are connected by turning pairs at their intersections, as shown in below fig. The pin at A is constrained to move along the circumference of a circle with the fixed diameter OP, by means of the link O_1A.

$$AC = CB = BD = DA;\ OC = OD;\ \text{and } OO_1 = O_1A$$

It may be proved that the product OA × OB remains constant, when the link O_1A rotates. Join CD to bisect AB at R. Now from right angled triangles ORC and BRC, we have

$$OC^2 = OR^2 + RC^2 \quad ...(i)$$

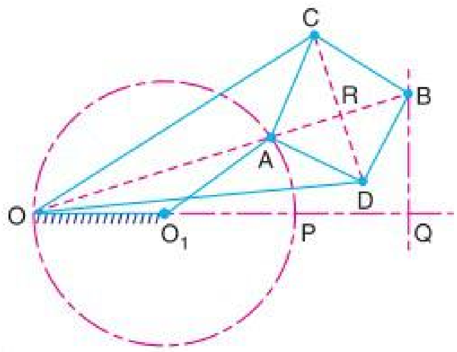
and

$$BC^2 = RB^2 + RC^2 \quad ...(ii)$$

Subtracting equation *(ii)* from *(i)*, we have

$$OC^2 - BC^2 = OR^2 - RB^2$$
$$= (OR + RB)(OR - RB)$$
$$= OB \times OA$$

Since *OC* and *BC* are of constant length, therefore the product *OB* × *OA* remains constant. Hence the point *B* traces a straight path perpendicular to the diameter *OP*.



Peaucellier mechanism.

Hart's mechanism. This mechanism requires only six links as compared with the eight links required by the Peaucellier mechanism. It consists of a fixed link OO_1 and other straight links O_1A, FC, CD, DE and EF are connected by turning pairs at their points of intersection, as shown in below fig. The links FC and DE are equal in length and the lengths of the links CD and EF are also equal. The points O, A and B divide the links FC, CD and EF in the same ratio. A little consideration will show that BOCE is a trapezium and OA and OB are respectively parallel to FD and CE.

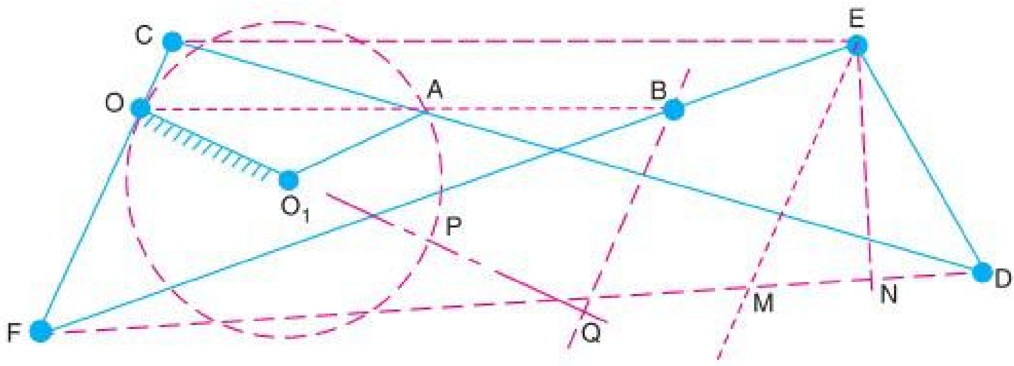
Hence OAB is a straight line. It may be proved now that the product OA × OB is constant.

From similar triangles *CFE* and *OFB*,

$$\frac{CE}{FC} = \frac{OB}{OF} \quad \text{or} \quad OB = \frac{CE \times OF}{FC} \quad ...(i)$$

and from similar triangles *FCD* and *OCA*

$$\frac{FD}{FC} = \frac{OA}{OC} \quad \text{or} \quad OA = \frac{FD \times OC}{FC} \quad ...(ii)$$



Hart's mechanism.

Multiplying equations (*i*) and (*ii*), we have

$$OA \times OB = \frac{FD \times OC}{FC} \times \frac{CE \times OF}{FC} = FD \times CE \times \frac{OC \times OF}{FC^2}$$

Since the lengths of OC, OF and FC are fixed, therefore

$$OA \times OB = FD \times CE \times \text{constant} \quad ...(iii)$$

$$...\left(\text{substituting } \frac{OC \times OF}{FC^2} = \text{constant}\right)$$

Now from point E, draw EM parallel to CF and EN perpendicular to FD. Therefore

$$FD \times CE = FD \times FM \quad ...(\because CE = FM)$$

$$= (FN + ND)(FN - MN) = FN^2 - ND^2 \quad ...(\because MN = ND)$$

$$= (FE^2 - NE^2) - (ED^2 - NE^2)$$

...(From right angled triangles FEN and EDN)

$$= FE^2 - ED^2 = \text{constant} \quad ...(iv)$$

...($\because$ Length FE and ED are fixed)

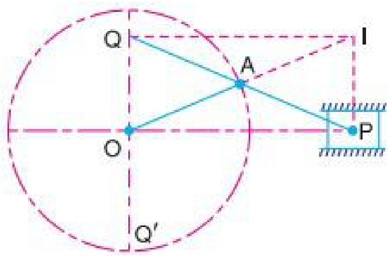
From equations (*iii*) and (*iv*),

$$OA \times OB = \text{constant}$$

It therefore follows that if the mechanism is pivoted about O as a fixed point and the point A is constrained to move on a circle with centre O_1, then the point B will trace a straight line perpendicular to the diameter OP produced.

Exact Straight Line Motion Consisting of One Sliding Pair-Scott Russell's Mechanism:

It consists of a fixed member and moving member P of a sliding pair as shown in below fig. The straight link PAQ is connected by turning pairs to the link OA and the link P. The link OA rotates about O. A little consideration will show that the mechanism OAP is same as that of the reciprocating engine mechanism in which OA is the crank and PA is the connecting rod.



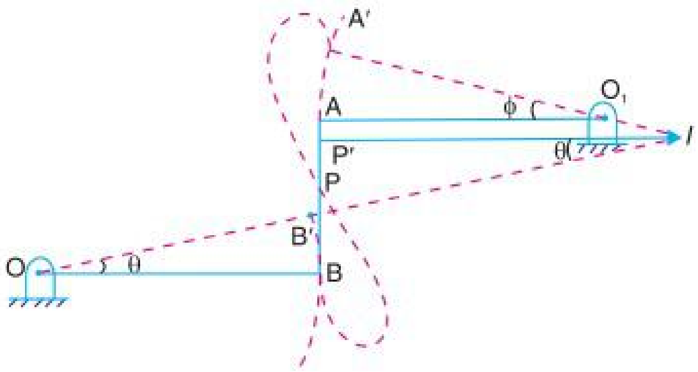
Scott Russell's mechanism.

In above fig., A is the middle point of PQ and OA=AP=AQ. The instantaneous centre for the link PAQ lies at I in OA produced and is such that IP is perpendicular to OP. Join IQ. Then Q moves along the perpendicular to IQ. Since OPIQ is a rectangle and IQ is perpendicular to OQ, therefore Q moves along the vertical line OQ for all positions of QP. Hence Q traces the straight line OQ′. If OA makes one complete revolution, then P will oscillate along the line OP through a distance 2 OA on each side of O and Q will oscillate along OQ′ through the same distance 2 OA above and below O. Thus, the locus of Q is a copy of the locus of P.

Approximate Straight Line Motion Mechanisms:

The approximate straight line motion mechanisms are the modifications of the four-bar chain mechanisms.

Watt's mechanism. It is a crossed four bar chain mechanism and was used by Watt for his early steam engines to guide the piston rod in a cylinder to have an approximate straight line motion.



Watt's mechanism.

A little consideration will show that in the initial mean position of the mechanism, the instantaneous centre of the link BA lies at infinity. Therefore the motion of the point P is along the vertical line BA. Let OB′ A′O_1 be the new position of the mechanism after the links OB and O_1A are displaced through an angle θ and φ respectively. The instantaneous centre now lies at I. Since the angles θ and φ are very small, therefore

$$\text{arc } BB' = \text{arc } AA' \text{ or } OB \times \theta = O_1A \times \varphi \qquad \ldots(i)$$

∴ $OB / O_1A = \phi / \theta$

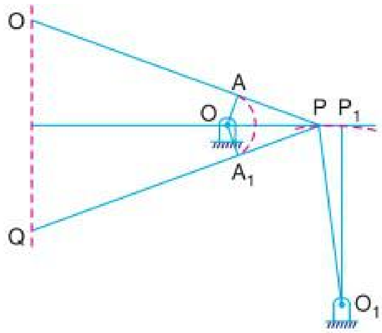
Also $A'P' = IP' \times \phi$, and $B'P' = IP' \times \theta$

∴ $A'P' / B'P' = \phi / \theta \qquad \ldots(ii)$

From equations (*i*) and (*ii*),

$$\frac{OB}{O_1A} = \frac{A'P'}{B'P'} = \frac{AP}{BP} \quad \text{or} \quad \frac{O_1A}{OB} = \frac{PB}{PA}$$

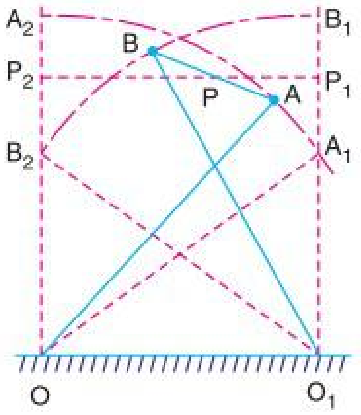
Grasshopper mechanism. This mechanism is a modification of modified Scott-Russel's mechanism with the difference that the point P does not slide along a straight line, but moves in a circular arc with centre O.



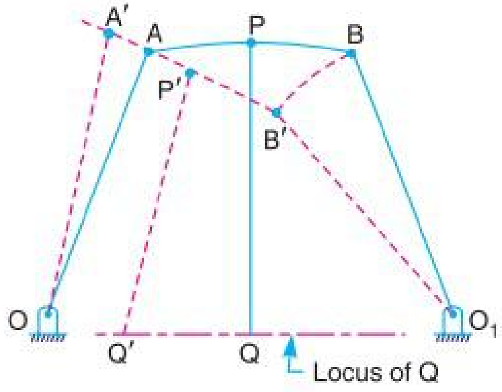
Grasshopper mechanism.

It is a four bar mechanism and all the pairs are turning pairs as shown in above fig. In this mechanism, the centres O and O_1 are fixed. The link OA oscillates about O through an angle AOA_1 which causes the pin P to move along a circular arc with O_1 as centre and O_1P as radius. For small angular displacements of OP on each side of the horizontal, the point Q on the extension of the link PA traces out an approximately a straight path QQ′, if the lengths are such that $OA = (AP)^2/AQ$.

Tchebicheff's mechanism. It is a four bar mechanism in which the crossed links OA and O_1B are of equal length, as shown in below fig. The point P, which is the mid-point of AB traces out an approximately straight line parallel to OO_1. The proportions of the links are, usually, such that point P is exactly above O or O1 in the extreme positions of the mechanism i.e. when B A lies along OA or when B A lies along BO_1. It may be noted that the point P will lie on a straight line parallel to OO_1 , in the two extreme positions and in the mid position, if the lengths of the links are in proportions $AB : OO_1 : OA = 1 : 2 : 2.5$.



Tchebicheff's mechanism



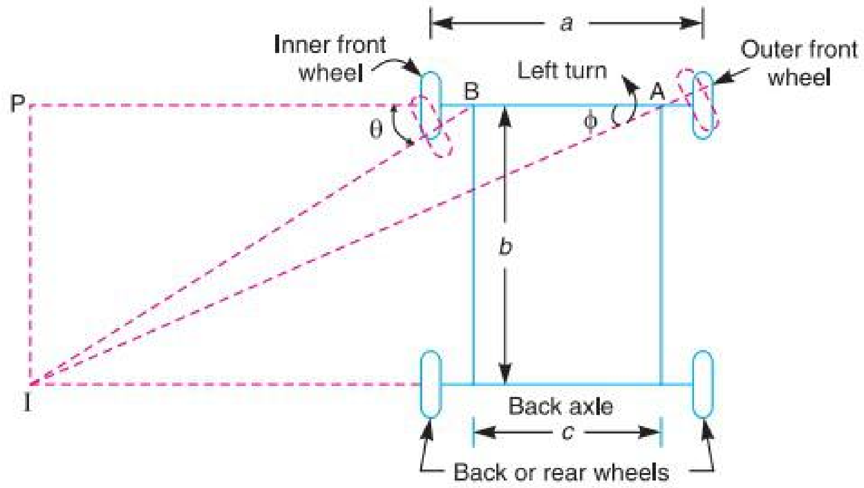
Roberts mechanism

Roberts mechanism. It is also a four bar chain mechanism, which, in its mean position, has the form of a trapezium. The links OA and O_1B are of equal length and OO_1 is fixed. A bar PQ is rigidly attached to the link A B at its middle point P.

A little consideration will show that if the mechanism is displaced as shown by the dotted lines in above fig., the point Q will trace out an approximately straight line.

Steering Gear Mechanism:

The steering gear mechanism is used for changing the direction of two or more of the wheel axles with reference to the chassis, so as to move the automobile in any desired path. Usually the two back wheels have a common axis, which is fixed in direction with reference to the chassis and the steering is done by means of the front wheels.



Steering gear mechanism

In order to avoid skidding (i.e. slipping of the wheels sideways), the two front wheels must turn about the same instantaneous centre I which lies on the axis of the back wheels. If the instantaneous centre of the two front wheels do not coincide with the instantaneous centre of the back wheels, the skidding on the front or back wheels will definitely take place, which will cause more wear and tear of the tyres.

Thus, the condition for correct steering is that all the four wheels must turn about the same instantaneous centre. The axis of the inner wheel makes a larger turning angle θ than the angle φ subtended by the axis of outer wheel.

Let a = Wheel track,

b = Wheel base, and

c = Distance between the pivots A and B of the front axle.

Now from triangle IBP,

$$\cot\theta = \frac{BP}{IP}$$

and from triangle *IAP*,

$$\cot\phi = \frac{AP}{IP} = \frac{AB + BP}{IP} = \frac{AB}{IP} + \frac{BP}{IP} = \frac{c}{b} + \cot\theta \qquad ...(\because IP = b)$$

$$\therefore \cot\phi - \cot\theta = c / b$$

This is the fundamental equation for correct steering. If this condition is satisfied, there will be no skidding of the wheels, when the vehicle takes a turn.

Davis Steering Gear:

It is an exact steering gear mechanism. The slotted links A M and BH are attached to the front wheel axle, which turn on pivots A and B respectively. The rod CD is constrained to move in the direction of its length, by the sliding members at P and Q. These constraints are connected to the slotted link A M and BH by a sliding and a turning pair at each end. The steering is affected by moving CD to the right or left of its normal position. C'D′ shows the position of CD for turning to the left.

Let a = Vertical distance between A B and CD,

b = Wheel base,

d = Horizontal distance between AC and BD,

c = Distance between the pivots A and B of the front axle.

x = Distance moved by AC to AC′ = CC′ = DD′, and

α = Angle of inclination of the links AC and BD, to the vertical.

From triangle A A′C′,

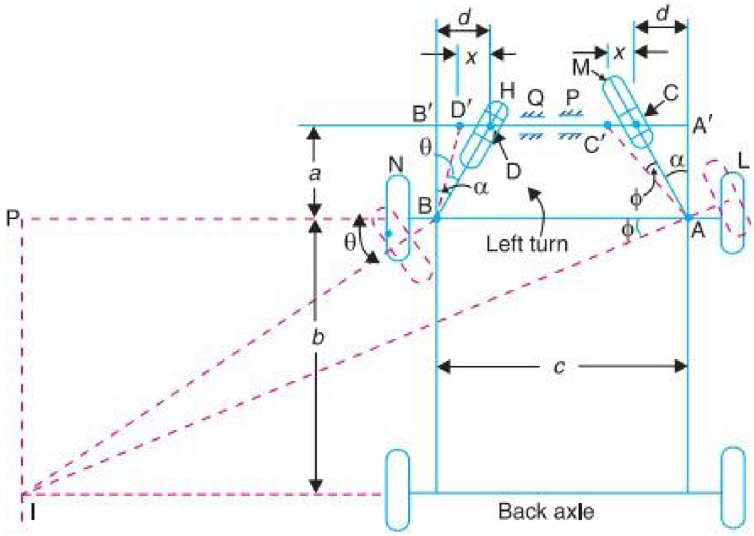
$$\tan(\alpha + \phi) = \frac{A'C'}{AA'} = \frac{d + x}{a} \qquad ...(i)$$

From triangle $A A'C$,

$$\tan\alpha = \frac{A'C}{AA'} = \frac{d}{a} \qquad ...(ii)$$

From triangle $BB'D'$,

$$\tan(\alpha - \theta) = \frac{B'D'}{BB'} = \frac{d - x}{a} \qquad ...(iii)$$



We know that $\tan(\alpha + \phi) = \dfrac{\tan\alpha + \tan\phi}{1 - \tan\alpha . \tan\phi}$

or $$\frac{d + x}{a} = \frac{d/a + \tan\phi}{1 - d/a \times \tan\phi} = \frac{d + a\tan\phi}{a - d\tan\phi}$$

...[From equations (*i*) and (*ii*)]

$$(d + x)(a - d\tan\phi) = a(d + a\tan\phi)$$

$$a.\,d - d^2\tan\phi + a.\,x - d.x\tan\phi = a.d + a^2\tan\phi$$

$$\tan\phi\,(a^2 + d^2 + d.x) = ax \quad \text{or} \quad \tan\phi = \frac{a.x}{a^2 + d^2 + d.x} \qquad ...(iv)$$

Similarly, from $\tan(\alpha - \theta) = \dfrac{d - x}{a}$, we get

$$\tan\theta = \frac{ax}{a^2 + d^2 - d.x} \qquad ...(v)$$

We know that for correct steering,

$$\cot\phi - \cot\theta = \frac{c}{b} \quad \text{or} \quad \frac{1}{\tan\phi} - \frac{1}{\tan\theta} = \frac{c}{b}$$

$$\frac{a^2 + d^2 + d.x}{a.x} - \frac{a^2 + d^2 - d.x}{a.x} = \frac{c}{b}$$

...[From equations (*iv*) and (*v*)]

$$\text{or} \qquad \frac{2d.x}{a.x} = \frac{c}{b} \qquad \text{or} \qquad \frac{2d}{a} = \frac{c}{b}$$

$$\therefore \qquad 2\tan\alpha = \frac{c}{b} \qquad \text{or} \qquad \tan\alpha = \frac{c}{2b} \qquad \ldots(\because d/a = \tan\alpha)$$

P) In a Davis steering gear, the distance between the pivots of the front axle is 1.2 metres and the wheel base is 2.7 metres. Find the inclination of the track arm to the longitudinal axis of the car, when it is moving along a straight path.

Solution. Given : $c = 1.2$ m ; $b = 2.7$ m

Let α = Inclination of the track arm to the longitudinal axis.

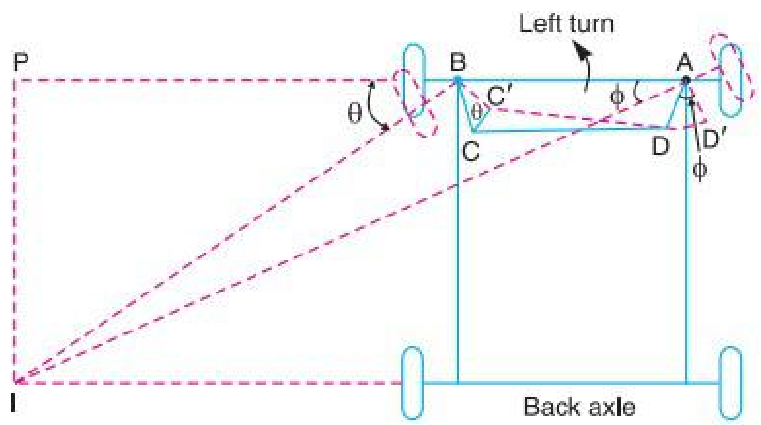
We know that $\tan\alpha = \dfrac{c}{2b} = \dfrac{1.2}{2 \times 2.7} = 0.222$ or $\alpha = 12.5°$ **Ans.**

Ackerman Steering Gear:

The Ackerman steering gear mechanism is much simpler than Davis gear. The difference between the Ackerman and Davis steering gears are:

1. The whole mechanism of the Ackerman steering gear is on back of the front wheels; whereas in Davis steering gear, it is in front of the wheels.

2. The Ackerman steering gear consists of turning pairs, whereas Davis steering gear consists of sliding members.



The shorter links BC and AD are of equal length and are connected by hinge joints with front wheel axles. The longer links A B and CD are of unequal length. The following are the only three positions for correct steering.

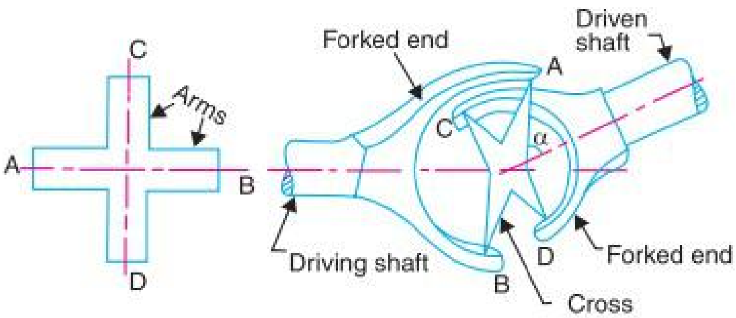
1. When the vehicle moves along a straight path, the longer links A B and CD are parallel and the shorter links BC and AD are equally inclined to the longitudinal axis of the vehicle, as shown by firm lines in above fig.

2. When the vehicle is steering to the left, the position of the gear is shown by dotted lines in above fig. In this position, the lines of the front wheel axle intersect on the back wheel axle at I, for correct steering.

3. When the vehicle is steering to the right, the similar position may be obtained.

Universal Or Hooke's Joint:

A Hooke's joint is used to connect two shafts, which are intersecting at a small angle, as shown in below fig. The end of each shaft is forked to U-type and each fork provides two bearings for the arms of a cross. The arms of the cross are perpendicular to each other. The motion is transmitted from the driving shaft to driven shaft through a cross. The inclination of the two shafts may be constant, but in actual practice it varies, when the motion is transmitted. The main application of the Universal or Hooke's joint is found in the transmission from the gear box to the differential or back axle of the automobiles. It is also used for transmission of power to different spindles of multiple drilling machine. It is also used as a knee joint in milling machines.



Double Hooke's Joint:

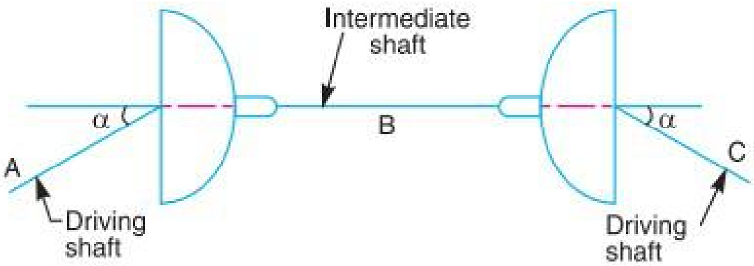
The velocity of the driven shaft is not constant, but varies from maximum to minimum values. In order to have a constant velocity ratio of the driving and driven shafts, an intermediate shaft with a Hooke's joint at each end as shown in below fig. is used. This type of joint is known as double Hooke's joint.

Let the driving, intermediate and driven shafts, in the same time, rotate through angles θ, φ and γ from the position.

Now for shafts A and B, $\tan\theta = \tan\varphi \cdot \cos\alpha$ ...(i)

and for shafts B and C, $\tan\gamma = \tan\varphi \cdot \cos\alpha$ ...(ii)

From equations (i) and (ii), we see that $\theta = \gamma$ or $\omega_A = \omega_C$.



This shows that the speed of the driving and driven shaft is constant. In other words, this joint gives a velocity ratio equal to unity, if

1. The axes of the driving and driven shafts are in the same plane, and
2. The driving and driven shafts make equal angles with the intermediate shaft.

Ratio of the Shafts Velocities:

The top and front views connecting the two shafts by a universal joint are shown in Fig. Let the initial position of the cross be such that both arms lie in the plane of the paper in front view, while the arm AB attached to the driving shaft lies in the plane containing the axes of the two shafts.

Let the driving shaft rotates through an angle θ, so that the arm AB moves in a circle to a new position A_1B_1 as shown in front view.

A little consideration will show that the arm CD will also move in a circle of the same size. This circle when projected in the plane of paper appears to be an ellipse.

Therefore the arm CD takes new position C_1D_1 on the ellipse, at an angle θ. But the true angle must be on the circular path.

To find the true angle, project the point C_1 horizontally to intersect the circle at C_2.

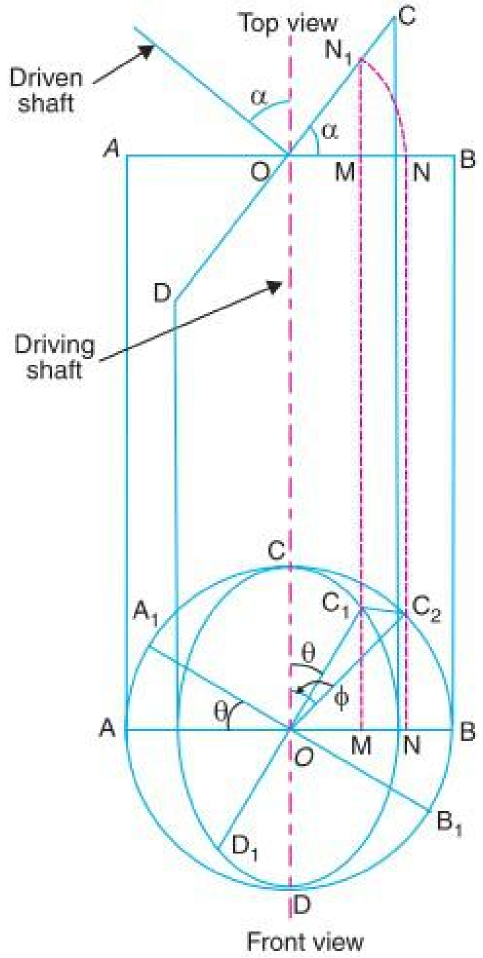
Thus when the driving shaft turns through an angle θ, the driven shaft turns through an angle φ.

In triangle OC_1M, $\angle OC_1M = \theta$

$$\therefore \quad \tan\theta = \frac{OM}{MC_1} \quad \ldots(i)$$

and in triangle OC_2N, $\angle OC_2N = \phi$

$$\therefore \quad \tan\phi = \frac{ON}{NC_2} = \frac{ON}{MC_1} \quad \ldots(\because NC_2 = MC_1) \quad \ldots(ii)$$



Dividing equation (*i*) by (*ii*),

$$\frac{\tan\theta}{\tan\phi} = \frac{OM}{MC_1} \times \frac{MC_1}{ON} = \frac{OM}{ON}$$

But $\quad OM = ON_1 \cos\alpha = ON \cos\alpha$

...(where α = Angle of inclination of the driving and driven shafts)

$$\therefore \quad \frac{\tan\theta}{\tan\phi} = \frac{ON\cos\alpha}{ON} = \cos\alpha$$

or $$\tan\theta = \tan\phi \,.\, \cos\alpha \qquad \text{...}(iii)$$

Let $\quad \omega =$ Angular velocity of the driving shaft $= d\theta / dt$

$\quad \omega_1 =$ Angular velocity of the driven shaft $= d\phi / dt$

Differentiating both sides of equation (*iii*),

$$\sec^2\theta \times d\theta / dt = \cos\alpha \,.\, \sec^2\phi \times d\phi / dt$$

$$\sec^2\theta \times \omega = \cos\alpha \,.\, \sec^2\phi \times \omega_1$$

$$\therefore \quad \frac{\omega_1}{\omega} = \frac{\sec^2\theta}{\cos\alpha.\sec^2\phi} = \frac{1}{\cos^2\theta.\cos\alpha.\sec^2\phi} \qquad \text{...}(iv)$$

We know that $\sec^2\phi = 1 + \tan^2\phi = 1 + \dfrac{\tan^2\theta}{\cos^2\alpha}$ ...[From equation (*iii*)]

$$= 1 + \frac{\sin^2\theta}{\cos^2\theta.\cos^2\alpha} = \frac{\cos^2\theta.\cos^2\alpha + \sin^2\theta}{\cos^2\theta.\cos^2\alpha}$$

$$= \frac{\cos^2\theta(1-\sin^2\alpha)+\sin^2\theta}{\cos^2\theta.\cos^2\alpha} = \frac{\cos^2\theta - \cos^2\theta.\sin^2\alpha + \sin^2\theta}{\cos^2\theta.\cos^2\alpha}$$

$$= \frac{1-\cos^2\theta.\sin^2\alpha}{\cos^2\theta.\cos^2\alpha} \quad ...(\because \cos^2\theta + \sin^2\theta = 1)$$

Substituting this value of $\sec^2\phi$ in equation (*iv*), we have veloity ratio,

$$\frac{\omega_1}{\omega} = \frac{1}{\cos^2\theta.\cos\alpha} \times \frac{\cos^2\theta.\cos^2\alpha}{1-\cos^2\theta.\sin^2\alpha} = \frac{\cos\alpha}{1-\cos^2\theta.\sin^2\alpha} \quad ...(v)$$

Note: If N = Speed of the driving shaft in r.p.m., and

N_1 = Speed of the driven shaft in r.p.m.

Then the equation (*v*) may also be written as

$$\frac{N_1}{N} = \frac{\cos\alpha}{1-\cos^2\theta.\sin^2\alpha}.$$

P) Two shafts are connected by a Hooke's joint. The driving shaft revolves uniformly at 500 r.p.m. If the total permissible variation in speed of the driven shaft is not to exceed ± 6% of the mean speed, find the greatest permissible angle between the centre lines of the shafts.

Solution. Given : $N = 500$ r.p.m. or $\omega = 2\pi \times 500 / 60 = 52.4$ rad/s

Let α = Greatest permissible angle between the centre lines of the shafts.

Since the variation in speed of the driven shaft is ± 6% of the mean speed (*i.e.* speed of the driving speed), therefore total fluctuation of speed of the driven shaft,

$$q = 12\ \%\text{ of mean speed } (\omega) = 0.12\,\omega$$

We know that maximum or total fluctuation of speed of the driven shaft (q),

$$0.12\,\omega = \omega\left(\frac{1-\cos^2\alpha}{\cos\alpha}\right) \quad \text{or} \quad \cos^2\alpha + 0.12\cos\alpha - 1 = 0$$

and $$\cos\alpha = \frac{-0.12 \pm \sqrt{(0.12)^2+4}}{2} = \frac{-0.12 \pm 2.0036}{2} = 0.9418$$

...(Taking + sign)

$\alpha = 19.64°$ **Ans.**

P) The angle between the axes of two shafts connected by Hooke's joint is 18°. Determine the angle turned through by the driving shaft when the velocity ratio is maximum and unity.

Given: $\alpha = 98°$

Let θ = Angle turned through by the driving shaft.

When the velocity ratio is maximum

We know that velocity ratio,

$$\frac{\omega_1}{\omega} = \frac{\cos\alpha}{1 - \cos^2\theta . \sin^2\alpha}$$

The velocity ratio will be maximum when $\cos^2\theta$ is minimum, *i.e.* when

$$\cos^2\theta = 1 \quad \text{or} \quad \text{when } \theta = 0° \quad \text{or} \quad 180° \quad \textbf{Ans.}$$

When the velocity ratio is unity

The velocity ratio (ω / ω_1) will be unity, when

$$1 - \cos^2\theta . \sin^2\alpha = \cos\alpha \qquad \text{or} \qquad \cos^2\theta = \frac{1 - \cos\alpha}{\sin^2\alpha}$$

$$\therefore \qquad \cos\theta = \pm\sqrt{\frac{1 - \cos\alpha}{\sin^2\alpha}} = \pm\sqrt{\frac{1 - \cos\alpha}{1 - \cos^2\alpha}} = \pm\sqrt{\frac{1}{1 + \cos\alpha}}$$

$$= \pm\sqrt{\frac{1}{1 + \cos 18°}} = \pm\sqrt{\frac{1}{1 + 0.9510}} = \pm 0.7159$$

$$\therefore \qquad \theta = 44.3° \quad \text{or} \quad 135.7° \quad \textbf{Ans.}$$