Unit-1
Three Hinged Arches.

An arch is a curved girder which takes the transverse loads. The Vertical loads makes an angle with the normal face on the arch. and hence the arch is subjected to the following three resisting forces,

1. Thrust
2. Shear force
3. Bending Moment.

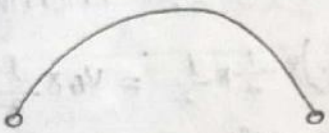
Based on the number of hinges, arches are divided as follows.

1. Three hinged Arch.



$D_s = 0$

2. Two hinged Arch.



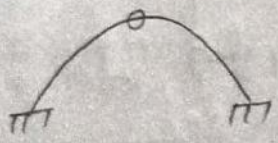
$D_s = 1$

3. fixed Arch



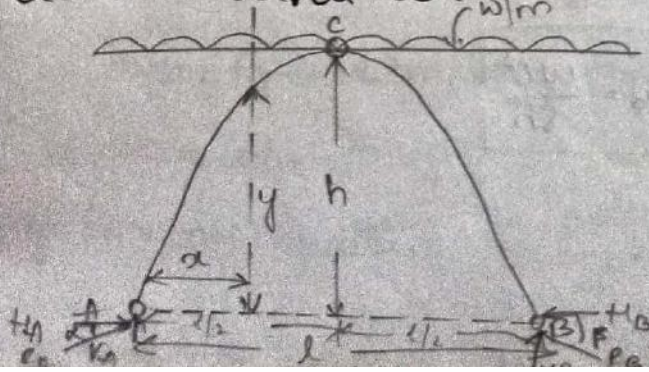
$D_s = 3$

4. Single hinged Arch.



$D_s = 1$

Analysis of three hinged Arch:



Scanned with CamScanner

Step ①

$\Sigma V = 0$

$V_A + V_B = w \times l$

Step ②

$\Sigma M_A = 0$

anti clockwise moments = clockwise moments

$V_B \times l = w \times l \times \frac{l}{2}$ $\quad \frac{wl^2}{2}$

$\therefore V_B = \frac{wl}{2}$

$V_A = wl - V_B$

$= wl - \frac{wl}{2}$

$\therefore V_A = \frac{wl}{2}$

Step ③

$\Sigma M_C = 0$ (Left Part due to symmetry)

anti clockwise moments = clockwise moments

$(H_A \times h) + (w \times \frac{l}{2}) \times \frac{1}{2} \times \frac{1}{2} = V_A \times \frac{l}{2}$.

$(H_A \times h) + \frac{wl^2}{8} = \frac{wl^2}{4}$

$H_A \times h = \frac{wl^2}{4} - \frac{wl^2}{8}$

$H_A \cdot h = \frac{wl^2}{8}$

***

$H_A = \frac{wl^2}{8h}$

Step ④

$\Sigma H = 0$

$H_A = H_B$

$\therefore H_B = \frac{wl^2}{8h}$

Scanned with CamScanner

Step ⑤

Resultant force

$$R_A = \sqrt{V_A^2 + H_A^2}$$

$$R_B = \sqrt{V_B^2 + H_B^2}$$

$$\tan\alpha = \frac{V_A}{H_A}$$

$$\therefore \alpha = \tan^{-1}\left(\frac{V_A}{H_A}\right)$$

$$\tan\beta = \frac{V_B}{H_B}$$

$$\therefore \beta = \tan^{-1}\left(\frac{V_B}{H_B}\right)$$

imp

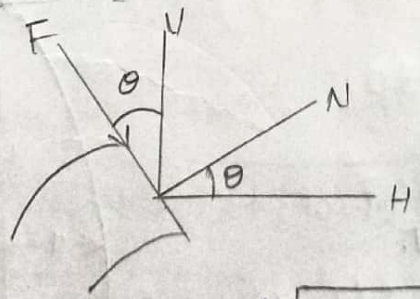
Step ⑥ Equation of Parabola.

$$y = \frac{4h}{l^2}x(l-x)$$

imp

Step ⑦

Radial shear and Normal thrust



Radial shear at any Point, $F = V\cos\theta - H\sin\theta$

Normal thrust, $N = V\sin\theta + H\cos\theta$

Where,

V = shear force under Considered Point

H = Horizontal thrust.

using Parabolic equation, $y = \frac{4h}{l^2}x(l-x)$

Scanned with CamScanner

$$y = \frac{4h}{l^2}(lx - x^2)$$

$$\tan\theta = \frac{dy}{dx} = \frac{d}{dx}\frac{4h}{l^2}(lx - x^2)$$

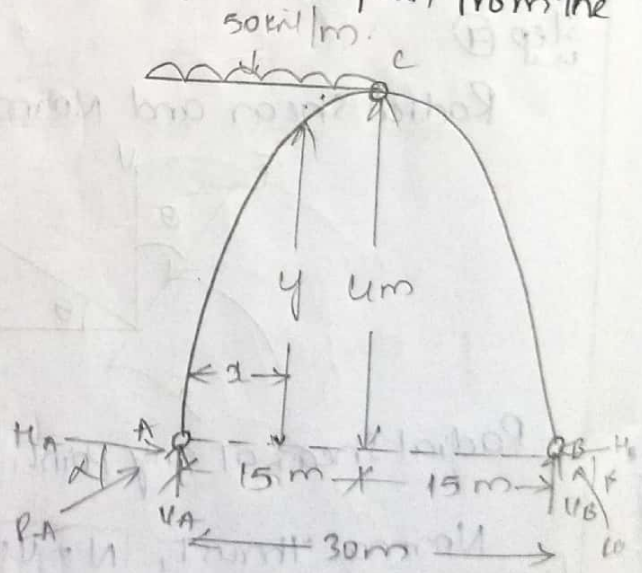
$$\boxed{\tan\theta = \frac{4h}{l^2}(l - 2x)}$$

Bending Moment at a section is calculated as follows

$$M_{x-x} = (V_A \cdot x) - (H_A \cdot y) - \left(w \cdot x \cdot \frac{x}{2}\right)$$

Problems

1. A three hinged Parabolic arch has a span of 30m and a central rise 4m and it is subjected to a U.D.L of 50 KN/m in the left hand span determine,
   (i) Resultant reactions and support and direction.
   (ii) Maximum Sagging Bending moment
   (iii) Maximum Hogging Bending moment
   (iv) Radial shear and normal thrust at quarter span from the left support



Step ①

$$\Sigma V = 0$$

$$V_A + V_B = wl$$

$$= 50 \times 15$$

$$V_A + V_B = 750 \text{ KN}.$$

Step ②

$$\Sigma M_A = 0$$

$$a_{cm} = c_m$$

$$V_B \times 30 = \frac{wl^2}{2}$$

$$V_B \times 30 = \frac{50 \times 15^2}{2}$$

$$\boxed{V_B = 187.5 \text{ KN}}$$

$$V_A + V_B = 750$$

Scanned with CamScanner

$V_A + 187.5 = 750$

$V_A = 562.5\ KN.$

Step ③

$\Sigma M_c = 0$

$acm = cm$

$(H_A \times h) + (\omega \times \frac{l}{2}) \times \frac{1}{2} \times \frac{1}{2} = V_A \times \frac{l}{2}$.

$(H_A \times 4) + (50 \times \frac{4}{2}) \times \frac{1}{2} \times \frac{4}{2} = 562.5 \times \frac{15}{2}$

Step ③

$\Sigma M_c = 0$ (from Right)

$acm = cm$

$V_B \times 15 = H_B \times 4$

$187.5 \times 15 = H_B \times 4$

$H_B = 703.12 KN$

Step ④

$\Sigma H = 0$

$H_A = H_B$

$\therefore H_A = 703.12 KN$

Step ⑤ Resultant reactions

$R_A = \sqrt{V_A^2 + H_A^2}$

$= \sqrt{(562.5)^2 + (703.12)^2}$

$R_A = 900.43 KN$

$R_B = \sqrt{V_B^2 + H_B^2}$

$= \sqrt{(187.5)^2 + (703.12)^2}$

$R_B = 727.69 KN$

Scanned with CamScanner

$$\alpha = \tan^{-1}\left(\frac{V_A}{H_A}\right)$$

$$= \tan^{-1}\left(\frac{562.5}{703.12}\right)$$

$$= 38.6^\circ$$

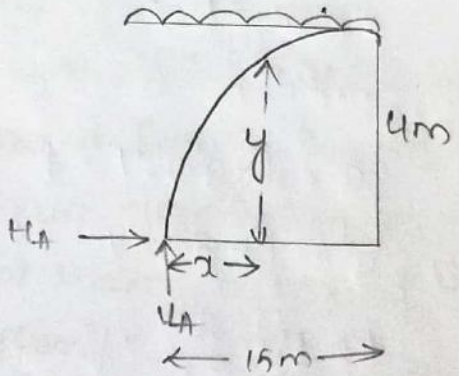
$$\beta = \tan^{-1}\left[\frac{V_B}{H_B}\right]$$

$$= \tan^{-1}\left(\frac{187.5}{703.12}\right)$$

$$\beta = 14.9^\circ$$

(ii) Maximum Sagging Moment (Left Part) loadey side

Paduthundho adhi Sagging lekapothe Hogging.



$$M_{x\text{-}x} = (V_A \cdot x) - (H_A \cdot y) - \left(w \cdot x \cdot \frac{x}{2}\right)$$

$$= (V_A \cdot x) - H_A \cdot y = \frac{50x^2}{2}$$

$$= V_A \cdot x - H_A \cdot y - 25x^2$$

Parabolic eqn, $y = \frac{4h}{l^2} x(l-x)$

$$= \frac{4 \times 4}{30^2} x(30-x)$$

$$= \frac{16}{900}(30x - x^2)$$

$$y = 0.54x - 0.018x^2$$

Substitute 'y' value in "$M_{x\text{-}x}$"

Scanned with CamScanner

$M_{x\text{-}x} = 562.5x - 703.12(0.54x - 0.018x^2) - 25x^2$

$= 562.5x - 379.68x + 12.65x^2 - 25x^2$

$M_{x\text{-}x} = 182.82x - 12.34x^2$

We know, maximum bending moment occurs at a point where shear force is zero

$SF = 0$

$\frac{dm_{x\text{-}x}}{dx} = 0$

$\frac{d}{dx}[182.82x - 12.3x^2] = 0$

$182.82 - 12.34 \cdot 2x = 0$

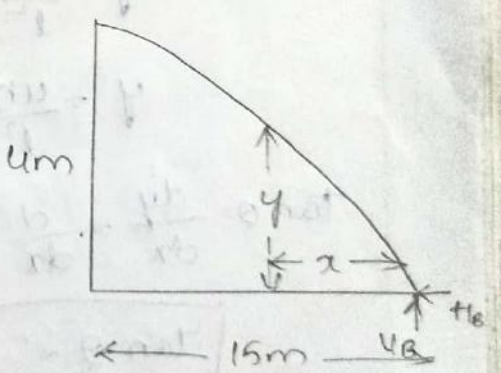
$x = 7.4m$

max. sagging moment,

$M_{x\text{-}x} = 182.82(7.4) - 12.34(7.4)^2$

$M_{x\text{-}x} = 677.13 KN\text{-}m$

(iii) Maximum tlogging Moment (Right Part)



$M_{x\text{-}x} = V_B \cdot x - (H_B \cdot y)$

$y = \frac{4h}{l^2} x(l - x)$

$= \frac{4h}{l^2}(lx - x^2)$

$= \frac{4 \times 4}{30^2}(30x - x^2)$

$y = 0.54x - 0.018x^2$

Substitute 'y' in $M_{x\text{-}x}$

$M_{x\text{-}x} = 187.5x - 703.12(0.54x - 0.018x^2)$

$= 187.5x - 379.68x + 12.65x^2$

$M_{x\text{-}x} = -192.18x + 12.65x^2$

Scanned with CamScanner

We know,

$$\frac{dM_{x-x}}{dx} = 0.$$

$$\frac{d}{dx}\left[-192.18x + 12.65x^2\right] = 0.$$

$$-192.18 + 12.65 \times 2x = 0$$

$$\boxed{x = 7.59\text{m}}$$

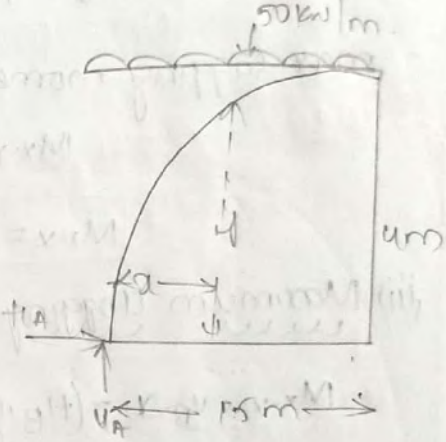
$$M_{x-x} = -192.18(7.59) + 12.65(7.59)^2$$

$$\boxed{M_{x-x} = -729.9\ \text{KN-m}}$$

Q) Radial shear and Normal thrust at quarter span from left

$$\therefore x = \frac{30}{4}$$

$$x = 7.5\text{m}$$



$$F = V\cos\theta - H\sin\theta$$

$$N = V\sin\theta + H\cos\theta$$

$$y = \frac{4h}{l^2}x(l-x)$$

$$y = \frac{4h}{l^2}(lx - x^2)$$

$$\tan\theta = \frac{dy}{dx} = \frac{d}{dx}\left[\frac{4h}{l^2}(lx - x^2)\right]$$

$$\boxed{\tan\theta = \frac{4h}{l^2}(l - 2x)}$$

$$= \frac{4 \times 4}{30^2}\left[30 - 2(7.5)\right]$$

$$\theta = 14.9^\circ$$

$$V = V_A - (50 \times 7.5)$$

$$= 562.5 - (50 \times 7.5)$$

$$V = 187.5\ \text{KN}.$$

Scanned with CamScanner

$F = V\cos\theta - H\sin\theta$

$= 187.5\cos(14.9°) - 703.12\sin(14.9)$

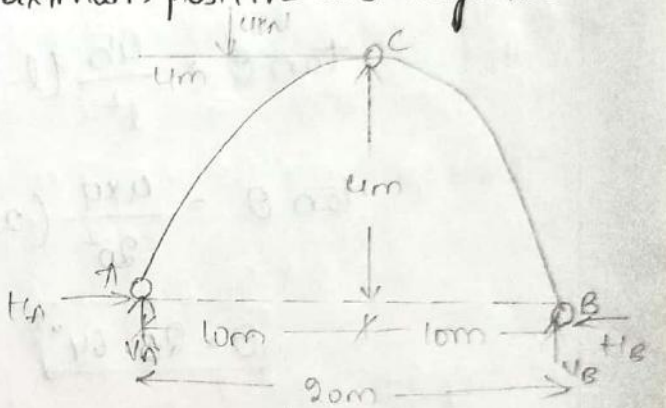
$F = 0.4\,kN$

$N = V\sin\theta + H\cos\theta$

$= 187.5\sin(14.9°) + 703.12\cos(14.9°)$

$N = 727.69\,kN.$

2. A three hinged Parabolic arch of 20m span and 4m central rise carries a point load of 4kN from the left hand hinge at a distance of 4m. Calculate normal thrust and Radial shear under the load. Also calculate maximum positive and negative bending moment.



Step ① $\Sigma V = 0$

$V_A + V_B = 4$

Step ② $\Sigma M_A = 0$

$acm = cm$

$V_B \times 20 = 4 \times 4$

$V_B = 0.8\,kN.$

$V_A + V_B = 4$

$V_A + 0.8 = 4$

$V_A = 3.2\,kN$

Step ③ $\Sigma M_C = 0$ (right Part)

$acm = cm$

$V_B \times 10 = H_B \times 4$

$0.8 \times 10 = H_B \times 4$

$H_B = 2\,kN$

Step ④ $\Sigma H = 0$

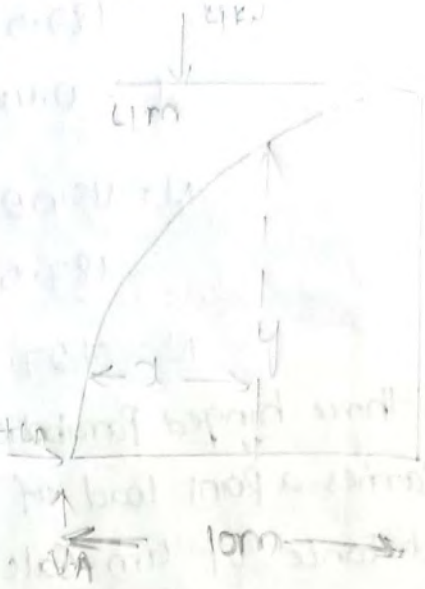
$\therefore H_A = H_B$

$H_A = 2\,kN$

Scanned with CamScanner

(ii) Normal thrust and radial shear under the load.

$x = 4m$



$F = V\cos\theta - H\sin\theta$

$N = V\sin\theta + H\cos\theta$

$y = \frac{4h}{l^2} x(l-x)$

$= \frac{4h}{l^2}(lx - x^2)$

$\tan\theta = \frac{dy}{dx} = \frac{d}{dx}\left[\frac{4h}{l^2}(lx - x^2)\right]$

$\tan\theta = \frac{4h}{l^2}(l - 2x)$

$\tan\theta = \frac{4 \times 4}{20^2}(20 - 2(4))$

$\theta = 25.64°$

$V = V_A - 4$

$= 3.2 - 4$

$V = -0.8\,kN$

$F = V\cos\theta - H\sin\theta$

$= -0.8\cos(25.64) - 2\sin(25.64)$

$F = -1.58\,kN$

$N = V\sin\theta + H\cos\theta$

$= -0.8\sin(25.64) + 2\cos(25.64)$

$N = 1.45\,kN$

Scanned with CamScanner

(ii) Maximum Sagging Bending Moment (left Part)

Max. BM occurs under the point load.

$x = 4m$.

$M_{x-x} = (V_A \cdot x) - (H_A \cdot y) - (w \times 0)$

$= (3.2 \times 4) - (8 \times y) - (4 \times 0)$

$= 12.8 - 2y$

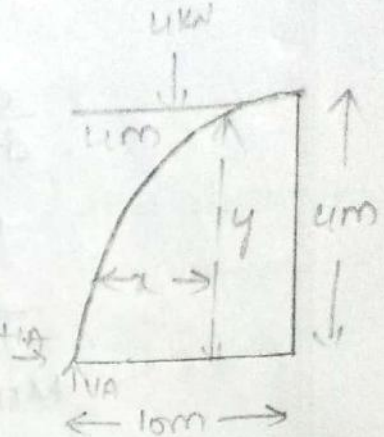
$y = \frac{4h}{l^2} x(l-x)$

$= \frac{4 \times 4}{20^2} \times 4(20 \times -4)$

$y = 2.56$

$M_{xx} = 12.8 - 2(2.56)$

$M_{xx} = 7.68\ KN-m$



(iii) Maximum Hogging Bending Moment (Right Part)

$M_{xx} = V_B \cdot x - (H_B \cdot y)$

~~$x = 4m$~~

~~$M_{xx} = (0.8 \times 4) - (2 \times 2.56)$~~

~~$M_{xx} = -1.92\ KN-m$~~

$y = \frac{4h}{l^2} x(l-x)$

$y = \frac{4h}{l^2}(lx - x^2)$

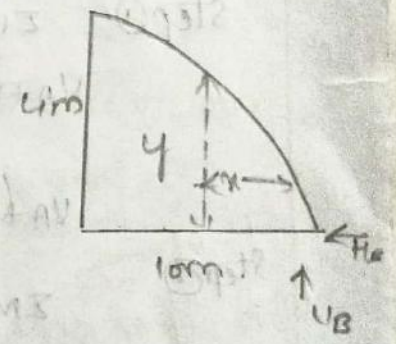
$= \frac{4 \times 4}{20^2}(20x - x^2)$

$y = 0.8x - 0.04x^2$

$M_{xx} = 0.8x - 2 \times (0.8x - 0.04x^2)$

$= 0.8x - 1.6x + 0.08x^2$

$M_{xx} = -0.8x + 0.08x^2$



Scanned with CamScanner

We know, max BM occurs where SF=0.

$$\frac{dM_{xx}}{dx}=0$$

$$\frac{d}{dx}\left[-0.8x+0.08x^2\right]=0$$

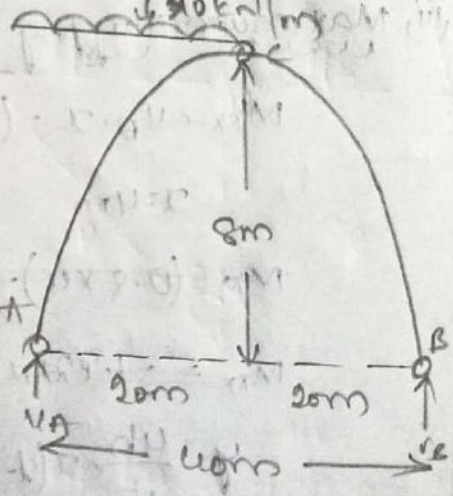
$$-0.8+0.08\times 2x=0$$

$$0.8+0.08x \quad x=5m$$

$$M_{xx}=-0.8\times 5+0.08\times 5^2$$

$$M_{xx}=-2\,kN\,m$$

3. A symmetrical three hinged Parabolic arch, of 40m span and central rise 8m carries a UDL of 30kN/m over the left half of the span. Calculate the reactions at the supports, also Calculate the bending moment, shear force and normal thrust at a distance of 10m from the left support.



Step ① $\Sigma V=0$

$$V_A+V_B=wl$$

$$=30\times 20$$

$$V_A+V_B=600\,kN$$

Step ② $\Sigma M_A=0$

$$a_{cm}=c_m$$

$$V_B\times 40=\frac{30\times 20^2}{2}$$

$$V_B=300$$

$$V_B=150\,kN$$

$$V_A+V_B=600$$

$$V_A+150=600$$

$$V_A=450\,kN$$

Step ③ $\Sigma M_C=0$ (right Part)

$$a_{cm}=c_m$$

$$V_B\times 20=H_B\times 8$$

$$150\times 20=H_B\times 8$$

Scanned with CamScanner

$H_B = 375\ KN$.

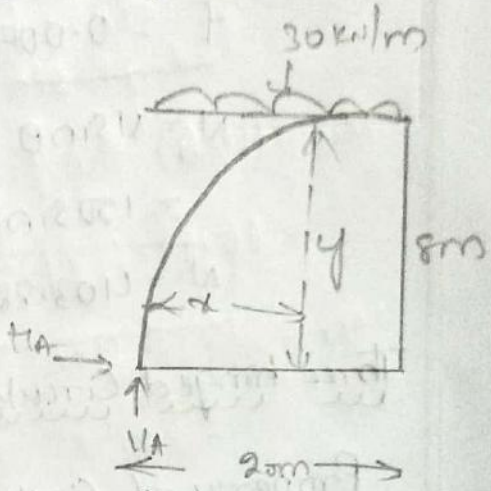
Step ④

$\Sigma H = 0$

$H_A = H_B$

$H_A = 375\ KN$.

(ii) Maximum sagging Bending moment (left Part) at a distance of 10m from left support.



$x = 10m$.

$M_{xx} = (V_A \cdot x) - (H_A \cdot y) - (\omega x \cdot \frac{x}{2})$

$y = \frac{4h}{l^2}(lx - x^2)$

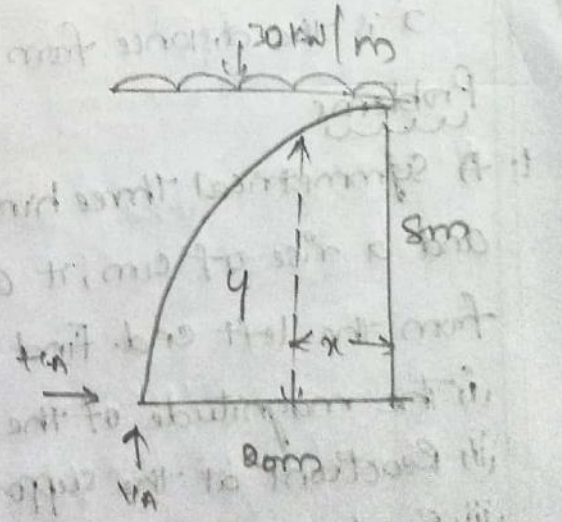
$= \frac{4 \times 8}{40^2}(40 \times 10 - 10^2)$

$y = 6m$

$M_{xx} = (450 \times 10) - (375 \times 6) - (30 \times 10 \times \frac{10}{2})$

$M_{xx} = 750\ KN\text{-}m$

(iii) Normal thrust and Radial shear force at a distance of 10m from left support



$x = 10m$

$f = V\cos\theta - H\sin\theta$

$N = V\sin\theta + H\cos\theta$

$y = \frac{4h}{l^2}(lx - x^2)$

$= \frac{4 \times 8}{40^2}(40 \times 10 - 10^2)$

$y = 6m$.

$\tan\theta = \frac{4h}{l^2}(l - 2x)$

$\tan\theta = \frac{4 \times 8}{40^2}(40 - 2(10))$

$\theta = 21.8°$

Scanned with CamScanner

$V = V_A - (30 \times 10)$

$= 450 - (30 \times 10)$

$V = 150\ KN.$

$F = V\cos\theta - H\sin\theta$

$= 150\cos(21.8) - 375\sin(21.8)$

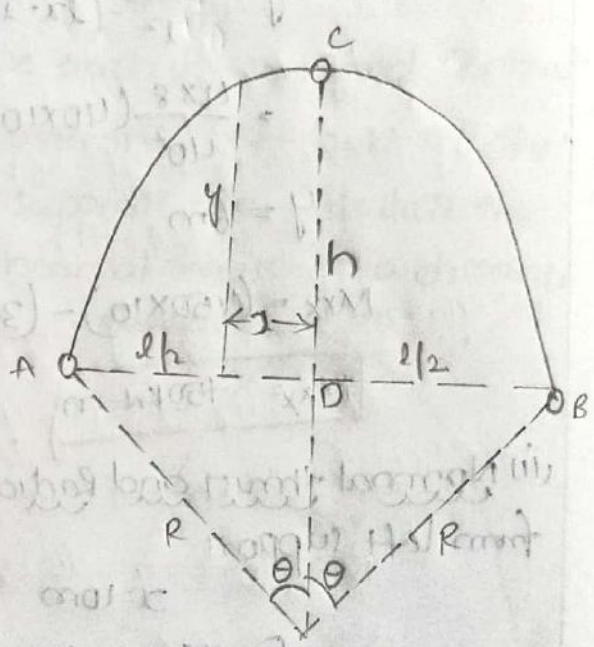
$\boxed{F = 0.009\ KN}$

$N = V\sin\theta + H\cos\theta$

$= 150\sin(21.8) + 375\cos(21.8)$

$\boxed{N = 403.88\ KN.}$

Three hinged Circular Arch



Property of Circle is,

$$h(2R-h) = \left(\frac{l}{2}\right)^2$$

$$R^2 = \{y + (R-h)\}^2 + x^2$$

Where,

'x' is the distance from "D"

Problems

1. A Symmetrical three hinged circular arch has a span of 16m and a rise of 4m, it carries a vertical load of 16 KN at 4m from the left end. Find

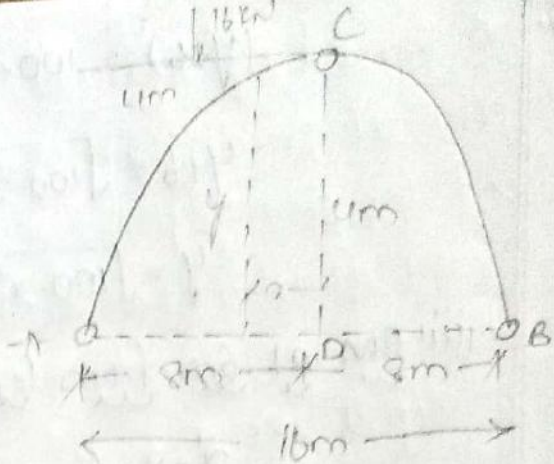
(i) the magnitude of the thrust at the supports.

(ii) Reactions at the supports

(iii) Bending Moment at 6m from the left hinge.

(iv) Maximum positive and negative moments.

Scanned with CamScanner



Step ① $\Sigma V = 0$

$V_A + V_B = 16$

Step ② $\Sigma M_A = 0$

$a_{cm} = c_m$

$V_B \times 16 = 16 \times 4$

$V_B = 4kN.$

$V_A + V_B = 16$

$V_A + 4 = 16$

$V_A = 12kN$

Step ③ $\Sigma M_C = 0$ (right Part)

$a_{cm} = c_m$

$V_B \times 8 = H_B \times 4$

$4 \times 8 = H_B \times 4$

$H_B = 8kN$

Step ④ $\Sigma H = 0$

$H_A = H_B$

$H_A = 8kN.$

We know,

$h(2R-h) = \left(\frac{l}{2}\right)^2$

$4(2R-4) = \left(\frac{16}{2}\right)^2$

$R = 10m$

$R^2 = \{y + (R-h)\}^2 + x^2$

$10^2 = \{y + (10-4)\}^2 + x^2$

$100 = (y+6)^2 + x^2$

$100 - x^2 = (y+6)^2$

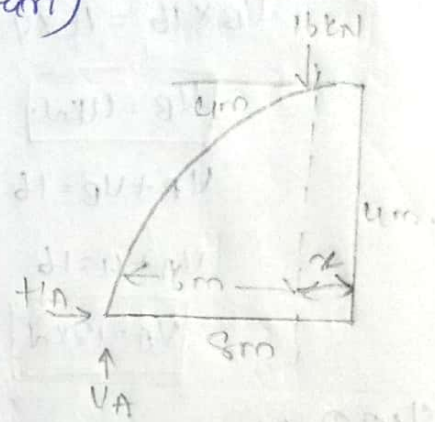
Scanned with CamScanner

$$(y+6)^2 = 100 - x^2$$

$$y+6 = \sqrt{100-x^2}$$

$$y = \sqrt{100-x^2} - 6$$

(iii) BM at 6m from left hinge (left Part)



$$x = 8-6$$

$$x = 2m.$$

$$M_{xx} = (V_A \cdot 6) - (H_A \cdot y) - (16 \times 2)$$

$$= (12 \times 6) - (8 \times y) - 32.$$

$$M_{xx} = 72 - 8y - 32$$

$$y = \sqrt{100-x^2} - 6$$

$$= \sqrt{100-2^2} - 6$$

$$y = 3.79m$$

Substitute 'y' value in M_{xx}

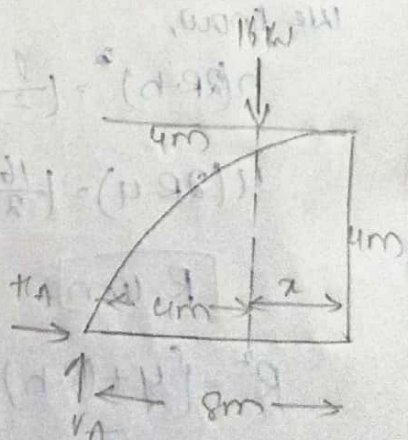
$$M_{xx} = 72 - 8y - 32$$

$$= 72 - 8(3.79) - 32$$

$$M_{xx} = 9.68 kN-m$$

(iv) Maximum Sagging BM (left Part)

Max BM occurs under the load



$$x = 8-4$$

$$= 4m$$

$$M_{xx} = (V_A \cdot 4) - (H_A \cdot y) - (16 \times 0)$$

$$= (12 \times 4) - 8xy$$

$$M_{xx} = 48 - 8y$$

$$y = \sqrt{100-x^2} - 6$$

$$= \sqrt{100-4^2} - 6$$

$$y = 3.16m$$

Scanned with CamScanner

$= 48 - 8y$

$= 48 - 8(3.16)$

$M_{xx} = 22.72$ kN-m

Maximum Hogging Bending Moment (Right Part)

$M_{xx} = (R_B \times (8-x)) + R_B \times y$

$= 4 \times (8-x) - 8y$

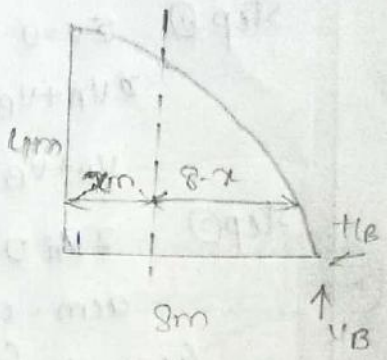
$M_{xx} = 32 - 4x - 8y$

$y = \sqrt{100 - x^2} - 6$

$M_{xx} = 32 - 4x - 8\left[\sqrt{100-x^2} - 6\right]$

$= 32 - 4x - 8\sqrt{100-x^2} + 48$

$= 80 - 4x - 8\sqrt{100-x^2}$



We know max BM occurs at SF = 0.

$$\frac{dm_{xx}}{dx} = 0$$

$$\frac{d}{dx}\left(32 - 4x - 8\sqrt{100-x^2} + 48\right) = 0$$

$$\boxed{\frac{d}{dx}\sqrt{1-x^2} = \frac{-x}{\sqrt{1-x^2}}}$$

$$\frac{d}{dx}\ 0 - 4 - 8\frac{-x}{\sqrt{100-x^2}} + 0 = 0$$

$x = 4.47$m

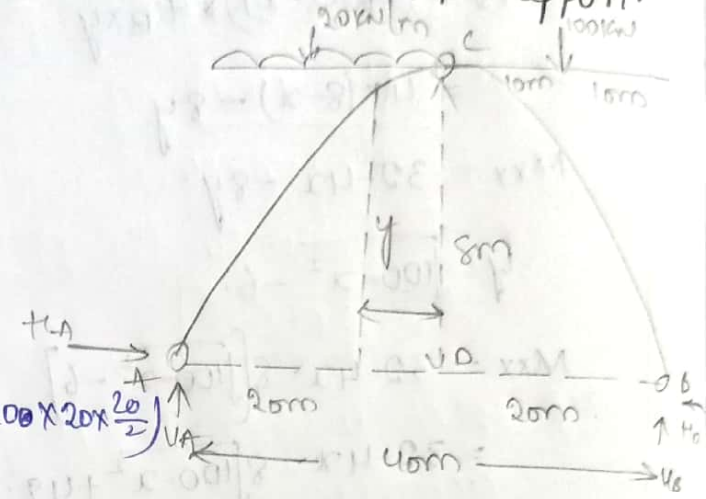
$M_{xx} = 32 - 4x - 8y$

$= 32 - 4(4.47) - 8(2.94)$

$M_{BX} = -9.4$ kN-m

Scanned with CamScanner

2. A three hinged circular arch has a span of 40m and a central rise of 8m. It carries a UDL of 20KN/m over the left half of the span together with a concentrated load of 100KN at right quarter span point. Find the reactions at the supports, normal thrust and shear at a section 10m from left support.



Step ① $\Sigma V = 0$

$V_A + V_B = (20 \times 20) + 100$

$V_A + V_B = 500 KN.$

Step ② $\Sigma M_A = 0$

acm = cm.

$(V_B \times 40) = \left[100 \times (10+20)\right] + \left(200 \times 20 \times \frac{20}{2}\right)$

$V_B = 175 KN.$

$V_A + V_B = 500$

$V_A + 175 = 500$

$V_A = 325 KN.$

Step ③ $\Sigma M_C = 0$ (Right Part)

acm = cm

$(V_B \times 20) = (H_B \times 8) + (100 \times 10)$

$H_B = 312.5 KN.$

Step ④ $\Sigma H = 0$

$H_A = H_B$

$H_A = 312.5 KN$

We know,

$h(2R-h) = \left(\frac{l}{2}\right)^2$

$8(2R-8) = \left(\frac{40}{2}\right)^2$

$R = 29m$

Scanned with CamScanner

$$R^2 = \{y + (R-h)^2\}^2 + x^2$$

$$29^2 = \{y + (29-8)\}^2 + x^2.$$

$$841 = (y+21)^2 + x^2.$$

$$(y+21)^2 = 841 - x^2$$

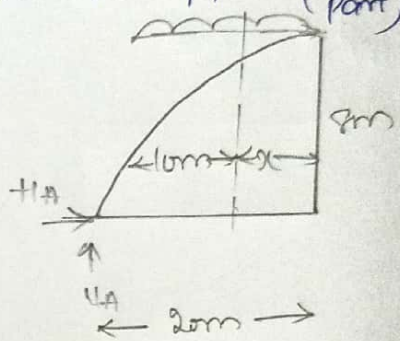
$$y + 21 = \sqrt{841 - x^2}$$

$$y = \sqrt{841 - x^2} - 21.$$

Normal Thrust and Radial shear at 10m from left support (left part)

$$x = 20 - 10$$

$$x = 10m$$



$$F = V\cos\theta - H\sin\theta$$

$$N = V\sin\theta + H\cos\theta.$$

$$y = \sqrt{841 - x^2} - 21.$$

$$\tan\theta = \frac{dy}{dx}\left(\sqrt{841 - x^2} - 21\right)$$

$$\tan\theta = \frac{-x}{\sqrt{841 - x^2}} - 0$$

$$\boxed{\theta = -20.17^\circ}$$

$$V = V_A - (20 \times 10)$$

$$= 325 - (20 \times 10)$$

$$\boxed{V = 125}$$

$$F = V\cos\theta - H\sin\theta$$

$$= 125\cos(-20.17^\circ) - 312.5\sin(-20.17^\circ)$$

$$\boxed{F = 228.08\ kN.}$$

$$N = V\sin\theta + H\cos\theta$$

$$= 125\sin(-20.17^\circ) + 312.5\cos(-20.17^\circ)$$

$$\boxed{N = 250.23\ kN}$$

Scanned with CamScanner

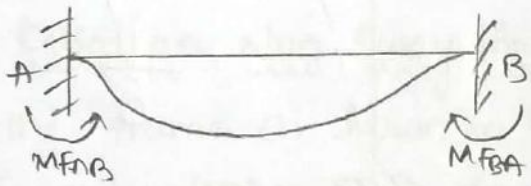
Unit-II
Part-1

Slope Deflection Method

Slope Deflection equations expressed the final end moments of each member interms of:

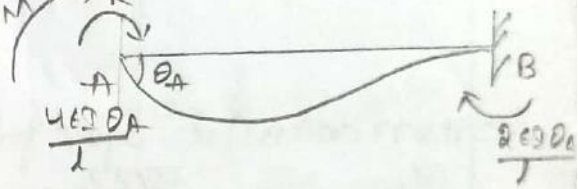
1. Fixed end Moments due to external loads.
2. Moments due to rotation at left support.
3. Moments due to rotation at right support.
4. Moments due to transverse displacement b/w the two supports.

Case-①

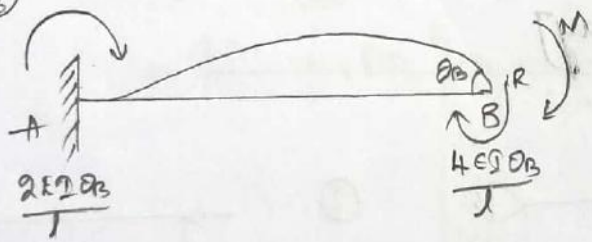


Case-②

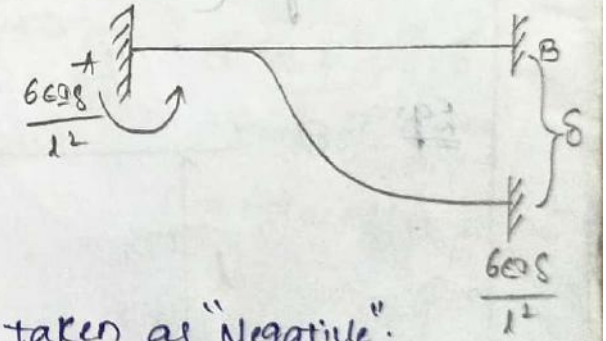
R → Rotation
M → Moment



Case-③



Case-④



Note:

Anti clockwise moments should be taken as "Negative".

Clockwise moments should be taken as "Positive".

Procedure:

1. Determine fixed end Moments
2. Determine the moments dure to rotation and transverse displacement
3. ~~Write~~ down joint equilibrium equations ($\Sigma M_B = 0$).
4. If the frame has support side sways, write down the horizontal [shear] equilibrium equations.

Scanned with CamScanner

5. Using equilibrium equations, Determine the unknown displacements [$\theta_A, \theta_B, \ldots$ (or) δ]

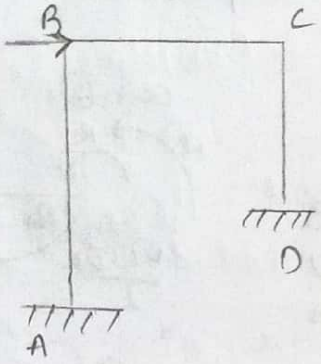
6. Using the displacements, Determine the final end moments [$M_{AB}, M_{BA}, \ldots$]

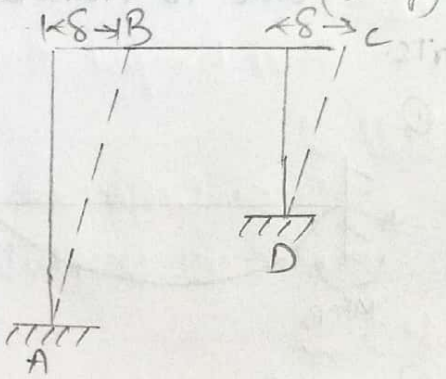
Frames

Sway :-

The side movement of the end of a column in a frame is called sway.

Eg:

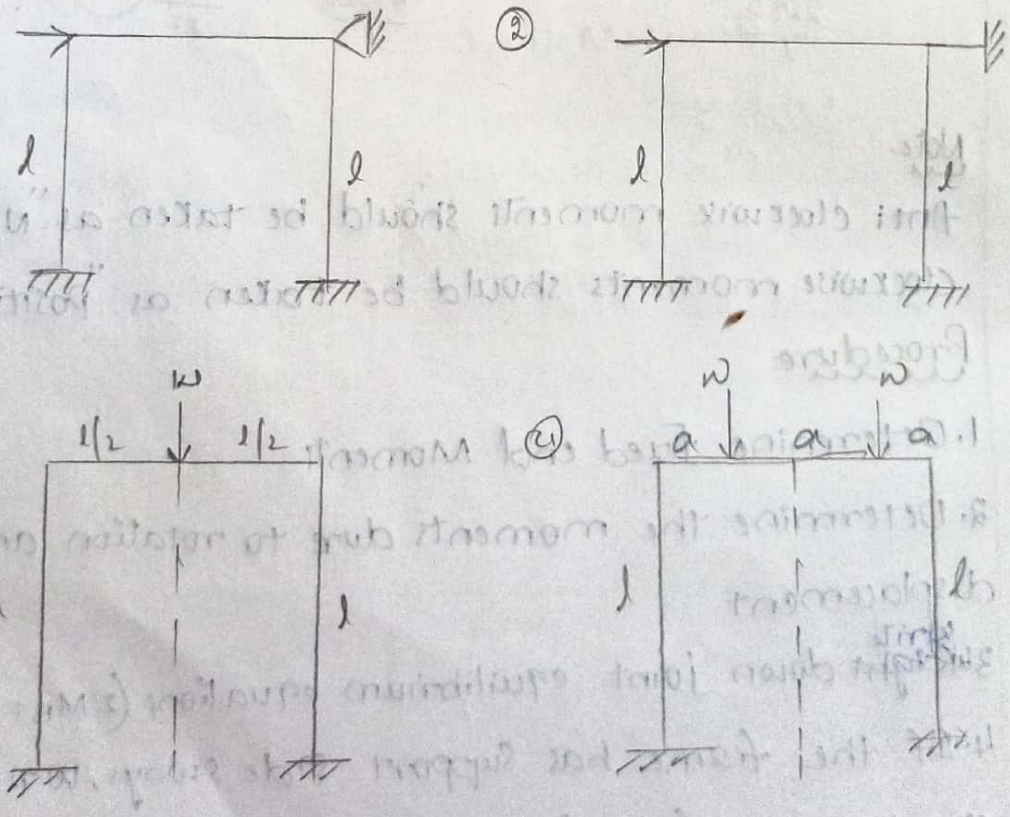




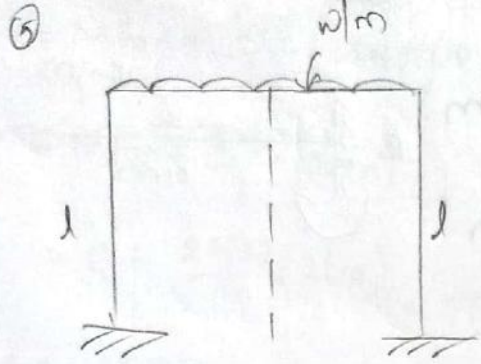
Non-sway :- [without sway]

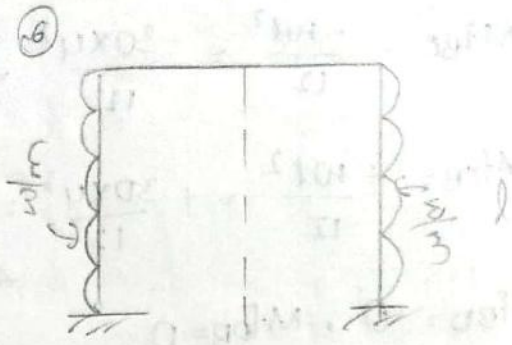
Eg:

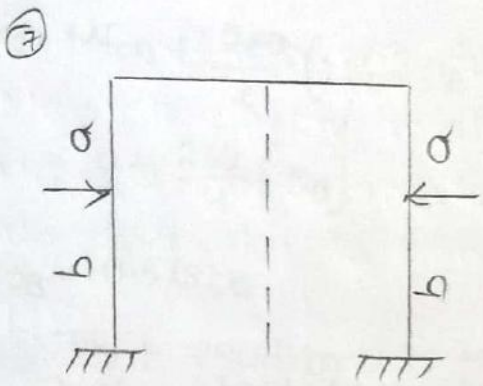
① ② ③ ④

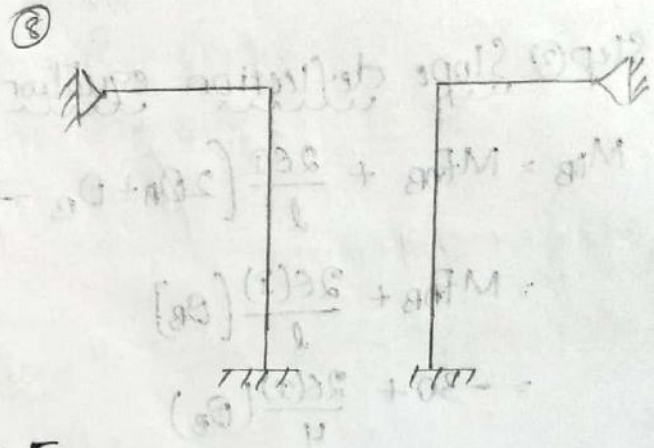


Scanned with CamScanner



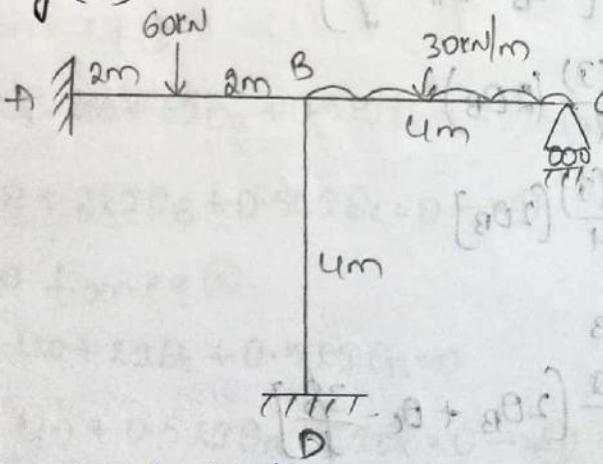






Problems based on Non-Sway Frames

Analyse the frame as shown in fig. by slope deflection method. Flexural rigidity (EI) is same for all members.



Non-sway

$\theta_A, \theta_B, \theta_C, \phi_D, \delta$ ⁰

$\Sigma M_B = 0$ (Internal Joint)

$M_{BA} + M_{BC} + M_{BD} = 0 \rightarrow (1)$

$M_{CB} = 0 \rightarrow (2)$ (simply supported)

Bending Moment due to loads

Span AB, $M = \frac{Wl}{4} = \frac{60 \times 4}{4} = 60$ kN-m

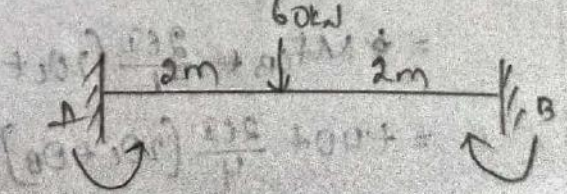
Span BC, $M = \frac{Wl^2}{8} = \frac{30 \times 4^2}{8} = 60$ kN-m

Span BD, $M = 0$

Step ① Fixed end Moment

$M_{FAB} = -\frac{Wl}{8} = -\frac{60 \times 4}{8} = -30$ kN-m

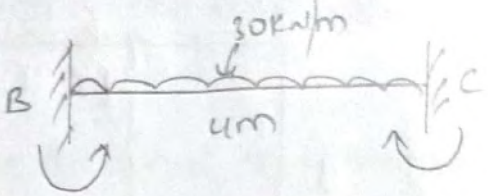
$M_{FBA} = +\frac{Wl}{8} = +\frac{60 \times 4}{8} = +30$ kN-m



Scanned with CamScanner

$$M_{FBC} = \frac{-wl^2}{12} = \frac{-30\times4^2}{12} = -40\,kN\text{-}m$$

$$M_{FCB} = +\frac{wl^2}{12} = +\frac{30\times4^2}{12} = +40\,kN\text{-}m$$



$M_{FBD} = 0$, $M_{FDB} = 0$



Step ② Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right]$$

$$= M_{FAB} + \frac{2E(I)}{l}\left[\theta_B\right]$$

$$= -30 + \frac{2E(I)}{4}(\theta_B)$$

$$M_{AB} = -30 + 0.5EI\theta_B$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\delta}{l}\right]$$

$$= M_{FBA} + \frac{2E(I)}{l}(2\theta_B)$$

$$= +30 + \frac{2E(I)}{4}\left[2\theta_B\right]$$

$$M_{BA} = 30 + EI\theta_B$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[2\theta_B + \theta_C - \frac{3\delta}{l}\right]$$

$$= M_{FBC} + \frac{2E(I)}{l}\left[2\theta_B + \theta_C\right]$$

$$= -40 + \frac{2E(I)}{4}\left[2\theta_B + \theta_C\right]$$

$$M_{BC} = -40 + EI\theta_B + 0.5EI\theta_C$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left[2\theta_C + \theta_B - \frac{3\delta}{l}\right]$$

$$= M_{FCB} + \frac{2EI}{l}\left[2\theta_C + \theta_B\right]$$

$$= +40 + \frac{2EI}{4}\left[2\theta_C + \theta_B\right]$$

$$M_{CB} = 40 + EI\theta_C + 0.5EI\theta_B$$

Scanned with CamScanner

$$M_{BD} = M_{FBD} + \frac{2EI}{l}\left[2\theta_B + \theta_D - \frac{3\Delta}{l}\right]$$

$$= M_{FBD} + \frac{2(EI)}{l}[2\theta_B]$$

$$= 0 + \frac{2(EI)}{4}(2\theta_B)$$

$$M_{BD} = EI\theta_B$$

$$M_{DB} = M_{FDB} + \frac{2EI}{l}\left[2\theta_D + \theta_B - \frac{3\Delta}{l}\right]$$

$$= 0 + \frac{2EI}{4}[\theta_B]$$

$$M_{DB} = 0.5EI\theta_B$$

Step ③ Joint equilibrium equations

$$\Sigma M_B = 0$$

$$M_{BA} + M_{BC} + M_{BD} = 0 \longrightarrow ①$$

$$M_{CB} = 0 \longrightarrow ②$$

From eq ①

$$30 + EI\theta_B - 40 + EI\theta_B + 0.5EI\theta_C + EI\theta_B = 0$$

$$-10 + 3EI\theta_B + 0.5EI\theta_C = 0 \longrightarrow ③$$

From eq ②

$$40 + EI\theta_C + 0.5EI\theta_B = 0$$

$$40 + 0.5EI\theta_B + EI\theta_C = 0 \longrightarrow ④$$

From eq ③ & ④

$$3EI\theta_B + 0.5EI\theta_C = 10$$

$$0.5EI\theta_B + EI\theta_C = -40$$

$$\theta_B = \frac{10.9}{EI}, \quad \theta_C = \frac{-45.45}{EI}$$

Scanned with CamScanner

Step ① Final end Moment

$$M_{AB} = -30 + \frac{2EI}{4}(\theta_B)$$

$$= -30 + \frac{2EI}{4}\left[\frac{10.9}{EI}\right]$$

$$= -24.55 \text{ KN-m}$$

$$M_{BA} = 30 + \frac{2EI}{4}(2\theta_B)$$

$$= 30 + \frac{2EI}{4}\left[2 \times \frac{10.9}{EI}\right]$$

$$M_{BA} = 40.9 \text{ KN-m}$$

$$M_{BC} = -40 + EI\theta_B + 0.5EI\theta_C$$

$$= -40 + EI\left[\frac{10.9}{EI}\right] + 0.5EI\left[\frac{-45.45}{EI}\right]$$

$$M_{BC} = -51.83 \text{ KN-m}$$

$$M_{CB} = 40 + \frac{2EI}{4}\left[2\theta_C + \theta_B\right]$$

$$= 40 + EI\theta_C + 0.5EI\theta_B$$

$$= 40 + EI\left[\frac{-45.45}{EI}\right] + 0.5EI\left[\frac{10.9}{EI}\right]$$

$$M_{CB} = 0$$

$$M_{BD} = EI\theta_B$$

$$= EI\left[\frac{10.9}{EI}\right]$$

$$M_{BD} = 10.9 \text{ KN-m}$$

$$M_{DB} = 0.5EI\theta_B$$

$$= 0.5EI\left(\frac{10.9}{EI}\right)$$

$$M_{DB} = 5.45 \text{ KN-m}$$

Check ① $\Sigma M_B = 0$

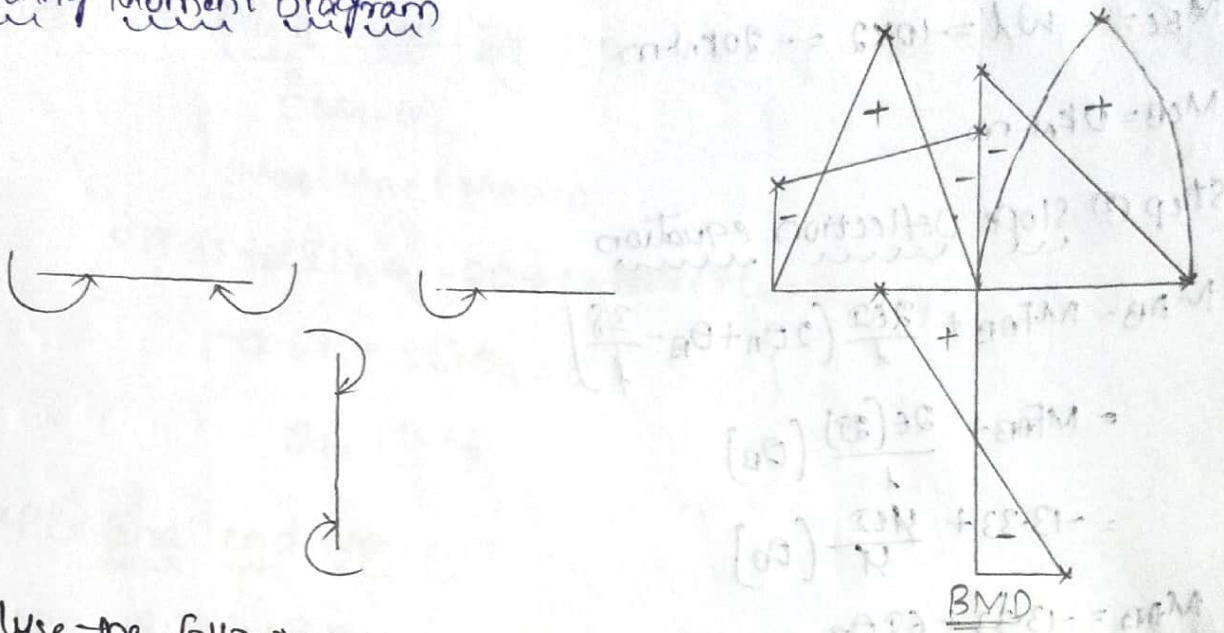
$$M_{BA} + M_{BC} + M_{BD} = 0$$

Check ②

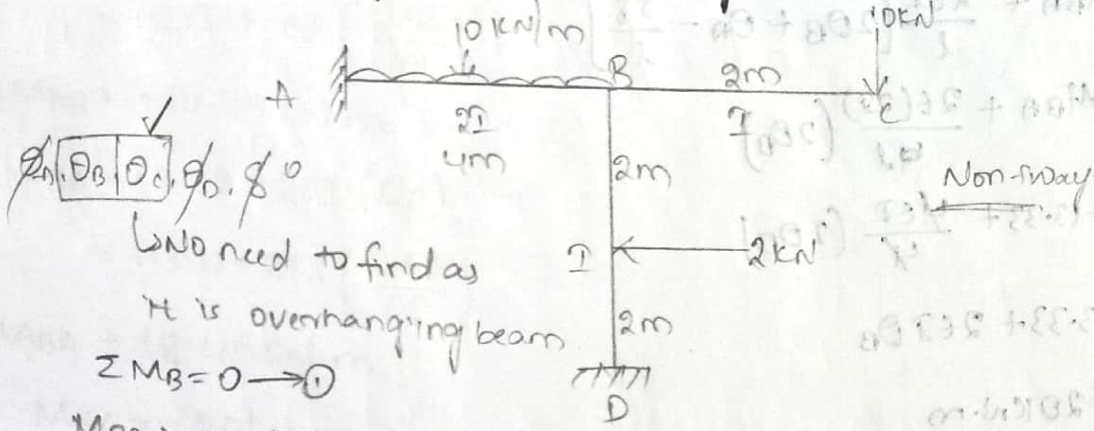
$$M_{CB} = 0.$$

Scanned with CamScanner

Bending Moment Diagram



2. Analyse the following structure using slope Deflection method.



↳No need to find as it is overhanging beam

$\Sigma M_B = 0 \rightarrow (1)$

$M_{BA} + M_{BC} + M_{BD} = 0.$

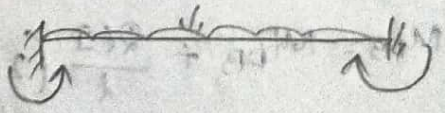
Bending Moment due to loads

Span AB, $M_{AB} = \frac{Wl^2}{8} = \frac{10 \times 4^2}{8} = 20\ KN\text{-}m$

Span BD, $M_{BD} = \frac{Wl}{4} = \frac{2 \times 4}{4} = 2\ KN\text{-}m.$

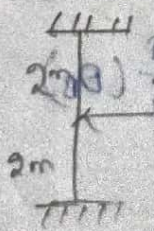
Step (1) Fixed end Moment

$MF_{AB} = \frac{-Wl^2}{12} = \frac{-10 \times 4^2}{12} = -13.33\ KN\text{-}m$



$MF_{BA} = \frac{+Wl^2}{12} = \frac{+10 \times 4^2}{12} = +13.33\ KN\text{-}m.$

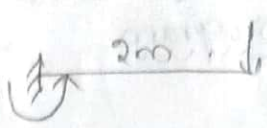
$MF_{BD} = \frac{-Wl}{8} = \frac{-2 \times 4}{8} = -1\ KN\text{-}m$



$MF_{DB} = \frac{+Wl}{8} = \frac{+2 \times 4}{8} = +1\ KN\text{-}m.$

Scanned with CamScanner

$M_{BC} = -wl = -10 \times 2 = -20 \text{ kN-m}$



$M_{CB} = 0 \text{ kN-m}$

Step ② Slope Deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right)$$

$$= M_{FAB} + \frac{2E(2I)}{l}(\theta_B)$$

$$= -13.33 + \frac{4EI}{4}(\theta_B)$$

$$M_{AB} = -13.33 + EI\theta_B$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\delta}{l}\right)$$

$$= M_{FBA} + \frac{2E(2I)}{4l}(2\theta_B)$$

$$= +13.33 + \frac{4EI}{4}(2\theta_B)$$

$$M_{BA} = 13.33 + 2EI\theta_B$$

$M_{BC} = -20 \text{ kN-m}$

$M_{CB} = 0 \text{ kN-m}.$

$$M_{BD} = M_{FBD} + \frac{2EI}{l}\left(2\theta_B + \theta_D - \frac{3\delta}{l}\right)$$

$$= M_{FBD} + \frac{2EI}{l}(2\theta_B)$$

$$= -1 + \frac{2EI}{2l}(2\theta_B)$$

$$M_{BD} = -1 + 2EI\theta_B$$

$$M_{DB} = M_{FDB} + \frac{2EI}{l}\left(2\theta_D + \theta_B - \frac{3\delta}{l}\right)$$

$$= M_{FDB} + \frac{2EI}{l}(\theta_B)$$

$$= 1 + \frac{2EI}{4}(\theta_B)$$

$$M_{DB} = 1 + 0.5EI\theta_B$$

Scanned with CamScanner

Step ③ Joint Equilibrium Equation

$$\Sigma M_B = 0$$

$$M_{BA} + M_{BC} + M_{BD} = 0.$$

$$+13.33 + 2EI\theta_B + -20 + 1 + EI\theta_B = 0$$

$$-5.67 + 3EI\theta_B = 0$$

$$\theta_B = \frac{2.56}{EI}$$

Step ④ Final end Moments

$$M_{AB} = -13.33 + EI\theta_B$$

$$= -13.33 + EI\left[\frac{2.56}{EI}\right]$$

$$M_{AB} = -10.77 \text{ Kn-m}$$

$$M_{BA} = 13.33 + 2EI(\theta_B)$$

$$= +13.33 + 2EI\left(\frac{2.56}{EI}\right)$$

$$M_{BA} = 18.45 \text{ Kn-m}$$

$$M_{BC} = -20 \text{ Kn-m}$$

$$M_{CB} = 0 \text{ Kn-m}$$

$$M_{BD} = -1 + EI\theta_B$$

$$= -1 + EI\left[\frac{2.56}{EI}\right]$$

$$M_{BD} = 1.56 \text{ Kn-m}$$

$$M_{DB} = 1 + 0.5EI\theta_B$$

$$= 1 + 0.5EI\left(\frac{2.56}{EI}\right)$$

$$M_{DB} = 2.28 \text{ Kn-m}.$$

Check

$$\Sigma M_B = 0$$

$$M_{BA} + M_{BC} + M_{BD} = 0$$

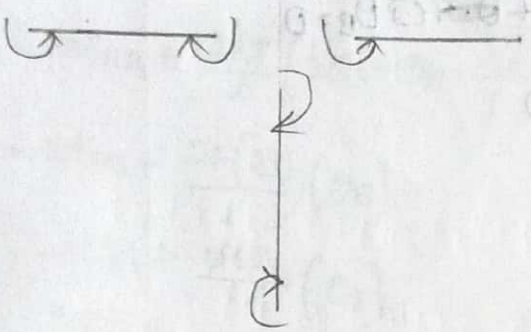
$$18.45 + -20 + 1.56 = 0$$

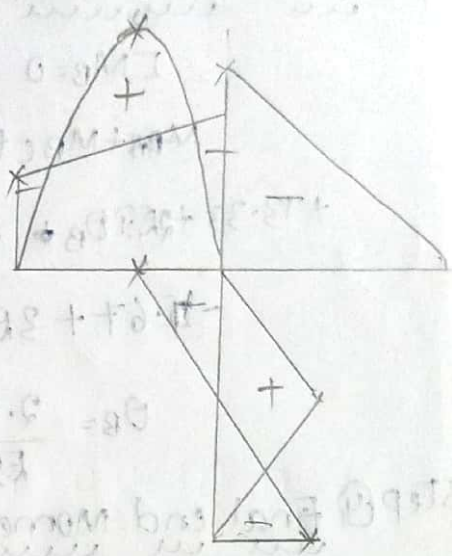
$$0.01 \approx 0$$

Scanned with CamScanner

Bending Moment Diagram

FBD

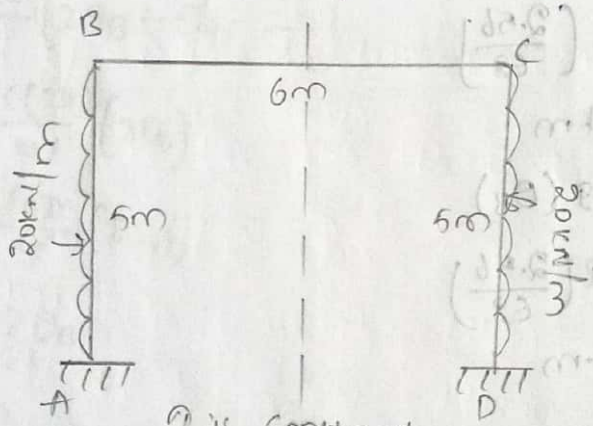




BMD

3. Analyse the following frame using slope deflection Method.

$\phi_A, \theta_B, \theta_C, \phi_D$



Non-sway
(Due to symmetry)

Bending Moment due to loads

Span, AB

$$M_{AB} = \frac{wl^2}{8} = \frac{+20\times5^2}{8} = 62.5\ \text{KNm}$$

$M_{BC} = 0$

Span, CD.

$$M_{CD} = \frac{wl^2}{8} = \frac{+20\times5^2}{8} = 62.5\ \text{KNm}$$

Step ① Fixed end Moment

$$M_{FAB} = \frac{-wl^2}{8\times12} = \frac{-20\times5^2}{12} = -41.67\ \text{KNm}$$

$$M_{FBA} = \frac{+wl^2}{8\times12} = \frac{+20\times5^2}{12} = +41.67\ \text{KNm}$$

$M_{FBC} = 0$

$M_{FCB} = 0$

$$M_{FCD} = \frac{-wl^2}{12} = \frac{-20\times5^2}{12} = -41.67\ \text{KNm}$$

Scanned with CamScanner

$M_{FDC} = \frac{+Wl^2}{12} = +\frac{20 \times 9^2}{12} = +41.67\ KN\text{-}m$

Step② Slope Deflection equation

$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\psi}{l}\right]$

$= M_{FAB} + \frac{2EI}{l}[\theta_B]$

$= -41.67 + \frac{2EI}{5}[\theta_B]$

$M_{AB} = -41.67 + 0.4EI\theta_B$

$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\psi}{l}\right]$

$= +41.67 + \frac{2EI}{5}[2\theta_B]$

$M_{BA} = +41.67 + 0.8EI\theta_B$

$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[2\theta_B + \theta_C - \frac{3\psi}{l}\right]$

$= 0 + \frac{2EI}{6}[2\theta_B + \theta_C]$

$M_{BC} = 0 + 0.33EI(2\theta_B + \theta_C)$

$M_{CB} = M_{FCB} + \frac{2EI}{l}\left[2\theta_C + \theta_B - \frac{3\psi}{l}\right]$

$= 0 + \frac{2EI}{6}(2\theta_C + \theta_B)$

$M_{CB} = 0 + 0.33EI(2\theta_C + \theta_B)$

$M_{CD} = M_{FCD} + \frac{2EI}{l}\left[2\theta_C + \theta_D - \frac{3\psi}{l}\right]$

$= -41.67 + \frac{2EI}{5}[2\theta_C]$

$M_{CD} = -41.67 + 0.8EI\theta_{BC}$

$M_{DC} = M_{FDC} + \frac{2EI}{l}\left[2\theta_D + \theta_C - \frac{3\psi}{l}\right]$

$= +41.67 + \frac{2EI}{5}[\theta_C]$

$M_{DC} = 41.67 + 0.4EI\theta_C$

Scanned with CamScanner

Step ③ Joint Equilibrium Equation

$\Sigma M_B = 0 \rightarrow (1)$

$M_{BA} + M_{BC} = 0$

$\Sigma M_C = 0 \rightarrow (2)$

$M_{CB} + M_{CD} = 0$

from eq (1)

$41.67 + 0.8EI\theta_B + 0 + 0.69EI\theta_B + 0.33EI\theta_C = 0$

$41.67 + 1.47EI\theta_B + 0.33EI\theta_C = 0 \rightarrow (3)$

from eq (2)

$0 + 0.33EI\theta_B + 0.69EI\theta_C - 41.67 + 0.8EI\theta_C$

$-41.67 + 0.33EI\theta_B + 1.47EI\theta_C = 0 \rightarrow (4)$

from eq (3) & (4)

$1.47EI\theta_B + 0.33EI\theta_C = -41.67$

$0.33EI\theta_B + 1.47EI\theta_C = +41.67$

$\theta_B = \frac{-36.55}{EI}$, $\theta_C = \frac{+36.55}{EI}$

Step ④ Final end Moment

$M_{AB} = -41.67 + 0.4EI\theta_B$

$= -41.67 + 0.4EI\left(\frac{-36.55}{EI}\right)$

$M_{AB} = -56.29 kN.m$

$M_{BA} = 41.67 + 0.8EI\theta_B$

$= -41.67 + 0.8EI\left(\frac{-36.55}{EI}\right)$

$M_{BA} = +12.43 kN.m$

$M_{BC} = 0 + 0.33EI(2\theta_B + \theta_C)$

$= 0 + 0.33EI\left(\left(2 \times \frac{-36.55}{EI}\right) + \left(\frac{36.55}{EI}\right)\right)$

$M_{BC} = -12.06 kN.m$ (12.427)

Scanned with CamScanner

$M_{CB} = 0 + 0.33 \times 2 (2\theta_C + \theta_B)$

$= 0 + 0.33 \times 2 \left[2\left(\frac{36.55}{EI}\right) + \left(\frac{-36.55}{EI}\right) \right]$

$M_{CB} = +12.06 \text{ kN-m}$ (12.497)

$M_{CD} = -41.67 + 0.8 EI \theta_C$

$= -41.67 + 0.8 EI \left[\frac{36.55}{EI}\right]$

$M_{CD} = -12.43 \text{ kN-m}$

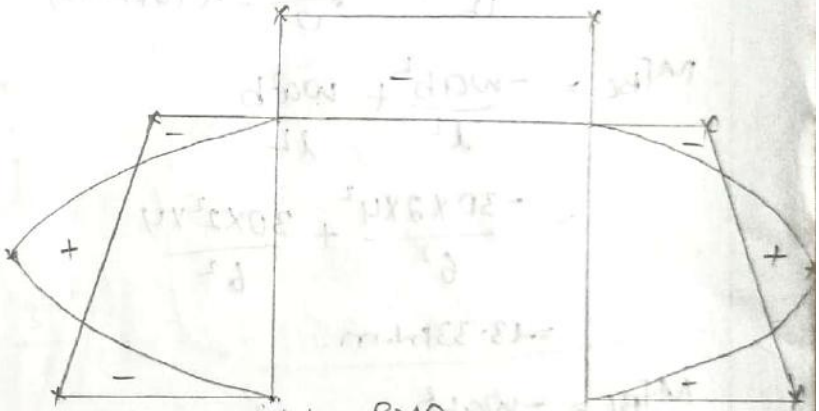
$M_{DC} = 41.67 + 0.4 EI \theta_C$

$M_{DC} = +56.29 \text{ kN-m}$

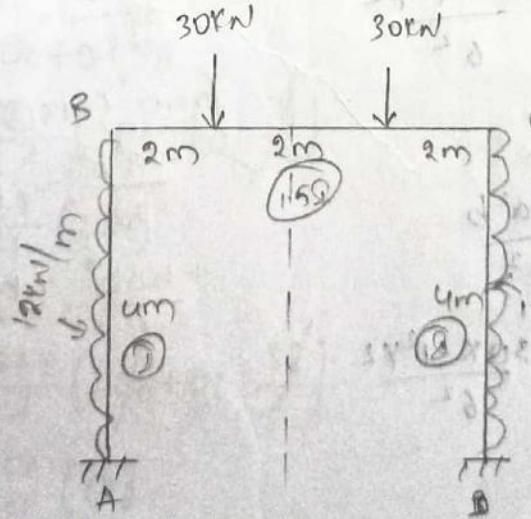
Bending Moment Diagram

FBD





4. Analyse the following frame using slope Deflection Method.



Nonsway

(due to symmetry)

$\theta_A = \theta_B = \theta_C = \theta_D = 0$

$\Sigma M_B = 0 \Rightarrow m_{BA} + m_{BC} = 0$

$\Sigma M_C = 0 \Rightarrow m_{CB} + m_{CD} = 0$

Bending moment due to loads

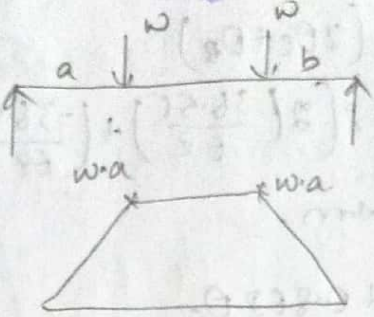
Span AB, $M = \frac{wl^2}{8} = \frac{12 \times 4^2}{8} = 24 \text{ kN-m}$

Scanned with CamScanner

Span BC, $M = w \cdot a$

$= 30 \times 2$

$= 60 \text{ KN-m}$



Span CD, $M = \frac{wl^2}{8}$

$= \frac{12 \times 4^2}{8} = 24 \text{ KN-m}$.

Step ① Fixed end Moment

$M_{FAB} = \frac{-wl^2}{12} = \frac{-12 \times 4^2}{12} = -16 \text{ KN-m}$

$M_{FBA} = \frac{+wl^2}{12} = \frac{+12 \times 4^2}{12} = +16 \text{ KN-m}$.

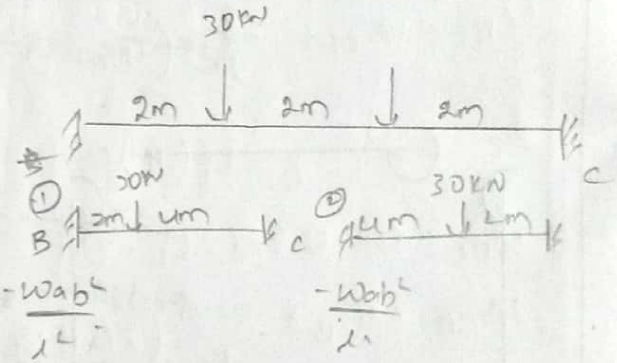
$M_{FCD} = \frac{-wl^2}{12} = \frac{-12 \times 4^2}{12} = -16 \text{ KN-m}$

$M_{FDC} = \frac{+wl^2}{12} = \frac{+12 \times 4^2}{12} = +16 \text{ KN-m}$

$M_{FBC} = \frac{-wab^2}{l^2} + \frac{wa^2b}{l^2}$

$= \frac{-30 \times 2 \times 4^2}{6^2} + \frac{30 \times 2^2 \times 4}{6^2}$

$= -13.33 \text{ KN-m}$.



$M_{FBC} = \frac{-wab^2}{l^2} + \frac{wa^2b}{l^2}$

$= \frac{-30 \times 2 \times 4^2}{6^2} - \frac{30 \times 4^2 \times 2}{6^2}$

$= -40 \text{ KN-m}$

$M_{FCB} = \frac{+wa^2b}{l^2} + \frac{wa^2b}{l^2}$

$= \frac{30 \times 2^2 \times 4}{6^2} + \frac{30 \times 4^2 \times 2}{6^2}$

$= +40 \text{ KN-m}$.

From case ①

$\frac{-wab^2}{l^2} = \frac{-30 \times 2 \times 4^2}{6^2} = -26.67$

$\frac{+wa^2b}{l^2} = \frac{+30 \times 2^2 \times 4}{6^2} = +13.33$

From case ②

$\frac{-wa^2b^2}{l^2} = \frac{-30 \times 4 \times 2^2}{6^2} = -13.33$

$\frac{+wab^2}{l^2} = \frac{+30 \times 4^2 \times 2}{6^2} = +26.67$

Scanned with CamScanner

Step ② Slope Deflection Equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\psi}{l}\right)$$

$$= M_{FAB} + \frac{2E(2I)}{l}[\theta_B]$$

$$= -16 + \frac{2EI}{4}[\theta_B]$$

$$M_{AB} = -16 + 0.5EI\theta_B$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\psi}{l}\right)$$

$$= M_{FBA} + \frac{2E(2I)}{l}(2\theta_B)$$

$$= +16 + \frac{2EI}{4}(2\theta_B)$$

$$M_{BA} = 16 + EI\theta_B$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\psi}{l}\right)$$

$$= -40 + \frac{2E(1.5I)}{6}(2\theta_B + \theta_C)$$

$$= -40 + 0.5EI(2\theta_B + \theta_C)$$

$$M_{BC} = -40 + EI\theta_B + 0.5EI\theta_C$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\psi}{l}\right)$$

$$= +40 + \frac{2E(1.5I)}{6}(2\theta_C + \theta_B)$$

$$= 40 + 0.5EI(2\theta_C + \theta_B)$$

$$M_{CB} = 40 + EI\theta_C + 0.5EI\theta_B$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}\left(2\theta_C + \theta_D - \frac{3\psi}{l}\right)$$

$$= -16 + \frac{2EI}{4}(2\theta_C)$$

$$M_{CD} = -16 + EI\theta_C$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}\left(2\theta_D + \theta_C - \frac{3\psi}{l}\right)$$

$$= +16 + \frac{2EI}{4}(\theta_C)$$

$$M_{DC} = 16 + 0.5EI\theta_C$$

Scanned with CamScanner

Step ③ Joint Equilibrium Equation

$\Sigma M_B = 0$

$M_{BA} + M_{BC} = 0 \rightarrow (1)$

$\Sigma M_C = 0$

$M_{CB} + M_{CD} = 0 \rightarrow (2)$

from eq (1)

$16 + 6\theta_B - 4\theta_B + 0.5(6\theta_C) = 0$

$-24 + 6\theta_B + 4\theta_B + 0.5(6\theta_C) = 0$

$2(6\theta_B + 4\theta_B) + 0.5(6\theta_C) = +24 \rightarrow (3)$

from eq (2)

$40 + 6\theta_C + 0.5(6\theta_B - 16(6\theta_C) = 0$

$24 + 0.5(6\theta_B + 2(6\theta_C = 0$

$0.5(6\theta_B + 2(6\theta_C = -24 \rightarrow (4)$

from eq (3) & (4)

$2(6\theta_B + 0.5(6\theta_C = 24$

$0.5(6\theta_B + 2(6\theta_C = -24$

$\theta_B = \frac{+16}{6}$, $\theta_C = \frac{-16}{6}$

Step ④ Final end Moment

$M_{AB} = -16 + 0.5(6\theta_B$

$= -16 + 0.5(6\left[\frac{16}{6}\right]$

$M_{AB} = -8\,kN\text{-}m$

$M_{BA} = 16 + 6\theta_B$

$= 16 + 6\left(\frac{16}{6}\right)$

$M_{BA} = 32\,kN\text{-}m$

$M_{BC} = -40 + 6\theta_B + 0.5(6\theta_C$

$= -40 + 6\left[\frac{16}{6}\right] + 0.5(6\left[\frac{-16}{6}\right]$

$M_{BC} = -32\,kN\text{-}m$

Scanned with CamScanner

$M_{CB} = 40 + 2EI\theta_C + 0.5(2EI\theta_B)$

$= 40 + 2EI\left[\frac{-16}{2EI}\right] + 0.5\,2EI\left[\frac{16}{2EI}\right]$

$M_{CB} = +32\ kN\text{-}m$.

$M_{CD} = -16 + 2EI\theta_C$

$= -16 + 2EI\left(\frac{-16}{2EI}\right)$

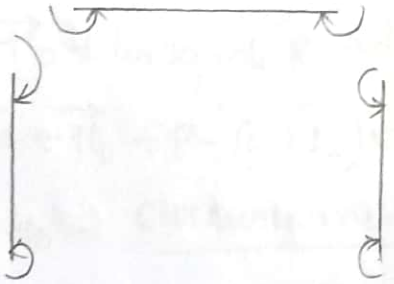
$M_{CD} = -32\ kN\text{-}m$

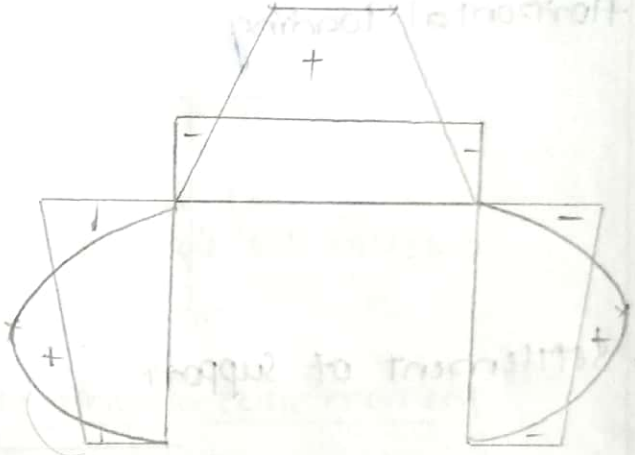
$M_{DC} = 16 + 0.5\,2EI\theta_C$

$= 16 + 0.5\,2EI\left(\frac{-16}{2EI}\right)$

$M_{DC} = +8\ kN\text{-}m$

Bending Moment Diagram

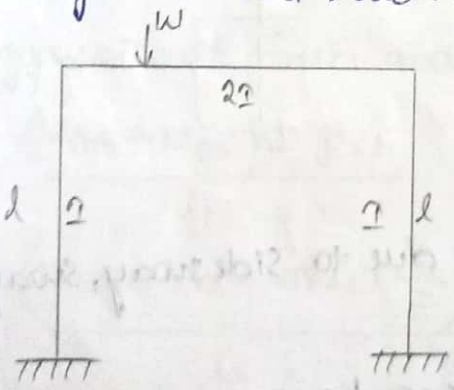




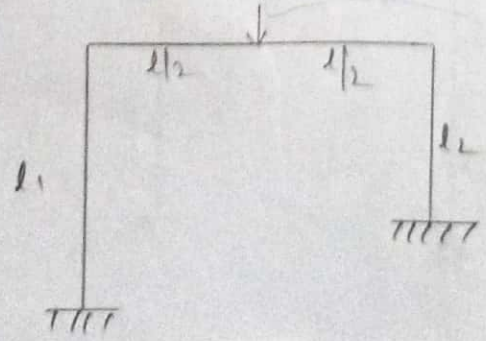
BMD

Total Frames with side sway

1. Eccentric (or) Unsymmetrical Loading:

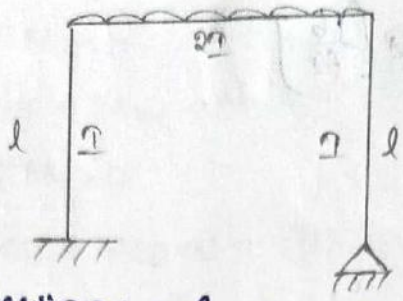


2. Unsymmetrical shape of frame

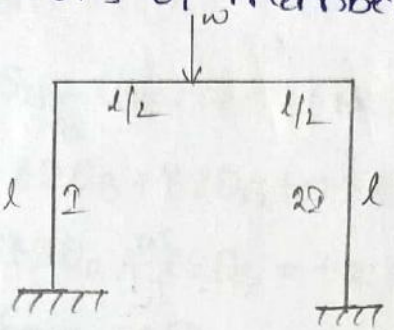


Scanned with CamScanner

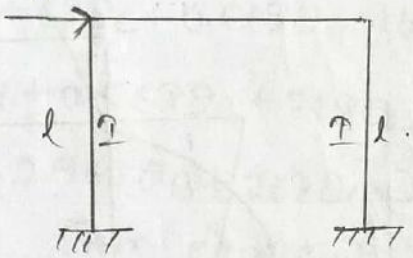
3. Different end conditions of Support:



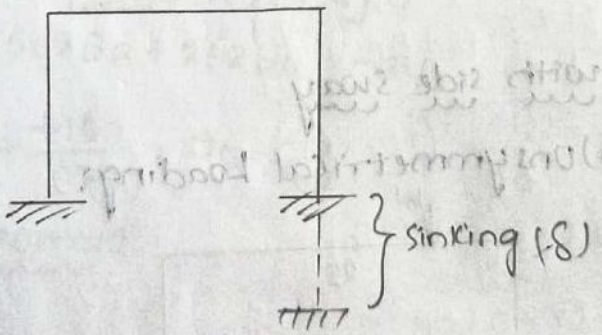
4. Non-uniform sections of members:



5. Horizontal loading



6. Settlement of Support



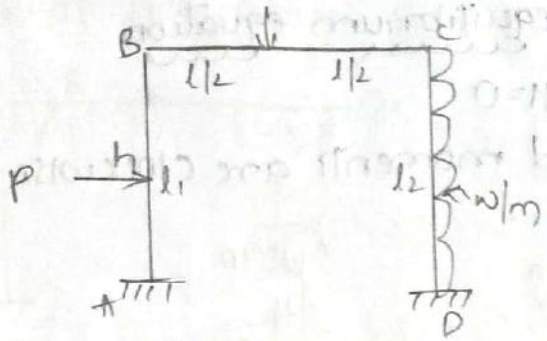
Note:

*In the Portal frames due to sidesway, Sway (Δ) becomes additional unknown.

*It is evaluated by using horizontal (or) shear equilibrium equation i.e., $\boxed{\Sigma H = 0}$

Scanned with CamScanner

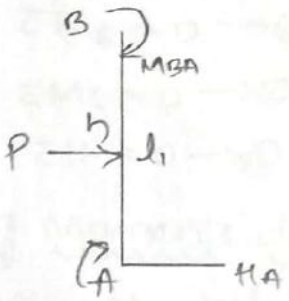
Example:1

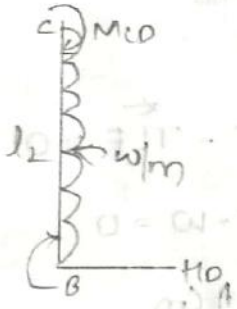


Horizontal (or) Shear Equilibrium

$\Sigma H = 0$

Assume, all end moments of a column are clockwise.





Standard form $\Sigma H = 0$

$H_A + H_D +$ Towards Right forces $-$ Towards left forces $= 0$

Eg: $\vec{H_A} + \vec{H_D} + P - (w \times l_2) = 0$

$$H_A = \frac{\text{Clockwise moment} - \text{Anticlockwise moment}}{h_{AB}}$$

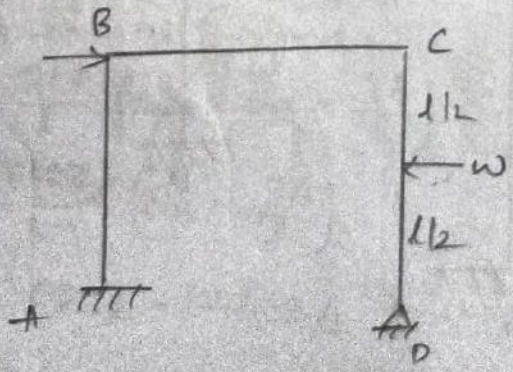
$$H_D = \frac{\text{Clockwise moment} - \text{Anticlockwise moment}}{h_{CD}}$$

Take the moments of forces about top of column.

Eg: $H_A = \frac{(M_{AB} + M_{BA}) - (P \times h)}{l_1}$

$$H_D = \frac{\left(M_{CD} + M_{DC} + w \times l_2 \times \frac{l_2}{2}\right) - 0}{l_2}$$

Example 2.

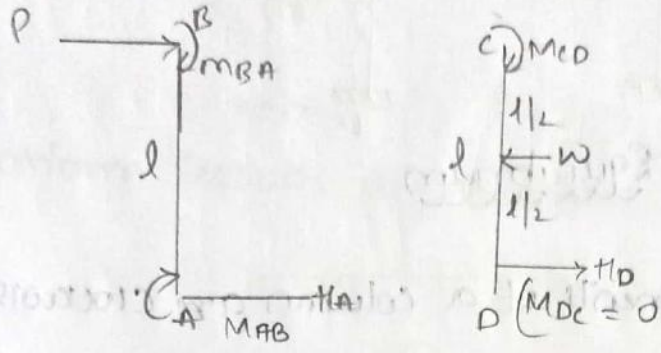


Scanned with CamScanner

Horizontal (or) shear equilibrium equation

$$\Sigma H = 0$$

Assume all the end moments are clockwise



$$\Sigma H = 0$$

$$\vec{H_A} + \vec{H_D} + \vec{TRF} - \vec{TLF} = 0$$

$$H_A + H_D + P - W = 0$$

$$H_A = \frac{M - A\bar{C}m}{h_{AB}}$$

$$H_D = \frac{\text{clockwise moment} - \text{anticlockwise}}{h_{CD}}$$

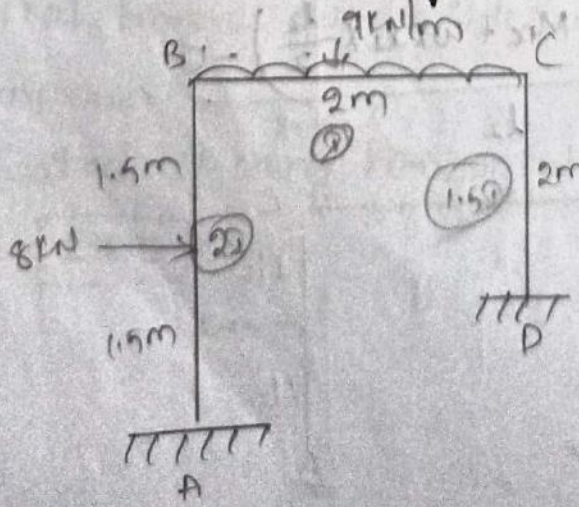
Take the moments of forces about top of the column

$$H_A = \frac{(M_{AB} + M_{BA}) - (P \times O)}{l}$$

$$H_D = \frac{\left(M_{CD} + \frac{wl}{2}\right) - O}{l}$$

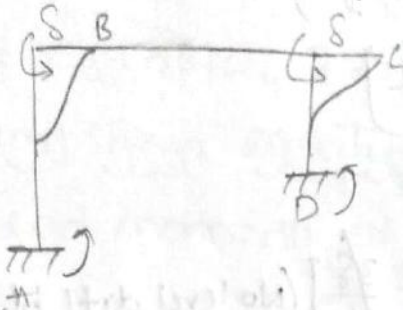
Problems Based on Sway Frames

1. Analyse the given frame using Slope Deflection Method.



Scanned with CamScanner

Assume it is Right side sway.



$\theta_A, \theta_B, \theta_C, \theta_D, \delta$

$\theta_B, \theta_C, \delta$

$\Sigma M_B = 0 \rightarrow (1)$

$\Sigma M_C = 0 \rightarrow (2)$

$\Sigma H = 0 \rightarrow (3)$

Bending Moments due to loads:

Span AB, $M = \frac{wl}{4} = \frac{8 \times 3}{4} = 6$ KNt-m

Span BC, $M = \frac{wl^2}{8} = \frac{9 \times 2^2}{8} = 4.5$ KNt-m

Span CD, $M = 0$

Step ① Fixed end moments:

$M_{FAB} = \frac{-wl}{8} = \frac{-8 \times 3}{8} = -3$ KNt-m.

$M_{FBA} = \frac{+wl}{8} = \frac{+8 \times 3}{8} = +3$ KNt-m

$M_{FBC} = \frac{-wl^2}{12} = \frac{-9 \times 2^2}{12} = -3$ KNt-m

$M_{FCB} = \frac{+wl^2}{12} = \frac{+9 \times 2^2}{12} = +3$ KNt-m

$M_{FCD} = M_{FDC} = 0$

Step ② Slope Deflection equation

$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\Delta}{l}\right]$

$= -3 + \frac{2E(2I)}{3}\left[\theta_B - \frac{3\Delta}{l}\right]$

$M_{AB} = -3 + 1.33EI\theta_B - 1.33EI\Delta$

Scanned with CamScanner

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\delta}{l}\right]$$

$$= 3 + \frac{2E(2I)}{3}\left[2\theta_B - \frac{3\delta}{l}\right]$$

$$M_{BA} = 3 + 2.66EI\theta_B - 1.33\delta EI$$

$$M_{BC} = M_{FBC} + \frac{2E(I)}{l}\left[2\theta_B + \theta_C - \frac{3\delta}{l}\right]$$ (No level diff b/w B & C)

$$= -3 + \frac{2EI}{2}\left[2\theta_B + \theta_C\right]$$

$$M_{BC} = -3 + 2EI\theta_B + \theta_C EI$$

$$M_{CB} = M_{FCB} + \frac{2E(I)}{l}\left[2\theta_C + \theta_B - \frac{3\delta}{l}\right]$$

$$= 3 + \frac{2EI}{2}\left[2\theta_C + \theta_B\right]$$

$$M_{CB} = 3 + 2EI\theta_C + EI\theta_B$$

$$M_{CD} = M_{FCD} + \frac{2E(I)}{l}\left[2\theta_C + \theta_D - \frac{3\delta}{l}\right]$$

$$= 0 + \frac{2E(1.5I)}{2}\left[2\theta_C - \frac{3\delta}{2}\right]$$

$$M_{CD} = 3EI\theta_C - 2.25EI\delta$$

$$M_{DC} = M_{FDC} + \frac{2E(I)}{l}\left[2\theta_D + \theta_C - \frac{3\delta}{l}\right]$$

$$= 0 + \frac{2E(1.5I)}{2}\left[\theta_C - \frac{3\delta}{2}\right]$$

$$M_{DC} = 1.5EI\theta_C - 2.25EI\delta.$$

Step ③ Joint Equilibrium Equation

$$\Sigma M_B = 0$$

$$M_{BA} + M_{BC} = 0$$

$$3 + 2.66EI\theta_B - 1.33EI\delta - 3 + 2EI\theta_B + \theta_C EI = 0$$

$$4.66EI\theta_B - 1.33EI\delta + \theta_C EI = 0$$

$$4.66EI\theta_B + \theta_C EI - 1.33EI\delta = 0 \rightarrow (1)$$

$$\Sigma M_C = 0$$

$$M_{CB} + M_{CD} = 0$$

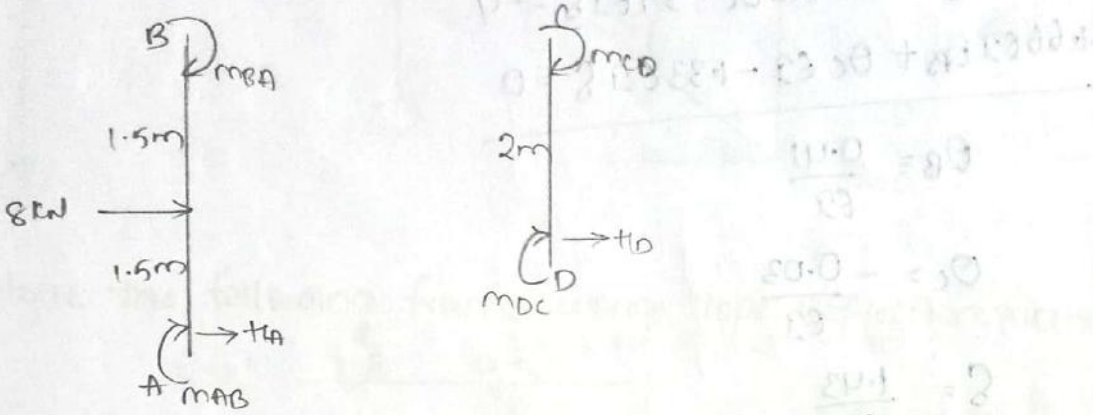
Scanned with CamScanner

$3 + 2EI\theta_C + EI\theta_B + 3EI\theta_C - 2.25EI\delta = 0$

$3 + EI\theta_B + 5EI\theta_C - 2.25EI\delta = 0 \rightarrow (2)$

Horizontal (or) shear equation ($\Sigma H = 0$)

Assume all end moments of column are clockwise.



$\Sigma H = 0$

$\overrightarrow{H_A} + \overrightarrow{H_D} + \text{Towards right forces} - \text{Towards left forces} = 0$

$H_A + H_D + 8 - 0 = 0$

$H_A = \dfrac{\text{Clockwise moment} - \text{Anticlockwise moment}}{h_{AB}}$

$H_D = \dfrac{\text{Clockwise moment} - \text{Anticlockwise moment}}{h_{CD}}$

Take the moments of forces of top of the column.

$H_A = \dfrac{(M_{AB} + M_{BA}) - (8 \times 1.5)}{3} = \dfrac{M_{AB} + M_{BA} - 12}{3}$

$= 0.33(M_{AB} + M_{BA}) - 4$

$H_D = \dfrac{(M_{CD} + M_{DC}) - 0}{2} = \dfrac{M_{CD} + M_{DC}}{2} = 0.5(M_{CD} + M_{DC})$

$\therefore H_A + H_D + 8 = 0$

$0.33M_{AB} + 0.33M_{BA} - 4 + 0.5M_{CD} + 0.5M_{DC} + 8 = 0$

$0.33(-3 + 1.33EI\theta_B - 1.33EI\delta) + 0.33(3 + 2.66EI\theta_B - 1.33EI\delta) - 4 + 0.5(3EI\theta_C - 2.25EI\delta) + 0.5[1.5EI\theta_C - 2.25EI\delta] + 8 = 0$

$-0.99 + 0.43EI\theta_B - 0.43EI\delta + 0.99 + 0.87EI\theta_B - 0.43EI\delta - 4 + 1.5EI\theta_C - 1.12EI\delta + 0.75EI\theta_C - 1.12EI\delta + 8 = 0$

Scanned with CamScanner

$4 + 1.3EI\theta_B + 2.25EI\theta_C - 3.1EI\delta = 0 \rightarrow (3)$

Solve (2) & (3)

$\theta_B EI + 5EI\theta_C - 2.25EI\delta = -3$

$1.3EI\theta_B + 2.25EI\theta_C - 3.1EI\delta = -4$

$4.66EI\theta_B + \theta_C EI - 1.33EI\delta = 0$

$\theta_B = \frac{0.41}{EI}$

$\theta_C = -\frac{0.03}{EI}$

$\delta = \frac{1.43}{EI}$

Step (4) Final end Moments

$M_{AB} = -3 + 1.33EI\left(\frac{0.41}{EI}\right) - 1.33EI\left(\frac{1.43}{EI}\right)$

$= -4.35$ Knt-m

$M_{BA} = 3 + 2.66EI\left(\frac{0.41}{EI}\right) - 1.33EI\left(\frac{1.43}{EI}\right)$

$= +2.18$ Knt-m

$M_{BC} = -3 + 2EI\left(\frac{0.41}{EI}\right) + EI\left(\frac{-0.03}{EI}\right)$

$= -2.21$ Knt-m

$M_{CB} = 3 + 2EI\left(\frac{-0.03}{EI}\right) + EI\left(\frac{0.41}{EI}\right)$

$= 3.35$ Knt-m

$M_{CD} = 3EI\left(\frac{-0.03}{EI}\right) - 2.25EI\left(\frac{1.43}{EI}\right)$

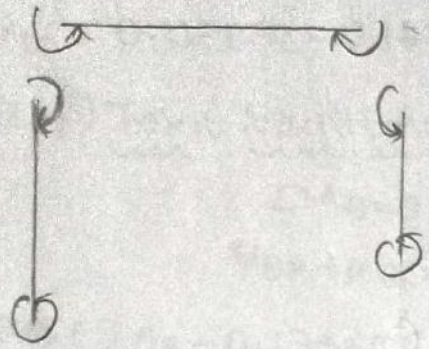
$= -3.30$ Knt-m.

$M_{DC} = 1.5EI\left(\frac{-0.03}{EI}\right) - 2.25EI\left(\frac{1.43}{EI}\right)$

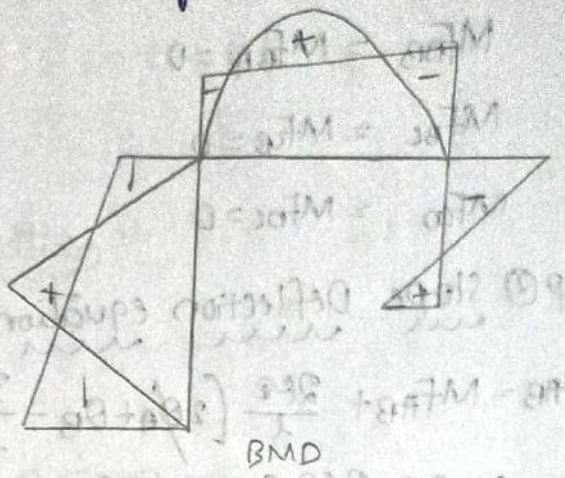
$= -3.26$ Knt-m

Scanned with CamScanner

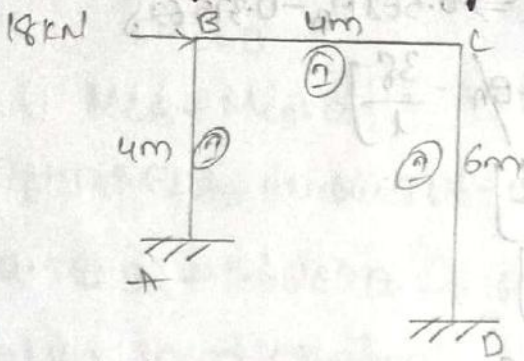
Free Bending Moment



Bending Moment Diagram.

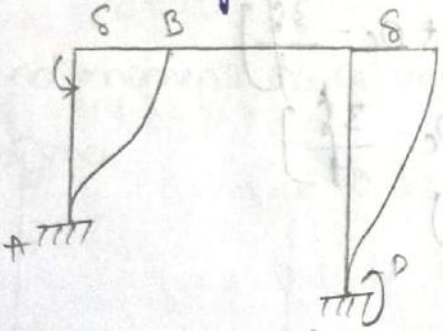


2. Analyse the following frame using slope Deflection method.



Sway
(Assume Right side Sway)

Assume right side sway.



$\theta_A, \theta_B, \theta_C, \theta_D$ and δ

$\boxed{\theta_B, \theta_C, \delta}$

$\Sigma M_B = 0 \rightarrow (1)$

$\Sigma M_C = 0 \rightarrow (2)$

$\Sigma H = 0 \rightarrow (3)$

Bending moment due to loads

Span AB, $M = 0$

Span BC, $M = 0$

Span CD, $M = 0$

Scanned with CamScanner

Step ① fixed end moment

$M_{FAB} = M_{FBA} = 0$

$M_{FBC} = M_{FCB} = 0$

$M_{FCD} = M_{FDC} = 0$

Step ② slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right]$$

$$= 0 + \frac{2EI}{4}\left[\theta_B - \frac{3\delta}{4}\right]$$

$$M_{AB} = 0.5EI\left[\theta_B - 0.75\delta\right] \Rightarrow 0.5EI\theta_B - 0.375EI\delta$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\delta}{l}\right]$$

$$= 0 + \frac{2EI}{4}\left[2\theta_B - \frac{3\delta}{4}\right]$$

$$= 0.5EI\left[2\theta_B - \frac{3\delta}{4}\right]$$

$$M_{BA} = EI\theta_B - 0.375EI\delta$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[2\theta_B + \theta_C - \frac{3\delta}{l}\right]$$

$$= 0 + \frac{2EI}{4}\left[2\theta_B + \theta_C - \frac{3\delta}{4}\right]$$

$$= 0.5EI\left[2\theta_B + \theta_C\right]$$

$$M_{BC} = EI\theta_B + 0.5EI\theta_C$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left[2\theta_C + \theta_B - \frac{3\delta}{l}\right]$$

$$= 0 + \frac{2EI}{4}\left[2\theta_C + \theta_B\right]$$

$$= 0.5EI\left(2\theta_C + \theta_B\right)$$

$$M_{CB} = EI\theta_C + 0.5EI\theta_B$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}\left[2\theta_C + \theta_D - \frac{3\delta}{l}\right]$$

$$= 0 + \frac{2EI}{6}\left[2\theta_C + \theta_D - \frac{3\delta}{l}\right]$$

$$= 0.33EI\left[2\theta_C - \frac{3\delta}{6}\right]$$

$$M_{CD} = 0.66EI\theta_C - 0.165EI\delta$$

Scanned with CamScanner

$M_{DC} = M_{FDC} + \frac{2EI}{l}\left[2\theta_D + \theta_C - \frac{3\delta}{l}\right]$

$= 0 + \frac{2EI}{6}\left[2\theta_C - \frac{3\delta}{l}\right]$

$M_{DC} = 0.33EI\theta_C - 0.167EI\delta$

Step ③ Joint Equilibrium equation

$\Sigma M_B = 0$

$M_{BA} + M_{BC} = 0$

$EI\theta_B - 0.375EI\delta + EI\theta_B + 0.5EI\theta_C = 0$

$2EI\theta_B + 0.5EI\theta_C - 0.375EI\delta = 0 \rightarrow (1)$

$\Sigma M_C = 0$

$M_{CB} + M_{CD} = 0$

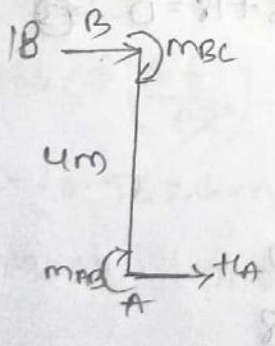
$EI\theta_C + 0.5EI\theta_B + 0.66EI\theta_C - 0.165EI\delta = 0.$

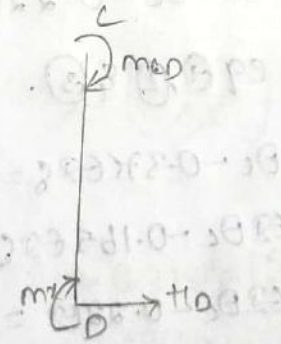
$0.5EI\theta_B + 1.66EI\theta_C - 0.165EI\delta = 0 \rightarrow (2).$

Horizontal (or) shear equation

$\Sigma H = 0$

Assume all end moments of column are clockwise.





$\Sigma H = 0$

$\vec{H_A} + \vec{H_D} + \text{Towards right forces} - \text{Towards left forces} = 0$

$H_A + H_D + 18 - 0 = 0$

$H_A = \frac{\text{Clockwise moments} - \text{Anti clockwise moments}}{h_{AB}}$

$H_D = \frac{\text{Clockwise moments} - \text{Anti clockwise moments}}{h_{CD}}$

Scanned with CamScanner

Take the moment of forces of top of the column.

$$H_A = \frac{(M_{AB} + M_{BA}) - (18 \times 0)}{4}$$

$$H_A = 0.25 (M_{AB} + M_{BA})$$

$$H_D = \frac{(M_{CD} + M_{DC}) - 0}{6}$$

$$H_D = 0.167 (m_{CD} + m_{DC})$$

$$\therefore H_A + H_D + 18 = 0.$$

$$0.25 (M_{AB} + m_{BA}) + 0.167 (M_{CD} + m_{DC}) + 18 = 0$$

$$0.25 \left[0.5 EI\theta_B - 0.375 EI\delta + EI\theta_B - 0.375 EI\delta \right] + 0.167 \left[0.66 EI\theta_C - 0.165 EI\delta + 0.33 EI\theta_C - 0.167 EI\delta \right] + 18 = 0.$$

$$0.125 EI\theta_B - 0.09 EI\delta + 0.25 EI\theta_B - 0.09 EI\delta + 0.11 EI\theta_C - 0.02 EI\delta + 0.05 EI\theta_C - 0.02 EI\delta + 18 = 0.$$

$$0.375 EI\theta_B + 0.16 EI\theta_C - 0.22 EI\delta + 18 = 0 \quad \text{--- (3)}$$

from eq (1) (2) & (3).

$$2 EI\theta_B + 0.5 EI\theta_C - 0.375 EI\delta = 0$$

$$0.5 EI\theta_B + 1.66 EI\theta_C - 0.165 EI\delta = 0$$

$$0.375 EI\theta_B + 0.16 EI\theta_C - 0.22 EI = -18$$

$$\theta_B = \frac{+21.59}{EI}$$

$$\theta_C = \frac{+5.69}{EI}$$

$$\delta = \frac{122.77}{EI}$$

Scanned with CamScanner

Step ④ Final end Moments

$M_{AB} = 0.5EI\theta_B - 0.375EI\delta$

$= 0.5EI\left(\frac{21.59}{EI}\right) - 0.375EI\left(\frac{122.27}{EI}\right)$

$M_{AB} = -35.05 \text{ Kn-m}$

$M_{BA} = EI\theta_B - 0.375EI\delta.$

$= EI\left(\frac{21.59}{EI}\right) - 0.375EI\left(\frac{122.77}{EI}\right)$

$= -24.44 \text{ Kn-m.}$

$M_{BC} = EI\theta_B + 0.5EI\theta_C$

$= EI\left(\frac{21.59}{EI}\right) + 0.5EI\left(\frac{5.69}{EI}\right)$

$M_{BC} = +24.39 \text{ Kn-m}$

$M_{CB} = EI\theta_C + 0.5EI\theta_B$

$= EI\left(\frac{5.69}{EI}\right) + 0.5EI\left(\frac{21.59}{EI}\right)$

$= +16.48 \text{ Kn-m}$

$M_{CD} = 0.66EI\theta_C - 0.165EI\delta$

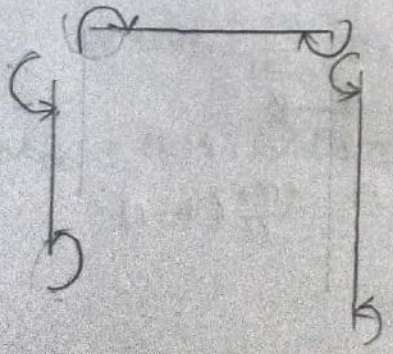
$= 0.66EI\left(\frac{5.69}{EI}\right) - 0.165EI\left(\frac{122.77}{EI}\right)$

$M_{CD} = -16.56 \text{ Kn-m}$

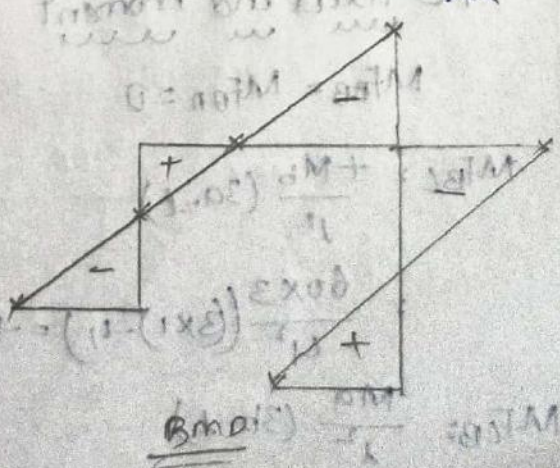
$M_{DC} = 0.33EI\theta_C - 0.165EI\delta \Rightarrow 0.33EI\left(\frac{5.69}{EI}\right) - 0.165EI\left(\frac{122.77}{EI}\right)$

$= -18.62.$

Free Bending moment

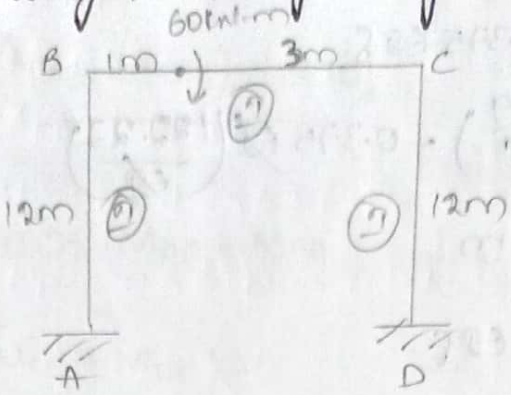


Bending moment Diagram.

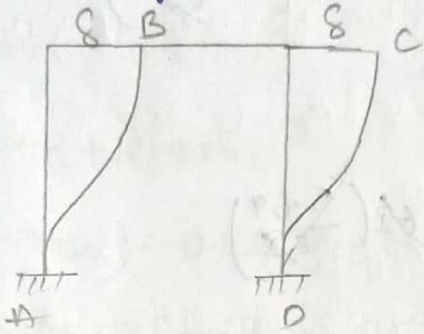


Scanned with CamScanner

3. Analyse the following frame by using slope Deflection Method.



Assume right side sway.



$\theta_A, \theta_B, \theta_C, \phi_D, \delta$

$\theta_B, \theta_C, \delta$

$\Sigma M_B = 0$

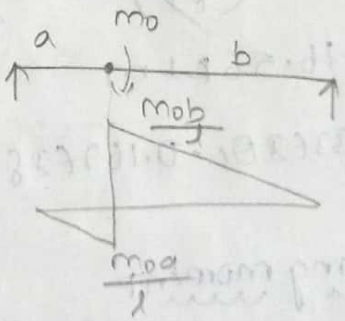
$\Sigma M_C = 0$

$\Sigma H = 0.$

Bending Moments due to loads.

Span AB, M = 0

Span BC, M =



$\frac{M_{oa}}{l} = \frac{60 \times 1}{4}$

$= 15 kN.m$

$\frac{M_{ob}}{l} = \frac{60 \times 3}{4}$

$= 45 kN.m$

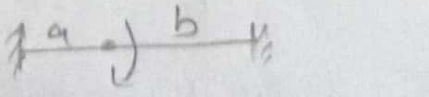
Span CD, M = 0

Step ① Fixed end moment

$M_{FAB} = M_{FBA} = 0$

$M_{FBC} = \frac{+M_b}{l^2}(3a - l)$

$= \frac{60 \times 3}{4^2}((3 \times 1) - 4) = -11.25$



$\frac{+m_b}{l^2}(3a - l) \quad \frac{+m_a}{l^2}(3b - l)$

$M_{FCB} = \frac{M_a}{l^2}(3b - l)$

Scanned with CamScanner

$= \frac{60 \times 1}{4^2}((3\times3)-4)$

$= +18.75 \text{ kN-m}$

$M_{FCD} = M_{FDC} = 0.$

Step ③ Slope Deflection equation

$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right]$

$= 0 + \frac{2EI}{12}\left(\theta_B - \frac{3\delta}{12}\right)$

$M_{AB} = 0.167EI\theta_B - 0.04EI\delta$

$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\delta}{l}\right]$

$= 0 + \frac{2EI}{12}\left[2\theta_B - \frac{3\delta}{12}\right]$

$M_{BA} = 0.33EI\theta_B - 0.04EI\delta$

$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[2\theta_B + \theta_C - \frac{3\delta}{l}\right]$

$= -11.25 + \frac{2EI}{4}\left[2\theta_B + \theta_C\right]$

$M_{BC} = -11.25 + EI\theta_B + 0.5EI\theta_C$

$M_{CB} = M_{FCB} + \frac{2EI}{l}\left[2\theta_C + \theta_B - \frac{3\delta}{l}\right]$

$= 18.75 + \frac{2EI}{4}\left[2\theta_C + \theta_B\right]$

$M_{CB} = 18.75 + EI\theta_C + 0.5EI\theta_B$

$M_{CD} = M_{FCD} + \frac{2EI}{l}\left[2\theta_C + \theta_D - \frac{3\delta}{l}\right]$

$= 0 + \frac{2EI}{12}\left[2\theta_C + \theta_D - \frac{3\delta}{12}\right]$

$M_{CD} = 0.33EI\theta_C - 0.04EI\delta$

$M_{DC} = M_{FDC} + \frac{2EI}{l}\left[2\theta_D + \theta_C - \frac{3\delta}{l}\right]$

$= 0 + \frac{2EI}{12}\left[\theta_C - \frac{3\delta}{12}\right]$

$M_{DC} = 0.167EI\theta_C - 0.04EI\delta$

Scanned with CamScanner

Step ③ Joint equilibrium equation

$$\Sigma M_B = 0$$

$$M_{BA} + M_{BC} = 0$$

$$0.33 EI\theta_B - 0.04 EI\delta - 11.25 + EI\theta_B + 0.5 EI\theta_C = 0$$

$$1.33 EI\theta_B + 0.5 EI\theta_C - 0.04 EI\delta = 11.25 \rightarrow (1)$$

$$\Sigma M_C = 0$$

$$M_{CB} + M_{CD} = 0$$

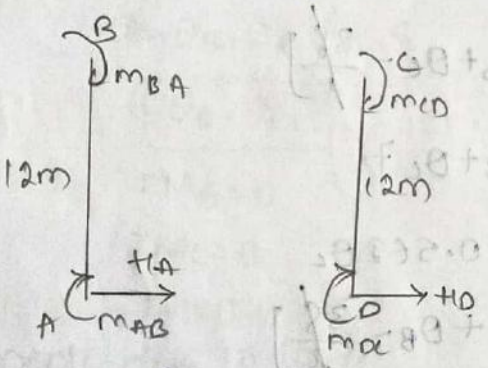
$$18.75 + EI\theta_C + 0.5 EI\theta_B + 0.33 EI\theta_C - 0.04 EI\delta = 0$$

$$0.5 EI\theta_B + 1.33 EI\theta_C - 0.04 EI\delta = -18.75 \rightarrow (2)$$

Horizontal (or) Shear eq^n

$$\Sigma F_x = 0$$

Assume all end moments of columns are clockwise.



$$\rightarrow H_A + \rightarrow H_D + \rightarrow TRF - \leftarrow TLF = 0$$

$$H_A + H_D + 0 - 0 = 0$$

$$H_A = \frac{Cm - Acm}{h_{AB}}$$

$$H_D = \frac{Cm - Acm}{h_{CD}}$$

Take the moment of forces of top of column

$$H_A = \frac{(m_{AB} + m_{BA}) - 0}{12}$$

$$= 0.08 (m_{AB} + m_{BA})$$

Scanned with CamScanner

$$H_D = \frac{(m_{CD} + m_{DC}) - 0}{12}$$

$$= 0.08 (m_{CD} + m_{DC})$$

$$H_A + H_D = 0.$$

$$0.08 (m_{AB} + m_{BA}) + 0.08 (m_{CD} + m_{DC}) = 0.$$

$$0.08 \left[0.167 EI\theta_B - 0.04 EI\delta + 0.33 EI\theta_B - 0.04 EI\delta\right] + 0.08 \left[0.33 EI\theta_C - 0.04 EI\delta + 0.167 EI\theta_C - 0.04 EI\delta\right] = 0$$

$$0.013 EI\theta_B - 0.003 EI\delta + 0.026 EI\theta_B - 0.003 EI\delta + 0.02 EI\theta_C - 0.003 EI\delta + 0.13 EI\theta_C - 0.003 EI\delta = 0.$$

$$0.039 EI\theta_B + 0.039 EI\theta_C - 0.012 EI\delta = 0 \rightarrow (3)$$

from eq (1), (2) & (3)

$$1.33 EI\theta_B + 0.5 EI\theta_C - 0.04 EI\delta = 11.25$$

$$0.5 EI\theta_B + 1.33 EI\theta_C - 0.04 EI\delta = -18.75$$

$$0.039 EI\theta_B + 0.039 EI\theta_C - 0.012 EI\delta = 0.$$

$$\theta_B = \frac{15.68}{EI}$$

$$\theta_C = \frac{-20.46}{EI}, \quad \delta = \frac{-15.52}{EI}$$

Step (4) Final end moments

$$M_{AB} = M_{FAB} + 0.167 EI\theta_B - 0.04 EI\delta$$

$$= 0.167 EI \left(\frac{15.68}{EI}\right) - 0.04 EI \left(\frac{-15.52}{EI}\right)$$

$$= +3.23 kN\text{-}m.$$

$$M_{BA} = 0.33 EI\theta_B - 0.04 EI\delta$$

$$= 0.33 EI \left(\frac{15.68}{EI}\right) - 0.04 EI \left(\frac{-15.52}{EI}\right)$$

$$= +5.49 kN\text{-}m$$

$$M_{BC} = -11.25 + EI\theta_B + 0.5 EI\theta_C$$

$$= -11.25 + EI \left(\frac{15.68}{EI}\right) + 0.5 EI \left(\frac{-20.46}{EI}\right)$$

$$= -5.65 kN\text{-}m.$$

Scanned with CamScanner

$M_{CB} = 18.75 + 2EI\theta_C + 0.5\,2EI\theta_B$

$= 18.75 + 2EI\left(\frac{-20.16}{2EI}\right) + 0.5\,2EI\left(\frac{15.68}{2EI}\right)$

$= 6.43$ KN-m

$M_{CD} = 0.33\,2EI\theta_C - 0.04\,2EI\delta$

$= 0.33\,2EI\left(\frac{-20.16}{2EI}\right) - 0.04\,2EI\left(\frac{-15.52}{2EI}\right)$

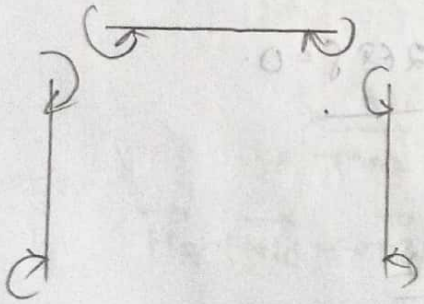
$= -6.03$ KN-m

$M_{DC} = 0.167\,2EI\theta_C - 0.04\,2EI\delta$

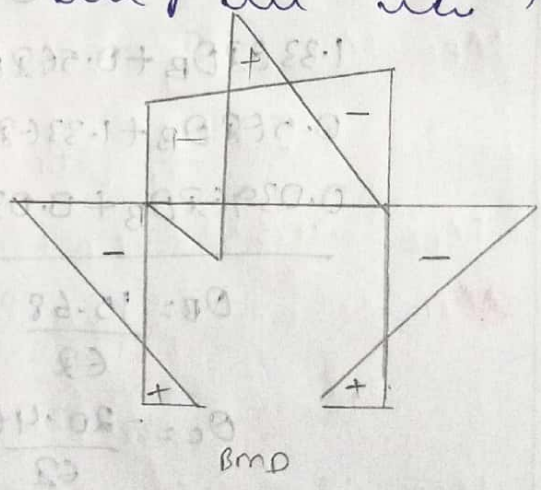
$= 0.167\,2EI\left(\frac{-20.16}{2EI}\right) - 0.04\,2EI\left(\frac{-15.52}{2EI}\right)$

$= -2.74$ KN-m.

Free Bending moment



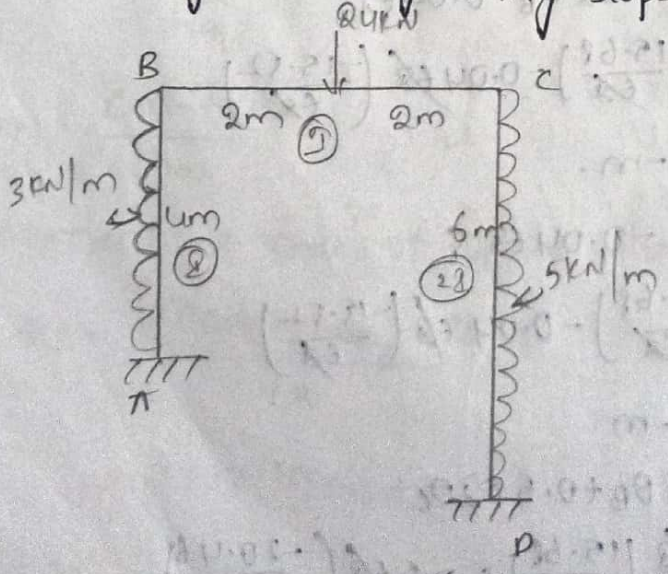
Bending Moment Diagram



BMD

H.W

4. Analyse the following frame by using slope Deflection method.



Scanned with CamScanner

Bending Moment due to loads

Span AB: $M = \frac{wl^2}{8} = \frac{3 \times 4^2}{8} = 6\ kN\text{-}m$

Span BC: $M = \frac{wl}{4} = \frac{24 \times 4}{4} = 24\ kN\text{-}m$

Span CD: $M = \frac{wl^2}{8} = \frac{5 \times 6^2}{8} = 22.5\ kN\text{-}m$

Step ① Fixed end Moment

$M_{FAB} = \frac{-wl^2}{12} = \frac{-3 \times 4^2}{12} = -4\ kN\text{-}m$

$M_{FBA} = +\frac{wl^2}{12} = \frac{+3 \times 4^2}{12} = +4\ kN\text{-}m$

$M_{FBC} = \frac{-wl}{8} = \frac{-24 \times 4}{8} = -12\ kN\text{-}m$

$M_{FCB} = \frac{+wl}{8} = \frac{+24 \times 4}{8} = +12\ kN\text{-}m$

$M_{FCD} = \frac{-wl^2}{12} = \frac{-5 \times 6^2}{12} = -15\ kN\text{-}m$

$M_{FDC} = \frac{+wl^2}{12} = \frac{+5 \times 6^2}{12} = +15\ kN\text{-}m$.

Step ② Slope Deflection Equation

$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right]$

$= -4 + \frac{2EI}{4}\left[\theta_B - \frac{3\delta}{4}\right]$

$M_{AB} = -4 + 0.5EI\theta_B - 0.37EI\delta$

$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\delta}{l}\right]$

$= +4 + \frac{2EI}{4}\left[2\theta_B - \frac{3\delta}{4}\right]$

$M_{BA} = 4 + EI\theta_B - 0.37EI\delta.$

$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[2\theta_B + \theta_C - \frac{3\delta}{l}\right]$

$= -12 + \frac{2EI}{4}\left[2\theta_B + \theta_C - \frac{3\delta}{4}\right]$

$M_{BC} = -12 + EI\theta_B + 0.5EI\theta_C - 0.37EI\delta$

$M_{CB} = M_{FCB} + \frac{2EI}{l}\left[2\theta_C + \theta_B - \frac{3\delta}{l}\right]$

$= 12 + \frac{2EI}{4}\left[2\theta_C + \theta_B\right]$

$M_{CB} = 12 + EI\theta_C + 0.5EI\theta_B$

Scanned with CamScanner

$$M_{CD} = M_{FCD} + \frac{2EI}{l}\left(2\theta_C + \theta_D - \frac{3\delta}{l}\right)$$

$$= -15 + \frac{2E(2I)}{6}\left(2\theta_C - \frac{3\delta}{6}\right)$$

$$M_{CD} = -15 + 1.33EI\theta_C - 0.33EI\delta$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}\left(2\theta_D + \theta_C - \frac{3\delta}{l}\right)$$

$$= 15 + \frac{2E(2I)}{6}\left(\theta_C - \frac{3\delta}{6}\right)$$

$$M_{DC} = 15 + 0.67EI\theta_C - 0.33EI\delta$$

Step ③

Joint Equilibrium Equation

$$\Sigma M_B = 0$$

$$M_{BA} + M_{BC} = 0$$

$$4 + EI\theta_B - 0.37EI\delta - 12 + EI\theta_B + 0.5EI\theta_C = 0$$

$$-8 + 2EI\theta_B + 0.5EI\theta_C - 0.37EI\delta = 0$$

$$2EI\theta_B + 0.5EI\theta_C - 0.37EI\delta = 8 \rightarrow (1)$$

$$\Sigma M_C = 0$$

$$M_{CB} + M_{CD} = 0$$

$$12 + EI\theta_C + 0.5EI\theta_B - 15 + 1.33EI\theta_C - 0.33EI\delta = 0$$

$$-3 + 0.5EI\theta_B + 2.33EI\theta_C - 0.33EI\delta = 0$$

$$0.5EI\theta_B + 2.33EI\theta_C - 0.33EI\delta = 3 \rightarrow (2)$$

Horizontal (or) Shear Equation

$$\Sigma H = 0$$

Assume all end moments of columns are clockwise

Scanned with CamScanner

Part-II

Moment Distribution Method

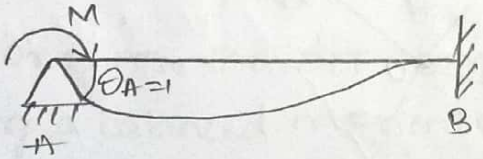
* This Method is introduced by Prof. "Hardy Cross" in 1932.
* It is an iterative method to find the final end moments.
* The basic Concepts that are included in this method are,
  1. Fixed end Moment
  2. Relative Stiffness
  3. Distribution factor
  4. Carry Over

1. Fixed end Moments

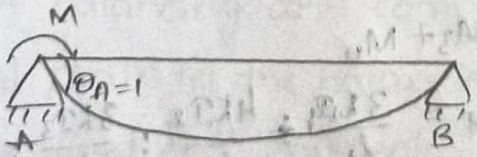
All members of a given frame are initially assumed fixed at both ends

2. Stiffness

It is the moment required to produce unit rotation at a specified joint in a member.



The Stiffness of the member whose far end is fixed is given by "$\frac{4EI}{l}$"



The Stiffness of the member whose far end is hinged is given by "$\frac{3EI}{l}$"

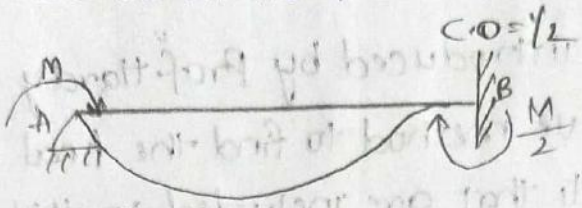
3. Carry over

The moment (M) at the joint 'A' will develope a moment of "$\frac{M}{2}$" at the joint 'B', when the joint 'B' is fixed

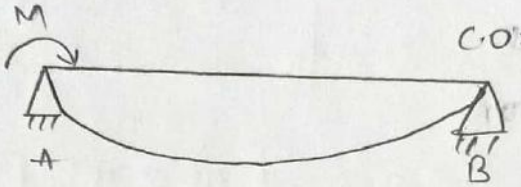
Scanned with CamScanner

*The factor '1/2' is called "carry over factor".

It is applicable when the far end is fixed



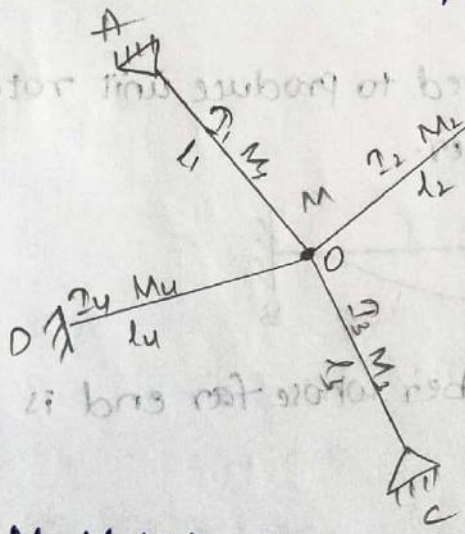
If the far end is hinged, then the carry over factor is "0"



4. Distribution factor

A Moment which is applied at a joint where several members are connected, then that moment will be distributed among the members interms of their stiffness.

Eg



$M = M_1 + M_2 + M_3 + M_4$

$M_1 : M_2 : M_3 : M_4 = \frac{3EI_1}{l_1} : \frac{4EI_2}{l_2} : \frac{3EI_3}{l_3} : \frac{4EI_4}{l_4}$

(Stiffness Proportion)

Divide by '4E'

$= \frac{3}{4}\cdot\frac{I_1}{l_1} : \frac{I_2}{l_2} : \frac{3}{4}\frac{I_3}{l_3} : \frac{I_4}{l_4}$ (Relative stiffness) (K)

$\therefore K_1 : K_2 : K_3 : K_4$

$\Sigma K = K_1 + K_2 + K_3 + K_4$

Scanned with CamScanner

* The relative stiffness of a member @ a joint whose far end is fixed is given by "$\frac{I}{l}$"
* The relative stiffness of a member whose far end is hinged is given by "$\frac{3}{4}\frac{I}{l}$"
* The Distribution factor is defined as the ratio of relative stiffness of a member to the total relative stiffness of all members meeting @ a joint.

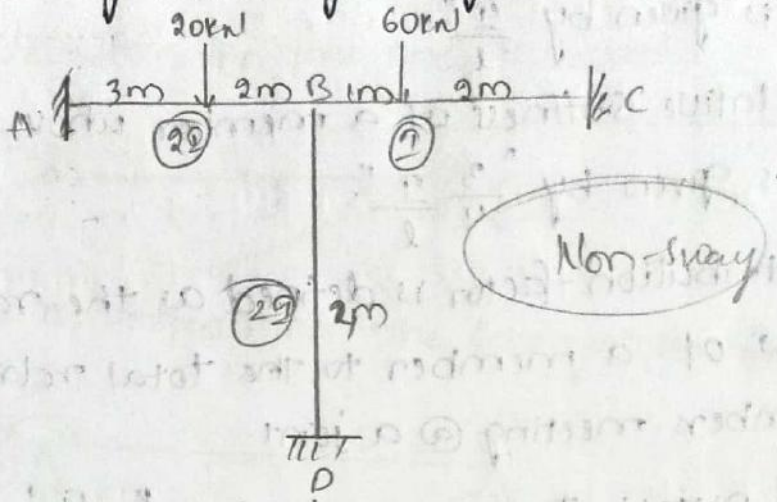
$\therefore$ Distribution factor (D.F) = $\frac{K}{\Sigma K}$.

Procedure

* Assume all the joints are fixed, then calculate fixed end Moments
* At simply supported end, balance the moment to get zero. Carry over the balanced moment. If the far end is fixed, Carry over moment is '$\frac{M}{2}$'
If the far end is hinged, the carry Moment is 'zero'.
* At the interior joints, the net moment should be made zero by applying a balanced moment. This balanced moment should be distributed to the members according to 'D.F'.
This distributed moment should be carry over to the following
* the Process of balancing and carry over shall be carried out at the internal joints till the balancing moments are negligable
* Find out the final end moments.

Scanned with CamScanner

1. Analyse the following frame by using Moment Distribution Method.



Bending Moment due to loads

Span AB, $M = \frac{wab}{l} = \frac{20 \times 3 \times 2}{5} = 24\ KNm$

Span BC, $M = \frac{wab}{l} = \frac{60 \times 1 \times 2}{3} = 40\ KNm$.

Span BD, $M = 0$

Step ① Fixed end Moment

$M_{FAB} = \frac{-wab^2}{l^2} = \frac{-20 \times 3 \times 2^2}{5^2} = -9.6\ KNm$

$M_{FBA} = \frac{+wa^2b}{l^2} = \frac{+20 \times 3^2 \times 2}{5^2} = +14.4\ KNm$.

$M_{FBC} = \frac{-wab^2}{l^2} = \frac{-60 \times 1 \times 2^2}{3^2} = -26.67\ KNm$

$M_{FCB} = \frac{+wa^2b}{l^2} = \frac{+60 \times 1^2 \times 2}{3^2} = +13.33\ KNm$.

$M_{FBD} = M_{FDB} = 0$.

Step ② Distribution factor $(D.F = \frac{K}{\Sigma K})$

Joint	Member	$K\ (\frac{I}{L})$	ΣK	$DF = \frac{K}{\Sigma K}$
B	BA	$\frac{2I}{5} = 0.4I$	$1.23I$	0.32
	BC	$\frac{I}{3} = 0.33I$		0.26
	BD	$\frac{2I}{4} = 0.5I$		$\frac{0.4I}{1.00}$

Scanned with CamScanner

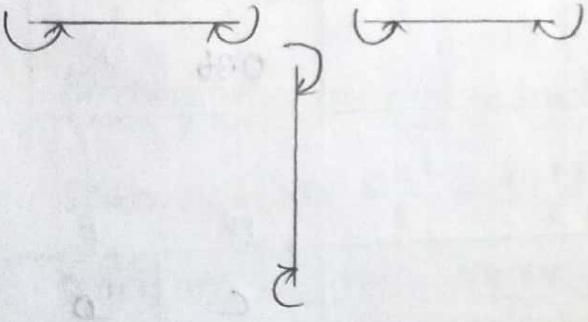
Step③ Moment Distribution

Joint	A	B			C	D
Member	AB	BA	BC	BD	CB	DB
D.F	–	0.32	0.26	0.42	–	–
FEM	-9.6	+14.4	-26.67	0	+13.33	0
Balance	–	+3.92	+3.19	+5.15	–	–
COM	3.92/2 = 1.96	–	–	–	3.19 → 1.59	5.15 → 2.57
final end Moment	-7.64	+18.32	-23.48	+5.15	+14.92	+2.57

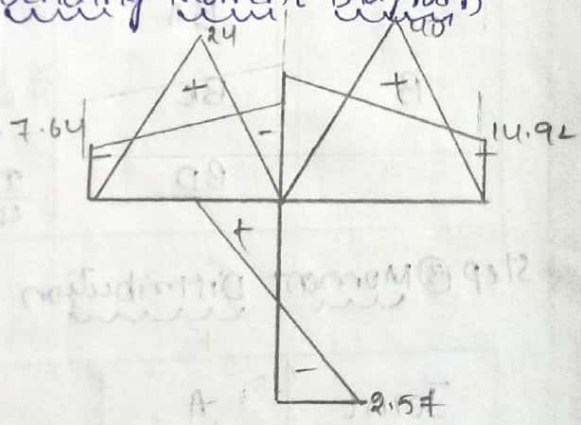
= 0 (check ✓)

B
+14.4
-26.67
+0
-12.27
Bal = +12.27
BA = 12.27 × 0.32 = +3.92
BC = 12.27 × 0.26 = 3.19
BD = 12.27 × 0.42 = 5.15

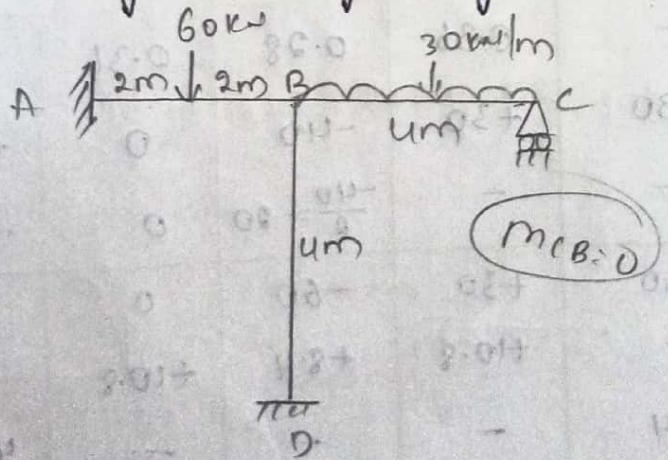
Free Body Diagram



Bending Moment Diagram



② Analyse the following frame by using Moment Distribution Method.



Bending Moment due to loads

Span AB, $M = \frac{wl}{4} = \frac{60 \times 4}{4} = 60 kNm$

Span BC, $M = \frac{wl^2}{8} = \frac{30 \times 4^2}{8} = 60 kNm$

Span BD, M = 0

Scanned with CamScanner

Step① Fixed end Moments

$M_{FAB} = \frac{-wl}{8} = \frac{-60\times4}{8} = -30\ kN\text{-}m$

$M_{FBA} = \frac{+wl}{8} = \frac{+60\times4}{8} = +30\ kN\text{-}m$

$M_{FBC} = \frac{-wl^2}{12} = \frac{-30\times4^2}{12} = -40\ kN\text{-}m$

$M_{FBD} = M_{FDB} = 0$

$M_{FCB} = \frac{+wl^2}{12} = \frac{+30\times4^2}{12} = +40\ kN\text{-}m$

Step② Distribution factor

Joint	Member	K	ΣK	$Df = \frac{K}{\Sigma K}$
B	BA	$\frac{I}{4} = 0.25I$	0.69I	0.36
	BC	$\frac{3}{4}\frac{I}{L} = \frac{3}{4}\cdot\frac{I}{4} = 0.18I$		0.28
	BD	$\frac{I}{4} = 0.25I$		0.36

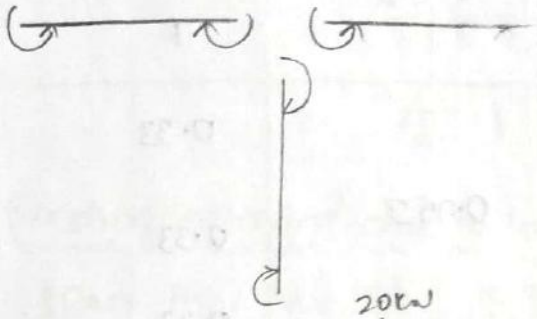
Step③ Moment Distribution

	F	I.J			H	F
Joint	A	B			C	D
Member	AB	BA	BC	BD	CB	DB
D.F	–	0.36	0.28	0.36	–	–
FEM	-30	+30	-40	0	+40	0
Balance & com	–	–	$\frac{-40}{2} = -20$	0	-40	–
Initial moment	-30	+30	-60	0	0	0
Balance	–	+10.8	+8.4	+10.8	0	–
Com	5.4	–	–	–	0	5.4
FEM	-24.6	+40.8	-51.6	+10.8	0	+5.4

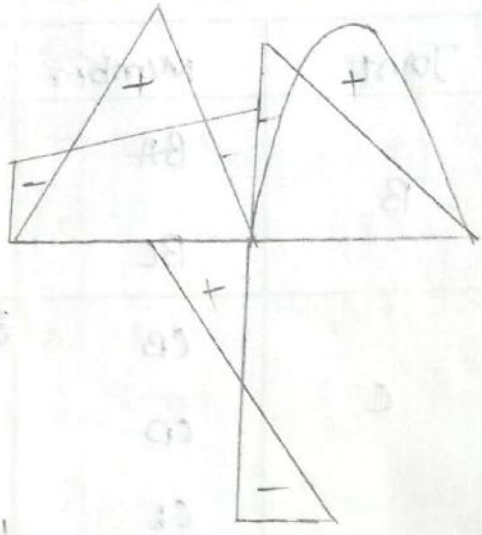
0 (Check ✓)

Scanned with CamScanner

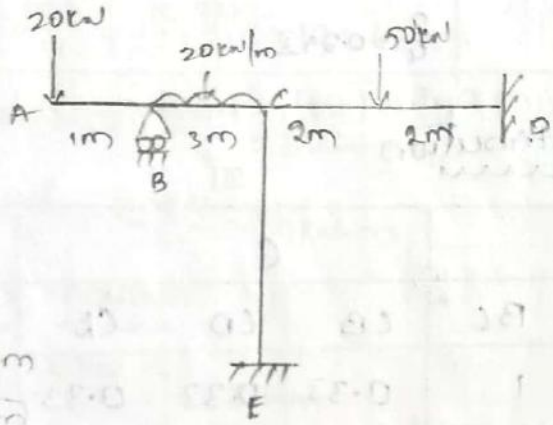
Free Body Diagram



Bending Moment Diagram



3.



Non-sway (single column)

$M_{AB} = (20 \times 0)$

$= 0$

$M_{BA} = +20 \times 1$

$= +20 \text{ kNm}$

$M_{BC} = -20 \text{ kNm}$

Bending Moment due to loads

Span BC, $M = \frac{wl^2}{8} = \frac{20 \times 3^2}{8} = 22.5 \text{ kNm}$.

Span CD, $M = \frac{wl}{4} = \frac{50 \times 4}{4} = 50 \text{ kNm}$

Span CE, $M = 0$

Step ① Fixed end Moment

$M_{AB} = (20 \times 0) = 0 \text{ kNm}$.

$M_{BA} = +(20 \times 1) = 20 \text{ kNm}$.

$M_{FBC} = \frac{-wl^2}{12} = \frac{-20 \times 3^2}{12} = -15 \text{ kNm}$

$M_{FCB} = \frac{+wl^2}{12} = \frac{+20 \times 3^2}{12} = +15 \text{ kNm}$

$M_{FCD} = \frac{-wl}{8} = \frac{-50 \times 4}{8} = -25 \text{ kNm}$

$M_{FDC} = \frac{+wl}{8} = \frac{+50 \times 4}{8} = +25 \text{ kNm}$

$M_{FCE} = M_{FEC} = 0$

Scanned with CamScanner

Step ② Distribution factor

** for the span CB, the far end B is a hinged support ; 'K' value should be taken as $\frac{3}{4}\frac{I}{L}$

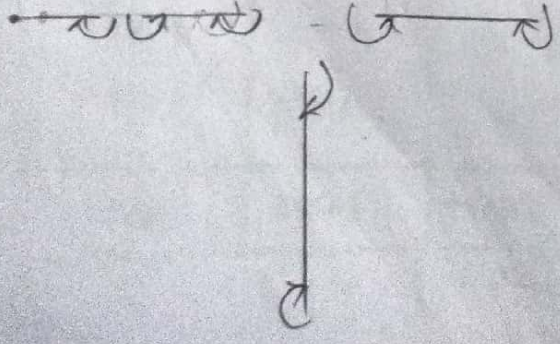
Joint	Member	K	ΣK	DF = $\frac{K}{\Sigma K}$
B	BA	0	0.33 I	0
	BC	$\frac{I}{3}$ = 0.33 I		1
C	CB	$\frac{3}{4}\cdot\frac{I}{L} = \frac{3}{4}\times\frac{I}{3}$ = 0.25 I	0.95 I	0.33
	CD	$\frac{I}{4}$ = 0.25 I		0.33
	CE	$\frac{I}{4}$ = 0.25 I		0.33

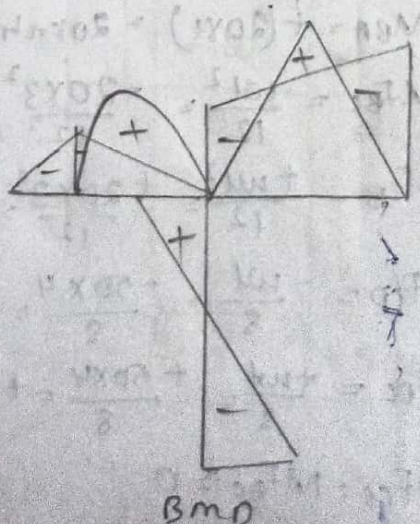
* for the overhanging portion (BA), K is always 'zero'

Step ③ Moment Distribution

Joint	B		C			D	E
Member	BA	BC	CB	CD	CE	DC	EC
DF	0	1	0.33	0.33	0.33	—	—
FEM	+20	−15	+15	−25	0	+25	0
Balance & CO	—	−5	$\frac{-5}{2}$ = −2.5	—	—	—	—
Initial moment	+20	−20	+12.5	−25	0	+25	0
Balance	—	—	4.125	4.125	4.125	—	—
CO	—	—	0	0	0	$\frac{4.125}{2}$ = 2.06	$\frac{4.125}{2}$ = 2.06
FEM	+20	−20	+16.65	−20.84	4.125	+27.06	+2.06

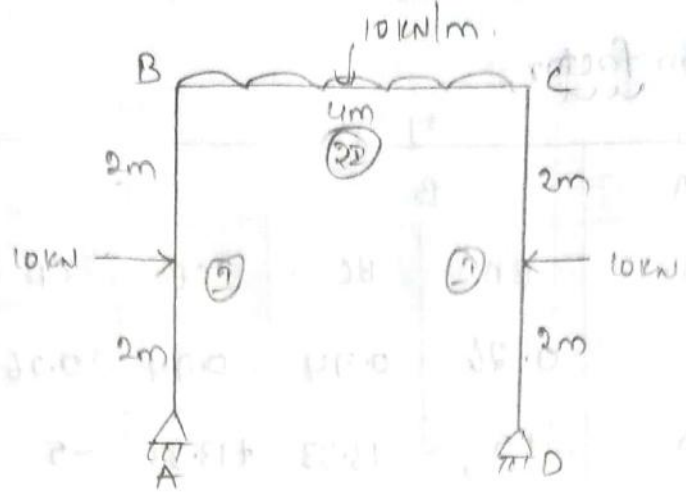
Free body diagram





Scanned with CamScanner

4.



Bending moment due to load,

Span AB, $M = \frac{wl}{4} = \frac{10 \times 4}{4} = 10 KNm$

Span BC, $M = \frac{wl^2}{8} = \frac{10 \times 4^2}{8} = 20 KNm$.

Span CD, $M = \frac{wl}{4} = \frac{10 \times 4}{4} = 10 KNm$

Step ① Fixed end moment

$M_{FAB} = \frac{-wl}{8} = \frac{-10 \times 4}{8} = -5 KNm$

$M_{FBA} = \frac{+wl}{8} = \frac{+10 \times 4}{8} = +5 KNm.$

$M_{FBC} = \frac{-wl^2}{12} = \frac{-10 \times 4^2}{12} = -13.33 KNm$

$M_{FCB} = \frac{+wl^2}{12} = \frac{+10 \times 4^2}{12} = +13.33 KNm$

$M_{FCD} = \frac{-wl}{8} = \frac{-10 \times 4}{8} = -5 KNm$

$M_{FDC} = \frac{+wl}{8} = \frac{+10 \times 4}{8} = +5 KNm.$

Step ② Distribution factor

Joint	Member	K	EK	DF = K/EK
B	BA	$\frac{3}{4} \cdot \frac{I}{2} = 0.18I$	0.68I	0.26
	BC	0.5I		0.74.
C	CB	0.5I	0.68I	0.74
	CD	0.18I		0.26

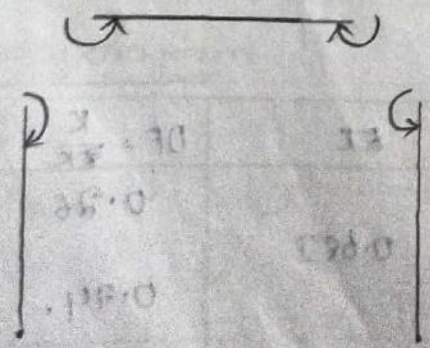
Scanned with CamScanner

Step ③ Moment Distribution:

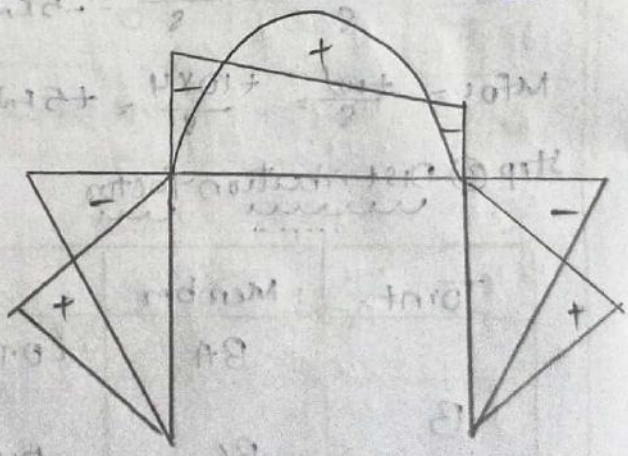
	A		IJ		IJ	H
Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	–	0.26	0.74	0.74	0.26	–
FEM	-5	+5	-13.33	+13.33	-5	+5
Balance & CO	+5	$\frac{5}{2}$ = 2.5	–	–	$\frac{-5}{2}$ -2.5	-5
Initial moment	0	7.5	-13.33	+13.33	-7.5	0
Balance	–	+1.52	+4.31	-4.31	-1.52	–
CO	0.	–	-2.15	2.15	–	+0
Balance	0	+0.56	+1.59	-1.59	-0.56	0
CO	0	0	-0.79	0.79	0	0
Balance	0	0.2	0.58	-0.58	-0.2	0
CO	0	0	-0.29	0.29	–	0
FEM	0	+9.78	-10.08	10.08	-9.78	0

=0 =0

Free Body Diagram



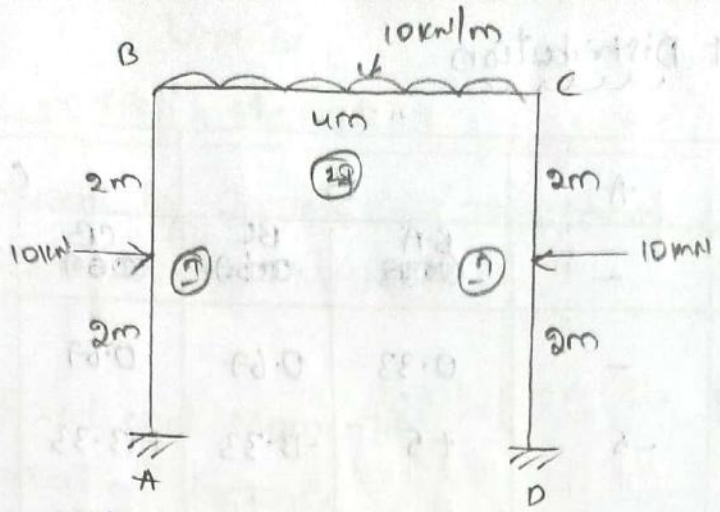
Bending Moment Diagram



BMD.

Scanned with CamScanner

4.
5.



Bending Moments due to loads

Span AB, $M = \frac{wl}{4} = \frac{10 \times 4}{4} = 10 \text{ KNm}$

Span BC, $M = \frac{wl^2}{8} = \frac{10 \times 4^2}{8} = 20 \text{ KNm}$

Span CD, $M = \frac{wl}{4} = \frac{10 \times 4}{4} = 10 \text{ KNm}$

Step ① fixed end moments

$M_{FAB} = \frac{-wl}{8} = \frac{-10 \times 4}{8} = -5 \text{ KNm}$

$M_{FBA} = \frac{+wl}{8} = \frac{+10 \times 4}{8} = +5 \text{ KNm}$

$M_{FBC} = \frac{-wl^2}{12} = \frac{-10 \times 4^2}{12} = -13.33 \text{ KNm}$

$M_{FCB} = \frac{+wl^2}{12} = \frac{+10 \times 4^2}{12} = +13.33 \text{ KNm}$

$M_{FCD} = \frac{-wl}{8} = \frac{-10 \times 4}{8} = -5 \text{ KNm}$

$M_{FDC} = \frac{+wl}{8} = \frac{+10 \times 4}{8} = +5 \text{ KNm}$

Step ② Distribution factor

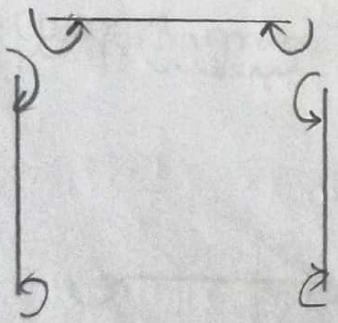
Joint	Member	K	ΣK	$DF = \frac{K}{\Sigma K}$
B	BA	$\frac{I}{4} = 0.25I$	$0.75I$	0.33
	BC	$\frac{2I}{4} = 0.5I$		0.69
C	CB	$\frac{2I}{4} = 0.5I$	$0.75I$	0.67
	CD	$\frac{I}{4} = 0.2(I)$		0.33

Scanned with CamScanner

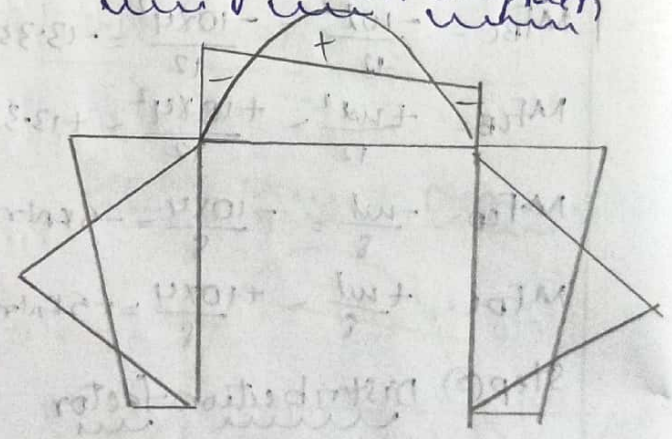
Step ③ Moment Distribution

	F	2T		2T		F
Joint	A	B		C		D
Member	–	BA	BC	CB	CD	DC
DF	–	0.33	0.67	0.67	0.33	–
FEM	-5	+5	-13.33	+13.33	-5	–
Balance	–	+2.7	5.58	-5.58	-2.75	–
CO	+1.37	0	+2.79	+2.79	–	-1.37
Balance	–	0.92	1.86	-1.86	-0.92	–
CO	0.46	–	-0.93	0.93	–	-0.46
Balance	–	0.3	+0.62	-0.62	-0.3	–
CO	0.15	–	-0.31	0.31	–	-0.15
FEM	-3.02	8.91	-9.97	+9.97	-9	+3.02

Free Body Diagram



Bending Moment Diagram



BMD

Scanned with CamScanner

Unit -III

Part - 1 - Stiffness Method (Displacement Method)

Degree of Kinematic Indeterminacy (or) Degree of Freedom [DF]

A structure is said to be kinematic indeterminate, if the no. of unknown displacements of the joints is more than the compartability equations.

Beams

* At an unyielding (without sinking) hinge (or) roller support, the degree of freedom is "1". At an unyielding hinge (or) roller support, we have "Rotational Displacement".
* Thus the degree of freedom of a continous beam with unyielding supports is equal to the number of hinge (or) roller supports.
* At fixed support, the degree of freedom is "zero"
* In this method the basic unknowns are displacements at the various joints. Hence the degree of freedom is to be identified first
* Co-ordinate number is assigned to each of the unknown displacement.
* In this analysis, the given structure should be made to a restrained structure.
* Let $P_{L1}, P_{L2}, P_{L3}, \ldots$ (P → forces, L → load) are the forces due to external loads in the restrained structure at the co-ordinates
* Let $P_1, P_2, P_3, \ldots$ are the final forces at the co-ordinates
* Let $\Delta_1, \Delta_2, \Delta_3, \ldots$ are the displacements at the co-ordinates
* Let [k] is known as stiffness matrix

Final equations

$$[\Delta] = [k]^{-1}\{[P] - [P_L]\}$$

Scanned with CamScanner

Procedure

1. Determine Degree of freedom $[D_k]$
2. Assign the co-ordinates to the unknown displacements.
3. Impose restrained in all co-ordinates to get a fully restrained structure.
4. Determine the forces due to loads at the cordinates

$$[P_L]$$

5. Determine the final forces at the the co-ordinate

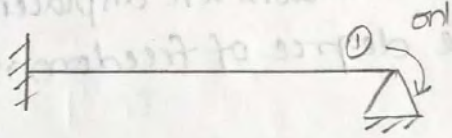
$$[P]$$

6. Determine the stiffness matrix $([k])$ by giving unit displacement at the cordinates.
7. Use final equation

$$[\Delta] = [k]^{-1}\{[P] - [P_L]\}$$

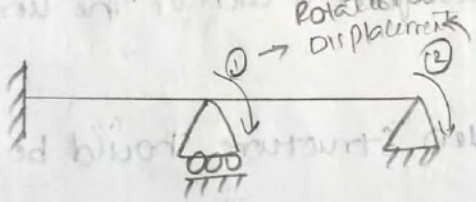
Cases

① only one hinge



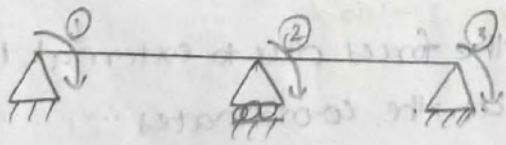
$D_k = 1$

②



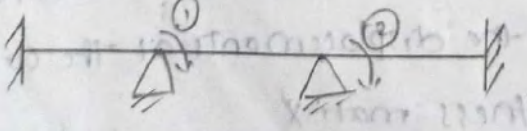
$D_k = 2.$

③



$D_k = 3.$

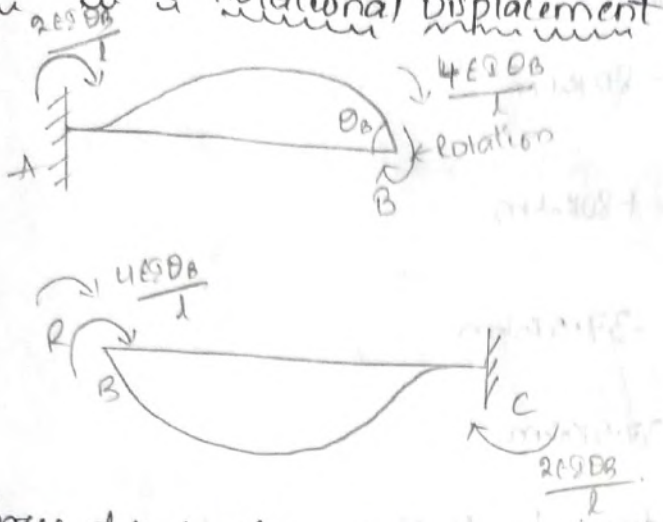
④



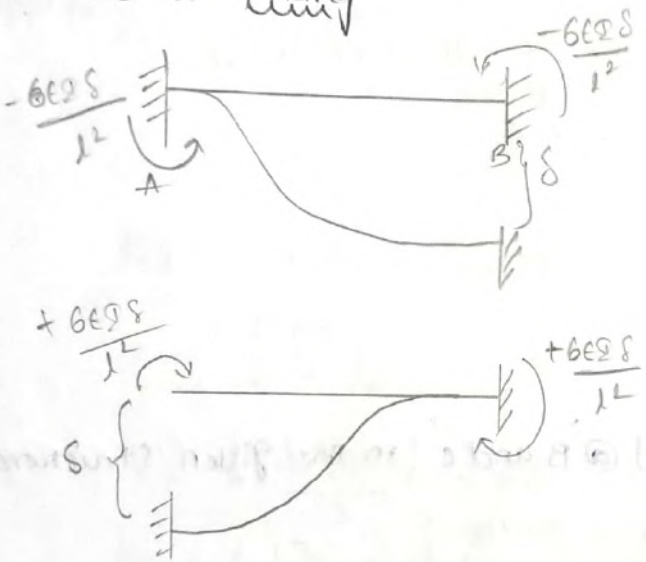
$D_k = 4.$

Scanned with CamScanner

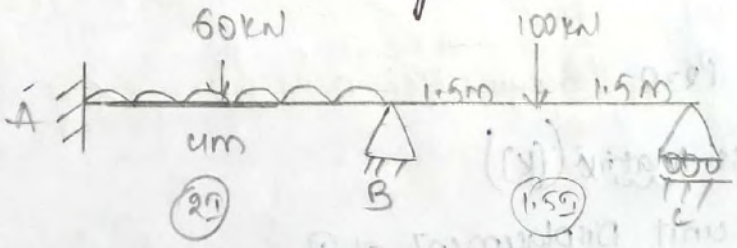
Forces due to Rotational Displacement



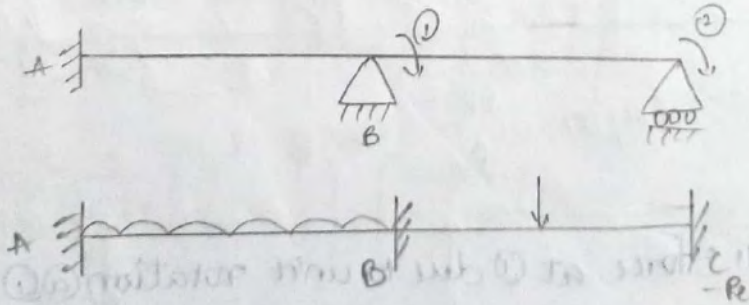
Forces due to sinking



1. Analyse the continous beam using stiffness ~~factor~~ Matrix method



Degree of freedom, DK = 2.



- Restrained structure

Scanned with CamScanner

Step① Fixed end Moments

$M_{FAB} = \frac{-wl^2}{12} = \frac{-60\times4^2}{12} = -80$ Kn-m

$M_{FBA} = \frac{+wl^2}{12} = \frac{+60\times4^2}{12} = +80$ Kn-m

$M_{FBC} = \frac{-wl}{4} = \frac{-100\times3}{4} = -37.5$ Kn-m

$M_{FCB} = \frac{+wl}{4} = \frac{+100\times3}{4} = +37.5$ Kn-m.

The force Matrix $[P_L]$ due to loads @ the co-ordinates is Calculated as follows. (As Per restrained structure)

$\therefore P_{1L} = M_{FBA} + M_{FBC}$

$= +80 - 37.5$

$= 42.5$ Kn-m

$P_{2L} = M_{FCB}$

$= +37.5$ Kn-m.

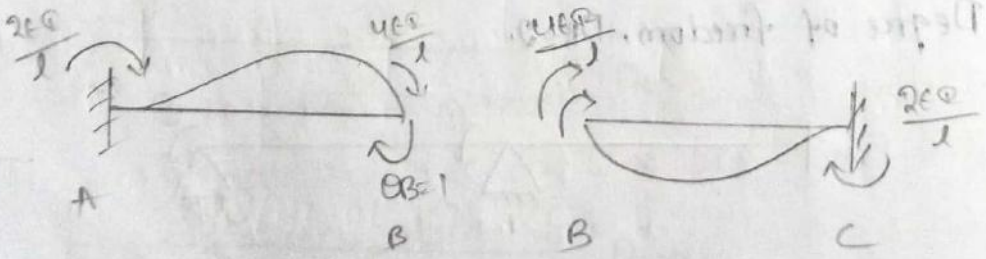
The final force Matrix $[P]$ @ B and C in the given structure is given by (As Per given structure)

$P_1 = 0$ $\left(\Sigma M_B = 0 \Rightarrow M_{BA} + M_{BC} = 0\right)$ Joint equilibrium equation.

$P_2 = 0$ (Because $M_{CB} = 0$ as it is Simply Support)

Stiffness Matrix $([K])$

Applying unit Displacement at ①



K_{11} = force at ① due to unit rotation @ ①

$= \left(\frac{4EC}{l}\right)_{AB} + \left(\frac{4EC}{l}\right)_{BC}$

Scanned with CamScanner

$= \frac{4E(2I)}{4} + \frac{4E(1.5I)}{3}$

$K_{11} = 4EI.$

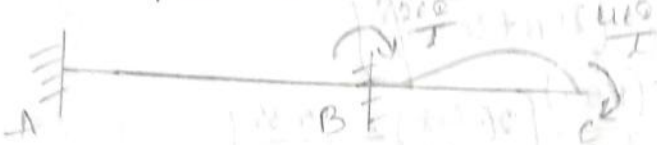
K_{21} = force at ② due to unit rotation at ①

$= \left[\frac{2EI}{l}\right]_{BC} +$

$- \frac{2E(1.5I)}{3}$

$K_{21} = EI.$

Applying unit displacement at ②



$K_{12} = K_{21}$

K_{12} = force at ① due to unit rotation at ②

$= \left[\frac{2EI}{l}\right]_{BC}$

$= \frac{2E(1.5I)}{3} = EI.$

K_{22} = force at ② due to unit rotation at ②

$= \left[\frac{4EI}{l}\right]_{BC}$

$= \frac{4E[1.5I]}{3} = 2EI.$

$[\Delta] = [K]^{-1}\{[P] - [P_L]\}$

$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} 4EI & EI \\ EI & 2EI \end{bmatrix}^{-1} \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 42.5 \\ 37.5 \end{bmatrix} \right\}$

$= \frac{1}{EI} \begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} -42.5 \\ -37.5 \end{bmatrix}$

$\theta_B = \frac{-6.78}{EI}$; $\theta_C = \frac{-15.3}{EI}$

Scanned with CamScanner

final end Moments

$$M_{AB} = M.F_{AB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3R}{l}\right]$$

$$= -80 + \frac{2E(2I)}{4}\left[-\frac{6.8}{E2}\right]$$

$$= -86.8 \text{ KN-m}$$

$$M_{BA} = M.F_{BA} + \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3R}{l}\right]$$

$$= +80 + \frac{2E(2I)}{4}\left[2\times\left(\frac{-6.8}{E2}\right)\right]$$

$$= 66.4 \text{ KN-m}$$

$$M_{BC} = M.F_{BC} + \frac{2EI}{l}\left[2\theta_B + \theta_C - \frac{3R}{l}\right]$$

$$= -37.5 + \frac{2E(1.5I)}{3}\left[2\left(\frac{-6.8}{E2}\right) + \frac{15-35}{E2}\right]$$

$$= -66.45 \text{ KN-m}$$

$M_{CB} = 0$ KN-m.

free Body Diagram

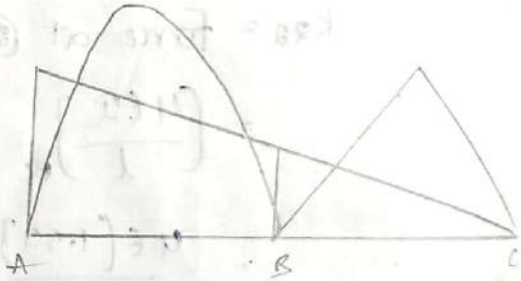


BM

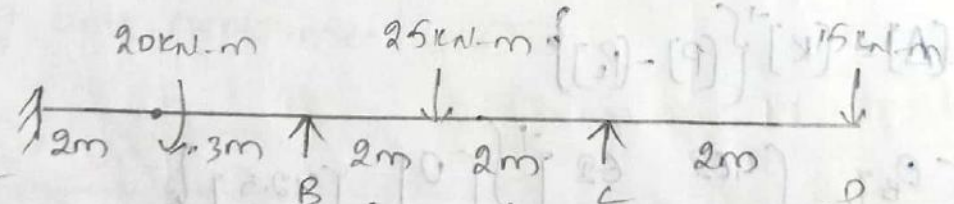
Span AB $= \frac{wl^2}{8} = \frac{60\times4^2}{8} = +120$ KN-m

Span BC $= \frac{wl}{4} = \frac{100\times3}{4} = 75$ KN-m.

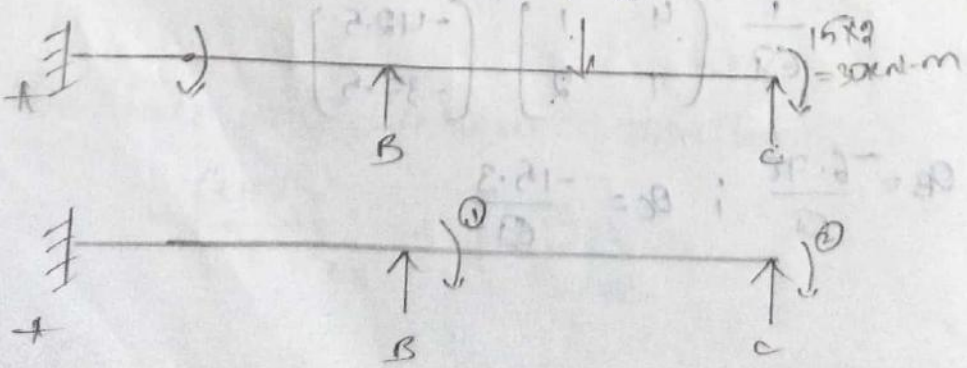
Bending Moment Diagram



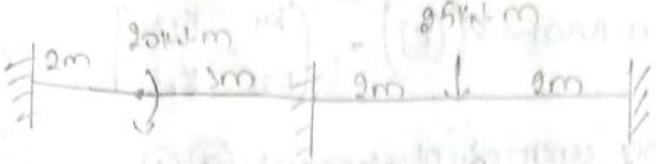
Q.



Degree of freedom $D_K = 2$.



Scanned with CamScanner



Restrained structure

Bending Moment

Span AB, $M = \frac{Mb}{l} = \frac{20 \times 3}{5} = +12 KN.m$

$m = \frac{ma}{l} = -\frac{20 \times 2}{5} = -8 KN.m.$

Span BC, $m = \frac{wl}{4} = \frac{25 \times 4}{4} = 25 KN.m.$

fixed end Moments

$M_{FAB} = 0 + \frac{Mb}{l^2}[3a - l]$

$= +\frac{20 \times 3}{5^2}[3(2) - 5]$

$= 2.4 KN.m$

$M_{FBA} = +\frac{Moa}{l^2}[3b - l] = +\frac{20 \times 2}{5^2}[3(3) - 5]$

$= +6.4 KN.m$

$M_{FBC} = -\frac{wl}{8} = -\frac{25 \times 4}{8} = -12.5 KN.m$

$M_{FCB} = +\frac{wl}{8} = +\frac{25 \times 4}{8} = +12.5 KN.m$

The force matrix due to loads @ the co-ordinates is calculated as follows $[P_L]$ in the restrained structure.

$P_{1L} = M_{FBA} + M_{FBC}$

$= +6.4 - 12.5$

$= -6.1 KN.m.$

$P_{2L} = M_{FCB}$

$= +12.5 KN.m$

The final force matrix $[P]$ @ the coordinates in the given beam is given as,

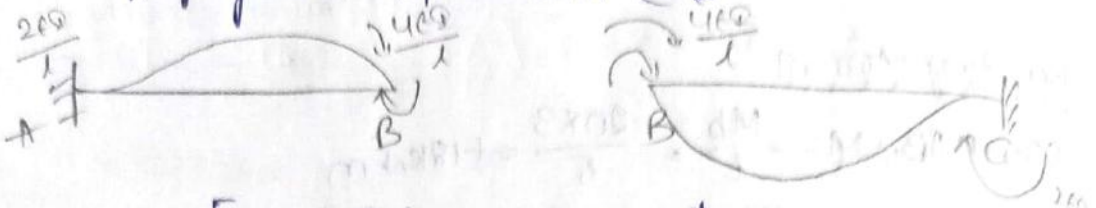
$P_1 = 0 \quad (\because M_B = 0)$

$P_2 = 30 KN.m$ (overhanging moment).

Scanned with CamScanner

Stiffness Matrix $([K]) = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$

Applying unit displacement @ ①



K_{11} = force @ ① due to unit rotation @ ①.

$$= \left[\frac{4EI}{l}\right]_{AB} + \left[\frac{4EI}{l}\right]_{BC}$$

$$= \frac{4EI}{5} + \frac{4EI}{4}$$

$$= 1.8EI.$$

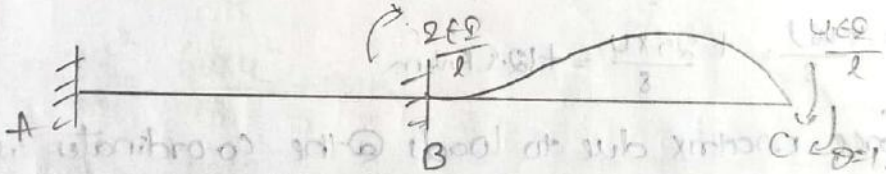
K_{21} = force @ ② due to unit rotation at ①

$$= \left[\frac{2EI}{l}\right]_{BC}$$

$$= \frac{2EI}{4}$$

$$= 0.5EI.$$

Applying unit displacement @ ②



K_{12} = Force @ ① due to unit rotation at ②

$$= \left[\frac{2EI}{l}\right]_{BC}$$

$$= \frac{2EI}{4}$$

$$= 0.5EI$$

$K_{21} = K_{12}$

K_{22} = force @ ② due to unit rotation at ②

$$= \left[\frac{4EI}{l}\right]_{BC}$$

$$= \frac{4EI}{4}$$

$$= EI.$$

Scanned with CamScanner

$$[\Delta] = [k]^{-1}\left([P] - [P_L]\right)$$

$$\begin{bmatrix}\theta_B \\ \theta_C\end{bmatrix} = \begin{pmatrix}1.8EI & 0.5EI \\ 0.5EI & EI\end{pmatrix}^{-1}\left\{\begin{bmatrix}0 \\ 30\end{bmatrix} - \begin{bmatrix}-6.1 \\ +12.5\end{bmatrix}\right\}$$

$$= \frac{1}{EI}\begin{bmatrix}1.8 & 0.5 \\ 0.5 & 1\end{bmatrix}\begin{bmatrix}+6.1 \\ +17.5\end{bmatrix}$$

$$\theta_B = \frac{-1.70}{EI} \; ; \; \theta_C = \frac{+18.35}{EI}$$

final end Moments

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\psi}{l}\right]$$

$$= 2.4 + \frac{2EI}{5}\left[\frac{-1.70}{EI}\right]$$

$$= 1.78 \text{ KNm}.$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\psi}{l}\right]$$

$$= 6.4 + \frac{2EI}{5}\left[2\left[\frac{-1.70}{EI}\right]\right]$$

$$= 5.04 \text{ KNm}.$$

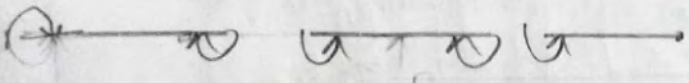
$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left[2\theta_B + \theta_C - \frac{3\psi}{l}\right]$$

$$= -12.5 + \frac{2EI}{4}\left[2\left[\frac{-1.70}{EI}\right] + \frac{18.35}{EI}\right]$$

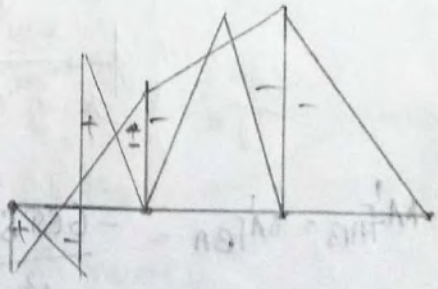
$$= -5.02 \text{ KNm}.$$

$$M_{CB} = 30 \text{ KNm}.$$

Free Body Diagram



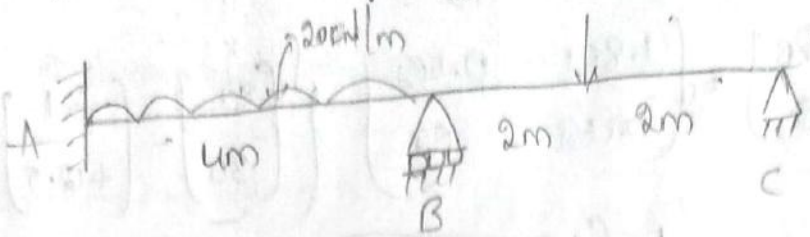
Bending Moment Diagram



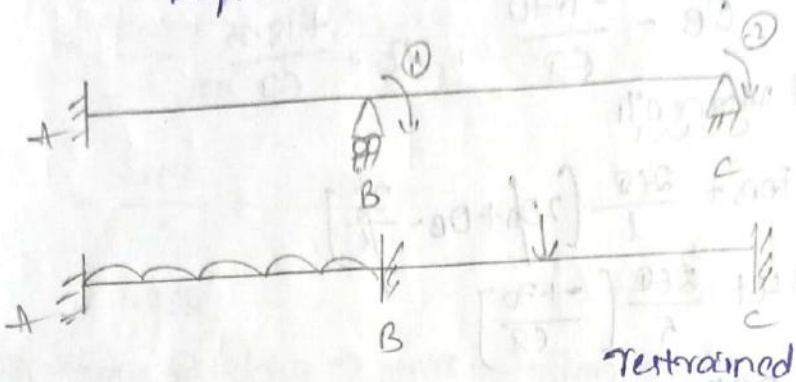
BMD

Scanned with CamScanner

MUP ★★★ Support 'B' sinks by $\frac{100}{EI}$



Degree of freedom: Dk = 2.



Restrained structure

fixed end moments

$$M_{FAB} = \frac{-wl^2}{12} = \frac{-20\times4^2}{12} = -26.67\ KN\text{-}m$$

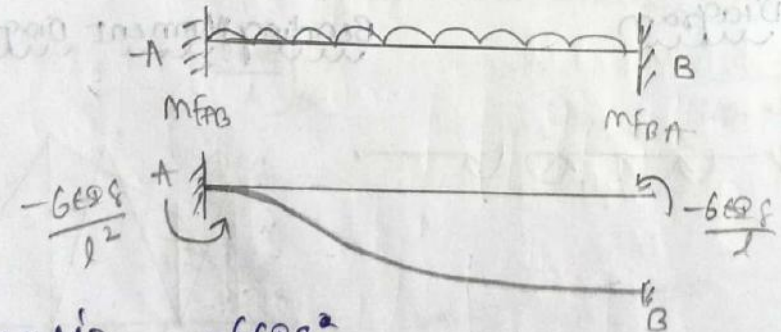
$$M_{FBA} = \frac{+wl^2}{12} = \frac{+20\times4^2}{12} = +26.67\ KN\text{-}m$$

$$m_{FBC} = \frac{-wl}{8} = \frac{-30\times4}{8} = -15\ KN\text{-}m$$

$$m_{FCB} = \frac{+wl}{8} = \frac{+30\times4}{8} = +15\ KN\text{-}m$$

The force matrix due to loads @ the coordinates is calculated as follows [R] in the restrained structure.

Span AB



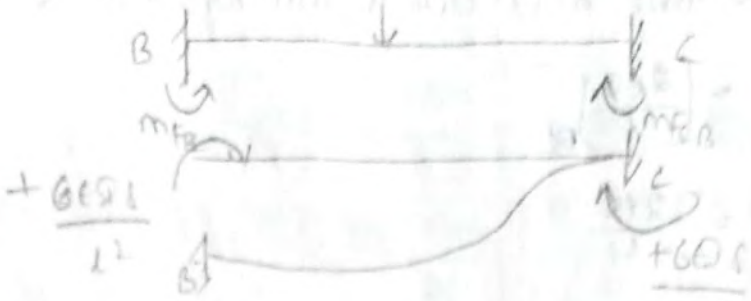
$$M'_{FAB} = M'_{FBA} = \frac{-6EIS^2}{l^2} = \frac{-6EI\times100^2}{4^2\times EI}$$

$$M'_{FAB} = -37.5\ KN\text{-}m$$

$$m'_{FBA} = -37.5\ KN\text{-}m.$$

Scanned with CamScanner

Span BC



$$M'_{FBC} = +\frac{6EI\delta}{l^2} = +\frac{6EI\times 100}{4^2\times EI}$$

$$= +37.5\ \text{KN-m}.$$

$$M'_{FCB} = +\frac{6EI\delta}{l^2} = +\frac{6EI\times 100}{4^2\times EI}$$

$$= +37.5\ \text{KN-m}.$$

$$\therefore P_{1L} = M_{FBA} + m_{FBC} + m'_{FBA} + m'_{FBC}$$

$$= +26.67 - 15 - 37.5 + 37.5$$

$$= +11.67\ \text{KN-m}.$$

$$P_{2L} = M_{FCB} + M'_{FCB}$$

$$= 15 + 37.5$$

$$= +52.5\ \text{KN-m}.$$

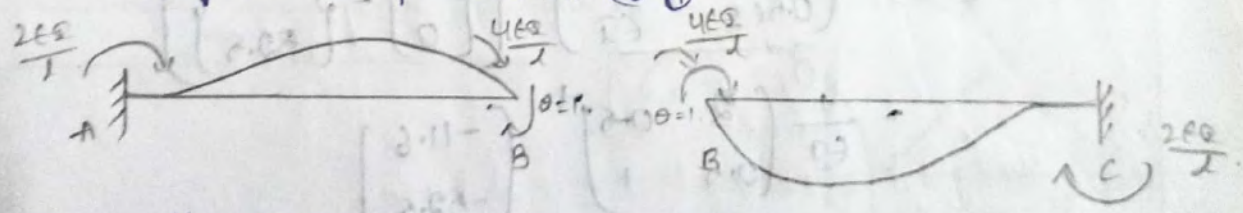
Final force matrix due [P] @ the co-ordinates in the given beam is given as.

$$P_1 = 0 \quad (\Sigma m_B = 0)$$

$$P_2 = 0 \quad (m_{CB} = 0)$$

Stiffness matrix $[K] = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$

Apply unit displacement @ ①



K_{11} = Force @ ① due to unit rotation at ①

$$= \left[\frac{4EI}{l}\right]_{BA} + \left(\frac{4EI}{L}\right)_{BC}$$

$$= \left[\frac{4EI}{4}\right] + \frac{4EI}{4} = 2EI.$$

Scanned with CamScanner

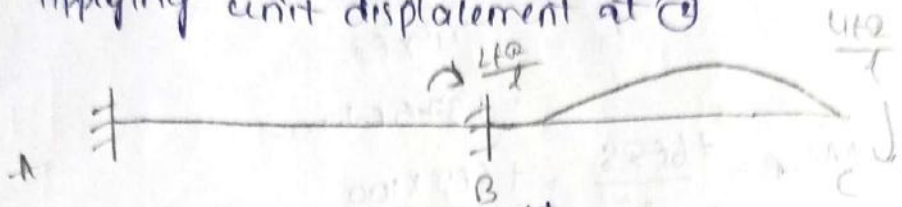
K_{21} = force at ② due to unit rotation at ①

$= \left[\frac{2EI}{L}\right]_{BC}$

$= \frac{2EI}{4}$

$= 0.5EI.$

Applying unit displacement at ②



K_{12} = force at ① due to unit rotation at ②

$= \left[\frac{2EI}{L}\right]_{BC}$

$= \frac{2EI}{4}$

$= 0.5EI.$

K_{22} = force at ② unit rotation at ②

$= \left[\frac{4EI}{L}\right]_{CB}$

$= \frac{4EI}{4}$

$= 1EI.$

$[\Delta] = [k]^{-1}([P] - [P_L])$

$$\begin{bmatrix}\theta_B \\ \theta_C\end{bmatrix} = \begin{bmatrix}2EI & 0.5EI \\ 0.5EI & EI\end{bmatrix}^{-1}\left(\begin{Bmatrix}0\\0\end{Bmatrix} - \begin{Bmatrix}11.6\\52.5\end{Bmatrix}\right)$$

$$= \frac{1}{EI}\begin{bmatrix}2 & 0.5 \\ 0.5 & 1\end{bmatrix}^{-1}\begin{bmatrix}-11.6\\-52.5\end{bmatrix}$$

$\theta_B = \frac{+8.37}{EI}$

$\theta_C = \frac{-56.68}{EI}$

Scanned with CamScanner

Final end Moments

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\psi}{l}\right)$$

$$= -26.67 + \frac{2EI}{4}\left[\frac{+8.37}{EI} - \frac{3\times100}{4}\right]$$

$$= -59.98 \text{ kN-m}.$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\psi}{l}\right)$$

$$= +26.67 + \frac{2EI}{4}\left(2\times\left[\frac{8.37}{EI}\right] - \frac{3\times100}{4}\right)$$

$$= -2.46 \text{ kN-m}.$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\psi}{l}\right)$$

$$= -15 + \frac{2EI}{4}\left(2\left[\frac{8.37}{EI}\right] + \frac{-5668}{EI} + \frac{3\times100}{4}\right)$$

clockwise moment due to in BC span

$$= +2.53 \text{ kN-m}.$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\psi}{l}\right)$$

$$= +15 + \frac{2EI}{4}\left[2\left[\frac{-5668}{EI}\right] - \left(\frac{8.37}{EI}\right) + \frac{3\times100}{4}\right]$$

$$= 0 \text{ kN-m}.$$

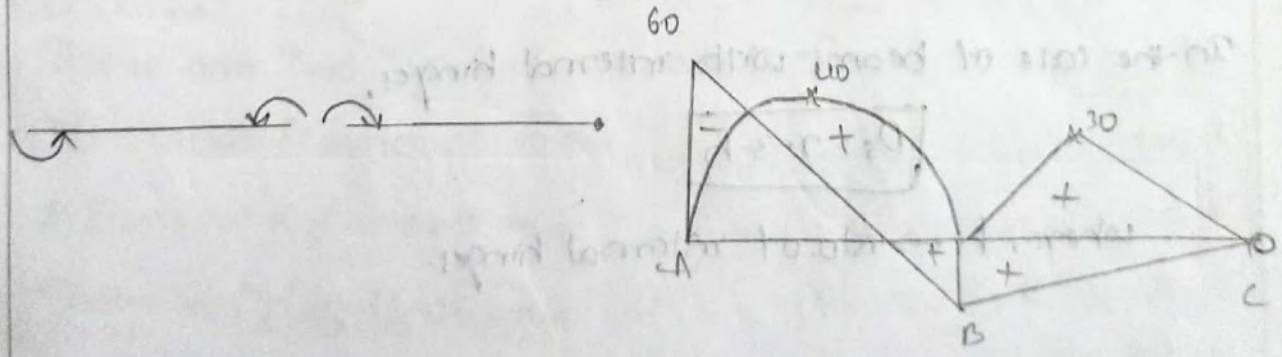
Bending moment

Span AB, $m = \frac{wl^2}{8} = \frac{20\times4^2}{8} = 40 \text{ kN-m}$

Span BC, $m = \frac{wl}{4} = \frac{30\times4}{4} = 30 \text{ kN-m}.$

Free BM

Bending Moment Diagram



BMD

Scanned with CamScanner

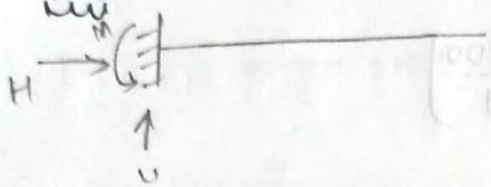
Part-1

Flexibility Matrix Method. [Force Method]

Degree of Static Indeterminacy (or) Degree of Redundancy (D_s)

If the number of unknown forces is greater than the available equations of equilibrium, then it is called degree of static Indeterminacy (or) Degree of Redundancy "D_s"

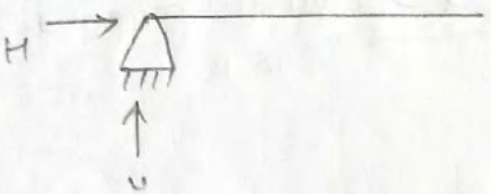
1. Fixed



Reactions

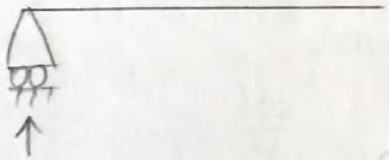
$r = 3$

2. Hinged (or) Pinned



$r = 2$

3. Roller Support



$r = 1$

In the case of beams without internal hinges (),

$$D_s = r - 3$$

Where, $r \rightarrow$ no. of reactions

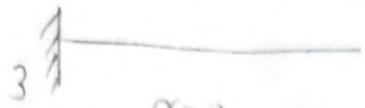
In the case of beams with internal hinges,

$$D_s = r - 3 - h$$

where, $h \rightarrow$ No. of internal hinges.

Scanned with CamScanner

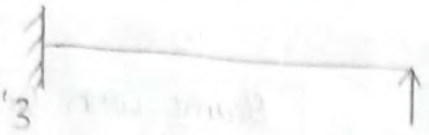
Cases

1. 

$r = 3$

$D_s = r - 3$

$= 3 - 3 = 0$

2. 

$r = 3 + 1 = 4$

$D_s = r - 3$

$= 4 - 3$

$= 1$

3. 

$r = 3 + 3 + 2 = 8$

$D_s = r - 3$

$= 8 - 3$

$= 5$

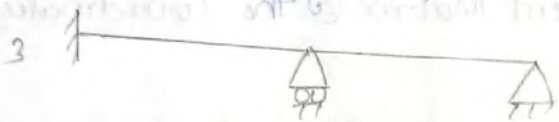
4. 

$r = 1 + 1 + 2 = 4$

$D_s = r - 3$

$= 4 - 3$

$= 1.$

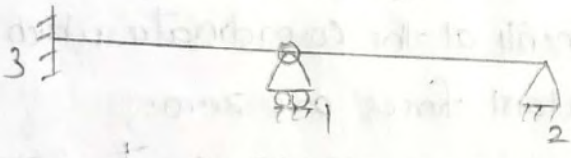
5. 

$r = 3 + 1 + 2 = 6$

$D_s = r - 3$

$= 6 - 3$

$= 3.$

6. 

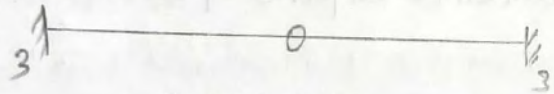
$r = 3 + 1 + 2 \quad ; h = 1$

$= 6$

$D_s = r - 3 - h$

$= 6 - 3 - 1$

$= 2.$

7. 

$r = 3 + 3 = 6 \quad ; h = 1$

$D_s = r - 3 - h$

$= 6 - 3 - 1$

$= 2.$

Redundant forces

There are two types of redundant forces:

1. Internal Redundant forces
2. External Redundant forces.

Internal Redundant forces

Moment forces are considered in this force.

External Redundant forces

Vertical forces are considered.

Scanned with CamScanner

*In the case of Non-sinking beams, consider internal as unknown forces.

*In the case of sinking beams, consider Vertical as unknown forces.

	Beams without sinking.	Beams with sinking.
Redundant forces	Internal	External.
Types of force	Moment type of force) M	Vertical force ↑F
Type of displacement	Rotation (θ)) θ	Deflection (δ) ↑

Let $[\Delta_L]$ be the displacement Matrix @ the coordinates due to loading.

Let $[\Delta]$ be the final displacement Matrix @ the co-ordinates.

* The net (or) final displacements at the coordinates which are assigned to internal redundant forces are zero.

* The net (or) final displacements at the coordinates which are assigned to external redundance depend upon the support conditions.

The above two Conditions are called Compatability Conditions

Let $\Delta[\delta]$ be the flexibility matrix.

Procedure:

1. Determine the degree of Redundancy (D_s).
2. Choose the redundance (internal (or) external)
3. Assign the coordinates to the redundance
4. Remove the restraints to the redundance to get released structure.
5. Determine $[\Delta_L]$ matrix at the Co-ordinates.

Scanned with CamScanner

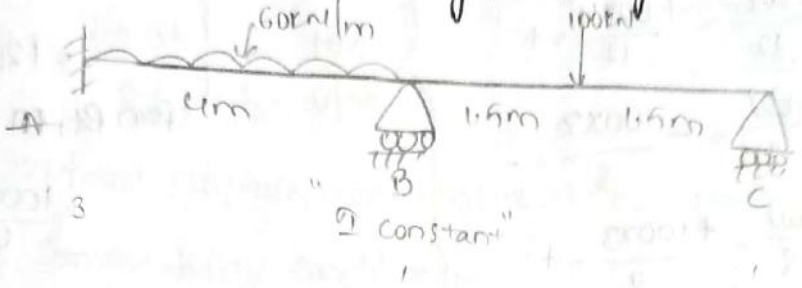
6. Determine [Δ] using compatability Conditions.

7. Determine [S] - flexibility matrix by applying unit force at the co ordinates.

8. Use the following equations.

$$[P] = [S]^{-1}\{[\Delta] - [\Delta_L]\}$$

1. Analyse the continous beam using Flexibility Matrix Method.



Reactions, $P_r = 3+1+1$

$= 5.$

$D_s = r - 3$

$= 5 - 3$

$D_s = 2.$

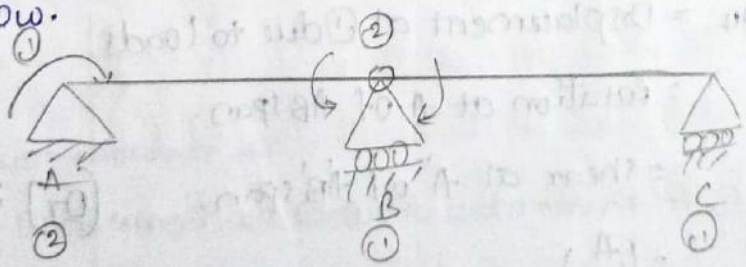
*Choose the redundant forces at "A" and "B". Because 'C' is the simply support where the moment is zero.

*Here fixed end Moment at "A" and internal moment at "B" are considered as Redundance.

*fixed end Moment at "A" is released by a hinged support.

*Internal Moment at "B" is realeased by providing an internal hinge at 'B'.

*The released structure with the assigned co-ordinates is shown below.



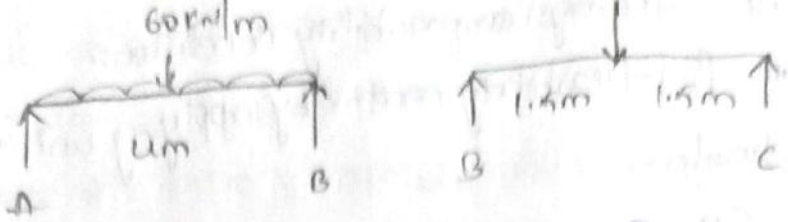
$r = 2+1+1 = 4$

$h = 1$

$D_s = r - 3 - h = 4 - 3 - 1$

$D_s = 0$

Scanned with CamScanner



Fixed end Moments

$$M_{FAB} = \frac{-wl^2}{12} = \frac{-60\times4^2}{12} = -$$

$$M_{FBA} = \frac{+wl^2}{12} = \frac{+60\times4^2}{12} = +$$

$$M_{FBC} = \frac{-wl}{8} = -\frac{100\times3}{8} = -$$

$$M_{FCB} = \frac{+wl}{8} = \frac{+100\times3}{8} = +$$

Not Required

Bending Moment

Span AB, $M = \frac{wl}{8}$

$= \frac{60\times4}{8}$

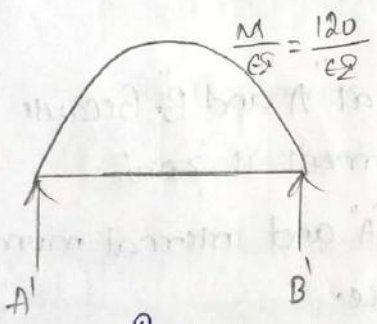
$= 120 kN.m$

Span BC, $m = \frac{wl}{4}$

$= \frac{100\times3}{4} = 75 kN.m$

The displacement matrix @ the coordinates due to loading $[\Delta_L]$ (Rotations @ A and B)

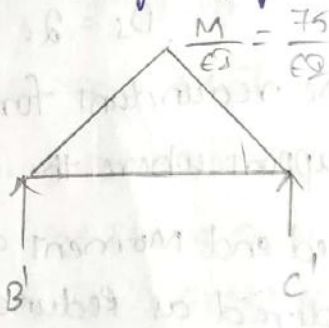
$[\Delta_L]$ will be Calculated using Conjugate Beam method.



$\frac{A}{2}$ $A = \frac{2}{3}\times b\times h$ $\frac{A}{2}$

$= \frac{2}{3}\times4\times\frac{120}{EI}$

$= \frac{320}{EI}$



$\frac{A}{2}$ $A = \frac{1}{2}\times b\times h$ $\frac{A}{2}$

$= \frac{1}{2}\times3\times\frac{75}{EI}$

$= \frac{112.5}{EI}$

Conjugate method.

Δ_{1L} = Displacement at ① due to loads

= Rotation at A of AB span.

= Shear at A' of A'B' span.

$= \left(\frac{A}{2}\right)_{A'B'}$

$= \frac{1}{2}\times\frac{320}{EI}$

$\Delta_{1L} = \frac{160}{EI}$

(Or) $\frac{wl^3}{24EI} = \frac{60\times4^3}{24EI}$

$= \frac{160}{EI}$.

Scanned with CamScanner

Δ_{2L} = Displacement at ② due to loads

= Rotation at B of AB and BC span

= Shear at A' of A'B' and B'C' span.

$= \left(\frac{A}{2}\right)_{A'B'} + \left(\frac{A}{2}\right)_{B'C'}$

$= \frac{320}{EI} + \frac{112.5}{EI}$

$\Delta_{2L} = \frac{216.25}{EI}$.

(or) $\frac{wl^3}{24EI} + \frac{wl^2}{16EI}$

$= \frac{60 \times 4^3}{24EI} + \frac{100 \times 3^2}{16EI}$

$= \frac{216.25}{EI}$.

The net (or) final displacement matrix at the coordinates $[\Delta]$

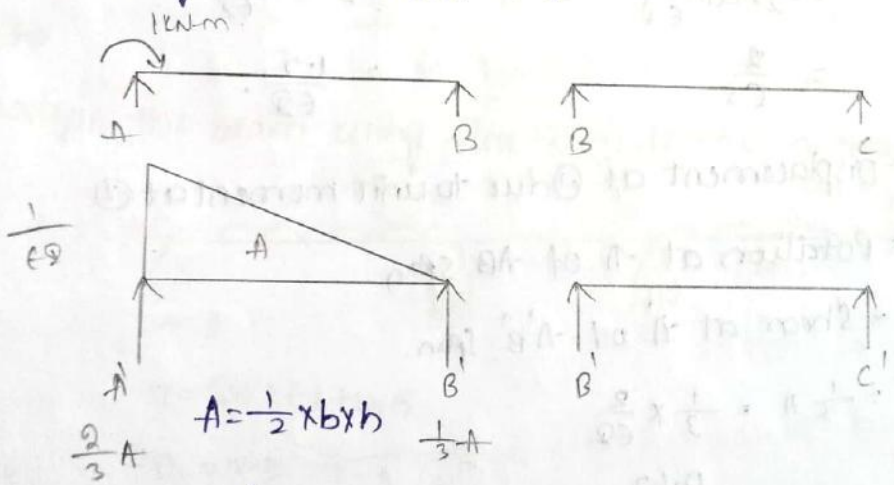
Using compatability conditions,

$\Delta_1 = 0$ and $\Delta_2 = 0$

Flexibility Matrix.

$$[\delta] = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}$$

Applying unit moment at ①.



$A = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 4 \times \frac{1}{EI}$.

$= \frac{2}{EI}$.

Applying unit moment at ②

δ_{11} = Displacement at ① due to unit moment at ①

= Rotation at A of AB

= Shear at A' of A'B'

$= \frac{2}{3} A = \frac{2}{3} \times \frac{2}{EI}$

$\delta_{11} = 1.33/EI$

Scanned with CamScanner

δ_{21} = Displacement at ② due to unit moment at ①

= Rotation at B of AB and BC

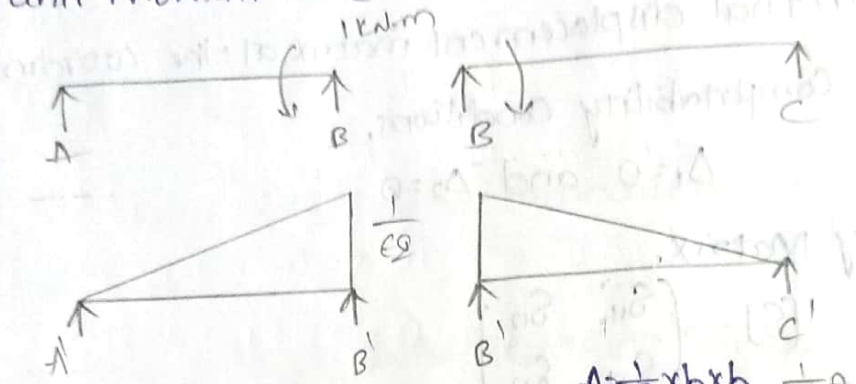
= Shear at B' of A'B' and B'C'

$= \left(\frac{1}{3} \times A\right)_{A'B'} + 0$

$= \frac{1}{3} \times \frac{2}{EI}$

$\delta_{21} = \frac{0.67}{EI}$

Applying unit moment at ②



$= \frac{1}{2} \times 1 \times \frac{1}{EI}$

$= \frac{2}{EI}$

$= \frac{1}{2} \times 3 \times \frac{1}{EI}$

$= \frac{1.5}{EI}$.

δ_{12} = Displacement at ① due to unit moment at ②

= Rotation at A of AB span

= Shear at A' of A'B' span.

$= \frac{1}{3} A = \frac{1}{3} \times \frac{2}{EI}$

$= \frac{0.67}{EI}$ $(\delta_{12} = \delta_{21})$

δ_{22} = Displacement at ② due to unit moment at ①

= Rotation at B of AB and BC span

= Shear at B' of A'B' and B'C' spans.

$= \left(\frac{2}{3} A\right)_{A'B'} + \left(\frac{2}{3} \times A\right)_{B'C'}$

$= \left(\frac{2}{3} \times \frac{2}{EI}\right) + \left(\frac{2}{3} \times \frac{1.5}{EI}\right)$

$\delta_{22} = \frac{2.33}{EI}$

Scanned with CamScanner

$$[P] = [S]^{-1}\{[\Delta] - [\Delta_L]\}$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} \frac{1.33}{EI} & \frac{0.67}{EI} \\ \frac{0.67}{EI} & \frac{2.33}{EI} \end{bmatrix}^{-1} \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{160}{EI} \\ \frac{216.25}{EI} \end{bmatrix} \right\}$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = EI \begin{bmatrix} 1.33 & 0.67 \\ 0.67 & 2.33 \end{bmatrix}^{-1} \begin{bmatrix} -\frac{160}{EI} \\ -\frac{216.25}{EI} \end{bmatrix}$$

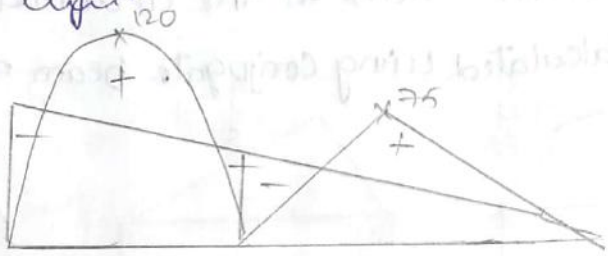
$P_1 = -86.0$ KN-m ; $P_2 = -68.08$ KN-m.

Change the direction of moments for the negative values of moments

Free BM

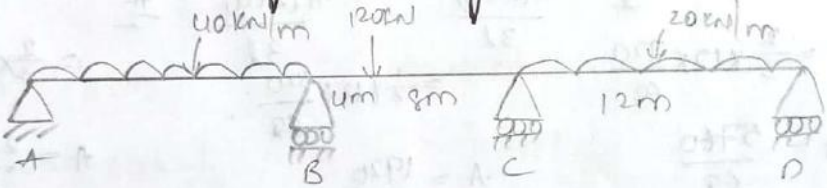


Bending Moment Diagram



BMD

2. Analyse the beam using Flexibility Matrix method.



$r = 2+1+1+1 = 5$

$D_s = r-3$

$= 5-3$

$D_s = 2.$

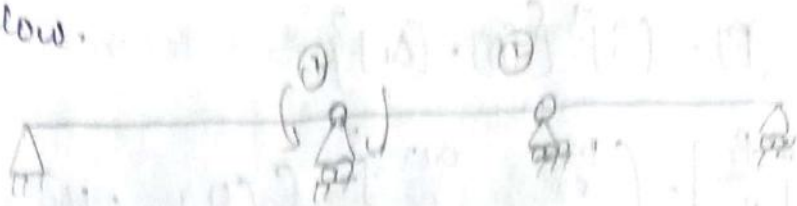
Choose the internal redundant forces @ B and C because 'A' and 'D' are simply supports where the final moment is zero.

Internal moment @ B and C are released by providing internal hinges.

The released structure with the assigned coordinates is

Scanned with CamScanner

shaon below.



released structure

r = 5, h = 2

DI = r - 3 - h

= 5 - 3 - 2

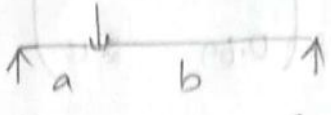
= 0



$M = \frac{wl^2}{8}$

$= \frac{40 \times 12^2}{8}$

$M = 720 \text{ KN-m}$



$M = \frac{Wab}{l}$

$= \frac{120 \times 4 \times 8}{12}$

$M = 320 \text{ KN-m}$

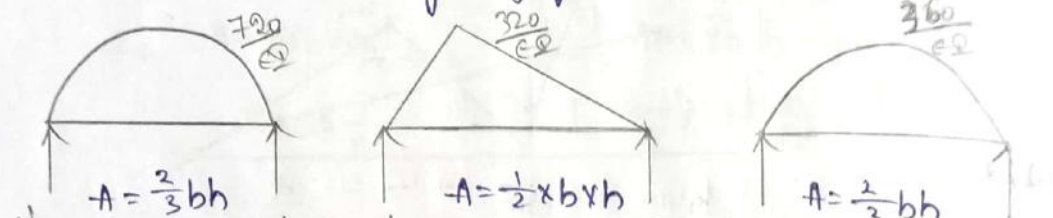


$M = \frac{wl^2}{8}$

$= \frac{20 \times 12^2}{8}$

$M = 360 \text{ KN-m}$

The displacement matrix at the co-ordinates due to loads $[\Delta]$ will be calculated using conjugate beam method.



$A = \frac{2}{3}bh$

$\frac{A}{2}$ $\frac{A}{2}$

$= \frac{2}{3} \times 12 \times \frac{720}{EI}$

$A = \frac{5760}{EI}$

$A = \frac{1}{2} \times b \times h$

$\frac{A(l+b)}{3l}$ $\frac{A(l+a)}{3l}$

$= \frac{1}{2} \times 12 \times \frac{320}{EI}$

$A = \frac{1920}{EI}$

$A = \frac{2}{3}bh$

$\frac{A}{2}$ $\frac{A}{2}$

$= \frac{2}{3} \times 12 \times \frac{720}{2} \times \frac{360}{EI}$

$A = \frac{2880}{EI}$

Δ_{1L} = displacement at ① due to loads

= Rotation @ B of AB and BC

= shear at B' of A'B' and B'C'

$= \left[\frac{A}{2}\right]_{A'B'} + \left[\frac{A(l+b)}{3l}\right]_{B'C'}$

$= \frac{1}{2} \times \frac{5760}{EI} + \frac{1}{3 \times 12}\left[\frac{1920}{EI} \times (12+8)\right]$

$\Delta_{1L} = \frac{3946.67}{EI}$

Scanned with CamScanner

Δ_{21} = displacement of B of B'C' and C'B'

$$= \left(\frac{A(l+a)}{3l}\right)_{B'C'} + \left(\frac{A}{2}\right)_{C'B'}$$

$$= \frac{1}{3\times12}\left[\frac{1920}{EI}(12+4)\right] + \left[\frac{1}{2}\times\frac{2880}{EI}\right]$$

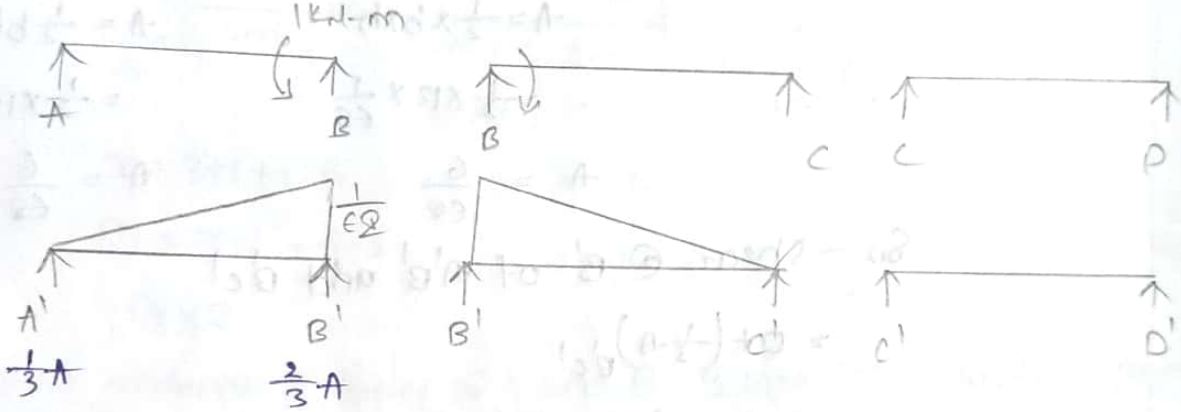
$$\Delta_{21} = \frac{2293.33}{EI}$$

The final displacement matrix at the coordinates [Δ] during compatability conditions are,

$$\Delta_1 = 0 \text{ and } \Delta_2 = 0.$$

Flexibility Matrix. $[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$

Applying unit moment at ①



$A = \frac{1}{2}bh$

$= \frac{1}{2}\times12\times\frac{1}{EI}$

$A = \frac{6}{EI}$

$A = \frac{1}{2}bh$

$= \frac{1}{2}\times12\times\frac{1}{EI}$

$A = \frac{6}{EI}$

S_{11} = shear of B' of A'B' and B'C'

$$= \left(\frac{2}{3}A\right)_{A'B'} + \left(\frac{2}{3}A\right)_{B'C'}$$

$$= \frac{2}{3}\times\frac{6}{EI} + \frac{2}{3}\times\frac{6}{EI}$$

$$S_{11} = \frac{8}{EI}$$

Scanned with CamScanner

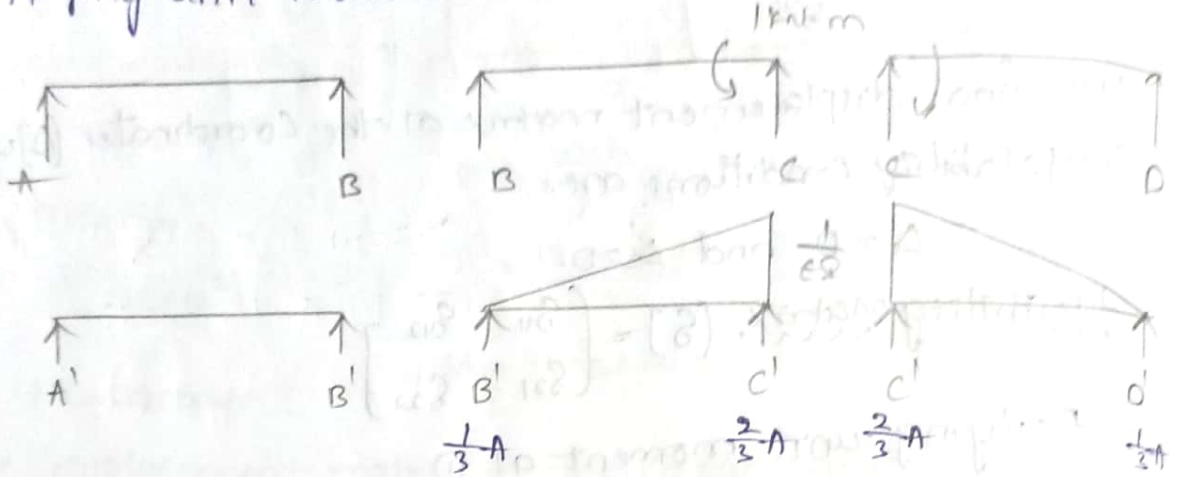
δ_{21} = shear at C' of B'C' and C'D

$= \left(\frac{1}{3}A\right)_{B'C'} + 0$

$= \frac{1}{3} \times \frac{6}{EI} = \frac{2}{EI}$

$\delta_{21} = \frac{2}{EI}$.

Applying unit moment at (2)



$A = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 12 \times \frac{1}{EI}$

$A = \frac{6}{EI}$

$A = \frac{1}{2} bh$

$= \frac{1}{2} \times 12 \times \frac{1}{EI}$

$A = \frac{6}{EI}$

δ_{12} = shear @ B' of A'B' and B'C'

$= 0 + \left(\frac{1}{3}A\right)_{B'C'}$

$= \frac{1}{3} \times \frac{6}{EI}$

$\delta_{12} = \frac{2}{EI}$

$\delta_{22} = \left(\frac{2}{3}A\right)_{B'C'} + \left(\frac{2}{3}A\right)_{C'D'}$

$= \frac{2}{3} \times \frac{6}{EI} + \frac{2}{3} \times \frac{6}{EI}$

$\delta_{22} = \frac{8}{EI}$

$[P] = [\delta]^{-1} \{[D] - [D_L]\}$

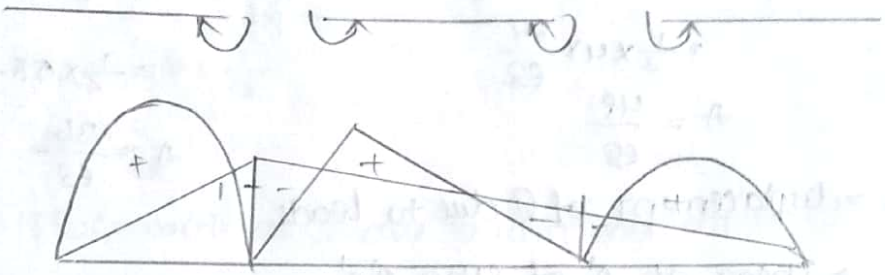
$= \begin{bmatrix} \frac{90}{EI} & \frac{2}{EI} \\ \frac{2}{EI} & \frac{8}{EI} \end{bmatrix}^{-1} \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{3946.67}{EI} \\ \frac{2283.33}{EI} \end{pmatrix} \right\}$

Scanned with CamScanner

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 8 & 2 \\ 2 & 8 \end{bmatrix}^{-1} \begin{pmatrix} \frac{-3946.69}{EI} \\ \frac{-2283.33}{EI} \end{pmatrix}$$

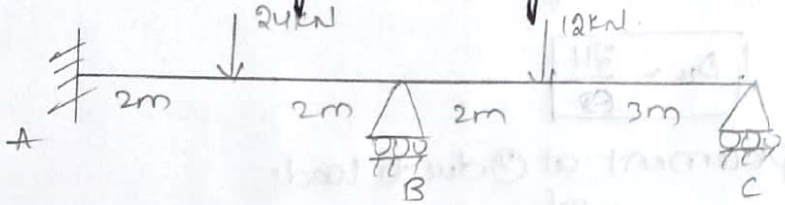
$P_1 = -449.7\,kN\text{-}m$, $P_2 = 194.22\,kN\text{-}m$.

Negative sign indicates that we need to change the direction of moment.



MID ***

3. Analyse the beam using flexibility matrix method. BMD

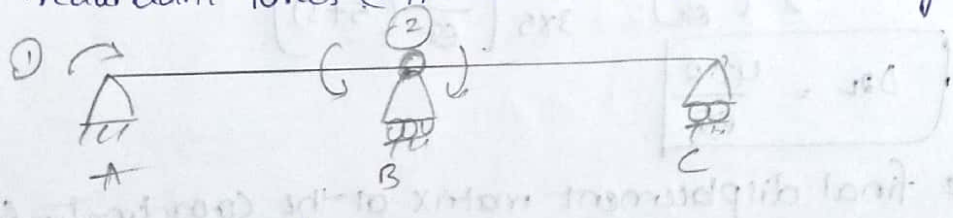


$r = 3+1+1 = 5$

$D_s = r-3 = 5-3$

$D_s = 2.$

* Choose redundant forces @ A and B. Because 'C' is simply supported.



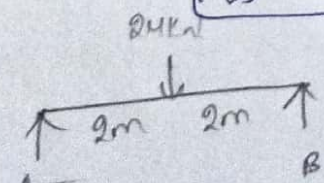
$r = 4$

$n = 1$

$D_s = r-3-n$

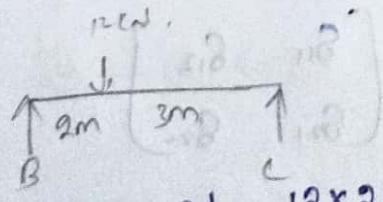
$= 4-3-1$

$D_s = 0$



$M = \frac{wl}{4} = \frac{24 \times 4}{4}$

$M = 24\,kN\text{-}m$

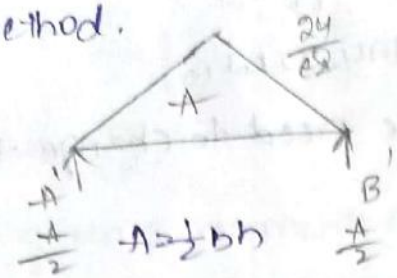


$M = \frac{wab}{l} = \frac{12 \times 2 \times 3}{5}$

$= 14.4\,kN\text{-}m$

Scanned with CamScanner

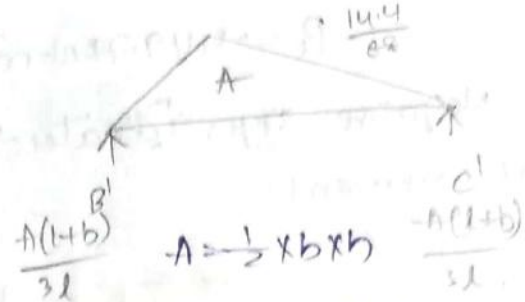
the displacement matrix at the co-ordinates due to loading [Δ] (rotation @ A and B) will be calculated using Conjugate beam method.



$A = \frac{1}{2} bh$

$= \frac{1}{2} \times 4 \times \frac{24}{EI}$

$A = \frac{48}{EI}$



$A = \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 5 \times \frac{14.4}{EI}$

$A = \frac{36}{EI}$

Δ_{1L} = displacement of ① due to loads

= shear at A' of span A'B'

$= \left(\frac{A}{2}\right)_{A'B'} = \frac{1}{2} \times \frac{48}{EI}$

$$\boxed{\Delta_{1L} = \frac{24}{EI}}$$

Δ_{2L} = displacement at ② due to loads

= shear at B' of A'B' and B'C' spans.

$$= \left(\frac{A}{2}\right)_{A'B'} + \left(\frac{A(1+b)}{3l}\right)_{B'C'}$$

$$= \frac{1}{2}\left[\frac{48}{EI}\right] + \frac{1}{3\times 5}\left[\frac{36}{EI}(5+3)\right]$$

$$\boxed{\Delta_{2L} = \frac{43.2}{EI}.}$$

The final displacement matrix at the coordinates [Δ], using compatability conditions.

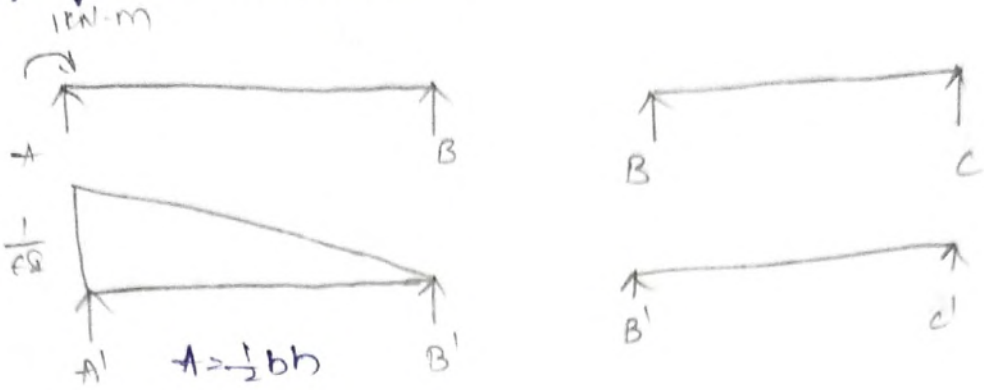
$\Delta_1 = 0$, $\Delta_2 = 0$.

Flexibility Matrix

$$[S] = \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix}$$

Scanned with CamScanner

Applying unit moment at ①



$A = \frac{1}{2} bh$

$\frac{2}{3}A$ $\frac{1}{3}A$

$= \frac{1}{2} \times 4 \times \frac{1}{EI}$

$A = \frac{2}{EI}$

δ_{11} = displacement at ① due to unit moment at ①

= shear

Scanned with CamScanner

Unit-IV

KANI's Method.

It is Introduced by "Gasper Kani". This method is an excellent extension of Slope Deflection Method.

Procedure:

1. Determine Fixed end Moments
2. Determine Rotation factors.

$$Rf = \frac{-1}{2} \cdot \frac{K}{\Sigma K}$$

3. Represent the fixed end moments above the horizontal line.
4. Balance the moment whenever it is required and carryover half of the balanced moment to the far end.
5. Calculate Sum of the fixed end moments at internal joints. Represent this value inside a box at each internal joint.
6. Represent rotation factors at both sides of the box for each internal joint.
7. Rotation ~~factors~~ Contribution at the external joints is assumed 'zero'.
8. Calculate rotation Contribution at each internal joint.

$$M'_{BA} = (\Sigma M_{FB} + M'_{AB} + M'_{CB}) R_{FBA}$$

where, $\Sigma M_{FB} = M_{FBA} + M_{FBC}$.

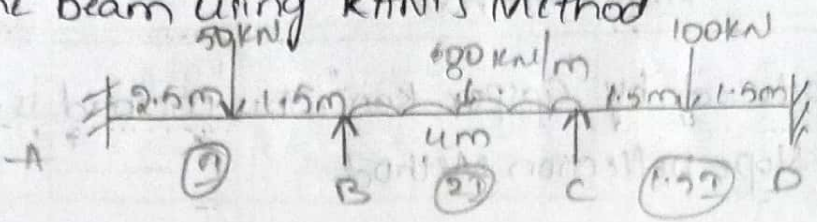
M'_{BA}, M'_{AB} & M'_{CB} are Rotation Contribution.

9. Iterate the process till the two consecutive rotation Contribution are same.
10. Determine the final end Moments.

Scanned with CamScanner

Problems

1. Analyse the beam using KANI's Method



Bending Moment due to loads

Span AB. $M = \frac{Wab}{l} = \frac{50 \times 2.5 \times 1.5}{4} = 46.8$ KNm

Span BC. $M = \frac{Wl^2}{8} = \frac{80 \times 4^2}{8} = 160$ KNm.

Span CD. $M = \frac{Wl}{4} = \frac{100 \times 3}{4} = 75$ KNm

Step ① Fixed end Moments

$M_{FAB} = -\frac{Wab^2}{l^2} = -\frac{50 \times 2.5 \times 1.5^2}{4^2} = -17.57$ KNm

$M_{FBA} = +\frac{Wa^2b}{l^2} = +\frac{50 \times 2.5^2 \times 1.5}{4^2} = +29.29$ KNm

$M_{FBC} = -\frac{Wl^2}{12} = -\frac{80 \times 4^2}{12} = -106.67$ KNm

$M_{FCB} = +\frac{Wl^2}{12} = +\frac{80 \times 4^2}{12} = +106.67$ KNm.

$M_{FCD} = -\frac{Wl}{8} = -\frac{100 \times 3}{8} = -37.5$ KNm

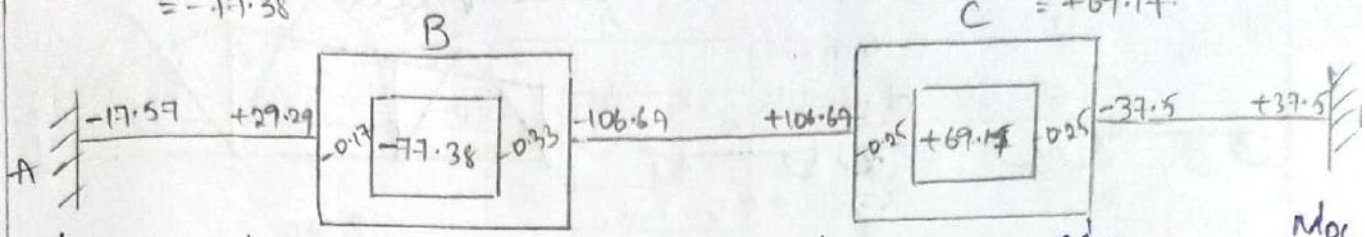
$M_{FDC} = +\frac{Wl}{8} = +\frac{100 \times 3}{8} = +37.5$ KNm.

Step ② Rotation factor

Joint	Member	K	ΣK	$RF = -\frac{1}{2} \cdot \frac{K}{\Sigma K}$	
B	BA	$\frac{I}{4} = 0.25I$	0.75I.	-0.167	-0.5
	BC	$\frac{2I}{4} = 0.5I$		-0.334	
C	CB	$\frac{2I}{4} = 0.5I$	I	-0.25	-0.5
	CD	$\frac{1.5I}{3} = 0.5I$		-0.25	

Scanned with CamScanner

Step ③ Rotation Contribution



M'_{AB}	M'_{BA}	M'_{BC}	M'_{CB}	M'_{CD}	M'_{DC}
0	+13.15	+25.53	-23.67	-23.67	0
0-77.38+0 = -77.38	+17.17	+33.34	-25.63	-25.63	
(-77.38)x(-0.17) = +13.15	+17.951	+33.09	-25.79	-25.79	
(-77.38)x(-0.33) = +25.53	+17.55	+34.04	-25.80	-25.80	

25.53+69.17+0 = 94.7

94.7x(-0.25) = -23.67

Step ④ Final End Moments

$M_{AB} = M_{FAB} + 2\theta_A + \theta_B \Rightarrow M_{FAB} + 2(M'_{AB}) + M'_{BA}$

$= -17.57 + 2(0 + 17.55) + 34.04$

$= -0.02$ Kn-m

$M_{BA} = M_{FBA} + 2\theta_B + \theta_A$

$= +29.29 + 2(17.55) + 0$

$= +64.39$ Kn-m

$M_{BC} = M_{FBC} + 2\theta_B + \theta_C$

$= -106.69 + 2(34.04) - 25.80$

$= -64.39$ Kn-m

$= 0$ (check ✓)

$M_{CB} = M_{FCB} + 2\theta_C + \theta_B$

$= +106.69 + 2(-25.80) + 34.04$

$= +89.11$ Kn-m

$M_{CD} = M_{FCD} + 2\theta_C + \theta_D$

$= -37.5 + 2(-25.80) + 0$

$= -89.1$ Kn-m

$= 0$ (check ✓)

$M_{DC} = M_{FDC} + 2\theta_D + \theta_C$

$= +37.5 + 2(0) - 25.80$

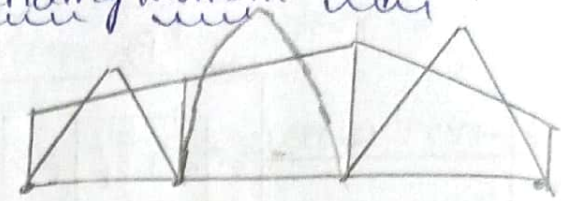
$= +11.7$ Kn-m.

Scanned with CamScanner

Free Body Diagram

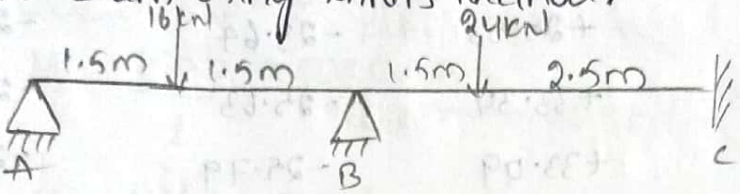


Bending Moment Diagram



BMD

2. Analyse the beam using KANI's Method.



I is Constant

Bending Moment due to loads:

Span AB, $M = \frac{wl}{4} = \frac{16 \times 3}{4} = 12 KN\text{-}m$

Span BC, $M = \frac{wab}{l} = \frac{24 \times 1.5 \times 2.5}{4} = 22.5 KN\text{-}m$.

Step ① fixed end moment

$M_{FAB} = -\frac{wl}{8} = \frac{-16 \times 3}{8} = -6 KN\text{-}m$

$M_{FBA} = +\frac{wl}{8} = +\frac{16 \times 3}{8} = +6 KN\text{-}m$

$M_{BC} = -\frac{wab^2}{l^2} = -\frac{24 \times 1.5 \times 2.5^2}{4^2} = -14.06 KN\text{-}m$.

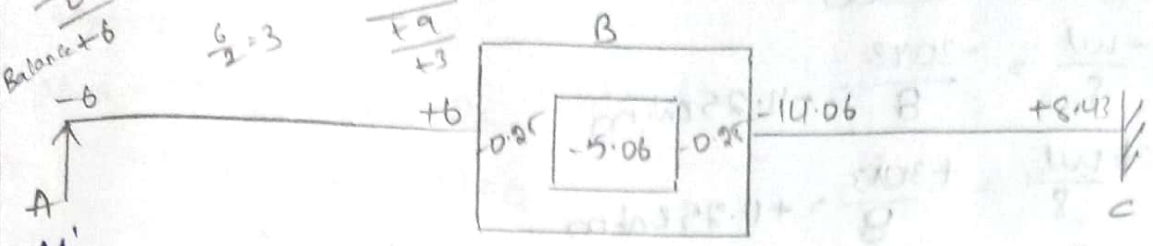
$M_{CB} = +\frac{wa^2b}{l^2} = +\frac{24 \times 1.5^2 \times 2.5}{4^2} = +8.43 KN\text{-}m$.

Step ② Rotation Factor

Joint	Member	K	ΣK	RF = $-\frac{1}{2} \cdot \frac{K}{\Sigma K}$
B	BA	$\frac{3}{4} \cdot \frac{I}{3} = 0.25I$	0.5I	-0.25
	BC	$\frac{I}{4} = 0.25I$		-0.25

Scanned with CamScanner

Step (3) Rotation Contribution



M'_{AB}	M'_{BA}	M'_{BC}	M'_{CB}
0	+1.26	+1.26.	0

Step (4) Final end Moments

$M_{AB} = MF_{AB} + 2\theta_A + \theta_B$ $\boxed{M_{AB} = 0}$

$= -6 + 2(0) + 1.26$

$=$

$M_{BA} = MF_{BA} + 2\theta_B + \theta_A$

$= +6 + 2(1.26) + 0$

$= 11.52$ KN-m

$M_{BC} = MF_{BC} + 2\theta_B + \theta_C$

$= -14.06 + 2(1.26) + 0$

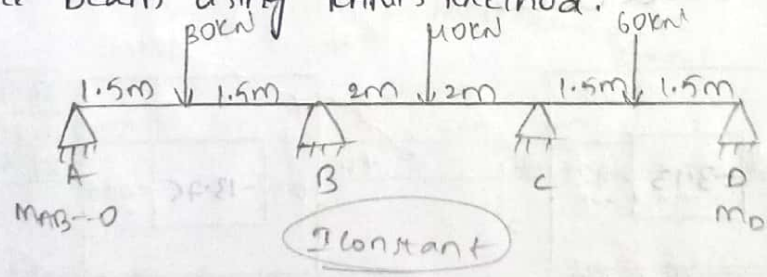
$= -11.54$ KNm

$M_{CB} = MF_{CB} + 2\theta_C + \theta_B$

$= +8.43 + 2(0) + 1.26$

$= +9.69$ KNm

3. Analyse the beam using KANI's Method.



Bending moment due to loads:

Span AB, $M = \frac{Wl}{4} = \frac{30 \times 3}{4} = 22.5$ KN-m

Span BC, $M = \frac{Wl}{4} = \frac{40 \times 4}{4} = 40$ KN-m

Span CD, $M = \frac{Wl}{4} = \frac{60 \times 3}{4} = 45$ KN-m

Scanned with CamScanner

Step ① Fixed end Moment

$M_{FAB} = \frac{-wl}{8} = \frac{-30\times3}{8} = -11.25\ kN\text{-}m$

$M_{FBA} = \frac{+wl}{8} = \frac{+30\times3}{8} = +11.25\ kN\text{-}m$

$M_{FBC} = \frac{-wl}{8} = \frac{-40\times4}{8} = -20\ kN\text{-}m$

$M_{FCB} = \frac{+wl}{8} = \frac{+40\times4}{8} = +20\ kN\text{-}m$

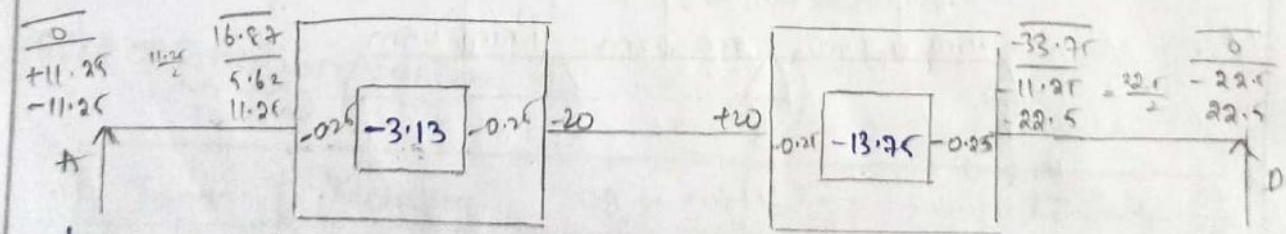
$M_{FBD} = \frac{-wl}{8} = \frac{-60\times3}{8} = -22.5\ kN\text{-}m$

$M_{DC} = \frac{+wl}{8} = \frac{+60\times3}{8} = +22.5\ kN\text{-}m$

Step ② Rotation-factor

Joint	Member	K	ΣK	$RF = -1 \cdot \frac{K}{\Sigma K}$
B	BA	$\frac{3}{4}\cdot\frac{I}{3} = 0.25I$	0.5I	-0.25
	BC (H is 3I)	$\frac{I}{4} = 0.25I$		-0.25
C	CB (H is 3I)	$\frac{I}{4} = 0.25I$	0.5I	-0.25
	CD	$\frac{3}{4}\cdot\frac{I}{3} = 0.25I$		-0.25

Step ③ Rotation Contribution



M'_{AB}	M'_{BA}	M'_{BC}	M'_{CB}	M'_{CD}	M'_{DC}
0	+0.78	+0.78	+3.24	+3.24	0
0	-0.02	-0.02	+3.44	+3.44	0
0	-0.07	-0.07	+3.45	+3.45	0
0	-0.08	-0.08	+3.45	+3.45	0

Scanned with CamScanner

Step (4) final end Moment

$M_{AB} = 0$

$M_{BA} = M_{FBA} + 2(\theta_B) + \theta_A$

$= 16.87 + 2(-0.08) + 0$

$= 16.71 KN\text{-}m$

$M_{BC} = M_{FBC} + 2(\theta_B) + \theta_C$

$= -20 + 2(-0.08) + 3.45$

$= -16.7 KN\text{-}m$

$M_{CB} = M_{FCB} + 2\theta_C + \theta_B$

$= 20 + 2(3.45) - 0.08$

$= 26.82 KN\text{-}m$

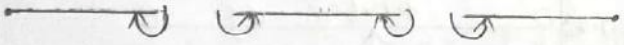
$M_{CD} = M_{FCD} + 2\theta_C + \theta_D$

$= -33.75 + 2(3.45) + 0$

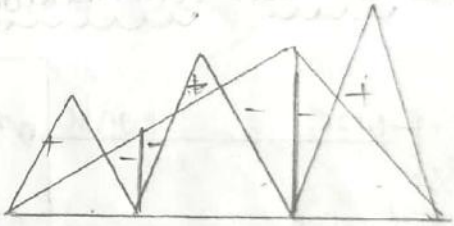
$= -26.82 KN\text{-}m$

$M_{DC} = 0$

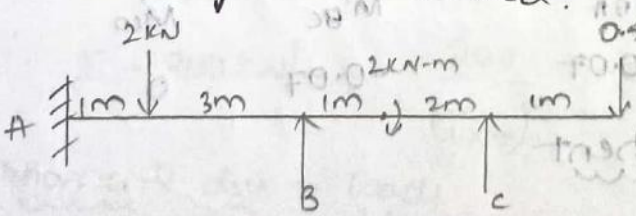
Free Body Diagram



Bending Moment Diagram



4. Analyse the beam by KANI's Method.



$M_{DC} = 0.5 \times D$

$M_{AB} = 0$

$M_{CD} = -(0.5 \times 1)$

$M_{DC} = 0.5$

Bending Moment due to loady

Span AB, $M = \frac{Wab}{l} = \frac{2 \times 1 \times 3}{4} = 1.5 KN\text{-}m$

Span BC, $M = \frac{Mab}{l} = \frac{2 \times 1}{3} = 0.67 KN\text{-}m$

$\frac{Mab}{l} = \frac{2 \times 2}{3} = 1.33 KN\text{-}m.$

Scanned with CamScanner

Step ① Fixed end Moments

$M_{FAB} = \frac{-wab^2}{l^2} = \frac{-2\times1\times3^2}{4^2} = -1.125\ KN\text{-}m$

$M_{FBA} = \frac{+wa^2b}{l^2} = \frac{+2\times1^2\times3}{4^2} = +0.375\ KN\text{-}m$

$M_{FBC} = +\frac{M_b}{l^2}(3a-l) = \frac{+2\times2}{3^2}[3(1)-3] = 0\ KN\text{-}m$

$M_{FCB} = \frac{+M_a}{l^2}(3b-l) = \frac{+2\times1}{3^2}[3(2)-3] = +0.69\ KN\text{-}m.$

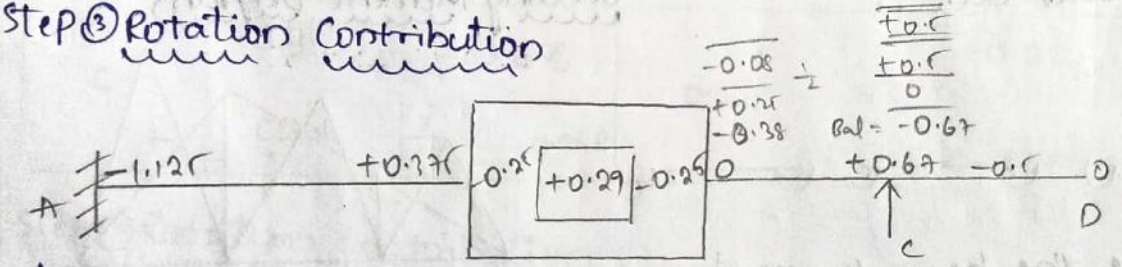
$M_{FCD} = -(0.5\times1) = -0.5\ KN\text{-}m$

$M_{DC} = (0.5\times0) = 0\ KN\text{-}m$

Step ② Rotation factor

Joint	Member	K	ΣK	$RF = -\frac{1}{2}\cdot\frac{K}{\Sigma K}$
B	BA	$\frac{I}{4} = 0.25I$	$0.5I$	-0.25
	BC	$\frac{3}{4}\cdot\frac{I}{3} = 0.25I$		-0.25

Step ③ Rotation Contribution



M'_{AB}	M'_{BA}	M'_{BC}	M'_{CD}
0	-0.07	-0.07	0

Step ④ Final end Moment

$M_{AB} = M_{FAB} + 2(\theta_A) + \theta_B$

$= -1.125 + 2(0) - 0.07$

$= -1.19\ KN\text{-}m.$

$M_{BA} = M_{FBA} + 2(\theta_B) + \theta_A$

$= 0.375 + 2(-0.09) + 0$

$= +0.235\ KN\text{-}m.$

Scanned with CamScanner

$M_{BC} = M_{FBC} + 2(\theta_B) + \theta_C$

$= -0.085 + 2(-0.09) + 0$

$= -0.225 \text{ KN-m}$

$M_{CB} = +0.5$

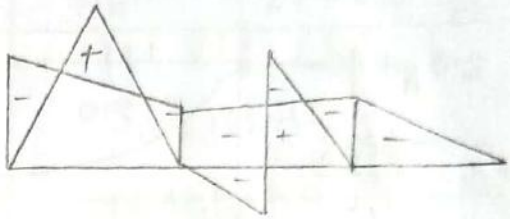
$M_{CD} = -0.5$

$M_{DC} = 0.$

Free Bending Diagram

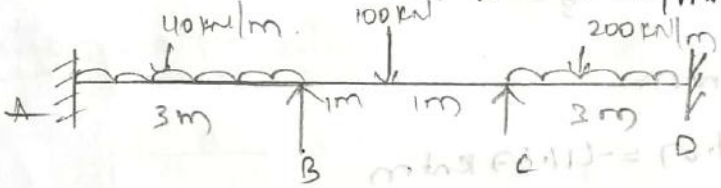


Bending Moment Diagram



BMD

**5* Analyse the beam by KANI's Method, if support B sinks by 2.5mm. Take $I = 3.5 \times 10^7 \text{ mm}^4$. and $E = 200 \text{ KN/mm}^2$



$\delta = 2.5\text{mm} = 2.5 \times 10^{-3} \text{m}$ $(1\text{m} = 1000\text{mm})$

$I = 3.5 \times 10^7 \text{ mm}^4$

$= 3.5 \times 10^7 \times \left(\frac{1}{1000}\right)^4$

$= 3.5 \times 10^{-5} \text{ m}^4$

$E = 200 \text{ KN/mm}^2 = \frac{200}{\left(\frac{1}{1000}\right)^2} = 200 \times 10^6 \text{ KN/m}^2$

Bending Moments due to loads

Span, AB, $M = \frac{wl^2}{8} = \frac{40 \times 3^2}{8} = 45 \text{ KN-m}.$

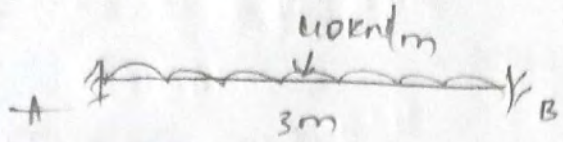
Span BC, $M = \frac{wl}{4} = \frac{100 \times 2}{4} = 50 \text{ KN-m}$

Span CD, $M = \frac{wl^2}{8} = \frac{200 \times 3^2}{8} = 225 \text{ KN-m}$

Scanned with CamScanner

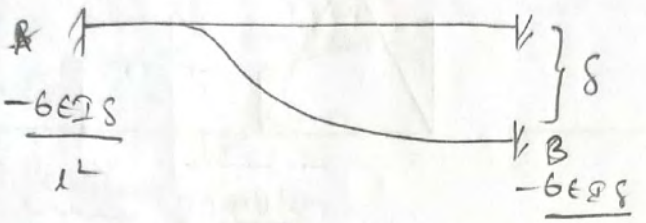
Step ① Fixed end Moments

①



$= \frac{-wl^2}{12} = \frac{-40 \times 3^2}{12} = -30\ kNm$

$= +\frac{wl^2}{12} = +\frac{40 \times 3^2}{12} = +30\ kNm$



$\frac{-6EI\delta}{l^2}$ $\quad$ $\frac{-6EI\delta}{l^2}$

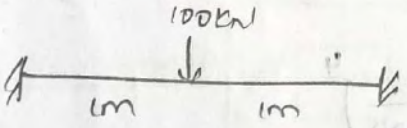
$= -\frac{6 \times 200 \times 10^6 \times 3.5 \times 10^{-5} \times 2.5 \times 10^{-3}}{3^2}$

$= -11.67\ kNm.$

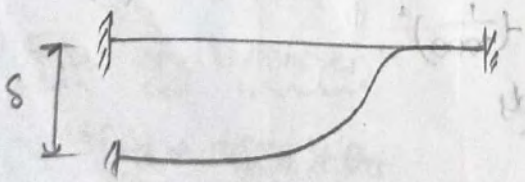
$M_{FAB} = -30 - 11.67 = -41.67\ kNm$

$M_{FBA} = +30 - 11.67 = +18.33\ kNm.$

②



$\frac{-wl}{8} = -25\ kNm$ $\quad$ $\frac{+wl}{8} = +25\ kNm$



$\frac{+6EI\delta}{l^2}$ $\quad$ $\frac{+6EI\delta}{l^2}$

$= \frac{6 \times 200 \times 10^6 \times 3.5 \times 10^{-5} \times 2.5 \times 10^{-3}}{2^2}$

$= +26.25$

Scanned with CamScanner

$M_{FBC} = -25 + 26.25 = +1.25$ kN-m

$M_{FCB} = +25 + 26.25 = 51.25$ kN-m

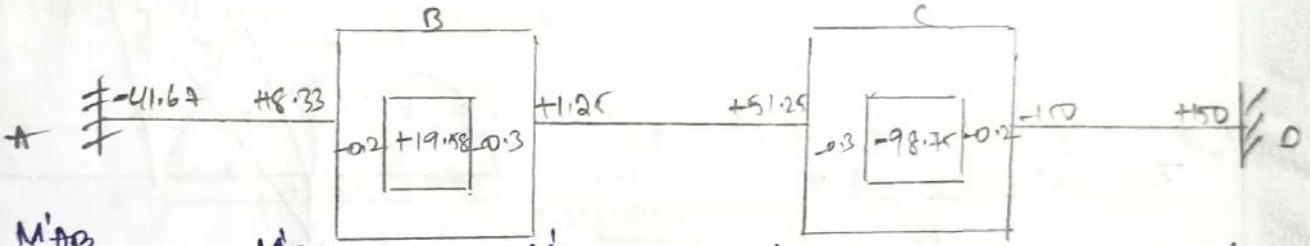
$M_{FCD} = \frac{-wl^2}{12} = \frac{-200 \times 3^2}{12} = -150$ kN-m

$M_{FDC} = \frac{+wl^2}{12} = \frac{+200 \times 3^2}{12} = +150$ kN-m

Step ② Rotation Factor

Joint	Member	K	ΣK	$Rf = -\frac{1}{2} \cdot \frac{K}{\Sigma K}$
B	BA	$\frac{I}{3} = 0.33I$	$0.83I$	-0.2
	BC	$\frac{I}{2} = 0.5I$		-0.3
C	CB	$\frac{I}{2} = 0.5I$	$0.83I$	-0.3
	CD	$\frac{I}{3} = 0.33I$		-0.2

Step ③ Rotation Contribution



M'_{AB}	M'_{BA}	M'_{BC}	M'_{CB}	M'_{CD}	M'_{DC}
0	-3.916	-5.874	+31.38	+20.92	0
	-10.19	-15.28	+34.20	+22.80	
	-10.75	-16.13	+34.46	+22.97	
(θ_A)	-10.8 (θ_B)	-16.21 (θ_C)	+34.48 θ_D	+22.99 (θ_C)	(θ_D)

Step ④ Final end Moments

$M_{AB} = M_{FAB} + 2\theta_A + \theta_B$

$= -41.67 + 2(0) - 10.8$

$= -52.47$ kN-m

$M_{BA} = +M_{FBA} + 2\theta_B + \theta_A$

$= 18.33 + 2(-10.8) + 0$

$= -3.27$ kN-m.

Scanned with CamScanner

$M_{FBC} = M_{FBC} + 2\theta_B + \theta_C$

$= 1.25 + 2(-16.21) + 34.48$

$= -3.31$ kN-m

$M_{CB} = M_{FCB} + 2\theta_C + \theta_B$

$= 51.25 + 2(34.48) - 16.21$

$= 104$ kN-m

$M_{CD} = M_{FCD} + 2\theta_C + \theta_D$

$= -150 + 2(22.99) + 0$

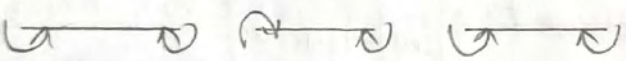
$= -104.02$ kN-m

$M_{DC} = M_{FDC} + 2\theta_D + \theta_C$

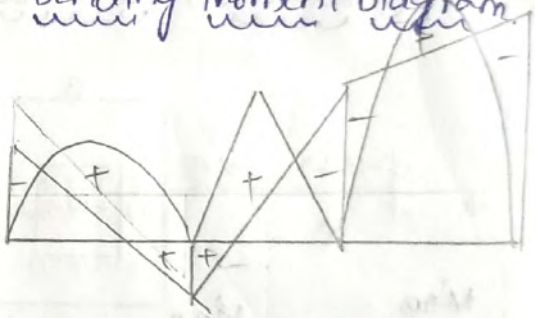
$= +150 + 2(0) + 22.99$

$= 172.99$ kN-m

Free Body Diagram

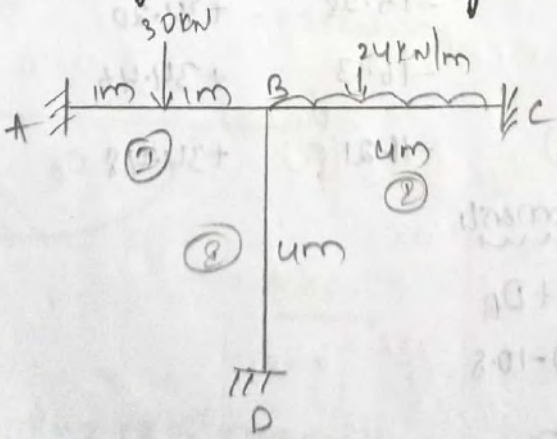


Bending Moment Diagram



BMD

6. Analyse the following structure using KANI's Method.



Bending Moment due to loads

Span AB, $M = \frac{Wl}{4} = \frac{30 \times 2}{4} = 15$ kN-m Span CD, $m = 0$.

Span BC, $M = \frac{wl^2}{8} = \frac{24 \times 4^2}{8} = 48$ kN-m

Scanned with CamScanner

Step ① Fixed end Moments

$M_{FAB} = \frac{-wl}{8} = \frac{-30 \times 2}{8} = -7.5$ Kn-m

$M_{FBA} = \frac{+wl}{8} = \frac{+30 \times 2}{8} = +7.5$ Kn-m

$M_{FBC} = \frac{-wl^2}{12} = \frac{-24 \times 4^2}{12} = -32$ Kn-m

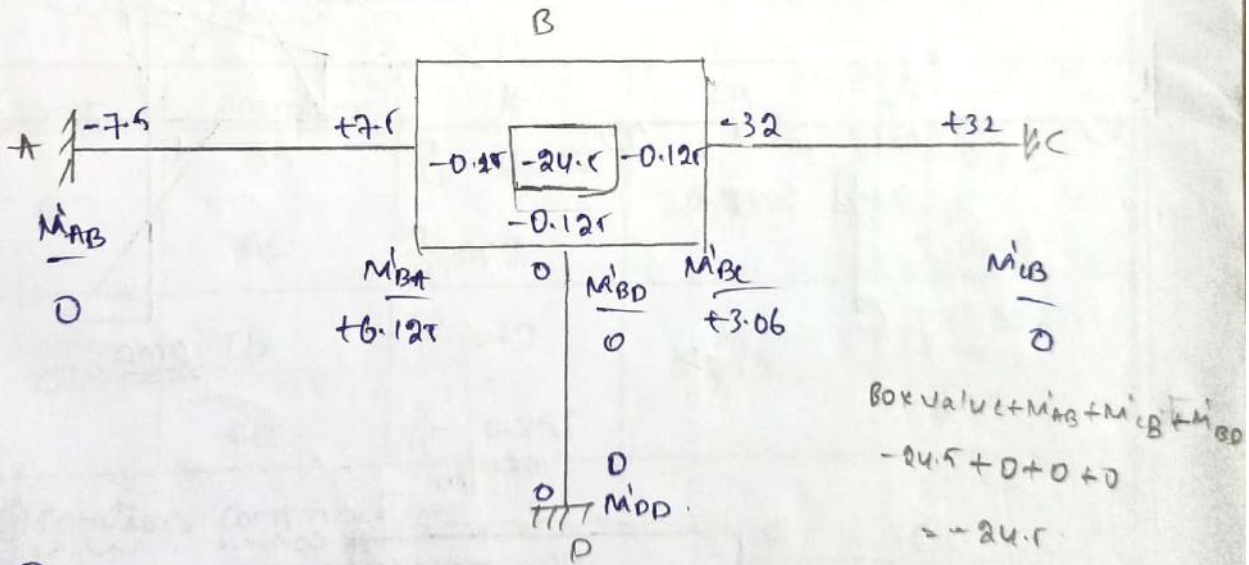
$M_{FCB} = \frac{+wl^2}{12} = \frac{+24 \times 4^2}{12} = +32$ Kn-m

$M_{FBD} = M_{FDB} = 0$

Step ② Rotation factor

Joint	Member	K	ΣK	$RF = -\frac{1}{2} \cdot \frac{K}{\Sigma K}$
B	BA	$\frac{1}{2} = 0.5$	1	-0.25
	BC	$\frac{1}{4} = 0.25$		-0.125
	BD	$\frac{1}{4} = 0.25$		-0.125

Step ③ Rotation contribution



Box value $= M'_{AB} + M'_{CB} + M'_{BD}$

$= -24.5 + 0 + 0 + 0$

$= -24.5$

Step ④ Final end moments

$M_{AB} = M_{FAB} + 2(M'_{AB}) + M'_{DB}$

$= -7.5 + 2(6.125) + 6.125$

$= -1.375$ Kn-m.

$M_{BA} = M_{FBA} + 2(0_B) + 0_A$

$= +7.5 + 2(6.125) + 0$

$= 19.75$ Kn-m.

Scanned with CamScanner

$M_{BC} = M_{FBC} + 2(D_B) + D_A$

$= -32 + 2(3.06) + 0$

$= -25.88$ Kn-m.

$M_{CB} = M_{FCB} + 2(D_C) + D_B$

$= 32 + 2(0) + 3.06$

$= 35.06$ Kn-m.

$M_{BD} = M_{FBD} + 2D_B + D_D$

$= 0 + 2(3.06) + 0$

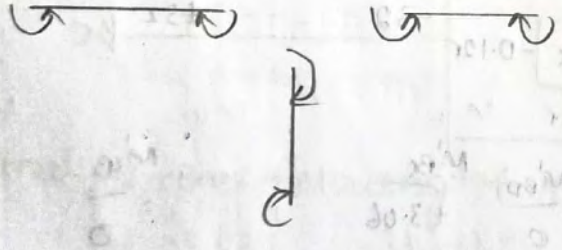
$= 6.12$ Kn-m.

$M_{DB} = M_{FDB} + 2D_D + D_B$

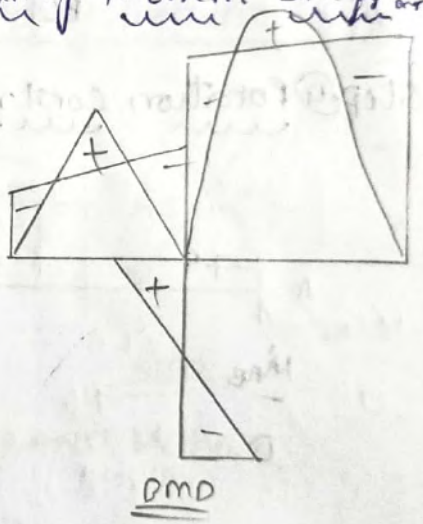
$= 0 + 2(0) + 3.06$

$= 3.06$ Kn-m.

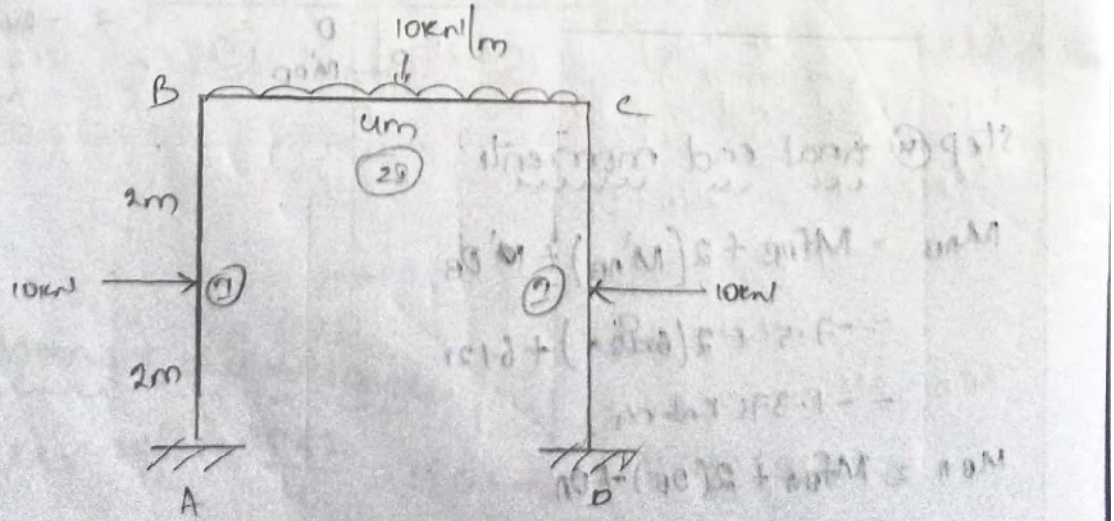
Free Bending Diagram



Bending Moment Diagram



7.



Scanned with CamScanner

Bending Moment due to loads

Span AB, $M = \frac{wl}{4} = \frac{10 \times 4}{4} = 10\ KN\text{-}m$

Span BL, $M = \frac{wl^2}{8} = \frac{10 \times 4^2}{8} = 20\ KNm.$

Span CD, $M = \frac{wl}{4} = \frac{10 \times 4}{4} = 10\ KNm$

Step ① Fixed end Moments

$M_{FAB} = -\frac{wl}{8} = -\frac{10 \times 4}{8} = -5\ KNm$

$M_{FBA} = +\frac{wl}{8} = +\frac{10 \times 4}{8} = +5\ KNm$

$M_{FBL} = -\frac{wl^2}{12} = -\frac{10 \times 4^2}{12} = -13.33\ KNm$

$M_{FCB} = +\frac{wl^2}{12} = +\frac{10 \times 4^2}{12} = +13.33\ KNm$

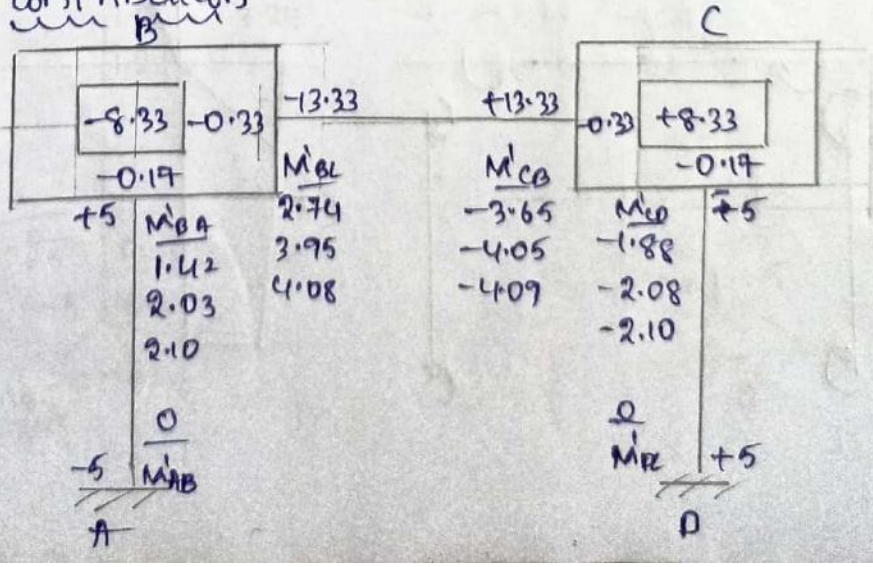
$M_{FCD} = -\frac{wl}{8} = -\frac{10 \times 4}{8} = -5\ KNm$

$M_{FDC} = +\frac{wl}{8} = +\frac{10 \times 4}{8} = +5\ KNm$

Step ② Rotation factor

Joint	Member	K	ΣK	$RF = -\frac{1}{2} \cdot \frac{K}{\Sigma K}$
B	BA	$\frac{I}{4} = 0.25I$	$0.75I$	-0.17
	BL	$\frac{2I}{4} = 0.5I$		-0.33
C	CB	$\frac{2I}{4} = 0.5I$	$0.75I$	-0.33
	CD	$\frac{I}{4} = 0.25I$		-0.17

Step ③ Rotation contribution



Scanned with CamScanner

Step ④ Final end moments

$M_{AB} = M^{\prime}_{FAB} + 2\theta_A + \theta_B$

$= -5 + 2(0) + 2.10$

$= -2.9 \text{ Kn-m}$

$M_{BA} = M_{FBA} + 2\theta_B + \theta_A$

$= +5 + 2(2.10) + 0$

$= 9.2 \text{ Kn-m}.$

$M_{BC} = M_{FBC} + 2\theta_B + \theta_C$

$= -13.33 + 2(4.08) - 4.09$

$= -9.26 \text{ Kn-m}$

$M_{CB} = M_{FCB} + 2\theta_C + \theta_B$

$= +13.33 + 2(-4.09) + 4.08$

$= 9.23 \text{ Kn-m}$

$M_{CD} = M_{FCD} + 2\theta_C + \theta_D$

$= -5 + 2(-2.10) + 0$

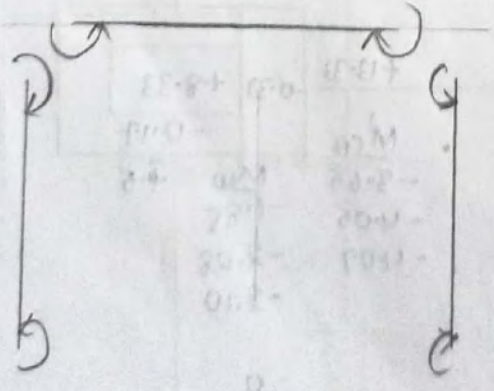
$= -9.2 \text{ Kn-m}$

$M_{DC} = M_{FDC} + 2\theta_D + \theta_C$

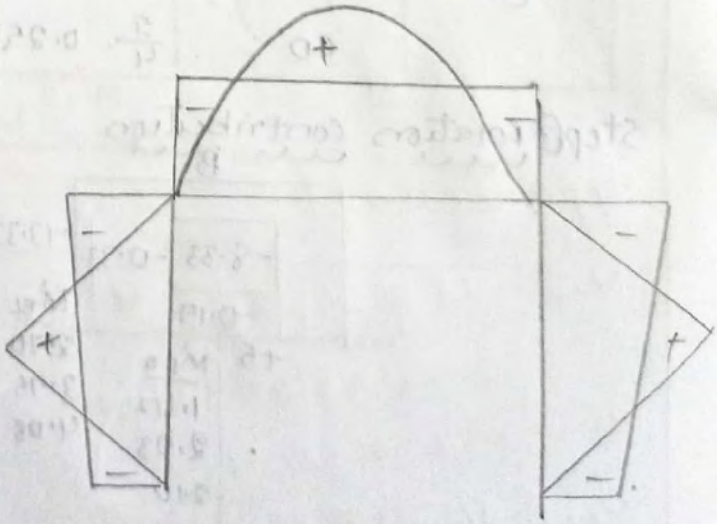
$= +5 + 2(0) - 2.10$

$= 2.9 \text{ Kn-m}.$

Free Body Diagram



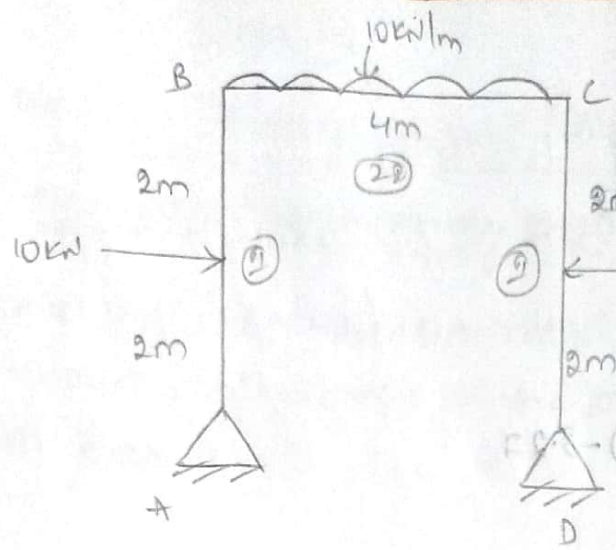
Bending Moment Diagram



BMD

Scanned with CamScanner

8.



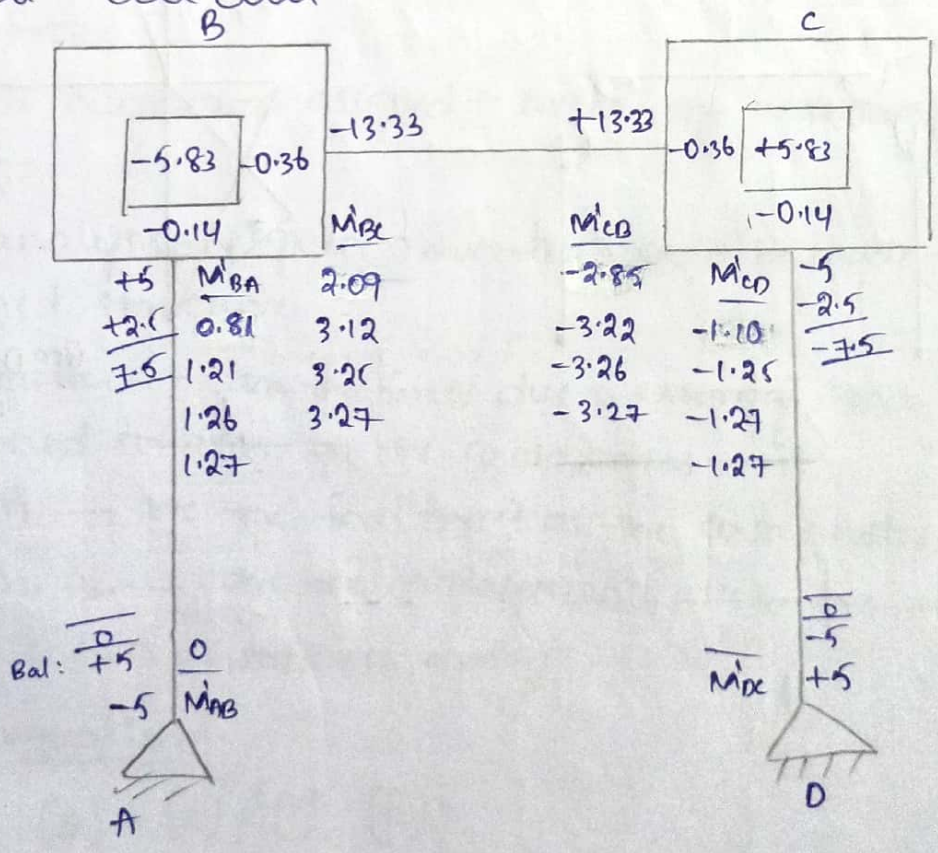
→ compulsory question in MID

Bending Moment and step ① is same as Previous problem

Step ② Rotation factor

Joint	Member	K	ΣK	Rf = $-\frac{1}{2}\cdot\frac{K}{\Sigma K}$
B	BA	$\frac{3}{4}\cdot\frac{I}{L} = 0.1872$	0.6877	-0.14
	BC	$\frac{I}{L} = 0.5I$		-0.36
C	CB	$\frac{I}{L} = 0.5I$	0.6877.	-0.36
	CD	$\frac{3}{4}\frac{I}{L} = 0.187$		-0.14

Step ③ Rotation Contribution



Scanned with CamScanner

$M_{AB} = 0$

$M_{BA} = M_{FBA} + 2\theta_B + \theta_A$

$= +5 + 2(1.27) + 0$

$= 10.54 \text{ kN-m}$

$M_{BC} = M_{FBC} + 2\theta_B + \theta_C$

$= -13.33 + 2(3.27) - 3.27$

$= -10.06 \text{ kN-m}$

$M_{CB} = M_{FCB} + 2\theta_C + \theta_B$

$= +13.33 + 2(-3.27) + 3.27$

$= +10.06 \text{ kN-m}$.

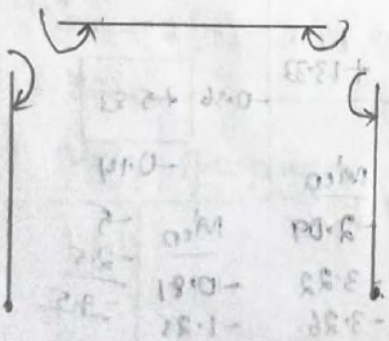
$M_{CD} = M_{FCD} + 2\theta_C + \theta_D$

$= -7.5 + 2(-1.27) + 0$

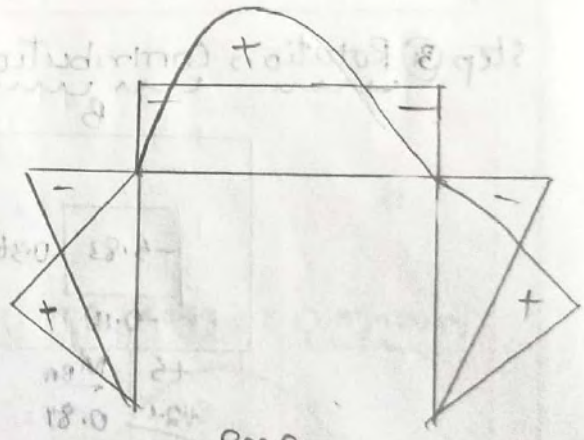
$= -10.04 \text{ kN-m}$

$M_{DC} = 0$

Free Body Diagram



Bending Moment Diagram



BMD

Scanned with CamScanner

14/12/2022.

Unit-V

Plastic Analysis.

* Plastic design (or) analysis is used for designing steel structures based on Collapse Load (or) Ultimate load.
* In this analysis the stress distribution is always of rectangular Profile.
* The moment developed at collapse condition due to collapse load is called plastic moment (Mp).
* The plastic design of steel structures is more economical than the elastic method. because 100% utilization of material takes place.

There are three theorems,

1. Lower Bound Theorem (Static Theorem)
2. Upper Bound Theorem (Kinematic / Mechanism Theorem)
3. Uniqueness Theorem.

Kinematic Theorem:

In this theorem, it is assumed that the external load is more than or equal to collapse load. Hence, external Moment is more than or equal to plastic moment.

The structure must develop minimum number of plastic hinges to form mechanism.

In this Method external work done = Internal energy.

External work done = $W \times \Delta$

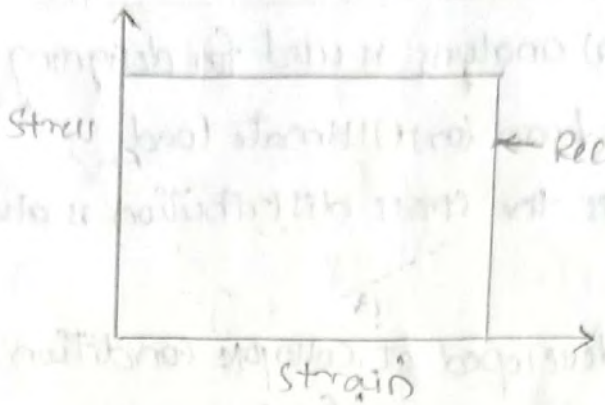
Internal energy = $M_p \theta$.

Assumptions.

* The material is homogenous and isotrophic.
* The plane section remains same even after bending.
* Compressive and Tensile forces are equal.
* Compressive and Tensile stresses are equal.

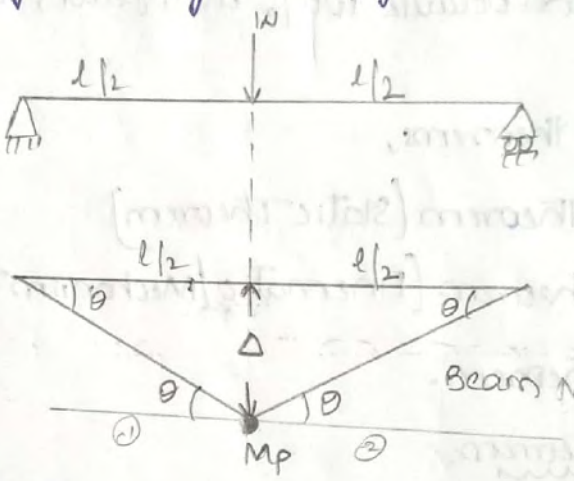
Scanned with CamScanner

Idealized stress-strain Diagram (Plastic stage only)



In the plastic stage the stress-strain diagram is not linear but it is always having rectangular profile.

Case(i)



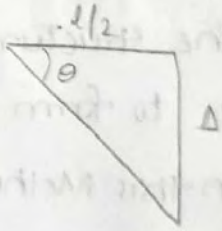
External work done = Internal energy

W_u → ultimate load

$$W_u \cdot \Delta = M_p\theta + M_p\theta$$

$$W_u \cdot \Delta = 2M_p\theta.$$

$$\tan\theta = \frac{\Delta}{l/2}$$



As θ is very small.

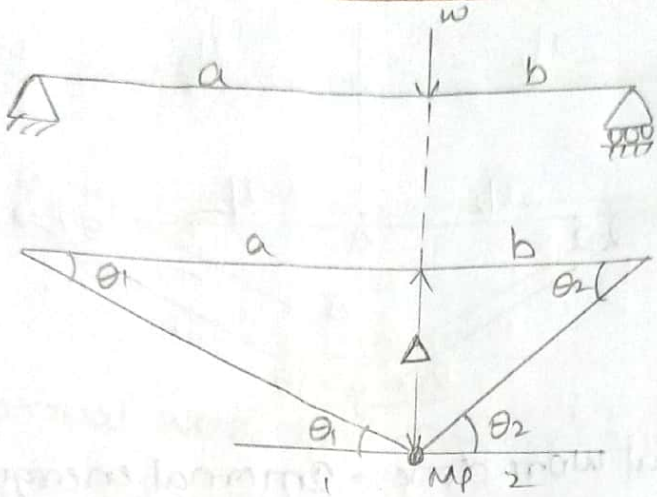
$$\theta = \frac{\Delta}{l/2}$$

$$\Delta = \theta \cdot l/2$$

where, $W_u = W$.

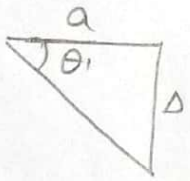
Scanned with CamScanner

Case (ii)



External Work done = Internal energy.

$$W_u \Delta = M_p\theta_1 + M_p\theta_2.$$

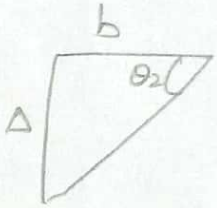


$$\theta_1 = \frac{\Delta}{a}$$

$$\Delta = a\theta_1$$

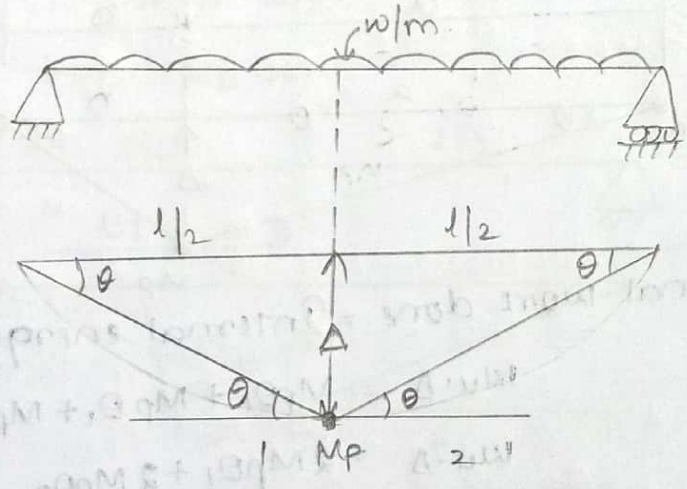
$$\theta_2 = \frac{\Delta}{b}$$

$$\theta_2 = \frac{a\theta_1}{b}$$



Where $W_u = W$.

Case (iii)



External Work done = Internal energy.

$$\frac{1}{2} \cdot W_u \Delta = M_p\theta + M_p\theta$$

$$= 2M_p\theta$$

$$\theta = \frac{\Delta}{l/2}$$

as θ is very small.

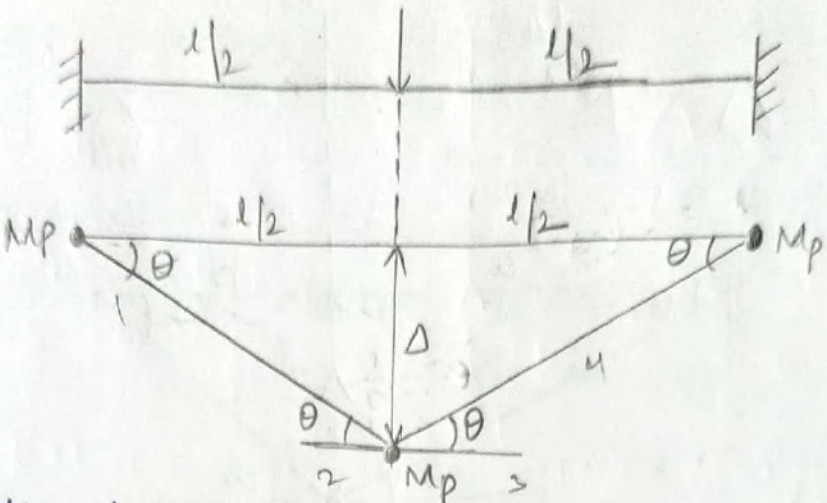
$$\theta = \frac{\Delta}{l/2}$$

$$\Delta = \theta \cdot l/2$$

Where, $W_u = W$KL.

Scanned with CamScanner

Case(iii)



External work done = Internal energy.

$$W_u \cdot \Delta = M_p\theta + M_p\theta + M_p\theta + M_p\theta$$

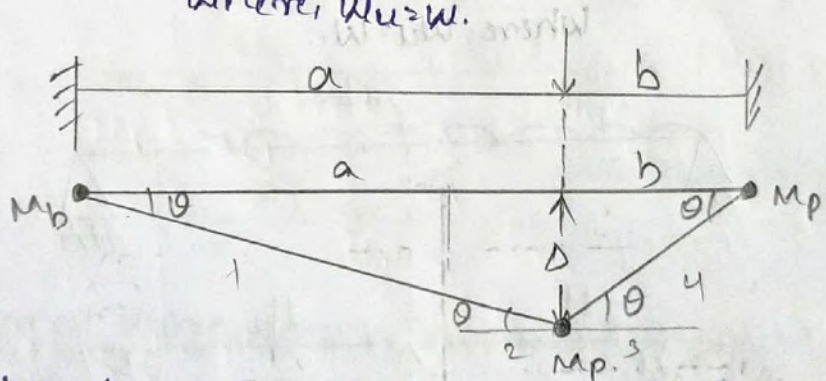
$$W_u \cdot \Delta = 4M_p\theta.$$

$$\theta = \frac{\Delta}{l/2}$$

$$\Delta = \theta \cdot \frac{l}{2}$$

Where, $W_u = W$.

Case(iv)



External work done = Internal energy.

$$W_u \cdot \Delta = M_p\theta_1 + M_p\theta_1 + M_p\theta_2 + M_p\theta_2$$

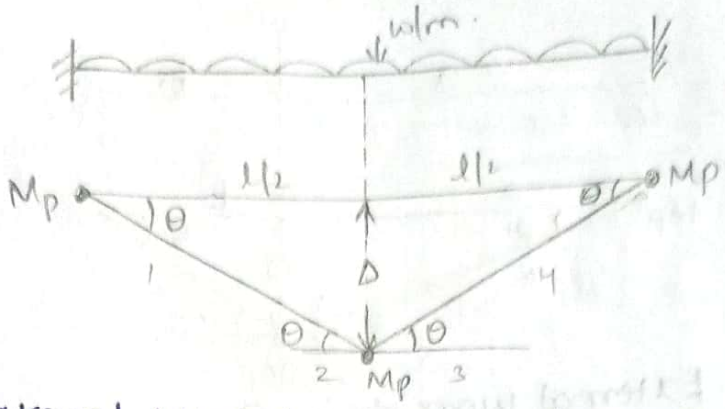
$$W_u \cdot \Delta = 2M_p\theta_1 + 2M_p\theta_2.$$

$$\theta_1 = \frac{\Delta}{a}, \quad \Delta = a\theta_1 \qquad \theta_2 = \frac{\Delta}{b}, \quad \theta_2 = \frac{a\theta_1}{b}$$

Where $W_u = W$.

Scanned with CamScanner

Case (vi)



External work done = Internal energy

$$\frac{1}{2} w_u \cdot \Delta = M_p\theta + M_p\theta + M_p\theta + M_p\theta$$

$$\frac{1}{2} w_u \cdot \Delta = 4M_p\theta$$

$$\theta = \frac{\Delta}{l/2}$$

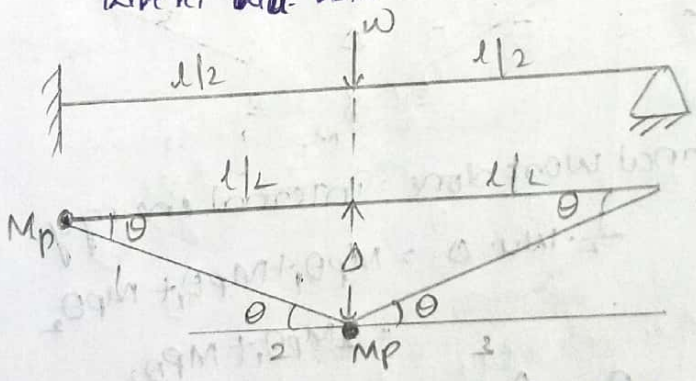
~~as~~ θ is very small.

$$\theta = \frac{\Delta}{l/2}$$

$$\Delta = \theta \cdot l/2.$$

where, $w_u = wxl$

Case (vii)



External work done = Internal energy.

$$w_u \cdot \Delta = M_p\theta + M_p\theta + M_p\theta.$$

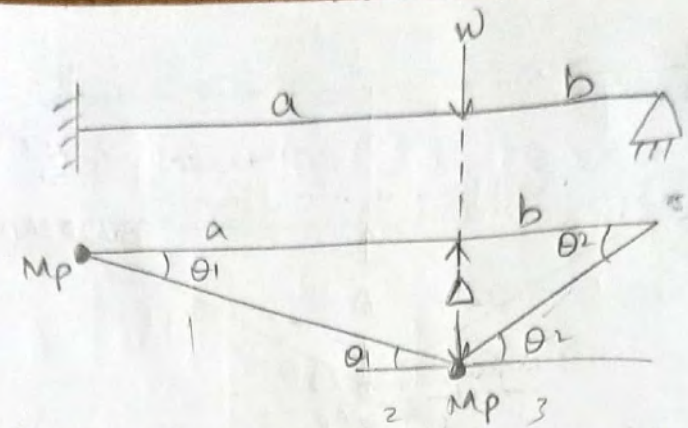
$$\theta = \frac{\Delta}{l/2}$$

$$\Delta = \theta \cdot \frac{l}{2}$$

where, $w_u = w$.

Scanned with CamScanner

Case (viii)



External work done = Internal energy.

$W_u \Delta = M_p\theta_1 + M_p\theta_1 + M_p\theta_2$

$= 2M_p\theta_1 + M_p\theta_2$.

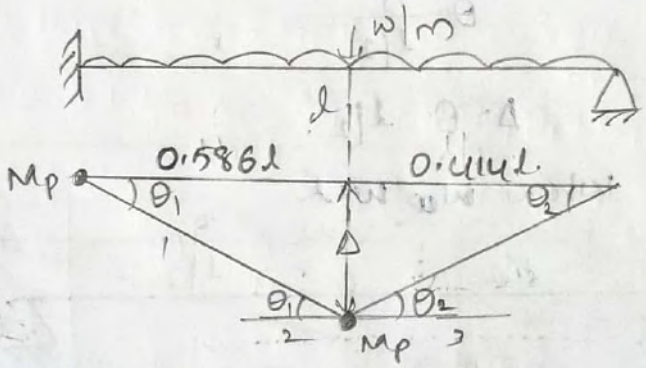
$\theta_1 = \frac{\Delta}{a}$ | $\theta_2 = \frac{\Delta}{b}$

$\Delta = \theta_1 a$ | $\theta_2 = \frac{\theta_1 a}{b}$

where, $W_u = W$.

***

Case (ix)



External work done = Internal energy.

$\frac{1}{2} \cdot W_u \cdot \Delta = M_p\theta_1 + M_p\theta_1 + M_p\theta_2$

$= 2M_p\theta_1 + M_p\theta_2$

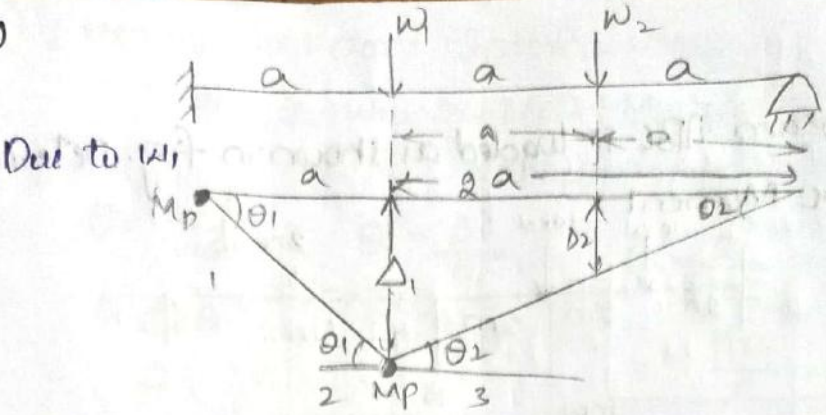
$\theta_1 = \frac{\Delta}{0.586l}$ $\quad \theta_2 = \frac{\Delta}{0.414l}$

$\Delta = \theta_1 \times 0.586l$ $\quad \theta_2 = \frac{0.586l\,\theta_1}{0.414l}$

where, $W_u = wl$.

Scanned with CamScanner

Case (x)



External work done = Internal energy.

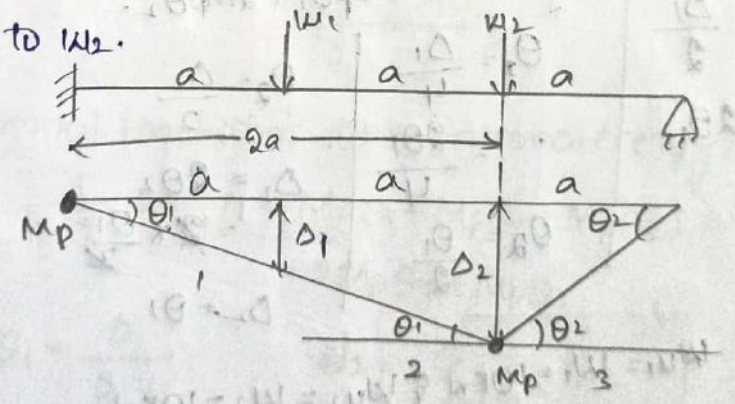
$W_{u1} * \Delta_1 + W_{u2} \Delta_2 = M_p\theta_1 + M_p\theta_1 + M_p\theta_2$

$W_{u1} \cdot \Delta_2 + W_{u2} \Delta_2 = 2M_p\theta_1 + M_p\theta_2$.

$\theta_1 = \frac{\Delta_1}{a}$	$\theta_2 = \frac{\Delta_1}{2a}$	$\theta_2 = \frac{\Delta_2}{a}$
$\Delta_1 = a\theta_1$	$= \frac{a\theta_1}{2a}$	$\Delta_2 = a\theta_2$
	$\theta_2 = \frac{\theta_1}{2}$	$\Delta_2 = \frac{a\theta_1}{2}$

where, $W_{u1} = W_1$ & $W_{u2} = W_2$.

Due to W_2.



External work done = Internal energy.

$W_{u1} \cdot \Delta_1 + W_{u2} \cdot \Delta_2 = M_p\theta_1 + M_p\theta_1 + M_p\theta_2$

$W_{u1} \cdot \Delta + W_{u2} \cdot \Delta_2 = 2M_p\theta_1 + M_p\theta_2$.

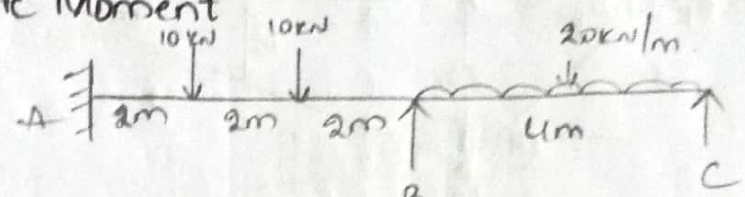
$\theta_1 = \frac{\Delta_1}{a}$	$\theta_1 = \frac{\Delta_2}{2a}$	$\theta_2 = \frac{\Delta_2}{a}$
$\Delta_1 = \theta_1 a$	$\Delta_2 = 2a \cdot \theta_1$	$\theta_2 = \frac{2a\theta_1}{a} = 2\theta_1$

where, $W_{u1} = W_1$ & $W_{u2} = W_2$.

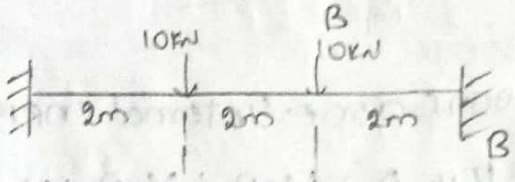
Scanned with CamScanner

Problems

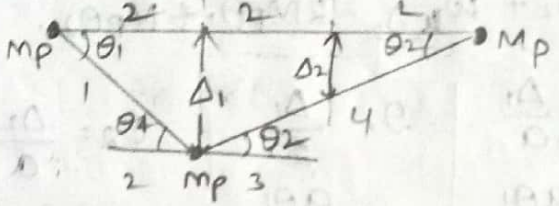
1. A continous beam ABC is loaded as shown in fig. Determine the required plastic Moment



Span AB



First load



External work done = Internal energy.

$W_{u1}\Delta_1 + W_{u2}\Delta_2 = M_p\theta_1 + \theta_p M_p\theta_1 + M_p\theta_2 + M_p\theta_2$

$= 2M_p\theta_1 + 2M_p\theta_2$.

$\theta_1 = \frac{\Delta_1}{2}$

$\Delta_1 = 2\theta_1$

$\theta_2 = \frac{\Delta_1}{4}$

$= \frac{2\theta_1}{4}$

$\theta_2 = \frac{\theta_1}{2}$

$\theta_2 = \frac{\Delta_2}{2}$

$\Delta_2 = 2\theta_2$

$= 2 \times \frac{\theta_1}{2}$

$\Delta_2 = \theta_1$

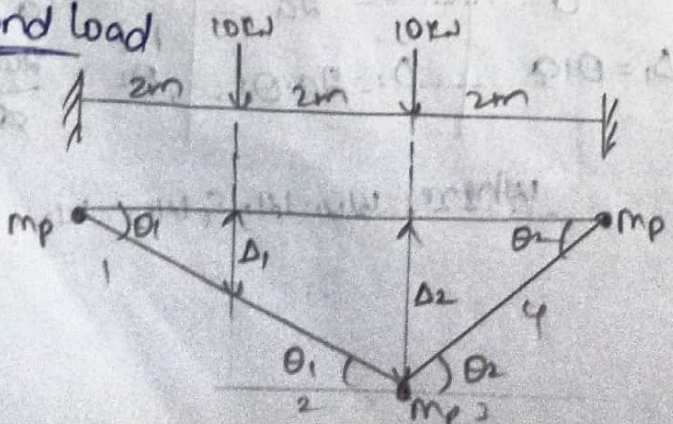
$W_{u1} = W_1 = 10KN$ & $W_{u2} = W_2 = 10KN$.

$(10 \times 2\theta_1) + (10 \times \theta_1) = 2M_p\theta_1 + 2M_p \times \frac{\theta_1}{2}$

$20\theta_1 + 10\theta_1$

$M_p = 10 KNm$

Due to second load



Scanned with CamScanner

External Work done = Internal energy:

$W_{u1} \cdot \Delta_1 + W_{u2} \cdot \Delta_2 = M_p\theta_1 + M_p\theta_1 + M_p\theta_2 + M_p\theta_2$

$= 2M_p\theta_1 + 2M_p\theta_2$.

$\theta_1 = \frac{\Delta_1}{2}$ | $\theta_1 = \frac{\Delta_2}{4}$ | $\theta_2 = \frac{\Delta_2}{2}$

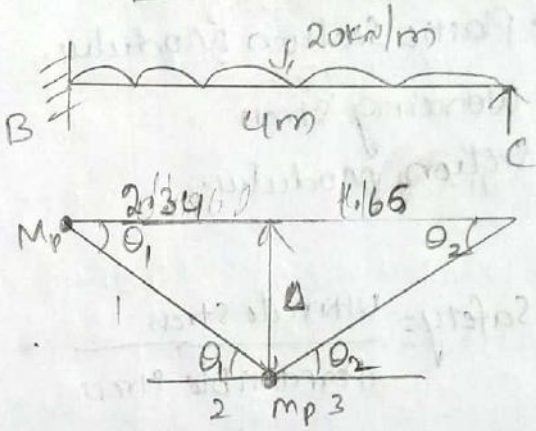
$\Delta_1 = 2\theta$ | $\Delta_2 = 4\theta_1$ | $= \frac{4\theta_1}{2} = 2\theta_1$

$W_{u1} = W_{L1} = 10$ KN & $W_{u2} = W_{L2} = 10$ KN.

$(10 \times 2\theta_1) + (10 \times 4\theta_1) = 2M_p\theta_1 + 2M_p 2\theta_1$

$M_p = 10$ KN-m

Span BC



External ~~load~~ work done = Internal energy.

$\frac{1}{2} \cdot W_u \cdot \Delta = M_p\theta_1 + M_p\theta_1 + M_p\theta_2$

$= 2M_p\theta_1 + M_p\theta_2$.

$\theta_1 = \frac{\Delta}{2.34}$ $\quad$ $\theta_2 = \frac{\Delta}{1.65}$

$\Delta = 2.34\theta_1$ $\quad$ $\theta_2 = \frac{2.34\theta_1}{1.65} = 1.42\theta_1$.

where, $W_u = w \times l$.

$\frac{1}{2} \times 20 \times 4 \times 2.34\theta_1 = 2M_p\theta_1 + M_p 1.42\theta_1$

$M_p = 27.37$ KN-m.

Take the highest "Mp" value from the above,

$M_p = 27.37$ KN-m

Scanned with CamScanner

2. Load factor (λ):

It is defined as the ratio of plastic moment to the working moment.

$$\therefore \lambda = \frac{M_p}{M_w}$$

$M_p = f_y \times Z_p$

$M_w = \sigma \times Z$ ($\frac{M}{I} = \frac{\sigma}{y}$, $M = \sigma \times \frac{I}{y}$, $M = \sigma \times Z$)

$$\lambda = \frac{f_y \times Z_p}{\sigma \times Z}$$

Where, $f_y \rightarrow$ Yield stress

$Z_p \rightarrow$ Plastic Section Modulus.

$\sigma \rightarrow$ Working stress

$Z \rightarrow$ Section Modulus.

We known,

$$\text{Factor of Safety} = \frac{\text{Ultimate stress}}{\text{Permissible stress}} = \frac{f_y}{\sigma}$$

The ratio $\frac{Z_p}{Z}$ is called shape factor (S).

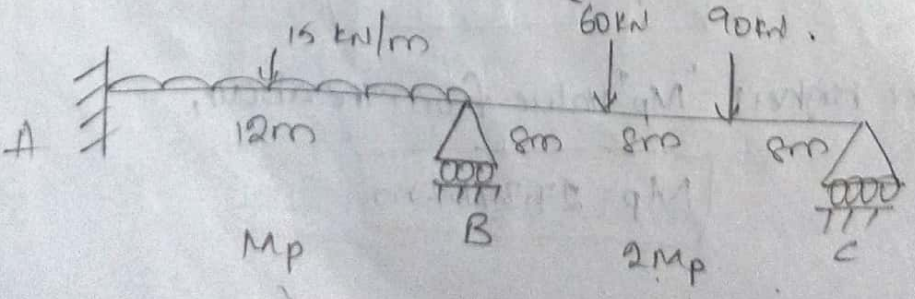
* The ratio b/w Plastic Section Modulus to the Section Modulus is known as "Shape Factor".

$$\therefore S = \frac{Z_p}{Z}$$

$$\therefore \lambda = FOS \times S$$ factor of safety x Shape factor

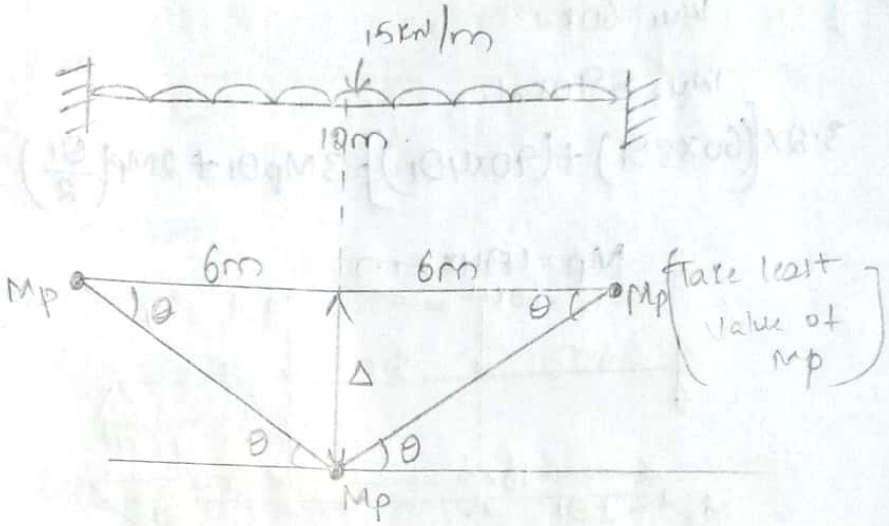
***

2. A Continous beam ABC is loaded as shown in fig. Determine the required M_p, if the load factor is 3.2



Scanned with CamScanner

Span AB



External work done = Internal energy.

$$\lambda \cdot \frac{1}{2} \cdot W_u \cdot \Delta = M_p\theta + M_p\theta + M_p\theta + M_p\theta$$

$$\lambda \cdot \frac{1}{2} W_u \cdot \Delta = 4M_p\theta$$

$$\theta = \frac{\Delta}{6}$$

$$\Delta = 6\theta$$

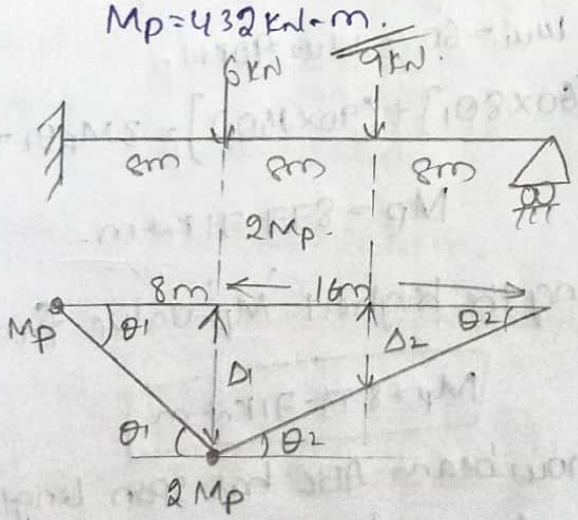
where $W_u = \frac{wxl}{}$ ~~15 KN/m~~ $= 15 \times 12 = 180$ KN/m.

$$\lambda \cdot \frac{1}{2} \times 180 \cdot 6\theta = 4M_p\theta$$

$$3.2 \times \frac{1}{2} \times 180 \times 6\theta = 4M_p\theta$$

$$M_p = 432 \text{ KN-m}.$$

Span CB



External work done = Internal energy.

$$\lambda \cdot [W_{u1} \Delta_1 + W_{u2} \cdot \Delta_2] = M_p\theta_1 + 2M_p\theta_1 + 2M_p\theta_2$$

$$3.2 \times [W_{u1} \Delta_1 + W_{u2} \cdot \Delta_2] = 3M_p\theta_1 + 2M_p\theta_2.$$

$\theta_1 = \frac{\Delta_1}{8}$	$\theta_2 = \frac{\Delta_1}{16}$	$\theta_2 = \frac{\Delta_2}{8}$
$\Delta_1 = 8\theta_1$	$\theta_2 = \frac{8\theta_1}{16} = \frac{\theta_1}{2}$	$\Delta_2 = 8\theta_2$
		$\Delta_2 = 4\theta_1$

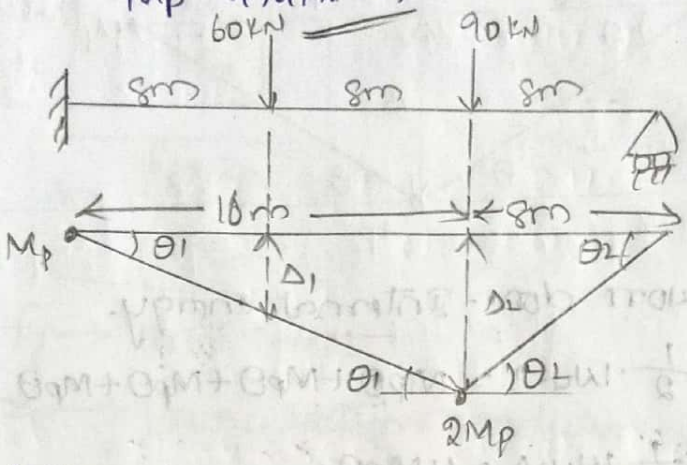
Scanned with CamScanner

$W_{u1} = 60\ KN$

$W_{u2} = 90\ KN.$

$$3.2\left[(60\times 8\theta_1)+(90\times 4\theta_1)\right] = 3M_p\theta_1 + 2M_p\left(\frac{\theta_1}{2}\right)$$

$M_p = 444\ KN\text{-}m$



External work done = Internal energy.

$$3.\left[W_{u1}\Delta_1 + W_{u2}\Delta_2\right] = M_p\theta_1 + 2M_p\theta_1 + 2M_p\theta_2$$

$$3.2\left[W_{u1}\Delta_1 + W_{u2}\Delta_2\right] = 3M_p\theta_1 + 2M_p\theta_2.$$

$\theta_1 = \frac{\Delta_1}{8}$ | $\frac{\Delta_1}{8} = \frac{\Delta_2}{16}$ | $\theta_2 = \frac{\Delta_2}{8}$

$\Delta_1 = 8\theta_1$ | $\Delta_2 = 16\theta_1$ | $\theta_2 = \frac{16\theta_1}{8} = 2\theta_1.$

$W_{u1} = 60$, $W_{u2} = 90\ KN.$

$$3.2\left[(60\times 8\theta_1)+(90\times 16\theta_1)\right] = 3M_p\theta_1 + 2M_p(2\theta_1)$$

$$M_p = 877.71\ KN\text{-}m.$$

Taking the highest 'Mp' value from above

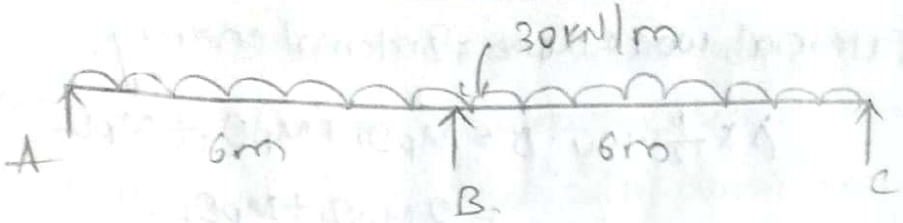
MID

$$M_p = 877.71\ KN\text{-}m.$$

***

A two span continous beam ABC has span lengths AB = 6m & BC = 6m and carries a UDL of 30 KN/m completely covering the spans AB & BC. A and C are simply supported. The load factor is 1.80 and shape factor is 1.15 for the given I-section. Find the section Modulus needed. Assume Yield stress of the material as 250 MPa.

Scanned with CamScanner



$\lambda = 1.80$

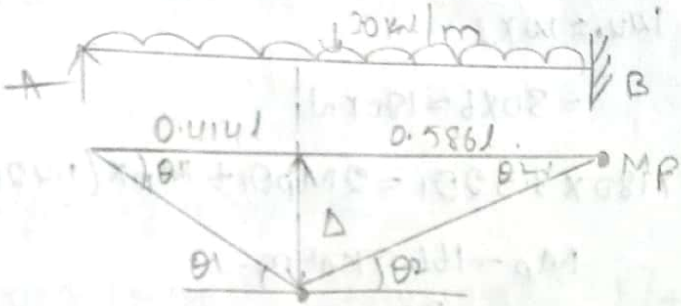
$S = 1.15$

$f_y = 250 MPa$

$= 250 N/mm^2$

Section Modulus, Z = ?

Span AB



External work done = Internal energy.

$\lambda \cdot \frac{1}{2} \cdot wu \cdot \Delta = M_p\theta_1 + M_p\theta_2 + M_p\theta_2$

$= M_p\theta_1 + 2M_p\theta_2$

$\theta_1 = \frac{\Delta}{0.414l}$

$\Delta = 0.414 \times 6 \times \theta_1$

$\Delta_1 = 2.484\theta_1$

$\theta_2 = \frac{\Delta_1}{0.586l}$

$= \frac{2.48\theta_1}{0.586 \times 6}$

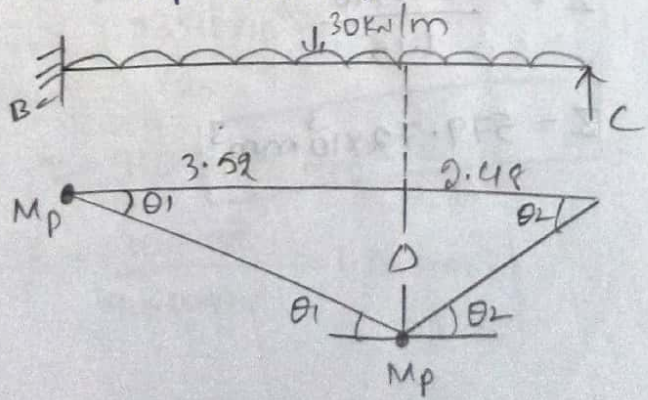
$\theta_2 = 0.74\theta_1$

~~Load 30Kn~~ $wu \times l = 30 \times 6 = 180 KN.m$.

$1.8 \times \frac{1}{2} \times 180 \times 2.48\theta_1 = M_p\theta_1 + 2M_p(0.74\theta_1)$

$M_p = 162 KNm$

Span CB



Scanned with CamScanner

External work done = Internal energy.

$\lambda \times \frac{1}{2} \cdot w_u \cdot \Delta = M_p\theta_1 + M_p\theta_1 + M_p\theta_2$

$= 2M_p\theta_1 + M_p\theta_2$.

$\theta_1 = \frac{\Delta}{3.52}$ $\quad \theta_2 = \frac{\Delta}{0.414 \times 2}$

$\Delta = 3.52\theta_1$ $\quad = \frac{3.52\theta_1}{2.48}$

$\theta_2 = 1.42\theta_1$.

$1.8 \times \frac{1}{2} \times 180 \times 3.52\theta_1 = 2M_p\theta_1 + M_p \times (1.42\theta_1)$

$M_p = 166.7$ kN-m.

Taking the highest 'mp' value of above

$M_p = 166.67$ kN-m $= 166.67 \times 10^3 \times 10^3$ N-mm.

Section Modulus, $S = \frac{Z_p}{Z}$

$M_p = f_y \times Z_p$

~~$166\,166.710^6 = 250 \times Z_p$~~

$166.7 \times 10^6 = 250 \times Z_p$

$Z_p = 666.68 \times 10^3$ mm^3

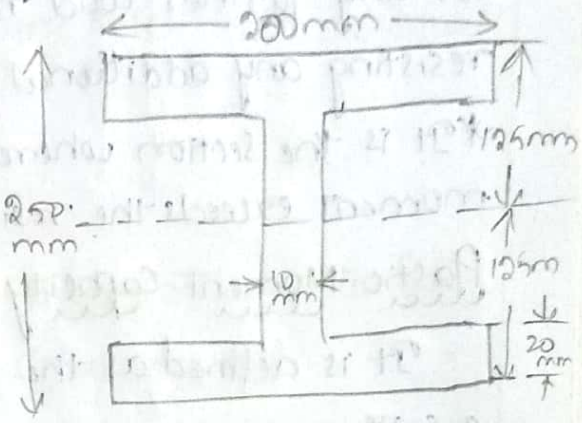
$S = \frac{Z_p}{Z}$

$Z = \frac{666.68 \times 10^3}{1.15}$

$Z = 579.72 \times 10^3$ mm^3

Scanned with CamScanner

A Mild steel I-Section 200mm wide and 250mm deep has a beam flange thickness of 20mm and a web thickness of 10mm. Calculate the shape factor and find fully plastic moment if $\sigma_y = 252 N/mm^2$



(i) Equal area axis (Plastic N.A.)

Since the given section is symmetrical about x-x axis, the equal area axis will be at half of the overall depths. i.e. 125mm.

(ii) Plastic Section Modulus (Z_p)

$$Z_p = a_1y_1 + a_2y_2 + a_3y_3 + a_4y_4$$

$$= \left[(200\times20)\left(125-\frac{20}{2}\right)\right] + \left[(10\times105)\left(\frac{105}{2}\right)\right] + \left[(10\times105)\left(\frac{105}{2}\right)\right] + \left[(200\times20)\left(125-\frac{20}{2}\right)\right]$$

$$Z_p = 10.3\times10^5 mm^3$$

From plastic Neutral Axis.

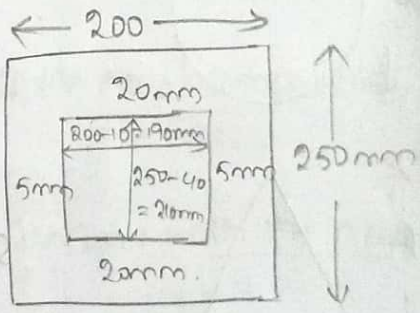
(iii) Calculation of Z

$$Z = \frac{I_{xx}}{y_{max}}$$

$$I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$= \frac{200\times250^3}{12} - \frac{190\times210^3}{12}$$

$$= 113.78\times10^6 mm^4$$



$$y_{max} = \frac{D}{2} = \frac{250}{2} = 125mm.$$

$$Z = \frac{I_{xx}}{y_{max}} = \frac{113.78\times10^6}{125}$$

$$Z = 910.24\times10^3 mm^3$$

$$S = \frac{Z_p}{Z} = \frac{10.3\times10^5}{910.24\times10^3} = 1.131 mm^3$$

(iv) fully plastic moment

$$M_p = Z_p \times f_y$$

$$= 10.3\times10^5 \times 252$$

$$M_p = 259.56\times10^6 N\text{-}mm.$$

Scanned with CamScanner

Plastic ~~Limit~~ hinge:

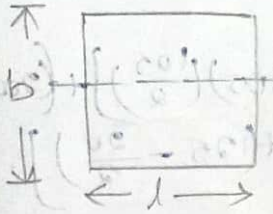
It is the section at which all the fibres yield, because for any further load rotation takes place at the section without resisting any additional moment

*It is the section where rotation takes place when the applying moment exceeds the resisting moment

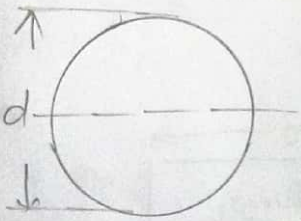
Plastic Moment Capacity (M_P)

It is defined as the moment which forms plastic hinge at a section where all the fibres are subjected to yield.

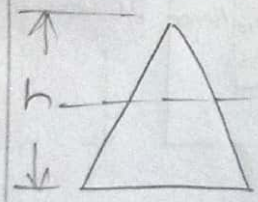
Shape of Factors



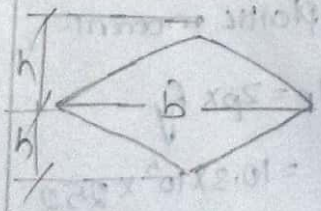
S = 1.5



S = 1.698



S = 2.343



S = 2.0

Scanned with CamScanner

Lower Bound Theorem (2nd Static Theorem)

It is assumed that the external load less than or equal to collapsable load. i.e., the external moment is less than or equal to Plastic Moment.

Uniqueness Theorem

The static and kinematic theorem's can be combined to form a theorem which gives unique value of collapse load.

Collapse load (or) Ultimate load

The load at which sufficient number of plastic hinges are formed in a structure

Neutral Axis

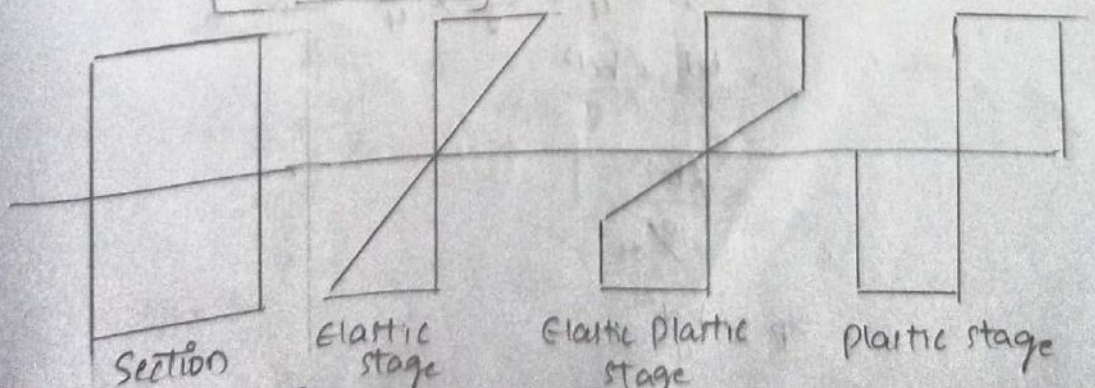
The line of intersection of the neutral layer with the c/s is known as "Neutral Axis". It is the axis where the layers will neither extend nor contrast.

Plastic Neutral Axis (Equal Area Axis)

It is the location of axis which results in equal compressive and tensile forces at fully plastic stage.

* for symmetrical sections equal Area axis coincides with the neutral axis.
* At fully plastic stage area above and below equal area axis is same.

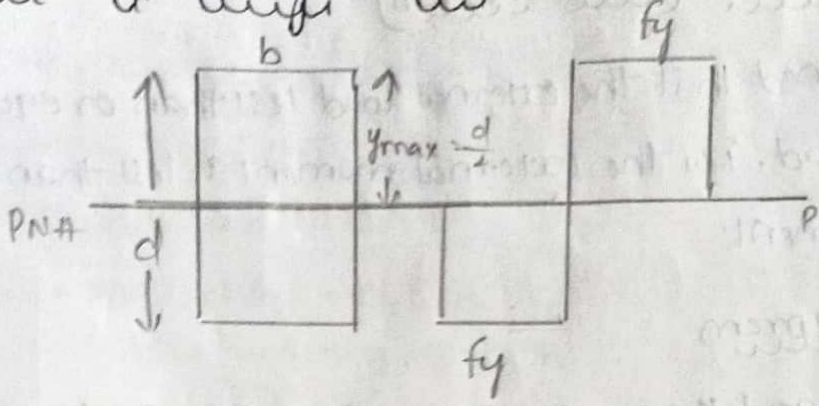
$A_1 = A_2$
(Top) (Bottom)



Stress Distribution Diagram.

Scanned with CamScanner

Shape Factor of Rectangular Section



$$S = \frac{Z_p}{Z}$$

$$Z = \frac{I}{y_{max}}$$

$$I = \frac{bd^3}{12}$$

$$y_{max} = \frac{d}{2}$$

$$Z = \frac{\frac{bd^3}{12}}{\frac{d}{2}}$$

$$Z = \frac{bd^2}{6}$$

$$Z_p = A_1 y_1 + A_2 y_2$$

We know, $A_1 = A_2 = \frac{A}{2}$

$$A_1 = b \times \frac{d}{2} \qquad A_2 = b \times \frac{d}{2}$$

$$y_1 = \frac{1}{2} \times \frac{d}{2} \qquad y_2 = \frac{1}{2} \times \frac{d}{2}$$

(from plastic Neutral Axis) $= \frac{d}{4}$ (from PNA) $= \frac{d}{4}$

$$Z_p = A_1 y_1 + A_2 y_2$$

$$= \frac{bd}{2} \times \frac{d}{4} + \frac{bd}{2} \times \frac{d}{4}$$

$$= \frac{bd^2}{8} + \frac{bd^2}{8}$$

$$= \frac{2bd^2}{8/4}$$

$$Z_p = \frac{bd^2}{4}$$

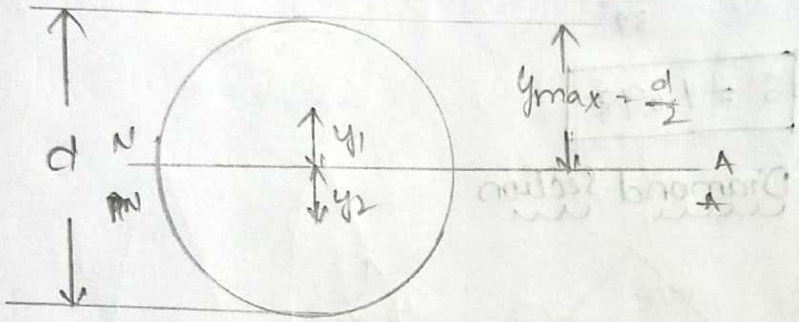
Scanned with CamScanner

$$\therefore S = \frac{Zp}{Z} = \frac{\frac{bd^2}{4}}{\frac{bd^2}{6}}$$

$$= \frac{bd^2}{4} \times \frac{6}{bd^2}$$

$$\boxed{\therefore S = 1.5}$$

Shape factor of circular section



$$S = \frac{Zp}{Z}$$

$$Z = \frac{I}{y_{max}}$$

$$I = \frac{\pi d^4}{64}$$

$$y_{max} = \frac{d}{2}.$$

$$Z = \frac{\frac{\pi d^4}{64}}{\frac{d}{2}}$$

$$Z = \frac{\pi d^3}{32}$$

$$Zp = A_1 y_1 + A_2 y_2$$

We know, $A_1 = A_2 = \frac{A}{2}$.

$$A = \frac{\pi d^2}{4}$$

$$\therefore A_1 = \frac{1}{2} \times \frac{\pi d^2}{4} \qquad A_2 = \frac{1}{2} \times \frac{\pi d^2}{4}$$

$$= \frac{\pi d^2}{8} \qquad = \frac{\pi d^2}{8}$$

$r = \frac{D}{2}$

$$y_1 = \frac{4r}{3\pi} \qquad y_2 = \frac{4r}{3\pi}$$

$$= \frac{4D}{2 \times 3\pi} = \frac{2D}{3\pi} \qquad = \frac{4D}{2 \times 3\pi} = \frac{2D}{3\pi}$$

Scanned with CamScanner

$$Z_p = A_1 y_1 + A_2 y_2$$

$$= \frac{\pi d^2}{8} \times \frac{2d}{3\pi} + \frac{\pi d^2}{8} + \frac{2d}{3\pi}$$

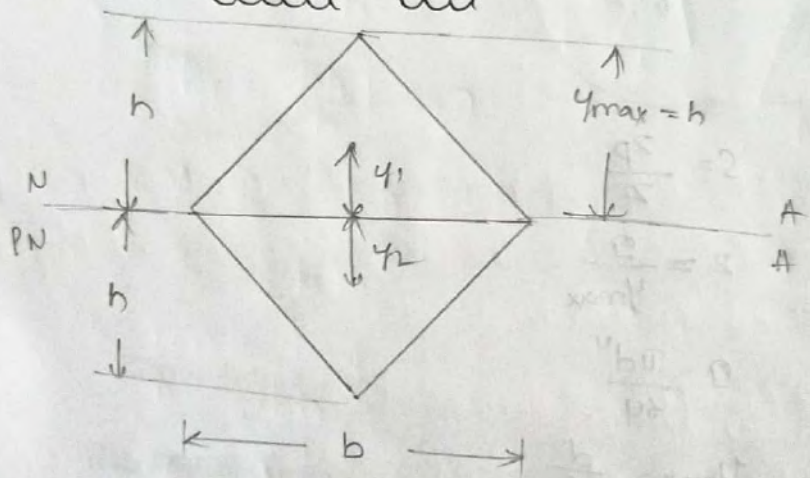
$$= 2 \times \frac{\pi d^2}{8} \times \frac{2d}{3\pi}$$

$$Z_p = \frac{d^3}{6}$$

$$S = \frac{Z_p}{Z} = \frac{\frac{d^3}{6}}{\frac{\pi d^4}{32}}$$

$$\boxed{S = 1.698}$$

Shape factor of Diamond Section



$$S = \frac{Z_p}{Z}$$

$$Z = \frac{I}{y_{max}}$$

$$I = 2 \times \frac{bh^3}{12}$$

M.I of trainangle about the base $= \frac{bh^3}{12}$

$$y_{max} = h.$$

$$Z = \frac{2 \times \frac{bh^3}{12}}{h}$$

$$Z = \frac{bh^2}{6}$$

Scanned with CamScanner

$$Z_p = A_1 y_1 + A_2 y_2$$

$$A_1 = A_2 = \frac{A}{2}$$

$$A_1 = \frac{1}{2} \times b \times h \qquad A_2 = \frac{1}{2} \times b \times h$$

$$y_1 = \frac{h}{3} \qquad y_2 = \frac{h}{3}$$

$$\therefore Z_p = \left(\frac{1}{2} bh \times \frac{h}{3}\right) + \left(\frac{1}{2} bh \times \frac{h}{3}\right)$$

$$= \frac{1}{2} bh \left(\frac{h}{3} + \frac{h}{3}\right)$$

$$= \frac{1}{2} bh \times \frac{2h}{3}$$

$$Z_p = \frac{bh^2}{3}$$

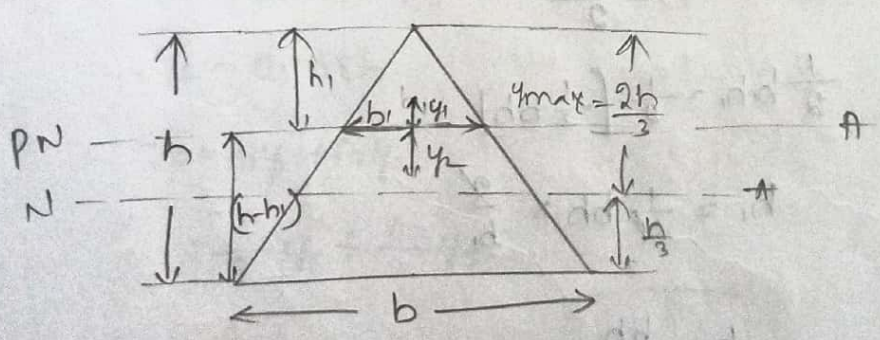
$$S = \frac{Z_p}{Z} = \frac{\frac{bh^2}{3}}{\frac{bh^2}{6}}$$

$$= \frac{bh^2}{3} \times \frac{6}{bh^2}$$

$$\boxed{S = 2}$$

***** MUD

Shape factor of Triangular Section



$$S_p = \frac{Z_p}{Z}$$

$$Z = \frac{I}{y_{max}}$$

$$I = I_{N.A} = \frac{bh^3}{36}$$

$$y_{max} = \frac{2h}{3}$$

Scanned with CamScanner

$$Z = \frac{I}{y_{max}}$$

$$= \frac{\frac{bh^3}{36}}{\frac{2h}{3}} = \frac{bh^3}{36} \times \frac{3}{2h}$$

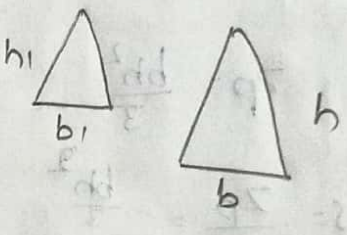
$$Z = \frac{bh^2}{24}$$

$$Z_p = A_1 y_1 + A_2 y_2$$

$$A_1 = A_2 = \frac{A}{2}.$$

Using Similar triangles,

$$\frac{b_1}{b} = \frac{h_1}{h}$$



$$\boxed{b_1 = \frac{bh_1}{h}} \rightarrow (1)$$

A_1 = Area of small triangle.

$$= \frac{1}{2} b_1 h_1$$

We know,

$$A = \frac{1}{2} bh$$

$$A_1 = \frac{A}{2}.$$

$$\frac{1}{2} b_1 h_1 = \frac{\frac{1}{2} bh}{2}$$

$$\frac{1}{2} b_1 h_1 = \frac{1}{2} \cdot \left[\frac{1}{2} bh\right]$$

$$h_1 = \frac{1}{4} bh \times \frac{2}{b_1}$$

$$\boxed{h_1 = \frac{bh}{2b_1}}$$

$$\boxed{b_1 = \frac{bh}{2h_1}} \rightarrow (2)$$

Scanned with CamScanner

Equate eq① & eq②

$$\frac{bh_1}{h} = \frac{bh}{2h_1}$$

$$2h_1^2 = h^2$$

$$h_1^2 = \frac{h^2}{2}$$

$$\boxed{h_1 = \frac{h}{\sqrt{2}}}$$

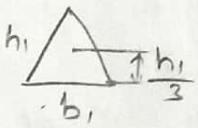
Sq+ Substitute 'h_1' value in eq①

$$b_1 = \frac{b}{h} \times \frac{h}{\sqrt{2}}$$

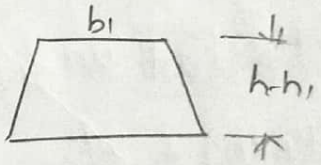
$$\boxed{b_1 = \frac{b}{\sqrt{2}}}$$

$$y_1 = \frac{h_1}{3}$$

$$y_1 = \frac{h}{3\sqrt{2}}$$



$$y_2 = \frac{a+2b}{a+b}\left(\frac{h}{3}\right)$$



$$= \left(\frac{b_1' + 2b}{b_1' + b}\right)\left(\frac{h-h_1}{3}\right)$$

$$y_2 = 0.158h$$

$$Z_p = A_1 y_1 + A_2 y_2$$

$$= \frac{A}{2} y_1 + \frac{A_2}{2} y_2$$

$$= \frac{A}{2}[y_1 + y_2]$$

$$= \frac{1}{2} \times \frac{1}{2} \times bh\left[\frac{h}{3\sqrt{2}} + 0.1548h\right]$$

$$Z_p = 0.097bh^2$$

Scanned with CamScanner

$$S = \frac{2p}{z}$$

$$= \frac{0.097h^2b}{\frac{bh^2}{24}}$$

$$\boxed{S = 2.343}$$

Scanned with CamScanner