ANNAMACHARYA
INSTITUTE OF TECHNOLOGY AND SCIENCES
(AUTONOMOUS)

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Venkatapuram Village, Renigunta Mandal, Tirupati, Andhra Pradesh-517520.

Department of Computer Science and Engineering

Academic Year 2023-24

II. B.Tech I Semster

Discrete Mathematical Structures

(Common to CSE,CIC,AIDS,AIML,CSE(DS))
(20ABS9914)

Prepared By

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Course Code	Discrete Mathematical Structures (common to CSE,CIC,AIDS,AIML,CSE(DS))	L	T	P	C
20ABS9914		3	0	0	3
Pre-requisite	**Basic Mathematics**	**Semester**			**II-I**

Course Objectives:

Introduce the concepts of mathematical logic and gain knowledge in sets, relations and functions and Solve problems using counting techniques and combinatorics and to introduce generating functions and recurrence relations. Use Graph Theory for solving real world problems.

Course Outcomes (CO):

CO1: Make use of mathematical logic to solve problems
CO2: Analyse the concepts and perform the operations related to sets, relations and functions.
CO3: Identify basic counting techniques to solve combinatorial problems.
CO4: evaluate solutions by using recurrence relations
CO5: utilize Graph Theory in solving computer science problems

UNIT – I	**Mathematical Logic**	9 Hrs

Introduction, Statements and Notation, Connectives, Well-formed formulas, Tautology, Duality law, Equivalence, Implication, Normal Forms, Functionally complete set of connectives, Inference Theory of Statement Calculus, Predicate Calculus, Inference theory of Predicate Calculus.

UNIT – II	**Set theory**	9 Hrs

Basic Concepts of Set Theory, Relations and Ordering, The Principle of Inclusion- Exclusion, Pigeon hole principle and its application, Functions composition of functions, Inverse Functions, Recursive Functions, Lattices and its properties. Algebraic structures: Algebraic systems-Examples and General Properties, Semi groups and Monoids, groups, sub groups, homomorphism, Isomorphism.

UNIT – III	**Elementary Combinatorics**	9 Hrs

Basics of Counting, Combinations and Permutations, Enumeration of Combinations and Permutations, Enumerating Combinations and Permutations with Repetitions, Enumerating Permutations with Constrained Repetitions, Binomial Coefficients, The Binomial and Multinomial Theorems.

UNIT – IV	**Recurrence Relations**	9 Hrs

Generating Functions of Sequences, Calculating Coefficients of Generating Functions,Recurrence relations, Solving Recurrence Relations by Substitution and Generating functions, The Method of Characteristic roots, Solutions of Inhomogeneous Recurrence Relations.

UNIT – V	**Graphs**	9 Hrs

Basic Concepts, Isomorphism and Sub-graphs, Trees and their Properties, Spanning Trees,Directed Trees, Binary Trees, Planar Graphs, Euler's Formula, Multigraphs and Euler Circuits, Hamiltonian Graphs, Chromatic Numbers, The Four Color Problem

Textbooks:

1. Joe L. Mott, Abraham Kandel and Theodore P. Baker, Discrete Mathematics for Computer Scientists & Mathematicians, 2nd Edition, Pearson Education.
2. J.P. Tremblay and R. Manohar, Discrete Mathematical Structures with Applications to Computer Science, Tata McGraw Hill, 2002.

Reference Books:

1. Kenneth H. Rosen, Discrete Mathematics and its Applications with Combinatorics and Graph Theory, 7th Edition, McGraw Hill Education (India) Private Limited.
2. Graph Theory with Applications to Engineering and Computer Science by Narsingh Deo.

Online Learning Resources:

http://www.cs.yale.edu/homes/aspnes/classes/202/notes.pdf

Mapping of course outcomes with program outcomes

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2
CO1	3													
CO2	3													
CO3	3													
CO4	3													
CO5	3													

(Levels of Correlation, viz., 1-Low, 2-Moderate, 3 High)

preposition (or) sentence (or) statement:

A preposition is a declarative sentence which is in the given contest can be set to be either 'true' or 'false' but not both.

(or)

Every statement is a sentence but all sentence are not sentence.

Negation:

A statement obtained by inserting the word 'NOT' at an appropriate place in a given statement is called the negation. It is denoted by ($\sim$) or ($\neg$). The negation of the statement 'p' is denoted by $\sim p$ or $\neg p$.

Eg: p: 2 is an even number

$\sim p$: 2 is not an even number.

p	$\sim p$
T	F
F	T

Conjunction:

A compound statements obtained by combaining to given preposition (statements) by inserting the word 'AND' it is denoted by the symbol '$\wedge$' and read as 'AND'. The conjunction of two statements 'p' and 'q' is denoted by '$p \wedge q$'.

Eg: p: Ramu went to school

q: Raghu went to school

P∧Q : Ramu and Raghu went to school.

Truth table :

If p and Q $(P \wedge Q)$ is true 'T' when p is true and Q is true otherwise false.

$2^n = 2^2 = 4$

$2^n = 2^3 = 8$

P	Q	P∧Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	R
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Disjunction :

A compound statement obtained by combining two given statements by inserting the 'OR' in between them is called the Disjunction it is denoted by the symbol 'v' and read as 'OR'. The statements are p and Q is denoted by 'PvQ'.

Eg : P : I will buy a computer.

Q : I will buy a car

PvQ : I will buy a computer or a car

Truth table :

If 'PvQ' is false when p is false and Q is false, otherwise true

p	q	
T	T	T
T	F	T
F	T	T
F	F	F

Conditional or Implication:

A compound statements obtained by combining two given statements by using the word "IF" and "THEN" at an appropriate place is called conditional or implication. It is denoted by the symbol $\Rightarrow$ and reads as IF and THEN or implies. The condition of two statements "p" and "q" is denoted by $p \Rightarrow q$.

Eg: p: Ramya works hard.

q: Ramya will pass the exam.

$p \Rightarrow q$: IF Ramya works hard THEN she will pass the exam.

Truth table:

In the conditional $p \Rightarrow q$ is false when p is true and q is false, otherwise true.

The conditional $q \Rightarrow p$ is false when p is false and q is true, otherwise true.

p	q	$p \Rightarrow q$	$q \Rightarrow p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Bi implication: A compound statement obtained by combining two given statement by inserting a word if and only if. It is denoted by the symbol <=> ⇉ and read as double implies.

Eg: p: Two lines are parallel

Q: They have same slope

p<=>Q: Two lines are parallel if and only if they have same slope

Truth table:

If p<=>Q is 'true' when p is true and Q is true or p is false and Q is false otherwise, false.

p	Q	p<=>Q
T	T	T
T	F	F
F	T	F
F	F	T

Exclusive Disjunction: A compound statement obtained by combining two given statements by inserting a word OR in the exclusive sense. we required that the compou statement pVQ to be True only when p is true OR Q is true but not both. the exclusive OR is denoted by the symbol $\underline{V}$.

Truth table:

If (p $\underline{V}$ Q) p exclusive OR is true when p is true or Q is true but not both true (or) p is false or Q is

False, otherwise F true False.

P	Q	P ⊻ Q
T	T	F
T	F	T
F	T	T
F	F	F

If p ⊻ R is true when p is

construct the truth table for $(P \wedge Q) \Rightarrow (P \vee Q)$

P	Q	$P \wedge Q$	$P \vee Q$	$(P \wedge Q) \Rightarrow (P \vee Q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

$(P \Rightarrow Q) \vee (P \Leftrightarrow \sim Q)$

P	Q	$\sim Q$	$P \Rightarrow Q$	$P \Leftrightarrow \sim Q$	$(P \Rightarrow Q) \vee (P \Leftrightarrow$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	T	T	T
F	F	T	T	F	T

$(P \Rightarrow Q) \vee [\sim (P \Leftrightarrow \sim Q)]$

P	Q	$\sim Q$	$P \Rightarrow Q$	$P \Leftrightarrow \sim Q$	$\sim (P \Leftrightarrow \sim Q)$	$(P \Rightarrow Q) \vee [\sim (P \Leftrightarrow \sim$
T	T	F	T	F	T	T
T	F	T	F	T	F	F
F	T	F	T	T	F	T
F	[illegible]	T	T	F	T	T

$(p \vee \sim Q) \Leftrightarrow (\sim p \wedge \sim Q)$

p	Q	$\sim p$	$\sim Q$	$(p \vee \sim Q)$	$(\sim p \wedge \sim Q)$	$(p \vee \sim Q) \Leftrightarrow (\sim p \wedge \sim Q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	F	F	T
F	F	T	T	T	T	T

$\sim p \Rightarrow Q \quad (p \veebar Q) \wedge (p \Rightarrow Q)$

p	Q	$p \veebar Q$	$p \Rightarrow Q$	$(p \veebar Q) \wedge (p \Rightarrow Q)$
T	T	T	F	F
T	F	T	F	F
F	T	T	T	T
F	F	F	F	F

$(p \Rightarrow \sim Q) \Leftrightarrow (\sim p \veebar Q)$

p	Q	$\sim p$	$\sim Q$	$(p \Rightarrow \sim Q)$	$(\sim p \veebar Q)$	$(p \Rightarrow \sim Q) \Leftrightarrow (\sim p \veebar Q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

$(p \veebar Q) \wedge (p \Rightarrow Q)$

p	Q	$p \veebar Q$	$p \Rightarrow Q$	$(p \veebar Q) \wedge (p \Rightarrow Q)$
T	T	F	T	
T	F	T	F	
F	T	T	T	
[illegible]			T	

$P \Rightarrow (Q \Rightarrow R)$

P	Q	R	$Q \Rightarrow R$	$P \Rightarrow (Q \Rightarrow R)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

$(P \vee R) \Rightarrow (\sim P \wedge Q)$

P	Q	R	$\sim P$	$P \vee R$	$\sim P \wedge Q$	$(P \vee R) \Rightarrow (\sim P \wedge Q)$
T	T	T	F	T	F	F
T	T	F	F	T	F	F
T	F	T	F	T	F	F
T	F	F	F	T	F	F
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	F	F
F	F	F	T	F	F	T

$(\sim P \Leftrightarrow \sim R) \Rightarrow (P \veebar Q)$

P	Q	R	$\sim P$	$\sim R$	$\sim P \Leftrightarrow \sim R$	$P \veebar Q$	$(\sim P \Leftrightarrow \sim R) \Rightarrow (P \veebar$
T	T	T	F	F	T	F	F
T	T	F	F	T	F	F	T
T	F	T	F	F	T	T	T
T	F	F	F	T	F	T	T
F	T	T	T	F	F	T	T
F	T	F	T	T	T	T	T
F	F	T	T	F	F	F	T
[illegible]	F	F	T	T	T	F	F

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$[\sim(P\wedge Q)\vee\sim R] \vee [(\sim P\wedge Q)\vee\sim R]\wedge S$

P	$\sim P$	Q	R	$\sim R$	$(P\wedge Q)$	$\sim(P\wedge Q)$	$[\sim(P\wedge Q)\vee\sim R]$	$(\sim P\wedge Q)$	$[(\sim P\wedge Q)\vee\sim R]$	S	$[(\sim P\wedge Q)\vee\sim R]\wedge S$	$[\sim(P\wedge Q)\vee\sim R] \vee [(\sim P\wedge Q)\vee\sim R]\wedge S$
T	F	T	T	F	T	F	F	F	F	T	F	F
T	F	T	T	F	T	F	F	F	F	F	F	F
T	F	T	F	T	T	F	T	F	T	T	T	T
T	F	T	F	T	T	F	T	F	T	F	F	T
T	F	F	T	F	F	T	T	F	F	T	F	T
T	F	F	T	F	F	T	T	F	F	F	F	T
T	F	F	F	T	F	T	T	F	T	T	T	T
T	F	F	F	T	F	T	T	F	T	F	F	T
F	T	T	T	F	F	T	T	T	T	T	T	T
F	T	T	T	F	F	T	T	T	T	F	F	T
F	T	T	F	T	F	T	T	T	T	T	T	T
F	T	T	F	T	F	T	T	T	T	F	F	T
F	T	F	T	F	F	T	T	F	F	T	F	T
F	T	F	T	F	F	T	T	F	F	F	F	T
F	T	F	F	T	F	T	T	F	T	T	T	T
F	T	F	F	T	F	T	T	F	T	F	F	T

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$(P \land Q) \Rightarrow (R \Leftrightarrow S)$

P	Q	R	S	$P \land Q$	$R \Leftrightarrow S$	$(P \land Q) \Rightarrow (R \Leftrightarrow S)$
T	T	T	T	T	T	T
T	T	T	F	T	F	F
T	T	F	T	T	F	F
T	T	F	F	T	T	T
T	F	T	T	F	T	T
T	F	T	F	F	F	T
T	F	F	T	F	F	T
T	F	F	F	F	T	T
F	T	T	T	F	T	T
F	T	T	F	F	F	T
F	T	F	T	F	F	T
F	T	F	F	F	T	T
F	F	T	T	F	T	T
F	F	T	F	F	F	T
F	F	F	T	F	F	T
F	F	F	F	F	T	T

$P \land Q$

Tautology (T_0) :

" A compound statement which is true for all possible truth values of its statements is called a tautology. It is denoted by T_0.

contradiction F_0 :

A compound statement which is falls for all possible truth values of its statements is called contradiction. It is denoted by F_0.

contingency :

A compound statement that can be true or false is called contingency.

Eg :-

$p \vee \sim p$

p	$\sim p$	$p \vee \sim p$
T	F	T
T	F	T

$p \vee \sim p$ is a tautology

$p \wedge \sim p$

p	$\sim p$	$p \wedge \sim p$
T	F	F
T	F	F

$p \wedge \sim p$ is contradiction.

*

1) prove that $[(p \Rightarrow Q) \wedge (Q \Rightarrow R)] \vee [(p \Leftrightarrow Q)]$ is tautology

2) prove that $(p \veebar Q) \vee (p \Leftrightarrow Q)$ is tautology

3) prove that $(p \veebar Q) \wedge (p \Leftrightarrow Q)$ is contradiction

4) prove that $(p \veebar Q) \wedge (p \Rightarrow Q)$ is contingency

① $[(P \Rightarrow Q) \wedge (Q \Rightarrow R)]$ [illegible]

P	Q	R	$P \Rightarrow Q$	$Q \Rightarrow R$	$(P \Rightarrow Q) \wedge (Q \Rightarrow R)$	$P \Leftrightarrow Q$	Z
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	[illegible]
T	F	T	F	T	F	F	F
T	F	F	F	T	F	F	F
F	T	T	T	T	T	F	[illegible]
F	T	F	T	F	F	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

② $[P \veebar Q] \vee (P \Leftrightarrow Q)$

P	Q	$P \veebar Q$	$P \Leftrightarrow Q$	$(P \veebar Q) \vee (P \Leftrightarrow Q)$
T	T	F	T	T
T	F	T	F	T
F	T	T	F	T
F	F	F	T	T

3) $(P \veebar Q) \wedge (P \Leftrightarrow Q)$

P	Q	$P \veebar Q$	$P \Leftrightarrow Q$	$(P \veebar Q) \wedge (P \Leftrightarrow Q)$
T	T	F	T	F
T	F	T	F	F
F	T	T	F	F
F	F	F	T	F

4) $(P \veebar Q) \wedge (P \Rightarrow Q)$

P	Q	$P \veebar Q$	$P \Rightarrow Q$	$(P \veebar Q) \wedge (P \Rightarrow Q)$
T	T	F	T	F
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

Tautological implications :-

implication ($\Rightarrow$)

Recall the definition of the conditional statement and truth table for any statement formula $P \Rightarrow Q$

Converse implication:

The statement $Q \Rightarrow P$ is called the converse implication.

Inverse implication:

The statement $\sim P \Rightarrow \sim Q$ is called the Inverse implication.

Contra positive implication:

The statement $\sim Q \Rightarrow \sim P$ is called the contra positive implication.

Eg:

P	Q	$P \Rightarrow Q$	$\sim P$	$\sim Q$	converse $Q \Rightarrow P$	Inverse $\sim P \Rightarrow \sim Q$	contra +ve $\sim Q \Rightarrow \sim P$
T	T	T	F	F	T	T	T
T	F	F	F	T	T	T	F
F	T	T	T	F	F	F	T
F	F	T	T	T	T	T	T

In above,

+ $(P \Rightarrow Q) \equiv (\sim Q \Rightarrow \sim P)$ or $(P \Rightarrow Q) \Leftrightarrow (\sim Q \Rightarrow \sim P)$
* $(Q \Rightarrow P) \equiv (\sim P \Rightarrow \sim Q)$ o $(Q \Rightarrow P) \Leftrightarrow (\sim P \Rightarrow \sim Q)$

1. Define converse, contra-positive and inverse of implications and then prove that $(P \Rightarrow Q) \Rightarrow (\sim Q \Rightarrow \sim P)$ is a Tautology

P	Q	$\sim P$	$\sim Q$	$P \Rightarrow Q$	$\sim Q \Rightarrow \sim P$	Z
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

⇒ logically equivalence :-

Two programs propositions/statements p and Q are said to be logically equivalent (or) simply equivalent if they have identical truth values. It is denoted by $P \equiv Q$ (or) $P \Leftrightarrow Q$.

algebra of statements (or) -formula for equivalence Replacement (or) alternative method (or) substraction method

1. Idempotent law
   * $p \vee p \equiv p$
   * $p \wedge p \equiv p$
2. associative law
   * $p \wedge (Q \wedge R) \equiv (p \wedge Q) \wedge R$
   * $p \vee (Q \vee R) \equiv (p \vee Q) \vee R$
3. Commutative law
   * $p \vee Q \equiv Q \vee p$
   * $p \wedge Q \equiv Q \wedge p$
4. complement law
   * $p \vee \sim p \equiv T$
   * $p \wedge \sim p \equiv F$
   * $p \wedge \sim T \equiv F$
   * $\sim F \equiv T$ ***
5. Demorgans law
   * $\sim (p \vee Q) \equiv \sim p \wedge \sim Q$
   * $\sim (p \wedge Q) \equiv \sim p \vee \sim Q$
6. Distributive law
   * $p \vee (Q \wedge R) \equiv (p \vee Q) \wedge (p \vee R)$
   * $p \wedge (Q \vee R) \equiv (p \wedge Q) \vee (p \wedge R)$
7. Identity law
   * $p \vee F \equiv p$
   * $p \vee T \equiv T$
   * $p \wedge F \equiv F$
   * $p \wedge T \equiv p$
8. Double negation law
   * $\sim(\sim p) \equiv p$
9. Inverse law
   * $p \vee \sim p \equiv T_0$
   * $p \wedge \sim p \equiv F_0$
10. Domination law
    * $p \vee T_0 \equiv T_0$
    * $p \wedge F_0 \equiv F_0$
11. Absorption law
    * $p \vee (p \wedge Q) \equiv p$
    * $p \wedge (p \vee Q) \equiv p$

* Implication Law :

1) $P \Rightarrow Q \equiv \sim P \vee Q$

2) $\sim(P \Rightarrow Q) \equiv P \wedge \sim Q$

3) $P \Leftrightarrow Q \equiv (P \Rightarrow Q) \wedge (Q \Rightarrow P)$

13)

$R \vee (P \wedge \sim P) \equiv R$

$R \wedge (P \vee \sim P) \equiv R$

14) $R \vee (P \vee \sim P) \equiv T_0$

$R \wedge (P \wedge \sim P) \equiv F_0$

1) Show that $[\sim P \wedge (\sim Q \wedge R)] \vee (Q \wedge R) \vee (P \wedge R) \equiv R$

L.H.S $\Rightarrow$ $[\sim P \wedge (\sim Q \wedge R)] \vee (Q \wedge R) \vee (P \wedge R) \equiv$

$\equiv [(\sim P \wedge \sim Q) \wedge R] \vee [(Q \wedge R) \vee (P \wedge R)]$ ($\because$ associative law)

$\equiv [(\sim P \wedge \sim Q) \wedge R] \vee [R \wedge (Q \vee P)]$ ($\because$ Distributive law)

$\equiv [\sim(P \vee Q) \wedge R] \vee [R \wedge (Q \vee P)]$ ($\because$ Demorgan's law)

$\equiv R \wedge [\sim(P \vee Q) \vee (Q \vee P)]$ ($\because$ Distributive law)

$\equiv R \wedge [\sim(P \vee Q) \vee (P \vee Q)]$ ($\because$ commutative law)

$\equiv R \wedge T_0$ ($\because$ inverse law)

$\equiv R$ (Identity law)

2) $P \Leftrightarrow Q \equiv (P \Rightarrow Q) \wedge (Q \Rightarrow P)$

L.H.S

$\rightarrow P \Leftrightarrow Q \equiv$

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2) $p \Leftrightarrow q \equiv (p \vee q) \Rightarrow (p \wedge q)$

L.H.S

$p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$ ($\because$ Implication law)

$\equiv (\sim p \vee q) \wedge (\sim q \vee p)$ ($\because$ implication law)

$\equiv [(\sim p \vee q) \wedge (\sim q)] \vee [(\sim p \vee q) \wedge p]$ ($\because$ distributive law

$\equiv [\sim q \wedge (\sim p \vee q)] \vee [p \wedge (\sim p \vee q)]$ ($\because$ commutative law

$\equiv [(\sim q \wedge \sim p) \vee (\sim q \wedge q)] \vee [(p \wedge \sim p) \vee (p \wedge q)]$ ($\because$ distributive law

$\equiv [\sim (q \vee p) \vee (\sim q \wedge q)] \vee [(p \wedge \sim p) \vee (p \wedge q)]$ ($\because$ Demorgan's

$\equiv [\sim (q \vee p) \vee F_0] \vee [F_0 \vee (p \wedge q)]$ ($\because$ Inverse law)

$\equiv [\sim (q \vee p)] \vee [(p \wedge q)]$ ($\because$ Identity law)

$\equiv [\sim (p \vee q) \vee (p \wedge q)]$ ($\because$ commutative law)

$\equiv (p \vee q) \Rightarrow (p \wedge q)$ ($\because$ Implication law)

$\therefore p \Leftrightarrow q \equiv (p \vee q) \Rightarrow (p \wedge q)$

3) P.T $p \Rightarrow (q \Rightarrow p) \equiv \sim p \Rightarrow (p \Rightarrow q)$

L.H.S

$p \Rightarrow (q \Rightarrow p) \equiv p \Rightarrow (\sim q \vee p)$ ($\because$ Implication law)

$\equiv \sim p \vee (\sim q \vee p)$ ($\because$ implication law)

$\equiv (\sim p \vee \sim q) \vee p$ ($\because$ Associative law)

$\equiv p \vee (\sim p \vee \sim q)$ ($\because$ commutative)

$\equiv (p \vee \sim p) \vee \sim q$ ($\because$ Associative)

$\equiv T_0 \vee \sim q$ (Inverse)

$\equiv T_0$ ($\because$ Domination law)

R.H.S

$\sim p \Rightarrow (p \Rightarrow q) \equiv \sim p \Rightarrow (\sim p \vee q)$ ($\because$ Implication

$\equiv \sim(\sim p) \vee (\sim p \vee q)$ ($\because$ Implication)

$\equiv p \vee (\sim p \vee Q)$

$\equiv (p \vee \sim p) \vee Q$ ($\because$ Associative law)

$\equiv T_0 \vee Q$ ($\because$ Inverse law

$\equiv T_0$ ($\because$ Domination law

$\therefore$ $p \Rightarrow (Q \Rightarrow p) \equiv \sim p \Rightarrow (p \Rightarrow Q)$

4) S.T $[(p \vee Q) \wedge \sim[\sim p \wedge (\sim Q \vee \sim R)]] \vee (\sim p \wedge \sim Q) \vee (\sim p \wedge \sim R)$

Is tautology?

$\Rightarrow \sim p \wedge \sim R \equiv \sim(p \vee R)$ (" Demo organs law")

$\sim p \wedge \sim Q \equiv \sim(p \vee Q)$ (")

$\Rightarrow (\sim p \wedge \sim Q) \vee (\sim p \wedge \sim R) \equiv [\sim(p \vee Q)] \vee [\sim(p \vee R)]$

$\equiv \sim[(p \vee Q) \underset{\wedge}{\vee} (p \vee R)]$ ($\because$ Demorgan

$\Rightarrow \sim(\sim p \wedge (\sim Q \vee \sim R)) \equiv p \vee \sim(\sim Q \vee \sim R)$

$\equiv p \vee \sim[\sim(Q \wedge R)]$ ($\because$ Demorgan's)

$\equiv p \vee (Q \wedge R)$

$\Rightarrow (p \vee Q) \wedge \sim[\sim p \wedge (\sim Q \vee \sim R)] \equiv (p \vee Q) \wedge [p \vee (Q \wedge R)]$ distrib

$\equiv (p \vee Q) \wedge [(p \vee Q) \wedge (p \vee R)]$ (Demo organ's)

$\equiv [(p \vee Q) \wedge (p \vee Q)] \wedge (p \vee R)$ (Associative law)

$\equiv (p \vee Q) \wedge (p \vee R)$ ($\because$ Idempotent)

$\equiv T$

Now

$[(p \vee Q) \wedge \sim[\sim p \wedge (\sim Q \vee \sim R)]] \vee (\sim p \wedge \sim Q) \vee (\sim p \wedge \sim R)$

(Inverse law)

$\equiv T_0$ (or)

$\Rightarrow [(p \vee Q) \wedge [(p \vee Q) \wedge (p \wedge R)]] \vee [\sim(p \vee Q) \vee \sim(p \vee R)]$

$\Rightarrow [(p \vee Q) \wedge (p \vee (Q \wedge R))] \vee \sim[(p \vee Q) \wedge (p \vee R)]$

$\Rightarrow [(p \vee Q) \wedge [(p \vee Q) \wedge (p \vee R)]] \vee \sim[(p \vee Q) \wedge (p \vee R)]$

$\Rightarrow [(p \vee Q) \wedge (p \vee Q)] \wedge (p$

$\vee \sim[(p \vee Q) \wedge (p \vee R$

$\Rightarrow [(p \vee Q) \wedge (p \vee R)] \vee$

$\sim[(p \vee Q) \wedge (p \vee R$

$\equiv T_0$

5) $(P \Rightarrow Q) \Rightarrow R$ and $P \Rightarrow (Q \Rightarrow R)$ logically equivalent justify your answer. by using logical equivalence and by using truth table ?

$(P \Rightarrow Q) \Rightarrow R$ truth table

P	Q	R	$P \Rightarrow Q$	$(P \Rightarrow Q) \Rightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	F
F	F	T	T	T
F	F	F	T	F

$P \Rightarrow (Q \Rightarrow R)$ truth table

P	Q	R	$Q \Rightarrow R$	$P \Rightarrow (Q \Rightarrow R)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

logical equivalence :

$(P \Rightarrow Q) \Rightarrow R \equiv P \Rightarrow (Q \Rightarrow R)$

$(P \Rightarrow Q) \Rightarrow R$

$(\sim P \vee Q) \Rightarrow R$ (Implication law)

$\sim(\sim P \vee Q) \vee R$

$(P \wedge \sim Q) \vee R$

P Q

$P \Rightarrow (Q \Rightarrow R) \equiv$

$P \Rightarrow (\sim Q \vee R)$

$\equiv \sim P \vee (\sim Q \vee R)$

$\equiv (\sim P \vee \sim Q) \vee R$

$\equiv \sim (P \wedge Q) \vee R$

Both are not equal $(P \Rightarrow Q) \Rightarrow R \neq P \Rightarrow (Q \Rightarrow R)$

6) Show the following implication without constructing truth table

1) $(P \wedge Q) \vee (\sim P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$ is tautology

ii) $\sim [P \vee (Q \wedge R)] \wedge [(P \vee Q) \wedge (P \vee R)]$ F_0

iii) $[P \wedge (P \Rightarrow Q)] \Rightarrow Q$ is tautology

iv) $(P \Rightarrow Q) \wedge (R \Rightarrow Q) \equiv (P \vee R) \Rightarrow Q$

v) $P \vee [P \wedge (P \vee Q)] \Leftrightarrow P$

1) $(P \wedge Q) \vee (\sim P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

$\equiv [Q \wedge (P \vee \sim P)] \vee [\sim Q \wedge (P \vee \sim P)]$ (∴ distributive law

(∴ complement law

$\equiv (Q \wedge (T)) \vee (\sim Q \wedge (T))$

$\equiv T \wedge (Q \vee \sim Q)$

$\equiv T_0 \wedge T_0 = T_0$

(or)

$\equiv (P \wedge Q) \vee (Q \Rightarrow P) \vee \sim Q \wedge (P \vee \sim P)$

$\equiv [(P \wedge Q) \vee (\sim Q \vee P)] \vee \sim Q \wedge T_0$

$\equiv P \vee (Q \vee \sim Q) \vee \sim Q \wedge T_0$

$\equiv P \vee T_0 \vee \sim Q \wedge T_0 = T_0 \vee T_0 \equiv T_0$

2) $\sim [P \vee (Q \wedge R)] \wedge [(P \vee Q) \wedge (P \vee R)]$

$\equiv \sim [(P \vee Q) \wedge (P \vee R)] \wedge [(P \vee Q) \wedge (P \vee R)]$

$\equiv \sim [P \vee Q] \vee \sim [P \vee R] \wedge [P \vee (Q \wedge R)]$

$\equiv \sim [P \vee (Q \wedge R)] \wedge [P \vee (Q \wedge R)]$

$\equiv T$

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4) $(P \Rightarrow Q) \wedge (R \Rightarrow Q) \equiv (P \vee R) \Rightarrow Q$

L.H.S $(P \Rightarrow Q) \wedge (R \Rightarrow Q) \equiv (\sim P \vee Q) \wedge (\sim R \vee Q)$

$\equiv Q \vee (\sim P \wedge \sim R)$

$\equiv Q \vee \sim (P \vee R)$

R.H.S $(P \vee R) \Rightarrow Q \equiv \sim (P \vee R) \vee Q$ $\therefore$ Im

$\equiv Q \vee (\sim P \wedge \sim R)$ $\therefore$ demorg

$\equiv Q \vee \sim (P \vee R)$

$\therefore$ L.H.S = R.H.S

$(P \Rightarrow Q) \wedge (R \Rightarrow Q) \equiv (P \vee R) \Rightarrow Q$

5) $P \vee [P \wedge (P \vee Q)] \Leftrightarrow P$

$P \vee P \Leftrightarrow P$ $\therefore$ absorption law

$P \Leftrightarrow P$ $\therefore$ Idempotent law

ii) $\sim[p\vee(Q\wedge R)]\wedge[p\vee(Q\wedge R)]$ (∴ distr

$P = p\vee(Q\wedge R)$

$= \sim p\wedge p$

$= F_0$ (∴ Inverse law

iii) $[p\wedge(p\Rightarrow Q)]\Rightarrow Q$

$p\wedge(\sim p\vee Q)\Rightarrow Q$

$(p\wedge\sim p)\vee(p\wedge Q)\Rightarrow Q$

$\equiv T_0\vee(p\wedge Q)\Rightarrow Q$

$\equiv T_0\vee[\sim(p\wedge Q)\vee Q]$

$\equiv T_0\vee[\sim p\vee(\sim Q\vee Q)]$

$\equiv T_0\vee[\sim p\vee T_0]$

$= T_0$

∴ $[p\wedge(p\Rightarrow Q)]\Rightarrow Q$ is tautology

Application of prepositional logics

* 1) Tanslating English sentence to symbolic form
* 2) System specification
3) Boolean
4) logical curcuits
5) logical puzzles

1. Translating english sentence to symbolic form:

* Convert english sentence to symbolic form by using prepositional(statement) logic.

* Identify preposition and respect using prepositional logic

* Determine appropriate logical conection.

Eg:- If I get the book then ~~and~~ I began to repeat

Sol: If I get the book then I began to read.

Sol: P : I get the book

Q : I begin to read

The symbolic form is $P \Rightarrow Q$

Eg: If either Ramu prefers tea or Ravi prefers coffee, then Seetha prefers milk.

Sol: P : ~~If~~ Ramu prefers tea

Q : Ravi prefers coffee

R : Seetha prefers milk.

The symbolic form:

$(P \vee Q) \Rightarrow R$

2. System specification:

Translating sentence of natural language into logical expression is required for hardware

Software system.

Eg: The automated reply cannot be sent when file system is full.

P : The automated reply can be sent

Q : The file system is full

The symbolic form : $Q \Rightarrow \sim P$

1) precedence of logical operators

operator	precedence
$\sim$	1
$\wedge$	2
$\vee$	3
$\Rightarrow$	4
$\Leftrightarrow$	5

logic and Bit operations

Statement	Bit
T	1
F	0

Eg: $(P \wedge Q) \Rightarrow (P \vee Q)$

P	Q	$P \wedge Q$	$P \vee Q$	$(P \wedge Q) \Rightarrow (P \vee Q)$
1	1	1	1	1
1	0	0	1	1
0	1	0	1	1
0	0	0	0	1

*** predicates and quantifieds :

predicates:

. predicate describes something about one or more objects

Note :* Generally predicates are denoted by upper case letter (A, B, C . . . X, Y, Z) and objects are denoted by lower case letters (a, b, c . . . x, y, z)

* Any statement obtain of type small 'p'. is Q but Q is predicate and 'p' is object. so we represent Q(p)

Eg: 1) $\underset{p\ \text{object}}{\underline{\text{Jack}}}$ is $\underset{S}{\underline{\text{taller than}}}$ $\underset{q}{\underline{\text{Ramu}}}$

we can write symbolic form $S(p,q)$

2) $\underset{m}{\underline{\text{Naveen}}}$ $\underset{Q}{\underline{\text{sits}}}$ between $\underset{n}{\underline{\text{madhu}}}$ and $\underset{r}{\underline{\text{Ravi}}}$

$Q(m, n, r)$

Note: * The order in which the names appear in the statement and the predecate should be taken in that order only

* If **S** is an n places predecate and $\alpha_1, \alpha_2 . . . . \alpha_r$ are names of object then $S(\alpha_1, \alpha_2, \alpha_3 . . . . \alpha_n)$ is a statement.

Eg: Let $p(x)$ derote the statement $x>3$ what are the truth values of $p(4)$ and $p(2)$.

Given $p(x) = x>3$, Here p is a predicate and x is object.

$p(4) = 4>3$, which statement is true.

$p(2) = 2>3$, which statement is false

Eg: 2

Let $p(x, y)$ is denoted by the statement $x = y+3$. what are the truth value of $Q(1,2)$, $Q(3,0)$.

Sol: Given statement is $x = y+3$, $p(x, y)$

here p is predict and x, y are objects

$P(1,2) = p(2+3, 2)$

$= p(5,2)$

$Q(1,2) \Rightarrow x = y+3$

$Q(1,2) \Rightarrow 1 = 2+3$

$1 = 5$

The given statement is false

$Q(3,0) \Rightarrow 3 = 0+3$

$3 = 3$

which statement is true

Quantifiers:

In predecate calculas each statement contains a word indicating quantity such as all, everyone, some and one such words are called quantifiers.

Quantifiers are classified into two types

1) universal quantifiers 2) Existencial quantifiers

1) Universal quantifiers:

$\forall x\, p(x)$ - is true
$p(x)$ is true for every value of x
$\forall x\, p(x)$ - false
$p(x)$ is false for every value of x

It is used for the case of for all, for each, for every. (or)

The symbol for all ($\forall$) a been used to denote the sentence for all and for every in logic. these sentence are regarded as equivalence sentence. The sentence for each and for every are also taken as equivalente to sentence.

The symol $\forall$ (for all) is used to denote all of these statements. each of this equivalence sentence is called the universal quantifiers

*Eg: All squares are rectangle

sol: Let S denotes the set of all squares and x is set of all rectangles. That symbolically written as $\forall x \in S$. where $p(x)$ is a open statement.

Existential quantifiers:

Existential quantifiers is used for when a statemen is true for some values given in the universe of ~~discours~~ discourse (limit of the domain, Retricted Domain of the quantifiers)

It is denoted by the symbol $\exists$. The existential quantifiers of $p(x)$ is the statement. There exist

There exist some x in the universe of discour such that $p(x)$ and it is denoted by the symbol $\exists x\, p(x)$

Note!

$\exists x\, p(x)$ is true

* when $p(x)$ is true for atleast one value of x in the universe of discourse.
* when $p(x)$ is false for every x in the universe of discourse

Note! Universal quantifiers represent words are
$\forall$ for all, for every, for each, every-thing,
each thing, there exist, there is a atleast, there is an, there is some.

$\forall Q$	$\exists Q$
for all	There exist
for every	There is a atleast
for each	there is an
every thing	there is some
each thing	

free and Bound variables (Binding variable):

Given a formula containing a part of the form $\forall x\, p(x)$ (or) $\exists x\, p(x)$ such a part is called x bound part of the formulae any occurence of x is an x bound part of a formula is called a bound accurance by it while any accurance of x (or) any variable that is not a bound accurance is called a free accurance and the formula $p(x)$ either in $\forall x\, p(x)$ (or) $\exists x\, p(x)$. is described as the scope of the quantifier.

Eg:-
$\forall x\, p(x,y)$

Here $p(x,y)$ is the scope of the quantifier and accurance is x bounded accurence ~~of x~~. occurence of y is free accurence.

Eg: $\forall x\, [p(x) \Rightarrow (\exists y)\, R(x,y)]$

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Here $p(x) \Rightarrow \exists(y) R(x,y)$ is the scope of the quantifier is and occurence of x, y is bounded occurence of x. and occurence of y is bounded occurence

Indicate the variables that are free & bound also scope of the quantifier $\forall x(p(x)\ Q(x)) \Rightarrow \forall x(p(x) \wedge Q(x))$

The scope of the first quantifier p(x) and Q(x) and the occurence of variable x is bounded occurence

The scope of the second quantifier p(x) ..

and the occurence of variable x is bounded occurence

The universe of discourse

we can limit the domain of the quantifiers by modifying the notation in a bit.

Ex:- $\forall x<0\ (x^2>0)$ (Domain - Real no).

Meaning of the above statement

A) the squares of the -ve real no. is +ve.

precidence of quantifiers :-

The quantifiers $\forall, \exists$ have higher precidence then all logical operators from prepositional calculus.

Ex:- $\forall x p(x) \vee Q(x)$ is the disjunction of $\forall x p(x)$ and $Q(x)$.

Negating quantifiers :-

consider the following statement.

→ Every student in the SV university has studied discrete mathematics.

Domain :- Every student in the SV university

p(x) :- x has studied discrete mathematics

Symbolic form :- $\forall x p(x)$.

The negation of this statement.

1. It is not the case that every student in SV university has studied discrete mathematics. $\sim(\forall x\, p(x))$

(or)

there is some students in the SV univ. who has not studied discrete mathematics.

The symbolic form of this statement is
$\exists x \sim p(x)$.

similarly,

$\sim(\forall x\, p(x)) \equiv \exists x . \sim p(x)$

$\sim(\exists x\, p(x)) \equiv \forall x . \sim p(x)$

this is called Demorgan's law of quantifiers.

logical equivalence involving quantifiers:-

statements involving predicates and quantifiers are logical equivalent if and only if they have the same truth values no matter which predicates are substitute into this statements and which domain of discourse is used for the variables in this prepositions functions.

we use the notation S≡T indicate that two statements S and T involving predicates and quantifiers are logically equivalent.

logical equivalence

$\exists x\,(p(x) \vee Q(x)) \equiv \exists x\, p(x) \vee \exists x\, Q(x)$

$\forall x\,(p(x) \wedge Q(x)) \equiv \forall x\, p(x) \wedge \forall x\, Q(x)$

Demorgan's law

$\sim(\forall x\, p(x)) \equiv \exists x . \sim p(x)$

$\sim(\exists x . p(x)) \equiv \forall x . \sim p(x)$

proof:-

$\forall x (p(x) \wedge Q(x)) \equiv \forall x\, p(x) \wedge \forall x\, Q(x)$

Domain x : All students of SV university

$p(x)$: x has studied discrete mathematics

$Q(x)$: x has scored more than 60% marks in exam.

LHS $\Rightarrow \forall x [p(x) \wedge Q(x)] \equiv$ Every student in SV university has studied discrete mathematics and has scored more than 60% marks in exam

RHS $\Rightarrow \forall x\, p(x) \wedge \forall x\, Q(x) \equiv$ Every student in SV university has studied discrete mathematics and every student in SV university has scored more than 60% marks in exam.

Translating english sentence to logical expression

Let us suppose we have to understand translate the following english sentence into an equivalent logical expression.

1. statement : for each integer x their exist an integer y such that $x+y=0$

   for each integer $x \Rightarrow \forall x$

   there exist an integer $y = \exists y$

   predicate $x+y=0 \Rightarrow p(x,y)$

   The symbolic form

   $\forall x \exists y\, p(x,y)$

2. Statement : For all integers x and y such that $xy=yx$

   for each integer $x \Rightarrow \forall x$

   for each integer $y = \forall y$

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predicate $xy = yx \Rightarrow p(x,y)$

The symbolic form $\forall x \forall y \cdot p(x,y)$

Nested quantifiers

Two quantifiers are set to be nested quantifiers. If one quantifier is within the scope of the other quantifier.

Eg:- $\forall x \exists y (x+y=0)$, domain = real numbers.

for every real number x. there exists a real number y such that $x+y=0$.

These statement that every real number as an additive inverse.

Note! Anything within the scope of the quantifier can be thought of as a prepositional functional.

$\forall x \exists y \, p(x,y) \Rightarrow \forall x \, Q(x)$

Here, $Q(x) = \exists y \, p(x,y)$

Different combinations of nested quantifiers.

1. $\forall x \forall y . p(x,y)$
2. $\exists x \exists y \, p(x,y)$
3. $\forall x \exists y \, p(x,y)$
4. $\exists x \forall y \, p(x,y)$

Order of quantifiers:

The order of quantifiers is important unless all the quantifiers are universal quantifiers or All the quantifiers are existential quantifiers.

1. $\forall x \forall y \; p(x,y) = \forall y \forall x \; p(x,y)$

2. $\exists x \exists y \; p(x,y) = \exists y \exists x \; p(x,y)$

3. $\forall x \exists y \; p(x,y) \neq \exists y \forall x \; p(x,y)$

4. $\exists x \forall y \; p(x,y) \neq \forall y \exists x \; p(x,y)$

Negating (Negation) of Nested quantifiers

The Negation of multiple nested quantifiers predicate formulas may be optained by applying the rules for negation from the left to right

*** $\sim[\forall x. \exists y. p(x,y)] \equiv \exists x. \forall y \sim p(x,y)$

** Normal forms

1. Elementary product ($\wedge$) :-

A product of the variables and their negations in a formula is called an elementary product

Eg: If p and Q are two variables then the elementary products of p and Q is $(p \wedge \sim Q)$, $(\sim p \wedge Q)$, $(\sim p \wedge \sim Q)$.

2. Elementary sum($\vee$) :-

A sum of the variables and their negations is called the elementary sum.

Eg: If p and Q are two variables then the elementary sum of p and Q are $(p \vee \sim Q)$, $(\sim p \vee Q)$, $(\sim p \vee \sim Q)$

Definition of normal form:

Converting the given statement formula into any one of the standard forms (elementary product, elementary sum). is called the normal form or canonical form.

Normal forms are classified into two types they are : 1. Disjunctive Normal Form (DNF)

2. Conjunctive Normal form (CNF)

Disjunctive Normal Form (DNF)

A formula which is equivalent to the given formula and which consist of a sum of elementary products is called a Disjunctive Normal Form

Eg:- $(p \wedge Q) \vee (\sim p \wedge Q) \vee (\sim p \wedge \sim Q) \vee (p \wedge \sim Q)$

procedure to the DNF :-

* Remove all implication and Bi-implication by equivalent expressions containing connectives $(\wedge, \vee, \sim)$.

* Elemenate negation before sums and product by using double negation and demorgans law.

Eg: $\sim(\sim p) \equiv p$

$\sim(p \wedge Q) = \sim p \vee \sim Q$, $\sim(p \vee Q) = \sim p \wedge \sim Q$

* Apply the distribution law until a sum of elementary products obtained.

$$p \wedge (Q \vee R) \equiv (p \wedge Q) \vee (p \wedge R)$$

* Note :

Disjunctive normal-form need not to be unique

problems :

1. Find the DNF of $p \wedge (p \Rightarrow Q)$

$\therefore \; p \wedge (p \Rightarrow Q) \equiv p \wedge (\sim p \vee Q)$

$\equiv (p \wedge \sim p) \vee (p \wedge Q)$ ($\therefore$ Sum of elements of product)

2. Write an equivalent DNF for the equation.

$p \vee [\sim p \Rightarrow (Q \vee (Q \Rightarrow \sim R))]$

$\equiv p \vee [\sim p \Rightarrow (Q \vee (\sim Q \vee \sim R))]$ $\therefore$ implication law

$\equiv p \vee [\sim p \Rightarrow ((Q \vee \sim Q) \vee \sim R)]$ $\therefore$ Associative law

$\equiv p \vee [\sim p \Rightarrow (T \vee \sim R)]$ $\therefore$ complement law

$\equiv p \vee [\sim p \Rightarrow T]$ $\therefore$ identity law

$\equiv p \vee [\sim(\sim p) \vee T]$ $\therefore$ identity law

$\equiv p \vee T$

3. write an equivalent DNF for statement $\sim [(p \vee Q) \Leftrightarrow (p \wedge Q)]$

Given that $\sim [(p \vee Q) \Leftrightarrow (p \wedge Q)]$

$\equiv [\sim (p \vee Q) \wedge (p \wedge Q)] \vee [(p \vee Q) \wedge \sim (p \wedge Q)]$

$$\boxed{\sim (p \Leftrightarrow Q) \equiv (\sim p \wedge Q) \vee (p \wedge \sim Q)}$$

$\equiv [(\sim p \wedge \sim Q) \wedge (p \wedge Q)] \vee [(p \vee Q) \wedge (\sim p \vee \sim Q)]$ $\therefore$ Demorgan's law

$\equiv [(\sim p \wedge \sim Q \wedge p) \wedge Q] \vee [(p \vee Q) \wedge \sim p] \vee [(p \vee Q) \wedge \sim Q]$ $\therefore$ Associative law

$\therefore$ Distributive law

$\equiv [(F \wedge \sim Q) \wedge Q] \vee [(P \wedge \sim P) \vee (Q \wedge \sim P)] \vee [(P \wedge \sim Q) \vee (Q \wedge \sim Q)]$

$P \vee P \to P$ $\quad \therefore$ Distributive law

$\equiv [F \wedge Q] \vee [F \vee (Q \wedge \sim P)] \vee [(P \wedge \sim Q) \vee F]$ ($\because$ complement law)

$\equiv (F \wedge Q) \vee [(Q \wedge \sim P) \vee (P \wedge \sim Q) \vee F]$ $\quad \therefore$ complement law

$\equiv F \vee [(Q \wedge \sim P) \vee (P \wedge \sim Q)]$ $\quad \therefore$ complement law

$\equiv (Q \wedge \sim P) \vee (P \wedge \sim Q)$

which is DNF of given statement

4. $P \Rightarrow (P \Rightarrow Q) \wedge \sim(\sim Q \vee \sim P)$

* Conjunctive Normal form (CNF) :-

A formula which is equivalent to a given formula and which consists of a product of elementary sums is called a conjunctive Normal form of the given formula.

Eg:- 1) $p \wedge (p \vee Q) \wedge (\sim p \vee Q)$

2) $(p \vee \sim Q \vee R) \wedge (p \vee \sim Q \vee \sim R)$

problems :-

1) Find the CNF of $p \wedge (p \Rightarrow Q)$

$p \wedge (p \Rightarrow Q) \equiv p \wedge (\sim p \vee Q)$

which is the CNF of given statement.

2) obtain conjunctive Normal form of this statement

$\sim(p \vee Q) \Leftrightarrow (p \wedge Q)$

sol:

$\sim(p \vee Q) \Leftrightarrow (p \wedge Q)$

$\equiv [\sim(p \vee Q) \Rightarrow (p \wedge Q)] \wedge [(p \wedge Q) \Rightarrow \sim(p \vee Q)]$

$\boxed{\therefore\ p \Leftrightarrow Q \equiv (p \Rightarrow Q) \wedge (Q \Rightarrow p)}$

$\equiv [\sim(\sim(p \vee Q)) \vee (p \wedge Q)] \wedge [\sim(p \wedge Q) \vee \sim(p \vee Q)]$ $\therefore$ implication law

$\equiv [(p \vee Q) \cancel{\wedge} (p \wedge Q)] \wedge [\sim((p \wedge Q) \wedge (p \vee Q))]$ $\therefore$ Demorgan's law

$\equiv [(p \vee Q \vee p) \wedge (p \vee Q \vee Q)] \wedge [\sim((p \wedge Q \wedge p) \vee (p \wedge Q \wedge Q))]$

$\therefore$ Distributive law

$\equiv [(p \vee Q) \wedge (p \vee Q)] \wedge [\sim((p \wedge Q) \vee (p \wedge Q))]$

$\therefore$ Associative law

$\equiv (p \vee Q) \wedge \sim(p \wedge Q)$

$\equiv (p \vee Q) \wedge (\sim p \cancel{\wedge} \sim Q)$

which is the CNF of given statement.

* obtain CNF of the following statement $[(p \Rightarrow Q) \wedge \sim Q]$.

Sol: Given that

$[(p \Rightarrow Q) \wedge \sim Q] \Rightarrow \sim p \equiv [(\sim p \vee Q) \wedge \sim Q] \Rightarrow \sim p$ (∴ implic

$\equiv \sim[(\sim p \vee Q) \wedge \sim Q] \vee \sim p$ (∴ implication law)

$\equiv [(\sim(\sim p) \vee \sim Q) \vee Q] \vee \sim p$ (∴ Demorgan's law)

$\equiv [(p \wedge \sim Q) \vee Q] \vee \sim p$ (∴ Double negation law)

$\equiv [(p \vee Q) \wedge (\sim Q \vee Q)] \vee \sim p$ (∴ Distributive law)

Q R p

$\equiv [(p \vee Q) \vee \sim p] \wedge [(\sim Q \vee Q) \vee \sim p]$ (Distributive law)

which is the given statement

* obtain C.N.F of the following statement

$[(p \Rightarrow Q) \wedge \sim p] \Rightarrow \sim Q$

Hint $(p \vee p \vee \sim Q) \wedge (\sim Q \vee p \vee \sim Q$

Given that $(p \Rightarrow Q) \wedge \sim p \Rightarrow \sim Q$

$(\sim p \vee Q) \wedge \sim p \Rightarrow \sim Q$ (implication law)

$\sim[(\sim p \vee Q) \wedge \sim p] \vee \sim Q$ (" ")

$((p \wedge \sim Q) \vee p) \vee \sim Q$ double ne

$(p \vee p) \wedge (p \vee \sim p) \vee \sim Q$ (distributive law)

$[\sim Q \vee (p \vee p)] \wedge [\sim Q \vee (p \vee \sim Q)]$ "

principle of Disjunctive Normal form :-

Minterms :-

Let p, Q be statement variables. Let us construct all possible formulae which consists of conjunction of p or its Negation and conjunction of Q or its Negation which $p\wedge Q$, $\sim p\wedge Q$, $p\wedge\sim Q$, $\sim p\wedge\sim Q$. These formulas are called minterms(or) Boolean conjunction of p and Q

* Note :

1) Minterms of 2 variables are $2^2=4$ (p, Q)

2) Minterms of 3-variables p, Q, R are $2^3=8$ which are $(p\wedge Q\wedge R)$, $(\sim p\wedge Q\wedge R)$, $(p\wedge\sim Q\wedge R)$, $(p\wedge Q\wedge\sim R)$, $(\sim p\wedge\sim Q\wedge$ $(p\wedge\sim Q\wedge\sim R)$, $(\sim p\wedge Q\wedge\sim R)$, $(\sim p\wedge\sim Q\wedge\sim R)$

3) Every minterm is an elementary product but every elementary product need not to be minterm

Definition :

An equivalent formula consisting of disjunction of minterms only is called a principle Disjunction Normal form.

Eg:

1) $(p\wedge Q)\vee(p\wedge\sim Q)\vee(\sim p\wedge Q)\vee(\sim p\wedge\sim Q)$ is a PDNF of two variables p and Q.

2) $(p\wedge Q)\vee(p\wedge\sim Q\wedge R)$ is not a PDNF

NOTE :

1) principle Disjunction Normal Form is unique

2) Every PDNF is a DNF but converse need not to be true

3) There are two methods to obtain a PDNF which are (1) Using truth table method (2) Replacement method.

problems :-

1) find PDNF of $p \Rightarrow Q$

Truth table method of $p \Rightarrow Q$

p	Q	$p \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

From the above table PDNF is $(p \wedge Q) \vee (\sim p \wedge Q) \vee (\sim p \wedge \sim Q)$

2) Find the PDNF of $p \vee Q$

Truth table for $p \vee Q$

p	Q	$p \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

from the above table PDNF is
$(p \wedge Q) \vee (p \wedge \sim Q) \vee (\sim p \wedge Q)$

** Replace method :

We need to follow the steps

1) First Replace the conditionals and Bi-conditionals By using equivalence formulas

2) The negations are applied to the variables by De-morgan's law followed by the applications of Distributive law.

$\sim (p \wedge Q) \equiv \sim p \vee \sim Q$

$\sim (p \vee Q) = \sim p \wedge \sim Q$

3) Any elementary products which are contradictions to be dropped

4) Minterms are applied in the Disjunctions by using missing functions factors

Eg:- $p \vee (p \wedge \sim Q) \equiv (p \wedge T) \vee (p \wedge \sim Q)$

$\equiv (p \wedge (Q \vee \sim Q)) \vee (p \wedge \sim Q)$

$\equiv (p \wedge Q) \vee (p \wedge \sim Q) \vee (p \wedge \sim Q)$

4) Identical minterms appearing in the Disjunction are to be dropped.

Note:-

If two formulas are equivalent then both must have identical proof PDNF

* problems :-

sum of minimum terms

1) obtain the PDNF for the following formulas(statements

$p \Rightarrow Q$.

$p \Rightarrow Q \equiv \sim p \vee Q$ ($\because$ implication law)

$\equiv (\sim p \wedge T) \vee (Q \wedge T)$

$\equiv [\sim p \wedge (Q \vee \sim Q)] \vee [Q \wedge (p \vee \sim p)]$ ($\because$ inverse/complementary law)

$\equiv (\sim p \wedge Q) \vee (\sim p \wedge \sim Q) \vee (Q \wedge p) \vee (Q \wedge \sim p)$

($\because$ Distributive law)

2) $P \Leftrightarrow Q$

Given that $p \Leftrightarrow Q$

$P \Leftrightarrow Q \equiv (p \Rightarrow Q) \wedge (Q \Rightarrow p)$ implication

$\boxed{\therefore P \Leftrightarrow Q \equiv (P \Rightarrow Q) \wedge (Q \Rightarrow P)}$

$\equiv (\sim p \vee Q) \wedge (\sim Q \vee p)$

$\equiv [((\sim p \vee Q) \wedge \sim Q) \vee ((\sim p \vee Q) \wedge p)]$ Distributive law

$\equiv [(\sim p \wedge \sim Q) \vee (Q \wedge \sim Q)] \vee [(\sim p \wedge p) \vee (Q \wedge p)]$ "

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$\equiv (\sim p \wedge \sim Q) \vee (Q \wedge p)$ rule 4

3) obtain the PDNF for the following formulas

$(p \wedge Q) \vee (\sim p \wedge R) \vee (Q \wedge R)$

Given that

$(p \wedge Q) \vee (\sim p \wedge R) \vee (Q \wedge R)$

$\equiv (p \wedge Q \wedge T) \vee (\sim p \wedge T \wedge R) \vee (T \wedge Q \wedge R)$

$\equiv [p \wedge Q \wedge (R \vee \sim R)] \vee [\sim p \wedge (Q \vee \sim Q) \wedge R] \vee [(\sim p \vee p) \wedge Q \wedge R]$

$\equiv (p \wedge Q \wedge R) \vee (p \wedge Q \wedge \sim R) \vee [\sim p \wedge [(Q \wedge R) \vee (\sim Q \wedge R)]$

$\vee (\sim p \wedge Q \wedge R) \vee (p \wedge Q \wedge R)$ ∴ Distributive law

$\equiv (p \wedge Q \wedge R) \vee (p \wedge Q \wedge \sim R) \vee [(\sim p \wedge Q \wedge R) \vee (\sim p \wedge \sim Q \wedge R)] \vee$

$(\sim p \wedge Q \wedge R) \vee (p \wedge Q \wedge R)$ ∴ Distributive law

$\equiv (p \wedge Q \wedge R) \vee (p \wedge Q \wedge \sim R) \vee (\sim p \wedge Q \wedge R) \vee (\sim p \wedge \sim Q \wedge R)$ (∴ Idempotent law)

∴ which is the required PDNF

4) $\sim p \vee Q \Leftrightarrow (\sim p \wedge T) \vee (T \wedge Q)$. H.W

Sol: $(p \wedge Q) \vee (\sim p \wedge Q) \vee (\sim p \wedge \sim Q)$

L.H.S $\sim p \vee Q = (\sim p \wedge T) \vee (Q \wedge T)$

$= [\sim p \wedge (Q \vee \sim Q)] \vee [Q \wedge (p \vee \sim p)]$

$= [(\sim p \wedge Q) \vee (\sim p \wedge \sim Q)] \vee [(Q \wedge p) \vee (Q \wedge \sim p)]$

$= (Q \wedge \sim p) \vee (\sim p \wedge \sim Q) \vee (Q \wedge p)$

R.H.S $= (\sim p \wedge T) \vee (T \wedge Q)$

$= [\sim p \wedge (Q \vee \sim Q)] \vee [(p \vee \sim p) \wedge Q]$

$= [(\sim p \wedge Q) \vee (\sim p \wedge \sim Q)] \vee [(Q \wedge p) \vee (Q \wedge \sim p)]$

$= (Q \wedge \sim p) \vee (Q \wedge p) \vee (\sim p \wedge \sim Q)$

∴ Both are same

∴ $\sim p \vee Q \Leftrightarrow (\sim p \wedge T) \vee (T \wedge Q)$

* Show the following are equivalent formulas for PDNF

i) $p \vee (p \wedge Q) \equiv p$ ii) $p \vee (\sim p \wedge Q) \equiv p \vee Q$

i) Given that

$p \vee (p \wedge Q) \equiv p$

L.H.S $p \vee (p \wedge Q) \equiv (p \wedge T) \vee (p \wedge Q)$

$\equiv [p \wedge (Q \vee \sim Q)] \vee (p \wedge Q)$

$\equiv (p \wedge Q) \vee (p \wedge \sim Q) \vee (p \wedge Q)$ ∴ Distributive law

$\equiv (p \wedge Q) \vee (p \wedge \sim Q)$ ∴ Idempotent law

PDNF of $p \vee (p \wedge Q)$ is $(p \wedge Q) \vee (p \wedge \sim Q)$ — ①

R.H.S $p \equiv (p \wedge T)$

$\equiv [p \wedge (Q \vee \sim Q)]$

$\equiv (p \wedge Q) \vee (p \wedge \sim Q)$

PDNF of p is $(p \wedge Q) \vee (p \wedge \sim Q)$ — ②

from eqn(1) and eqn(2) the PDNF of $p \vee (p \wedge Q)$ and p are same.

Hence $p \vee (p \wedge Q) = p$

* Maxterms :-

A maxterms consists of disjunction in which each variable and its Negation but not both appears only once.

Example :

1) For two variables p and Q the number of max terms are $2^2 = 4$ which are $p \vee Q$, $p \vee \sim Q$, $\sim p \vee Q$, $\sim p \vee \sim Q$

2) From three variables p, Q and R are the number of maxterms are

$2^3 = 8$ which are $(p \vee Q \vee R)$, $(\sim p \vee Q \vee R)$, $(p \vee \sim Q \vee R)$, $(p \vee Q \vee \sim R)$, $(\sim p \vee \sim Q \vee R)$, $(p \vee \sim Q \vee \sim R)$, $(\sim p \vee Q \vee \sim R)$, $(\sim p \vee \sim Q \vee \sim R)$

* NOTE :

Imp: The Duals of minterms are called maxterms

product of elementary summing

* principle conjunctive Normal form (PCNF) :-

principle conjunctive Normal form of a given formula can be defined as an equivalent formula consists of conjunction of maxterms only. This is also called product of elementary sums cannonical.

Eg:- $(P \vee Q) \wedge (P \vee \sim Q) \vee (\sim P \vee Q)$

Note :

1) The process for obtaining PCNF is similar to the process of PDNF
2) The PCNF is unique.
3) Every compound proposition which is not a tautology have equivalent PCNF ($\Rightarrow$, $\Leftrightarrow$)
4) If the compound proposition which is not contradiction then its PCNF will contains all possible maxterms of its components.

problems :- using truth table

* The truth table for formula 'S' is given for in following determine its PDNF and PCNF

P	Q	R	S (assumption)
T	T	T	T - D
T	T	F	T - D
T	F	T	F
T	F	F	T - D
F	T	T	F
F	T	F	F
F	F	T	T - D
F	F	F	F

The PDNF :-
$(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (P \wedge \sim Q \wedge \sim R) \vee (\sim P \wedge \sim Q \wedge R)$

The PCNF :-
$(P \vee \sim Q \vee R) \wedge (\sim P \vee Q \vee R) \wedge (\sim P \vee Q \vee \sim R) \wedge (\sim P \vee \sim Q \vee \sim R)$

Find the PCNF of $P \Leftrightarrow Q$ (using truth table)

P	Q	$P \Leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

* NOTE :
For obtaining PCNF of formula 'S', one can also construct the PCNF of $\sim S$ and then apply $(\sim)$ Negation

14) obtain the PCNF of a formula 'S' is $(\sim P \Rightarrow R) \wedge (Q \Leftrightarrow P)$ and also find PDNF of S.

Given that
$(\sim P \Rightarrow R) \wedge (Q \Leftrightarrow P) = (\sim(\sim P) \vee R) \wedge [(Q \Rightarrow P) \wedge (P \Rightarrow Q)]$
∵ implication law

$\equiv (P \vee R) \wedge [(\sim Q \vee P) \wedge (\sim P \vee Q)]$

$\equiv (P \vee R \vee F) \wedge [(\sim Q \vee P \vee F) \wedge (\sim P \vee Q \vee F)]$

$\equiv [P \vee R \vee (Q \wedge \sim Q)] \wedge [(\sim Q \vee P \vee (R \wedge \sim R)) \wedge (\sim P \vee Q \vee (R \wedge \sim R))]$

$\equiv (P \vee R \vee Q) \wedge (P \vee R \vee \sim Q) \wedge [(\sim Q \vee P \vee R) \wedge (\sim Q \vee P \vee \sim R) \wedge [(\sim P \vee Q \vee R) \wedge (\sim P \vee Q \vee \sim R)]]$ distri

$\equiv (P \vee Q \vee R) \wedge (P \vee \sim Q \vee R) \wedge (P \vee \sim Q \vee \sim R) \wedge (P \vee \sim Q \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge (\sim P \vee Q \vee \sim R)$ (Idempotent law)

$\equiv$ ~~$(P \vee Q \vee R) \wedge (P \vee \sim Q \vee R)$~~ $\wedge [(\sim Q \vee P \vee R) \wedge (\sim Q \vee P \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge (\sim P \vee Q \vee \sim R)]$

$\equiv (p \vee Q \vee R) \wedge (p \vee \sim Q \vee R) \wedge (p \wedge \sim Q \vee \sim R) \wedge (\sim p \vee Q \vee R) \wedge$

$\cancel{(\sim p \vee Q \vee R)} \wedge (\sim p \vee Q \vee \sim R)$

which is the required PCNF of $\sim S$

Now the conjunctive Normal form is can be obtained by written the conjunction of remaining maxterms

$2^3 = 8 \Rightarrow 8 - 5 = 3$

$\therefore (\sim p \vee \sim Q \vee \sim R) \wedge (\sim p \vee \sim Q \vee R) \wedge (p \vee Q \vee \sim R)$ then

the considering the $\sim(\sim S)$

we obtain PDNF of S

$\sim(\sim S) = \sim\left[(\sim p \vee \sim Q \vee \sim R) \wedge (\sim p \vee \sim Q \vee R) \wedge (p \vee Q \vee \sim R)\right]$

$\equiv \sim(\sim p \vee \sim Q \vee \sim R) \vee \sim(\sim p \vee \sim Q \vee R) \vee \sim(p \vee Q \vee \sim R)$

$= (p \wedge Q \wedge R) \vee (p \wedge Q \wedge \sim R) \vee (\sim p \wedge \sim Q \wedge R)$

which is the required PDNF.

1)* obtain PCNF of $(p \wedge Q) \vee (\sim p \wedge Q) \vee (p \wedge \sim Q)$

2)* obtain PDNF, PCNF for the following and which of the formula are tautology

i) $Q \wedge (p \vee \sim Q)$ ii) $(Q \Rightarrow p) \wedge (\sim p \wedge Q)$

3)* find the PDNF and PCNF of

i) $\sim(p \vee Q)$ ii) $\sim(p \Rightarrow Q)$

imp

** 4) obtain the PDNF and PCNF of the formula $p \vee [\sim p \Rightarrow (Q \vee (\sim Q \Rightarrow R))]$

(1) $(p \wedge Q) \vee (\sim p \wedge Q) \vee (p \wedge \sim Q)$

$(p \wedge Q) \equiv (p \vee F) \wedge (Q \vee F)$

$= [p \vee (Q \wedge \sim Q)] \wedge [Q \vee (p \wedge \sim p)]$

$= [(p \vee Q) \wedge (p \vee \sim Q)] \wedge [(Q \vee p) \wedge (Q \vee \sim p)]$

$= (p \vee Q) \wedge (p \vee \sim Q) \wedge (\sim p \vee Q)$

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$(\sim p\wedge Q) = (\sim p\vee F)\wedge(Q\vee F)$

$= (\sim p\vee(Q\wedge\sim Q))\wedge(Q\vee(p\wedge\sim p))$

$= [(\sim p\vee Q)\wedge(\sim p\vee\sim Q)]\wedge[(Q\vee p)\wedge(Q\vee\sim p)]$

$= (\sim p\vee Q)\wedge(\sim p\vee\sim Q)\wedge(Q\vee\sim p)$

$(p\wedge\sim Q) = (p\vee F)\wedge(\sim Q\vee F)$

$= [p\vee(Q\wedge\sim Q)]\wedge[\sim Q\vee(p\wedge\sim p)]$

$= [(p\vee Q)\wedge(p\vee\sim Q)]\wedge[(\sim Q\vee p)\wedge(\sim Q\vee\sim p)]$

$= (p\vee Q)\wedge(p\vee\sim Q)\wedge(\sim p\vee\sim Q)$

$\Rightarrow [(p\vee Q)\wedge(p\vee\sim Q)\wedge(\sim p\vee Q)]\vee[(\sim p\vee Q)\wedge(p\vee Q)\wedge(\sim p\vee\sim Q)]$

$\vee[(p\vee Q)\wedge(p\vee\sim Q)\wedge(\sim p\vee\sim Q)]$

2) (i) $Q\wedge(p\vee\sim Q)$

PDNF $\Rightarrow (Q\wedge T)\wedge[(p\wedge T)\vee(\sim Q\wedge T)]$

$\Rightarrow [Q\wedge(p\vee\sim p)]\wedge[(p\wedge(Q\vee\sim Q))\vee(\sim Q\wedge(p\vee\sim p))]$

$= [(Q\wedge p)\vee(Q\wedge\sim p)]\wedge[(p\wedge Q)\vee(p\wedge\sim Q)\vee(\sim Q\wedge p)\vee(\sim Q\wedge\sim p)]$

$= (p\wedge Q)\vee(\sim p\wedge Q)\vee(p\wedge\sim Q)\vee(\sim p\wedge\sim Q)$

PCNF $\Rightarrow Q\wedge(p\vee\sim Q)$

$= (Q\vee F)\wedge[(p\vee F)\vee(\sim Q\vee F)]$

$= [Q\vee(p\wedge\sim p)]\wedge[(p\vee(Q\wedge\sim Q))\vee(\sim Q\vee(p\wedge\sim p))]$

$= [(Q\vee p)\wedge(Q\vee\sim p)]\wedge[(p\vee Q)\wedge(p\vee\sim Q)]\vee[(\sim Q\vee p)\wedge(\sim Q\vee\sim p)]$

$= (p\vee Q)\wedge(\sim p\vee Q)\wedge(p\vee\sim Q)\wedge(\sim p\vee\sim Q)$

(ii) $(Q\Rightarrow p)\wedge(\sim p\wedge Q)$

PDNF $\Rightarrow (\sim Q\vee p)\wedge(\sim p\wedge Q)$

$= [(\sim Q\wedge T)\vee(p\wedge T)]\wedge[(\sim p\wedge T)\wedge(Q\wedge T)]$

$= [\sim Q\wedge(p\vee\sim p)]\vee[p\wedge(Q\vee\sim Q)]\wedge[\sim p\wedge(Q\vee\sim Q)]\wedge$
$(Q\wedge(p\vee\sim p))]$

$= [(\sim Q\wedge p)\vee(\sim Q\wedge\sim p)]\vee[(p\wedge Q)\vee(p\wedge\sim Q)]\wedge[(\sim p\wedge Q)\wedge(\sim p\wedge\sim Q)]$

$\wedge[(Q\wedge p)\vee(Q\wedge\sim p)]$

$= [(p\wedge \sim Q)\vee(p\wedge Q)\vee(\sim p\wedge \sim Q)] \wedge [(\sim p\wedge Q)\vee(p\wedge Q)\vee(\sim p\wedge \sim$

$= (p\wedge \sim Q)\vee(p\wedge Q)\vee(\sim p\wedge \sim Q)$

PCNF :- $(Q\Rightarrow p)\wedge(\sim p\wedge Q)$

$= (\sim Q\vee p)\wedge(\sim p\wedge Q)$

$= [(\sim Q\vee F)\vee(p\vee F)]\wedge[(\sim p\vee F)\wedge(Q\vee F)]$

$= [\sim Q\vee(p\wedge\sim p)]\vee(p\vee(Q\wedge\sim Q)]\wedge[(\sim p\vee(p\wedge\sim p))\wedge(Q\vee(p\wedge\sim$

$= [(\sim Q\vee p)\wedge(\sim Q\vee\sim p)]\vee[(p\vee Q)\wedge(p\vee\sim Q)]\wedge[(\sim p\vee p)\wedge(\sim p\wedge\sim p)$

$\wedge[(Q\vee p)\wedge(Q\vee\sim p)]$

(3)(i) $\sim(p\vee Q)$

PDNF : $\sim(p\vee Q) = \sim p\wedge\sim Q$

$\Rightarrow = (\sim p\wedge T)\wedge(\sim Q\wedge T)$

$= [\sim p\wedge(Q\vee\sim Q)]\wedge[\sim Q\wedge(p\vee\sim p)]$

$= [(\sim p\wedge Q)\vee(\sim p\wedge\sim Q)]\wedge[(\sim Q\wedge p)\vee(\sim Q\wedge\sim p)]$

$= [\underbrace{(\sim p\wedge Q)}_{Q}\vee\underbrace{(\sim p\wedge\sim Q)}_{R}]\wedge\underbrace{(\sim Q\wedge p)}_{P}$

$= [(p\wedge\sim Q)\wedge(\sim p\wedge Q)]\vee[(p\wedge\sim Q)\wedge(\sim p\wedge\sim Q)]$

$= (p\wedge\sim Q)\wedge(\sim p\wedge Q)\wedge(\sim p\wedge\sim Q)$

PCNF $\Rightarrow \sim(p\vee Q) = \sim p\wedge\sim Q$

$= (\sim p\vee F)\wedge(\sim Q\vee F)$

$= [\sim p\vee(Q\wedge\sim Q)]\wedge[\sim Q\vee(p\wedge\sim p)]$

$= [(\sim p\vee Q)\wedge(\sim p\vee\sim Q)]\wedge[(\sim Q\vee p)\wedge\underbrace{(\sim Q\vee\sim p)}_{P}]$ $\quad (p\wedge Q)\wedge R = p\wedge(Q\wedge R)$

$= (\sim p\vee\sim Q)\wedge(\sim p\vee Q)\wedge(p\vee\sim Q)$

(ii) $\sim(p\Rightarrow Q)$

PDNF :- $\sim(p\Rightarrow Q) = \sim[\sim p\vee Q] = p\wedge\sim Q = (p\wedge T)\wedge(\sim Q\wedge T)$

$= [p\wedge(Q\vee\sim Q)]\wedge[\sim Q\wedge(p\vee\sim p)]$

$= [(p\wedge Q)\vee(p\wedge\sim Q)]\wedge[(\sim Q\wedge p)\vee(\sim Q\wedge\sim p)]$

$= (p\wedge Q)\vee(p\wedge\sim Q)\vee(\sim p\wedge\sim Q)$

PCNF : $\sim(p\Rightarrow Q) = \sim[\sim p\vee Q] = (p\wedge\sim Q) = (p\vee F)\wedge(\sim Q\vee F)$

$= [p \vee (Q \wedge \sim Q)] \wedge [\sim Q \vee (p \wedge \sim p)]$

$= [(p \vee Q) \wedge (p \vee \sim Q)] \wedge [(\sim Q \vee p) \wedge (\sim Q \vee \sim p)]$

$= (p \vee Q) \wedge (p \vee \sim Q) \wedge (\sim p \vee \sim Q)$

1)
(ii) $p \vee (\sim p \wedge Q) \equiv p \vee Q$

L.H.S $\Rightarrow p \vee (\sim p \wedge Q)$

$= (p \wedge T) \vee (\sim p \wedge Q)$

$= [p \wedge (Q \vee \sim Q)] \vee (\sim p \wedge Q)$

$= [(p \wedge Q) \vee (p \wedge \sim Q)] \vee (\sim p \wedge Q)$

$= (p \wedge Q) \vee (p \wedge \sim Q) \vee (\sim p \wedge Q)$ — ①

R.H.S $\Rightarrow p \vee Q$

$= (p \wedge T) \vee (Q \wedge T)$

$= [p \wedge (Q \vee \sim Q)] \vee [Q \wedge (p \vee \sim p)]$

$= [(p \wedge Q) \vee (p \wedge \sim Q)] \vee [(Q \wedge p) \vee (Q \wedge \sim p)]$

$= (p \wedge Q) \vee (p \wedge \sim Q) \vee (\sim p \wedge Q)$ — ②

from ① and ②

L.H.S = R.H.S

$\therefore p \vee (\sim p \wedge Q) \equiv p \vee Q$

Inference theory for calculus :

The main function of logic is to provide rules of inference to inform a conclusion from certain premises. The theory associated with rules of inference is known as inference theory

Deduction (or) formal proof :

If a conclusion is derived from a set of premises by using accepted rules of reasoning then such a process of derivation is called a deduction or a formal proof and the argument is called a valid argument (or) conclusion is called a valid conclusion.

Note :

* Consider a set of ~~premises~~ statement $H_1, H_2, H_3 \ldots H_n$ and C then compound proposition of $H_1 \wedge H_2 \wedge H_3 \ldots H_n$ is called a argument.

where $H_1, H_2 \ldots H_n$ are called premises or assumptions (or) Hypothesis of the argument and C is called conclusion of arguments

* To determine whether conclusion logically follows from the given premises, we use the following two methods

1) Truth table

2. Rules of inference method.

Definition:

Let A and B be two statements formula, we say that 'B' logically from 'A' or 'B' is a valid conclusion from the premises 'A' iff $A \Rightarrow B$ is a tautology problems (using truth table) :-

1) Determine whether the conclusion 'C' follows logically from Hypothesis (premises / assumptions) H_1 and H_2

i) $H_1 : P \Rightarrow Q$, $H_2 : P$, $C : Q$

P	Q	$P \Rightarrow Q$ (H_1)	$H_1 \wedge H_2$	$H_1 \wedge H_2 \Rightarrow C$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

$\therefore H_1 \wedge H_2 \Rightarrow C$ is a tautology

$\therefore$ 'C' is valid conclusion.

2) $H_1 : P \Rightarrow Q$, $H_2 : \sim P$, $C : Q$

$\sim P$	P	Q	$P \Rightarrow Q$	$H_1 \wedge H_2$	$H_1 \wedge H_2 \Rightarrow C$
F	T	T	T	F	T
F	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	T	F

$\therefore H_1 \wedge H_2 \Rightarrow$ is not a tautology.

'c' is not a valid conclusion.

3) $H_1 : \sim P$, $H_2 : P \Leftrightarrow Q$, $C : \sim(P \wedge Q)$

4) $H_1 : P \Rightarrow Q$, $H_2 : \sim(P \wedge Q)$, $C : \sim P$

5) $H_1 : \sim Q$, $H_2 : P \Leftrightarrow Q$, $C : \sim P$

6) $H_1 : P \Rightarrow Q$, $H_2 : Q \Rightarrow R$, $C : P \Rightarrow R$

7) $H_1 : \sim P \vee Q$, $H_2 : \sim(Q \wedge \sim R)$, $H_3 : \sim R$, $C : \sim P$

* Rules of Inference :-

We now describe the process of derivation by which one demonstrates that a particular formula is a valid consequence of a given set of premises. Before we do this, we give two rules of inference which are called rules of P and rules T
Transformation

* Rules of inferences are :-

We know describe

1) Rule-P :- A premise may be introduced at any point in the derivation

2) Rule-T :- A formula 'S' may be introduced in derivation if 'S' is a tautologically implied by one or more of preceding formula's in the derivation.

* Implications :

I_1 : $P \wedge Q \Rightarrow P$

I_2 : $P \wedge Q \Rightarrow Q$

I_3 : $P \Rightarrow P \vee Q$

I_4 : $Q \Rightarrow P \vee Q$

I_5 : $\sim P \Rightarrow (P \Rightarrow Q)$

I_6 : $Q \Rightarrow (P \Rightarrow Q)$

I_7 : $\sim (P \Rightarrow Q) \Rightarrow P$

I_8 : $\sim (P \Rightarrow Q) \Rightarrow \sim Q$

I_9 : $P, Q \Rightarrow P \wedge Q$

I_{10} : $\sim P, P \vee Q \Rightarrow Q$

I_{11} : $P, P \Rightarrow Q \Rightarrow Q$

I_{12} : $\sim Q, P \Rightarrow Q \Rightarrow \sim P$

I_{13} : $P \Rightarrow Q, Q \Rightarrow R \Rightarrow P \Rightarrow R$

I_{14} : $P \vee Q, P \Rightarrow R, Q \Rightarrow R \Rightarrow R$.

Valid and Invalid arguments

An argument with premises $P_1, P_2, P_3 \ldots P_n$ and conclusion 'c' is said to be valid if whenever each of premises $P_1, P_2, \ldots P_n$ are true, then the conclusion 'c' is likewise true.

In otherwords the argument $(P_1 \wedge P_2 \wedge P_3 \ldots \wedge P_n) \Rightarrow c$ is valid.

The premises are always taken to be true where as the conclusion may be true (or) false. The conclusion is true only in case of a valid argument.

* Some of the rules are listed below :-

Rule of Inference	Tautology	Name
P $P \Rightarrow Q$ $\therefore Q$	$P \wedge (P \Rightarrow Q) \Rightarrow Q$	Modus ponens
$\sim Q$ $P \Rightarrow Q$ $\therefore \sim P$	$(\sim Q) \wedge (P \Rightarrow Q) \Rightarrow \sim P$	Modus tollens
$P \Rightarrow Q$ $Q \Rightarrow R$ $\therefore P \Rightarrow R$	$(P \Rightarrow Q) \wedge (Q \Rightarrow R) \Rightarrow P \Rightarrow R$	Hypothetical syllogism

$p \vee q$ $\sim p$ $\therefore q$	$(p \vee q) \wedge \sim p \Rightarrow q$	disjunctive syllogism
p $\therefore p \vee q$	$p \Rightarrow (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \Rightarrow p$	Simplification
p q $\therefore p \wedge q$	$(p) \wedge (q) \Rightarrow (p \wedge q)$	conjunction
$p \vee q$ $\sim p \vee R$ $\therefore q \vee R$	$(p \vee q) \wedge (\sim p \vee R) \Rightarrow q \vee R$	Resolution

★ Verify the following arguments valid or invalid:

1) If Sachin hits a century then he gets a free car
Sachin hits a century
∴ Sachin gets a free car

Sol:- p : Sachin hits a century
q : Sachin gets a free car.

Given argument is $\quad \dfrac{\begin{matrix} p \Rightarrow q \\ p \end{matrix}}{\therefore q}$ (modus ponens)

q : Sachin gets a free car.

2) If Sachin hits a century then he gets a free car
Sachin does not get free car
∴ Sachin has not hit a century

Sol:- P : Sachin hits a century
Q : Sachin gets a free car.
$\sim Q$: Sachin does not get free car.

Given argument is
$$\begin{array}{l} P \Rightarrow Q \\ \underline{\sim Q} \\ \therefore \sim P \end{array}$$
(modus tollens)

Sachin has not hit a century.

* I will become famous or I will not become a dancer. I will become a famous dancer. I will b-
∴ I will become famous

Sol:- P : I will become famous
$\sim Q$: I will not become a dancer
Q : I will become a dancer.

Given argument is
$$\begin{array}{l} P \vee \sim Q \\ \underline{Q} \\ P \end{array}$$
(modus ponens)

P: I will become a famous

* If it is rains today then we will not have a barbeque today. If we donot have a barbeque today then we will have a barbeque tomorrow.
∴ Show that if it rains today then we will have a barbeque tomorrow.

P : It is raining today
Q : we will not have a barbeque today
R : we will have a barbeque tomorrow

Given argument is
$$\begin{array}{l} P \Rightarrow Q \\ \underline{Q \Rightarrow R} \\ P \Rightarrow R \end{array}$$
(hypothetical syllogism)

* Determine that R is a valid inference from the premises $P \Rightarrow Q$, $Q \Rightarrow R$ and P

{1}	(1)	P	Rule P
{2}	(2)	$P \Rightarrow Q$	Rule P
{1,2}	(3)	Q	Rule-T (I_{11}) ($P, P \Rightarrow Q \Rightarrow Q$)
{4}	(4)	$Q \Rightarrow R$	Rule-P
{1,2,4}	(5)	R	Rule-T (I_{11}) ($Q, Q \Rightarrow R \Rightarrow R$)

(or)

{1}	(1)	$P \Rightarrow Q$	Rule-P premises
{2}	(2)	$Q \Rightarrow R$	Rule-P
{1,2}	(3)	$P \Rightarrow R$	Rule-T(I_{13}) ($P \Rightarrow Q, Q \Rightarrow R \Rightarrow P \Rightarrow R$) Transform
{4}	(4)	P	Rule-P
{1,2,4}	(5)	R	Rule-T, I_{11} ($P \Rightarrow R, P \Rightarrow R$)

R is a valid conclusion.

* Show that $\sim P$ logically follows from the premises, $\sim(P \wedge \sim Q)$, $\sim Q \vee R$, $\sim R$

{1}	(1)	$\sim(P \wedge \sim Q)$	Rule-P
{1}	(2)	$\sim P \vee Q$	Rule T (Demorgan's law
{1}	(3)	$P \Rightarrow Q$	Rule-T (implication law
{4}	(4)	$\sim Q \vee R$	Rule-P
{4}	(5)	$Q \Rightarrow R$	Rule-T (implication law
{1,4}	(6)	$P \Rightarrow R$	Rule-T(I_{13})($P \Rightarrow Q$, $Q \Rightarrow R$,

$\{1,4\}$	(7)	$\sim R \Rightarrow \sim P$	Rule-T ([illegible])
$\{8\}$	(8)	$\sim R$	Rule-P
$\{1,4,8\}$	(9)	$\sim P$	Rule-T $(I_{11})(P, P \Rightarrow Q)$.

$\sim P$ is a valid conclusion.

* Show that $R \vee S$ follows logically from the premises $C \vee D$, $(C \vee D) \Rightarrow \sim H$, $\sim H \Rightarrow (A \wedge \sim B)$ and $(A \wedge \sim B) \Rightarrow R \vee S$

operatory sets		premises	inferences
$\{1\}$	(1)	$C \vee D$	Rule-P
$\{2\}$	(2)	$(C \vee D) \Rightarrow \sim H$	Rule-P
$\{1,2\}$	(3)	$\sim H$	Rule-T (I_{11}), $(P, P \Rightarrow Q \Rightarrow Q)$
$\{4\}$	(4)	$\sim H \Rightarrow (A \wedge \sim B)$	Rule-P
$\{1,2,4\}$	(5)	$A \wedge \sim B$	Rule-T $(I_{11})(P, P \Rightarrow Q \Rightarrow Q)$
$\{6\}$	(6)	$(A \wedge \sim B) \Rightarrow R \vee S$	Rule-P
$\{1,2,4\}$	(5)	$A \wedge \sim B$	~~Rule-T $(I_{11})(P, P \Rightarrow Q$~~ $\Rightarrow Q)$
$\{6\}$	(6)	$(A \wedge \sim B) \Rightarrow R \vee S$	Rule-P
$\{1,2,4,6\}$	(7)	$R \vee S$	Rule-T $(I_{11})(P, P \Rightarrow Q \Rightarrow Q)$

$R \vee S$ logically follows from the given premises.

* Show that $S \vee R$ is tautology follows from the premises $(P \vee Q)$, $(P \Rightarrow R)$ & $(Q \Rightarrow S)$

$\{1\}$	(1)	$P \vee Q$	Rule-P
$\{1\}$	(2)	$\sim(\sim P) \vee Q$	Rule-T (Double negation)
$\{1\}$	(3)	$\sim P \Rightarrow Q$	Rule-T (implication law)
$\{4\}$	(4)	$Q \Rightarrow S$	Rule-P

{1,4}	(5)	$\sim P \Rightarrow S$	Rule-T (I_{13}) $(P \Rightarrow Q, Q \Rightarrow R \Rightarrow P \Rightarrow R)$
{1,4}	(6)	$\sim S \Rightarrow P$	Rule-T $(P \Rightarrow Q \Rightarrow \sim Q \Rightarrow \sim P)$
{7}	(7)	$P \Rightarrow R$	Rule-P
{1,4,7}	(8)	$\sim S \Rightarrow R$	Rule-T (I_{13}) $(P \Rightarrow Q, Q \Rightarrow R \Rightarrow P \Rightarrow R)$
{1,4,7}	(9)	$\sim(\sim S) \vee R$	Rule-T (Implication)
{1,4,7}	(10)	$S \vee R$	Rule-T (Double negation)

$S \vee R$ is a valid conclusion

4. S.T $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \Rightarrow R$, $P \Rightarrow M$ and $\sim M$

{1}	(1)	$P \Rightarrow M$	Rule-P
{2}	(2)	$\sim M$	Rule-P
{1,2}	(3)	$\sim P$	Rule-T (I_{12}) $(\sim Q, P \Rightarrow Q \Rightarrow \sim P)$
{4}	(4)	$P \vee Q$	rule-P
{1,2,4}	(5)	Q	Rule-T (I_{10}) $(\sim P, P \vee Q \Rightarrow Q)$
{6}	(6)	$Q \Rightarrow R$	Rule-P
{1,2,4,6}	(7)	R	Rule-T (I_{11}) $(Q, Q \Rightarrow R \Rightarrow R)$
{1,2,4,6}	(8)	$R \wedge (P \vee Q)$	Rule-T (I_9) $(P, Q \Rightarrow P \wedge Q)$

Connectives NAND and NOR

The word NAND is a combination of NOT and AND while the word NOR is the combination of NOT and OR

where

NOT stands for negation ($\sim$)

NAND stands for conjunction ($\wedge$)

NOR stands for disjunction ($\vee$)

* The connective NAND is denoted by the symbol $\uparrow$

$P \uparrow Q \equiv \sim(P \wedge Q)$

* The connective NOR is denoted by the symbol $\downarrow$

$P \downarrow Q \equiv \sim(P \vee Q)$

1) For any two propositions prove the following.

i) $\sim(P \downarrow Q) \equiv \sim P \uparrow \sim Q$

L.H.S $\Rightarrow$

$$\begin{aligned} \sim(P \downarrow Q) &\equiv \sim[\sim(P \vee Q)] \\ &\equiv \sim[\sim P \wedge \sim Q] \\ &\equiv \sim P \uparrow \sim Q \end{aligned}$$

ii) $\sim(P \uparrow Q) \equiv \sim P \downarrow \sim Q$

L.H.S $\Rightarrow$

$$\begin{aligned} \sim(P \uparrow Q) &\equiv \sim(\sim(P \wedge Q)) \\ &\equiv \sim[\sim P \vee \sim Q] \\ &\equiv \sim P \downarrow \sim Q \end{aligned}$$

2) For any three propositions P, Q, R prove that

i) $[P \uparrow (Q \uparrow R)] \equiv \sim P \vee (Q \wedge R)$

L.H.S $\Rightarrow$

$$\begin{aligned} P \uparrow (Q \uparrow R) &\equiv \sim [P \wedge (Q \uparrow R)] \\ &\equiv \sim[P \wedge \sim(Q \wedge R)] \\ &\equiv \sim P \vee (Q \wedge R) \end{aligned}$$

ii) $(P \uparrow Q) \uparrow R \equiv (P \wedge Q) \vee \sim R$

iii) $P \downarrow (Q \downarrow R) \equiv \sim P \wedge (Q \vee R)$

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ii) $(P \uparrow Q) \uparrow R \equiv (P \wedge Q) \vee \sim R$.

L.H.S

$$(P \uparrow Q) \uparrow R \equiv \sim(P \wedge Q) \uparrow R$$

$$\equiv \sim[\sim(P \wedge Q) \wedge R]$$

$$\equiv (P \wedge Q) \vee \sim R$$

$\therefore (P \uparrow Q) \uparrow R \equiv (P \wedge Q) \vee \sim R$

(iii) $P \downarrow (Q \downarrow R) \equiv \sim P \wedge (Q \vee R)$

L.H.S

$$P \downarrow (Q \downarrow R) \equiv P \downarrow [\sim(Q \vee R)]$$

$$\equiv \sim[P \vee \sim(Q \vee R)]$$

$$\equiv \sim P \wedge (Q \vee R)$$

$\therefore P \downarrow (Q \downarrow R) \equiv \sim P \wedge (Q \vee R)$

problems using truth table $H_1 \wedge H_2 \Rightarrow C$

3) $H_1 : \sim P$, $H_2 : P \Leftrightarrow Q$, $C : \sim(P \wedge Q)$

P	$\sim P$	Q	$H_2 : P \Leftrightarrow Q$	$(P \wedge Q)$	$\sim(P \wedge Q)$	$H_1 \wedge H_2$	$H_1 \wedge H_2 \Rightarrow C$
T	F	T	T	T	F	F	T
T	F	F	F	F	T	F	T
F	T	T	F	F	T	F	T
F	T	F	T	F	T	T	T

$\therefore H_1 \wedge H_2 \Rightarrow C$ is a tautology

$\therefore$ 'C' is valid conclusion.

4. $H_1 : P \Rightarrow Q$ $H_2 : \sim(P \wedge Q)$, $C : \sim P$

P	Q	$\sim P$	$P \Rightarrow Q$	$P \wedge Q$	$\sim(P \wedge Q)$	$H_1 \wedge H_2$	$H_1 \wedge H_2 \Rightarrow C$
T	T	F	T	T	F	F	T
T	F	F	F	F	T	F	T
F	T	T	T	F	T	T	T
F	F	T	T	F	T	T	T

$\therefore$ $H_1 \wedge H_2$ is a tautology

$\therefore$ 'c' is valid conclusion

5. $H_1 : \sim Q$ $H_2 : P \Rightarrow Q$, $C : \sim P$

P	Q	$\sim P$	$\sim Q$	$P \Rightarrow Q$	$H_1 \wedge H_2$	$H_1 \wedge H_2 \Rightarrow C$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

$\therefore$ $H_1 \wedge H_2$ is a tautology

$\therefore$ 'c' is valid conclusion.

6. $H_1 : P \Rightarrow Q$ $H_2 : Q \Rightarrow R$ $C : P \Rightarrow R$

P	Q	R	$P \Rightarrow Q$	$Q \Rightarrow R$	$H_1 \wedge H_2$	$P \Rightarrow R$	$H_1 \wedge H_2 \Rightarrow C$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$\therefore$ $H_1 \wedge H_2$ is a tautology.

$\therefore$ c is valid conclusion.

7. $H_1 : \sim p \vee Q$, $H_2 : \sim(Q \wedge \sim R)$, $H_3 : \sim R$; $C : \sim p$

P	Q	R	$\sim p$	$\sim Q$	H_1 $\sim p \vee Q$	$Q \wedge \sim R$	H_2 $\sim(Q \wedge \sim R)$	H_3 $\sim R$	$H_1 H_2$	$H_1 \wedge H_2 \wedge H_3$	$H_1 \wedge H_2$
T	T	T	F	F	T	F	T	F	T	F	T
T	T	F	F	F	T	T	F	T	F	F	T
T	F	T	F	T	F	F	T	F	F	F	T
T	F	F	F	T	F	F	F	T	F	F	T
F	T	T	T	F	T	F	T	F	T	F	T
F	F	F	T	F	T	T	F	T	F	F	T
F	F	T	T	T	T	F	T	F	T	F	T
F	F	F	T	T	T	F	T	T	T	T	T

$\therefore$ $H_1 \wedge H_2 \wedge H_3 \Rightarrow C$ is a tautology.

$\therefore$ 'C' is a valid conclusion.

21/10 2. Set Theory

Set :-

A set is a collection of well defined objects elements.

for Eg:- $A = \{a, b, c, d\}$

Finite set:-

A set having countable no of elements is called finite set.

Eg:- $A = \{1, 2, 3, 4\}$

Infinite set :-

A set having uncountable no. of elements is called infinite set.

Eg:- $N = \{1, 2, 3, \ldots\}$

Single set:-

A set having only one element is called single set.

Eg:- $X = \{2\}$ → even prime num.

Null set / Empty set:-($\emptyset$)

A set which does not contain any element is called null set or empty set.

Eg:- $\emptyset / = \{ \}$

Equal set :-($A = B$)

Two sets are said to be equal iff $A \subseteq B$ Then $A \subseteq B$.

Subset:-

Let A and B are two non empty sets, The set A is called subset of B iff every element of a A is in element of B.

If $A \subset B$, Then B is called superset of A $(B \supset A)$

Power set:-

If 'S' is any set then The family of all The subsets of S is called The power set.

→ It is denoted by P(S)

If A is a finite set of n elements. Then The no.of subsets of A is 2^n.

Eg: ① $A = \{a\}$

subset of A are $\{a\}, \{\}$..

② $A = \{a, b\}$

$\{a\}, \{b\}, \{ab\}, \{\}$

③ $X = \{a, b, c\}$

$\{a\}, \{b\}, \{c\}, \{a,b\} \{bc\} \{ca\} \{a, b, c\}, \{\}$

Universal set:-

The set theory of all sets under discussion are assumed to be The subset of The fixed large Set is called universal set.

→ It is denoted by U or $\mathcal{U}$

Eg:- $U = \{1, 2, 3, ~~4~~, 5, 7, 8, 9, 10\}$

Union of sets:-

Consider 2 sets A, B Then The set consisting of all elements That belongs to A or B, or in both A and B is called The union of A and B.

→ It is denoted by $A \cup B$

Intersection of sets:-

let A and B are two non empty sets. The intersection of A and B is the set of all elements which are in both A and B.

→ It is denoted by $A \cap B$

Complement of a set:-

let A be any set. The complement of A is the set of elements that belongs to universe set but do not belongs to A.

$A \in U \; , \; U \notin A.$

If U is the universal set then the complement of A is $U - A$ (or) $U - A$.

It is denoted by A^c (or) A' (or) $\bar{A}$.

22/10/22

Laws of Set Theory:

① Commutative law:

i) $A \cup B = B \cup A$

ii) $A \cap B = B \cap A$

② Associative Law:

i) $A \cup (B \cup C) = (A \cup B) \cup C$

ii) $A \cap (B \cap C) = (A \cap B) \cap C$

③ Distributive Law:

i) $A \cap (B \cap C) = (A \cap B) \cup (A \cup C)$

ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

④ Idempotent Law:

i) $A \cup A = A$

ii) $A \cap A = A$

(5) Identity law:

i) $A \cup \phi = A$

ii) $A \cap U = A$

(6) Laws of double Complement:

i. $\overline{(\overline{A})} = A$ (or) $(A^c)^c = A$ (or) $(A')' = A$

(7) Inverse law:

i) $A \cup \overline{A} = U$

ii) $A \cap \overline{A} = \phi$

(8) Demorgan's law:

i) $(\overline{A \cup B}) = \overline{A} \cap \overline{B}$ (or)

ii) $(A \cap B)^c = A^c \cup B^c$

(9) Domination laws:

i) $A \cup U = U$

ii) $A \cap \phi = \phi$

(10) Absorption law:

i) $A \cup (A \cap B) = A$

ii) $A \cap (A \cup B) = A$

1) Show that $A \cap (B \cap C) = (A \cap B) \cap C$

Sol: Let x be the any arbitary element of $A \cap (B \cap C)$

LHS $A \cap (B \cap C) = x \in A \cap (B \cap C)$

$= x \in A$ and $x \in (B \cap C)$

$= (x \in A$ and $x \in B)$ and $x \in C$

$= x \in (A \cap B)$ and $x \in C$

$= x \in (A \cap B) \cap C$

$A \cap (B \cap C) = (A \cap B) \cap C$

⑦ S.T $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

Let x be the orbitary element of $A\cup(B\cap C)$

LHS

$A\cup(B\cap C) = x \in [A\cup(B\cap C)]$

$\Rightarrow x \in A$ or $x \in (B\cap C)$

$\Rightarrow x \in A$ or $(x \in B$ and $x \in C)$

$\Rightarrow (x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C)$

$\Rightarrow x \in (A\cup B)$ and $x \in A\cup C$

$\Rightarrow x \in (A\cup B)\cap(A\cup C)$

$\therefore A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

⑧ Let A, B be any two sets then P.T

i) $(A\cup B)^c = A^c\cap B^c$ ii) $(A\cap B)^c = A^c\cup B^c$

i) proof: Let x be the any orbitary element of $(A\cup B)^c$

LHS:

$(A\cup B)^c = x \in (A\cup B)^c$ $\langle A^c = U - A\rangle$

$\Rightarrow x \in [U - (A\cup B)]$

$\Rightarrow x \in U$ and $x \notin (A\cup B)$

$\Rightarrow x \in U$ and $[x \notin A$ or $x \in B]$

$\Rightarrow (x \in U$ and $x \notin A)$ and $(x \in U$ and $x \notin B)$

$\Rightarrow (x \in U - A)$ and $x \in (U - B)$

$\Rightarrow x \in A^c$ and $x \in B^c$

$\Rightarrow x \in A^c \cap B^c$

ii) Let x be the orbitary element of $(A\cap B)^c$

LHS

$(A\cap B)^c = x \in (A\cap B)^c$

$\Rightarrow x \in [U - (A\cap B)]$

$\Rightarrow x \in U$ and $x \notin (A\cap B)$

$\Rightarrow x \in U$ and $(x \notin A$ or $x \notin B)$

$\Rightarrow (x \in \mathcal{U}$ and $x \notin A)$ or $(x \in \mathcal{U}$ and $x \notin B)$

$\Rightarrow (x \in \mathcal{U} - A)$ or $(x \in \mathcal{U} - B)$

$\Rightarrow x \in A^c$ or $x \in B^c$

$\Rightarrow x \in A^c \cup B^c$

$(A \cap B)^c = A^c \cup B^c$ //

25/10

4) prove that $A - (B \cup C) \cancel{\cap (A - C)} = (A - B) \cap (A - C)$

proof:-

LHS :

$$A - (B \cup C) = x \in [A - (B \cup C)]$$
$$= x \in A \text{ and } x \notin (B \cup C)$$
$$= x \in A \text{ and } (x \notin B \text{ and } x \notin C)$$
$$= (x \in A \text{ and } x \notin B) \text{ and } (x \in A \text{ and } x \notin C)$$
$$= x \in (A - B) \text{ and } x \in (A - C)$$
$$= x \in (A - B) \cap (A - C)$$

$\therefore A - (B \cup C) = (A - B) \cap (A - C)$

5) prove that $A - (B \cap C) = (A - B) \cup (A - C)$

proof:- let 'x' be the arbitary elt of $A - (B \cap C)$

LHS

$$A - (B \cap C) = x \in [A - (B \cap C)]$$
$$= x \in A \text{ and } x \notin (B \cap C)$$
$$= x \in A \text{ and } (x \notin B \text{ or } x \notin C)$$
$$= (x \in A \text{ and } x \notin B) \text{ or } (x \in A \text{ and } x \notin C)$$
$$= x \in (A - B) \text{ or } x \in (A - C)$$
$$= x \in (A - B) \cup (A - C)$$

$\therefore A - (B \cap C) = (A - B) \cup (A - C)$

6) let $S_1 = \{1,2,3\}$, $S_2 = \{3,4,5,6\}$ Then find
(i) $S_1 \cup S_2$ (ii) $S_1 \cap S_2$ (iii) $S_1 - S_2$ (iv) $S_2 - S_1$

1) $S_1 \cup S_2 = \{1,2,3\} \cup \{3,4,5,6\}$
$= \{1,2,3,4,5,6\}$

2) $S_1 \cap S_2 = \{1,2,3\} \cap \{3,4,5,6\}$
$= \{3\}$

3) $S_1 - S_2 = \{1,2,\cancel{3}\} - \{\cancel{3},4,5,6\}$
$= \{1,2\}$

4) $S_2 - S_1 = \{\cancel{3},4,5,6\} - \{1,2,\cancel{3}\}$
$= \{4,5,6\}$

7) If $A = \{2,3,4\}$ $B = \{3,4,5,6\}$ $C = \{2,4,6,8\}$ Then Verify

① $A \cap (B-C) = (A \cap B) - (A \cap C)$

LHS $B - C = \{3,4,5,6\} - \{2,4,6,8\}$
$= \{3,5\}$

$A \cap (B-C) = \{2,3,4\} \cap \{3,5\}$
$= \{3\}$

$A \cap B = \{2,3,4\} \cap \{3,4,5,6\}$
$= \{3,4\}$

$A \cap C = \{2,3,4\} \cap \{2,4,6,8\}$
$= \{2,4\}$

$(A \cap B) - (A \cap C) = \{3,4\} - \{2,4\} = \{3\}$

$\therefore$ LHS = RHS.

8) Let $U = \{1,2,3,4,5,6,7,8,9,10\}$, $A = \{1,3,5\}$

$B = \{2,4,6,8\}$ $C\{2,5,10\}$ then verify

① $(A \cap B)^c = A^c \cup B^c$

② $(A \cup B)^c = A^c \cap B^c$

③ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

④ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

① $(A \cap B)^c = A^c \cup B^c$

$A \cap B = \{1,3,5\} \cap \{2,4,6,8\}$

$= \{\ \}$

② $(A \cap B)^c = U - (A \cap B)$

$= \{1,2,3,4,5,6,7,8,9,10\} - \{\ \}$

$= \{1,2,3,4,5,6,7,8,9,10\}$

$A^c = U - A$

$= \{\cancel{1},2,\cancel{3},4,\cancel{5},6,7,8,9,10\} - \{\cancel{1},\cancel{3},\cancel{5}\}$

$= \{2,4,6,7,8,9,10\}$

$B^c = U - B = \{1,\cancel{2},3,\cancel{4},5,\cancel{6},7,\cancel{8},9,10\} - \{\cancel{2},\cancel{4},\cancel{6},\cancel{8}\}$

$= \{1,3,5,7,9,10\}$

$A^c \cup B^c = \{2,4,6,7,8,9,10\} \cup \{1,3,5,7,9,10\}$

$= \{1,2,3,4,5,6,7,8,9,10\}$

$\therefore (A \cap B)^c = A^c \cup B^c$

③ $B \cup C = \{2,4,6,8\} \cup \{2,5,10\}$

$= \{2,4,5,6,8,10\}$

$A \cap (B \cup C) = \{1,3,5\} \cap \{2,4,5,6,8,10\}$

$= \{5\}$

$A \cap B = \{1, 3, 5\} \cap \{2, 4, 6, 8\}$

$= \{\ \}$

$A \cap C = \{1, 3, 5\} \cap \{2, 5, 10\}$

$= \{5\}$

$(A \cap B) \cup (A \cap C) = \{\ \} \cup \{5\}$

$= \{5\}$

$\therefore$ LHS = RHS

(4)

$B \cap C = \{2, 4, 6, 8\} \cap \{2, 5, 10\}$

$= \{2\}$

$A \cup (B \cap C) = \{1, 3, 5\} \cup \{2\}$

$= \{1, 2, 3, 5\}$

$A \cup B = \{1, 3, 5\} \cup \{2, 4, 6, 8\}$

$= \{1, 2, 3, 4, 5, 6, 8\}$

$A \cup C = \{1, 3, 5\} \cup \{2, 5, 10\}$

$= \{1, 2, 3, 5, 10\}$

$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6, 8\} \cap \{1, 2, 3, 5, 10\}$

$= \{1, 2, 3, 5\}$

$\therefore A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$

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26/10

Computer representation of sets :-

Bit string method can be used to represent sets to the Computer by storing their elements in an order manner since set operations like union, intersection, difference etc. Take large amount of time for searching their elements, therefore an orbitary ordering of the elements of the universal set to store the elements is commonly used to represent sets. Suppose an universal set 'U' = $\{x_1, x_2, x_3, --- x_n\}$ has 'n' elements. Then its subsets can be represented with a bit string of length 'n'.

A Bit string is a string over the alphabet the set is $\{0,1\}$. If the set 'A' is subset of 'U' then it is represented by bitstring method. where ith bit of string is one when $x_i \in A$ and 0 when $x_i \notin A$. this rule permits us to represent an universal set of length 'n' in the computer assignment either 0 or 1 to each location of A[K]. of the array specifies a unique subsets of U.

Eg:- ① $U = \{1,2,3,4,5,6,7\}$ be a universal set

$A = \{1,3,5\}$ $B = \{2,5\}$

$\therefore A = \{1,0,1,0,1,0,0\}$ $B = \{0,1,0,0,1,0,0\}$

② $U = \{1,2,3,4,5,6\}$ be a universal set

and $A = \{1,3\}$ $B = \{3,5,6\}$

$\therefore A = \{1,0,1,0,0,0\}$ $B = \{0,0,1,0,1,1,\}$

③ If $U = \{1,2,3,4,5,6\}$

$A = \{1,2,3,4\}$ and $B = \{3,4,5,6\}$

find the bit strings for A and B and use them to find union, intersection. also find A^c and B^c.

Sol:- length of universal set U = 6.

$A = \{1,1,1,1,0,0\}$ $A \cup B = \{1,2,3,4\} \cup \{3,4,5,6\}$

$B = \{0,0,1,1,1,1\}$ $= \{1,2,3,4,5,6\}$

$= \{1,1,1,1,1,1\}$

$A \cap B = \{1,2,3,4\} \cap \{3,4,5,6\}$

$= \{3,4\}$

$A \cap B = \{0,0,1,1,0,0\}$

$A^c = U - A$

$= \{1,2,3,4,5,6\} - \{1,2,3,4\}$

$= \{5,6\}$

$A^c = \{0,0,0,0,1,1\}$

$B^c = U - B$

$= \{1,2,3,4,5,6\} - \{3,4,5,6\}$

$= \{1,2\}$

$B^c = \{1,1,0,0,0,0\}$.

② If $U = \{1,2,3,4,5,6,7\}$ Then find set specified by each of the following bit strings.

(1) 1010100 $= \{1,3,5\}$

(2) 0101010 $= \{2,4,6\}$

(3) 0011001 $= \{3,4,7\}$

(4) 1110001 $= \{1,2,3,7\}$

28/10

The Inclusion and Exclusion principle:-

The no. of elements in a finite set is called the inclusion and exclusion principle or Cardinal no. of set 'A' is denoted by $n(A)$.

Eg:- Q) If $A = \{1,2,3\}$ Then find $n(A)$

$n(A) = 3$.

Formulas:-

1) $n(A \cup B) \leq n(A) + n(B)$, when $n(A \cap B) = \emptyset$

2) $n(A \cap B) \leq \min[n(A), n(B)]$

3) $n(A \Delta B) = n(A \oplus B) = n(A) + n(B) = n(A \cap B)$

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④ $n(A-B) \geq n(A) - n(B)$

⑤ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

⑥ $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

⑦ if A and B are disjoint sets, Then $n(A \cap B) = \emptyset$

$$n(A \cup B) = n(A) + n(B)$$

⑧ $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

⑨ If A, B and C are mutually disjoints sets, Then.

$n(A \cup B \cup C) = n(A) + n(B) + n(C)$

⑩ $n(A^C) = n(\mu) - n(A)$

⑪ $n(A) = n[(A \cap B^C) \cup (A \cap B)] = n[(A-B) \cup (A \cap B)]$

$= n(A-B) + n(A \cap B)$

$= n(A \cap B)^c + n(A \cap B)$

⑫ $n(B) = n(B-A) + n(A \cap B)$

⑬ $n(A \cup B) = n(A-B) + n(A \cap B) + n(B-A)$

1) Out of 450 students in a school 193 students read science and 200 students read commerce and 80 students read neither. Find out how many read both.

Sol:- $n(\mu) = n(T) = 450$.

$n(S) = 193$

$n(C) = 200$

$\langle A^c = \mu - A \rangle$

$n(S^c \cap C^c) = 80$

$n(S \cup C)^c = 80$

$n(A^c) = n(\mu) - n(A)$

$n(S \cup C)^c = n(\mu) - n(S \cup C)$

$80 = 450 - n(S \cup C)$

$n(S \cup C) = 450 - 80 = 370$.

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$$n(S \cap C) = n(S) + n(C) - n(S \cup C)$$
$$= 193 + 200 - 370$$
$$= 393 - 370$$
$$= 23$$

$$\boxed{n(S \cap C) = 23}$$

2) A group of 20 persons, 10 are interested in music 7 are in photography, 4 are interested in swimming for that more 4 are interested in both music and photography; 3 are interested in music and swimming, 2 are interested in photography and swimming, one ~~are~~ is interested in swimming, photography and music. How many are interested in photography but not in music and swimming.

$n(U) = 20$ $\quad$ $n(M \cap P) = 4$

$n(M) = 10$ $\quad$ $n(M \cap S) = 3$

$n(P) = 7$ $\quad$ $n(P \cap S) = 2$

$n(S) = 4$ $\quad$ $n(M \cap P \cap S) = 1$

$$n(P \cap S^c \cap M^c) = ?$$

$$\boxed{n[A \cap (B \cup C)^c] = n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)}$$

$$n(P \cap S^c \cap M^c) = n[P \cap (S \cup M)^c] + n(P \cap S \cap M)$$

$\because n(A \cap B^c) = n(A) - n(A \cap B)$

$$= n(P) - n(P \cap (S \cup M)) + n(P \cap S \cap M)$$
$$= n(P) - n[(P \cap S) \cup (P \cap M)] + n(P \cap S \cap M)$$
$$= n(P) - [n(P \cap S) + n(P \cap M) - n[(P \cap S) \cap (P \cap M)]] + n(P \cap S \cap M)$$
$$= n(P) - n(P \cap S) - n(P \cap M) + n(P \cap S \cap M)$$
$$= 7 - 2 - 4 + 1$$
$$= 2$$

Relation :-

The set of ordered pairs is called a relation.

Cartesian product of the sets :-

If A and B are two non empty sets than the set of all distinct or different order pairs whose first number belongs to A and second number belongs to B is called a cartesian product of A and B. It is denoted by AXB.

$\therefore A \times B = \{(a,b) : a \in A, b \in B\}$

Eg:- If $A = \{1,2,3\}$ and $B = \{2,3\}$, prove that $A \times B \neq B \times A$. Also find $n(A \times B)$

Sol:- $A = \{1,2,3\}$ and $B = \{2,3\}$

$A \times B = \{(1,2)\ (2,2)\ (3,2)\ (2,3)\ (1,3)\ (3,3)\}$

$B \times A = \{(2,1)\ (3,1)\ (2,2)\ (3,2)\ (2,3)\ (3,3)\}$

$\therefore A \times B \neq B \times A$

$n(A \times B) = 6$

Cartesian product of n sets :-

By the definition of cartesian product or cross product to more than two sets $A_1, A_2, A_3, --- A_n$ are n sets, the set of ordered pairs $(a_1, a_2, a_3, --- a_n)$ $a_1 \in A_1$; $a_2 \in A_2$, $a_3 \in A_3$ --- $a_n \in A_n$ is called the cartesian product of $A_1, A_2, A_3, --- A_n$ and it is denoted by $A_1 \times A_2 \times A_3 \times --- \times A_n = \prod_{i=1}^{n} A_i$

Binary Relation :-

Let A and B are two non empty sets then the binary relation R from A to B is defined to be a subsets of AXB Symbollicaly $R: A \to B$, where $a \in A$ and $b \in B$. iff $R \subseteq A \times B$ and $(a,b) \in R$ Then we say that a is related to b and we write aRb. If a is not related to b and we write $a \not R b$

eg: let $A \times B = \{(1,2)(1,4)(2,2)(2,4)\}$

$R = \{(1,2)(2,4)(2,1)\}$ state whether R is a from A to B or not.

Sol:- $R = \{(1,2)(2,4)(2,1)\}$

$A \times B = \{(1,2)(1,4)(2,2)(2,4)\}$

$\therefore$ R is not the subset of $A \times B$ because $(2,1) \notin A \times B$.

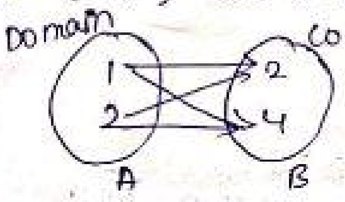
$\therefore$ R is not related to $A \to B$

$A \not R B$

Domain and Range of a relation:-

If R is a relation from A to B then the set of elements in a are related to some elem in B is called the domain of R and set B is called the co-domain of R.

$A = \{1,2\}$ $B = \{2,4\}$



3/10

Set Operations on Relations:-

All binary relations are set of order pairs, Therefore set of operations can be carry subsets.

let R and S be two relations, then few two relations defined as

1) Intersection of R and S : $x(R \cap S)y = (xRy) \cap (xSy)$

2) union of R and S : $x(R \cup S)y = (xRy) \cup (xSy)$

3) difference of R and S : $x(R-S)y = (xRy) - (xSy)$

4) Complement of R : $x(R')y = xR'y$

1) If $A = \{2,3,5\}$, $B\{6,8,10\}$, $C = \{2,3\}$, $D = \{8,10\}$ are four non-empty sets suppose a relation R from A to B is defined as $R = \{(2,6)(2,8)(3,10)\}$ and the relation S from C to D is defined as $S = \{(2,8)(3,10)\}$ then find.

1.) $R \cup S$ 5.) $\overline{S}$

2.) $R \cap S$

3.) $R - S$

4.) $\overline{R}$.

Sol:- Given that

$A = \{2,3,5\}$, $B = \{6,8,10\}$, $C = \{2,3\}$, $D = \{8,10\}$

$A \times B = \{(2,6)\ (2,8)\ (2,10)\ (3,6)\ (3,8)\ (3,10)\ (5,6)\ (5,8)\ (5,10)\}$

$C \times D = \{(2,8)\ (2,10)\ (3,8)\ (3,10)\}$

$R = \{(2,6)\ (2,8)\ (3,10)\}$ & $S = \{(2,8)\ (3,10)\}$

① $R \cup S = \{(2,6)\ (2,8)\ (3,10)\} \cup \{(2,8)\ (3,10)\}$

$= \{(2,6)\ (2,8)\ (3,10)\}$

② $R \cap S = \{(2,6)\ (2,8)\ (3,10)\} \cap \{(2,8)\ (3,10)\}$

$= \{(2,8)\ (3,10)\}$

③ $R - S = \{(2,6)\ \cancel{(2,8)}\ \cancel{(3,10)}\} - \{\cancel{(2,8)}\ \cancel{(3,10)}\}$

$= \{(2,6)\}$

④ $\overline{R} = (A \times B) - R$

$= \{\cancel{(2,6)}\ \cancel{(2,8)}\ (2,10)\ (3,6)\ (3,8)\ \cancel{(3,10)}\ (5,6)\ (5,8)\ (5,10)\}$

$- \{\cancel{(2,6)}\ \cancel{(2,8)}\ \cancel{(3,10)}\}$

$\overline{R} = \{(2,10)\ (3,6)\ (3,8)\ (5,6)\ (5,8)\ (5,10)\}$

⑤ $\overline{S} = (C \times D) - S$

$= \{\cancel{(2,8)}\ (2,10)\ (3,8)\ \cancel{(3,10)}\} - \{\cancel{(2,8)}\ \cancel{(3,10)}\}$

$= \{(2,10)\ (3,8)\}$

Types of Relations:-

1) Inverse Relation:-

Let R be a relation from A to B. The inverse relation is a relation from B to A and it is denoted by 'R^{-1}'.

$\therefore R^{-1} = \{(y,x) : x \in A, y \in B; (x,y) \in R\}$

$x R y \Leftrightarrow y R^{-1} x$

2) Identity relation:-

Let A be a set, Then The relation R in a set denoted by I_A is said to be identity relation or diagonal $I_A = \{(x,y) : x \in A \text{ and } y \in B, x = y\}$

Eg:- A $\{a,b,c\}$

$I_A = \{(a,a), (b,b), (c,c)\}$

3) Universal Relation:-

A relation R in a set A said to be universal relation if $R = A \times A$

Eg:- if $A = \{2,3\}$ then $R = A \times A$

$= \{(2,2)\ (2,3)\ (3,3)\ (3,2)\}$

4) Void relation:-

A relation 'R' in a set A is said to be a void relation provided R is null set.

$R = \{\ \}$

1/11

Properties of relations:-

1) Reflexive Relation:-

A Relation R on a set A is Reflexive if and only if each element of in A is related to itself i.e aRa, $\forall a \in A$

Eg:- $A = \{4,5,6\}$

$\therefore R = \{(4,4)\ (5,5)\ (6,6)\}$

2) Symmetric Relation:-

A Relation R on a set A is said to be symmetric iff $\forall (a,b) \in R$ ie $(a,b) \in R \Leftrightarrow (b,a) \in R$,

$$\boxed{aRb = bRa}$$

The necessary and sufficient condition for a relation R to be symmetric is $R = R^{-1}$

Eg:- $A = \{1,2\}$ then $R = \{(1,2)\}$
$R^{-1} = \{(2,1)\}$

$\therefore R \neq R^{-1}$

~~3> Transitive:-~~
~~A relation~~

3> Anti-Symmetric:-

A relation R on a set A then R is an anti-symmetric iff ~~A R B and BRA then A = B~~ aRb and $bRa \Rightarrow a=b$ for ~~A,B E~~ $a,b \in A$ i.e $(a,b) \in R$, $(b,a) \in R \Rightarrow a=b$

It is evident that the relation R ~~The Re~~ on a set is anti-symmetric $R \cap R^{-1} \subseteq I_A$ where I_A denotes identity relation.

Eg:- let $A = \{1,2,3,4\}$ then $R = \{(1,1)(2,3)(3,2)\}$

and $R^{-1} = \{(1,1)(3,2)(2,3)\}$

$\therefore R \cap R^{-1} = \{(1,1)(2,3)(3,2)\}$

$I_A = \{(1,1),(2,2)(3,3)(4,4)\}$

$\therefore$ R is not Anti symmetric

4> Transitive:-

A Relation R on a set A is said to be transitive iff $\forall a,b,c \in R$, $\boxed{aRb \text{ and } bRc \Rightarrow aRc}$

i.e $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$

5> Equivalence Relation:-

A Relation R on a set A is said to be equivalence relation iff it satisfies the following three conditions / properties

1. R is Reflexive, aRa
2. R is Symmetric, $aRb = bRa$
3. R is Transitive, $aRb \& bRc \rightarrow aRc$

6> Compatibility:- A Relation R on a set A is said to be compatibility relation iff it satisfies the following 2 conditions.

1> R is Reflective 2> R is Symmetric.

1) ~~let A =~~ Given an example of a relation that is reflexive but neither symmetric nor transitive.

Sol:- Let $A = \{1,2,3\}$ and R is defined as

$R = \{ (1,1)\ (1,2)\ (2,3)\ (2,2)\ (3,3) \}$

Hence, R is reflexive

since $(a,a) \in R \ \forall a \in A$

It is not symmetric

since $(1,2) \in R$ but $(2,1) \notin R$

$aRb \Leftrightarrow bRa$

It is also transitive

since $(1,2) \in R$ and $(2,3) \in R \Rightarrow (1,3) \notin R$.

$(a,b) \in R$ and $(c,b) \in R \Rightarrow (a,b) \in R$

aRb and $bRc \Rightarrow aRc$

2) The relation R on a set 's' of all real numbers is defined as aRb if and only if $1+ab>0$ show that relation is reflexive and symmetric but not transiti

Sol:- let 'a' be any real number

(i) Hence $1+ab : 1+a\cdot a = 1+a^2 > 0$

$\therefore$ $aRa \ \forall a \in S$

$\therefore$ R is a reflexive $= (a,a) \in R$

(ii) Let $a,b \in S$ then $aRb \Rightarrow 1+ab>0$

$1+ba>0$

$aRb = bRa$

$\therefore$ R is symmetric

(iii) Let $1, -1/2$ and -4.

now $1+ab = 1+(1)(-1/2) = 1/2 > 0$

$\therefore 1R(-1/2)$

$1+bc = 1+ab = 1+(-\frac{1}{2})(-4) = 3 > 0$

$\therefore (-1/2)\ R\ (-4)$

now

$1+ab = 1+ca = 1+(-4)(1)$

$= -3 < 0$

$1+ab<0$

$(-4) \not R (1)$

$\therefore$ R is not transitive

3) Let $A = \{1,2,3,4,5,6,7\}$ and $R = \{(x,y) : x-y$ is divisible by $3\}$ Show that R is an equivalence relation

Sol:- Given $A = \{1,2,3,4,5,6,7\}$ and

$R = \{(x,y) : x-y$ is divisible by $3\}$

(i) Reflexive:-

There exist an element $x \in A$ such that

$x-y = x-x$ is divisible by 3.

This show that $(x,x) \in R \ \forall\ x \in A$

$\therefore$ R is reflexive

(ii) Symmetric:-

If $x,y \in A$ and $(x,y) \in R$.

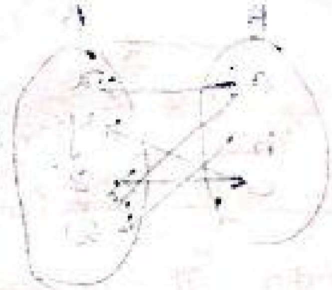
This means $x-y$ is divisible by 3.

Then, $x-y = 3m$; m is any integer

$\Rightarrow y-x = 3m_2$; m_2 is any integer

$y-x$ is divisible by 3

R is symmetric.



(iii) Transitive:-

let $x,y,z \in A$

also let $x-y = 3m_1$

$y-z = 3m_2$; m_1 & m_2 are integers

$\Rightarrow x-z = 3(m_1+m_2)$; m_1+m_2 is integer

So, $x-z$ is divisible by 3

$\therefore$ R is transitive

Hence, R is an equivalence relation.

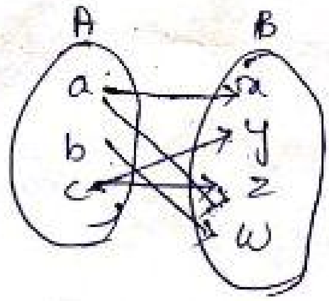
4) Representation of Relations:-

1) **Relation as an arrow diagram and tabular:-**

A Relation can also be represented in a tabu (or) graphical form. These helps us understandin a clear idea of the situation under considerati

Eg:- 1) A flowchart helps developing a program f comp solving the problems.

2) Let $A = \{a, b, c\}$ be a set of students of a and $B = \{x, y, z, w\}$ be a set of compani that come for campus interviews for selection of the students for jobs. We might have relation R_1 from A to B $(R_1: A \to B)$ to describe that the companies interviews with the students and the relation R_2 from A to B $(R_2: A \to B)$ to describe the jobs f offer to students by the companies the element of the both relations $R = \{(x,a)\ (y,c)\ (z,a)\ (z,c)$
$(w,b)\}$



tabular diagram:-

$R = \{(x,a)\ (y,c)\ (z,a)\ (z,c)\ (w,$

	x	y	z	w
a	(a,x)	(a,y)	(a,z)	(a,w)
b	(b,x)	(b,y)	(b,z)	(b,w)
c	(c,x)	(c,y)	(c,z)	(c,w)

	x	y	z	w
a	1	0	1	0
b	0	0	0	1
c	0	1	1	0

2) Relation as a directed graph or digraph

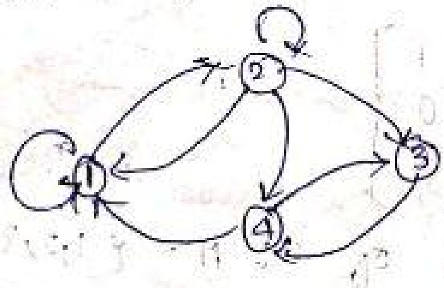
5/11

R be a relation from A to itself (R: A → A).
Draw a small circles for each element of A. and label the circle with the corresponding elements of A. These circles are called the vertices or nodes of the graph. Draw an arrow from the vertex a_i to the vertex b_j iff a_i is related to b_j. This type of graph of a relation R is called a directed graph or digraph. Let A be a non empty set. A directed graph G of A is made up of the elements of A called the vertices or nodes of A, and the subset E of AXA that contains the directed edges or arcs of G. The set A is called vertex set of G and E is called edge set of G. G = (A,E) is denoted the graph.

If (a,b) ∈ A and (a,b) ∈ E then there is an edge from A to B. Vertex a is called origin/source edge. and b is called terminus / terminating vertex

If a ≠ b then (a,b) ≠ (b,a) and an edge of the form (a,a) – loop

Ex:- let A = {1,2,3,4} and R = {(1,1) (1,2) (2,1) (2,2) (2,3) (2,4) (3,4) (4,1) (4,3)}



In the above graph R, the indegree of the vertex is the no of edges terminating at the vertex, and the outdegree of the vertex is no. of edges leaving the vertex.

vertex	indegree	outdegree
1	3	2
2	2	4
3	2	1
4	2	2

3) Relation as matrix (or) Boolean matrix (or) adjacent matrix:-

Consider a relation R from a finite set $A = \{a_1, a_2, a_3, \ldots, a_m\}$ to $B = \{b_1, b_2, \ldots, b_n\}$ containing m and n elements respectively, we define relation matrix $M_R = [m_{ij}]_{m \times n}$, for all whose elements are given by

$$m_{ij} = \begin{cases} 1, & \text{if } a_i R b_j \\ 0 & \text{otherwise} \end{cases}$$

The matrix M_R is called Relation or Boolean matrix.

Eg:-1) let $A = \{a_1, a_2, a_3\}$ and $B = \{b_1, b_2, b_3, b_4\}$ be two finite sets also let the relation defined b/w them is $R = \{(a_1,b_1)\ (a_1,b_4)\ (a_2 b_2)\ (a_2 b_3)\ (a_3 b_1)(a_3$

Sol:- Given $A = \{a_1, a_2, a_3\}$ and $B = \{b_1, b_2, b_3, b_4\}$

$R = \{(a_1 b_1)\ (a_1 b_4)\ (a_2 b_2)\ (a_2 b_3)\ (a_3 b_1)\ (a_3 b_2)\}$

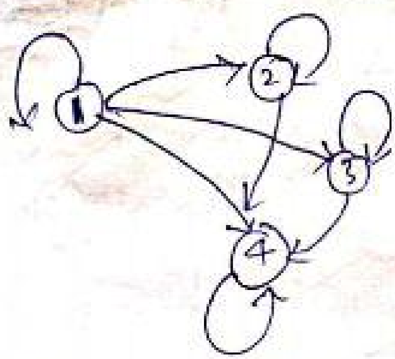
R (A \ B)	b_1	b_2	b_3	b_4
a_1	$(a_1 b_1)$	$(a_1 b_2)$	$(a_1 b_3)$	$(a_1 b_4)$
a_2	$(a_2 b_1)$	$(a_2 b_2)$	$(a_2 b_3)$	$(a_2 b_4)$
a_3	$(a_3 b_1)$	$(a_3 b_2)$	$(a_3 b_3)$	$(a_3 b_4)$

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

2) let R be the relation of set $A = \{1,2,3,4\}$ defined by $R = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (2,2)\ (2,4)\ (3,3)\ (3,4)\ (4,4)\}$ construct the matrix and digraph of R.

Sol:-

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Vertex	indegree	outdegree
1	1	4
2	2	2
3	2	2
4	4	1

7/11

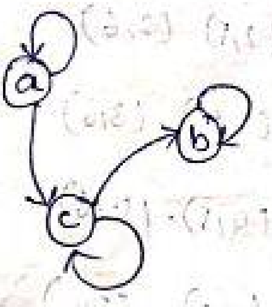
3) let $A = \{a, b, c\}$ be a non empty set and R be the relation on A that has the matrix

$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & \phi & 1 \end{bmatrix}$ construct the digraph of R and list out indegrees, outdegrees of all vertices.

Sol:- Given $A = \{a, b, c\}$

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow R = \{(a,a)(a,c)(b,b)(c,b)(c,c)\}$$

digraph G:-



Vertex	indegree	outdegree
a	1	2
b	2	ϕ
c	2	2

4) Let $A = \{1, 2, 3, 4, 5, 6\}$ and R be the Relation of A defined by aRb iff a is "a multiple of b". Represent the relation 'R' as a matrix and draw its digraph.

$R = \{(1,1)\ (2,1)\ (2,2)\ (3,1)\ (3,3)\ (4,1)\ (4,2)\ (4,4)$
$(5,1)\ (5,5)\ (6,1)\ (6,2)\ (6,3)\ (6,6)\}$

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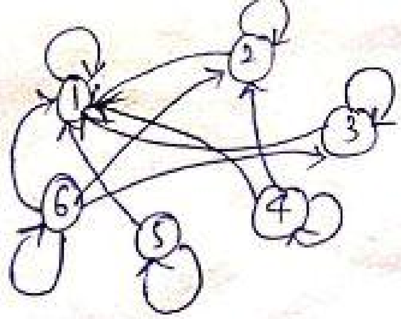
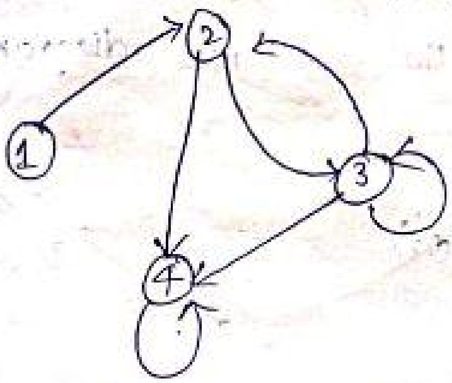


Vertex	indegree	outdegree
1	6	1
2	3	2
3	2	2
4	1	3
5	1	2
6	1	4

R	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}_{6\times 6}$$

5) Find the relation R, write matrix ~~and draw the digraph~~



$R = \{ (1,2)\ (2,3)\ (2,4)\ (3,2)\ (3,3)\ (3,4)\ (4,4) \}$

$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times4}.$$

8/11

Partial order relations:-

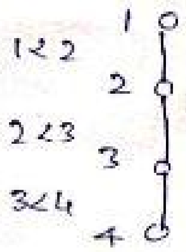
A binary relation 'R' on a set 'p' is called partially ordered relation (or) partial ordering in p. iff 'R' is Reflexive, Anti-symmetric and transitive. It is denoted by the symbol '$\leq$'. If $\leq$ is a partially ordered relation in p then the order pair $(P, \leq)$ is called partially ordered set. (or) poset.

Hasse diagram:-

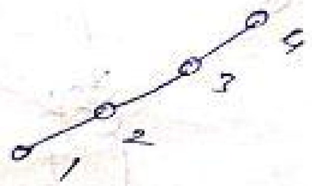
A partially ordering $\leq$ on a set 'p' can be represented by means of a diagram known as Hasse diagram of $(P, \leq)$. In such a diagram each element is represented by a small circle. The circle for $x \in P$ is drawn b/w the circle for $y \in P$. If $x < y$ and a line is drawn b/w x and y. If $x < y$ but does not connect 'x', Then x and y are not connected directly single line. ~~how ever~~ However, They are connected to one or more elements of 'p'. It is possible to obtain the set of order pairs in $\leq$ from such a diagrams.

Eg: 1) let $P = \{1,2,3,4\}$ and $\leq$ be the relation less than or equal to, Then the Hasse diagram is

Sol:

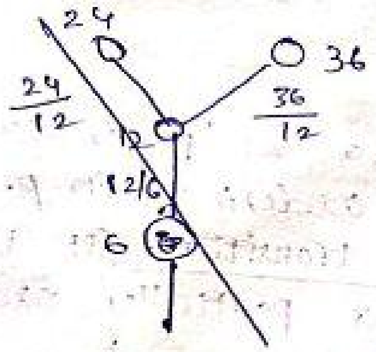


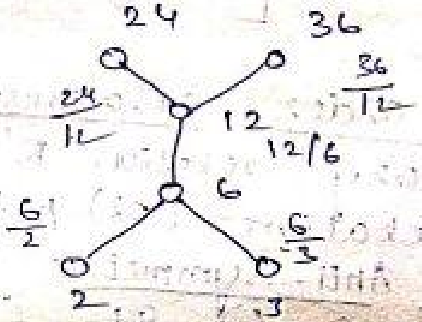
(or)



2) Let $x = \{2,3,6,12,24,36\}$ and the relation $\leq$ be such that $x \leq y$ if x divides y. Draw the hasse diagram.

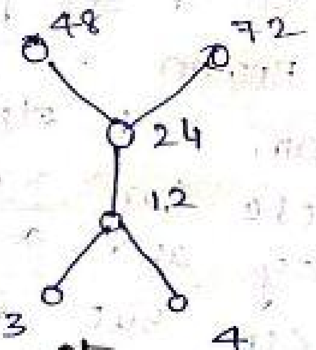
Sol:





3) Let $P = \{3,4,12,24,48,72\}$ and the relation be defined on P such that $a \leq b$, if "a divides b"

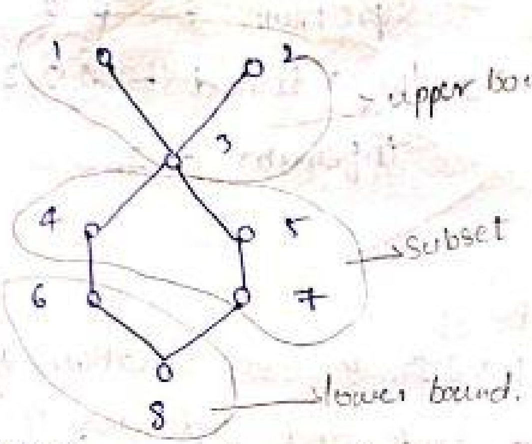
Sol:-



9/11

4) Hasse diagram of poset $S = \{1,2,3,4,5,6,7,8\}$ is given below. if $A = \{4,5,7\}$ is a subset of S find the upper and lower bounds, supremum and infimum of A.

Sol: Given $S = \{1,2,3,4,5,6,7,8\}$
$A = \{4,5,7\}$



Supremum → no. of elements in upper bound

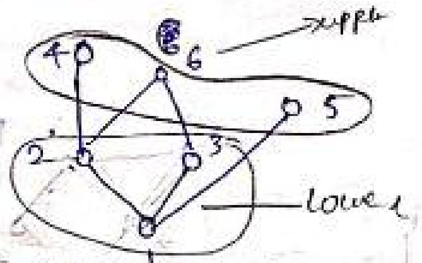
infimum → no. of elements in lower bound

upper bounds = $\{1,2,3\}$
supremum = 3
lower bound = $\{6,8\}$
infimum = 2

5) Let $P = \{1,2,3,4,5,6\}$ and the relation $\leq$ such that $x \leq y$ if x divides y draw the hasse diagram. Find upper and lower bounds, supremum and infimum of x and subset $A = \{4,5\}$

Sol:- Given $x = \{1,2,3,4,5,6\}$
$A = \{4,5\}$

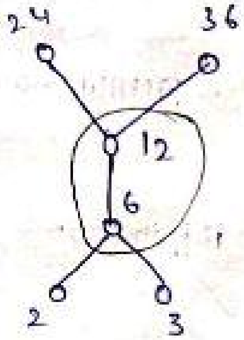
hasse diagram



upper bound = $\{6\}$
Supremum = 1
lower bound = $\{1,2,3\}$
infremum = 3.

6) let $R = \{2,3,6,12,24,36\}$ and the relation $\leq$ such that $x \leq y$ of x divides y draw the hasse diagram. Find upper and lower bounds, Supremum and infimum of X and subset $A = \{6,12\}$.

Sol:-



upper bound $= \{24, 36\}$

Supremum $= \{2$

lower bound $= \{2, 3\}$

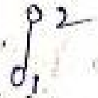
infimum $= 2$

5) let A be the set of factors of a particular +ve integer m and let ≤ be the relation divides i.e ≤ $= \{(x,y) / x \in A$ and $y \in A$ and x divides $y\}$. Draw the hasse diagram for

i) m = 2 ii) m = 6 iii) m = 30 iv) m = 12 v) m = 45

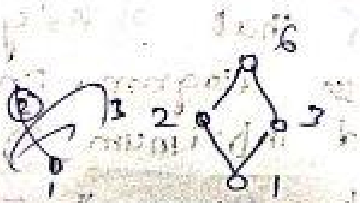
i) m = 2

$A = \{1, 2\}$



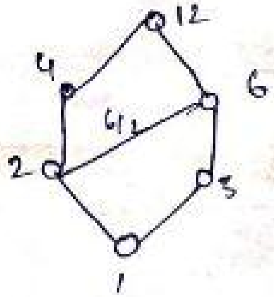
ii) m = 6

$A = \{1, 2, 3, 6\}$



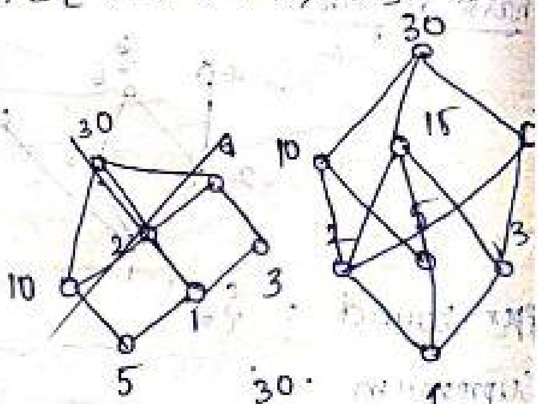
iv) m = 12

$A = \{1, 2, 3, 4, 6, 12\}$



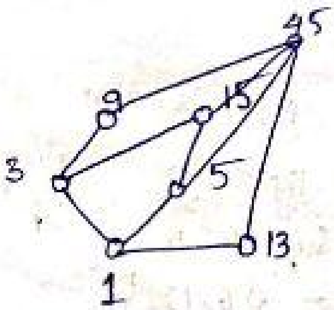
iii) m = 30

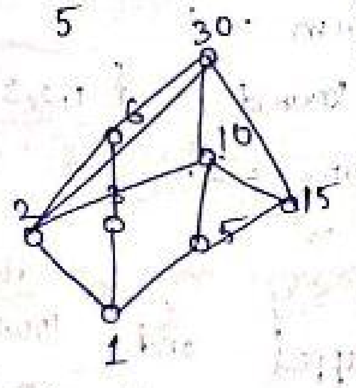
$A = \{1, 2, 3, 5, 6, 10, 15, 30\}$



v) m = 45

$A = \{1, 3, 5, 9, 13, 15, 45\}$





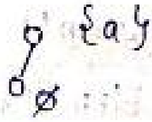
(8) Let A be a given finite set and P(A) is its power set. Let ≤ be inclusion relation on the set of P(A). Draw Hasse diagram of (P(A), ≤) for

(1) $A = \{a\}$ (2) $A = \{a, b\}$ (3) $A = \{a, b, c\}$

Sol:-

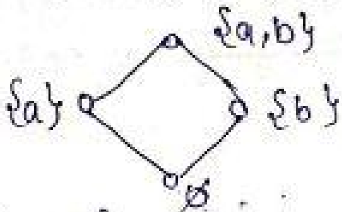
(1) $A = \{a\}$ $P(A) = 2^n = 2^1 = 2$

$P(A) = \{\emptyset, \{a\}\}$



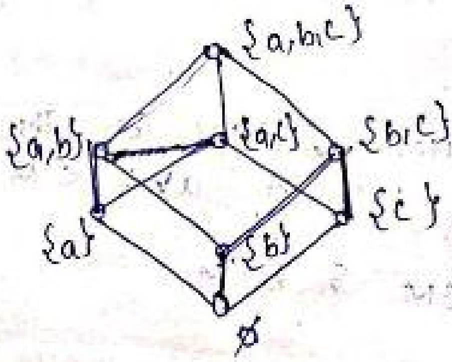
(2) $A = \{a, b\}$ $2^n = 2^2 = 4$

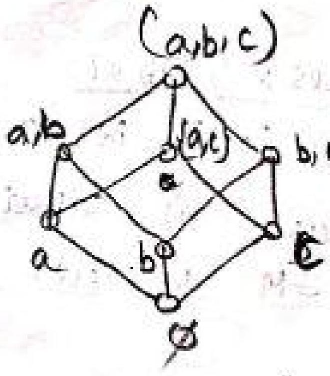
$P(A) = \{\emptyset\ \{a\}\ \{b\}\ \{ab\}\}$



(3) $A = \{a, b, c\}$

$P(A) = \{\emptyset\ \{a\}\ \{b\}\ \{c\}\ \{a,b\}\ \{b,c\}\ \{c,a\}\ \{abc\}\}$





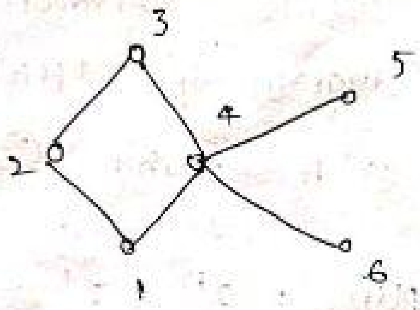
Lattice:-

A lattice is a partially ordered set (L, ≤) in which every pair of elements $a, b \in L$ has a greatest lower bound (GLB) and a least upper bound (LUB)

The GLB of a subset $(a, b) \subseteq L$ will be denoted by $a * b$. (a meet b) and LUB is denoted by $a \oplus b$ (a join b)

① Determine all minimal and maximal elements of The poset

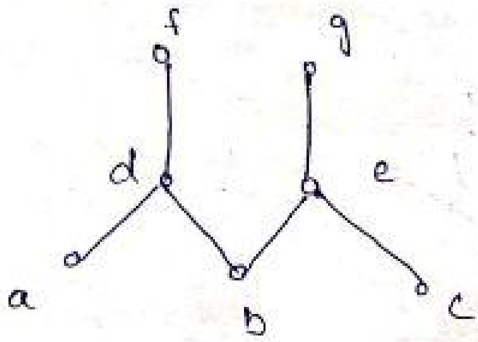
(i)



minimal Elements – {3,5} – (LUB)

maximal Elements – {1,6} – (GLB)

(ii)



LUB = maximal elements = {f,g}

GLB = minimal elements = {a,b,c}

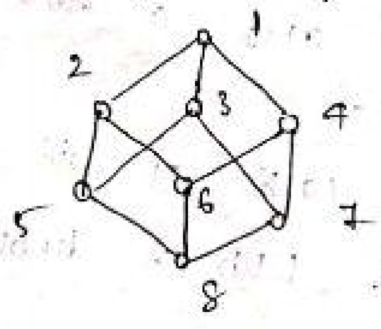
14/11/22

Sub lattices :- / sub set :-

Let (L,R) be a lattice & 'M' be The sub lattices (or) subset of 'L', if a ∨ b ∈ M and a ∧ b ∈ M wherever a ∈ M & b ∈ M.

Eg:-

1.) Consider the lattices (L,R) represented by The hasse diagram given below.



$L = \{1, 2, 3, 4, 5, 6, 7, 8\}$

Subset of L

$m_1 = \{1, 2, 4, 6\}$ & $m_2 = \{3, 5, 7, 8\}$

$m_3 = \{1, 2, 4, 8\}$

here (m_1, R) & (m_2, R) are the sublattices of L & (m_3, R) is not sub Lattice of L.

product of lattices:-

Consider the Lattices (L_1, R) & (L_2, R) Then $(L_1 \times L_2, R)$ is a poset and the product of partially ordered set defined by $(a, b)\, R\, (a', b')$ if $a R a'$ in L_1 and $b R b'$ in L_2.

properties of Lattices:

$\vee\ * -$ Meet
$\wedge\ \oplus -$ join

1.) Commutative property:-

i.) $a * b = b * a$
$a \vee b = b \vee a$

ii.) $a \oplus b = b \oplus a$
$a \wedge b = b \wedge a$

2.) Idempotent property:-

i.) $a * a = a$
$a \vee a = a$

ii.) $a \oplus a = a$
$a \wedge a = a$

3) Associate property:-

i.) $a * (b * c) = (a * b) * c$
$a \vee (b \vee c) = (a \vee b) \vee c$

ii.) $a \oplus (b \oplus c) = (a \oplus b) \oplus c$
$a \wedge (b \wedge c) = (a \wedge b) \wedge c$

4.) absorption property:-

i.) $a * (a \oplus b) = a$
$a \vee (a \wedge b) = a$

ii.) $a \oplus (a * b) = a$
$a \wedge (a \vee b) = a$.

Bounded Lattices:-

A lattice L is said to be bounded lattice if it has least element zero and greatest element one.

If L is bounded lattice, Then for any element $a \in L$ we have the following identities.

① $0 \le a \le 1$

② $a \vee 0 = a \quad a \wedge 0 = 0$

③ $a \wedge 1 = a \quad a \vee 1 = a$

Distributive Lattices:-

A lattice L is said to be distributive lattice if for any elements $a, b, c \in L$. Then we have the following identities....

① $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

② $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

Complemented Lattices:-

Let L be the bounded lattice, it has lower bou 0 and upper bound 1. An element 'x' in bounded lat 'L' is said to be Complement of it's another eleme $y \in L$ provided.

① $x \wedge y = 0$

② $x \vee y = 1$

The Complement of x of an element is $y \in L$ can also be denoted $\bar{x} / x' / x^c$.

Eg = $\bar{0} = 1$
$\bar{1} = 0$

Modular Lattices:-

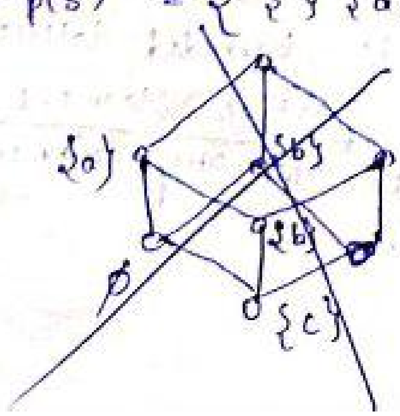
A lattice L is said to be modular lattice if $a \vee (b \wedge c) = a \wedge (b \vee c)$ and $a \le c \;\forall\, a, b, c \in L$

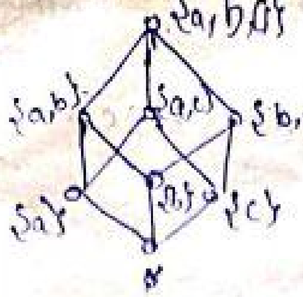
~~Let S = {a, b, c}~~

i) Let $S = \{a, b, c\}$ and $A = P(S)$ draw the hasse diagram of the poset A with partial order $\subseteq$.

Sol:- $S = \{a, b, c\}$

$A = P(S) = \{ \{\} \{a\} \{b\} \{c\} \{ab\} \{bc\} \{ca\} \{a,bc$





$\emptyset << \{a\}, \ \emptyset << \{b\} \quad \emptyset << \{c\}$

$\{a\} << \{a,b\}, \ \{b\} << \{a,b\}, \ \{c\} << \{b,c\}$

$\{b\} << \{b,c\}$

$\{a,b\} << \{a,b,c\}, \ \{a,c\} << \{a,b,c\}, \ \{b,c\} << \{a,b,c\}$

2) Let $(L, \leq)$ be a lattice for any $a,b,c \in L$ the following properties are called Isotopicity hold $b \leq c \Rightarrow$

$$\begin{cases} a * b \leq a * c \\ a \oplus b \leq a \oplus c \end{cases}$$

Proof:- we know that

$b \leq c$

$b' \leq c' \Leftrightarrow b' + c' = b'$

$a \leq b = a * b$

To prove $a * b \leq a * c$:-

$\{(a*b)\} * (a*c) = (a*b*a)*c$

$= (a*b)*c = (a*a*b)*c$

$= (a*a) + (b*c)$

$= a*b$

$a*b \leq a*c.$

The second statement is the dual of the first statement $a \oplus b \leq a \oplus c$.

$\underline{a \oplus b \leq a \oplus c}$

$(a \oplus b) \oplus (a \oplus c) \Rightarrow (a \oplus a) \oplus (b \oplus c)$

$= a \oplus b$

$= a \oplus c$

$a \oplus b \leq a \oplus c.$

15/11

Problems

1) The Complement of an element a in a bounded lattice if it exists, is unique.

Sol: let a_1 and a_2 be the Complements of $a \in L$

Then $a \vee a_1 = 1$, $a \vee a_2 = 1 \rightarrow ①$

$a \wedge a_1 = 0$, $a \wedge a_2 = 0 \rightarrow ②$

Now,

$a_1 = a_1 \vee 0$

$= a_1 \vee (a \wedge a_2)$ by ②

$= (a_1 \vee a) \wedge (a_1 \vee a_2)$

$= (a \vee a_1) \wedge (a_1 \vee a_2)$

$= 1 \wedge (a_1 \vee a_2)$ by ①

$\boxed{a_1 = a_1 \vee a_2}$

Now,

$a_2 = a_2 \vee 0$

$= a_2 \vee (a \wedge a_1)$ by ②

$= (a_2 \vee a) \wedge (a_2 \vee a_1)$

$= (a \vee a_2) \wedge (a_2 \vee a_1)$

$= 1 \wedge (a_2 \vee a_1)$

$\boxed{a_2 = a_2 \vee a_1}$

$\therefore \boxed{a_1 = a_2}$

The Complement of an elt a in a bounded lattice if it exist, it is unique.

2) prove that a and b are elements in bounded, distributive lattice and if a has a Complement a', Then $a \vee (a' \wedge b) = a \vee b$ and $a \wedge (a' \vee b) = a \wedge b$.

Sol: Given that a a... distributive and a'...

Now, we have to show

$a \vee (a' \wedge$ ...

① $a \vee (a' \wedge b) = a \vee b$

LHS

$a \vee (a' \wedge b) = (a \vee a'$ ...

$= 1 \wedge$ ...

$= a \vee b$

$\boxed{\therefore a \vee (a' \wedge b)}$ ...

② $a \wedge (a' \vee b) = a \wedge b$

LHS

$a \wedge (a' \vee b) =$ ...

=

=

$\boxed{a \wedge (a' \vee b}$ ...

3) if $(L, \leq)$ is a ... elt ..., Then for ...

(i) $a \vee 1 = 1$ and $a \wedge$ ...

(ii) $a \vee b \geq a$ and $a \wedge$ ...

Sol: let a be any ...

since 1 is the ...

and also $a \vee$ ...

a and 1 we have ...

$\therefore$ from ...

Sol: Given that a and b are the elts in bounded distributive and a' is the Complement of a.

Now, we have to show that

$$a \vee (a' \wedge b) = a \vee b \;\&\; a \wedge (a' \vee b) = a \wedge b.$$

① $\underline{a \vee (a' \wedge b) = a \vee b}$

LHS

$$a \vee (a' \wedge b) = (a \vee a') \wedge (a \vee b) \quad \{\because a \vee a' = 1\}$$
$$= 1 \wedge (a \vee b)$$
$$= a \vee b$$

$$\boxed{\therefore a \vee (a' \wedge b) = a \vee b}$$

② $a \wedge (a' \vee b) = a \wedge b$

LHS

$$a \wedge (a' \vee b) = (a \wedge a') \vee (a \wedge b)$$
$$= 0 \vee (a \wedge b) \quad \{a \wedge a' = 0\}$$
$$= a \wedge b$$

$$\boxed{a \wedge (a' \vee b) = a \wedge b}$$

3) if $(L, \leq)$ is a lattice with least elt 0, greatest elt 1, Then for any $a \in L$. show that

(i) $a \vee 1 = 1$ and $a \wedge 1 = a$

(ii) $a \vee 0 = a$ and $a \wedge 0 = 0$

Sol: let a be any elt of lattice L

since 1 is the greatest elt of lattice L

$$\therefore a \vee 1 \leq 1 \rightarrow ①$$

and also $a \vee 1$ is the supremum (or) LUB of a and 1 we have $1 \leq a \vee 1 \longrightarrow ②$

$\therefore$ from ① & ②

$$\boxed{a \vee 1 = 1}$$

for then since $a \wedge 1$ is the infremum (or) GLB of a & 1

$$a \wedge 1 \leq a \rightarrow ③$$

and also $a \leq a$ and $a \leq 1$

we have

$$a \leq a \wedge 1 \rightarrow ④$$

from ③ & ④

$$\boxed{a \wedge 1 = a}$$

(ii) let $a \wedge 0$ is least (or) minimum of elements a, and 0, so $a \wedge 0 \leq 0 \rightarrow ①$

and also $0 \leq a$, we get $0 \leq a \wedge 0 \rightarrow ②$

from ① & ② $\boxed{a \wedge 0 = 0}$

$a \vee 0$ is greatest (or) maximum of a and 0 so

$$a \vee 0 \leq a \rightarrow ③$$

and also $0 \leq a$, we get $a \leq a \vee 0 \rightarrow ④$

from ③ & ④ we get $\boxed{a \vee 0 = a}$

Every sub lattice of a distributive lattice is a sub lattice.

proof:- Let S be a sublattice of distributive lattice L.

let $a, b, c \in S$ then $a, b, c \in L$

then $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c) \in L$

Since 'S' is the closed in '$\wedge$' and '$\vee$', we have

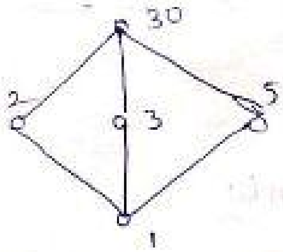
$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c) \in S$$

Hence S is a distributive lattice.

19/11

1) If $A = \{1, 2, 3, 5, 30\}$ and R is the divisibility relation prove that (A, R) is a lattice but not a distributive lattice.

Sol:- Given $A = \{1, 2, 3, 5, 30\}$ and R is the relation on A is divisibility relation which is poset.



Here we find that every two elts a & b of A has a LUB : $a \vee b$ in A

& GLB : $a \wedge b$ in A

indeed $a \vee b$ and $a \wedge b$ for all $a, b \in A$ are shown in the following tables

LUB (upper limits)

$\vee$	1	2	3	5	30
1	1	2	3	5	30
2	2	2	30	30	30
3	3	30	3	30	30
5	5	30	30	5	30
30	30	30	30	30	30

GLB (lower limits)

$\wedge$	1	2	3	5	30
1	1	1	1	1	1
2	1	2	1	1	2
3	1	1	3	1	3
5	1	1	1	5	5
30	1	2	3	5	30

Since $a \vee b$ and $a \wedge b$ are in A for every $a, b \in A$ we refer that the poset (A, R) is a lattice.

Further note that

$2 \vee (3 \wedge 5) = 2 \vee 1 = 2 \longrightarrow (1)$

$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

$(2 \vee 3) \wedge (2 \vee 5) = 30 \wedge 30 \longrightarrow (2)$

$= 30$

$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

$2 \neq 30$

Hence given lattice is not distributive lattice

This means that the distributive laws do not hold in this lattice.

Homeomorphism / Homo / Iso :-

let f / $(L_1, \leq_1)$ and $(L_2, \leq_2)$ be two posets.
If the function 'f' defined from L_1 to L_2 is called homeomorphism.

(Or)

[If $f: L_1 \to L_2$ such that $a \leq_1 b \Rightarrow f(a) \leq_2 f(b)$ $\forall\, a, b \in L$]

Then 1) f is one-one and onto

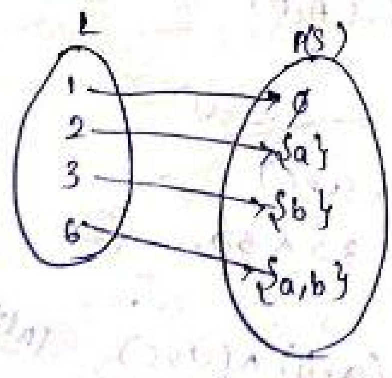
2) $f(a \vee b) = f(a) \vee f(b)$

$f(a \wedge b) = f(a) \wedge f(b)$

where L_1 is a lattice iff L_2 is a lattice, however f is one-one and onto from L_1 to L_2 Then for any element $a, b \in L_1$ and $f(a) \leq_2 f(b)$ in L_2

1) Show that the lattice $L = \{1, 2, 3, 6\}$ under divisibility relation and lattice $(P(S), \leq)$ where $S = \{a, b\}$ are homomorphism.

Sol:- Given that $L = \{1, 2, 3, 6\}$ under the divisibility relation and the lattice $(P(S), \leq)$ where $S = \{a, b\}$ we define a mapping $f: L \to P(S)$

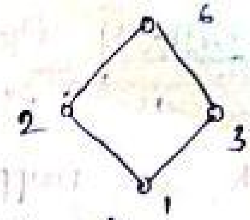


$f(1) = \emptyset$, $f(2) = \{a\}$ $f(3) = \{b\}$ $f(6) = \{a, b\}$

This implies that f is one-one and onto and also for all $a, b \in L$.

$f(a \vee b) = f(a) \vee f(b)$

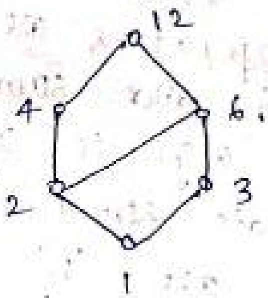
and $f(a \wedge b) = f(a) \wedge f(b)$



$\therefore$ f is a homomorphism and hence lattice L is homo. to lattice $(P(S), \leq)$

2) Let the lattice $L = \{1, 2, 3, 4, 6, 12\}$ consider the lattice $(L, /)$ and $(L, \leq)$ where '/' is the divisibility relation on L and L and $\leq$ is the relation 'L' show that Lattice $(L, /)$ and $(L, \leq)$ are not is isomorphism.

Sol:- Given that $L = \{1, 2, 3, 4, 6, 12\}$ be a Lattice we defined a mapping $f : (L, /) \to (L, \leq)$ such that



$3 \wedge 4 = 1 \in (L, /)$

$f(3 \wedge 4) = f(1)$

But $f(3 \wedge 4) = f(3)$ or $f(4)$ depends upond the values of $f(3)$ & $f(4)$

In any case

$f(3 \wedge 4) = f(3) \wedge f(4) = f(3)$ and $f(4)$

$f(3 \wedge 4) \neq f(3) \wedge f(4)$

$(L, /)$ and $(L, \leq)$ are not Isomorphism to each other.

II

Algebraic Structure:

Elementary operators:-

$+, -, \cdot, \div$ are called the elementary operators

Binary Operation:

Let 'S' be a non empty set, if $f: S \times S \to S$ is a mapping or function Then f is said to be binary Operation on 'S' i.e $\forall a, b \in S$ Then There exist an unique image $f(a,b) \in S$ and it is denoted by '$*$' (or) 'o'.

we observe that $+, -, \cdot$ are binary Operation in 'R' and $\div$ is not binary operation in 'R' i.e $\{ \frac{1}{0} \in R, \frac{1}{0} = \infty \notin R$

$R = |x \in R|, \quad -\infty < x < \infty.$

Algebraic system:-

Let S be a non empty set on ~~with~~ which one or more n-array ~~sa~~ operators are defined Then a system consisting of 'S' and some n-array Operators on S is called algebraic system (or) Simply algebra or Algebraic structure.

If $*_1, *_2, *_3 \ldots *_n$ are 'n' Operations on 'S' then The System $(S, *_1, *_2, *_3 \ldots *_n)$ is called an algebraic system.

Properties of binary expression:-

1) Closure property: A binary operation $*$ on a set 'S' is said to be closure property, if for every ~~a,b∈S~~

$\boxed{a, b \in S \text{ Then} \Rightarrow a * b \in S.}$

2) Associative:- A binary operation $*$ on a set 'S' is said to be associative property, if for every

$\forall a, b \in S \Rightarrow \boxed{a*(b*c) = (a*b)*c}$

3) Identity:- Let 's' be a non empty set and $*$ be The binary operation on 's' if There exist an element $e \in S$ such that

~~$a*e = a = e*a$~~

$\boxed{a*e = a = e*a}$ $\forall a \in S$ is called identity property.

4) Inverse property :-

Let $(S, *)$ be an algebraic structure with the identity element in 'S'. An element $a \in S$ is said to be invertable.

If there exist an elmt $x \in S$ such that

$$\boxed{a * x = e = x * a}$$

This property is called inverse property.

5.) Commutative :-

A binary operation '$*$' on a set 'S' is said to be Commutative property if for every $a, b \in S$ then

$$\boxed{a * b = b * a}$$

23/11

Group :- Let 'G' be a non-empty set & $*$ be the binary operation in 'G'. Then the algebraic structure $(G, *)$ is called a group. If it satisfies the following properties

1) closure property
2) Associative property
3) Identity property
4) Inverse property

Semi Group :- Let 'G' be a non-empty set & $*$ be the binary option in 'G'. Then the algebraic structure $(G, *)$ is called a semi group. If it satisfies the following properties

(1) closure property
(2) Associative property

Abelian group: Let 'G' be a non-empty set and $*$ be the binary operation on 'G'. Then the algebraic structure $(G, *)$ is called Abelian group if it satisfies the following properties

(1) closure property
(2) Associative property
(3) Identity property
(4) Inverse property
(5) Commutative property

Monoid group:- The semi group $(G, *)$ which has an identity element with respect to the binary operations $*$ is said to be a monoid and it is denoted by $(M, *)$

(OR)

An algebraic structure $(m, *)$ is called a monoid, if it satisfies the following properties.

(1) closure property
(2) Associative property
(3) Identity property

Problems

1) prove that the set $G = \{1, \omega, \omega^2\}$ (The set of cubic roots of unity)
$\{x / x^3 = 1\}$ forms an abelian group. we get the operation "$\cdot$" (multiplication)

Sol:- $G = \{1, \omega, \omega^2\}$

The set of cubic roots of units (i.e; $\omega^3 = 1$) Now to show that G forms an abelian group.

① closure property:- wt $1 \cdot \omega \in G$

$$1 \cdot \omega = \omega \in G$$

i.e; $a, b \in G \Rightarrow a * b \in G$.

$\therefore$ G satisfies closure property

② Associative property:-

wt $1, \omega, \omega^2 \in G$

$$(1 \cdot \omega) \cdot \omega^2 = 1 \cdot (\omega \cdot \omega^2)$$
$$\omega \cdot \omega^2 = 1 \cdot \omega^3$$
$$\omega^3 = \omega^3$$
$$1 = 1$$

$\therefore a, b, c \in G \Rightarrow (a * b) * c = a * (b * c) \in G$

$\therefore$ G satisfies Associative property

3) Identity property: we know that w.r.t the multiplication Identity element is "1".

$1.1 = 1$

$\omega.1 = \omega$

$\omega^2.1 = \omega^2$

i.e ; $a * e = a = e * a \ \forall \ a \in G$.

$\therefore$ G satisfies Identity property.

$a * e = a$

$a * x = e$

4) Inverse property:

Let $1, \omega, \omega^2 \in G$

$1.1 = 1$.

$\omega.\omega^2 = \omega^3 = 1$.

$\omega^2.\omega = \omega^3 = 1$.

$a * x = e = x * a$

$\therefore$ G satisfies Inverse property

5) Commutative property:-

Let $1, \omega, \omega^2 \in G$

$1, \omega^2 \in G$

$1.\omega^2 = \omega^2.1$

$\omega^2 = \omega^2$

$a, b \in G \Rightarrow a * b = b * a$

$\therefore$ G satisfies Commutative property

Hence, G forms an Abelian group.

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2) prove that the set $G = \{1, -1, i, -i\}$ form an abelian group w.r.t to the multiplication $(\cdot)$.

(The set of fourth roots of unity), $= \{x / x^4 = 1, i$

Sol:- Given $G = \{1, -1, i, -i\}$ & $i^2 = -1$

i) Closure property:-

Let $1, i \in G$

$1 \cdot i = i \in G$

i.e $a, b \in G \Rightarrow a * b \in G$

$\therefore$ G satisfies closure property.

ii) Associative property:-

Let $1, i, -i \in G$

$(1 \cdot i) \cdot (-i) = 1 \cdot (i \cdot -i)$

$i \cdot (-i) = 1 \cdot (-i)^2$

$\langle i^2 = -1 \rangle$ $\quad -i^2 = -i^2$

$-(-1) = -(-1)$

$1 = 1$

$a, b, c \in G$ Then $(a * b) * c = a * (b * c)$

$\therefore$ G satisfies Associative property.

iii) Identity property:

we know multiplication identity is 1

Let $1, -1, i, -i \in G$

$1 \cdot 1 = 1$

$-1 \cdot 1 = -1$

$i \cdot 1 = i$

$-i \cdot 1 = -i$

(i.e $a * e = a = e * a$)

$\therefore$ G satisfies identity property

iv) Inverse property:-

Let $1, -1, i, -i \in G$

$1 \cdot 1 = 1$

$-1 \cdot -1 = 1$

$i \cdot -i = -i^2 = 1$

$-i \cdot i = 1$

~~$a * e = e$~~ $a * x = e = x * a$

$\forall a \in G$

$\therefore$ G satisfies inverse property

v) Commutative property:-

Let $1, i \in G$

$1 \cdot i = i \cdot 1$

$i = i$

$a, b \in G \Rightarrow a * b = b * a$

$\therefore$ G satisfies Commutative property.

$\therefore$ G form ~~an~~ ~~a~~ an ~~abelium~~ abelian group.

3) prove that The set z (integers) forms an abelian group with the Operation is defined by $a \circ b = a+b+2$ for all $a, b \in z$

Sol:- Given $z = \{-\infty, \ldots -2, -1, 0, +1, +2 \ldots \infty\}$

and The operation $\boxed{\begin{array}{l} a \circ b = a+b+2 \\ a * b = a+b+2 \end{array}}$

to prove that z forms an abelian group

i> closure property:-

let $1, 5 \in z$

$a \circ b = a+b+2$

$1 \circ 5 = 1+5+2$

$= 8 \in z$.

$\therefore a, b \in z$ then $a \circ b \in z$

. z satisfies closure property

ii> Associative property:-

let $1, 2, 3 \in z$

$(a \circ b) \circ c = a \circ (b \circ c)$

$(a+b+2) \circ c = a \circ (2+3+2)$

$(1+2+2) \circ 3 = 1 \circ (7)$

$5 \circ 3 = 1+7+2$

$5+3+2 = 10$

$10 = 10$.

$\therefore$ z satisfies Associative property

iii> Identity property:-

we know multiplication identity is 1

by the identity $a \circ e = a$

$a \circ b = a$

$a+b+2 = a$

$a+\cancel{b}+2 = a \cancel{-a} = 0$

$\cancel{b = -2}$

$e+2 = a-a \Rightarrow e = -2$

i.e $a * e = a = e * a$

z satisfies Identity property

iv> Inverse property:-

x is inverse of 'a'.

$a \circ x = e$

$a+b+2 = e$

$a+x+2 = \cancel{4}-2$

$a+x = -4$

$\boxed{x = -4-a} \in z$

ie $a \circ x = e = x \circ a$

z satisfies inverse property

v> Commutative property:-

Let $-3, 6 \in z$

$a \circ b = b \circ a$

$-3+6+2 = 6-3+2$

$3+2 = 3+2$

$5 = 5$

$\therefore$ z satisfies Commutative property

$\therefore$ z form an abelian group.

Finite group:-

If the set G contains a finite no.of elements, Th The group $(G,*)$ is called a finite group otherwise $(G,*)$ is called an infinite group.

Order of group:-

The no.of elements in a finite group $(G,*)$ is called order of the group and it is denoted b $O(G)$ is a group Then $O(G) = 4$.

Addition Modulo m (or) Addition of residue classes:

Let $a, b \in z$ and m be the fixed positive intege if r is the remainder $\Rightarrow (0 \le r < m)$ when "(a+b) is divided by m" $\left(\frac{a+b}{m}\right)$ be defined by $\boxed{a +_m b =}$ a addition module m b

Eg:- ① $20 +_6 5 = 1$

$= \frac{25}{6}$ $\quad 6)25(4$, 24, 1

② $24 +_5 4 = 3$

$\frac{28}{5}$ $\quad 5)28(5$, 25, 3

3 is remainder when 28 divides by 5

Multiplication modulo p:-

Let a, b are integers and p be the fixed positive inte if a b divides by p such that r is the remainc $(0 \le r < p)$, we define $\boxed{a \times_p b = r}$ and Read as a multiplication modulo p b".

Eg: ① $2 \times_2 6 = 0$ $\quad 2)12(6$, 12, 0

$\frac{12}{2}$

0 is remainder when 12 divided by 2

② $20 \times_{20} 7 = 0$

$20)140(7$, 140, 0

③ $20 \times_6 5 = 4$

$6)100(16$, 6, 40, 36, 4

1.) prove that the set $G = \{0,1,2,3\}$ forms an abelian group wrt $\oplus_4$

Sol:- $G = \{0,1,2,3\}$ and the operation is wrt $\oplus_4$

$\oplus_4$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

$0+_4 0 = 0/4 = 0$	$1+_4 0 = 1/4 = 1$	$2+_4 0 = \frac{2}{4} = 2$
$0+_4 1 = 1/4 = 1$	$1+_4 1 = 2/4 = 2$	$2+_4 1 = 3/4 = 3$
$0+_4 2 = 2/4 = 2$	$1+_4 2 = 3/4 = 3$	$2+_4 2 = \frac{4}{4} = 0$
$0+_4 3 = 3/4 = 3$	$1+_4 3 = 4/4 = 0$	$2+_4 3 = \frac{5}{4} = 1$

$3+_4 0 = 3/4 = 3$

$3+_4 1 = \frac{4}{4} = 0$

$3+_4 2 = 5/4 = 1$

$3+_4 3 = 6/4 = 2$

② prove that set $G = \{1,2,3,4\}$ ~~$\{0,1,2,3,4,5\}$~~ forms an abelian group wrt X_5

Sol:- $G = \{1,2,3,4\}$

and the operation is wrt X_5

X_5	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

$1 \times_5 1 = 1/5 = 1$

H.W ① {0,1,2,3,4} ...

28/11

Subgroup:-

Let $(G,*)$ / $(G,\cdot)$ be a group and 'H' be a nonempty subset of 'G' such that $(H,*)$ / $(H,\cdot)$ is a group Then 'H' is called the subgroup of G.

Normal subgroup:-

A sub group 'H' of a group 'G' is said to be normal subgroup if $\forall x \in G$ and $h \in H$ then $xhx^{-1} \in H$ and it is denoted by $H \Delta G$ and read as 'H' is a normal subgroup of 'G'

Homomorphism group:-

A function or a mapping 'f' is said to be homomorphism between two groups $(G,\cdot) \to (G',*)$ Then

i> $f:(G,\cdot) \to (G',*)$ is a function

ii> $f:(a\cdot b) \to f(a) * f(b)\ \forall\ a,b \in G$,

$$f(a\cdot b) = f(a)\cdot f(b)$$

Homomorphism Into:-

Let (G,G') be two groups and f is a mapping from G into G', $\forall\ a,b \in G$

$f(a\cdot b) = f(a)\cdot f(b)$ Then 'f' is said to be homomorphism from G into G'.

Homomorphism Onto:-

Let 'G','G'' be two groups and 'f' is a mapping from G onto G, if $\forall\ a,b \in G$. Then

$f(a\cdot b) = f(a)\cdot f(b)$ Then 'f' is said to be homomorphism from G onto G'.

Theorems:-

i> prove that every subgroup of an abelian group G is a normal subgroup.

proof:-

Given 'G' is an abelian group.

Let 'N' be a sub group of 'G'

Now, to show 'N' is a normalsubgroup of G.

let $g \in G$ and $h \in N$ Then

by The definition

$$xhx^{-1} \in H$$

$$ghg^{-1} = (gh)g^{-1}$$
$$= (hg)g^{-1}$$
$$= h(gg^{-1}) \quad \langle gg^{-1} = e \to 1$$
multiplicative identity
$$= h.e.$$
$$\boxed{ghg^{-1} = h \in N}$$

$\therefore$ N is a normal subgroup of G. ($N \triangle G$)

Hence every subgroup of an abelian group G is a Normal sub group.

iii) A subgroup of a group G is a normal.
$\Rightarrow xhx^{-1} = H; \ \forall x \in G$

Sol: proof :- Given that G is a group and H is a subgroup of G. Let H is a normal subgroup of G. Now we have to show that

$$xhx^{-1} = H \ \forall x \in G$$

Since H is normal subgroup of G

i.e $xHx^{-1} \subseteq H \ \forall x \in G \longrightarrow (1)$

Since $x \in G \Rightarrow x^{-1} \in G$
$\Rightarrow x^{-1}Hx \subseteq H$
$\Rightarrow x(x^{-1}Hx) \subseteq xH$
$\Rightarrow (xx^{-1})(Hx) \subseteq xH$
$\Rightarrow e(Hx) \subseteq xH$
$\Rightarrow Hx \subseteq xH$
$\Rightarrow (Hx)x^{-1} \subseteq (xH)x^{-1}$
$\Rightarrow H(xx^{-1}) \subseteq xHx^{-1}$
$\Rightarrow H.e \subseteq xHx^{-1}$

from (1) & (2) we get $\Rightarrow H \subseteq xHx^{-1} \longrightarrow (2)$

$$\boxed{xHx^{-1} = H} \longrightarrow (3)$$

Conversly: Let us take $xHx^{-1} = H$

Now to show that H is a normal subgroup of G.

We know that every set is a subset of itself.

i.e $xHx^{-1} \subseteq xHx^{-1}$

$xHx^{-1} \subseteq H$ by ③

$\therefore$ H is a normal subgroup of G.

3) If M & N are two normal subgroup of a group then prove that MN is also normal subgroup of G.

Proof:-

Given that

M and N are two normal subgroups of G

Now we have to show that

MN is a normal subgroup of G.

for this $mn \in MN$ so that $m \in M$ & $n \in N$

Since M is a normal subgroup of G, Then we have

$$gmg^{-1} \in m \ \forall\ g \in G \rightarrow ①$$

$m \in M$

and also

Since N is a normal subgroup of G, then we have

$$gng^{-1} \in N \ \forall\ g \in G \rightarrow ②$$

$n \in N$

Let us take $g(mn)g^{-1} = (gm)(ng^{-1})$

$= (gm)e(ng^{-1})$

$= (gm)(g^{-1}g)(ng^{-1})$

$= (gmg^{-1})(gng^{-1})$

i.e $(gmg^{-1}) \cdot (gng^{-1}) \in mn$

$\therefore$ MN is a normal subgroup of G

4) If $f:(Z,+) \rightarrow (R^+,\cdot)$ is a function and $f(x) = e^x \; \forall \; x \in Z$ then prove that f is a homomorphism

proof:-

Given that $f:(Z,+) \rightarrow (R^+,\cdot)$ is a function

and $f(x) = e^x \; \forall \; x \in Z$

Now we have to prove

f is a homomorphism

by the definition of closure

let $x,y \in Z$

$$f(x) = e^x$$

$$f(x+y) = e^{x+y}$$

$$= e^x \cdot e^y$$

$$\therefore f(x+y) = f(x) \cdot f(y)$$

$\therefore$ Hence f is a homomorphism from $(Z,+)$ to $(R^+,\cdot)$

If $f:(Q^+,\cdot) \rightarrow (R,+)$ is a function and $f(x) = \log x \; \forall \; x \in Q^*$, then prove that f is a homomorphism.

proof:- Given that $f:(Q^*,\cdot) \rightarrow (R,+)$ is a function and $f(x) = \log x \; \forall \; x \in Q^*$

Now to show that f is ~~also~~ a Homomorphism

by the definition of closure property $x,y \in Q^*$

$$\Rightarrow x,y \in Q^* \Rightarrow x.y \in Q^*$$

$$f(x) = \log x$$

$$f(x.y) = \log(x.y)$$

$$= \log x + \log y$$

$$f(x.y) = f(x) + f(y)$$

$\therefore f$ is a homomorphism from $(Q^*,\cdot)$ to $(R,+)$

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Functions :- ($f: A \to B$)

A function 'f' from set A to set B associates to each element x in A, A unique element f(x) in B and is written as $f: A \to B$

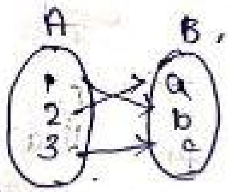
Types of functions:-

1) **One - One (or) injective :-**

Let $f: A \to B$ then 'f' is called an One-One function, if no. of two different elements in A have the same image. i.e different elements in 'A' have different elements in 'B'

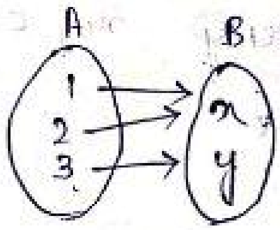
(or)

Let $f: A \to B$ be a function from 'A' to 'B'. If distinct elements of 'A' are mapped to distinct elements of B, Then f is called One-One or injective function.



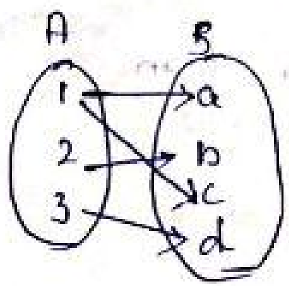
2) **Onto / Surjective:-**

A function $f: A \to B$ is said to be onto function if every element of 'B' is the image of 'some' elements of 'A' under 'f'.



3) **Bijective function:-**

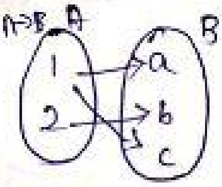
A function $f: A \to B$ is both injective and surjective, Then f is said to be bijective function



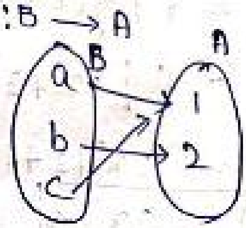
4) Inverse function:-

If $f: A \to B$ is a bijective of 'A' onto 'B' then the set $\{(b,a) \in B \times A, (a,b) \in f\}$ is a function on B into A. This function is called the inverse function of 'f' and it is denoted by f^{-1}.

Eg:- $f: A \to B$



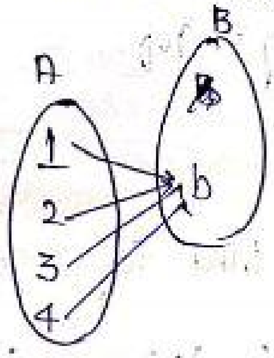
$f^{-1}: B \to A$



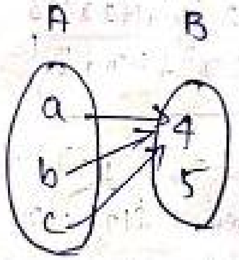
Constant function:-

→ A constant function is a function of the form $f(x) = b$ where 'b' is a number, ∴ $\boxed{y = f(x) = b}$

Eg:- 1)



2)



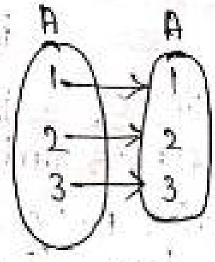
Identity function:-

Let 'A' be a non empty set and $f: A \to A$ be a mapping, if every element of 'A' is mapped into itself, then f is called an Identity function on A.

→ It is denoted by I_A

∴ $I_A = \{(a,a) / a \in A\}$

Eg:



Composition of a function:-

Let $f: A \to B$ and $g: B \to C$ are two mapping. Then the composition of two mappings f and g denoted by 'gof' is the mapping from 'A' to 'C' denoted by 'gof : A → C'.

$$gof = \{(a,c) / (a,b) \in f, (b,c) \in g\}$$

∴ $gof: A \to C$ is a mapping

$$\boxed{gof(a) = g[f(a)]} \text{ where } a \in A$$

In general the composition of functions is not commutative i.e $gof \neq fog$. Where f and g are functions.

for Eg: 1) Let $f: R \to R$ & $g: R \to R$ be defined by $f(x) = x+1$ and $g(x) = 2x^2+3$ Then find fog & gof.

Sol:- Given $f(x) = x+1$, $g(x) = 2x^2+3$

$$(fog)(x) = f[g(x)]$$
$$= f[2x^2+3]$$
$$= 2x^2+3+1$$
$$\boxed{(fog)(x) = 2x^2+4}$$

$$(gof)(x) = g[f(x)]$$
$$= g(x+1)$$
$$= 2(x+1)^2+3$$
$$= 2(x^2+1+2x)+3$$
$$\boxed{(gof)(x) = 2x^2+4x+5}$$

$\therefore fog \neq gof$

2) If $f(x) = x^2$, $g(x) = x+4$ Then find fog & gof.

Sol:- Given $f(x) = x^2$ $g(x) = x+4$

$$(fog)(x) = f[g(x)]$$
$$= f(x+4)$$
$$= (x+4)^2 = x^2+16+8x$$
$$\boxed{(fog)x = x^2+16+8x}$$

$$(gof)(x) = g[f(x)]$$
$$= g(x^2)$$
$$= x^2+4$$
$$\boxed{(gof)x = x^2+4}$$

$\therefore fog \neq gof$

Association of a function:-

If $f: A \to B$ and $g: B \to C$, $h: C \to D$ are three functions Then $gof: A \to C$ and $hog: B \to D$ are also functions we can form $(hog)of: A \to D$ and $ho(gof): A \to D$ assuming that $a \in A$, we have

$$[(hog)of](a) = [hog]\,f(a)$$
$$= h[g[f(a)]]$$
$$= h[(gof)a]$$
$$= ho(gof)(a)$$

$\therefore$ The composition of function is associative.

Q) 1) Let $f(x) = x+2$, $g(x) = x-2$ & $h(x) = 3x$, $\forall x \in R$
find i) gof ii) fog iii) hog iv) fogoh v) (hog)of
vi) ho(gof)

Sol:- Given $f(x) = (x+2)$, $g(x) = x-2$, $h(x) = 3x$

i) $[gof](x) = g[f(x)]$
$= g[x+2]$
$= x+\cancel{2}-\cancel{2}$
$\boxed{(gof)x = x}$

ii) $[fog](x) = f[g(x)]$
$= f(x-2)$
$= x-\cancel{2}+\cancel{2}$
$\boxed{(fog)x = x}$

iii) $[hog](x) = h[g(x)]$
$= h(x-2)$
$= 3(x-2)$
$\boxed{(hog)x = 3x-6}$

iv) $(fogoh)(x) = f[g[h(x)]]$
$= f[g(3x)]$
$= f[3x-2]$
$= 3x-\cancel{2}+\cancel{2}$
$\boxed{(fogoh)x = 3x}$

v) $((hog)of)(x) = hog[f(x)]$
$= h\{g[f(x)]\}$
$= h\{g[x+2]\}$
$= h[x+\cancel{2}-\cancel{2}]$
$= h(x)$
$= 3x.$

vi) $h(gof)(x)$
$= h[g[f(x)]]$
$= h[g[x+2]]$
$= h[x+\cancel{2}-\cancel{2}]$
$= h(x)$
$= 3x.$

20/12

Pigeon hole principal:-

If 'm' pigeons occupies 'n' pigeon holes, then atleast one pigeon hole must contain $\left(\frac{m-1}{n}\right)+1$ (or) more pigeons. $(m > n)$

1) If 7 cars carry 26 passengers, prove that atleast one car must have 4 or more passengers.

Sol:- Let given no.of cars (pigeon holes) $= n = 7$
no.of passengers (pigeons) $= m = 26$
by pigeon hole principle

$$\left(\frac{m-1}{n}\right)+1 = \frac{26-1}{7}+1$$

$$= \frac{25}{7}+1$$

$$= \frac{25+7}{7} = \frac{32}{7} = 4.5$$

Hence pigeon hole principle is Verified.

Atleast one car must Carry 4 or more passe

2) If 6 persons have a total of ₹ 2161 with the show that one or more of them must have atleast of ₹ 361.

Sol:- Given

total money (pigeons) = 2161

no. of persons (pigeon holes) = n = 6.

by using generalised pigeon hole principle.

$$\left(\frac{m-1}{n}\right)+1 = \left(\frac{2161-1}{6}\right)+1 = \frac{2160}{6}+1 = \frac{2166}{6}$$

Hence it is prove that $= 361$.

One or more of them must have atleast of ₹ 3

3) prove that 30 dictionaries in a library Contain a total of 61327 pages than atleast one of the dictionary must have atleast 2045 pages. (m>n)

Sol:- Let us consider

no. of dictionaries (pigeon holes) = n = 30

no. of pages (pigeon) = m = 61327

by pigeon hole principle

$$\left(\frac{m-1}{n}\right)+1 = \left(\frac{61327-1}{30}\right)+1 = \frac{61326}{30}+1$$

$$= 2045$$

Hence pigeon hole principle is proved.

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4) how many persons must choosen in order that atleast 5 of them will have birthdate in the same calender month.

Sol:- Let m be no.of persons.

n be no.of months in a year = n = 12

and also given atleast no.of persons who have their birthdate in the same month = 5

by generalized pigeon hole principle.

$$\left(\frac{m-1}{n}\right)+1 = 5 \Rightarrow \left(\frac{m-1}{12}\right)+1 = 5$$

$$\Rightarrow \frac{m-1+12}{12} = 5$$

$$\frac{m+11}{12} = 5$$

$$m+11 = 60$$

$$m = 60-11$$

$$\boxed{m = 49}$$

no. of pages $m = 49$.

5) Find the max. no.of students in a class to be sure that 4 out of them are born on the same month.

Sol:- $\cancel{n=4}$

$$\left(\frac{m-1}{12}\right)+1 = 4.$$

$$\frac{m-1+12}{12} = 4$$

$$\frac{m+11}{12} = 4$$

$$m+11 = 48$$

$$m = 48-11 = 37$$

$$\boxed{m = 37}$$

6) prove that in a set of 13 children atleast 2 have birthdays during the same month.

Sol:

$$\left(\frac{m-1}{12}\right)+1 = 2$$

$$\frac{m+11}{12} = 2$$

$$m+11 = 24$$

$$m = 24-11$$

$$\boxed{m = 13}$$

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Elementary Combinatorics

In daily lives, many a times one needs to find out the number of all possible outcomes for a Series of events.

for instance, in how many ways different 10 lettered PAN numbers Can be generated such that the five letters are Capital alphabets, the next four are digits and the last is again a Capital letter. for Solving these problems, mathematically theory of "Counting" are used.

Counting mainly encompasses(contains) "fundamental Counting rule", the "permutation rule", and the "Combination rule."

There are two types of Counting principles:

They are : (i) Sum Rule (or Disjunctive Rule)
(ii) product rule (or Sequential Rule)

The Sum Rule:

If an event 'A' Can occur in 'm' ways and another event 'B' Can occur in 'n' ways, and if these two events Cannot occur Simultaneously. Then A or B Can occur in m+n ways.

In general, if $E_1, E_2, \ldots, E_n$ are mutually exclusive events and E_1 Can happen n_1 ways, E_2 Can happen n_2 ways, ..., E_n Can happen n_n ways. Then one of the 'n' events Can occur in $n_1 + n_2 + \cdots + n_n$ ways.

EX:

1. If 8 male professor and 5 female professor teaching DMS then the Student Can choose professor in $8+5=13$ ways.

2. If there are 5 boys and 4 girls in a class, then there are $5+4=9$ ways of selecting one student (either a boy or a girl) as class representative.

3. A student can choose a computer project from one of three lists contain 23, 18, 10 possible projects. Then the number of possible projects are there to choose from are $23+18+10 = 51$.

4. How many ways can we get a sum of 4 or of 8 when two distinguishable dice are rolled? And how many ways can we get an even sum?

Sol:

i) we see that the outcomes (1,3), (2,2) and (3,1) are the only ones whose sum is 4. Thus, there are 3 ways to obtain the sum is 4.

Similarly, we obtain the sum 8 from the outcomes (2,6), (3,5), (4,4), (5,3) and (6,2). Thus, there are $3+5=8$ outcomes whose sum is 4 or 8.

5. From a well shefled pack of playing cards, find the following:

i) How many ways can we draw a heart or a spade?

ii) How many ways can we draw an ace or a king?

iii) How many ways can we draw a card numbered 2 through 10?

iv) How many way can we draw a numbered card or a king?

Sol: i) Since there are 13 hearts and 13 spades, we may draw a heart or a spade in $13+13=26$ ways.

ii) Since there are only 3 aces that are not hearts, we may draw a heart or an ace in $13+3=16$ ways.

iii) Since there are 9 cards numbered 2 through 10 in each of 4 suits (clubs, diamonds, hearts or spades) we may choose a numbered card in 36 ways.

(iv) we may choose a numbered card or a king in $36+4=40$ ways

NOTE:

In a deck, we have 52 Cards. And these Cards are distributed in 4 Suits.

1. Spades ♠
2. Diamonds ♦
3. Clubs ♣
4. Hearts ♥

Each Contain

Ace, two, three, four, five, Six, Seven, eight, nine, ten, Jack, queen and king.

The product Rule:

If an event occur in 'm' ways and a Second event Can occur in 'n' ways, and if the number of ways the Second event occurs doesnot depend upon how the first event occurs, then the two events Can occur Simultaneously in "mn" ways.

In general, if events $E_1, E_2, \ldots, E_n$ Can happen in $n_1, n_2, \ldots, n_n$ ways, then the Sequence of events E_1 first, followed by $E_2 \ldots$, followed by E_n Can happen in $n_1 \cdot n_2 \cdot \ldots \cdot n_n$ ways.

EX: 1. In class, there are 4 boys and 10 girls if a boy and a girl have to be chosen for the class representative, the students can hoose CR in $4 \times 10 = 40$ ways.

2. If 2 distinguishable dice are rolled then the first die Can fall (event E_1) in 6 ways and the Second can fall (event E_2) in 6 ways. Hence there are $6 \cdot 6 = 36$ ways.

③ How many different license plates are available if each plate contains a sequence of three letters followed by three digits.

Sol There are 26 choices for each of the three letters and 10 choices for each of the three digits.

There are a total of $26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 17576000$ possible license plates.

Combinations and permutations:

A "Combination" of n objects taken 'r' at a time called an unordered selection of r $(r \leq n)$ of the n-objects.

A "permutation" of n objects taken 'r' at a time called an ordered selection or arrangement of r of the 'n' objects.

Note: The order of the things is not considered in combinations, and the order of the things considered in Permutations.

The total number of permutations of n objects taken 'r' at a time is denoted by ${}^{n}P_{r}$ (or) $P(n,r)$.

$$ {}^{n}P_{r} = \frac{n!}{(n-r)!} = n(n-1)(n-2)\cdots(n-r+1) $$

Important Results:

1. ${}^{n}P_{n} = n!$
2. ${}^{n}P_{n-1} = {}^{n}P_{n}$
3. $0! = 1$

The number of Combination of n objects taken 'r' at a time is denoted by ^nC_r or $C(n,r)$ or $\binom{n}{r}$.

$$^nC_r = \frac{n!}{r!(n-r)!}$$

* $C(n,n) = 1$
* Relationship between ^nC_r and ^nP_r is : $r! \times {^nC_r} = {^nP_r}$.
* i) $C(n,r) = C(n,n-r)$ (ii) If $C(n,r) = C(n,s)$ then either $r=s$ or $r+s=n$.

Example:

1. Compute $p(8,5)$

Sol $p(8,5) = \frac{8!}{(8-5)!} = \frac{8!}{3!} = \frac{8\cdot7\cdot6\cdot5\cdot4\cdot3!}{3!} = 6720$

2. Compute n and r if $p(n,r) = 3024$

Sol $p(n,r) = \frac{n!}{(n-r)!}$

Since $p(n,r)$ is a product of Consecutive integers.

we write $p(n,r) = 3024 = 9\times8\times7\times6 = p(9,4)$

$\Rightarrow n=9, r=4$

3. Find n if $p(n-1,3) : p(n+1,3) = 5:12$

Sol: $p(n-1,3) : p(n+1,3) = 5:12$

$\Rightarrow 12\, p(n-1,3) = 5\, p(n+1,3)$

$\Rightarrow 12\,(n-1)(n-2)(n-3) = 5(n+1)n(n-1)$

$\Rightarrow 12\,(n-2)(n-3) = 5(n+1)n$

$\Rightarrow 12[n^2-5n+6] = 5(n^2+n)$

$\Rightarrow 12n^2-60n+72 = 5n^2+5n$

$\Rightarrow 7n^2-65n+72=0$

$n=8$ (or) $\frac{9}{7} = 1.2857$

Since 'n' is +ve integer, $n=\frac{9}{7}$ is rejected. $\therefore n=8$.

④ If $C(n,r) = 126$, find n.

Sol Since $C(n,r)$ is a positive integer, we write

$$C(n,r) = 126 = 63 \times 2 = 9 \times 7 \times 2 = \frac{9 \times 8 \times 7}{4} = \frac{9 \times 8 \times 7 \times 6}{6 \times 4}$$

$$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} = C(9,5)$$

$\therefore n = 9$

⑤ If $C(n,6) = C(n,10)$ find $C(n,8)$.

Sol Since $C(n,6) = C(n,10)$

$6 + 10 = n$

$\therefore n = 16$

$$C(n,8) = C(16,8) = \frac{16!}{8!\,8!} = 12870$$

⑥ How many ways Can a hand of 5 Cards to be selected from a deck of 52 Cards?

Sol

$$C(52,5) = \frac{52!}{47!\,5!}$$

⑦ How many Committees of 6 or more Can be chosen from 9 people.

Sol:

$$C(9,6) + C(9,7) + C(9,8) + C(9,9)$$

$$= \frac{9!}{6!\,3!} + \frac{9!}{7!\,2!} + \frac{9!}{8!\,1!} + \frac{9!}{9!\,0!}$$

$= 130.$

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Enumerating Combinations and permutations with Repetitions:

If repetition is allowed then the number of permutations of 'r' objects from a Set of 'n' objects is "n^r."

Example:

1. Consider the 6 digits number 2, 3, 4, 5, 6 and 8 and repetions of digits are allowed.

(a) How many 3 digit numbers Can be formed?

(b) How many 3 digit number must Contain the digit 5.

Sol (a) For a 3-digit number we have to fill up three places.

Since repetitions of the digits is allowed, each of the places Can be filled up in 6 ways.

Hence, the required 3-digit number is $6 \times 6 \times 6 = 6^3 = 216$

(b) Excluding the digit 5, the number of 3 digit numbers that Can be formed from the remaining 5 digits 2, 3, 4, 6 and 8 is $5 \times 5 \times 5 = 5^3 = 125$.

Hence the number must Contain the digit 5.

= Total 3 digit number − the number of 3 digit number that do not Contain 5.

$= 216 - 125$

$= 91$

2. How many four digit numbers Can be formed using the digits 0, 1, 2, 3, 4, 5 if

i) repetition of digits is not allowed

ii) repetition of digits is allowed.

Sol (i) In a four digit number 0 cannot appear in the thousand's place. So, thousand's place can be filled in 5 ways. (viz. 1,2,3,4,5) Since repetition of digits is not allowed and 0 can be used at hundred's place, so hundred's place can be filled in 5 ways. Now, any one of the remaining four digits can be used to fill up ten's place. So, ten's place can be filled in 4 ways. One's place can be filled from the remaining three digits in 3 ways.

Hence, the required number of numbers $= 5 \times 5 \times 4 \times 3 = 300$.

(ii) For a four-digit number we have to fill up four places and 0 cannot appear in the thousand's place. So, thousand's place can be filled in 5 ways. Since repetition of digits is allowed, so each of the remaining three places viz. hundred's, ten's and one's can be filled in 6 ways.

Hence, the required number of numbers $= 5 \times 6 \times 6 \times 6 = 1080$

3. A computer password consists of a letter of the alphabet followed by 4 or 5 digits. Find (a) the total number of passwords that can be formed, and (b) the number of passwords in which no digit repeats.

Sol: (a) Since there are 26 alphabets and 10 digits and the digits can be repeated, by product rule the number of 4-character passwords is $26 \times 10 \times 10 \times 10 = 26000$. Similarly the number of 5-character password is $26 \times 10 \times 10 \times 10 \times 10 = 260000$. Hence the total no. of passwords is $26000 + 260000 = 286000$.

(b) Since the digits are not repeated, the first digit after alphabet can be taken from any one out of 10, the second digit from remaining 9 digits and so on. Thus the no. of 4-character password is $26 \times 10 \times 9 \times 8 = 18720$.

and the number of 5-character password is $26 \times 10 \times 9 \times 8 \times 7$ (5)
$= 131040$ by the product rule. Hence, the total number of passwords is 149760.

Permutations of objects not all Distinct:

The number of permutations of 'n' objects in which 'p' objects are of one type, q objects are of Second type, r objects are of third type and rest are all distinct is

$$\frac{n!}{p!\,q!\,r!}$$

Example:

1. How many different words Can be formed with the letter of the word MISSISSIPPI?

Sol The total no. of words are $\frac{11!}{4!4!2!} = 34650$.

2. The number of arrangements of letters in the word ENGINEERING is

$$P(11; 3,3,2,2,1) = \frac{11!}{3!3!2!2!1!}$$

3. In how many different Arrangements of 28 books be given to 6 students so that 2 of the students will have 4 books each and the other 4 will have 5 books each?

Sol: $P(28; 4,4,5,5,5,5) = \frac{28!}{4!4!5!5!5!5!}$ ways.

4. Find the number of arrangements of letters in the word TALLAHASSEE.

Sol: $P(11; 3,2,2,2,1,1) = \frac{11!}{3!2!2!2!1!1!}$

⑤ How many arrangements can be made of the letters of the word

① Apple ② COMMERCE ③ PROGRAMMING ④ MATHEMATICS

Sol ① There are 5 letters in the word APPLE.

In which p's are 2.

∴ The no. of arrangements of 5 letters of which 2 are similar of one kind is $\frac{n!}{p!} = \frac{5!}{2!} = 60$.

aeiou

② COMMERCE = $\frac{8!}{2!\,2!\,2!}$

③ PROGRAMMING = $\frac{11!}{2!\,2!\,2!}$

④ MATHEMATICS = $\frac{11!}{2!\,2!\,2!}$

⑥ In how many ways a Committee of 5 members Can be Selected from 6 men and 5 women Consisting of 3 men and 2 women?

Sol 3 men out of 6 men Can be Selected in 6_{C_3} ways

2 women out of 5 women Can be Selected in 5_{C_2} ways.

By product rule: $6_{C_3} \times 5_{C_2} = 200$ ways

⑦ out of 5 men & 2 women a Committe of 3 is to be formed, in how many ways can it be formed. If atleast one women is to be included.

Sol There are 2 possible ways:

Total men = 5 & women = 2

① 2 men and 1 women

② 1 men and 2 women

∴ The no. of ways of selecting 2 men & 1 women is $^5C_2 \times {^2C_1}$
$= 20$

Similarly, the no. of ways of selecting 1 men & 2 women is
$^5C_1 \times {^2C_2} = 5$

∴ Required no. of ways of forming the Committee is $20+5=25$.

⑧ The question paper of Mathematics Contains two questions divided into two groups of 5 questions each. In how many ways can an examine answer six questions taking atleast two questions from each group.

Sol The examine can answer questions from two groups in following ways.

① 2 from first group and 4 from second group.

∴ The no. of ways of selecting the questions $= {^5C_2} \times {^5C_4} = 50$

② 3 from first group and 3 from second group.

∴ The no. of ways of selecting the questions $= {^5C_3} \times {^5C_3} = 100$

③ 4 from first group and 2 from Second group.

∴ The no. of ways of selecting the questions $= {^5C_4} \times {^5C_2}$
$= 50$.

∴ The required no. of ways $= 50+100+50 = 200$

① out of 9 girls and 15 boys. How many different Committees Can be formed each Consisting of 6 boys and 4 girls.

Sol: There are 9 girls and 15 boys then we can form 2 Committees such that each Consisting of 6Boys and 4 Girls.

i) To select 6 Boys out of 15 boys & 4 Girls out of 9 Girls.

The no. of ways to select 6B out of 15B is $^{15}C_6 = 5005$

The no. of ways to select 4G out of 9G is $^{9}C_4 = 126$.

By product rule, $^{15}C_6 \times {}^{9}C_4 = 6,30,630$ ways to form a Committee with 6 Boys & 4 Girls.

ii) After forming a 1st Committee, there are remaining 9 Boys & 5 Girls. In which we Can form Another 2nd Committee also.

i.e., we have to select again 6B out of 9B & 4G out of 5G.

∴ No. of ways of selecting 6 boys from 9 boys is $^{9}C_6$

No. of ways of selecting 4 Girls from 5 Girls is $^{5}C_4$.

∴ By product rule,

No. of ways of 2nd Committee is $^{9}C_6 \times {}^{5}C_4 = 420$.

∴ By Sum rule $630630 + 420 = 631050$.

Circular Permutation:

→ Clockwise and Anti clockwise orders are same:

Case i: * The Number of Circular permutations of a distinct items is $\frac{1}{2}[(n-1)!] = P_n$

If anti-clockwise and clockwise order of arrangements are not distinct. e.g. arrangements of beads in a necklace, arrangements of flowers in a garland etc.,

Ex:

1. In how many ways can 7 differently coloured beads be strung on a necklace?

Sol: Since the arrangement is circular, the direction of the arrangements need not be considered,

the number of ways required $= \frac{(7-1)!}{2} = 360$.

→ Clockwise and Anticlockwise orders are different:

Case ii: The number of circular permutations of 'n' objects taken all 'n' at a time is $(n-1)! = P_n$

Ex:

1. How many ways can 5 children arrange themselves in a ring.

Sol: $(n-1)! = (5-1)! = 4! = 24$ ways. [orders are different]

Problem:

1. Calculate circular permutation of 4 persons sitting around a round table considering

i) clockwise and anti clockwise orders as different and

ii) Clockwise and anticlockwise orders as same.

Sol: i) $n=4$,

$P_n = (n-1)! = (4-1)! = 3!$

ii) $P_n = \frac{1}{2}(n-1)!$

$P_4 = \frac{3!}{2} = 3.$

2. How many different arrangements of 8 balls are possible in a circle, given that the clockwise and anticlockwise arrangements are different?

Sol $P_n = (n-1)!$

$\therefore P_8 = (8-1)! = 7! = 5040.$ ways

3. How many different arrangements of 5 students are possible in a circle, given that the clockwise and anticlockwise arrangements are the same?

Sol $P_n = \frac{1}{2}(n-1)!$

$\therefore P_5 = \frac{1}{2}(5-1)! = \frac{24}{2} = 12.$

Combinations with Repetitions formula:

To find out the number of combinations when repetition is allowed.

$$\boxed{C(n,r) = \frac{(n+r-1)!}{r!\,(n-1)!}}$$

Here, n = total no. of objects in a set

r = no. of objects that can be selected from a set.

Example:

1. There are five colored balls in a pool. All balls are of different colors. In how many ways can we choose four pool balls?

Sol: The order in which the balls can be selected doesnot matter in this case. The selection of balls can be repeated.

Total no. of balls in the pool $n=5$

The no. of balls to be selected $r=4$.

we have $C(n,r) = \frac{(n+r-1)!}{r!\,(n-1)!}$

$$\therefore C(5,4) = \frac{(5+4-1)!}{4!\,(5-1)!} = \frac{8!}{4!\,4!} = 70 \text{ different ways.}$$

2. Maria has ten different candies. How many ways can six candies to be selected?

Sol: $$C(10,6) = \frac{(10+6-1)!}{6!\,9!} = \frac{15!}{6!\,9!} = 5005 \text{ ways}$$

3. Ali has seven different chocolates. How many ways can five chocolates be selected?

Sol: $$C(7,5) = \frac{(7+5-1)!}{5!\,6!} = \frac{11!}{5!\,6!} = 462 \text{ ways.}$$

Combinations without repetitions:

To find out the number of Combinations when repetitions are not allowed.

$$C(n,r) = \frac{n!}{(n-r)!\, r!}$$

Example:

1. The number of possible Combinations of 3 objects from 5.

$$C(5,3) = \frac{5!}{2!\, 3!} = 1.$$

2. A man will go on a trip for 3 days, So he will take with him 3 shirts, if he has 7 shirts, how many Combination of Shirts Can he take.

Sol

$$^{n}C_{r} = \frac{n!}{(n-r)!\, r!}$$

$$^{7}C_{3} = \frac{7!}{(7-3)!\, 3!} = 35 \text{ ways}$$

3. In a bucket there are 10 balls, every ball is numbered from 1 to 10, if Somebody pulls out 3 of this balls randomly, how many Combination of Could he take.

Sol

$$^{n}C_{r} = \frac{n!}{(n-r)!\, r!}$$

$$^{10}C_{3} = \frac{10!}{(10-3)!\, 3!}$$

$$\therefore\ ^{10}C_{3} = 120.$$

* The number of unordered choices of 'r' from 'n', with repetitions allowed is

$$\boxed{C(n+r-1, r)}$$

* The number of r-combinations of 'n' objects with unlimited repetitions. is
= The no. of ways of distributing 'r' similar balls into 'n' numbered boxes.

$$\boxed{C(n+r-1, n-1) = \frac{(n+r-1)!}{(n-1)!\, r!}}$$

* The number of solutions of $x_1+x_2 \cdots + x_n = r$ in non-negative integers x_i is $C(n+r-1, n)$

* The number of integral solutions of $x_1+x_2+\cdots+x_n = r$, where each $x_i > 0$. is $C(r-1, n-1)$.

* Suppose that $r_1, r_2, \ldots, r_n$ are integers.
Then the number of integral solutions of $x_1+x_2+\cdots+x_n = r$
where $x_1 \geq r_1, x_2 \geq r_2, \ldots, x_n \geq r_n$. is $C(n, n-r)$

Example:

1. How many solution does the equation $x_1+x_2+x_3 = 17$ have, where x_1, x_2, x_3 are non-negative integers?

Sol Here $n=17$, $r=3$.

Then $C(n+r-1, n) = C(17+3-1, 17) = C(19, 17) = C(19, 2)$
$= 171$

Each solution of the given Equation is equivalent to distribution of 17 identical balls. in 3 numbered boxes with repetitions, where x_i represents the number of balls in the i^{th} box.

② How many solutions are there of $x_1+x_2+x_3=17$ Subject to the Constraints $x_1 \geq 1$, $x_2 \geq 2$ & $x_3 \geq 3$.

Sol First we distribute 1 ball in box, 2 balls in box 2 and 3 balls in box 3.

The remaining 11 balls can be distributed in 3 boxes in

$C(11+3-1, 11) = C(13,11) = C(13,2) = 78$ ways which is the required no.of solutions.

(or)

put $x = 1+u$, $y = 2+v$ & $z = 3+w$.

The given Equation becomes $u+v+w=11$ and we seek in non negative integers u, v, w.

The no.of Solutions is therefore

$$C(11+3-1, 11) = C(13,11) = C(13,2) = 78.$$

③ In how many ways can a prize winner chose three CDS from the top ten if repeats are allowed?

Sol This is an unordered selection with repetition.

Here $n=10$ and $r=3$. Hence the no.of selection is

$C(10+3-1, 3) = C(12,3) = 220$.

Recurrence relation (R.R)

generating functions :-

* The generating function of a sequence $a_0, a_1, a_2, a_3 \ldots a_n$ of a real numbers is written as the series. ~~the~~ given below.

$G(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + \cdots + a_n z^n$

$$G(z) = \sum_{n=0}^{\infty} a_n z^n .$$

find the generating function for the sequence $1, 3, 3^2, 3^3 \ldots$ (or) find the generating function for the sequence $\{a_n\}$ with $a_n = 3^n$.

Sol :- given series $1, 3, 3^2, 3^3 \ldots$

$a_n = 3^n$,

The generating function of given series is $G(z) = \sum_{n=0}^{\infty} 3^n z^n$

Find the generating function for the sequence $1, 2, 3, 4$

Sol :- given series, $1, 2, 3, 4$

$a_n = n+1$.

The generating function for the given series is

$$G(z) = \sum_{n=0}^{\infty} (n+1) z^n.$$

find the generating function of the following sequences

(i) 0, 1, −2, 3, −4 - - - -

(ii) 0, 2, 6, 12, 20, 30, 42 - - - -

Sol:- (i) given series, 0, 1, −2, 3, −4, - - - -

$$a_n = (-1)^{n+1} \cdot n$$

The generating function for the given series is

$$G(z) = \sum_{n=0}^{\infty} (-1)^{n+1} n z^n$$

$(-1)^{n+1} \cdot n = 0 \quad n=0$

$(-1)^{1+1} \cdot 1 \quad (-1)^{n+1} \cdot n = 1 \quad n=1$

$(-1)^{2+1} \cdot 2 \quad (-1)^{n+1} \quad n = -2$

(ii) given series, 0, 2, 6, 12, 20, 30, 42,

$(-1)^{3+1} \cdot 3 \Rightarrow n = 3$

$(-1)^{4+1} \cdot 4 \Rightarrow n = -4$

$$a_n = \frac{2n(n+1)}{2}$$

the generating function for the given series is

$$G(z) = \sum_{n=0}^{\infty} \frac{2n(n+1)}{2} z^n$$

$n=0 \Rightarrow 0$

$n=1 \Rightarrow \frac{\cancel{2} \cdot 2}{\cancel{2}} = 2$

$n=2 \Rightarrow \frac{\cancel{2}(2)(3)}{\cancel{2}} = 6$

$n=3 \Rightarrow \frac{\cancel{2} \cdot 3 \cdot 4}{\cancel{2}} = 12$

$n=4 \Rightarrow \frac{\cancel{2} \cdot 4 \cdot 5}{\cancel{2}} = 20$

$n=5 \Rightarrow \frac{\cancel{2} \cdot 5 \cdot 6}{\cancel{2}} = 30$

$n=6 \Rightarrow \frac{\cancel{2} \cdot 6 \cdot 7}{\cancel{2}} = 42$

	Sequence (a_n)	generating function $G(z)$
①	a^n	$\frac{1}{1-az}$
②	ka^n	$\frac{k}{1-az}$
③	bna^n	$\frac{baz}{(1-az)^2}$
④	1	$\frac{1}{1-z}$
⑤	$n+1$	$\frac{1}{(1-z)^2}$
⑥	$\frac{1}{n!}$	e^z
⑦	$\frac{(-1)^{n+1}}{n}$	$\log(1+z) = z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4} + \dots$
⑧	${}^nC_k$	$(1+x)^n$
⑨	${}^nC_k\, a^n$	$(1+ax)^n$
⑩	${}^{n-k+1}C_k = {}^{n+k-1}C_{n-1}$	$\frac{1}{(1-x)^n}$
⑪	$(-)^k\, {}^{n+k-1}C_k = (-1)^k\, {}^{(n+k-1)}C_{n-1}$	$\frac{1}{(1-x)^n}$

problems :-

using generating function to solve the recurrence relation using generating function $a_n = 3a_{n-1} + 2$, $n \geq 1$ with $a_0 = 1$

Sol: given $a_n = 3a_{n-1} + 2$ $n \geq 1$ with $a_0 = 1$

Taking bothsides $\sum_{n=0}^{\infty} z^n$

$$\sum_{n=0}^{\infty} a_n z^n = 3\sum_{n=0}^{\infty} a_{n-1} z^n + 2\sum_{n=0}^{\infty} 1 z^n$$

$$\sum_{n=1}^{\infty} a_n z^n = 3z \sum_{n=1}^{\infty} a_{n-1} z^{n-1} + 2\sum_{n=1}^{\infty} z^n$$

$0 \cdot z^n$

a_0

$$(G(z) - a_0) = 3z\,G(z) + 2\frac{1}{(1-z)}$$

$$G(z) - 1 - 3z\,G(z) = \frac{2z}{1-z}$$

$$G(z)\,[1-3z] = \frac{2z}{1-z} + 1$$

$$G(z) = \frac{2z + 1 - z}{(1-z)(1-3z)}$$

$$G(z) = \frac{z+1}{(1-z)(1-3z)}$$

$3 \sum_{n=1} a_{n-1} \frac{z^n z}{z}$

$3z$ $\quad \frac{z^n z^{-1}}{z}$

z^{n-1}

$G(z) = a_0 + \boxed{a_1 z + \cdots}$

$G(z) - a_0 = a_1 z + a_2 z^2$

$G(z) - a_0 - a_1 z = a_2 z^2 -$

$$G(z) = \frac{z+1}{(1-z)(1-3z)} = \frac{A}{(1-z)} + \frac{B}{(1-3z)} \longrightarrow (1)$$

$$\frac{z+1}{\cancel{(1-z)(1-3z)}} = \frac{A(1-3z) + B(1-z)}{\cancel{(1-z)(1-3z)}}$$

$$z+1 = A(1-3z) + B(1-z) \longrightarrow (2)$$

put $z=1$ in eq (2) we get

$\cancel{2} = A(\cancel{-2}) + 0$

$\boxed{A = -1}$

put $z = \frac{1}{3}$ in eq (2) we get

$\frac{1}{3} + 1 = 0 + B(1 - \frac{1}{3})$

$\frac{\cancel{4}^{2}}{\cancel{3}} = B\frac{\cancel{2}}{\cancel{3}}$

$\boxed{B = 2}$

$$G(z) = \frac{z+1}{(1-z)(1-3z)} = \frac{-1}{1-z} + \frac{2}{1-3z}$$

$$G(z) = \frac{-1}{1-z} + \frac{2}{1-3z}$$

$$G(z) = -1\left(\frac{1}{1-z}\right) + 2\left(\frac{1}{1-3z}\right)$$

$$a_n = -1(1) + 2(3^n)$$

$$\boxed{a_n = -1 + 2(3^n)}$$

(2) Using the method of generating function to solve recurrence relation of

$a_n - 2a_{n-1} - 3a_{n-2} = 0,\ n \geq 2$; with $a_0 = 3, a_1 = 1$

Sol:- given,

$a_n - 2a_{n-1} - 3a_{n-2} = 0, n \geq 2,$

$$\sum_{n=2}^{\infty} a_n z^n - 2\sum_{n=2}^{\infty} a_{n-1} z^n - 3\sum_{n=2}^{\infty} a_{n-2} z^n = 0$$

$$\left(G(z)-a_0-a_1z\right) - 2z\sum_{n=2}^{\infty} a_{n-1}z^{n-1} - 3z^2\sum_{n=2}^{\infty} a_{n-2}z^{n-2} = 0$$

$$\left(G(z)-a_0-a_1z\right) - 2z\left(G(z)-a_0\right) - 3z^2 G(z) = 0$$

$$\left(G(z)-3-z\right) - 2z\left(G(z)-3\right) - 3z^2 G(z) = 0$$

$$G(z)\left[-3z^2-2z+1\right] - 3 - z + 6z = 0$$

$$G(z)\left[-3z^2-2z+1\right] - 3 + 5z = 0$$

$$G(z) = \frac{3-5z}{(-3z^2-2z+1)}$$

$$G(z) = \frac{3-5z}{(1+z)(1-3z)}$$

$$G(z) = \frac{3-5z}{(1+z)(1-3z)} = \frac{A}{(1+z)} + \frac{B}{(1-3z)} \longrightarrow (1)$$

$$\frac{3-5z}{(1+z)\cancel{(1-3z)}} = \frac{A(1-3z)+B(1+z)}{(1+z)\cancel{(1-3z)}}$$

$$3-5z = A(1-3z)+B(1+z) \longrightarrow (2)$$

put $z=-1$ in eq ②, we get

$$3-5(-1) = A(1-3(-1)) + B(1-1)$$

$$3+5 = A(1+3) + B(0)$$

$$\overset{2}{\cancel{8}} = A(\cancel{4})$$

$$\boxed{A=2}$$

Put $z = 1/3$ in eq ②, we get

$$3-5(1/3) = A(1-3(1/3)) + B(1+1/3)$$

$3 - 5/3 = A(1 - 3/5) + B($

$3 - 5/3 = A(1-1) + B(4/3)$

$\frac{9-5}{3} = B(4/3)$

$4/3 = B(4/3)$

$\boxed{B=1}$

$G(z) = \frac{2}{1+z} + \frac{1}{1-3z}$

$G(z) = 2\left(\frac{1}{1-(-z)}\right) + \frac{1}{1-3z}$

$a_n = 2(-1^n) + (3^n)$

$a_n = -2 + 3^n$

Recurrence relation :-

An Equation that Express a_n in terms of one or more of the previous terms of the sequence $a_0, a_1, a_2 - - - a_n$ is called a recurrence relation for the sequence $\{a_n\}$.

1) find the first five terms of the sequence define by each of the following recurrence relation and intial conditions

(i) $a_n = a_n^2 - 1$, $a_1 = 2$

(ii) $a_n = n a_{n-1} + n^2 a_{n-2}$ $a_0 = 1$, $a_1 = 1$

(iii) $a_n = a_{n-1} + a_{n-3}$ $a_0 = 1$, $a_1 = 2$, $a_2 = 0$

(i) given R.R is $a_n = a_{n-1}^2$

sol:

put $a = 2$

$a_2 = a_{2-1}^2$

$a_2 = a_1^2$

$a_2 = 4$

$a_3 = a_2^2 = 16$

$a_4 = a_3^2 = (16)^2 = 256$

$a_5 = a_4^2 = (256)^2 = (65536)$

$a_6 = a_5^2 = (65536)^2 = 4294967296.$

(ii) $a_n = n a_{n-1} + n^2 a_{n-2}$, $a_0 = 1$, $a_1 = 1$

given R.R is $a_n = n a_{n-1} + n^2 a_{n-2}$

put $n = 2$

$a_2 = 2 a_{2-1} + 2^2 a_{2-2}$

$a_2 = 2a_1 + 4 a_0$

$a_2 = 2(1) + 4(1)$

$a_2 = 2 + 4$

$a_2 = 6$

$\Rightarrow a_3 = 3 a_{3-1} + 3^2 a_{3-2}$

$= 3 a_2 + 9 a_1$

$= 3(6) + 9(1)$

$= 18 + 9 \Rightarrow 27$

$a_4 = 4a_{4-1} + (4)^2 a_{4-2}$

$= 4a_3 + 16a_2$

$= 4(27) + 16(6)$

$= 108 + 96$

$= 204$

$a_5 = 5a_{5-1} + (5)^2 a_{5-2}$

$= 5a_4 + 25a_3$

$= 5(204) + 25(27)$

$= 1020 + 675$

$a_5 = 1695.$

$a_6 = 6(a_5) + 36a_4$

$= 6(1695) + 36(204)$

$= 17514$

(iii) given RR is, $a_n = a_{n-1} + a_{n-3}$, $a_0 = 1, a_1 = 2, a_2 = 0$

put $n = 3$

$\rightarrow a_3 = a_{3-1} + a_{3-3}$

$a_3 = a_2 + a_0$

$a_3 = 0 + 1$

$a_3 = 1$

$\rightarrow a_4 = a_{4-1} + a_{4-3}$

$a_4 = a_3 + a_1$

$a_4 = 1 + 2$

$a_4 = 3$

$\rightarrow a_5 = a_{5-1} + a_{5-3}$

$= a_4 + a_2 \Rightarrow 3 + 0 \Rightarrow 3$

$\rightarrow a_6 = a_{6-1} + a_{6-3}$

$= a_5 + a_3$

$= 3 + 1$

$a_7 = 4$

$\rightarrow a_7 = a_{7-1} + a_{7-3}$

$= a_6 + a_4$

$= 4 + 3$

$a_7 = 7$

By using an iterative approach find the solutions to each of these recurrence relation with the given initial conditions

(i) $a_n = a_{n-1} + 2$, $a_0 = 3$

(ii) $a_n = a_{n-1} + n$, $a_0 = 1$

(iii) $a_n = a_{n-1} + 2n + 3$, $a_0 = 4$

(iv) $a_n = 3a_{n-1} + 1$, $a_0 = 1$

(i) given R.R is $a_n = a_{n-1} + 2$

put $n = 1$

$a_1 = a_0 + 2$

$a_1 = 3 + 2$

$a_1 = 5$

put $n = 2$

$a_2 = a_1 + 2$

$a_2 = 5 + 2$

$= 7$

put $n=3$

$a_3 = a_2 + 2$

$a_3 = 9$

$3+0\times2$

$3+1\times2$

$3+2\times2$

$3+3\times2$

$3+4\times2$

$3+n\cdot2$

$a_n = 3+2n$

$\rightarrow$ it will satisify from, 0,1,2

(ii) given, $a_n = a_{n-1}+n$

given, R.R is $a_n = a_{n-1}+n$

put $n=1$

$a_1 = a_{1-1}+1$

$= a_0+1$

$= 1+1$

$= 2$

put $n=2$

$a_2 = a_{2-1}+2$

$= a_1+2$

$= 2+2$

$= 4$

put $n=3$

$a_3 = a_{3-1}+3$

$a_3 = a_2+3$

$= 4+3$

$a_3 = 7$

put $n=4$

$a_4 = a_{4-1}+4$

$= a_3+4$

$= 7+4$

$= 11$

$1+\frac{n+1}{2}\cdot n$

$\Rightarrow$ put $n=1 \Rightarrow 1+\frac{1+1}{2}\cdot 1 \Rightarrow 2$

$\Rightarrow$ put $n=2 \Rightarrow 1+\frac{2+1}{2}\cdot 2 \Rightarrow 4$

$\Rightarrow$ put $n=3 \Rightarrow 1+\frac{3+1}{2}\cdot 3 \Rightarrow 7$

$\Rightarrow$ put $n=4 \Rightarrow 1+\frac{4+1}{2}\cdot 4 \Rightarrow 11$

$a_n = 1+\left(\frac{n+1}{2}\right)\cdot n$

(iii)

given R.R is $a_n = a_{n-1}+2n+3$, $a_0=4$

put $n=1$

$a_1 = a_{1-1}+2(1)+3$

$= a_0+2+3$

$= a_0+5$

$= 4+5$

$= 9$

put $n=2$

$a_2 = a_{2-1}+2(2)+3$

$= a_1+4+3$

$= a_1+7$

$= 9+7$

$a_2 = 16$

put $n=3$

$a_3 = a_{3-1}+2(3)+3$

$= a_2+6+3$

$= a_2+9 = 16+9$

$= 25.$

put $n=4$

$$a_4 = a_{4-1} + 2(4) + 3$$
$$= a_3 + 8 + 3$$
$$= a_3 + 11$$
$$= 25 + 11$$
$$= 36$$

$\Rightarrow a_0 = n^2 + 0\times4 + 4 \Rightarrow n=0$
$\Rightarrow a_1 = n^2 + 1\times4 + 4 \Rightarrow n=1$
$\Rightarrow a_2 = n^2 + 2\times4 + 4 \Rightarrow n=2$
$\Rightarrow a_3 = n^2 + 3\times4 + 4 \Rightarrow n=3$

$\vdots$

$$a_n = n^2 + n4 + 4$$
$$a_n = n^2 + 4n + 4$$

(iv) given R.R is $a_n = 3a_{n-1} + 1$, $a_0 = 1$

put $n=1$

$$a_1 = 3a_{1-1} + 1$$
$$= 3a_0 + 1$$
$$= 3(1) + 1$$
$$= 3 + 1$$
$$a_1 = 4$$

put $n=2$

$$a_2 = 3a_{2-1} + 1$$
$$= 3a_1 + 1$$
$$= 3(4) + 1$$
$$= 13$$

put $n=3$

$$a_3 = 3a_{3-1} + 1$$

$a_3 = 3a_2 + 1$

$= 3(13)+1$

$= 39+1$

$= 40$

put $n=4$

$a_4 = 3a_3+1$

$= 3(40)+1$

$= 121$

$\vdots$

$a_n = \dfrac{3^{n+1}-1}{2}$

put $n=0 \Rightarrow a_n = \dfrac{3^{n+1}-1}{2}$

$a_n = \dfrac{3^{0+1}-1}{2}$

$a_n = \dfrac{3-1}{2} = 1$

put $n=1 \Rightarrow a_n = \dfrac{3^{1+1}-1}{2}$

$= \dfrac{3^2-1}{2}$

$= \dfrac{9-1}{2}$

$= 4$

put $n=2 \Rightarrow a_n = \dfrac{3^{2+1}-1}{2}$

$= \dfrac{3^3-1}{2}$

$= \dfrac{27-1}{2}$

$= 13$

put $n=3 \Rightarrow a_n = \dfrac{3^{3+1}-1}{2}$

$a_n = \dfrac{3^4-1}{2}$

$a_n = \dfrac{80}{2}$

$a_n = 40$

put $n=4, \Rightarrow a_n = \dfrac{3^{4+1}-1}{2}$

$a_n = \dfrac{3^5-1}{2}$

$a_n = 121$

characteristic roots : consider the recurence relation $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k}$, where $c_1, c_2, c_3 \cdots c_k$ are real numbers,

→ The characteristic equation of Recurrence relation, $r^k - c_1 r^{k-1} - c_2 r^{k-2} - \cdots a_c + 0$

→ The solutions of characteristic equations are three types

(1) if roots are real & different then the solution is,

$$a_n = c_1 r_1^n + c_2 r_2^n$$

(2) if roots are real & equal then solution is,

$$a_n = (c_1 + c_2 n) r^n$$

(3) if roots are complex roots, then solution is,

$$a_n = r^n [c_1 \cos n\theta + c_2 \sin n\theta]$$

Problems :-

1. solve the Recurrence relation, $a_n = 5a_{n-1} + 6a_{n-2}$ for $n \geq 2$, $a_0 = 1$, $a_1 = 0$.

Sol :- given R.R is,

$$a_n = 5a_{n-1} + 6a_{n-2}$$

By simplifying,

$a_n - 5a_{n-1} + 6a_{n-2} = 0$

characteristic eq of R.R is $r^2 - 5r + 6 = 0$

roots, $r = 2, 3$

$\therefore$ the given roots are real and different, then the solution is

$$a_n = c_1 r_1^n + c_2 r_2^n$$

$$a_n = c_1(2)^n + c_2(3)^n$$

Now, put $n=0$

$$a_0 = c_1 2^0 + c_2 3^0$$

$$1 = c_1 + c_2 \longrightarrow (1)$$

Now, put $n=1$

$$a_1 = c_1 2^1 + c_2 3^1$$

$$0 = 2c_1 + 3c_2 \longrightarrow (2)$$

Now (1) & (2) becomes,

$$c_1 + c_2 = 1 \times 2$$

$$2c_1 + 3c_2 = 0 \times 1$$

$$\cancel{2c_1} + 2c_2 = 2$$

$$\cancel{2c_1} + 3c_2 = 0$$

$$- \quad - \quad -$$

$$-c_2 = 2$$

$$\boxed{c_2 = -2}$$

from, $-2 + c_1 = 1$

$$c_1 = 1 + 2$$

$$\boxed{c_1 = 3}$$

$\therefore$ $a_n = 3 \, 2^n - 2 \cdot 3^n$.

2) solve the recurrence relation of

$a_n - 6a_{n-1} + 9a_{n-2} = 0 \quad n \geq 2 \cdot a_0 = 5, a_1 = 12,$

$n \geq i = 2$

Sol :- Given $a_0 = 5$

$a_1 = 12$

R.R is $a_n - 6a_{n-1} + 9a_{n-2} = 0$

characteristic equation

$r^2 - 6r + 9 = 0$

$r^2 - 3r - 3r + 9 = 0$

$r(r-3) - 3(r-3) = 0$

$(r-3)(r-3) = 0$

roots $= 3, 3$

the given roots are real and equal

the solution will be

$a_n = (c_1 + c_2 n)3^n$

put $n=0$, $a_0 = (c_1 + c_2(0))3^0$

$5 = c_1 \longrightarrow (1)$

put $n=1$, $a_1 = (c_1 + c_2(1))3^1$

$12 = (c_1 + c_2)3 \longrightarrow (2)$

$c_1 = 5$

$5 = c_1$

$12 = (c_1 + c_2)3$

$15 + 3c_2 = 12$

$3c_2 = 12 - 15$

$3c_2 = -3$

$c_2 = -1$

$a_n = (5 - 1)3^n$

3) solve the recurrence relation :

$a_n = 8a_{n-1} - 16a_{n-2}$ for $n \geq 2$, $a_0 = 16$, $a_1 = 80$.

sol :- given R.R is

$a_n = 8a_{n-1} - 16a_{n-2}$

By simplifying,

$a_n - 8a_{n-1} + 16a_{n-2} = 0$

characteristic eq of R.R is $r^2 - 8r + 16 = 0$

$r^2 - 8r + 16 = 0$

$r^2 - 4r - 4r + 16 = 0$

$r(r-4) - 4(r-4) = 0$

$(r-4)(r-4) = 0$

$r = 4, 4$

The given roots are real and equal

the solution will be,

$a_n = (c_1 + c_2 n) 4^n$

put $n = 0$

$a_0 = (c_1 + c_2(0)) 4^0$

$16 = c_1$

put $n = 1$.

$a_1 = (c_1 + c_2(1)) 4^1$

$80 = (16 + c_2) 4$

$64 + 4c_2 = 80$

$4c_2 = 80 - 64$

$c_2 = 4$

$a_n = (16+4)4^n$

(4) solve the recurrence relation

$a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3}$ for $n = 3,4,5 \ldots$

with $a_0 = 3$, $a_1 = 6$, $a_2 = 0$

Sol:- given recurrence relation is

$$a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3}$$

By simplifying,

$$a_n - 2a_{n-1} + a_{n-2} + 2a_{n-3} = 0$$

characteristic of recurrence relation is

$$r^3 - 2r^2 - r + 2 = 0$$

roots $= 1, 2, -1$

The roots are real & different $r^2 - r - 2 = 0$

The solution will be

$r^2 - 2r + r - 2 = 0$

$r(r-2) + 1(r-2) = 0$

$$a_n = c_1 r_1^n + c_2 r_2^n + c_3 r_3^n$$

$(r-2)(r+1) = 0$

$$a_n = c_1(1)^n + c_2(2)^n + c_3(-1)^n$$

$r = 2, -1$

$$a_n = c_1(1)^n + c_2(-1)^n + c_3(2)^n$$

put $n = 0$

$$a_0 = c_1(1)^0 + c_2(-1)^0 + c_3(2)^0$$

$$a_0 = c_1 1^0 + c_2(-1)^0 + c_3(2)^0$$

$$3 = c_1 + c_2 + c_3 \longrightarrow (1)$$

put $n = 1$

$$a_1 = c_1(1)^1 + c_2(-1)^1 + c_3(2)^1$$

$a_1 =$

$$6 = c_1 - c_2 + 2c_3 \longrightarrow (2)$$

put $n=2$

$$a_2 = c_1(1)^2 + c_2(-1)^2 + c_2(2)^2 \longrightarrow (3)$$

$$0 = c_1 + c_2 + 4c_3 \longrightarrow (3)$$

solve (1) & (2)

$$\begin{aligned} c_1 + \cancel{c_2} + c_3 &= 3 \\ c_1 - \cancel{c_2} + 2c_3 &= 6 \\ \hline 2c_1 + 3c_3 &= 9 \longrightarrow (4) \end{aligned}$$

solve (2) & (3)

$$\begin{aligned} c_1 - \cancel{c_2} + 2c_3 &= 6 \\ c_1 + \cancel{c_2} + 4c_3 &= 0 \\ \hline 2c_1 + 6c_3 &= 6 \longrightarrow (5) \end{aligned}$$

solve (4) & (5)

$$\begin{aligned} 2c_1 + 3c_3 &= 9 \\ 2c_1 + 6c_3 &= 6 \\ \hline -3c_3 &= 3 \\ c_3 &= -1 \end{aligned}$$

sub c_3 value in (5)

$$2c_1 + 6(-1) = 6$$

$$2c_1 = 12$$

$$\boxed{c_1 = 6}$$

sub c_1 & c_3 in (2)

$$6 - c_2 - 2 = 6 \qquad \boxed{c_3 = -1}$$

$$-c_2 = 2$$

$$\boxed{c_2 = -2}$$

Solutions of inhomogenous recurrence relation

→ A linear inhomogenous or non homogenous recurrence realtion with constant coefficents of degree k is a recurrence relation of the form $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k} + G(n)$, where c_1, c_2 up to c_k are real numbers and G(n) is a function not identically zero depending only on (n)

Particular solution for G(n) :-

G(n)	P.I
① constant c	constant d
② linear function $(c_0 + c_1 n)$	$d_0 + d_1 k$
③ m^{th} degree polynomial $c_0 + c_1 n + c_2 n^2 + \cdots + c_m n^m$	m^{th} degree polynomial $d_0 + d_1 k + d_2 k^2 + \cdots + d_m k^m$
④ r^n $r \in R$	$d r^n$

1) Solve the recurrence relation

$a_n = 3a_{n-1} + 2^n, a_0 = 1, n \geq 1$

Sol :-

Given $a_n = 3a_{n-1} + 2^n$

it is a non homogenous linear Equation,

$$a_n - 3a_{n-1} = 2^n$$

general solution

$a_n - 3a_{n-1} = 0$

The characteristic Equation of given Eq,

$r - 3 = 0$

roots r

$r = 3$

The roots are real solution will be

$a_n = c_1 (3)^n$

put $n=0$

$a_0 = c_1 (3)^0$

$1 = c_1$

$\boxed{c_1 = 1}$

$a_n = (3)^n$

Now, we can PI,

$PI = 2^n$

$d2^n - 3d\,2^{n-1} = 2^n$

$2^n \left(d - \frac{3d}{2}\right) = 2^n$

$2d - 3d = 2$

$-d = 2$

$\boxed{d = -2}$

this is of the form dr^n

$dr^n = (-2)2^n \Rightarrow PI$

Now, $a_n = C_1 + PI$

$a_n = (3)^n + (-2)2^n$

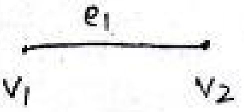
29-8-16 S.lakshmi...etha

VI. GRAPH THEORY

Graph:- A graph G has pair (V, E) where V is a non empty finite set whose elements are called vertices (nodes or points). E is a another set whose elements are called edges (lines).

The graph G with vertices V and edges E is written as $G = (V, E)$ (or) $G(V, E)$.

Ex:-



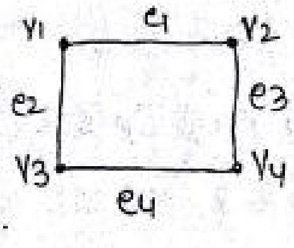
Here $V = \{v_1, v_2\}$, $E = \{e_1\}$

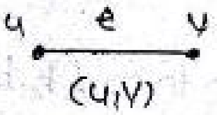
Note:- 1. If an edge $e \in E$ is associated with an ordered pair (u, v) where $(u, v) \in V$.

2. e(edge) is connected to u and v are called end points of e.

3. Any two vertices connected by an edge in a graph is called adjacent vertices.

4. Any two edges e_1 and e_2 are incident with a common point (or) vertex then they are called adjacent edges.



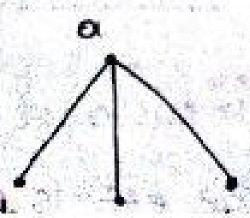


Here e has two adjacent vertices u and v.

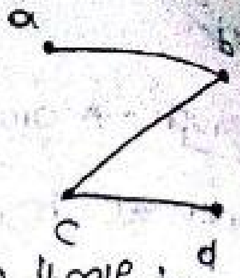
In the above graph v_1 has two adjacent edges e_1 and e_2 and v_1 denoted as incident vertex.

$V = \{a, b, c, d\}$ and $E = \{(a,b)(a,c)(a,d)\}$ draw the graph G.

Sol:-



2. construct the vertices and edges from given graph.



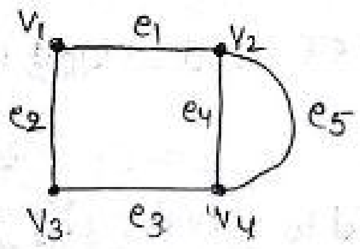
Sol:- the given graph $G = (V, E)$.

$V = \{a, b, c, d\}$ $E = \{(a,b)(b,c)(c,d)\}$

loop:- An edge of a graph G that join a node to itself is called a loop (or) self loop defined as $e_1 = (v_1, v_1)$

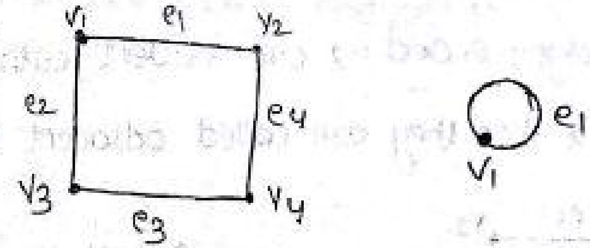
30-8-16

Multigraph:- If more than one line (edge) joining between two vertices are allowed in a graph then the graph is called multigraph. Ex:-



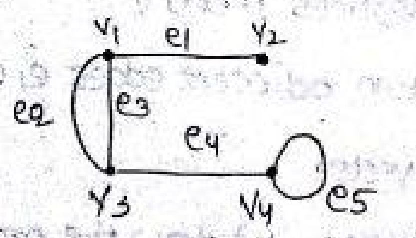
Simple graph:- A graph has neither loops nor multiple edges is called a simple graph.

Ex:-



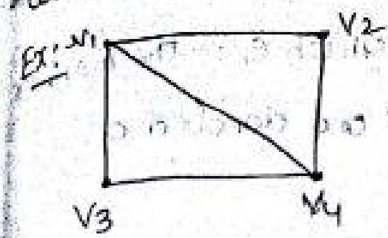
Psuedo graph:- A graph in which loops and multiple edges are allowed is called psuedo graph.

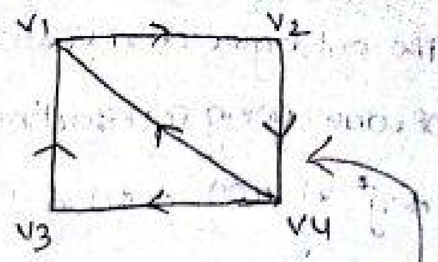
Ex:-



Directed and undirected graph:-

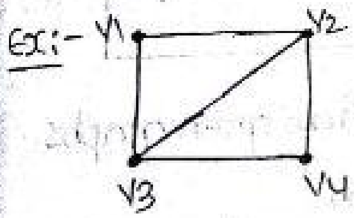
Undirected graph:- An undirected graph G has a set of vertices V and a set of edges E such that each edge $e \in E$ is associated with an unordered pair of vertices. ($e \in (v_i, v_j)$ and (v_j, v_i)).

Ex:- 

Ex:- 

directed graph:- A directed graph G has a set of vertices V and a set of edges E such that each edge $e \in E$ is associated with an ordered pair of vertices; means directions on each edge ($e \in (v_i, v_j)$)

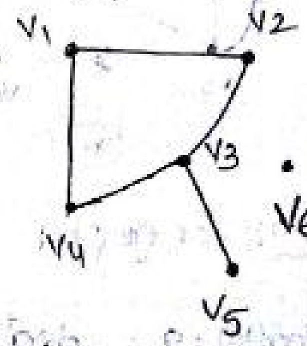
*Degree of a Vertex:- The degree of a vertex v of an undirected graph G is the no. of edges incident with it. The degree of that vertex denoted as deg(v) (or) d(v).

Ex:- 

$\deg(v_1) = 2$ $\quad$ $\deg(v_3) = 3$

$\deg(v_2) = 3$ $\quad$ $\deg(v_4) = 2$

1. construct degree of vertices from given diagram.



Sol:- $\deg(v_1) = 2$, $\deg(v_2) = 2$

$\deg(v_3) = 3$, $\deg(v_4) = 2$

$\deg(v_5) = 1$, $\deg(v_6) = 0$.

Note:- 1. The vertex degree '0' is called Isolated vertex.

2. The vertex degree '1' is called pendant vertex.

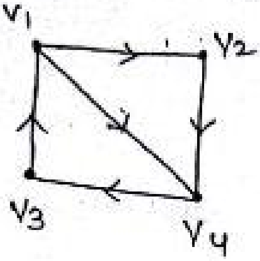
1/9/16

*In-degree and out-degree on directed graphs:-

The In-degree of a vertex v of a directed graph G is the number of edges receiving (or) ending (or) Coming at V and denoted as $\deg^-(V)$ (or) Indeg (V).

The outdegree of a vertex V of a directed graph G is the number of edges going (or) starting (or) sending at V and denoted as $\deg^+(V)$ (or) outdeg (V).

Ex:-



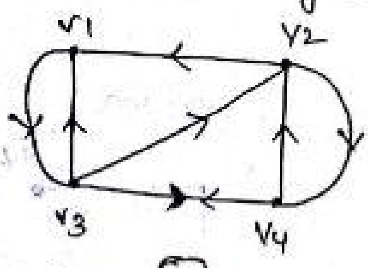
Indegree	out-degree
$\deg^-(V_1) = 1$	$\deg^+(V_1) = 2$
$\deg^-(V_2) = 1$	$\deg^+(V_2) = 1$
$\deg^-(V_3) = 1$	$\deg^+(V_3) = 1$
$\deg^-(V_4) = 2$	$\deg^+(V_4) = 1$

Note:- If $G = (V, e)$ is a directed graph with edge e then

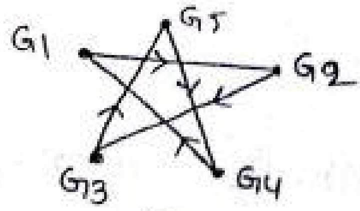
$$\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(V) = E$$

$$\boxed{\sum_{i=1}^{n} \deg(V_i) = 2E}$$

1. construct In-degrees and out-degrees from given graphs.



G1



G2

Sol:- for graph 1

$\deg^-(V_1) = 2$ $\quad \deg^+(V_1) = 1$

$\deg^-(V_2) = 2$ $\quad \deg^+(V_2) = 2$

$\deg^-(V_3) = 2$ $\quad \deg^+(V_3) = 2$

$\deg^-(V_4) = 1$ $\quad \deg^+(V_4) = 2$

for graph 2

$\deg^-(G_1) = 1$ $\quad \deg^+(G_1) = 1$

$\deg^-(G_2) = 1$ $\quad \deg^+(G_2) = 1$

$\deg^-(G_3) = 1$ $\quad \deg^+(G_3) = 1$

$\deg^-(G_4) = 1$ $\quad \deg^+(G_4) = 1$

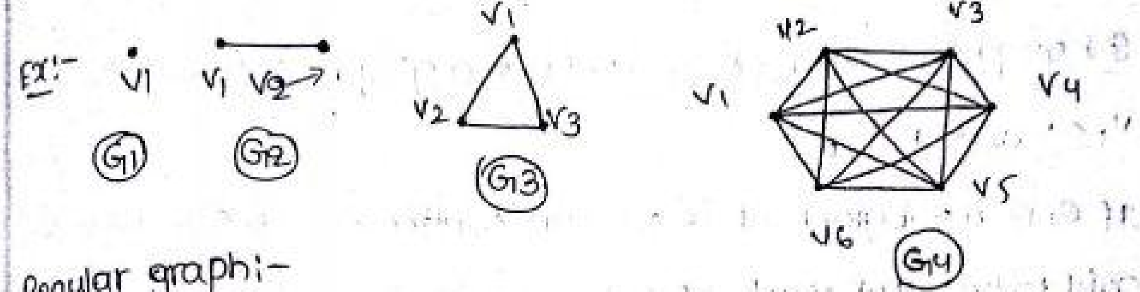
$\deg^-(G_5) = 1$ $\quad \deg^+(G_5) = 1$

Null graph:- A graph G in which node (or) vertex is isolated node is called a null graph. the vertex V has '0' edges.

Ex:- v_1 v_2

complete graph:- A simple graph G is said to be complete graph if every vertex in G is connected with every other vertex. i.e exactly one edge between pair of distinct vertices.

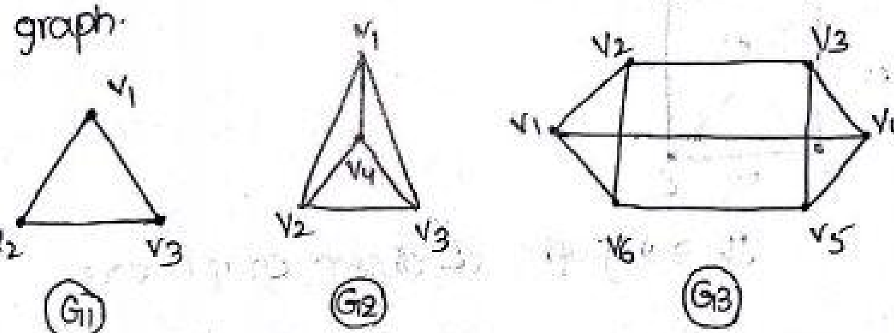
Ex:-



Regular graph:-

A graph G has all vertices of degree is equal is called a regular graph.

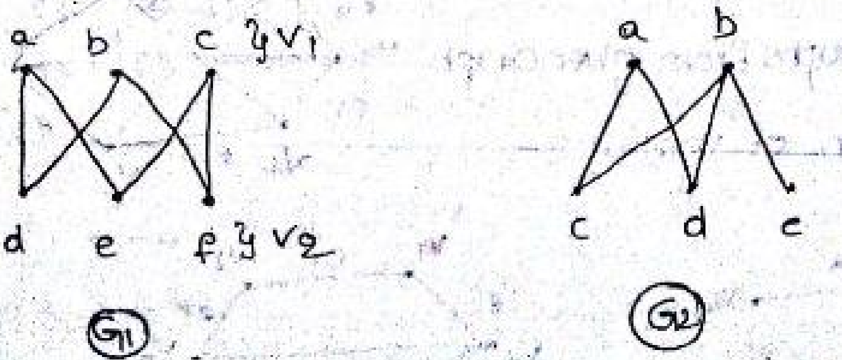
Ex:-

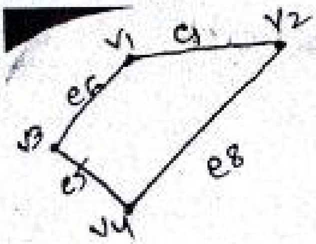


2.9-

*Bipartiate graph:- A graph $G=(V,E)$ is said to be Bipartiate graph if the vertex V can be divided into 2 disjoint subsets V_1 and V_2 such that every edge e connects from V_1 to V_2. No edge is connects either two vertices in V_1 (or) V_2 of G.

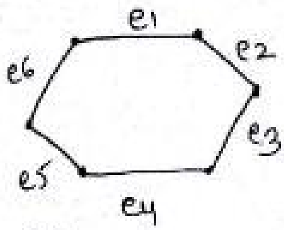
Ex:-

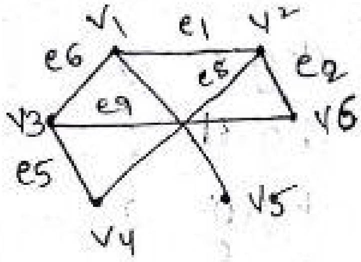




(V_5, V_6)

elimination of edges:-





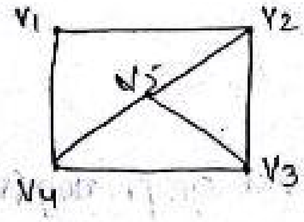
4-9-

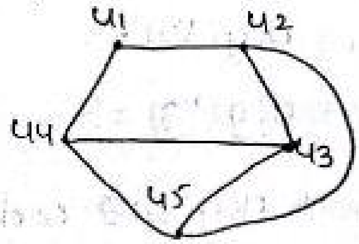
Isomorphism:- Two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are said to be Isomorphic if there exists a Bijection $f: V_1 \to V_2$ such that $(u_1, v_1) \in E_1$ (or) adjacent vertices in G_1. $(u_2, v_2) \in E_2$ are adjacent vertices in G_2.

degree of vertex in G_1 are equivalent to degree of Vertex in G_2. if the adjacent vertices degrees are equal in G_1 and G_2 Such that G_1 is isomorphic to G_2, then we write as $G_1 \cong G_2$.

(degrees of vertices are same).

Ex:-





$\deg(V_1) = 2 \Leftrightarrow \deg(u_1) = 2$

$\deg(V_2) = 3 \Leftrightarrow \deg(u_2) = 3$

$\deg(V_4) = 3 \Leftrightarrow \deg(u_4) = 3$

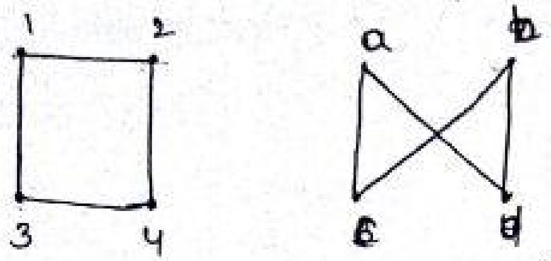
$\deg(V_3) = 3 \Leftrightarrow \deg(u_3) = 3$

$\deg(V_5) = 3 \Leftrightarrow \deg(u_5) = 3$

$\therefore$ The given two graphs are in isomorphism.

1. Show that the following graphs G and G' are isomorphic.

(a)



Sol:- $deg(1) = 2 \Leftrightarrow deg(a) = 2$

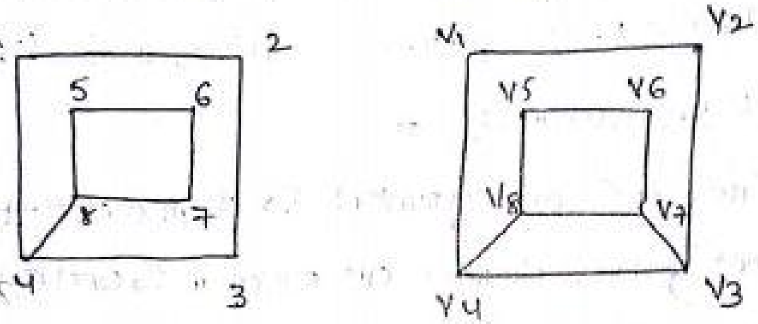
$deg(2) = 2 \Leftrightarrow deg(d) = 2$

$deg(3) = 2 \Leftrightarrow \cancel{deg(b) = 2}$ $deg(c) = 2$

$deg(4) = 2 \Leftrightarrow \cancel{deg(d) = 2}$ $deg(b) = 2$

∴ The given two graphs are isomorphism.

(b)



$deg(1) = 2 \Leftrightarrow deg(V_1) = 2$

$deg(2) = 2 \Leftrightarrow deg(V_2) = 2$

$deg(4) = 3 \Leftrightarrow deg(V_4) = 3$

$deg(3) = 2 \Leftrightarrow deg(V_3) = 3.$

In the first graph $deg(3) = 2$ and in second graph $deg(V_3) = 3$

∴ These two graphs are not Isomorphic.

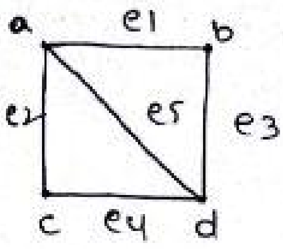
3) Matrix representation of a graph:- matrix representation of a graph has 2 types. 1. Adjacence matrix. 2. Incidence

1. Adjacence matrix:-

Let $G = (V, E)$ be a simple graph with n vertices ordered from v_1 to v_2 then the adjacence matrix $A_m = [a_{ij}]_{n \times n}$ of G is an $n \times n$ symmetric matrix defined by

$a_{ij} = \begin{cases} 1 & \text{When } v_i \text{ is adjacent to } v_j \\ 0 & \text{otherwise} \end{cases}$

Ex:-



$$A_m = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

9-9-10

2. Incidence matrix :-

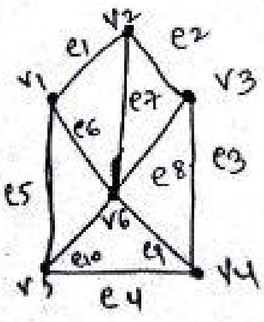
Let G be a graph with n vertices $V = \{v_1, v_2, v_3 - - - - v_n\}$ and $E = \{e_1, e_2 - - - - - e_m\}$ define $n \times m$ matrix $I_m = [a_{ij}]_{n \times m}$

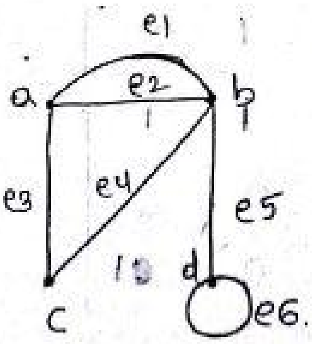
Where $a_{ij} = \begin{cases} 1 & \text{When } v_i \text{ is Incident with } e_j \\ 0 & \text{otherwise} \end{cases}$

Ex:-

$$A_m = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} \end{array}$$

1. find the adjacence and Incidence matrix from given graphs





Sol:- The adjacent matrix from first graph is $A_m =$

The Incidence " " " " is $I_m =$

$$A_m = \begin{array}{c|cccccc} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \hline v_1 & 0 & 1 & 0 & 0 & 1 & 1 \\ v_2 & 1 & 0 & 1 & 0 & 0 & 1 \\ v_3 & 0 & 1 & 0 & 1 & 0 & 1 \\ v_4 & 0 & 0 & 1 & 0 & 1 & 1 \\ v_5 & 1 & 0 & 0 & 1 & 0 & 1 \\ v_6 & 1 & 1 & 1 & 1 & 1 & 0 \end{array}$$

$$I_m = \begin{array}{c|cccccccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} \\ \hline v_1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ v_2 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ v_3 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ v_4 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ v_5 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ v_6 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{array}$$

The adjacent matrix to Second graph

$$A_m = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 2 & 1 & 0 \\ b & 2 & 0 & 1 & 1 \\ c & 1 & 1 & 0 & 0 \\ d & 0 & 1 & 0 & 1 \end{array}$$

The Incidence matrix to second graph is

$$
I_m = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}
\begin{array}{c}
\begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1
\end{bmatrix}
\end{array}
$$

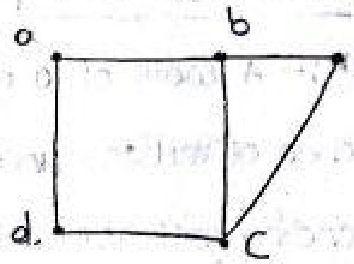
Draw the graph represented by the adjacence matrix.

$$
A_G = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}
\qquad
A_G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 & 1 \\ 1 & 2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}
$$

Sol:- The given adjacence matrix is

let $A_G =$

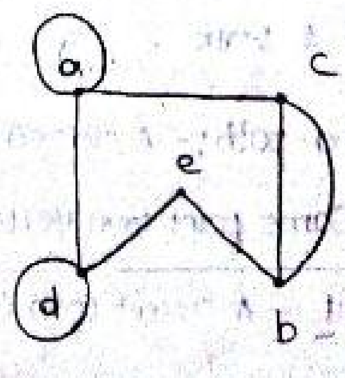
$$
\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}
\begin{array}{c}
\begin{array}{ccccc} a & b & c & d & e \end{array} \\
\begin{bmatrix}
0 & 1 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0
\end{bmatrix}
\end{array}
$$



The given adjacence matrix is

let $A_G =$

$$
\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}
\begin{array}{c}
\begin{array}{ccccc} a & b & c & d & e \end{array} \\
\begin{bmatrix}
1 & 0 & 1 & 1 & 0 \\
0 & 0 & 2 & 0 & 1 \\
1 & 2 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 0
\end{bmatrix}
\end{array}
$$

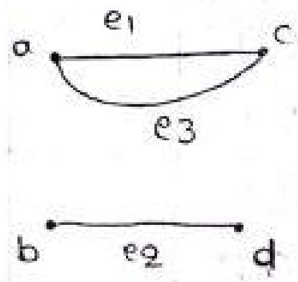


3. Draw the graph from given Incidence matrix

$$I_G = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Sol:- The given Incidence matrix is

$$I_G = \begin{array}{c|c} & e_1 \quad e_2 \quad e_3 \quad e_4 \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$



9-9-10

Paths and circuits:-

1. Walk:- A walk of a graph G is defined as an alternating sequence of vertices and edges. $v_0 e_0 v_1 e_1 \ldots\ldots e_n v_n$. starting and ending with vertices such that each line e_i is incident with V_i. A walk Joining V_0 and V_n is called V_0-V_n walk.

It contains only a single vertex such a walk is called trivial walk.

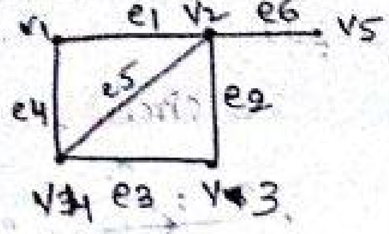
2. Trail:- A walk is called a Trail if all its edges are distinct.

3. path:- A Walk " " " path " " " vertices " "

4. closed path:- A closed path is a path that starts and ends at the same point (or) vertex.

5. Circuit:- A Circuit (or) cycle is defined as a closed path that does not contain repeated edges (distinct edges).

Ex:-



1. $v_1 e_1 v_2 e_2 v_3$

2. $v_1 e_1 v_2 e_2 v_3 e_3 v_4 e_5 v_2 e_2 v_3$

are walks.

3. $v_1 e_1 v_2 e_5 v_4 e_3 v_3 e_2 v_2 e_6 v_5$ is a trail.

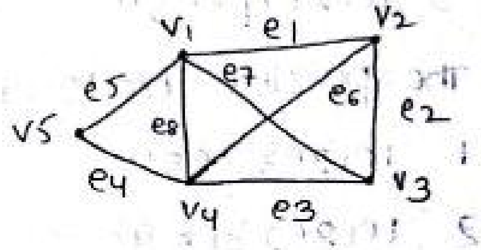
4. $v_1 e_4 v_4 e_3 v_3 e_2 v_2 e_6 v_5$ is a path.

5. $v_1 e_1 v_2 e_5 v_4 e_3 v_3 e_2 v_2 e_1 v_1$ is a closed path

6. $v_1 e_4 v_4 e_5 v_2 e_1 v_1$

7. $v_1 e_4 v_4 e_3 v_3 e_2 v_2 e_1 v_1$ are circuits.

1. Determine of the following sequences are circuits & paths from below graph.



1. $v_1 e_1 v_2 e_6 v_4 e_3 v_3 e_2 v_2$
2. $v_1 e_1 v_2 e_2 v_3 e_3 v_4 e_4 v_5$
3. $v_1 e_8 v_4 e_3 v_3 e_7 v_1 e_8 v_4$
4. $v_5 e_5 v_1 e_8 v_4 e_3 v_3 e_2 v_2 e_6 v_4 e_4 v_5$
5. $v_2 e_2 v_3 e_3 v_4 e_4 v_5 e_5 v_1 e_1 v_2$

Sol:- 1. vertex v_2 is repeated twice, so it is not a path.

Starting vertex v_1 and ending vertex v_2, so it is not circuit

2. Here all vertices are distinct, so it is a path.

Starting vertex v_1 ending vertex v_5, so it is not a circuit.

3. Here vertex v_1, v_4 are repeated, so it is not a path.

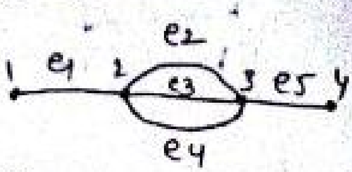
Starting vertex v_1 ending vertex v_4, so it is " " circuit

4. Here vertex v_5, v_4 are repeated, so it is not a path

Starting vertex v_5, ending vertex v_5, so it is a circuit.

5. Here V_2 is repeated, so it is not a path.

Starting vertex V_2, ending vertex V_2, so it is a circuit.



2. Let the graph G (i) How many paths are there from 1 to 4. (ii) How many trails

" " " " .

12-9-18

Sol:- The possible paths are from 1 to 4 is

1. $1e_12e_33e_54$
2. $1e_12e_23e_54$
3. $1e_12e_43e_54$

The possible trails are from 1 to 4 is

1. $1e_12e_33e_54$
2. $1e_12e_23e_54$
3. $1e_12e_43e_54$
4. $1e_12e_23e_32e_43e_54$
5. $1e_12e_23e_42e_33e_54$
6. $1e_12e_43e_22e_33e_54$
7. $1e_12e_43e_32e_23e_54$
8. $1e_12e_33e_42e_23e_54$
9. $1e_12e_33e_22e_43e_54$.

*Eulerian graph (or) Euler graph (or) Eulerian circuit:-

1. A trail in G is called an Eulerian trail (distinct edges).
2. It contains all vertices atleast once of G.
3. A closed Eulerian trail (starting and ending vertices same) is called Eulerian graph (or) Euler circuit.

Euler path:-

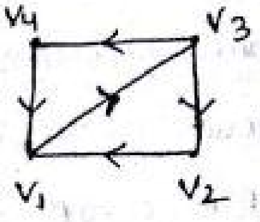
1. A path in a Graph G is called an Euler path if it includes every edge exactly once (distinct edges).
2. visit all vertices at least once.

Note:- 1. If G is a graph in which the degree of every vertex is even then it is possible to construct Euler circuit.

2. The graph G is a Euler path if atleast one degree of vertex is even.

3. If the given graph G is not a Euler circuit and path if and only if its vertices has odd degree

1. Determine whether the graph is Euler path (or) circuit.



Sol:- From a given graph

vertices $V = \{v_1, v_2, v_3, v_4\}$

Indegree to every vertex is

$deg^-(v_1) = 2$, $deg^-(v_2) = 1$, $deg^-(v_3) = 1$, $deg^-(v_4) = 1$

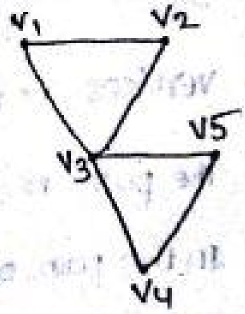
outdegree to every vertex is

$deg^+(v_1) = 1$, $deg^+(v_2) = 1$, $deg^+(v_3) = 2$, $deg^+(v_4) = 1$

Here vertex v_1 and v_3 has odd degree = 3

∴ The Euler path is $v_3 - v_2 - v_1 - v_3 - v_4 - v_1$.

2. from the given graph check whether Euler circuit or path.



Sol:- from given graph

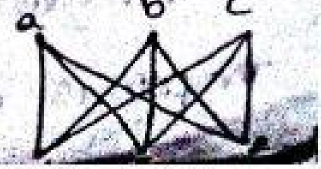
vertices $v = \{v_1, v_2, v_3, v_4, v_5\}$

$deg(v_1) = 2$, $deg(v_2) = 2$, $deg(v_3) = 4$

$deg(v_4) = 2$, $deg(v_5) = 2$

Here all the vertices have even degrees

∴ The Euler circuit is $v_1 - v_2 - v_3 - v_4 - v_5 - v_3 - v_1$.



Sol:- From given graph

Vertices $v = \{a, b, c, d, e, f\}$

deg(a) = 3, deg(b) = 3, deg(c) = 3

deg(d) = 3, deg(e) = 3, deg(f) = 3.

Here all the vertices have odd degrees = 3.

The path is a-d-c-f-b-e-a-f

Here b-d, e-c edges are not covered. So it is not a path.

So it is not a circuit.

13-9-18

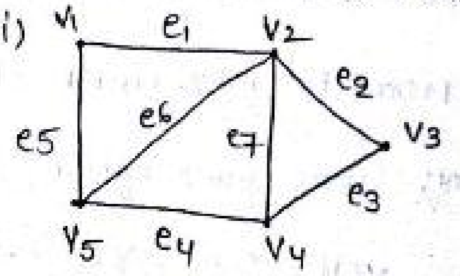
Hamaltonian graph:- A circuit in a graph G is called Hamoltonian circuit (or) graph.

2. If it contains each vertex in G exactly once except for the Starting and ending vertex that appears twice.

Hamaltonian path:- A Hamaltonian path is a path that contains all vertices of G where the endpoints (starting and ending vertices) may be distinct.

1. Determine which of the following graph is Hamaltonian circuit or path.

(i)



Sol:- (i) from given graph

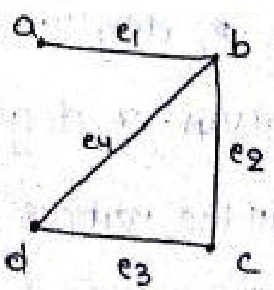
Vertices $V = \{v_1, v_2, v_3, v_4, v_5\}$

The path is $v_1 - v_2 - v_3 - v_4 - v_5 - v_1$.

In this path all vertices visited exactly once except starting & ending vertex.

∴ It is Hamaltonian circuit.

(ii)



(ii) from given graph vertices $V = \{a, b, c, d\}$

The path is a-b-c-d.

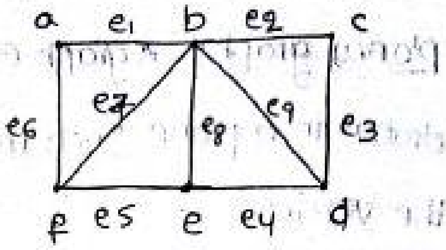
In this path all vertices visited exactly once and starting and ending vertices are distinct.

∴ It is a Hamaltonian path.

2. The given graph which is Hamaltonian circuit (or) Euler circuit

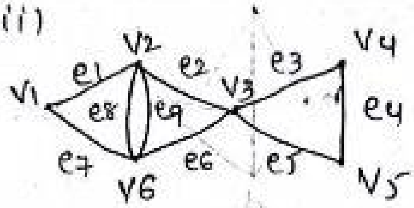
Sol:- (i) case 1:- from given graph

$V = \{a, b, c, d, e, f\}$

(i) 

The path is a-b-c-d-e-f-a

In this path all vertices visited exactly once except starting and ending vertex.

(ii) 

∴ It is a Hamaltonian circuit.

case 2:- from given graph degrees to all vertices

deg(a) = 2, deg(b) = 5, deg(c) = 2, deg(d) = 3, deg(e) = 3

deg(f) = 3.

degree of vertices b, d, e, f has odd degree. It is not possible to construct Euler circuit.

(ii) case 1:- from given graph $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$

The path is $v_1 - v_2$ - ~~[illegible]~~ - $v_6 - v_3 - v_4 - v_5$.

Here all vertices visited exactly once but starting and ending vertices are distinct.

∴ It is not a Hamaltonian circuit.

case 2:- from given graph degrees to all vertices

$deg(v_1) = 2$, $deg(v_2) = 4$, $deg(v_3) = 4$, $deg(v_4) = 2$

$deg(v_5) = 2$, $deg(v_6) = 4$

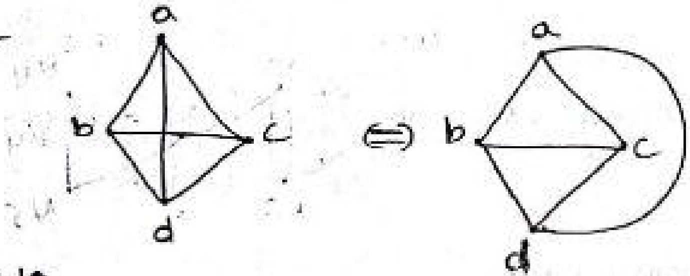
The path is $v_1 - v_2 - v_3 - v_4 - v_5 - v_3 - v_6 - v_2 - v_6 - v_1$.

Here all edges exactly once and visited all vertices. Starting and ending vertex is same.

∴ It is Euler circuit

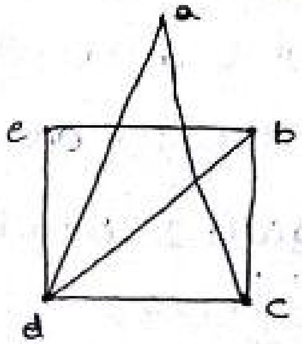
Planar graph:- A graph G is called a planar graph if it can be drawn in a plane such that no two edges intersect except at the vertices.

Ex:-

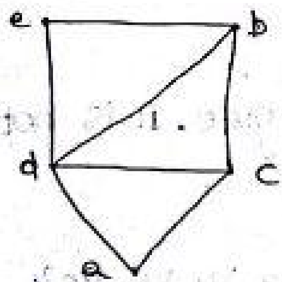


15-9-16

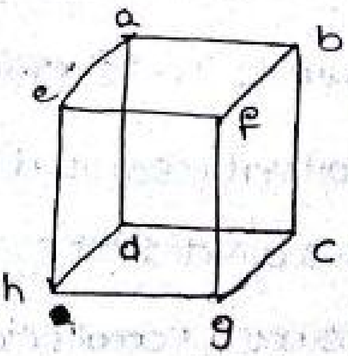
1. Determine the graph is it planar (or) not.



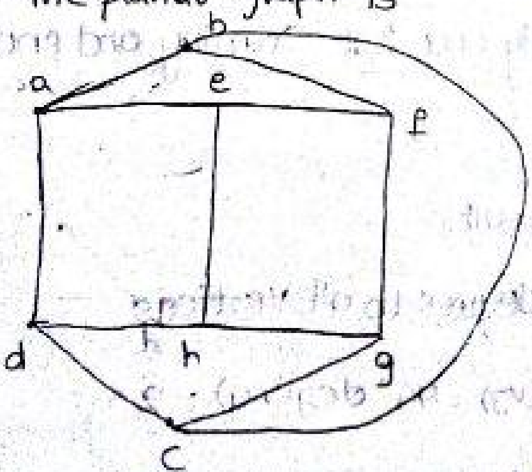
Sol:- The planar graph is



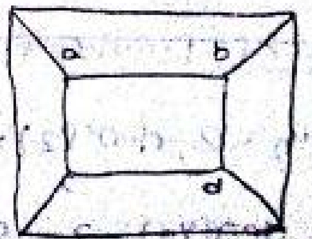
2. The given graph is planar (or) not.



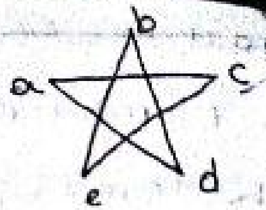
Sol:- The planar graph is



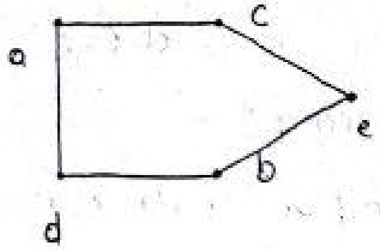
(or)



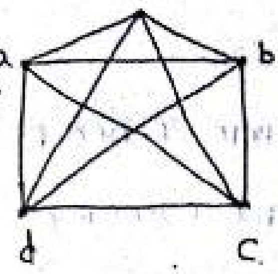
3. The given two graphs are planar or not.



sol:- The planar graph is



It is a planar graph from given graph.



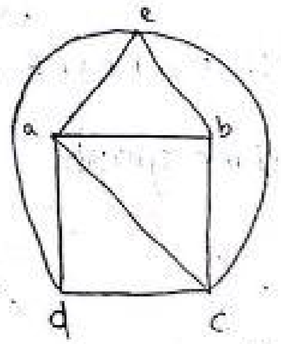
The planar graph is

e,d, b,c

d, a, e, c

a, b, d, e.

a, c, b, e



graph.

It is not possible to design the b to d edge. It is a nonplanar

Every complete graph and above are equal to 5 vertices

that is not possible to design the planar graph.

∴ These are non planar graphs.

~~Graph colouring~~ and ~~covering~~:-

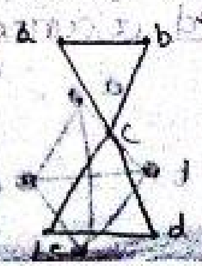
characteristics:- $nv = ne + nf$

where nv = number of vertices in a graph.

ne = " " edges

nf = " " faces (or) regions

nf:- It is the combination of Interior and exterior regions in a plane graph.



(1) no. of vertices $nv = 5$

" " edges $ne = 6$

no. of faces (or) regions from given graph n_f =

$R_1 = a-b-c-a$, $R_2 = c-d-e-c$

R_1 = The region bounded by the cycle $a-b-c-a$.

R_2 = " " " " " " $c-d-e-c$.

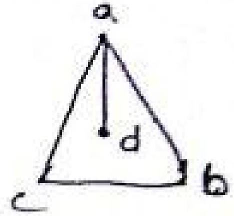
Here R_1 and R_2 are interior regions.

The exterior region R_3 = The plane graph outside path $a-b-c-d-e-c-a$.

$\therefore n_f = 3$.

The characteristics to given graph $n_v - n_e + n_f = 5-6+3 = 2$.

2. construct the characteristics to plane graph.



Sol:- no. of vertices $n_v = 4$

no. of edges $n_e = 4$

no. of faces (or) regions from given graph n_f =

$R_1 = d-a-c-b-a$.

Here R_1 is interior region

The exterior region R_3 = The plane outside path $a-c-b-a$

$\therefore n_f = 2$

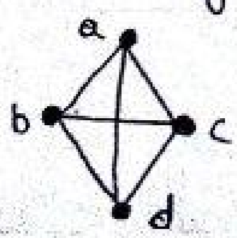
The characteristics to given graph $n_v - n_e + n_f = 4-4+2 = 2$.

8-9-19

Graph colouring and covering:-

Colouring:- An assignment of colours to the vertices of a graph and no two adjacent vertices get the same colour is called colouring of the graph (or) vertex colouring.

Ex:-



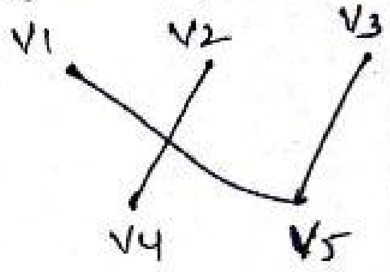
$f(a)$ = Red $f(c)$ = pink

$f(b)$ = Blue $f(d)$ = Yellow

chromatic number:- The chromatic number of a graph G is the minimum number of colours needed to colour the vertices of the graph G and denoted by $\chi(G)$.

$\therefore \chi(G) = 4$

1. Determine the chromatic number of given Bipartiate graph.



sol:- $\therefore f(V_1) = R, \ f(V_2) = R, \ f(V_3) = R$

$f(V_4) = B, \ f(V_5) = B$

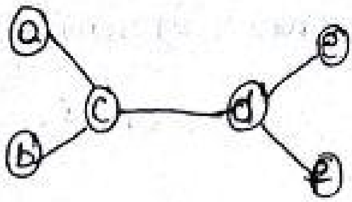
$\chi(G) = 2$

16-9-19

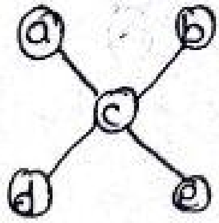
Trees

A tree is a simple graph G such that there is a unique simple undirected path between each pair of vertices in G.

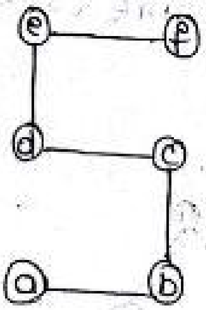
Ex:- 1.



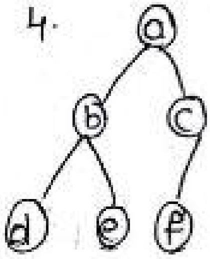
2.



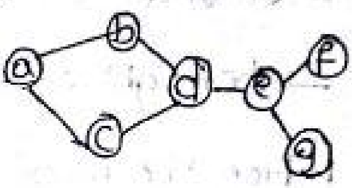
3.



4.



5.



Here 1, 2, 3, and 4 are trees. 5 is not a tree.

→ tree is denoted as 'T'.

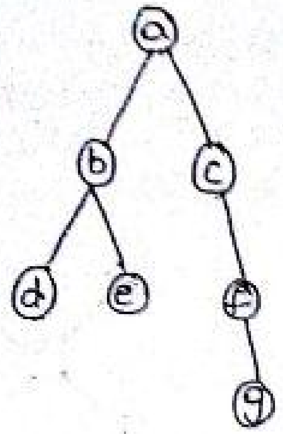
rooted tree:- A rooted tree is a tree in which a particular vertex is designed as the root (starting node (or) vertex).

→ If a vertex v of 'T' is a child vertex, if that vertex is a end vertex (or) exit vertex. Such that v_0 as a root node $(v_0, e_1, v_1, e_2 - - - - e_n v_n)$ and v_n is a child vertex in every path.

→ except root and child vertices remaining all vertices are interval (or) middle vertices.

→ The level of a vertex v in a tree is the length of simple path from the root. The height of a rooted tree is the maximum level number.

Ex:-



- In the above tree a is root node (or) vertex.

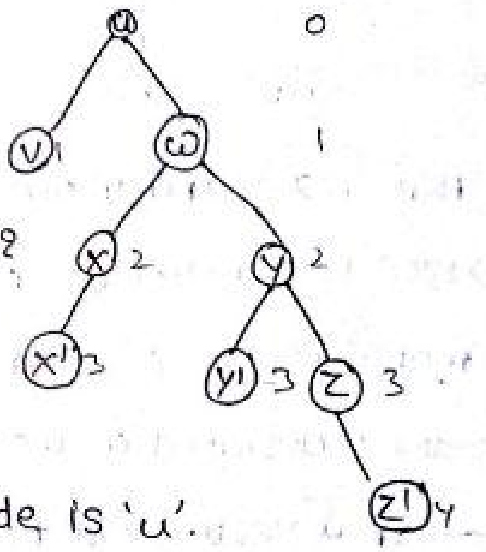
Here child nodes are d, e, and g.

Interval (or) terminal Vertices are b, c, and f.

→ a, b, c, d, e, f and g has 0, 1, 1, 2, 2, 2, 3 are levels respectively to all vertices.

→ The height of tree is '3'.

1. From the given rooted tree 'T'



(i) What is the root of T?

(ii) find the levels & interval vertices?

(iii) what are the levels of w and z?

(iv) find the childs of w and z?

Sol:- (i) from the given tree root node is 'u'.

(ii) The levels of given tree u, v, w, x, y, z, x', y', z'
0, 1, 1, 2, 2, 3, 3, 3, 4.

The Interval vertices are ~~u~~, w, x, y, z.

(iii) the level of w is 1 and level of z is 3.

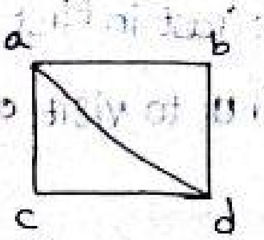
(iv) The childs of w and z are x', y', z' and z'.

Spanning tree:- A tree 'T' is a spanning tree of a graph G. T is a subgraph of G that contains all of the vertices of G.

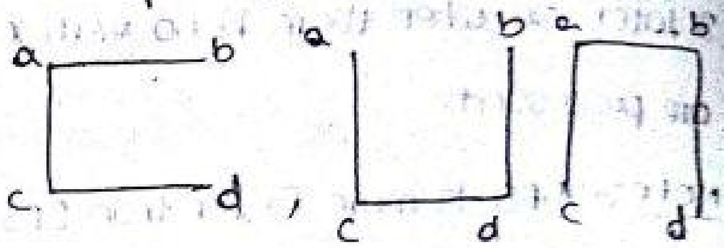
If G is a connected graph with n vertices and m edges, a spanning tree of G must have n-1 edges.

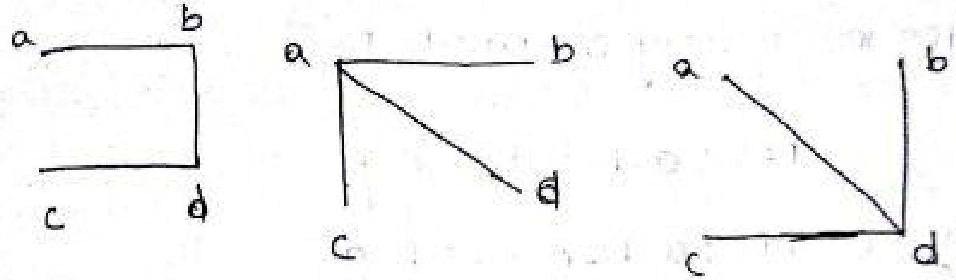
$\boxed{m=n-1}$

Ex:-

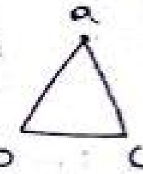


The spanning trees are

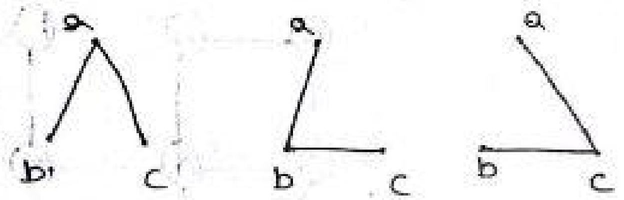




2. Determine the spanning trees from given graph



Sol:- The spanning trees are



BFS and DFS in spanning trees:-

BFS (Breadth first search):-

1. BFS starts traversal from root node and then explore the search in the level by level that is as possible from root node.

2. BFS can be done with the help of queue (First in first out).

3. The BFS works in a single level then visited vertices are removed from the queue until all vertices in tree.

DFS (Depth first search):-

1. DFS starts ~~from~~ the traversal from the root node and explore

the search for as possible from the root node that is depth wise

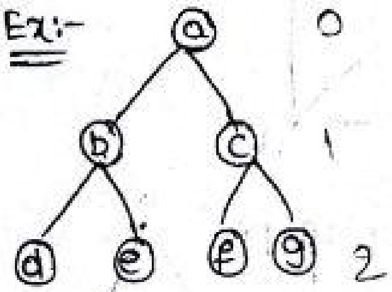
2. DFS can be done with the help of stack (last in first out).

3. Later on when there is no vertex further to visit all vertices are processed.

Note:- 1. DFS is more faster than BFS.

2. DFS requires less memory compare to BFS.

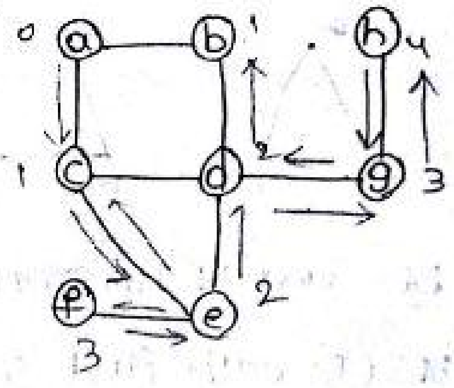
Ex:-



BFS : a-b-c-d-e-f-g.

DFS : a-b-d-e-c-f-g.

1. use BFS and DFS find the spanning tree for given graph

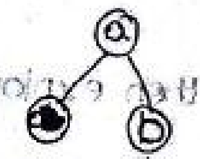


Sol:- BFS:-

1. The root node is a

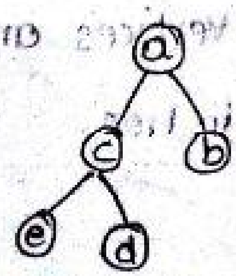
a

2. node a has two childs b and c.



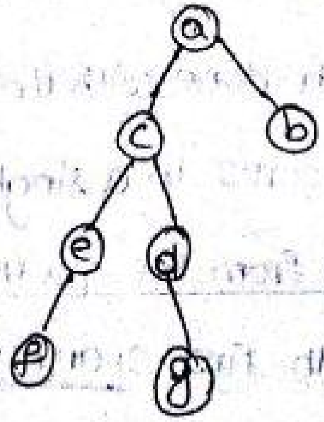
a-c-b

3. node c has 2 childs d and e and b has child d but it will be desired the cycle.



a-c-b-e-d

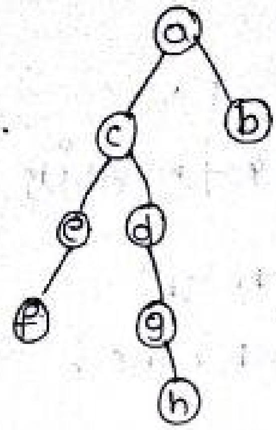
4. node e and d has child nodes f and g.



a-c-b-e-d-f-g

5. g has one child 'h'.

The BFS is a-c-b'-e-d-f-g-h.

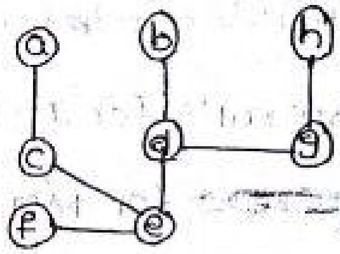


17-7-18

DFS:-

v=a	v=c	v=e	v=f	v=d
w={c,b}	ω={e,d,a}	ω={f,d,c}	ω={e}	ω={e,g,b,c}

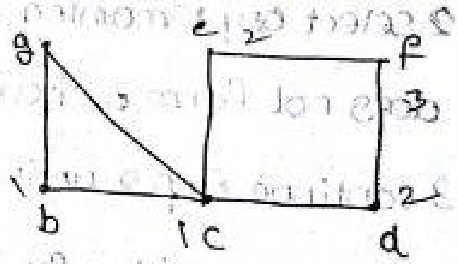
v=g	v=h	v=b
ω={h,d}	h={g}	w={d,a}



∴ The DFS is a-c-e-f-d-g-h-b.

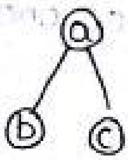
2. Determine BFS and DFS from the given graph

Sol:- BFS:-

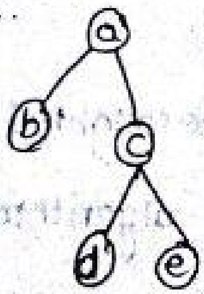


1. The root node is a (a)

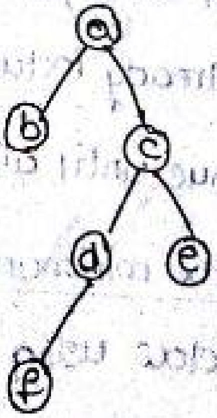
2. root node a has 2 childs b and c.



3. node b does not has one more newchild, c has two child nodes d and e.



4. node d has one new child node f.



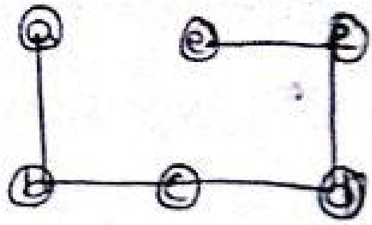
The BFS path is a-b-c-d-e-f.

DFS:-

v=a	v=b	v=c	v=d	v=f
W={b,c}	W={a,c}	W={d,e}	W={f,c}	W={d,e}

v=e
W={f,c}

∴ The DFS is
a-b-c-d-f-e.



MST (minimum Spanning tree):- A minimal spanning tree of g is a spanning tree [redacted] with minimum weight to constructing minimum spanning tree use following Techniques.

1. Kruskal's for MST 2. prim's for MST.

1. Prim's [redacted] for MST:- 1. Select any edge of minimum value that is not a loop this is the first edge of T.

2. select any remaining edge of G having minimal value that does not form a circuit with the edges already included in T.

3. continue step 2 until tree contains n-1 edges.

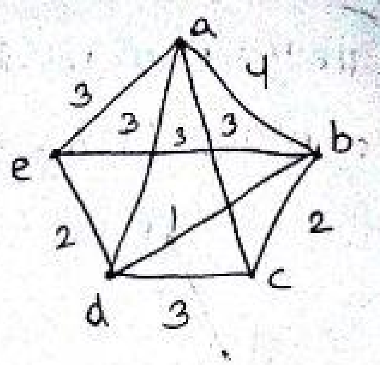
2. Kruskal's [redacted] algorithm for MST:- 1. select any vertex and choose the edge find minimum weight from G.

2. At each stage, choose the edge of smallest weight joining a vertex already included to vertex, not yet included.

3. continue until all vertices are included.

1. Find the minimum spanning tree of the weighted graph given below using kruskal's and prim's algorithm.

Sol:- Prim's [redacted] for MST:-

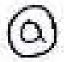


edges	{a,e}	{a,d}	{a,c}
weights	3	3	3

{a,b}	{b,e}	{b,d}	{b,c}
4	3	1	2

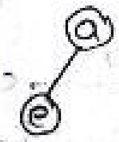
{c,d}	{d,e}
3	2

1. a is root node



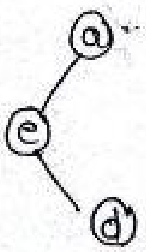
2. We have consider

{a,e} = 3



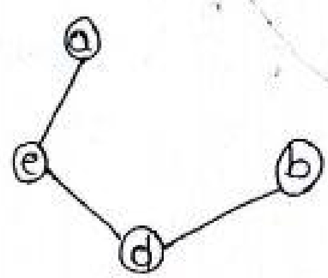
3. We have to consider

{e,d} = 2



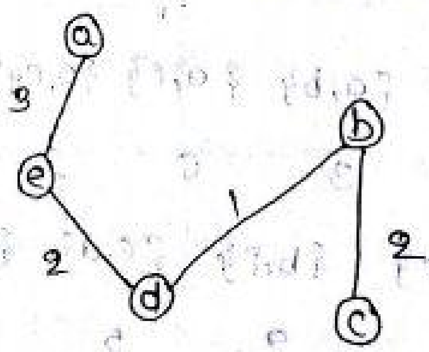
4. We have to consider

{d,b} = 1



5. We have to consider

{b,c} = 2



kroushkal's forms:-

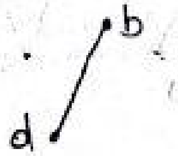
edges {a,e} {a,d} {a,c} {a,b} {b,e} {b,d} {b,c}

Weights 3 3 3 4 3 2

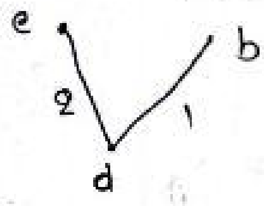
{c,d} {d,e}

3 2

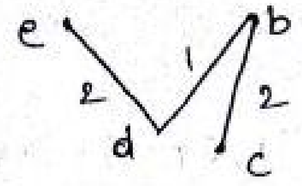
1. The minimum weight of all.nodes {b,d}.



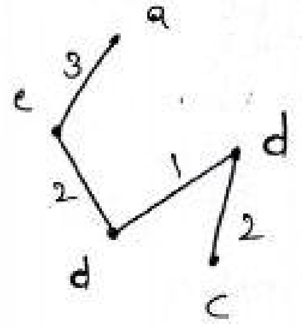
2. next {d,e} = 2



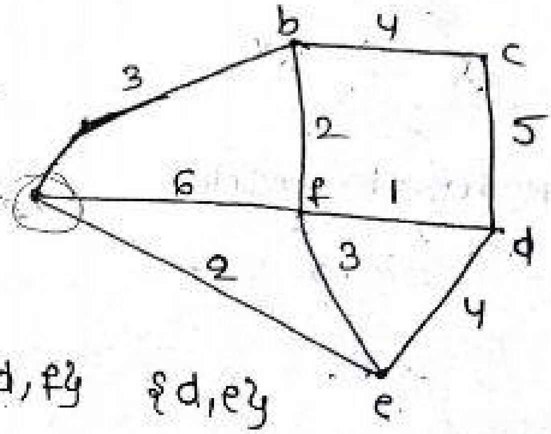
3. next minimum weight {b,c} = 2



4. next minimum weight {a,e} = 3



2. construct the minimum spanning tree using krushkal's and prim's algorithm.

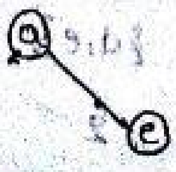


Sol:- Prim's algorithm:-

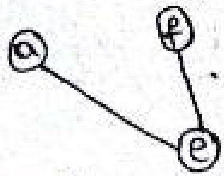
edges	{a,b}	{a,f}	{a,e}	{b,c}	{b,f}	{c,d}	{d,f}	{d,e}	{f,e}
weights	3	6	2	4	2	5	1	4	3

1. a is rootnode

2. We have consider {a,e} = 2

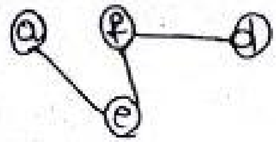


3. We have to consider $\{e, f\} = 3$

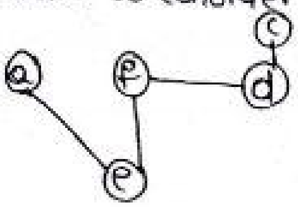


4. We have to consider

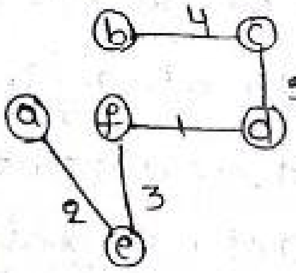
$\{f, d\} = 1$



5. We have to consider $\{d, e\} = 5$



6. We have to consider $\{c, b\} = 4$



Krusral's

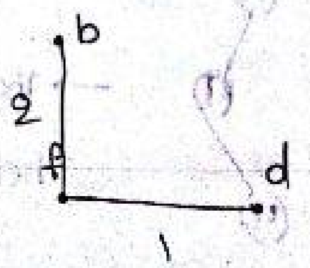
[redacted] algorithm :-

edges $\{a,b\}$ $\{a,f\}$ $\{a,e\}$ $\{b,c\}$ $\{b,f\}$ $\{c,d\}$ $\{d,f\}$,

weights 3 6 2 4 2 5 1

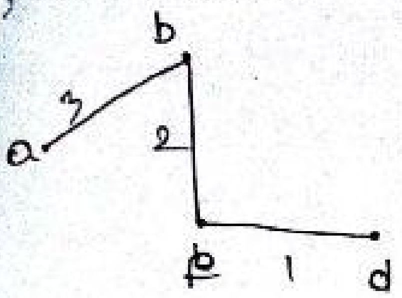
$\{d,e\}$. $\{f,e\}$: 4 3

1. The minimum weight of all nodes is $\{d, f\}$

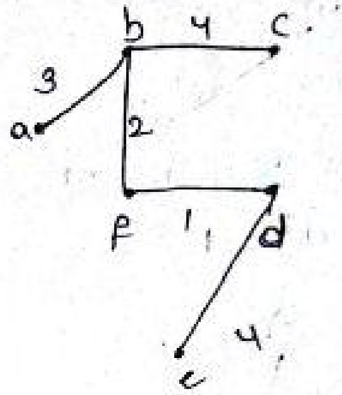


2. next $\{b, f\} = 2$

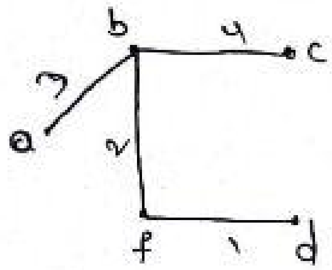
3. next {a,b} = 3



5. next {d,e} = 4.



4. next {b,c} = 4



19-9-18

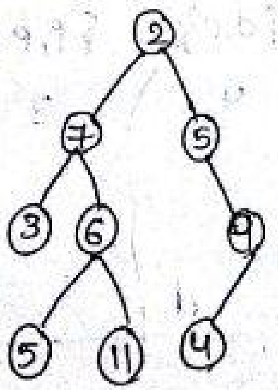
Binary tree:- 1. A rooted tree in which the children of each vertex are assigned a fixed ordering is called a Binary tree.

2. If either each vertex has no child, one child (or) two childs.

3. If a tree has one child then that child is designed as either leftchild (or) rightchild [but not both].

4. If a vertex (or) node has two children then the first child is designed as leftchild otherchild is designed as right child.

Ex:-



size is 9

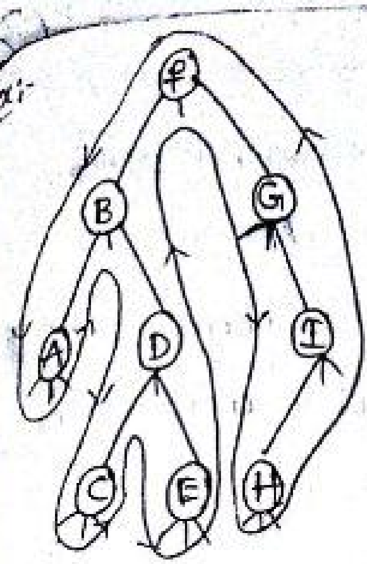
height h = 3.

→ The above tree size is '9' and height is '3'.

The root node is '2'.

The child nodes are 3, 5, 11, 4.

Ex:-

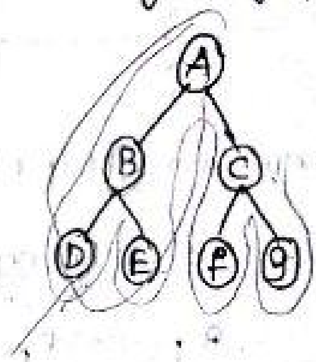


preorder:- FBADCEGIH

Inorder:- ABCDEFGHI

postorder:- ACEDBHIGF.

1. construct preorder, Inorder and postorder from given graph.



sol:- preorder:-

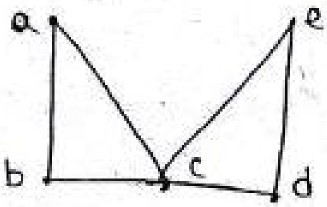
ABDECFg

Inorder:- ~~KBDEACFg~~ DBEAFCg

postorder:- DEBFgCA.

cut vertex:- A cut vertex of a connected graph 'G' is a vertex, ~~is~~ which is removal the number of components.

Ex:-

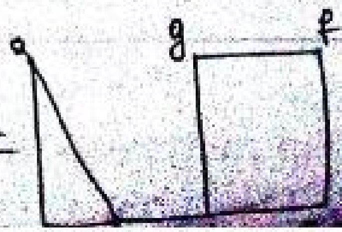


→ Here the vertex 'c' is a cut vertex.

→ If we are removing the vertex 'c'. It is dividing into two components {a,b} & {d,e}.

cut edge (Bridge):- A cut Bridge of a connected of 'G' is an edge which is removed increases the number of components.

Ex:-

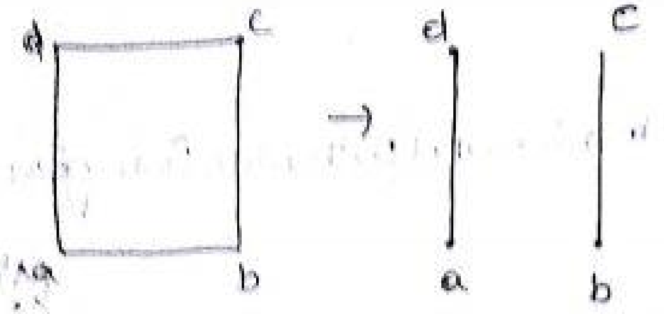


→ Here (c-d) edge cut edge (or) Bridge

→ If we are removing (c-d) edge, the graph is divided into two components.

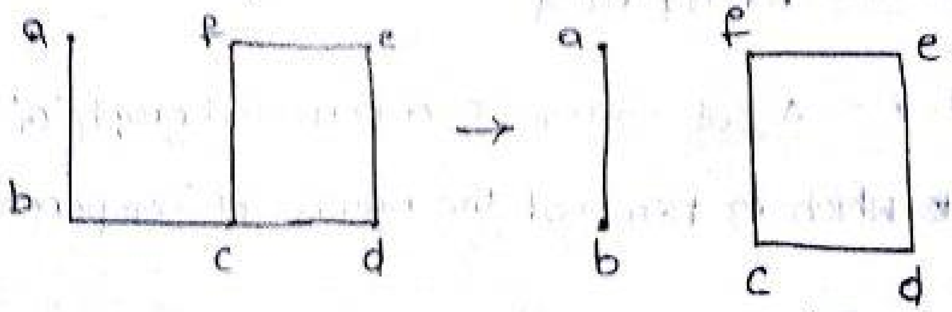
Cut set :- The set of all minimum number of edges of 'G' which is removal (or) disconnect a graph is a cut set of 'G'.

Ex:-



→ Here removing of two edges {(a-b), (c-d)}.

→ The graph dividing into two components



→ Here the cut sets is {(b-c)}