RADAR SYSTEMS

LECTURE NOTES

B.TECH

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IV Year B.Tech. ECE-I Sem
RADAR SYSTEMS

UNIT I
Basics of Radar: Introduction, Maximum Unambiguous Range, Radar Waveforms, Simple form of Radar Equation, Radar Block Diagram and Operation, Radar Frequencies and Applications, Prediction of Range Performance, Minimum Detectable Signal, Receiver Noise, Modified Radar Range Equation, Related Problems.
Radar Equation: SNR, Envelope Detector-False AlarmTime and Probability, Integration of Radar Pulses, Radar Cross Section of Targets (simple targets - sphere, cone- sphere), Transmitter Power, PRF and Range Ambiguities, System Losses (qualitative treatment), Related Problems.

UNIT-II

CW and Frequency Modulated Radar: Doppler Effect, CW Radar – Block Diagram, Isolation between Transmitter and Receiver, Non-zero IF Receiver, Receiver Bandwidth Requirements, Applications of CW radar, Related Problems.
FM-CW Radar: FM-CW Radar, Range and Doppler Measurement, Block Diagram and Characteristics (Approaching/ Receding Targets), FM-CW altimeter, Multiple Frequency CW Radar.

UNIT-III

MTI and Pulse Doppler Radar: Introduction, Principle, MTI Radar with - Power Amplifier Transmitter and Power Oscillator Transmitter, Delay Line Cancellers – Filter Characteristics, Blind Speeds, Double Cancellation, Staggered PRFs. Range Gated Doppler Filters. MTI Radar Parameters, Limitations to MTI Performance, MTI versus Pulse Doppler Radar.
Tracking Radar: Tracking with Radar, Sequential Lobing, Conical Scan, Monopulse Tracking Radar – Amplitude Comparison Monopulse (one- and two- coordinates), Angular Accuracy, Tracking in Range, Acquisition and Scanning Patterns, Comparison of Trackers.

UNIT-IV

Detection of Radar Signals in Noise: Introduction, Matched Filter Receiver – Response Characteristics and Derivation, Correlation Function and Cross-correlation Receiver, Efficiency of Non-matched Filters, Matched Filter with Non-white Noise.

UNIT-V

Radar Receivers: Noise Figure and Noise Temperature, Displays – types, Introduction to

Phased Array Antennas –Basic Concepts, Radiation Pattern, Beam Steering and Beam Width changes, Applications, Advantages and Limitations.

Electronic Warfare: Introduction to ESM, ECM and ECCM systems.

TEXT BOOK:

1. Introduction to Radar Systems – Merrill I. Skolnik, TMH Special Indian Edition, 2nd Edition, Tata McGraw-Hill, 2007.

REFERENCES:

1. Introduction to Radar Systems – Merrill I. Skolnik, 3rd Edition Tata McGraw-Hill, 2001.

2. Radar: Principles, Technology, Applications-Byron Edde, Pearson Education, 2004.

3. Principles of Modern Radar: Basic Principles-Mark A. Richards, James A. Scheer, William A. Holm, Yesdee,2013.

4. 'Radar Hand Book ' Ed. By M.I Skolnik, 2nd Edition, Tata McGraw Hill.

5. 'Understanding Radar Systems' by Simon Kinsley and Shaun Quegan, Scitech Publishing, McGraw-Hill.

UNIT-1

BASICS OF RADAR

- **Introduction**
- **Maximum Unambiguous Range**
- **Simple form of Radar Equation**
- **Radar Block Diagram and Operation**
- **Radar Frequencies and Applications**
- **Prediction of Range Performance**
- **Minimum Detectable Signal**
- **Receiver Noise**
- **Modified Radar Range Equation**

RADAR EQUATION

- **SNR**
- **Envelop Detector**
- **False Alarm time and Probability**
- **Integration of Radar Pulses**
- **Radar Cross Section of Targets (simple targets: sphere and cone sphere)**
- **Transmitter Power**
- **PRF and Range Ambiguities**
- **System Losses (qualitative treatment)**
  - ▪ **Important Formulae**
  - ▪ **Illustrative Problems**
  - ▪ **Questions from Previous Year Examinations**

BASICS OF RADAR

Introduction:
Basic principles and features:

- Radar is a contraction of the words **Ra**dio   **R**anging. Radar is an **D**etection **A**nd

  electromagnetic system for the detection and location of objects. It operates by transmitting a particular type of waveform, a pulse-modulated sine wave for example, and detects the nature of the echo signal.
- Radar can see through conditions such as darkness, haze, fog, rain, and snow which is not possible for human vision. In addition, radar has the advantage that it can measure the distance or range to the object.
- An elementary form of radar consists of a transmitting antenna emitting electromagnetic Radiation generated by an oscillator of some sort, a receiving antenna, and a signal receiver. A portion of the transmitted signal is intercepted by a reflecting object (target) and is reradiated in all directions. The receiving antenna collects the returned signal and delivers it to a receiver, where it is processed to detect the presence of the target and to extract its location and relative velocity. The distance to the target is determined by measuring the time taken for the Radar signal to travel to the target and back. The direction, or angular position, of the target is determined from the direction of arrival of the reflected wave front. The usual method of measuring the direction of arrival is with narrow antenna beams.
- If relative motion exists between target and radar, the shift in the carrier frequency of the reflected wave (Doppler Effect) is a measure of the target's relative (radial) velocity and may be used to distinguish moving targets from stationary objects. In radars which continuously track the movement of a target, a continuous indication of the rate of change of target position is also available.
- It was first developed as a detection device to warn the approach of hostile aircraft and for directing antiaircraft weapons. A well designed modern radar can extract more information from the target signal than merely range.

Measurement of Range:

- The most common radar waveform is a train of narrow, rectangular-shape pulses modulating a sine wave carrier.
- The distance, or range, to the target is determined by measuring the time T_R taken by the pulse to travel to the target and return.
- Since electromagnetic energy propagates at the speed of light **c (3×10^8 m/s)** the range **R** is given by : $R = cT_R/2$

- The factor 2 appears in the denominator because of the two-way propagation of radar. With the range **R** in kilometers or nautical miles, and T_R in microseconds, the above relation becomes: **R(km) = 0.15 X T_R (µS)**

(1 mile   = 0.8689 nautical mile or 1.6 km
   1 nautical mile   = 1.15078 miles   or 1.8412 km)

Maximum unambiguous range:

Once the transmitter pulse is emitted by the radar, sufficient time must elapse to allow any echo signals to return and be detected before the next pulse is transmitted. Therefore, the rate at which the pulses may be transmitted is determined by the longest range at which targets are expected. If the pulse repetition frequency is too high, echo signals from some targets might arrive after the transmission of the next pulse, and ambiguities in measuring range might result.
Echoes that arrive after the transmission of the next pulse are called s cond-time-around (or multiple-time-around) echoes. Such an echo would appear to be at a much shorter range than the actual. The range beyond which targets appear as second-time-around echoes (or the farthest target range that can be detected by a Radar without ambiguity) is called the ***maximum unambiguous range*** and is given by: $R_{unambig.} = C/2f_p$
Where f_p = pulse repetition frequency, in Hz. **(PRF)**

This can also be explained with the following simple relations.
- T_R is the time elapsed between transmission pulse and Echo pulse.

- $T_R = 2R/C$ where **R**= Range of target

- T_R increases with Range **R** and in extreme case Echo pulse merges with next Transmitted Pulse. Then T_R becomes equal to T_P Where T_P= Pulse repetition period

- $T_{R\,max} = T_P = 2\,R_{max}/C$ and so $R_{max} = CT_P/2 = C/2f_P = R_{unambig}$

- Therefore $R_{unambig}$ is directly proportional to the Pulse period T_P (or Inversely proportional to the PRF f_p)

Simple form of Radar Equation:

The radar equation
- Relates the range of a Radar to the characteristics of the transmitter, receiver, antenna, target, and environment.
- Useful as a means for determining the maximum measurable distance from the radar to the target
- It serves both as a tool for understanding radar operation and as a basis for radar design.

Derivation of the simple form of radar equation:

- If the power of the radar transmitter is denoted by P_t and if an isotropic antenna is used (one which radiates uniformly in all directions) the ***power*** density (watts per unit area) at a distance ***R***

from the radar is equal to the transmitter power divided by the surface area $4\pi R^2$ of an imaginary sphere of radius **R** with radar at its centre, or

Power density from anisotropic antenna = $P_t/4\pi R^2$

- Radars employ directive antennas to direct the radiated power P_t into some particular direction. The **gain G** of an antenna is a measure of the increased power radiated in the direction of the target as compared with the power that would have been radiated from an isotropic antenna. *It may be defined as the ratio of the maximum radiation intensity from the given antenna to the radiation intensity from a lossless, isotropic antenna with the same power input.* (The radiation intensity is the power radiated per unit solid angle in a given direction.)Then the power density at the target from an antenna with a transmitting gain **G** is given by

Power density from directive antenna = $P_t.G/4\pi R^2$

- The target intercepts a portion of the incident power and r radiates it in various directions. The measure of the amount of incident power intercepted by the target and reradiated back in the direction of the radar is denoted as the radar cross section **σ**, and is defined by the relation

Power density of echo signal at radar = $(P_t.G/4\pi R^2)(\sigma)/4\pi R^2$)

- The radar cross section **σ** has units of area.   f the particular target na captures a portion
  It is a characteristic and is a measure of its
  size as seen by the radar. The radar ante

  of the echo power. If the effective area of the receiving antenna is denoted as A_e, then the power P_r.received by the radar is given by

$$P_r = (P_t.G/4\pi R^2).(\sigma/4\pi R^2).\ A_e$$

$$= (P_t.G.\ A_e.\ \sigma)/\ (4\pi)^2.R^4$$

- The maximum radar range R_{max} is the distance beyond which the target cannot be detected. It occurs when the received echo signal power P_r just equals the minimum detectable signal S_{min}.

Therefore area of an antenna as:

$$R_{max} = [\ (P_t.G.\ A_e.\ \sigma)/\ (4\pi)^2.\ S_{min}\ ]^{1/4} \quad .....(1)$$

This is the fundamental form of the radar equation. Note that the important antenna parameters are the **transmitting gain** and the **receiving effective area**.
Antenna theory gives the relationship between the transmitting gain and the receiving effective

$$G = 4\pi A_e / \lambda^2$$

Since radars generally use the same antenna for both transmission and reception, the above relation between gain **G** and affective aperture area A_e can be substituted into the above equation, first for A_e and then for **G**, to give two other forms of the radar equation.

$$R_{max} = [(Pt . G^2 . \lambda^2 . \sigma) / (4\pi)^3 . S_{min}]^{1/4} \quad ......(2)$$

$$_e \quad R_{max} = [(Pt . A^2 . \sigma) / 4\pi . \lambda^2 . S_{min}]^{1/4} ....(3)$$

These three forms (eqs.1, 2, and 3) illustrate the need to be careful in the interpretation of the radar equation. For example, from Eq. (2) it might be thought that the range of radar varies as $\lambda^{1/2}$, but Eq. (3) indicates a $\lambda^{-1/2}$ relationship, and Eq. (1) shows the range to be independent of λ. The correct relationship depends on whether it is assumed the gain is constant or the effective area is constant with wavelength.

  Limitations of the simple form of Radar equation:

- Does not adequately describe the performance of practical radar.
- Many important factors that affect range are not explicitly included.
- In practice, the observed maximum radar ranges are usually much smaller than what would be predicted by the above equations, sometimes by as much as a factor oftwo.

There are many reasons for the failure of the simple radar equation to correlate with actual performance and these will be explained subsequently in the **modified Radar range equation**.

Radar block diagram and operation:

The operation of a typical pulse radar is described with the help of a simple block diagram shown in the figure below.The   a magnetron, that is
transmitter is an oscillator, such
as
"pulsed" (turned on and off) by the modulator to generate a repetitive train of pulses. The magnetron has been the most widely used of the various microwave generators for radar. A typical radar for the detection of aircraft at ranges of 100 or 200 nmi employs a peak power of the order of one megawatt, an average power of several kilowatts, a pulse width of several microseconds, and a pulse repetition frequency of several hundred pulses per second. A single antenna is generally used for both transmitting and receiving. The receiver must be protected from damage caused by the high power of the transmitter. This is the function of the duplexer. The duplexer also serves to channel the returned echo signals to the receiver and not to the transmitter. The duplexer consists of two gas-discharge device, one known as a TR (transmit-receive) and the other as ATR (anti-transmit-receive). The TR protects the receiver during transmission and the ATR directs the echo signal to the receiver during reception.

mixer and local oscillator (LO) convert the RF signal to an intermediate frequency IF. Typical IF amplifier center frequencies are 30 or 60 MHz and will have a bandwidth of the order of one megahertz.

The IF amplifier should be designed as a **matched filter** i.e**., its frequency-response function H(f) should maximize the peak-signal-to-mean-noise-power ratio at the output. This occurs when the magnitude of the frequency-response function |H(f)|is equal to the magnitude of the echo signal spectrum |S(f)|, and the phase spectrum of the matched filter is the negative of the phase spectrum of the echo signal.** In a radar whose signal waveform approximates a rectangular pulse, the conventional IF filter band pass characteristic approximates a matched filter when the product of the IF bandwidth **B** and the pulse width τ is of the order of unity, that is, **Bτ = 1**.
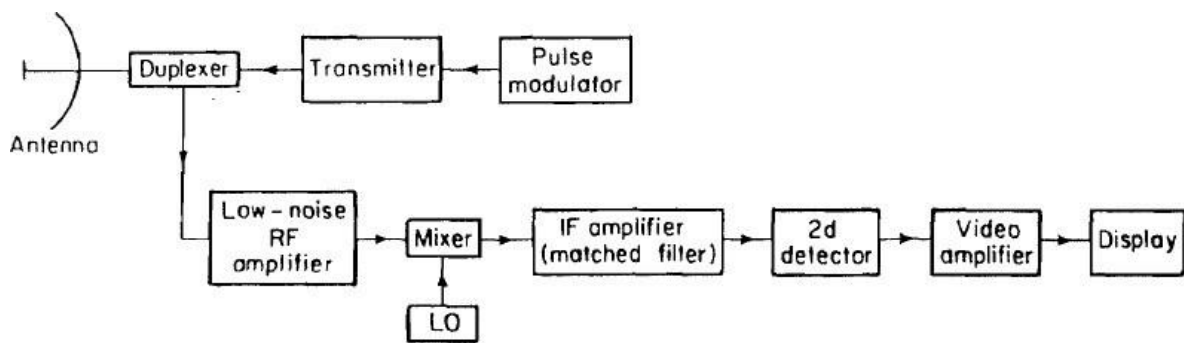

Fig 1.2: Block diagram of a pulse radar.

After maximizing the signal-to-noise ratio in the IF amplifier, the pulse modulation is extracted by the second detector and amplified by the video amplifier to a level where it can be properly displayed, usually on a cathode-ray tube (CRT). Timing signals are also supplied to the indicator to provide the range zero. Angle information is obtained from the pointing direction of the antenna.
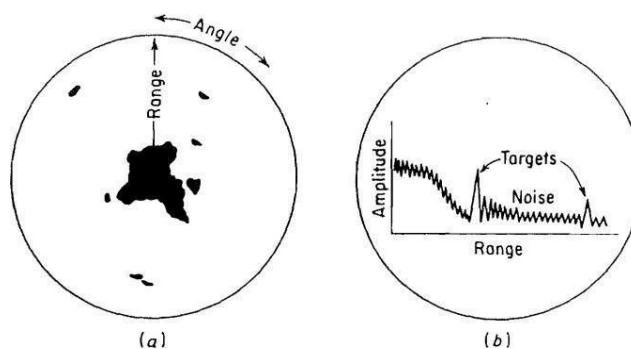

Fig 1.3(a) PPI presentation displaying Range vs. Angle (intensity modulation)
(b) A-scope presentation displaying Amplitude vs. Range (deflection modulation)

The most common form of cathode-ray tube display is the **P**lan **P**osition **I**ndicator, or **PPI (Fig. a)** which maps in polar coordinates the location of the target in azimuth and range. This is an intensity-modulated display in which the amplitude of the receiver output modulates the electron-beam intensity (z axis) as the electron beam is made to sweep outward from the center of the tube. The beam rotates in angle in response to the antenna position. Another form of display is the **A-scope,** shown in Fig. **b,** which plots target amplitude **(y axis)** vs. range **(x axis)**, for some fixed direction. This is a deflection-modulated display. It is more suited for tracking-radar application than for surveillance radar.

A common form of radar antenna is a reflector with a parabolic shape, fed (illuminated) from a point source at its focus. The parabolic reflector focuses the energy into a narrow beam, just as a searchlight or an automobile headlamp. The beam is scanned in space by mechanical pointing of the antenna.

Radar frequencies and applications:

Radar frequencies:

Conventional radars are operated at frequencies extending from about 220 MHz to 35 GHz**,** a spread of more than seven octaves. These are not necessarily the limits, since radars can be, and have been, operated at frequencies outside either end of this range.

The place of radar frequencies in the electromagnetic spectrum is shown in the figure below. Some of the nomenclature employed to designate the various frequency regions is also shown in this figure.
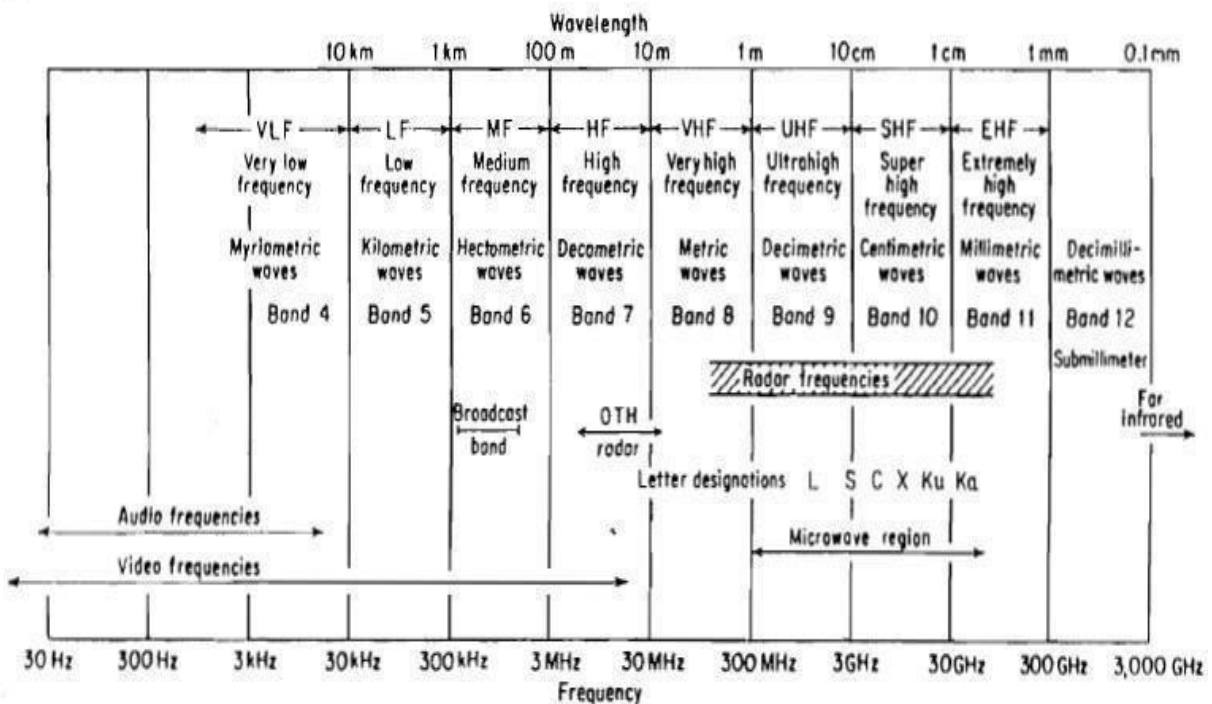
ELECROMAGNETIC SPECTRUM

Letter code designation of Radar frequencies:

Early in the development of radar, a letter code such as S, X, L, etc., was employed to designate Radar frequency bands. Although it's original purpose was to guard military secrecy, the designations were maintained, probably out of habit as well as the need for some convenient short nomenclature. This usage has continued and is now an accepted practice of radar engineers. The table below lists the radar-frequency letter-band nomenclature adopted by the **IEEE.** These are related to the specific bands assigned by the **I**nternational **T**elecommunications Union for radar. For example, although the nominal frequency range for L band is 1000 to 2000 MHz, a L-band radar is thought of as being confined within the region from 1215 to1400MHz since that is the extent of the assigned band.

Table 1.1: Standard radar-frequency letter-band nomenclature

Applications of Radar:

1. Military Use: Initial and important user of Radar

(i)Early warning of intruding enemy aircraft &missiles

(ii) Tracking hostile targets and providing location information to Air Defensesystems consisting of Tracking Radars controlling guns and missiles.

(iii) Battle field surveillance

(iv)Information Friend or Foe IFF

(v)Navigation of ships, aircraft, helicopter etc.

2. Civilian Use:

(i) Air Traffic Control (ATC)

Band designation	Nominal frequency range	Specific radiolocation (radar) bands based on ITU assignments for region 2
HF	3–30 MHz	
VHF	30–300 MHz	138–144 MHz
		216–225
UHF	300–1000 MHz	420–450 MHz
		890–942
L	1000–2000 MHz	1215–1400 MHz
S	2000–4000 MHz	2300–2500 MHz
		2700–3700
C	4000–8000 MHz	5250–5925 MHz
X	8000–12,000 MHz	8500–10,680 MHz
K_u	12.0–18 GHz	13.4–14.0 GHz
		15.7–17.7
K	18–27 GHz	24.05–24.25 GHz
K_a	27–40 GHz	33.4–36.0 GHz
mm	40–300 GHz	

All airports are equipped with ATC Radars, for safe landing and take-off and guiding of aircraft in bad weather and poor visibility conditions.

(ii) Aircraft Navigation

    (a) All aircrafts fitted with weather avoidance radars. These Radars give warning information to pilot about storms, snow precipitation etc. lying ahead of aircraft's path.

    (b) Radar is used as an altimeter to indicate the height of the aircraft or helicopter.

3. Maritime ship's safety and Navigation:

(i) Radar used to avoid collision of ships during poor visibility conditions (s orms, cyclones etc.)

(ii) Guide ships into seaports safely.

4. Meteorological Radar:

Used for weather warnings and forecasting. Provides sufficient advance information to civilian administration for evacuation of population in times cyclones, storms etc.

Prediction of Range Performance:

The simple form of Radar equation derived earlier expresses the maximum radar range R_{max} in terms of radar and target parameters:

$$R_{max}= \left[(P_t \cdot G \cdot A_e \cdot \sigma)/ (4\pi)^2 \cdot S_{min}\right]^{1/4}$$

Where P_t = transmitted power, watts

    G = antenna gain

    A_e = antenna effective aperture, m^2

    σ = radar cross section, m^2

S_{min} = minimum detectable signal, watts

All the parameters are to some extent under the control of the radar designer, except for the target cross section σ. The radar equation states that if long ranges are desired, The

- transmitted power must be large,
- The radiated energy must be concentrated into a narrow beam (high transmitting antenna gain),
- The received echo energy must be collected with a large antenna aperture (also synonymous with high gain) and
- The receiver must be sensitive to weak signals.

In practice, however, the simple radar equation does not predict the range performance of actual radar equipment to a satisfactory degree of accuracy. The predicted values of radar range are usually optimistic predicted.

This discrepancy is due to

- The failure of the above equation to explicitly include the various losses that can occur throughout the system or
- The loss in performance usually experienced when electronic equipment is operated in the field rather than under laboratory-type conditions and

- Another important factor i.e the statistical or unpredictable nature of several of the parameters in the radar equation.

The minimum detectable signal **S**min and the target cross section **σ** are both statistical in nature and must be expressed in statistical terms.

- Other statistical factors which do not appear explicitly in the simple radar equation but which have an effect on the radar performance are the meteorol gical conditions along the propagation path and the performance of the radar operator, if one is employed.

The statistical nature of these several parameters does not allow the maximum radar range to be described by a single number. Its specification must include a statement of the probability that the radar will detect a certain type of target at a particular range.

- *Hence in order to cover these aspects, the simple radar equati n will be **modified** to include most of the important factors that influence radar range performance.*

Minimum detectable signal:

- The ability of a radar receiver to detect a weak echo signal is limited by the noise energy that occupies the same portion of the frequency spectrum as does the signal energy and accompanies the signal.
- The weakest signal the receiver can detect is called the ***minimum detectable signal.*** It is difficult to define ***minimum detectable signal*** (MDS) because of its statistical nature and because the criterion for deciding whether a target is present or not is not too well defined.
- Detection is normally based on establishing a threshold level at the output of the receiver (as shown b the dotted line in the figure below.)Whenever Rx output signal which is a mixture of echo and noise crosses this threshold, then it is detected as a target. This is called ***threshold detection.***
- Consider the output of a typical radar receiver as a function of time as shown in the figure below which typically represents one sweep of the video output displayed on an A-scope.
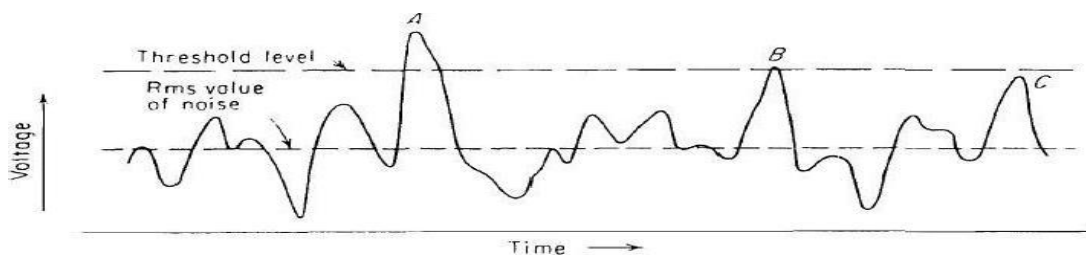

Fig 1.4: Typical envelope of the radar receiver output as a function of time. *A,* B, and *C* are three targets representing signal plus noise. A and B are valid detections, but C is a missed detection.

- The envelope has a fluctuating appearance due to the random nature of noise and consists of three targets A, B and C of different signal amplitudes.
  The signal at **A** is large which has much • without any difficulty and ambiguity. larger amplitude than t detection is possible the noise. Hence target
- Next consider the two signals at **B** and **C**, representing target echoes of equal amplitude. The noise voltage accompanying the signal at **B** is large enough so that the combination of signal plus noise exceeds the threshold and target detection is still possible.
- But for the target **C,** the noise is not as large and the resultant signal plus noise does not cross the threshold and hence target is not detected.
- *Threshold Level setting:* Weak signals such as **C** would not be lost if the threshold level were lower. But too low a threshold increases the likelihood that noise alone will rise above the threshold and is taken as target. Such an occurrence is called a *false alarm*. Therefore, if the threshold is set too low, false target indications are obtained, but if it is set too high, targets might be missed. The selection of the proper threshold level is a compromise that depends upon how important it is if a mistake is made either by
  1. Failing to recognize a signal that is present (*probability of a miss*) orby
  2. Falsely indicating the presence of a signal when it does not exist (*probability of a false alarm*)
- The signal-to noise ratio necessary to provide adequate detection is one of the important parameters that must be mpute *the minimum* determined in order to c
  detectable signal.

- Although the detection decision is usually based on measurements at the video output, it is easier to consider maximizing the signal-to-noise ratio at the output of the IF amplifier rather than in the video. The receiver may be considered linear up to the output of the IF. It is shown that maximizing the signal-to-noise ratio at the output of the IF is equivalent t maximizing the video output. The advantage of considering the signal-to-noise ratio at the IF is that the assumption of linearity may be made. It is also assumed that the IF filter characteristic approximates the matched filter, so that the output signal-to-noise ratio is maximized.

Receiver noise:

- Noise is unwanted electromagnetic energy which interferes with the ability of the receiver to detect the wanted signal thus limiting the receiver sensitivity.

It may originate within the receiver itself, or it may enter via the receiving antenna along with the desired signal. If the radar were to operate in a perfectly noise-free environment so that no external sources of noise accompanied the desired signal, and if the receiver itself were so perfect that it did not generate any excess noise, *there would still exist an unavoidable component of noise generated by the thermal motion of the conduction electrons in the ohmic*

<div align="center">*Available thermal-noise power* = **kTB$_n$**</div>

where **k** = Boltzmann's constant =**1.38 x 10^{-23}J/deg**. If the temperature **T** is taken to be290 K, which corresponds approximately to room temperature (62^{0}F), the factor **kT** is **4 x 10^{-21}W/Hz** of bandwidth. If the receiver circuitry were at some other temperature, the thermal-noise power would be correspondingly different.

- A receiver with a reactance input such as a parametric amplifier need not have any significant ohmic los . The limitation in this case is the thermal noise seen by the antenna and the ohmic losses in the transmission line.
- For radar receivers of the super heterodyne type (the type of receiver used for most radar applications), the receiver bandwidth is approximately that of the **intermediate- frequency** stages. It should be cautioned that the bandwidth **B$_n$** mentioned above is not
the 3-dB, or half-pow r, bandwidth commonly employed by electronic engineers. It is an integrated bandwidth and is given by:

$$B_n = \frac{\int_{-\infty}^{\infty} |H(f)|^2 \, df}{|H(f_0)|^2}$$

where **H (f)** = frequency-response characteristic of IF amplifier (filter) and **f$_o$** = frequency of maximum response (usually occurs at mid band).

- ***The bandwidth B$_n$ is called the noise bandwidth and is the bandwidth of an equivalent rectangular filter whose noise-power output is same as the filter with characteristic H(f).***It is not theoretically same as the 3-dB bandwidth. The 3-dB bandwidth is widely
used since it is easy   to measure. The measurement of   bandwidth however noise

involves a complete   knowledge of the response   **H(f).** The frequency- characteristic

response characteristics of many practical radar receivers are such that the 3 dB and the noise bandwidths do not differ appreciably. Therefore the 3-dB bandwidth may be used in many cases as an approximation to the noise bandwidth.
- The noise power in practical receivers is often greater than can be accounted for by thermal noise alone and is due to mechanisms other than the thermal agitation of the conduction electrons. The exact origin of the extra noise components is not important except to know that it exists. Whether the noise is generated by a thermal mechanism or by some other mechanism the total noise at the output of the receiver may be considered to be equal to the thermal-noise power obtained from an "ideal" receiver multiplied by a factor called the ***noise figure.***
- The noise figure **F$_n$** of a receiver is defined by the equation:

<div align="center">***F$_n$ = N$_o$/ kT$_o$ B$_n$G$_a$***</div>

= (Noise output of practical receiver) / (Noise output of ideal recei er at std. temp **T$_o$**)
Where **N$_o$** = noise output from receiver, and **G$_a$** = available gain. The standard
temperature **T$_o$** is taken to be 290 **K,** according to the Institute of Electrical and

Electronics Engineers definition. The noise N_o is measured over the linear portion of the receiver input-output characteristic, usually at the output of the IF amplifier before the nonlinear second detector. The receiver bandwidth B_n is that of the IF amplifier in most receivers. The available gain G_a is the ratio of the signal out S_o to the signal in S_i and

$kT_o B_n$ is the input noise N_i in an ideal receiver. The above equation may be rewritten as:

$$F_n = \frac{S_i/N_i}{S_o/N_o}$$

Therefore, the *noise figure* may be interpreted, as a measure of the degradation of signal-to noise-ratio as the signal passes through the receiver.

Modified radar equation:

Rearranging the above two equations for F_n, the input signal may be expressed as

$$S_i = \frac{kT_0 B_n F_n S_o}{N_o}$$

If the minimum detectable signal S_{min} is that value of S_i corresponding to the minimum ratio of output (IF) signal-to-noise ratio $(S_o/N_o)_{min}$ necessary for detection, then

$$S_{min} = kT_0 B_n F_n \left(\frac{S_o}{N_o}\right)_{min}$$

Substituting this expression for S_{min} into the earlier basic Radar equation results in the following form of the **modified radar equation**:

$$R_{max}^4 = \frac{P_t G A_e \sigma}{(4\pi)^2 k T_0 B_n F_n (S_o/N_o)_{min}}$$ ....(4)

RADAR EQUATION

Signal to Noise Ratio (SNR):

The results of statistical noise theory will be applied to obtain:
* *The signal-to-noise ratio at the output of the IF amplifier necessary to achieve a specified probability of detection without exceeding a specified probability of false alarm*.

The output signal-to-noise ratio thus obtained is substituted into the final modified radar equation, we have obtained earlier.

The details of system that is considered:
* **IF** amplifier with bandwidth B_{IF} followed by a second detector and a video amplifier with bandwidth B_v as shown in the figure below.
* The second detector and video amplifier are assumed to form an envelope detector, that is, one which rejects the carrier frequency but passes the modulation envelope.

- The video bandwidth B_v must be greater than $B_{IF}/2$ in order to pass all the videomodulation.
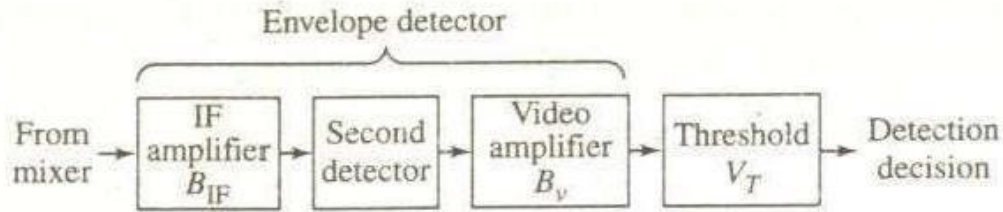

Figure 1.6: Envelope detector.

Step 1: To determine the Probability of false alarm when noise alone is assumed to be present as input to the receiver:

The noise entering the IF filter (the terms filter and amplifier are used interchangeably) is assumed to be Gaussian, with probability-density function given by

$$p(v) = \frac{1}{\sqrt{2\pi\psi_0}} \exp \frac{-v^2}{2\psi_0}$$

Where:
- **p(v) dv** is the probability of finding the noise voltage **v** between the values of **v** and **v + dv**
- ψ_0 is the variance, or mean-square value of the noise voltage, and the mean value of **v** is taken to be zero.

(Compare this with the Standard Probability density function of Gaussian noise

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \frac{-(x - x_0)^2}{2\sigma^2}$$

*With σ^2 replaced by ψ_0 and (x—x₀) replaced by **v** with mean value of zero)*

If Gaussian noise were passed through a narrowband IF filter whose Bandwidth is small compared with its mid band frequency-the probability density of the envelope of the noise voltage output is shown by **Rice** to be of the form of **Rayleigh** probability-density function

$$p(R) = \frac{R}{\psi_0} \exp \left(-\frac{R^2}{2\psi_0} \right) \quad \text{..........(6)}$$

where **R is the amplitude of the envelope of the filter output**. The probability that the envelope of the noise voltage will lie between the values of V_1 and V_2 is

$$\text{Probability } (V_1 < R < V_2) = \int_{V_1}^{V_2} \frac{R}{\psi_0} \exp \left(-\frac{R^2}{2\psi_0} \right) dR$$

The probability that the noise voltage envelope will exceed the voltage threshold V_T is

$$\text{Probability } (V_T < R < \infty) = \int_{V_T}^{\infty} \frac{R}{\psi_0} \exp\left(-\frac{R^2}{2\psi_0}\right) dR$$

$$= \exp\left(-\frac{V_T^2}{2\psi_0}\right) = P_{fa} \qquad \text{............(7)}$$

Whenever the voltage envelope exceeds the threshold V_T, a target is considered to have been detected. Since the probability of a false alarm is the probability that noise will cross the threshold, the above equation gives the probability of a false alarm, denoted by P_{fa}.

The probability of false alarm as given above by itself does not indicate that Radar is troubled by the false indications of Target. The time between the false alarms T_{FA} is a better measure of the effect of Noise on the Radar performance. (Explained with reference to the figure below)
The average time interval between crossings of the threshold by noise alone is defined as the *false- alarm time T_{FA}*

$$T_{fa} = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} T_k$$

Where T_K is the time between crossings of the threshold V_T by the noise envelope, when the slope of the crossing is positive.
The false-alarm probability may also be defined as the ratio of the duration of time the envelope is actually above the threshold to the total time it *could have been* above the threshold, i.e.

$$P_{fa} = \frac{\sum_{k=1}^{N} t_k}{\sum_{k=1}^{N} T_k} = \frac{\langle t_k \rangle_{av}}{\langle T_k \rangle_{av}} = \frac{1}{T_{fa} B}$$
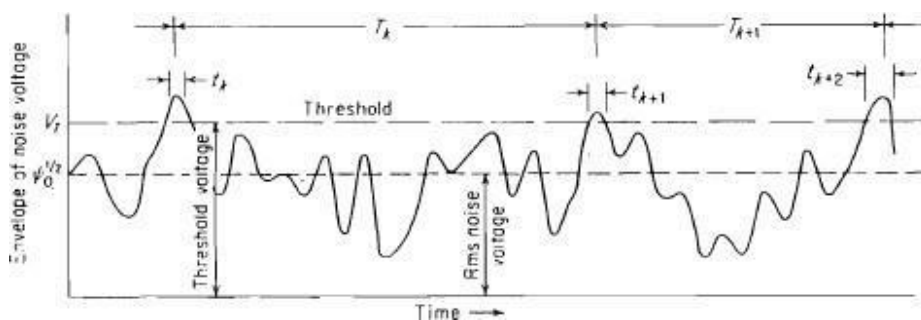
$$\text{............ (8)}$$

Fig 1.7: Envelope of receiver output illustrating false alarms due to noise.

Where t_K and T_K are shown in the Figure above. *The average duration of a noise pulse is approximately the reciprocal of the bandwidth B,* which in the case of the envelope detector is B_{IF}. Equating eqs. **7** and **8** we get

$$T_{fa} = \frac{1}{B_{IF}} \exp \frac{V_T^2}{2\psi_0}$$

..........(9)

A plot of the above equation is shown in the figure below with **(V_T^2/2 ψ_0)** as the abscissa. As can be seen, average time between false alarms T_{fa} is directly proportional to the Threshold to noise ratio and inversely proportional to the Bandwidth.
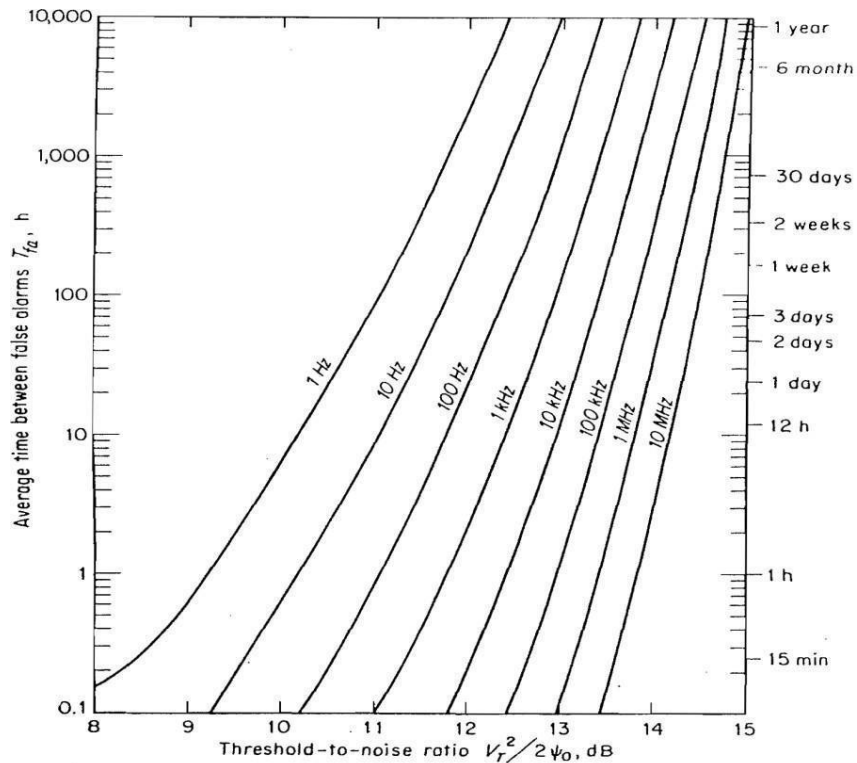

Fig. 1.8: Average time between false alarms as a function of the threshold level V_T and the receiver Bandwidth B. ψ_0 is the mean square noise voltage

Step 2 :
To determine Probability of detection when a sine wave signal is present along with noise:
Thus far, a receiver with only a noise input was discussed. Next, consider a sine-wave signal of amplitude **A** to be present along with noise at the input to the IF filters. The frequency of the signal is the same as the IF mid band frequency f_{IF}. The output of the envelope detector has a probability-density function given by

$$p_s(R) = \frac{R}{\psi_0} \exp\left(-\frac{R^2 + A^2}{2\psi_0}\right) I_0\left(\frac{RA}{\psi_0}\right)$$ ....(10)

where $I_0(Z)$ is the modified Bessel function of zero order and argument **Z.**

When the signal is absent, **A** = 0 and the above equation for **PDF** for signal plus noise reduces to the probability-density function for noise alone. This Equation is sometimes called the **Rice** probability- density function.

The probability that the signal will be detected (which is the **_probability of detection)_** is the same as the probability that the envelope **R** will exceed the predetermined threshold **V$_T$**. The probability of detection **P$_d$** is therefore:

$$P_d = \int_{V_T}^{\infty} p_s(R)\, dR = \int_{V_T}^{\infty} \frac{R}{\psi_0} \exp\left(-\frac{R^2 + A^2}{2\psi_0}\right) I_0\left(\frac{RA}{\psi_0}\right) dR$$

.........(11)

(After the expression of **PDF** for **P$_s$(R)[Eq. 10]** is substituted into the first part of the above equation we get the probability of detection as in **[eqn.11]).** But this equation cannot be evaluated by simple means, and numerical & empirical techniques or a series approximation must be used.

The expression for **P$_d$** given by equation **(11)** after series expansion is a function of the signal amplitude A, threshold voltage V$_T$,and mean noise power Ψ_0 . In Radar systems analysis, it is more convenient to use Signal to Noise power ratio (S/N) rather than signal to noise voltage ratio A/$\Psi_0^{½}$.These are related by:

$$\frac{A}{\psi_0^{1/2}} = \frac{\text{signal amplitude}}{\text{rms noise voltage}} = \frac{\sqrt{2}(\text{rms signal voltage})}{\text{rms noise voltage}} = \left(2\frac{\text{signal power}}{\text{noise power}}\right)^{1/2} = \left(\frac{2S}{N}\right)^{1/2}$$

The probability of detection P$_d$ can then be expressed in terms of S/N , and Threshold- Noise ratio V$_T^2$/2Ψ_0 .The probability of false alarm is also a function of V$_T^2$/2Ψ_0 as given by : **P$_{FA}$ = Exp(-V$_T^2$/2Ψ_0)**

The two expressions for P$_d$ and P$_{FA}$ can now be combined by eliminating the Threshold- Noise ratio **V$_T^2$/2Ψ_0**that is common in both expressions so as to get a single expression relating the probability of detection **P$_d$** ,Probability of false alarm **P$_{FA}$** and signal to Noise ratio **S/N**.The result is plotted in the figure below.

A much easier empirical formula developed by **_Albersheim_** for the relationship between **S/N,P$_{FA}$** and **P$_d$** is also given below :

S/N = A + 0.12AB + 1.7 B

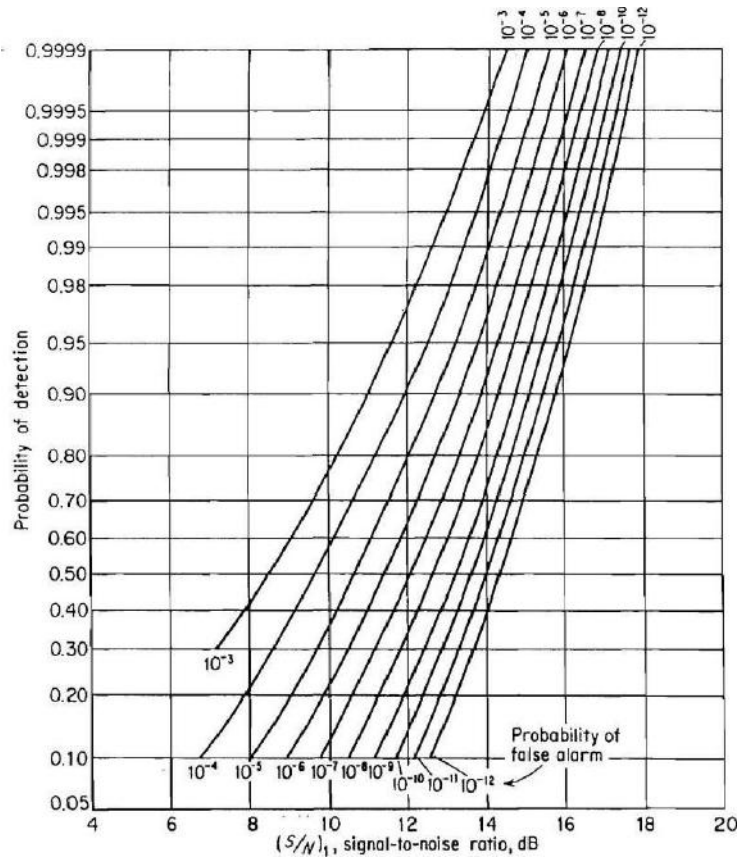**Where A = ln [0.62/ P$_{FA}$] and B = ln [P$_d$ /(1- P$_d$)]**

Fig. 1.9: Probability of detection for a sine wave in noise as a function of the signal-to-noise (power) ratio and the probability of false alarm

System design sequence:

- Both the false-alarm time**T$_{FA}$** and the detection probability**P$_d$**are specified by the system requirements.
- The radar designer computes the probability of the false alarm using the above **T$_{fa}$**& the relation

$$P_{fa} \quad = \quad 1/\ T_{fa}\ .\ B$$

- Then from the figure above or using the *Albersheim's* empirical equation given above the required signal-to-noise ratio to achieve the above **P$_{fa}$** & **P$_d$** is determined.

For example, suppose that the desired false-alarm time was **15 min** and the IF bandwidth was **1** MHz. This gives a false-alarm probability of **1.11 x 10^{-9}.** Figure above indicates that a signal-to-noise ratio of **13.1** dB is required to yield a **0.50** probability of detection, **14.7 dB** for **0.90**, and **16.5 dB** for **0.999**.

This is the signal-to-noise ratio that is to be used in the final modified Radar Equation we have obtained earlier.

$$R^4_{max} = \frac{P_t G A_e \sigma}{(4\pi)^2 k T_0 B_n F_n (S_o/N_o)_{min}}$$

Integration of Radar Pulses:

The relation between the signal to noise ratio, the probability of detection and the probability of false alarm as shown in the figure or as obtained using the **Albersheim's** empirical equation applies for a single pulse only. However, many pulses are usually returned from any target on each radar scan and
can be used to improve detection. The number of pulses   oint target as the radar
n_B returned from a antenna scans through its beam
width is

$$n_B = \theta_B \cdot f_P / \theta'_s \quad = \theta_B \cdot f_P / 6\,\omega_m$$

where θ_B= antenna beam width, deg
f_P= pulse repetition frequency, **Hz**
            θ'_s = antenna scann ng rate, deg/s
            ω_m= antenna scan rate, rpm

The process of summing all the radar echo pulses for the purpose of improving detection is called integration.
Integration may be accomplished in the radar receiver either before the second detector (in the IF) or after the second detector (in the video).

- Integration before the detector is called pre detection or coherent integration. In this the phase of the echo signal is to be preserved if full benefit is to be obtained from the summing process
- Integration after the detector is called post detection or non coherent integration. In this phase information is destroyed by the second detector. Hence post detection integration is not concerned with preserv ng RF phase. Due to this simplicity it is easier to implement in most applications, but is not as efficient as pre detection integration.

If *n* pulses, all of the same signal-to-noise ratio, were integrated by an ideal pre detection integrator, the resultant or integrated signal-to-noise (power) ratio would be exactly **n** times that of a single pulse. If the same *n* pulses were integrated by an ideal post detection device, the resultant signal-to-noise ratio would be less than **n** times that of a single pulse. ***This loss in integration efficiency is caused by the nonlinear action of the second detector, which converts some of the signal energy to noise energy in the rectification process.***

Due to its simplicity, Post detection integration is preferred many a times even though the integrated signal-to-noise ratio may not be as high as that of Pre-detection. An alert, trained operator viewing a properly designed cathode-ray tube display is a close approximation to the theoretical post detection integrator.
The efficiency of post detection integration relative to ideal pre-detection integration has been computed by **Marcum** when all pulses are of equal amplitude. The integration efficiency may be defined as follows:

$$E_i(n) = \frac{(S/N)_1}{n(S/N)_n}$$

Where n = number of pulses integrated

(S/N)₁= value of signal-to-noise ratio of a single pulse required to produce a given probability of detection(for **n = 1**)

(S/N)ₙ = value of signal-to-noise ratio per pulse required to produce the same probability of detection when **n** pulses (of equal amplitude) are integrated

The improvement in the signal-to-noise ratio when n pulses are integrated post detection is **n.E$_i$(n)** and is the integration-improvement factor. **It may also be thought of as the effective number of pulses integrated by the post detection integrator**. The improvement with ideal pre detection integration would be equal to **n**. Integration loss in decibels is defined as **L$_i$(n) = 10 log [1/E$_i$(n)].**
  The integration-improvement factor (or the integration loss) is not a sensitive function of either the probability of detection or the probability of false alarm.

The radar equation with n pulses integrated can be written

$$R^4_{max} = \frac{P_t G A_e \sigma}{(4\pi)^2 k T_0 B_n F_n (S/N)_n}$$

where the parameters are the same as in the earlier Radar equation except that **(S/N)ₙ,** is the signal-to- noise ratio of one of the **n** equal pulses that are integrated to produce the required probability of detection for a specified probability of false alarm. Substituting the equation for integration efficiency

$$E_i(n) = \frac{(S/N)_1}{n(S/N)_n}$$

into the above Radar equation gives the final modified Radar equation including integration efficiency.

$$R^4_{max} = \frac{P_t G A_e \sigma n E_i(n)}{(4\pi)^2 k T_0 B_n F_n (S/N)_1}$$

Radar Cross Section of Targets:

The radar cross section of a target is the (fictional) area intercepting that amount of power which when scattered equally in all directions, produces an echo at the radar equal to that from the target. Or in other terms

$$\sigma = \frac{\text{power reflected toward source/unit solid angle}}{\text{incident power density}/4\pi}$$

$$= \lim_{R \to \infty} 4\pi R^2 \left| \frac{E_r}{E_i} \right|^2$$

Where **R**= distance between radar and target

E_r= strength of reflected field at radar

E_i= strength of incident field attarget

For most common types of radar targets such as aircraft, ships, and terrain, the radar cross section does not necessarily bear a simple relationship to the physical area, except that the larger the target size, the larger will be the cross section.

Scattering and ***diffraction:*** are variations of the same physical process. When an object scatters an electromagnetic wave, the scattered field is defined as the difference between the total field in the presence of the object and the field that would exist if the object were absent (but with the sources unchanged). On the other hand, the diffracted field is the total field in the presence of the object. With radar backscatter, the two fie ds are the same, and one may talk about scattering and diffraction interchangeably.

Radar cross section of a simple sphere: is shown in the figure below as a function of its circumference measured in wavelengths.(**2πa/λ** where ***a*** is the radius of the sphere and **λ** is the wavelength). The plot consists of three regions.

  1. Rayleigh Region:
  - The region where the size of the sphere is small compared with the wavelength (**2πa/λ 1)** is called the **Rayleigh** region.
  - The Rayleigh scattering region is of interest to the radar engineer because the cross sections of raindrops and other meteorological particles fall within this region at the usual radar frequencies. **2. Optical region**:
  - It is at the other extreme from the **Rayleigh** region where the dimensions of the sphere are large compared with the wavelength **(2πa/λ 1).** For radar cross section large
  2πa/λ,

    the approaches the optical cross section **πa²**.
  3. Mie or Resonance region:

  - Between the optical and the Rayleigh region is the ***Mie,*** or resonance, region. The cross section is oscillatory with frequency within this region. The maximum value is 5.6 dB greater than the optical value, while the value of the first null is 5.5 dB below the optic l value. (The theoretical values of the maxima and minima may vary according to the method of calculation employed.
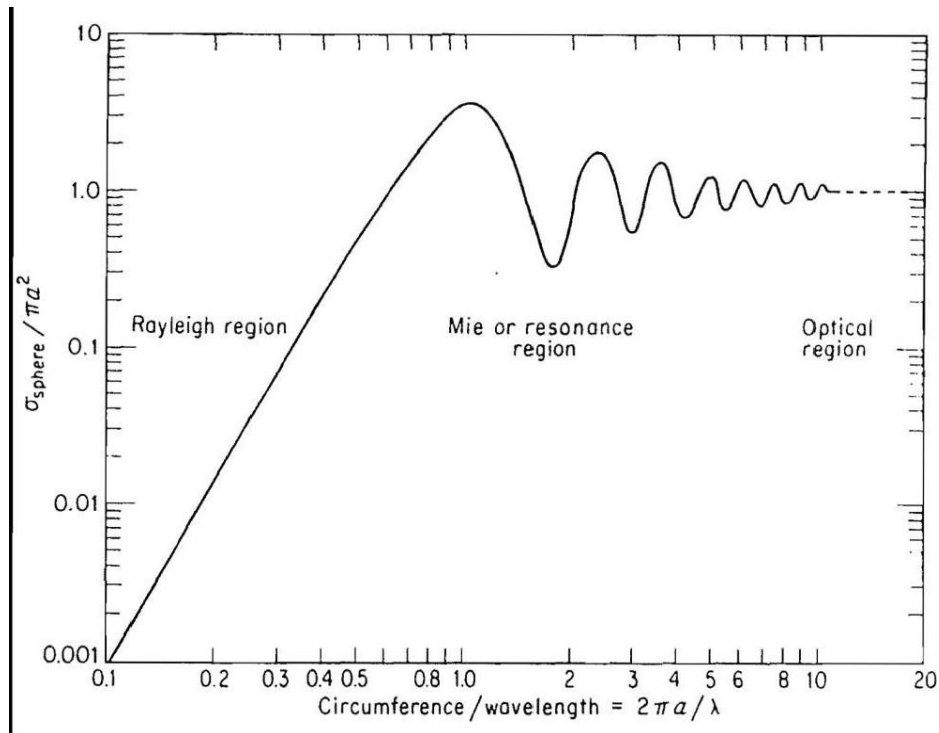
Figure 1.10: Radar cross section of the sphere. a = radius; λ= wavelength.

Since the sphere is a sphere no matter from what aspect it is viewed, its cross section will not be aspect-sensitive. The cross section of other objects, however, will depend upon the direction as viewed by the radar. (Aspect angle)

Radar cross section of a cone-sphere:

- An interesting radar scattering object is the cone-sphere, a cone whose base is capped with a sphere such that the first derivatives of the contours of the cone and sphere are equal at the joint. Figure below is a plot of the nose-on radar cross section. The cross section of the cone- sphere from the vicinity of the nose-on direction is quite low.
- Scattering from any object occurs from discontinuities. The discontinuities, and hence the backscattering, of the cone-sphere are from the tip and from the join between the cone and the sphere.
- The nose-on radar cross section is small and decreases as the square of the wavelength. The cross section is small over a relatively large angular region. **A** large **specular***(having qualities of a mirror)*return is obtained when the cone-sphere is viewed at near perpendicular incidence to the cone surface, i.e., when θ= 90 - α, where α= cone half angle. From the rear half of the cone- sphere, the radar cross section is approximately that of thesphere.

- The nose-on cross section of the cone-sphere varies, but its maximum value is approximately **0.4λ^2** and its minimum is **0.01λ^2** for a wide range of half-angles for frequencies above the Rayleigh region. The null spacing is also relatively insensitive to the cone half-angle.
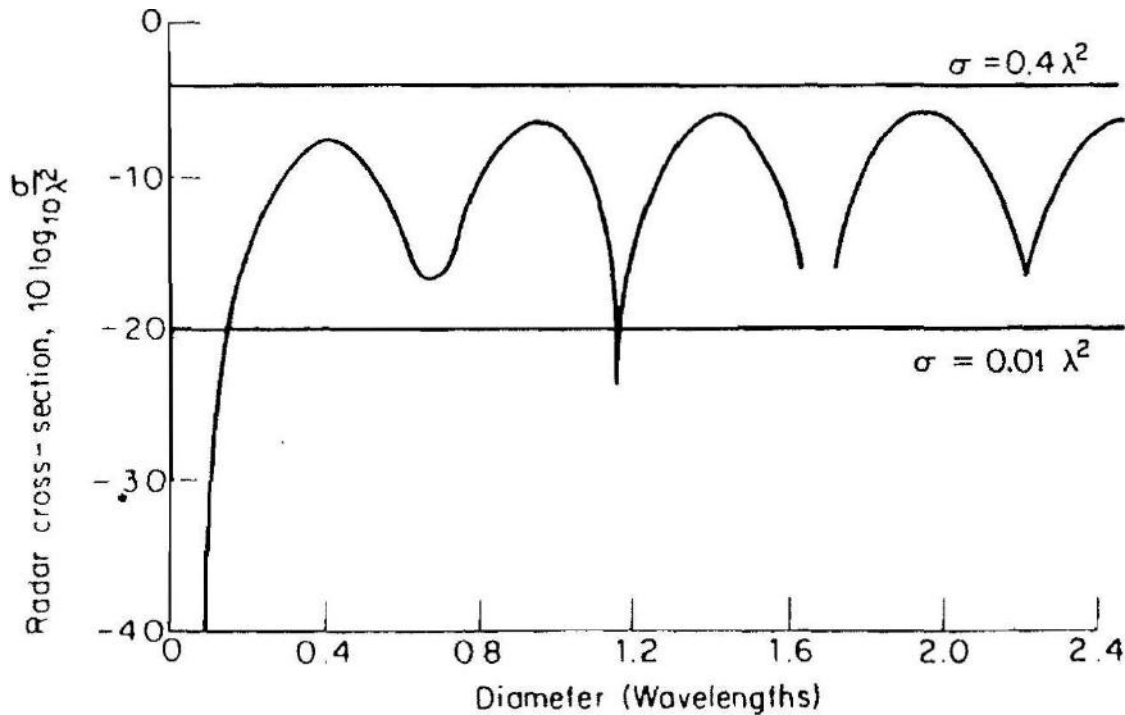
Figure 1.11: Radar cross section of a cone sphere with 15^0 half angle as a function of the diameter in Wave lengths.

- In order to realize in practice the very low theoretical values of the radar cross section for a cone sphere, the tip of the cone must be sharp and not rounded, the surface must be smooth (roughness small compared to a wavelength), the join between the cone and the sphere must have a continuous first derivative, and there must be no holes, windows, or protuberances on the surface.

- Shaping of the target, as with the cone-sphere, is a good method for reducing the radar cross section. Materials such as carbon-fiber composites, which are sometimes used in aerospace applications, can further reduce the radar cross   as compared with that
section of targets produced by highly reflecting
metallic materials.

Transmitter Power:

The peak power: The power P_t in the radar equation is called the   power. This is not the
peak

instantaneous peak power of a sine wave. It is the power averaged over that carrier-frequency cycle which occurs at the maximum power of the pulse.

The average radar power P_{av}: It is defined as the average transmitter power oover the pulse-repetition period. If the transmitted waveform is a train of rectangular pulses of width τ and pulse-repetition period $T_P = 1/f_P$, then the average power is related to the peak power by

$$P_{\mathrm{av}} = \frac{P_t \tau}{T_p} = P_t \tau f_p$$

Duty cycle: The ratio P_{av}/P_t, τ/T_P, or $\tau.f_P$ is called the duty cycle of the radar. A pulse radar for detection of aircraft might have typically a duty cycle of 0.001, while a CW radar which transmits continuously has a duty cycle of unity.

Writing the radar equation in terms of the average power rather than the peak power, we get

$$R_{\max}^4 = \frac{P_{\mathrm{av}} G A_e \sigma n E_i(n)}{(4\pi)^2 k T_0 F_n (B_n \tau)(S/N)_1 f_p}$$

The bandwidth and the pulse width are grouped together since the product of the two is usually of the order of unity in most pulse-radar applications.

Pulse Repetition Frequencies and Range Ambiguities:

- The pulse repetition frequency (**prf**) is determined primarily by the maximum range at which targets are expected. If the **prf** is made too high, the likelihood of obtaining target echoes from the wrong pulse transmission is increased. Echo signals received after an interval exceeding the pulse-repetition period are called ***multiple time around echoes.***

- Consider the three targets labeled *A, B,* and **C** in the figure(a) below. Target **A** is located within the maximum unambiguous range *Runamb* **[= C.T$_P$ /2]** of the radar, target **B** is at a distance greater than *Runamb* but less than *2Runamb* and the target **C** is greater than **2R_{unamb}** but less than

  3R_{unamb} The appearance of the three targets on an A-scope is shown in the figure (b)below. The multiple-time-around echoes on the A-scope cannot be distinguished from proper target echoes actually within the maximum unambiguous range. Only the range measured for target *A* is correct; those for **B** and **C** are not.

- One method of distinguishing multiple-time-around echoes from unambiguous echoes is to operate with a varying pulse repetition frequency. The echo signal from an unambiguous range target will appear at the same place on the A-scope on each sweep no matter whether the **prf** is modulated or not. However, echoes from multiple-time-around targets will be spread over a finite range as shown in the figure (c) below. The number of separate pulse repetition frequencies will depend upon the degree of the multiple time around targets. Second-time targets need only two separate repetition frequencies in order to be resolved.

Time (or range) →

(a)

Range →

(b)

Range →

(c)
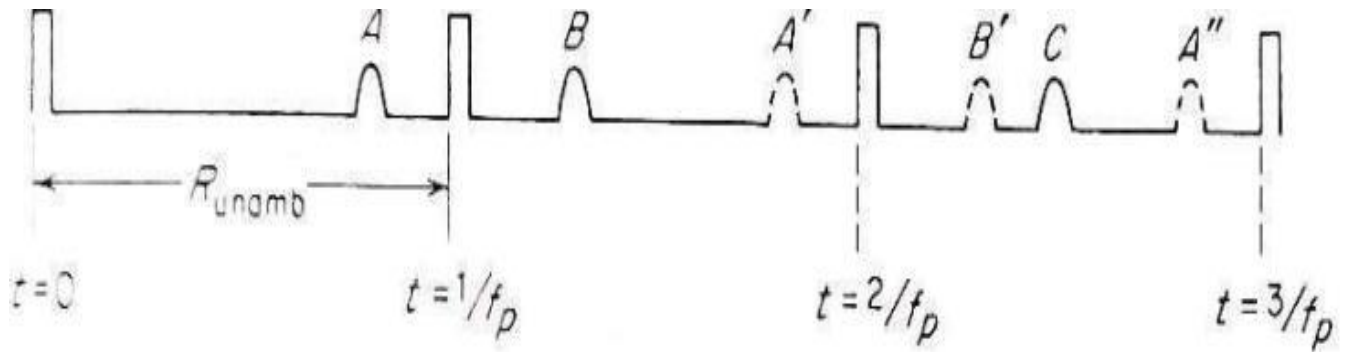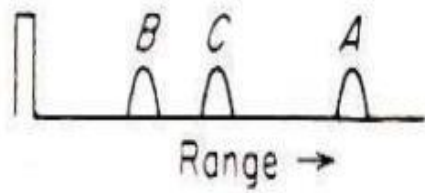
Fig. 1.12: Multiple-time-around echoes that give rise to ambiguities in range. (a) Three targets A, B and C, where A is within R_unamb, and B and Care multiple-time-around targets(b)the appearance ofthe three targets on the A-scope (c) appearance of the three targets on the A-scope with a changingprf.

System Losses:

- The losses in a radar system reduce the signal-to-noise ratio at the receiver output. They are two kinds, predictable with certain precision beforehand and unpredictable. The antenna beam- shape loss, collapsing loss, and losses in the microwave plumbing are examples of losses which are predictable if the system configuration is are real and cannot be known. These losses ignored.

- Losses not readily subject to calculation and which are less predictable include those due to field degradation and to operator fatigue or lack of operator motivation. They are subject to considerable variation and uncertainty.

Plumbing loss: This is loss in the transmission lines which connects the transmitter output to the antenna. (Cables and waveguides). At the lower radar frequencies the transmission line introduces little loss, unless its length is exceptionally long. At higher radar frequencies, loss/attenuation will not be small and has to be taken into account.

Connector losses: In addition to the losses in the transmission line itself, additional losses occurs at each connection or bend in the line and at the antenna **rotary joint** if used. Connector losses are usually small, but if the connection is poorly made, it can contribute significant attenuation. If the same transmission line is used for both receiving and transmission, the loss to be inserted in the radar equation is ***twice*** the one-way loss.

Duplexer loss: The signal suffers attenuation as it passes through the duplexer. Generally, the greater the isolation required from the duplexer on transmission, the larger will be the insertion loss. Insertion loss means the loss introduced when the component is inserted into the transmission line. For a typical duplexer it might be of the order of 1 dB.
In S-band (3000 MHz) radar, for example, the typical plumbing losses will be as follows:

100 ft of RG-113/U A1 waveguide transmission line (two-way):	*1.0 dB*
Loss due to poor connections (estimate):	*0.5 dB*
Rotary-joint loss:	*0.4 dB*
Duplexer loss:	*1.5 dB*
Total plumbing loss:	*3.4 dB*

Beam-shape loss: The antenna gain that appears in the radar equation was assumed to be a constant equal to the maximum value. But in reality the train of pulses returned from a target with scanning radar is modulated in amplitude by the shape of the antenna beam. To properly take into account the pulse- train modulation caused by the beam shape, the computations of the probability of detection (as explained earlier) would have to be performed assuming a modulated train of pulses rather than constant-amplitude pulses. But since this computation is difficult, a beam-shape loss is added to the radar equation and a

maximum gain is employed in the radar equation rather than a gain that changes pulse to pulse.

Scanning loss: When the antenna scans rapidly enough , the gain on transmit is not the same as the gain on receive. An additional loss has to be computed, called the scanning loss. The technique for computing scanning loss is similar in principle to that for computing beam-shape loss. Scanning loss is important for rapid-scan antennas or for very long range radars such as those designed to view extraterrestrial objects.

Collapsing loss: If the radar were to integrate additional noise samples along with the wanted Signal-to-noise pulses, the added noise results in degradation called the **collapsing loss.**

Non ideal equipment: The **transmitter power** in the radar equation was assumed to be the specified output power (either peak or average). However, all transmitting tubes are not uniform in quality, and even any individual tube performance will not be same throughout its useful life. Also, the power is not uniform over the operating band of frequencies. Thus, for one reason or another, the transmitted power may be other than the design value. To allow for this variation, a loss factor of about 2 dB is introduced.

Receiver noise figure also varies over the operating frequency band. Thus, if the best noise figure over the band is used in the radar equation, a loss factor has to be introduced to account for its poorer value elsewhere in the frequency band.

If the receiver is not the exact matched filter for the transmitted waveform, a loss in Signal-to-noise ratio will occur.

A typical value of loss for a non-matched receiver might be about **1 db.** Because of the exponential relation between the false-alarm time and the threshold level a slight change in the threshold can cause a significant change in the false alarm time. In practice, therefore, it may be necessary to set the threshold level slightly higher than calculated so as to insure a tolerable false alarm time in the event of circuit instabilities. This increase in the threshold is equivalent to a loss.

Operator loss: An alert, motivated, and well-trained operator performs as described by theory. However, when distracted, tired, overloaded, or not properly trained, operator performance will decrease. The resulting loss in system performance is called operator loss.

Field degradation: When a radar system is operated under laboratory conditions by engineering personnel and experienced technicians, the above mentioned losses give a realistic description of the performance of the radar. However, when a radar is operated under field conditions the performance usually deteriorates even more than that can be accounted for by the above losses. **To minimize field degradation** Radars should be designed with built-in automatic performance-monitoring equipment. Careful observation of performance-monitoring instruments and timely preventative maintenance will **minimize field degradation.**

There are many causes of loss and inefficiency in a Radar. Although each of them may be small, the sum total can result in a significant reduction in radar performance. It is important to understand the origins of these losses, not only for better predictions of radar range, but also for the purpose of keeping them to a minimum by careful radar design.

Important formulae:

- **Range of a Radar R = cT_R / 2**
- **Maximum unambiguous Range R $_{unambig.}$ = C /$2f_p$**
- **Basic Radar equation : R_{max} =   [(Pt .G. A_e. σ)/ $(4π)^2$. S_{min} $]^{1/4}$**
- **Gain of an antenna G = 4π A_e /$λ^2$**
- **Noise figure of a receiver F_n = N_o/ kT_o $B_n$$G_a$ Also given by**

$$F_n = \frac{S_i/N_i}{S_o/N_o}$$

- **If the minimum detectable signal S_{min} is that value of S_i corresponding to the minimum ratio of output (IF) signal-to-noise ratio (S_o/N_o)$_{min}$ necessary for detection, then**

$$S_{min} = kT_0 B_n F_n \left(\frac{S_o}{N_o}\right)_{min}$$

- **And modified Maximum possible range in terms of the IF amplifier output signal to noise ratio and noise figure**

$$R^4_{max} = \frac{P_t G A_e \sigma}{(4\pi)^2 k T_0 B_n F_n (S_o/N_o)_{min}}$$

- **Relationships between Probability of false alarm, Probability of detection and S/N :**
    - P_{fa} =   1/ T_{fa} . B

    - P_{FA} = Exp(-V_T^2/$2ψ_0$).

$$T_{fa} = \frac{1}{B_{IF}} \exp \frac{V_T^2}{2\psi_0}$$

    - ***Albersheim*** empirical relationship between **S/N, P_{FA} and P_d:**

        S/N = A + 0.12AB + 1.7 B

        Where A = ln [0.62/ P_{FA}] and B = ln [P_d /(1- P_d)] •

The radar equation with *n* pulses integrated :

$$R^4_{max} = \frac{P_t G A_e \sigma}{(4\pi)^2 k T_0 B_n F_n (S/N)_n}$$

- **Integration efficiency :**

$$E_i(n) = \frac{(S/N)_1}{n(S/N)_n}$$

- **Final Radar equation including Integration efficiency with n pulses integrated:**

$$R_{max}^4 = \frac{P_t G A_e \sigma n E_i(n)}{(4\pi)^2 k T_0 B_n F_n (S/N)_1}$$

Illustrative Problems:

Example1: A certain Radar has PRF of 1250 pulses per second. What is the maximum unambiguous range? Max. Unambiguous Range is given by

$$R_{unambig.} = C /2f_p$$
$$R_{unambig.} = 3x10^8/2x1250 \text{ mtrs} = 120X10^3 \text{mts} = 120 \text{Kms}$$

Example 2: A ship board radar has 0.9 micro sec transmitted pulse width. Two small boats in the same direction are separated in range by 150 mts. Will the radar detect the two boats as two targets?

Radar Range Resolution: The range resolution of a Radar is its ability to distinguish two closely spaced targets along the same line of sight (LOS). The Range resolution is a function of the pulse length, where the pulse length $L_P = c \times \tau/2$ (Two way range corresponding to the pulse width)

Radar Range resolution = $3x10^8 x 0.9 x 10^{-6}/2$ = 135 mtrs.

Since the boats are at 150 Mts. apart, which is greater than the range Resolution of 135mtrs., the radar can detect the 2 boats as 2 separate targets.

Example 3: A Pulse Radar transmits a peak power of 1 Mega Watt. It has a PRT equal to 1000 micro sec and the transmitted pulse width is 1 micro sec. Calculate (i)Maximum unambiguous range (ii) Average Power (iii)Duty Cycle (iv) Energy transmitted & (v)Bandwidth

(i) Maximum unambiguous range = $c.T_P /2 = 3x10^8 x 1000 x 10^{-6}/2 = 150x10^3$ mtrs = 150 Kms

(ii) Average Power = $P_P \times \tau / T_P = 1x10^6 \, 1x10^{-6} /1000x10^{-6} = 1000$ watts = 1kw

(iii)Duty Cycle = τ / T_P = $1x10^{-6} /1000x10^{-6}$ = 0.001

(iv) Energy transmitted   = $P_P \times \tau$ (Peak power xTime) = $1x10^6 \, x1x10^{-6}$ = 1

Joule (v)Bandwidth = $1/\tau$ = $1/10^{-6}$ =   1Mhz

Example4: The Bandwidth of I.F. Amplifier in a Radar Receiver is 1 MHZ. If the Threshold to noise ratio is 12.8 dB determine the False Alarm Time.

T_{fa} = False Alarm Time $T_{fa} = [1/B_{IF}]$ Exp $V_\tau^2/2\psi_0$ where B_{IF} = 1X 10^6 HZ

Threshold to Noise Ratio = 12.8 dB

i.e. 10 $Log_{10}[V_T^2/2\psi_0]$ = **12.8db**

∴**$V_T^2/2\psi_0$ = Antilog$_{10}$ [12.8/10] =   19.05**

∴T_{fa}= 1/(1X 10^6) Exp 19.05 =   187633284/10^6=   187.6 Seconds

Example5: The probability density of the envelope of the noise voltage output is given by the Rayleigh probability-density function

$$p(R) = \frac{R}{\psi_0} \exp\left(-\frac{R^2}{2\psi_0}\right)$$

where R is the amplitude of the envelope of the filter output for R   ≥ 0. If P_{fa} needed is   ≤ 10^{-5}. Determine the Threshold Level.

The probability of false alarm P_{FA} in terms of the threshold voltage level is given by :

P_{FA}= Exp(-$V_T^2/2\psi_0$) = 10^{-5}

 Taking logarithms on both the sides we get

- 5 Log $_e$10 = (-$V_T^2/2\psi_0$

) 5 x 2.3026 = ($V_T^2/2\psi_0$)

V_T^2 = 11.5 x 2 ψ_0

V_T = $\sqrt{23}$ x $\sqrt{\psi_0}$= 4.8 x $\sqrt{\psi_0}$

Example 6: The bandwidth of an IF amplifier is 1 MHz and the average false-alarm time that could be tolerated is 15 min. Find the probability of a false alarm.
The relationship between average false-alarm time T_{FA}, probability of a false alarm P_{FA} and the IF bandwidth B is given by :
P_{fa} =   1/ T_{fa} . B

Substituting **B** = 1 MHz ie 10^6 and **T_{fa}**=15 mnts. i.e. 900 secs. we get PFA=1.11 X 10^{-9}

Example 7: What is the ratio of threshold voltage to the rms value of the noise voltage necessary to achieve this false-alarm time?
This is found out using the relationship **P_{FA} = Exp (-$V_T^2/2\psi_0$)**

from which the ratio of Threshold voltage to rms value of the noise voltage is given by
$V_T/\sqrt{\psi_0}$ = $\sqrt{2 \ln(1/P_{fa})}$ = $\sqrt{2 \ln 9X10^8}$ = 6.45 = 16.2dB

Example 8: Typical parameters for a ground-based search radar are : 1. Pulse repetition frequency :300 Hz, 2. Beam width : 1.5^0, and 3. Antenna scan rate: 5 rpm (30^0/s). Find out the number of pulses returned from a point target on each scan.

Solution : The number of pulses retuned from a point target on each scan $n_B$is given by:

$$n_B = \theta_B . f_P / \theta'_s = \theta_B . f_P / 6 \ \omega_m$$

Substituting the above values we get : $n_B = 1.5 \times 300 / 30 = 15$

Questions from Previous Year Examinations:

1.(a) Derive Radar range equation in terms of MDS (minimum detectable signal) (b) What is maximum unambiguous range? How is it related with PRF?

2.(a) Explain the various system losses in a Radar (b) The bandwidth of The IF amplifier in a Radar is 1 Mhz and the threshold noise ratio is 13 db. Determine the false alarm time.

3.(a) Explain the basic principles of Radar and discuss about various parameters which improve the performance of the Radar (b) Discuss about Radar frequencies and list out the Applications of Radars.

4.(a) In a Radar receiver the mean noise voltage is 80 mv and the IF BW is 1 Mhz. If thetolerable false alarm time is 25 mnts., calculate the threshold voltage level and the probability of false alarm. (b) Bring out the advantages of Integration of Radar pulses.

5 (a) Discuss about the factors that influence the prediction of Radar range. (b) Define noise bandwidth of a radar receiver. How does it differ from 3-dB band width? Obtain the expression for minimum detectable signal in terms of noise bandwidth, noise figure and other relevant parameters. [8+8]

6. (a) Write the simplified version of radar range equation and explain how this equation does not adequately describe the performance of practical radar? (b) What are the specific bands assigned by the ITU for the Radar? What the corresponding frequencies? [8+8]

7. (a) Explain how the Radar is used to measure the range of a target? (b) Draw the block diagram of the pulse radar and explain the function of each block. [8+8]

8. (a) A low power, short range radar is solid-state throughout, including a low-noise RF amplifier which gives it an overall noise figure of 4.77dB. If the antenna diameter is 1m, the IF bandwidth is 500kHz, the operating frequency is 8 GHz and the radar set is supposed to be capable of detecting targets of $5m^2$ cross sectional area at a maximum distance of 12 km, what must be the peak transmitted pulse power? (b) The average false alarm time is a more significant parameter than the false alarm probability. Give the reasons. (c) Why post detection integration is not as efficient as pre-detection integration of radar pulses? [8+4+4]

CW and Frequency Modulated Radar

CW and Frequency Modulated Radar

- Doppler Effect
- CW Radar-Block Diagram
- Isolation between Transmitter and Receiver
- Non-zero IF receiver
- Receiver Bandwidth Requirements
- Applications of CW radar
- Related Problems

- FM-CW radar
- Range and Doppler measurement
- Block diagram and Characteristics
- FM-CW altimeter
- Multiple Frequency CW radar

Doppler Effect

- Doppler effect implies that the frequency of a wave when transmitted by the source is not necessarily the same as the frequency of the transmitted wave when picked by the receiver.

- The received frequency depends upon the relative motion between the transmitter and receiver.

- If transmitter and receiver both are moving towards each other the received frequency higher, this is true even one is moving.

- If they are moving apart the received signal frequency decreases and if both are stationary, the frequency remains the same. This change in frequency is known as *Doppler shift*.

- Doppler shift depends upon the relative velocity between radar and target

- If **R** is the distance from the radar to target, the total number of wavelengths λ contained in the two-way path between the radar and the target are **2R/λ.**

- Each wavelength corresponds to a phase change of **2π** radians. The total phase change in the two way propagation path is then

$$\phi = 2\pi * \frac{2R}{\lambda} = 4\pi R/\lambda$$

- If the target is in motion relative to the radar, R is changing and so will the phase. Differentiating ϕ w.r.t time gives the rate of change of phase, which is the angular frequency

$$w_d = \frac{d\phi}{dt} = \frac{4\pi}{\lambda}\frac{dR}{dt} = \frac{4\pi v_r}{\lambda} = 2\pi f_d$$

- $v_r = \frac{dR}{dt}$ is the radial velocity or rate of change range with time

- $w_d = 2\pi f_d$ is the rate of change of ϕ with time is the angular frequency, where f_d is the Doppler frequency shift.

$$f_d = \frac{2v_r}{\lambda} = \frac{2fv_r}{c}$$

- The relative velocity may be written as $V_r = V.\cos\theta$ where V is the target speed and θ is angle made by the target trajectory and the line joining radar and target. When $\theta=0$ the Doppler frequency is maximum. The Doppler is zero when the trajectory is perpendicular to the radar line of sight $(\theta= 90^0)$.
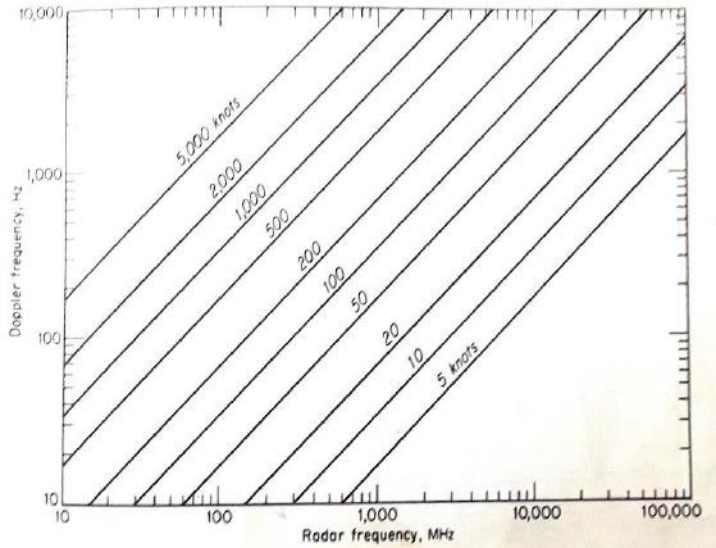

Figure: Doppler frequency f_d as a function of radar frequency and target relative velocity

CW RADAR

- It is possible to detect moving targets by radiating unmodulated Continuous wave energy instead of radiating in the form of pulses. Continuous Wave radars makes use of Doppler effect for target speed measurements.

Figure : (a) Block diagram of a Simple CW radar
(b) Response characteristic of the doppler filter

- Consider the simple CW radar shown the block diagram. **T**he transmitter generates a continuous (unmodulated) oscillation of frequency f_0 which is radiated by the antenna.

- A portion of the radiated energy is intercepted by the target and is scattered, some of it in the direction of the radar, where it is collected by the receiving antenna.

- If the target is in motion with a velocity $\mathbf{V_r}$ relative to the radar, the received signal will be shifted in frequency from the transmitted frequency f_0 by an amount $\pm f_d$ as given by the equation $$f_d = \frac{2v_r}{\lambda} = \frac{2fv_r}{c}$$

- *The plus sign associated with the Doppler frequency shift applies if the distance between target and radar is decreasing (approaching target) that is, when the received signal frequency is greater than the transmitted signal frequency. The minus sign applies if the distance is increasing (receding target).*

- The received echo signal at a frequency $f_0 \pm f_d$ enters the radar via the antenna and is heterodyned in the detector (mixer) with a portion of the transmitter signal $\mathbf{f_o}$ to produce a Doppler beat note of frequency $\mathbf{f_d}$. The sign of $\mathbf{f_d}$ is lost in this process.

- The beat frequency amplifier eliminates the echoes from stationary targets and amplifies the Doppler echo signal.

- The low-frequency cutoff must be high enough to reject the d-c component caused by stationary targets, but yet it must be low enough to pass the smallest Doppler frequency expected. Sometimes both conditions cannot be met simultaneously and a compromise is necessary. The upper cutoff frequency is selected to pass the highest Doppler frequency expected.

Advantages of CW Radar

- CW Doppler radar has no blind speed.

- CW Doppler radar is capable of giving accurate measurements of relative velocities.

- CW Doppler radars are always on, they need low power and are compact in size.

- They can be used for small to large range with high degree of efficiency and accuracy.

- The performance of radar is not affected by stationary object.

Disadvantages of CW Doppler radar

- The maximum range of CW Doppler radar is limited by the power that radar can radiate.

- The target range can not be calculated by CW Doppler radar.

- There is possibility of ambiguous results when number of targets are more.
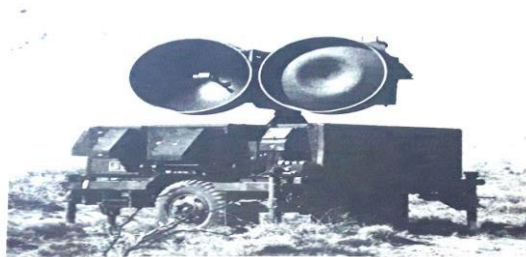
Applications of CW Radar

- CW Doppler radars are used where only velocity information is of intrest and actual range is not needed. E.g: in LAW and Enforcement radar applications

- Measuring motion of wave on water level.

- Runway monitors.

- Cricket ball speed measurement.

Isolation between transmitter and receiver

- A single antenna serves the purpose of both transmission and reception in the simple CW radar. In principle, a single antenna is sufficient as the necessary isolation is obtained by the separation in frequency (as a result of doppler effect), in practice there is considerable transmitter leakage.

- However, there are two reasons why the amount of transmitter leakage power should be kept at a low value.

    1. The maximum power the receiver input circuitry can withstand, without being physically damaged or having its sensitivity reduced, is quite low.

    2. The transmitter noise which enters the receiver from the transmitter reduces receiver sensitivity.

- The amount of isolation required depends on the transmitter power and the accompanying transmitter noise as well as the ruggedness and sensitivity of the receiver.

- *For example, If the safe value of power which might be applied to a receiver is 10mW and if the transmitter power is 1 kW, the isolation between transmitter and receiver must be at least 50 dB.*

- In long range CW applications, it is the level of the noise accompanying the transmitter leakage signal, rather than the damage this leakage might cause to the receiver circuitry, which determines the amount of isolation required.

- The amount of isolation which can be readily achieved between the arms of practical hybrid junctions such as the magic-T, rat race, or short-slot coupler is of the order of 20 to 30 dB.

- In some instances, when extreme precision is exercised, an isolation of perhaps 60 dB or more might be achieved. But one limitation of the hybrid junction is the 6-dB loss in overall performance.

- The largest isolations are obtained with two antennas one for transmission, the other for reception-physically separated from one another. Isolations of the order of 80 dB or more are possible with high-gain antennas. The more directive the antenna beam and the greater the spacing between antennas, the greater will be the isolation.

- The separate antennas of the AN/MPQ-46 CW tracker-illuminator of the Hawk missile system are shown

- The correct degree of isolation between transmitters and receivers can be implemented by one of two methods:

  1) Use two antennas, physically separated by a given distance, or;

  2) Use the appropriate duplexer with a single-antenna system.
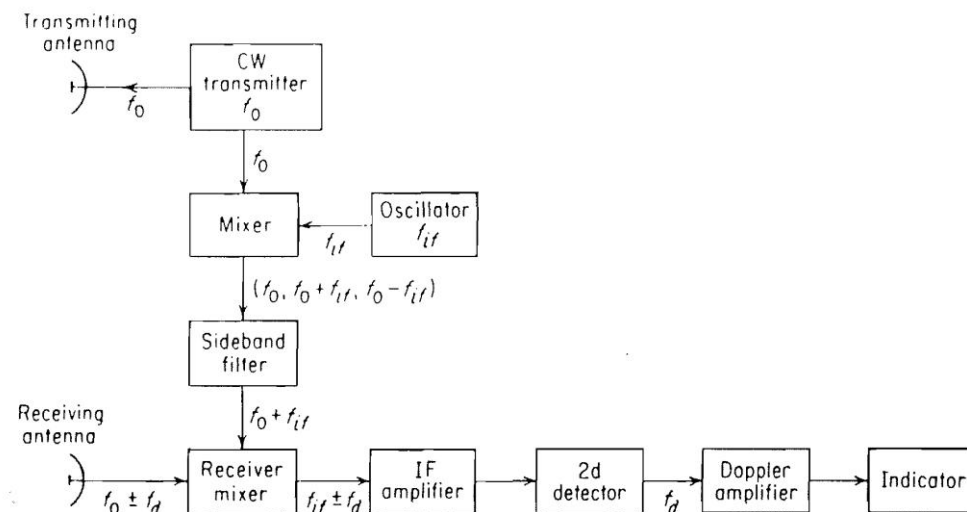
Intermediate-frequency receiver

Limitation of Zero IF receiver:

- Receivers of super heterodyne receiver type are also called homodyne receivers, or super heterodyne receivers with zero IF.

- However, this simpler receiver is not very sensitive because of increased noise at the lower intermediate frequencies caused by flicker effect.

- Flicker-effect noise occurs in semiconductor devices such as diode detectors and cathodes of vacuum tubes. The noise power produced by the flicker effect varies as **$1/f^\alpha$ where α is approximately unity.**

- At the lower range of frequencies (audio or video region), where the Doppler frequencies usually are found, the detector of the CW receiver can introduce a considerable amount of flicker noise, resulting in reduced receiver sensitivity.

- For short-range, low-power, applications this decrease in sensitivity might be tolerated

- But for maximum efficiency with CW radar, the reduction in sensitivity caused by the simple Doppler receiver with zero IF cannot be tolerated.

Non zero IF Receiver:

- Flicker effect noise reduces the receiver sensitivity of a CW Radar with zero IF (Simple Doppler radar). In order to increase the sensitivity and efficiency we go for CW Radar with Non-zero IF.

- Figure above shows the block diagram of a CW radar whose receiver operates with a nonzero IF. Separate antennas are shown for transmission and reception.

- Instead of the usual local oscillator found in the conventional super heterodyne receiver, the local oscillator (or reference signal) is derived in the receiver from a portion of the transmitted signal mixed with a locally generated signal of frequency equal to that of the receiver IF.

- Since the output of the mixer Consists of two sidebands on either side of the carrier plus higher harmonics, a narrow band filter selects one of the sidebands as the reference signal.

- The improvement in receiver sensitivity with an intermediate-frequency super heterodyne might be as much as 30 Db.

Limitations of CW radar with Non Zero IF

- False targets

- Unable to detect the range of the target

Receiver bandwidth requirements

- Bandwidth B, BW or Δf is the difference between the upper and lower cut-off frequencies of a radar receiver, and is typically measured in hertz.

- In case of a baseband channel or video signal, the bandwidth is equal to its upper cut-off frequency. In a Radar receiver the bandwidth is mostly determined by the IF filter stages.

- IF amplifier should be wide enough to pass the expected range of Doppler frequencies.

- Usually expected range of Doppler frequencies will be much higher than the frequency spectrum occupied by the signal energy . So a wide band amplifier is needed.

- which result in an increase in noise and a lowering of the receiver sensitivity and S/N.

- If the frequency of the Doppler-shifted echo signal are known beforehand, narrowband filter-that is just wide enough to reduce the excess noise without eliminating a significant amount of signal energy might be used.

- If the received waveform were a sine wave of infinite duration, its frequency spectrum would be a delta function as shown in the figure (a) below and the receiver bandwidth would be infinitesimal.

- But a sine wave of infinite duration and an infinitesimal bandwidth cannot occur in nature. The more normal situation is an echo signal which is a sine wave of finite duration.

- The frequency spectrum of a finite-duration sine wave has a shape of the form

$$[\sin\pi(f-f0)\delta]/\pi(f-f0)]$$

where f0 and δ are the frequency and duration of the sine wave, respectively, and f is the frequency variable over which the spectrum is plotted (Fig b).

Figure 3.5 Frequency spectrum of CW oscillation of (*a*) infinite duration and (*b*) finite duration.

Filter Banks in CW radar Receiver

- A bank of narrowband filter is required to measure the frequency of echo signals. The filter bank also increases signal to noise ratio of radar receiver.
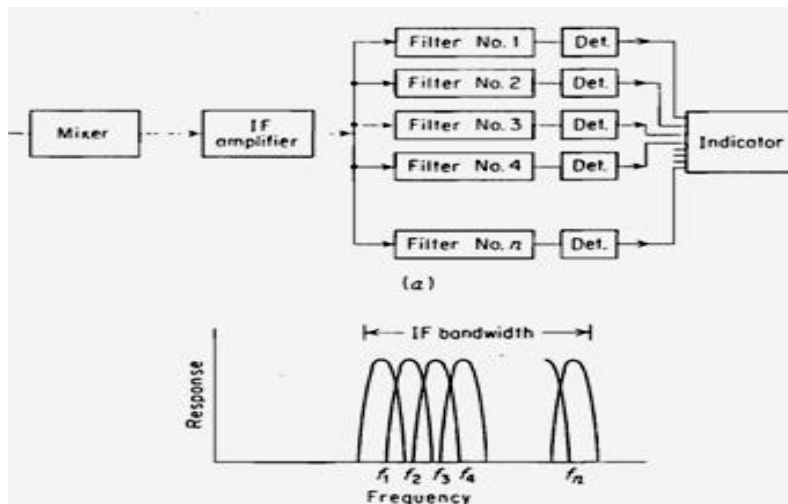
Figure: (a) Block diagram of IF Doppler filter bank (b) frequency-response characteristic of Doppler filter bank.

- BW of each filter is wide enough to accept the signal energy. But not so wide to accept the noise.
- The more the filters used less will be the SNR loss and less chance of missing a target.
- The ability to measure the magnitude of Doppler frequency and improvement in signal to noise ratio is better in IF filter bank than in video filter bank.
- Also the sign of Doppler shift (+ or -) is available which is not present in video filter bank.
- Each filter of filter bank has different bandwidth.

Sign of the radial velocity

- In many applications of CW radar it is of interest to know if the target is approaching or receding. This might be determined with separate filters located on either side of the intermediate frequency.

- If the echo-signal frequency lies below the carrier, then the target is receding; whereas if the echo frequency is greater that the carrier, then the target is approaching.
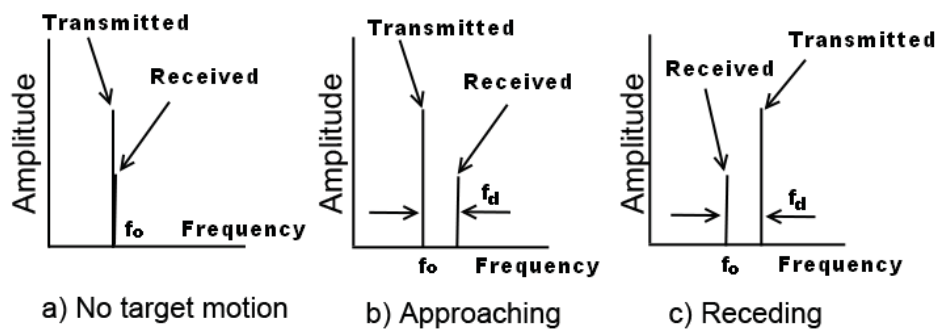

Figure: Spectra of received signals. (a) No Doppler shift, no relative target motion; (b) approaching target; (c) receding target.

- However, the Doppler-frequency spectrum "folds over" in the video because of the action of the detector, and hence the information about whether the doppler shift is positive or negative is lost. But it is possible to determine its sign from a technique borrowed from single-sideband communication.

- If the transmitter signal is given by,

$$E_t = E_o \cos w_o t$$

- The echo signal from the moving target will be,

$$E_r = K_1 E_o \cos [(w_o + w_d)t + \varphi]$$

where, E_o = *amplitude of the transmitted signal*

K_1 = *a constant determined from the radar equation*

w_o = *angular frequency of transmitted signal, rad/sec*

w_d = *dopper angular frequency shift, rad/sec*

φ = *a constant phase shift, which depends upon the range of initial detection (i.e., distance between the radar and the target)*

- The sign of the Doppler frequency, and therefore the direction of target motion, may be found by splitting the received signal into two channels as shown

- In channel A the signal is processed as in a simple CW radar. The receiver signal and a portion of the transmitter signal heterodyne in the detector (mixer) to yield a difference signal,

$$E_A = K_2 E_0 cos(\pm w_d t + \varphi)$$

- The channel B has $\pi/2$ phase delay introduced in the reference signal. The output of the channel B mixer is

$$E_B = K_2 E_0 \, cos(\pm w_d t + \varphi + \pi/2)$$

- If the target is approaching (positive doppler),the outputs from the two channels are,

$$E_A = K_2 E_0 \, cos(w_d t + \varphi)$$

$$E_B = K_2 E_0 \, cos(w_d t + \varphi + \pi/2)$$

on the other hand, if the target is receding (negative doppler),

$$E_A(-) = K_2 E_0 \, cos(w_d t - \varphi)$$

$$E_B(-) = K_2 E_0 \, cos(w_d t - \varphi - \pi/2)$$

- The sign of *w_d and the direction of the target's motion may be determined* according to whether the output of channel B leads or lags the output of channel A.

- One method of determining the relative phase relationship between the two channels is to apply the outputs to a *synchronous two-phase motor.* The direction of motor rotation is an indication of the direction of the target motion.

Applications of CW radar

- Police speed monitor
- Rate-of-climb meter (During aircraft take off)
- Vehicle counting
- As a replacement for "5th wheel speedometer" in vehicle testing
- Antilock braking system
- Collision avoidance
- In railways as speedometer instead of tachometer
- Advance warning system for approaching targets
- Docking speed measurement of large ships
- Intruder alarms
- Measurement of velocity of missiles, baseball etc

FM CW RADAR

- FM CW radar is capable of measuring the relative velocity and the range of the target with the expense of bandwidth.

- The inability of the simple CW radar to measure range is related to the relatively narrow spectrum (bandwidth) of its transmitted waveform.

- By providing timing marks into the Tx signal the time of transmission and the time of return can be calculated. This will increase the bandwidth

- More distinct the timing, more accurate the result will be and more broader will the Tx spectrum

- The spectrum of a CW transmission can be broadened by the application of modulation, either amplitude. frequency, or phase.

- An example of an amplitude modulation is the pulse radar. The narrower the pulse, the more accurate the measurement of range and the broader the transmitted spectrum

- A widely used technique to broaden the spectrum of CW radar is **to frequency-modulate** the carrier. The timing mark is the changing frequency.

Range and Doppler measurement:

- In the frequency-modulated CW radar (abbreviated FM-CW), the transmitter frequency is changed as a function of time.

- Assume that the transmitter frequency increases linearly with time, as shown by the solid line in the next slide.

- If there is a reflecting object at a distance **R**, an echo signal will return after a time **T = 2R/C.** The dashed line in the figure represents the echo signal.

- If the echo signal is heterodyned with a portion of the transmitter signal in a nonlinear element such as a diode, a beat note *fb will be produced.*

- If there is no Doppler frequency shift, the beat note (difference frequency) is a measure of the target's range and *fb = fr where fr is the beat frequency due only* to the target's range.

- If the rate of change of the carrier frequency is f_0 **(dot)**, then the beat frequency is given by

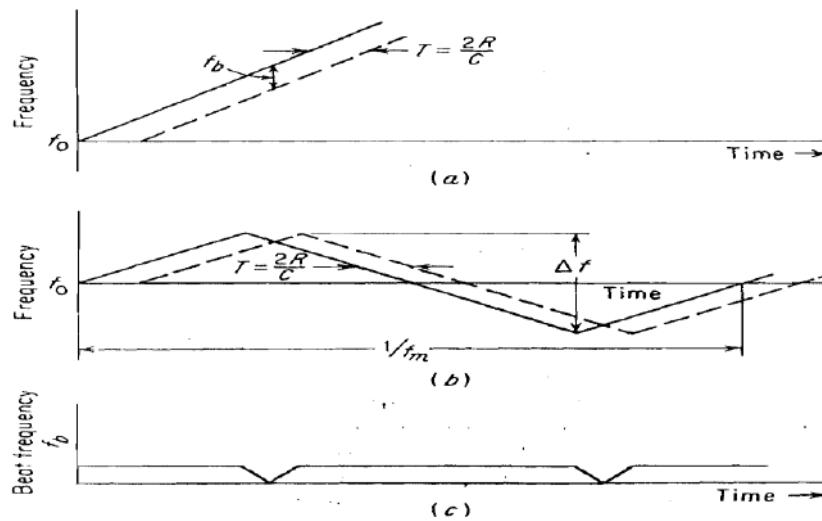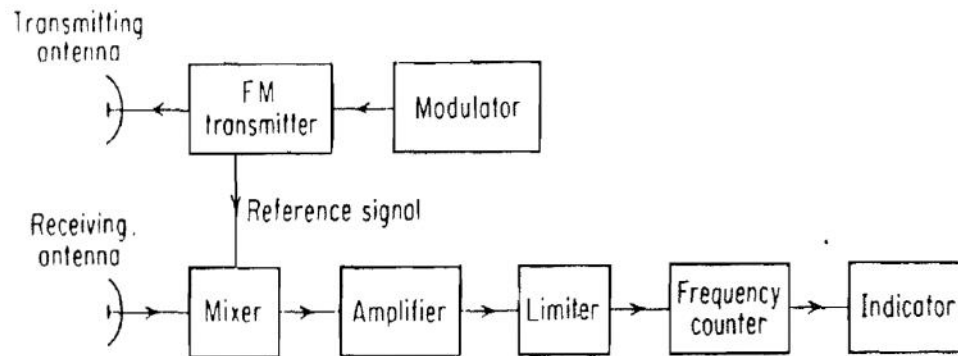$$f_r = \dot{f_0} T = \frac{2R}{c} \dot{f_0}$$

Figure: Frequency-time relation-ships in FM-CW radar. Solid curve represents transmitted signal; dashed curve represents echo. (a) Linear frequency modulation; (b)triangular frequency modulation; (c) beat note of (b).

- In any practical CW radar, the frequency cannot be continually changed in one direction. Periodicity in the modulation is necessary, as in the triangular frequency-modulation waveform shown in the figure (b). The modulation need not necessarily be triangular. It can be saw tooth, sinusoidal, or some other shape.

- The resulting beat frequency as a function of time is shown in figure(c) for triangular modulation.

- If the frequency is modulated at a rate f_m over a range Δf the beat frequency is

$$f_r = (2R/C).2f_m.\Delta f = 4Rf_m.\Delta f /C$$

- Thus the measurement of the beat frequency determines the range R.

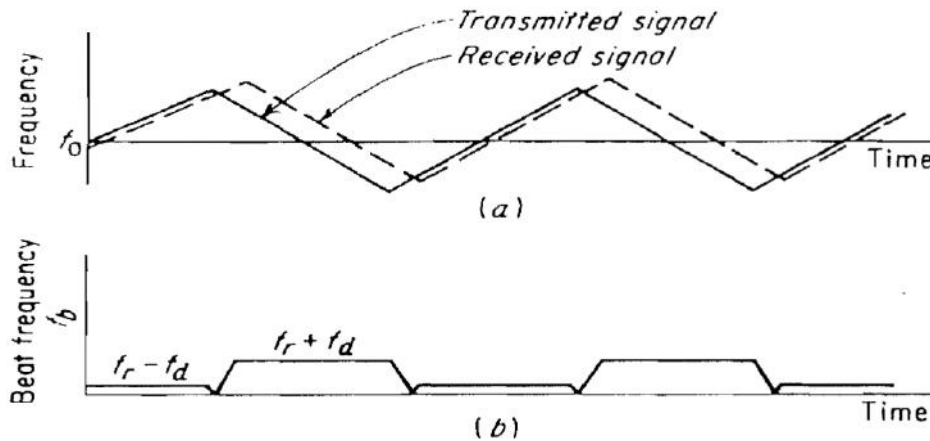# Block diagram illustrating the principle of the FM-CW radar

- A portion of the transmitter signal acts as the reference signal required to produce the beat frequency. It is introduced directly into the receiver via a cable or other direct connection.

- Ideally the isolation between transmitting and receiving antennas is made sufficiently large so as to reduce to a negligible level the transmitter leakage signal which arrives at the receiver via the coupling between antennas.

- The beat frequency is amplified and limited to remove any amplitude fluctuations.

- The frequency of the amplitude-limited beat note is measured with a cycle-counting frequency meter calibrated in distance.

- If the target is not stationary Doppler frequency shift will be superimposed on the FM range beat note and an erroneous range measurement results.

- The Doppler frequency shift causes the frequency-time plot of the echo signal to be shifted up or down as shown in the figure (a). On one portion of the frequency-modulation cycle, the beat frequency (Fig. b) is increased by the Doppler shift, while on the other portion, it is decreased.

- If for example, the target is approaching the radar, the beat frequency $f_b(\text{up})$ produced during the increasing or up will be the difference between the beat frequency due to the range f_r and the Doppler frequency shift f_d. Similarly, on the decreasing portion, the beat frequency $f_b(\text{down})$ is the sum of the two

$$f_b(\text{up}) = f_r - f_d$$
$$f_b(\text{down}) = f_r + f_d$$

- The beat frequency due to range f_r can be calculated as

$$\tfrac{1}{2}[f_b(\text{up}) + f_b(\text{down})] = f_r$$

FM Altimeter

- The FM-CW radar principle is used in the aircraft radio altimeter to measure height above the surface of the earth.

- Relatively short ranges of altimeters permit Low Tx power and low antenna gain.

- Since the relative motion between the aircraft and ground is small, the effect of the Doppler frequency shift may usually be neglected.

- Frequency range: 4.2 to 4.4 GHz (reserved for altimeters)

- Solid state Tx is used here.

- High sensitive super-heterodyne Rx is preferred for better sensitivity and stability
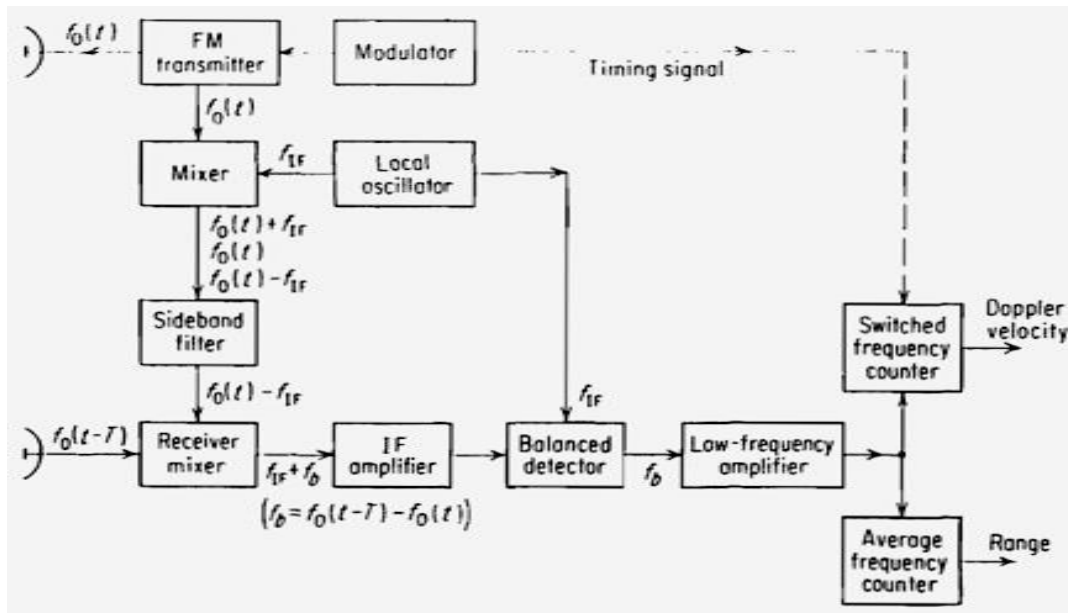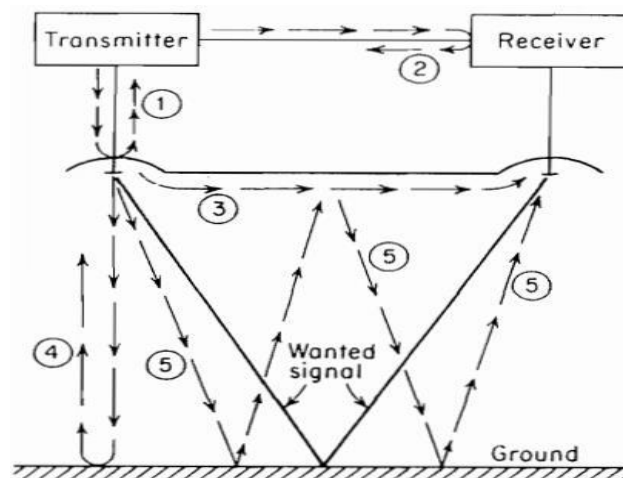
Figure: Block diagram of a FM-CW radar using sideband super heterodyne receiver

- A portion of the frequency-modulated transmitted signal is applied to a mixer along with the oscillator signal.

- The selection of the local-oscillator frequency is a bit different from that in the usual super heterodyne receiver. The local-oscillator frequency f_{IF} is the same as the intermediate frequency used in the receiver.

- The output of the mixer consists of the varying transmitter frequency $f_o(t)$ plus two sideband frequencies, one on either side of $f_o(t)$ and separated from $f_o(t)$ by the local-oscillator frequency f_{IF}.

- The filter selects the lower sideband, $f_o(t) - f_{IF}$ and rejects the carrier and the upper sideband. The sideband filter must have sufficient bandwidth to pass the modulation, but not the carrier or other sideband. The filtered sideband serves the function of the local oscillator.

- When an echo signal is present, the output of the receiver mixer is an IF signal of frequency $(f_{IF}+f_b)$ where f_b is composed of the range frequency f_r and the Doppler velocity frequency f_d.

- The IF signal is amplified and applied to the balanced detector along with the local-oscillator signal f_{IF}.

- The output of the detector contains the beat frequency (range frequency and the Doppler velocity frequency), which is amplified to a level where it can actuate the frequency-measuring circuits.

- The output of the low-frequency amplifier is divided into two channels: one feeds an average-frequency counter to determine the range, and the other feeds a switched frequency counter to determine the Doppler velocity (assuming $f_r > f_d$).

- Only the averaging frequency counter need be used in an altimeter application.

- A target at short range will generally result in a strong signal at low frequency, while one at long range will result in a weak signal at high frequency. Therefore the frequency characteristic of the low frequency amplifier in the FM-CW radar may be used to provide attenuation at the low frequencies corresponding to short ranges and large echo signals. Less attenuation is applied to the higher frequencies, where the echo signals are weaker.

- **Unwanted signals in FM altimeter:**

1. The reflection of the transmitted signals at the antenna caused by impedance mismatch.

2. The standing-wave pattern on the cable feeding the reference signal to the receiver, due to poor mixer match.

3. The leakage signal entering the receiver via coupling between transmitter and receiver antennas. This can limit the ultimate receiver sensitivity, especially at high altitudes.

3. The interference due to power being reflected back to the transmitter, causing a change in the impedance seen by the transmitter. This is usually important only at low altitudes. It can be reduced by an attenuator introduced in the transmission line at low altitude or by a directional coupler or an isolator.

4. The double-bounce signal.

Advantages of FM-CW Radar

- Range can be measured by simple broadening of frequency spectrum.

- FM modulation is easy to generate than linear modulation.

- Synchronization is not required as in multiple frequency CW radar.

- For measuring range, single frequency is required.

Disadvantages of FM-CW Radar

- FM CW radar can be used to detect single targets only.

- Accuracy of FM CW radar is less compared to Multiple frequency radar.

- Measurement of range is more difficult when FM signal is non uniform or mixer is not operating in linear region.

Multiple-frequency CW Radar

- Consider a CW radar radiating a single-frequency sine wave of the form $\sin 2\pi f_o t$

- The signal travels to the target at a range **R** and returns to the radar after a time **T = 2R/c** where **c** is the velocity of propagation.

- The echo signal received at the radar is $sin\ [2\pi f_o(t-T)]$.

- If the transmitted and received signals are compared in a phase detector, the output **is** proportional to the phase difference between the two and is given by :

$$\Delta\varphi = 2\pi f_0 T = 4\pi f_o R/c.$$

- The phase difference may therefore be used as a measure of the range, or

$$R = \frac{c\ \Delta\phi}{4\pi f_0} = \frac{\lambda}{4\pi}\Delta\phi$$

- However, the measurement of the phase difference $\Delta\varphi$ is unambiguous only if $\Delta\varphi$ does not exceed 2π radians. Substituting $\Delta\varphi = 2\pi$ into above eqn gives the maximum unambiguous range as $\lambda/2.$

- Unambiguous range may be extended considerably by utilizing two separate CW signals differing slightly in frequency.

- The transmitted waveform is assumed to consist of two continuous sine waves of frequency f_1 and f_2 separated by an amount Δf.

- For convenience, the amplitudes of all signals are set equal to unity.

- The voltage waveforms of the two components of the transmitted signal v_{1T} and v_{2T} may be written as:

$$v_{1T} = \sin\left(2\pi f_1 t + \phi_1\right)$$
$$v_{2T} = \sin\left(2\pi f_2 t + \phi_2\right)$$

where Ø1 and Ø2 are arbitrary (constant) phase angles.

- The echo signal is shifted in frequency by the Doppler effect. The form of the Doppler-shifted signals corresponding to the two frequencies f_1 and f_2 are:

$$v_{1R} = \sin\left[2\pi(f_1 \pm f_{d1})t - \frac{4\pi f_1 R_0}{c} + \phi_1\right]$$

$$v_{2R} = \sin\left[2\pi(f_2 \pm f_{d2})t - \frac{4\pi f_2 R_0}{c} + \phi_2\right]$$

Where R_0 = range to target at a particular time $t = t_o$

f_{d1} = Doppler frequency shift associated with frequency f_1

f_{d2} = Doppler frequency shift associated with frequency f_2

- Since the two RF frequencies f_1 and f_2 are approximately the same (that is $f_2 = f_1 + \Delta f$, where $\Delta f \lll f_1$) the Doppler frequency shifts f_{d1} and f_{d2} can be assumed to be equal to each other.

- Therefore we may write $f_{d1} = f_{d2} = f_d$

- The receiver separates the two components of the echo signal and heterodynes each received signal component with the corresponding transmitted waveform and extracts the two Doppler-frequency components given below:

$$v_{1D} = \sin\left(\pm 2\pi f_d t - \frac{4\pi f_1 R_0}{c}\right)$$

$$v_{2D} = \sin\left(\pm 2\pi f_d t - \frac{4\pi f_2 R_0}{c}\right)$$

- The phase difference between these two components is

$$\Delta\phi = \frac{4\pi(f_2 - f_1)R_0}{c} = \frac{4\pi \, \Delta f R_0}{c}$$

Hence

$$R_0 = \frac{c \, \Delta\phi}{4\pi \, \Delta f}$$

- which is the same as that of R in 37th slide with Δf substituted in place of f_0.
- The two-frequency CW technique for measuring range was described as using the Doppler frequency shift.
- A large difference in frequency between the two transmitted signals improves the accuracy of the range measurement since large Δf means a proportionately large change in $\Delta\emptyset$ for a given range.
- However, there is a limit to the value of Δf since $\Delta\emptyset$ cannot be greater than 2π radians if the range is to remain unambiguous.
- The maximum unambiguous range R_{unamb} is:

$$R_{unamb} = \frac{c}{2 \, \Delta f}$$

- Therefore Δf must be less than $c/2R_{unamb}$. Note that when Δf is replaced by the pulse repetition rate, above eq gives the maximum unambiguous range of a pulse radar.
- The two-frequency CW radar is essentially a single-target radar since only one phase difference can be measured at a time. If more than one target is present, the echo signal becomes complicated and the meaning of the phase measurement becomes doubtful.
- The theoretical accuracy with which range can be measured with the two-frequency CW radar can be found and it can be shown that the theoretical rms range error is

$$\delta R = \frac{c}{4\pi \, \Delta f \, (2E/N_0)^{1/2}}$$

- Where E = energy contained in received signal and
- No = noise power per hertz of bandwidth.

The above Equation indicates that the greater the separation Δf between the two frequencies, the lesser will be the **rms** error.

- Therefore the frequency difference must not be too large if unambiguous measurements are to be made.

- The selection of **Δf** represents a compromise between the requirements of accuracy and ambiguity.

- Both accurate and unambiguous range measurements can be made by transmitting three or more frequencies instead of just two.

- For example, if the three frequencies f_1, f_2 and f_3 are such that $f_3 - f_1 = k(f_2 - f_1)$ where **k** is a factor of the order of 10 or 20, the pair of frequencies f_3, f_1 (with greater Δf) gives an ambiguous but accurate range measurernent while the pair of frequencies f_2, f_1(with lesser Δf) resolve the ambiguities in the measurement of Range.

- Likewise, if further accuracy is required a fourth frequency can be transmitted and its ambiguities resolved by the less accurate but higher unambiguous measurement obtained from the three frequencies, f1, f2 and f3.

- As more frequencies are added the spectrum and target resolution approach that obtained with a pulse or an **FM-CW** waveform.

- The multiple-frequency CW radar technique has been applied to the accurate measurement of distance in surveying and in missile guidance. The ***Tellurometer is the name given to a*** portable electronic surveying instrument which is based on this principle.

- In addition to its use in surveying, the multiple CW frequency method of measuring range has been applied in

   1. Range-instrumentation radar for the measurement of the distance to a transponder-equipped missile,

   2. The distance to satellites,

   3. In satellite navigation systems based on range measurement

   4. For detecting the presence of an obstacle in the path of a moving automobile by measuring the distance,

   5. The Doppler velocity, and the sign of the Doppler (whether the target is approaching or receding).

- **Example1: Determine the Range and Doppler velocity of an approaching target using a triangular modulation FMCW Radar. Given : Beat frequency fb(up) = 15KHz and fb (down) = 25KHz , modulating frequency : 1MHz, Δf : 1KHz and Operating frequency : 3Ghz**

Solution:

We know fr = ½[fb(up)+ fb (down)] = ½(15+25) = 20 Khz

fd = ½[fb (down) - fb(up)] = ½(25-15) = 5 Khz

The Range R in terms of fr , fm and Δf is given by : R = c fr / 4fm.Δf = (3x10^8)20x10^3 / 4(1x10^6x1x10^3) mtrs = 1500 mtrs = 1.5 Kms
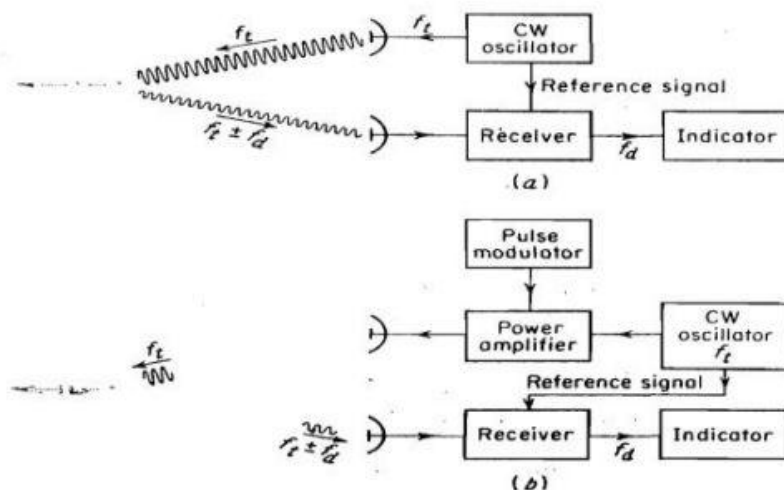
MTI AND PULSE DOPPLER RADAR

Introduction

- The Doppler frequency shift $[f_d = 2V_r / \lambda]$ produced by a moving target also used in a pulse radar just as in the CW radar, to determine the relative velocity of a target or to separate desired moving targets from undesired stationary objects (clutter).

- Pulse radar that utilizes the Doppler frequency shift as a means of discriminating moving targets from fixed targets is called a **MTI** (moving target indication) or a **pulse Doppler** radar.

- The two are based on the same physical principle, but in practice there are differences between **MTI** and **Pulse Doppler** radar.

- The MTI radar, usually operates with ambiguous Doppler measurement (so-called **blind speeds**) but with unambiguous range measurement (no second-time around echoes).

- A pulse Doppler radar operates with ambiguous range measurement but with unambiguous Doppler measurement. Its pulse repetition frequency is usually high enough to operate with unambiguous Doppler (no Blind speeds) but at the expense of range ambiguities.

- The discussion in this chapter, mostly is based on the **MTI Radar**, but much of what applies to MTI can be extended to **Pulse Doppler Radar** as well.

- **MTI** is a necessity in high-quality air-surveillance radars that operate in the presence of clutter.

- Its design is more challenging than that of a simple pulse radar or a simple CW radar.

- A **MTI** capability adds to a radar's cost and complexity.

- The basic MTI concepts were introduced during World War 2, and most of the signal processing theory on which **MTI** (and **pulse Doppler**) radar depends was formulated during the mid-1950s.

- However, the implementation of theory to practice was speeded up only subsequently after the availability of the necessary signal-processing technology.

- It took almost twenty years for the full capabilities offered by MTI signal-processing theory to be converted into practical and economical Radar equipment.

- The chief factor that made this possible was the development of reliable, small, and inexpensive digital processing hardware.

Principle of Operation

- **A** simple CW radar studied earlier is shown in Fig.1(a). It consists of a transmitter, receiver, indicator, and the necessary antennas.

- In principle, the CW radar converted into a pulse radar as shown in Fig.1(b) by providing a power amplifier and a modulator to turn the amplifier on and off for the purpose of generating pulses.

- The chief difference between the pulse radar of Fig. 1(b) and the one studied earlier is that a small portion of the CW oscillator power that generates the transmitted pulses is diverted to the receiver to take the place of the local oscillator.

- this CW signal also acts as the **coherent reference** needed to detect the Doppler frequency shift.

- By **coherent** it means that the phase of the transmitted signal is preserved in the reference signal.

- The reference signal is the distinguishing feature of **coherent MTI radar**.

Figure 1: (a) Simple CW Radar (b) Pulse Radar using Doppler Information

If the CW oscillator voltage is represented as $A_1 \sin 2\pi f_t t$ where A_1 is the amplitude and f_t the carrier frequency

Then the reference signal is: $\qquad V_{ref} = A_2 \sin 2\pi f_t t \qquad \qquad \qquad$ ....(1)

- And the Doppler-shifted echo-signal voltage is:

$$V_{echo} = A_3 \sin \left[2\pi(f_t \pm f_d)t - \frac{4\pi f_t R_0}{c} \right] \qquad \qquad ....(2)$$

Where $\quad A_2 \;=$ amplitude of reference signal

$\qquad \quad A_3 \;=$ amplitude of signal received from a target at a range R_0

$\qquad \quad f_d \;=$ Doppler frequency shift

$\qquad \quad t \;=$ time

$\qquad \quad c \;=$ velocity of propagation

The reference signal and the target echo signal are heterodyned in the mixer stage of the receiver. Only the low-frequency (difference-frequency) component from the mixer is of interest and is a voltage given by:

$$V_{diff} = A_4 \sin \left(2\pi f_d t - \frac{4\pi f_t R_0}{c} \right) \qquad \qquad ....(3)$$

- Note that the equations (1) to (3) above represent sine wave carriers upon which the pulse modulation is imposed.

- For stationary targets the Doppler frequency shift will be zero and hence V_{diff} will not vary with time and may take on any constant value from $+A_4$ to $-A_4$ including zero.

- However, when the target is in motion relative to the radar f_d has a value other than zero and the voltage corresponding to the difference frequency from the mixer [Eq. (3)] will be a function of time.

- An example of the output from the mixer when the Doppler frequency f_d is large compared with the reciprocal of the pulse width is shown in Fig.2(b).

- If, on the other hand f_d is small compared with the reciprocal of the pulse duration, the pulses will be modulated with an amplitude given by Eq. (3) [Fig. 2(c)] and many pulses will be needed to extract the Doppler information.

- The case illustrated in Fig. 2(c) is more typical of aircraft-detection radar, while the waveform of Fig. 2(b) might be more applicable to a radar used for the detection of extraterrestrial targets such as ballistic missiles or satellites.

- The video signals shown in Fig.2 are called **bipolar**, since they contain both positive and negative amplitudes.
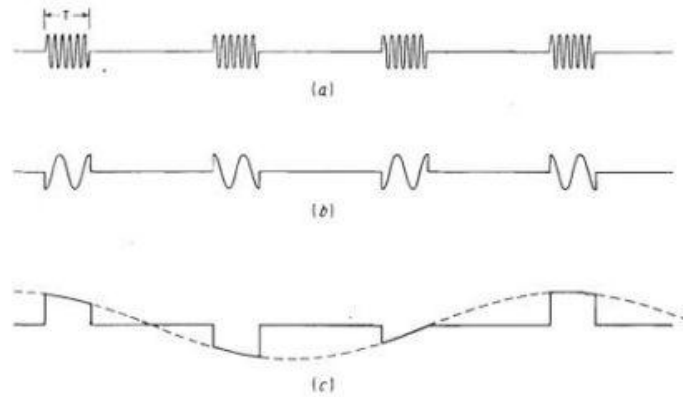

Figure 2 (a) RF or IFecho pulse train (b) video pulse train for Doppler frequency $f_d > 1/\tau$ (c) video pulse train for Doppler frequency $f_d < 1/\tau$.

- Moving targets may be distinguished from stationary targets by observing the video output on an A-scope (amplitude vs. range).

- A single sweep on an A-scope might appear as in Fig. 3 (a) shown below. This sweep shows several fixed targets and two moving targets indicated by the two arrows.

- On the basis of a single sweep, moving targets cannot be distinguished from fixed targets.

- Successive A-scope sweeps (pulse-repetition intervals) are shown in Fig. 3 (a) to (e).

- Echoes from fixed targets remain constant throughout, but echoes from moving targets vary in amplitude from sweep to sweep at a rate corresponding to the Doppler frequency.

- The superposition of the successive A-scope sweeps is shown in Fig. 3(f). The moving targets produce, with time, a" butterfly" effect on the A-scope.
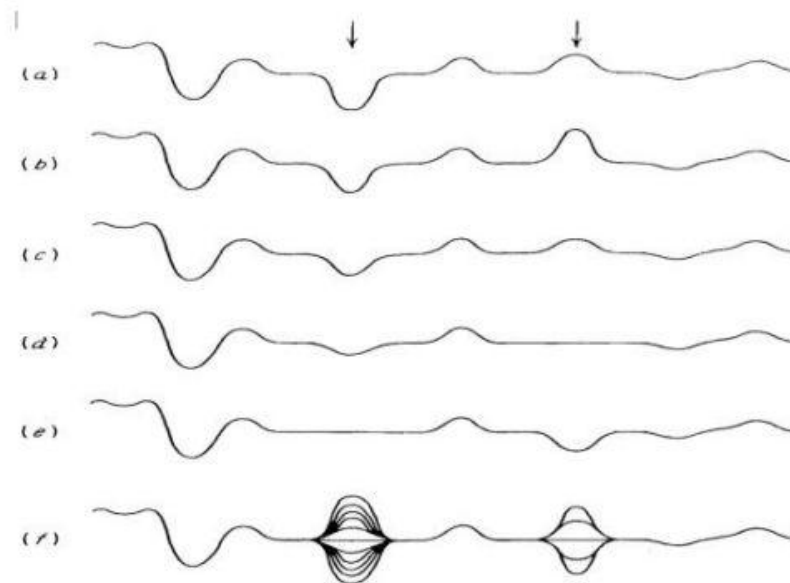
Figure 3 (a-e) Successive sweeps of a MTI radar A-scope display (echo amplitude as a function of time) (f) superposition of many sweeps : arrows indicate position of moving targets.

Delay-line canceler

- Although the butterfly effect is suitable for recognizing moving targets on an A-scope, it is not appropriate for display on the PPI. One method commonly employed to extract Doppler information in a form suitable for display on the PPI scope is with a delay-line canceler.

- The delay-line canceler acts as a filter to eliminate the d-c component of fixed targets and to pass the a-c components of moving targets.

- The video portion of the receiver is divided into two channels. One is a normal video channel. In the other, the video signal experiences a time delay equal to one pulse-repetition period (equal to the reciprocal of the pulse repetition frequency).

- The outputs from the two channels are subtracted from one another.

- The fixed targets with unchanging amplitudes from pulse to pulse are canceled on subtraction. However, the amplitudes of the moving-target echoes are not constant from pulse to pulse and subtraction results in an uncanceled residue.

• The output of the subtraction circuit is a bipolar   video just as was the input.

• Before bipolar video can intensity-modulate a PPI display it must be converted into unipotential voltages (unipolar video) by a full-wave rectifier.
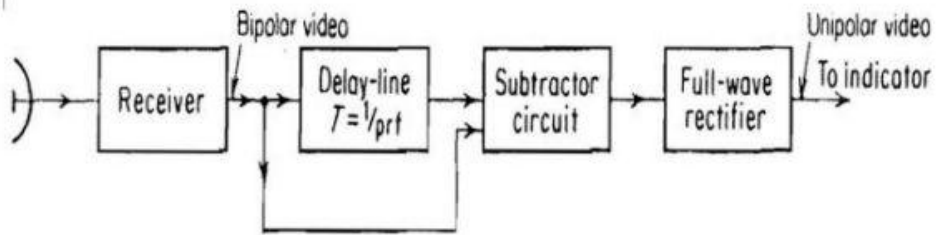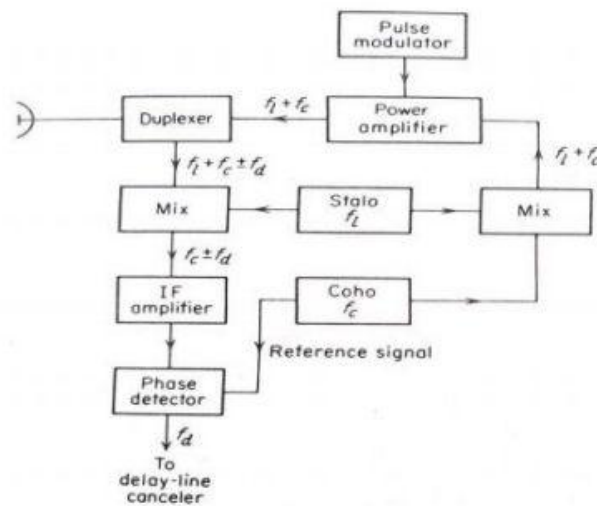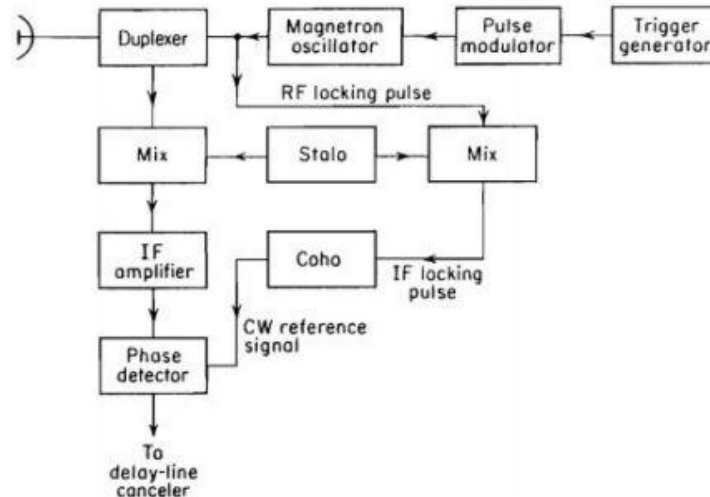

Figure 4 : MTI Receiver with delay-line canceler

MTI Radar with Power Amplifier

- The simple MTI radar shown in Fig. 1(b) is not the most typical. The block diagram of a more common MTI radar employing a power amplifier is shown above.

- The significant difference between this MTI configuration and that of **Fig. 1(b)** is the manner in which the reference signal is generated.

- In this diagram, the coherent reference is supplied by an oscillator called the **coho**, which stands for coherent oscillator.

- The **coho** is a stable oscillator whose frequency is the same as the intermediate frequency used in the receiver.

- In addition to providing the reference signal, the output of the **coho** is also mixed with the local-oscillator frequency f_1. This local oscillator also must be a stable oscillator and is called **stalo**, stands for stable local oscillator.

- The RF echo signal is heterodyned with the stalo signal to produce the IF just as in the conventional super heterodyne receiver.

- The **stalo**, **coho** and the mixer in which they are mixed are called Receiver-Exciter because of the dual role they serve both the receiver and the transmitter.

- The phase of the **stalo** influences the phase of the transmitted signal, any **stalo** phase shift is canceled on reception because the **stalo** that generates the transmitted signal also acts as the local oscillator in the receiver.

- The reference signal from the **coho** and the IF echo signal are both fed into a mixer called the **Phase detector.**

- The phase detector differs from the normal amplitude detector since its output is proportional to the phase difference between the two input signals.

- Any one of a number of transmitting-tube types might be used as the power amplifier. These include the triode, tetrode, klystron, traveling-wave tube, and the crossed-field amplifier.

- A transmitter which consists of a stable low-power oscillator followed by a power amplifier is sometimes called **MOPA**, which stands for Master-Oscillator Power Amplifier.

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MTI Radar with Power Oscillator Transmitter

- Before the development of the klystron amplifier, the only high-power transmitter available at microwave frequencies for radar application was the magnetron oscillator.

- In an oscillator, the phase of the RF bears no relationship from pulse to pulse. For this reason, the reference signal cannot be generated by a continuously running oscillator.

- However, a coherent reference signal may be readily obtained with the power oscillator by readjusting the phase of the **coho** at the beginning of each sweep according to the phase of the transmitted pulse.

- The phase of the **coho** is locked to the phase of the transmitted pulse each time a pulse is generated.

- A block diagram of an MTI radar (with a power oscillator) is shown in below slide.

- **A** portion of the transmitted signal is mixed with the **stalo** output to produce an **IF** beat signal whose phase is directly related to the phase of the transmitter.

- This IF pulse is applied to the **coho** and causes the phase of the **coho** CW oscillation to "lock" in step with the phase of the IF reference pulse.

- The phase of the **coho** is then related to the phase of the transmitted phase and may be used as the reference signal for echoes received from that particular transmitted pulse.

- Upon the next transmission, another **IF** locking pulse is generated to relock the phase of the CW **coho** until the next locking pulse comes along.

Delay-line canceler

- The simple MTI delay-line canceller shown earlier is an example of a time-domain filter.

- The capability of this device depends on the quality of the medium used as the delay line. The delay line must introduce a time delay equal to the pulse repetition interval.

- For typical ground-based air surveillance radars this will be several milliseconds.

- Delay times of this magnitude cannot be achieved with practical electromagnetic transmission lines.

- By converting the electromagnetic signal to an acoustic signal, it is possible to utilize delay lines of a reasonable physical length since the velocity of propagation of acoustic waves is about 10^{-5} that of electromagnetic waves.

- After the necessary delay is introduced by the acoustic line, the signal is converted back to an electromagnetic signal for further processing.

- The early acoustic delay lines developed during World War 2 used liquid delay lines filled with either water or mercury. Liquid delay lines were large and inconvenient to use. They were replaced in themid-1950s by the solid fused-quartz delay line that used multiple internal reflections to obtain a compact device.

- These analog acoustic delay lines were, in turn replaced in the early 1970s by storage devices based on digital computer technology. The use of digital delay lines requires that the output of the MTI receiver phase-detector be quantized into a sequence of digital words.

- One of the advantages of a time-domain delay-line canceller as compared to the more conventional frequency-domain filter is that a single network operates at all ranges and does not require a separate filter for each range resolution cell.

- Frequency-domain Doppler filter banks are of interest in some forms of MTI and Pulse-Doppler radar.

Filter Characteristics of the Delay Line Canceller

- The delay-line canceler acts as a filter which rejects the d-c component of clutter. Because of its periodic nature, the filter also rejects energy in the vicinity of the pulse repetition frequency and its harmonics.

- The video signal of Eq.(3)received from a particular target at a range R_0 is

$$V_1 = k \sin (2\pi f_d t - \phi_0) \qquad \ldots\ldots\ldots\ldots\ldots\ldots 4$$

where $\varnothing_0$ = phase shift and k = amplitude of video signal.

- The signal from the previous transmission, which is delayed by a time T = pulse repetition interval, is

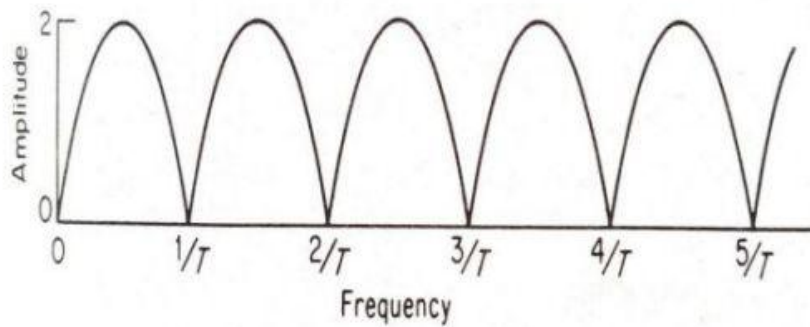$$V_2 = k \sin [2\pi f_d(t - T) - \varnothing_0] \quad \text{---------------- 5}$$

- The output from the subtractor is

$$V = V_1 - V_2 = 2k \sin(\pi f_d T) \cos [2 \pi f_d(t - T/2) - \varnothing o] \quad \text{------------ 6}$$

$$\sin a \pm \sin b = 2 \sin \{1/2 (a \pm b)\} \cos \{1/2 (a \mp b)\}$$

- It is assumed that the gain through the delay-line canceller is unity. The output from the canceller consists of a cosine wave at the Doppler frequency & with an amplitude $2k \sin \pi f_d T$.

- Thus, the amplitude of the canceled video output is a function of the Doppler frequency shift and the pulse-repetition interval, or prf.

- The magnitude of the relative frequency-response of the delay-line canceller [ratio of the amplitude of the output from the delay-line canceller, $2k \sin (\pi f_d T)$ to the amplitude of the normal radar video k] is shown below.

Frequency response of the single delay-line canceller: T = delay time $= 1/f_p$

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Blind speeds

- The response of the single-delay-line canceller will be zero whenever the argument ($\pi f_d T$) in the amplitude factor of **Eq. (6)** is 0, π, 2π, ..., etc., or when

$$f_d = \frac{n}{T} = nf_p \qquad \dots\dots\dots (7)$$

where $n = 0, 1, 2, \ldots,$ and f_p = pulse repetition frequency.

- The delay-line canceller not only eliminates the d-c component caused by clutter ($n = 0$), but unfortunately it also rejects any moving target whose Doppler frequency happens to be the same as the prf or a multiple thereof.

- Those relative target velocities which result in zero MTI response are called **blind speeds** and are given by

$$v_n = \frac{n\lambda}{2T} = \frac{n\lambda f_p}{2} \qquad n = 1, 2, 3, \ldots \qquad \dots\dots\dots (8)$$

- where v_n is the nth blind speed. If λ is measured in meters, f_p in Hz, and the relative velocity in knots, the blind speeds are :

$$v_n = \frac{n\lambda f_p}{1.02} \approx n\lambda f_p \qquad \text{------------}(9)$$

- The blind speeds are one of the limitations of pulse MTI radar which do not occur with CW radar. They are present in pulse radar because Doppler is measured by discrete samples (pulses) at the prf rather than continuously.

- Based on eq **9** there are four methods for reducing the determental effects of blind speeds:

  1. Operate the radar at long wavelengths

  2. Operate with a high pulse repetition frequency

  3. Operate with more that one PRF (Staggered prf's)

  4. Operate with more than one RF frequency

- Unfortunately, there are usually constraints other than blind speeds which determine the wavelength and the pulse repetition frequency. Therefore, blind speeds might not be easy to avoid.

- The possible solution for the blind speed is keep the first blind speed out of the expected range of Doppler frequency

Double cancellation

- The frequency response of a single-delay-line canceller (Fig. **7**) does not always have as broad a clutter-rejection null as might be desired in the vicinity of d-c.

- The clutter-rejection notches may be widened by passing the output of the delay-line canceller through a second delay-line canceller as shown in **Fig. (9) below.**

- The output of the two single-delay line cancellers in cascade is the square of that from a single canceller. Thus the frequency response is ($4 \sin^2 \pi f_d T$).

- The configuration of **Fig. 9 (a)** is called a double-delay-line canceller, or simply a *double canceller.*

- The relative response of the double canceller compared with that of a single-delay-line canceller is shown in Fig. **10.**

- The finite width of the clutter spectrum is also shown (hatched) in this figure so as to illustrate the additional cancellation of clutter offered by the double canceller.

- The two-delay-line configuration of Fig.**9 (b)** has the same frequency-response characteristic as the double-delay-line canceller.

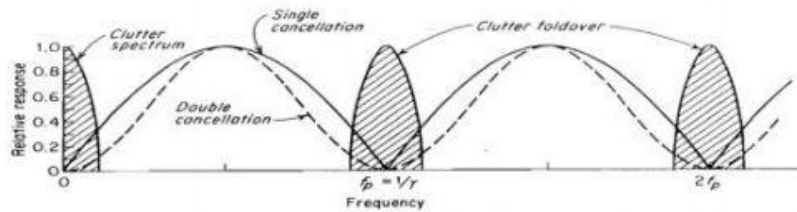**Figure 9 : (a) Double-delay-line canceller (b) three-pulse canceller**
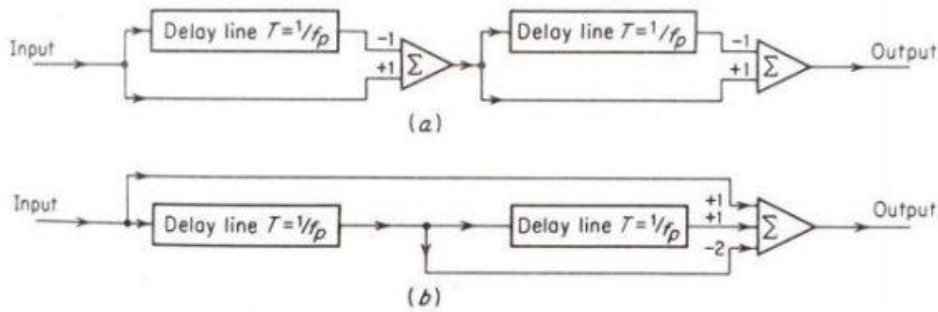

Figure (10): Relative frequency response of the single-delay-line canceller (solid curve) and the double delay-line canceller (dashed curve). Shaded area represents clutter spectrum.

An example of the composite (average) response of an MTI radar operating with two separate pulse repetition frequencies on a time-shared basis is shown below.

- Zero frequency response occurs only when the blind speeds of both radars coincides.
- The disadvantage is that the region of low sensitivity might appear. (that means may not detect the weak signals)
- As closer the ration of T1:T2 is unity, the lower the lower the value of the first blind speed.
- But first null in the vicinity of fd=1/T1 becomes deeper.

- Figure below shows the response of a five-pulse stagger (four periods) that might be used with a long-range air traffic control radar.

Figure 12: Frequency response of a five-pulse (four-period) stagger.

- If the periods of the staggered waveforms have the relationship $n_1/T_1 = n_2/T_2 = \ldots\ldots = n_N/T_N$, where $n_1, n_2, \ldots, n_N$ are integers, and if v_B is equal to the first blind speed of a non-staggered waveform with a constant period equal to the average period $T_{av} = (T_1 + T_2 + \ldots T_N)/N$ then the first blind speed v_1 is given by :

$$\frac{v_1}{v_B} = \frac{n_1 + n_2 + \cdots + n_N}{N}$$
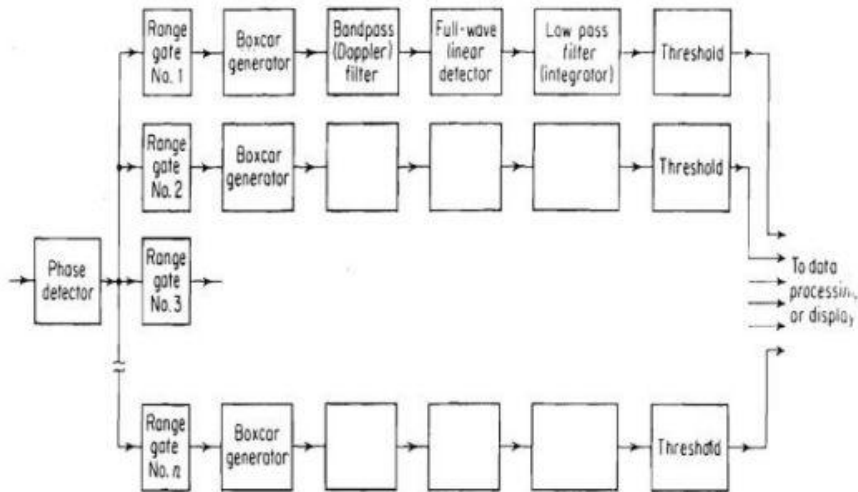
- A disadvantage of the staggered prf is its inability to cancel second-time-around clutter echoes. Such clutter does not appear at the same range from pulse to pulse and thus produces un canceled residue.

- Second-time-around clutter echoes can be removed by use of a constant prf, providing there is pulse-to-pulse coherence as in the power amplifier form of MTI

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Range-gated Doppler filters

- The delay-line canceller, which can be considered as a time-domain filter, is widely used in MTI radar to separate moving targets from stationary clutter.

- It is also possible to employ the more common frequency-domain band pass filters of conventional design in MTI radar to separate the Doppler-frequency-shifted targets.

- However the filter configuration would be more complex, than the single, narrow-band pass filter.

- The narrowband filter "smears" the input pulse since the impulse response is approximately the reciprocal of the filter bandwidth.

- This smearing destroys the range resolution.

- If more than one target is present they cannot be resolved.

- Even if only one target is present, the noise from the other range cells that do not contain the target will interfere with the desired target signal.

- The result is a reduction in sensitivity due to a collapsing loss. (This Loss Results When Radar Integrates Additional Noise Samples Along with Wanted (S/N) Pulses)

- The loss of the range information and the collapsing loss may be eliminated by first quantizing the range (time) into small intervals. This process is called *range gating*.

- The width of the range gates depends upon the range accuracy desired and the complexity which can be tolerated, but they are usually of the order of the pulse width.

- Range resolution is established by gating.

- Once the radar return is quantized into range intervals, the output from each gate may be applied to a narrowband filter.

- A collapsing loss does not take place since noise from the other range intervals is excluded.
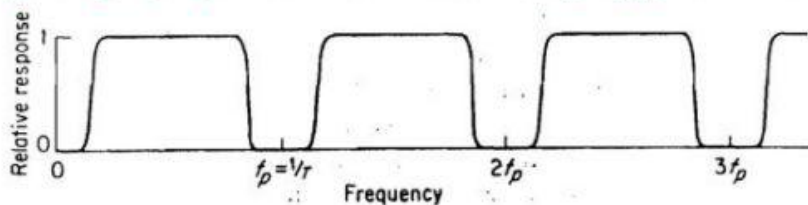
16

Figure 13: Block diagram of MTI radar using range gates and filters

- A block diagram of the video of an MTI radar with multiple range gates followed by clutter-rejection filters is shown below.

- The output of the phase detector is sampled sequentially by the range gates. Each range gate opens in sequence just long enough to sample the voltage of the video waveform corresponding to a different range interval in space low.

- The range gate acts as a switch or a gate which opens and closes at the proper time

- The range gates are activated once each pulse-repetition interval.

- The output for a stationary target is a series of pulses of constant amplitude.

- An echo from a moving target produces a series of pulses which vary in amplitude according to the Doppler frequency.

- The output of the range gates is stretched in a circuit called the *boxcar generator,* or *sample-and-hold* circuit, whose purpose is to aid in the filtering and detection process by emphasizing the fundamental of the modulation frequency and eliminating harmonics of the pulse repetition frequency.

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- The clutter rejection filter is a band pass filter whose bandwidth depends upon the extent of the expected clutter spectrum.

- Following the Doppler filter is a full-wave linear detector and an integrator (a low-pass filter). The purpose of the detector is to convert the bipolar video to unipolar video.

- **The** output of the integrator is applied to a threshold-detection circuit. Only those signals which cross the threshold are reported as targets.

- Following the threshold detector, the outputs from each of the range channels must be properly combined for display on the PPI or A-scope or for any other appropriate indicating or data-processing device.

- The CRT display from this type of MTI radar appears "cleaner" than the display from a normal MTI radar, not only because of better clutter rejection, but also because the threshold device eliminates many of the unwanted false alarms due to noise.

Figure Frequency-response characteristic of an MTI using range gates and filters.

The band pass filter can be designed with a variable low-frequency cutoff that can be selected to conform to the prevailing clutter conditions.

A variable lower cutoff might be advantageous when the width of the clutter spectrum changes with time as when the radar receives unwanted echoes from birds.

A relatively wide notch at zero frequency is needed to remove moving birds.

If the notch were set wide enough to remove the birds, it might be wider than necessary for ordinary clutter and desired targets might be removed.

Since the appearance of birds varies with the time of day and the season, it is important that the width of the notch be controlled according to the local conditions.

- MTI radar using range gates and filters is usually more complex than an MTI with a single-delay-line canceller.

- The additional complexity is justified in those applications where good MTI performance and the flexibility of the range gates and filter MTI are desired.

- The better MTI performance results from the better match between the clutter filter characteristic and the clutter spectrum.

Limitations to MTI Performance

The improvement in signal-to-clutter ratio of an MTI is affected by factors other than the design of the Doppler signal processor such as:

- Instabilities of the transmitter and receiver

- physical motions of the clutter

- Finite time on target (or scanning modulation)

- And limiting in the receiver

- Before discussing these limitations, we shall study the *related definitions*

Definitions related to MTI Performance

MTI improvement factor: The signal-to-clutter ratio at the output of the MTI system divided by the signal-to-clutter ratio at the input, averaged uniformly over all target radial velocities of interest.

With respect to Doppler frequency, the Improvement factor can be expressed as:

$$I_f = (S/C)_{out} * (S/C)_{in} = \frac{C_{in}}{C_{out}} G_{av}$$

C_{in} = strength of clutter at clutter filter input
C_{out} = strength of clutter at clutter filter output
G_{av} = average filter gain for moving targets

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Subclutter Visibility (SCV):

It describes the radar's ability to detect non-stationary targets embedded in a strong clutter background with a given signal-to-clutter ratio (SCR), for some specified probabilities of detection and false alarm. It is often used as a measure of the effectiveness of moving-target indicator radar, equal to the ratio of the signal from a fixed target that can be canceled to the signal from a just visible moving target.

The subclutter visibility is expressed as the ratio of the improvement factor to the minimum MTI output given signal-to-clutter ratio (SCR) required for proper detection for a given probability of detection.

$$SCV = \frac{I_f}{(SCR)_{out}}$$

SCV = Sub-Clutter Visibility SCR = Signal-to-Clutter Ratio

Interclutter visibility(ICV)

If a radar system can resolve the areas of strong and weak clutter within its field of view, then the phrase Interclutter visibility (ICV) describes the ability to recognize the moving target which occur in clear resolution cells between strong clutter patches.

Clutter visibility factor: The signal-to-clutter ratio, after cancellation or Doppler filtering that provides stated probabilities of detection and false alarm.

Clutter attenuation: The ratio of clutter power at the canceller input to the clutter residue at the output, normalized to the attenuation of a single pulse passing through the unprocessed channel of the canceller. (The clutter residue is the clutter power remaining at the output of a MTI system.)

Cancellation ratio: The ratio of canceller voltage amplification for the fixed-target echoes received with a fixed antenna, to the gain for a single pulse passing through the unprocessed channel of the canceller.

Equipment instabilities :

- Pulse-to-pulse changes in the amplitude, frequency, or phase of the transmitter signal, changes in the stalo or coho oscillators in the receiver, jitter in the timing of the pulse transmission, variations in the time delay through the delay lines, and changes in the pulse width can cause the apparent frequency spectrum from perfectly stationary clutter to broaden and thereby lower the improvement factor of an MTI radar.

- The stability of the equipment in MTI radar must be considerably better than that of an ordinary radar. It can limit the performance of MTI radar if sufficient care is not taken in design, construction, and maintenance.

Internal fluctuation of clutter :

- Although clutter targets such as buildings, water towers, bare hills. or mountains produce echo signals that are constant in both phase and amplitude as a function of time, there are many types of clutter that cannot be considered as absolutely stationary.

- Echoes from trees, vegetation, sea, rain fluctuate with time, and these fluctuations can limit the performance of MTI radar.

Antenna scanning modulation

- As the antenna scans by a target, it observes the target for a finite time equal to : $t_o = n_B / f_P = \theta_B / \theta_S$ where n_B = number of hits received, fp = pulse repetition frequency, θ_B, θ_S = antenna beamwidth and antenna scanning rate.

- The received pulse train of finite duration t_o has a frequency spectrum (which can be found by taking the Fourier transform of the waveform) whose width is proportional to l/to.

- Therefore, even if the clutter were perfectly stationary, there will still be a finite width to the clutter spectrum because of the finite time on target.

- If the clutter spectrum is too wide because the observation time is too short, it will affect the improvement factor. This limitation has sometimes been called scanning fluctuations or scanning modulation.

Limiting in MTI Radar:

- A limiter is usually employed in the IF amplifier just before the MTI processor to prevent the residue from large clutter echoes from saturating the display. Ideally a MTI radar should reduce the clutter to a level comparable to receiver noise.

- However, when the *MTI improvement factor* is not great enough to reduce the clutter sufficiently, the clutter residue will appear on the display and prevent the detection of aircraft targets whose cross sections are larger than the clutter residue. This condition may be prevented by setting the limit level L, relative to the noise N, equal to the MTI improvement factor I; or L/N = I.

- If the limit level relative to noise is set higher than the improvement factor. Clutter residue obscures part of the display. If it is set too low, there may be a "black hole" effect on the display. The limiter provides a constant false alarm rate (CFAR) and is essential to usable MTI Performance.

- Unfortunately, nonlinear devices such as limiters have side-effects that can degrade performance. Limiters cause the spectrum of strong clutter to spread into the canceller pass- band, and result in the generation of additional residue that can significantly degrade MTI performance as compared with a perfect linear system.

Benefits or advantages of MTI Radar

- MTI radar can distinguish between moving target and stationary target.
- It uses low PRF (Pulse Repetition Frequency) to avoid range ambiguities.
- MTI principle is used in air surveillance radar which operates in presence of clutter.
- It is simpler compare to pulse doppler radar.
- Antenna bandwidth is high.
- It is economical.
- It does not require waveforms with multiple PRF.
- It is preferred at UHF frequencies.

Drawbacks or disadvantages of MTI Radar

- Blind speed does not get detected by pulse MTI radar. Blind speed is defined as magnitude of radial component of velocity of target when moving target appears as stationary target.
- They can have doppler ambiguities.

MTI VS PULSE DOPPLER RADAR

- A Pulse radar that extracts the Doppler frequency shift for the purpose of detecting moving targets in the presence of clutter is either a **MTI Radar** or a **Pulse Doppler Radar**.

- The distinction between them is based on the fact that in a sampled measurement system like a pulse Radar, ambiguities arise in measuring both the Doppler frequency (relative velocity) and the Range (time delay).

- Range ambiguities are avoided with a **low** sampling rate (low pulse repetition frequency), and Doppler frequency ambiguities are avoided with a **high** sampling rate.

- However, in most radar applications the sampling rate, or pulse repetition frequency, cannot be selected to avoid both types of measurement ambiguities.

- Therefore, a compromise must be made and the nature of the compromise generally determines whether the radar is called an **MTI** or a **Pulse Doppler Radar**.

- MTI usually refers to a Radar in which the pulse repetition frequency is chosen low enough to avoid ambiguities in range (no multiple-time-around echoes) but with the consequence that the frequency measurement is ambiguous and results in blind speeds.

- The pulse Doppler radar, on the other hand, has a high pulse repetition frequency that avoids blind speeds, but it experiences ambiguities in range.

- The pulse Doppler radar is more likely to use range-gated Doppler filter-banks than delay-line cancellers. Also, a power amplifier such as a klystron is more likely to be used than a power oscillator like the magnetron.

- A pulse Doppler radar operates at a higher duty cycle than does an MTI.

- Although it is difficult to generalize, the MTI radar seems to be the more widely used of the two, but pulse Doppler radar is usually more capable of reducing clutter.

TRACKING RADAR

Tracking with Radar

Sequential Lobing

Conical Scan

Monopulse Tracking Radar

Amplitude Comparison Monopulse (One and two coordinate)

Phase Comparison Monopulse

Tracking in Range

Acquisition and Scanning Patterns

Comparison of Trackers

Tracking With Radar

- Tracking is the process of continuously maintaining the antenna beam on the target and also the echo signal within the range gate.

- The radar which detects target and determines location as well as predict its trajectory path as well as its future coordinates is known as tracking radar.

- Based on the measured coordinates error signal will be generated.

- Antenna should be moved based on error signal to maintain the target within the beam.

- Use pencil beam

- The figure below mentions block diagram of simple **tracking radar**.

- As shown tracking operation in the radar depends upon angular information. very narrow antenna beam is used here which will track one target object at one time. This can be performed using range gating and doppler filtering module.

- Range tracking is carried out using timing control unit. Doppler tracking is carried out using Doppler gating unit.

Simple Tracking Radar
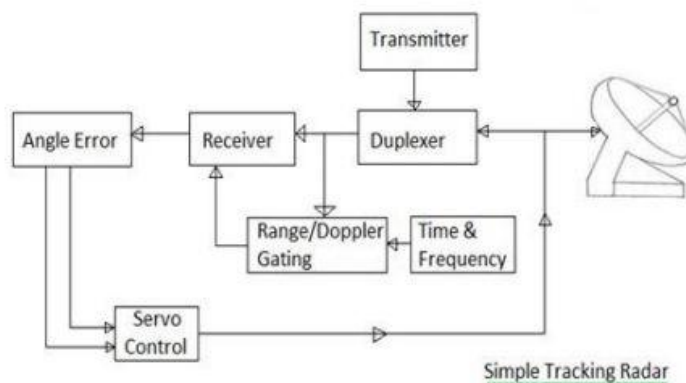
- The angle error signal is provided as input for servo motor based control system. This servo system will steer the antenna as per error input and hence will track the target.

- The various methods for generating the error signal are classified as sequential lobing, conical scan, and simultaneous lobing or Monopulse.

- The data available from a tracking radar may be presented on a cathode-ray-tube (CRT) display

Tracking Radar Types

Following are the types of tracking radar:

- STT Radar (Single Target Tracking Radar)
- ADT Radar (Automatic Detection and Tracking Radar)
- TWS Radar (Track While Scan Radar)
- Phased Array Tracking Radar
- Monopulse Tracking Radar

STT Radar (Single Target Tracking Radar)

- Tracks a single target at fast data
- High Data rate – 10 obs/sec.
- Employs a closed loop servo system to keep the error signal small.
- Application – tracking of aircraft/missile targets

ADT Radar (Automatic Detection and Tracking Radar)

- This Tracking is preferred in air surveillance tracking radar.
- Lower data rate than STT.
- Can track hundreds/ a few thousand targets simultaneously.
- Tracking is open loop i.e antenna position is not controlled by data processing.

TWS Radar (Track While Scan Radar)

- This rapidly scans a limited angular sector to maintain tracks with a moderate data rate on more than one target within the coverage of antenna (another name for ADT).
- Scans a limited angular sector to maintain tracks – **simultaneous track & search**
- Data rate : moderate
- Can track a number of targets.
- Equivalent of track while scan is **ADT**
- TWS systems are used for air-defense radars, air craft landing radars and in air borne intercept radars to track multiple objects

Phased Array Tracking Radar

- A large number of targets can be held in track

- This is done on time sharing basis

- Beam is electronically switched from one angular position to another in a few microseconds.

- It combines the rapid update rate of a single target tracker with the ability of ADT to hold many targets in track

- High data rate (like in STT)

- The cost is very very high.

- These radars are used in air defense weapon systems.

Angle Tracking

- When a target is approaching, the antenna is to be moved continuously to track the target
- To determine the direction in which the antenna beam needs to be moved, a measurement has to be made at two different beam positions.
- Below figure shows the basic principle of continuous angle tracking
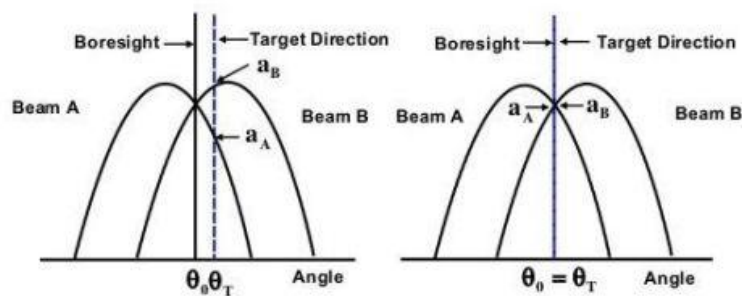
- Two overlapping antenna patterns that crossover at the boresight direction θ_0

- A target is located in this example to the right of the boresight at an angle θ_T

- The amplitude a_B of the target echo in beam B is larger than the amplitude a_A in the beam A

- Which indicates that the two beams should moved to the right to bring the target to the boresight position.

- If you want to track the target the boresight is always maintained in the direction of the target.

Methods to extract error signal may be classified as

- Sequential lobing
- Conical scan
  1. Time shares a single beam
  2. Antenna beam is switched between two positions
- Simultaneous lobing or monopulse

  1. More than one simultaneous beam is used for tracking
  2. Usually 4 simultaneous beams used for 2-dimensional tracking

| Single beam on time sharing basis.	Multiple beam.
Sequential lobing Radar and Conical scan Radar	Simultaneous lobing or monopulse Radar]
Simpler	Complex
One antenna	Multiple antennas
Less equipment	More equipments
Not accurate	Accurate
No of pulses are required to extract the error signal	Single pulse is used to determine the angular error.
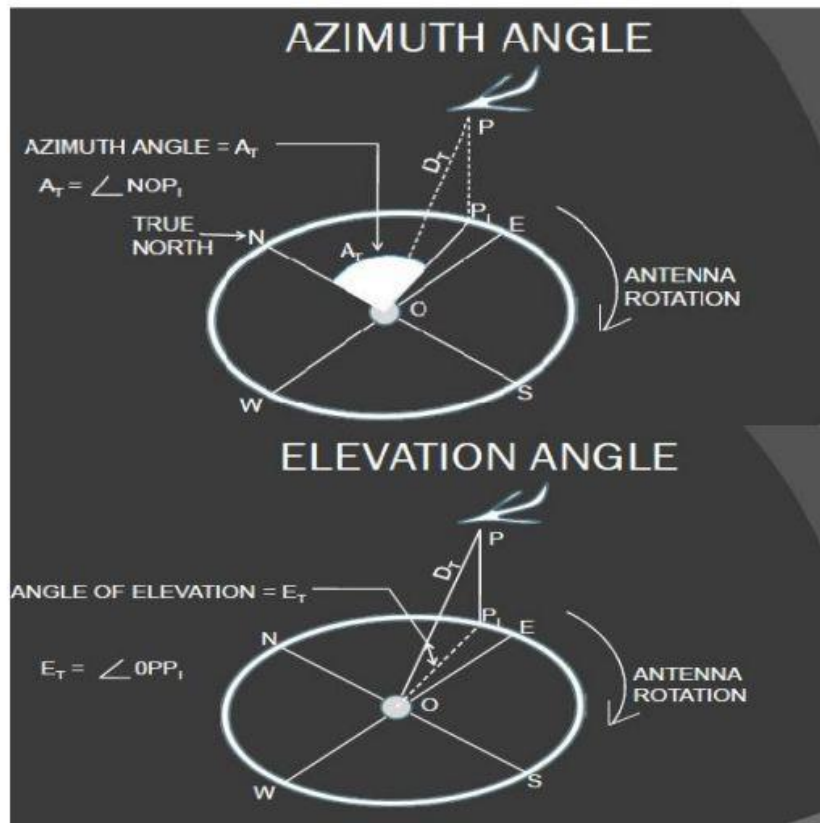
Sequential Lobing

- The antenna pattern commonly employed with tracking radars is the symmetrical pencil beam in which the, elevation and azimuth beam widths are approximately equal.

- Actually the difference between the target position and the reference direction is the angular error.

- The tracking radar attempts to position the antenna to make the angular error zero. When the angular error is zero, the target is located along the reference direction.

- One method of obtaining the direction and the magnitude of the angular error in one coordinate is switching the single antenna beam between two squinted angular positions. This is called as lobe switching, sequential switching or sequential lobing.

- The error signal is obtained from a target not on the switching axis.

- The direction in which to move the beam to bring the target on boresight is found by observing which beam position has the larger signal.

Fig 1-a is a polar representation of the antenna beam (minus the side lobes) in the two switched positions. A plot in rectangular coordinates is shown in **Fig.1-b,** and the error signal obtained from a target not on the switching axis (reference direction) is shown in **Fig.1-c.**
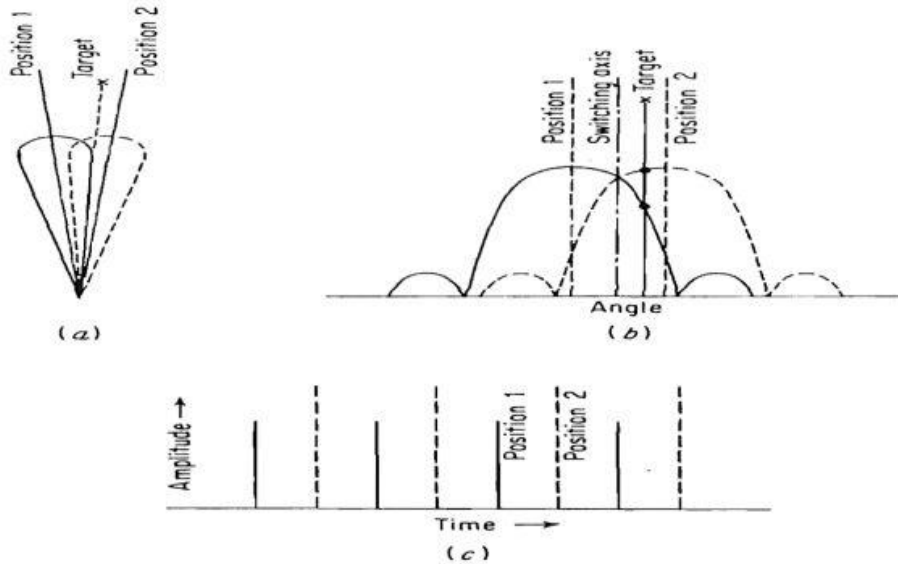
- When the echo signals in the two beam positions are equal, the target is n axis and its direction is that of the switching axis.

- If orthogonal angle information is needed, two more switching positions are needed.

- So, two dimensional sequentially lobing radar might consists of four feed horns illuminating a single reflector antenna.

- An improvement over this can be a single squinted feed which could be rotated continuously to obtain angle measurements in two coordinates . This results in conical scan.

- One of the limitations of a simple unswitched non-scanning pencil-beam antenna is that the angle accuracy can be no better than the size of the antenna beam width.

- An important feature of sequential lobing (as well as the other tracking techniques to be discussed) is that the target-position accuracy can be far better than that given by the antenna beam width.

Conical Scan
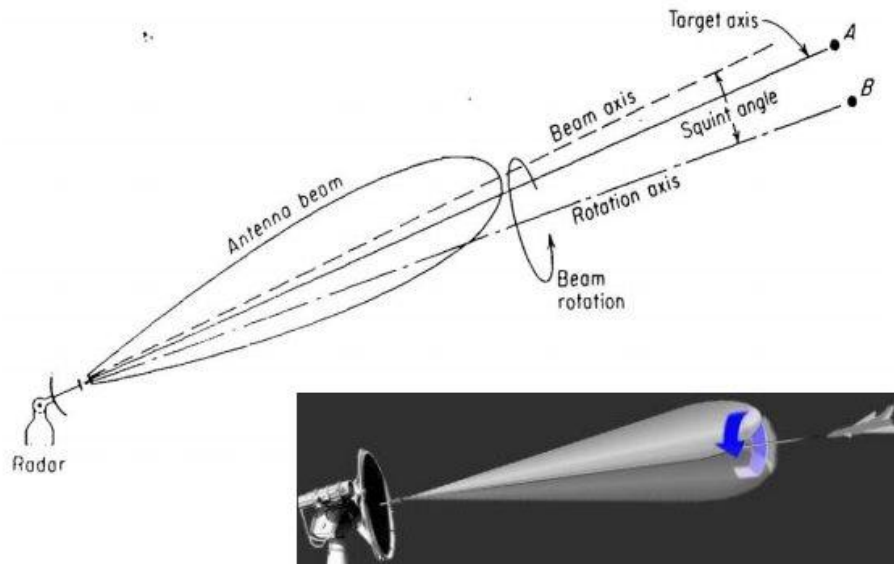
- The angle between the axis of rotation and the axis of the antenna beam is called the Squint Angle.

- Consider a target at position A.

- The echo signal amplitude will be modulated at a frequency equal to the rotation frequency of the beam.

- The amplitude of the echo signal modulation will depend upon the shape of the antenna pattern, the squint angle and the angle between the target line of sight & the rotation axis. This amplitude of the echo signal will be modulated ata frequency equal to the beam rotation frequency (conical Scan frequency).

- The phase of the modulation depends on the angle between the target and the rotation axis.

- The conical scan modulation is extracted from the echo signal and applied to a servo-control system which continually positions the antenna rotation axis in the direction of the target. [Note that two servos are required because the tracking is required in two-dimensions.

- When the antenna is on target, as in B of **Fig. 2,** the line of sight to the target and the rotation axis coincide, and the conical-scan modulation is zero.

- Two servo motors are required, one for azimuth and the other for elevation

Block diagram of conical scan tracking radar
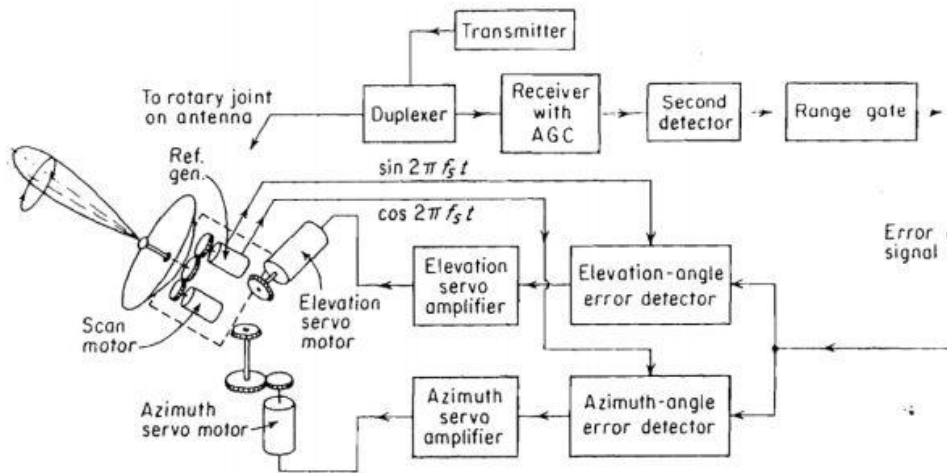
- The antenna is mounted so that it can be positioned in both azimuth and elevation by separate m

- Redirection of beam i) Rotating feed ii) Nutating feed.

- When the feed is designed to maintain the plane of polarization as it rotates about the axis, it is called ' nutating feed'.

- A rotating feed is one which causes the plane of polarization to rotate.

- The nutating feed is preferred over the rotating feed since a rotating polarization can cause the amplitude of the target echo signal to change with time even for a stationary target on axis.

- A change in amplitude caused by a modulated echo signal can result in degraded angle tracking accuracy.

- The nutating feed is more complicated than the rotating feed.

- A typical conical scan rotation speed might be in the vicinity of 30 rev/sec.

- The same motor that provides the conical-scan rotation of the antenna beam also drives a two-phase reference generator with two sinusoidal outputs **90⁰** apart in phase.

- These two outputs serve as a reference to extract the elevation and azimuth errors.

- The received echo signal is fed to the receiver from the antenna via two rotary joints (not shown in the block diagram). One rotary joint permits motion in azimuth; the other, in elevation.

- The receiver is conventional super heterodyne except for features related to the conical scan tracking.

- The error signal is extracted in the video after the second detector.

- Range gating eliminates noise and excludes other targets.

- The error signal from the range gate is compared with both the elevation and azimuth reference signals in the angle error detectors.

- The angle error outputs are amplified and used to drive the antenna elevation and azimuth servo motors.

- The video signal is a pulse train modulated by the conical scan frequency.

- It is usually convenient to stretch the pulses before low pass filtering so as to increase the energy at the conical scan frequency to perform A/D conversion.

- This pulse stretching is accomplished by a sample-and hold circuit which also known as boxcar generator.

Fig 4: (a) Pulse train with conical scan modulation (b)same pulse train after passing through boxcar generator.(stretching by a sample and hold circuit)
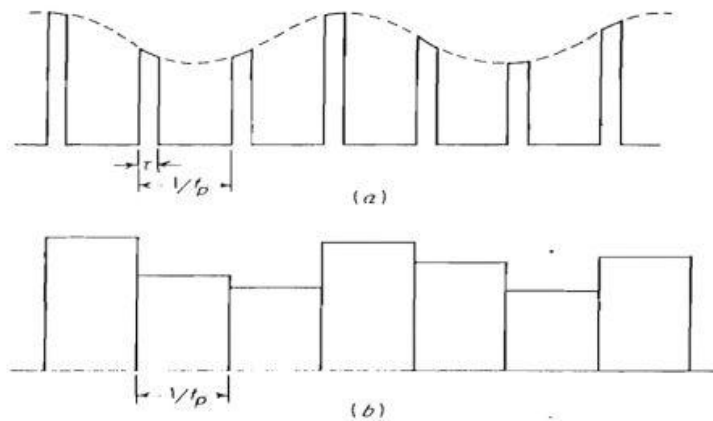
- PRF must be sufficiently large compared to conical scan frequency for proper filtering and avoiding inaccuracy of the angle measurement.

- The PRF must be atleast four times of conical scan frequency but normally 10 times.

Automatic Gain Control (AGC) :

- The echo-signal amplitude at the tracking-radar receiver will not be constant but will vary with time. The three major causes of variation in amplitude are:

  - The inverse-fourth-power relationship between the echo signal and range

  - The conical scan modulation (angle-error signal) and

  - Amplitude fluctuations in the target cross Section.

- The function of the automatic gain control (AGC) is to maintain the d-c level of the receiver output constant and to smooth or eliminate as much of the noise like amplitude fluctuations as possible without disturbing the extraction of the desired error signal at the conical-scan frequency.

- AGC is also important for avoiding saturation by large signals which could cause the loss of the scanning modulation and the accompanying error signal.

An example of the AGC portion of a tracking-radar receiver is shown in Fig

- A portion of the video-amplifier output is passed through a low-pass or smoothing filter and fed back to control the gain of the IF amplifier.

- The larger the video output, the larger will be the feedback signal and the greater will be the gain reduction.
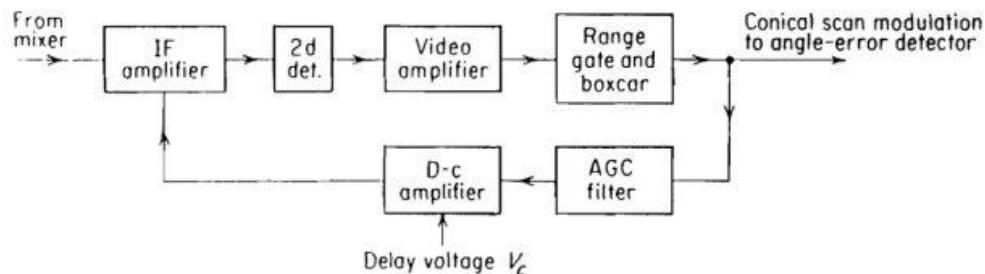
Figure: Block diagram of the AGC portion of a tracking-radar receiver

•The filter in the AGC loop should pass all frequencies from direct current to just below the conical-scan-modulation frequency.

•The loop gain of the AGC filter measured at the conical-scan frequency should be low so that the error signal will not be affected by AGC action.

•The phase shift of this filter must be small if its phase characteristic is not to influence the error signal.

•A phase change of the error signal is equivalent to a rotation of the reference axes and introduces cross coupling, or "cross talk," between the elevation and azimuth angle-tracking loops.

- Cross talk affects the stability of the tracking and might result in an unwanted nutating motion of the antenna.

- In conventional tracking radar applications, the phase change introduced by the feedback-loop filter should be less than 10^0 and in some applications, it should be as little as 2^0.

- For this reason, a filter with a sharp attenuation characteristic in the vicinity of the conical-scan frequency might not be desirable because of the relatively large amount of phase shift which it would introduce.

Other considerations:

- In both the sequential-lobing and conical-scan techniques, the measurement of the angle error in two orthogonal coordinates (azimuth and elevation) requires that a minimum of three pulses be processed.

- In practice, however, the minimum number of pulses in sequential lobing is usually four-one per quadrant. Although a conical scan radar can also be operated with only four pulses per revolution, it is more usual to have ten or more per revolution. This allows the modulation due to the angle error to be more that of a continuous sine wave.

- Thus, the **PRF** is usually at least an order of magnitude greater than the conical-scan frequency.

- The scan frequency also must be at least an order of magnitude greater than the tracking bandwidth.

- A conical-scan-on-receive-only (COSRO) tracking radar radiates a non-scanning transmit beam, but receives with a conical scanning beam to extract the angle error. The analogous operation with sequential lobing is called lobe-on-receive-only (LORO).

DISADVANTAGES

Sequential lobing
- Angle accuracy can be no better than the size of the antenna beamwidth.
- Variation in echo strength on a pulse-by-pulse basis changes the signal level thereby reducing tracking accuracy
- The antenna gain is less than the peak gain in beam axis direction, reducing maximum range that can be measured

Conical scan
- The antenna scan rate is limited by the scanning mechanism (mechanical or electronic)
- Sensitive to target modulation
- Mechanical vibration and wear and tear due to rotating feed
- It creates confusion by rapid changes in signal strength

MONOPULSE TRACKING RADAR

- Pulse-to-pulse amplitude fluctuations of the echo signal have no effect on tracking accuracy if the angular measurement is made on the basis of one pulse rather than many.

- There are several methods by which angle-error information might be obtained with only a single pulse.

- More than one antenna beam is used simultaneously in these methods, in contrast to the conical-scan or lobe-switching tracker, which utilizes one antenna beam on a time-shared basis.

- The angle of arrival of the echo signal may be determined in a single-pulse system by measuring the relative phase or the relative amplitude of the echo pulse received in each beam.

- The names **simultaneous lobing** and **monopulse** are used to describe those tracking techniques which derive angle-error information on the basis of a single pulse.

- Most popular monopulse is – Amplitude Comparison Monopulse
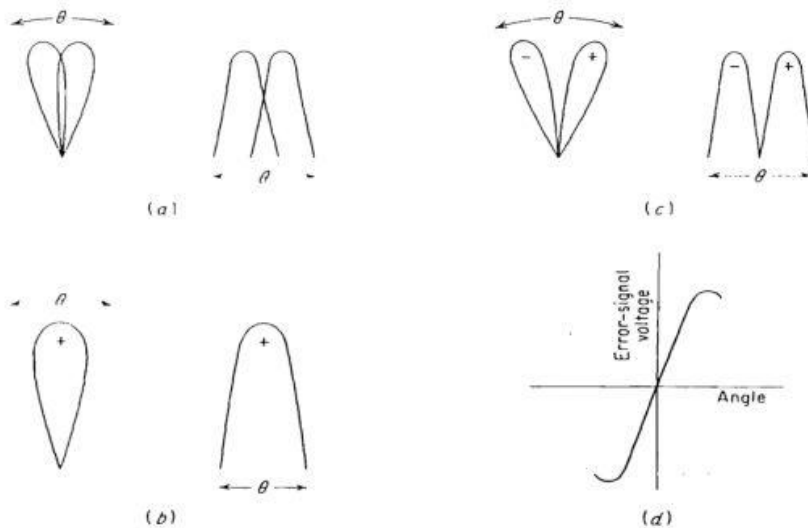
Amplitude Comparison Monopulse

Figure: Monopulse antenna patterns and error signal. Left-hand diagrams in (a-c) are in polar coordinates. Right-hand diagrams are in rectangular coordinates. (a) Overlapping antenna patterns (b) sum pattern (c) difference pattern (d) product (error) signal.

- In this technique, the RF signals received from two offset antenna beams are combined so that both the sum and the difference signals are obtained simultaneously.

- The sum and difference signals are multiplied in a **phase-sensitive detector** to obtain both the magnitude and the direction of the error signal.

- All the information necessary to determine the angular error is obtained on the basis of a single pulse; hence the name **monopulse**.

- The amplitude-comparison monopulse employs two overlapping antenna patterns to obtain the angular error in one coordinate.

- The sum of the two antenna patterns of **Fig (a)** is shown in **Fig (b),** and the difference in **Fig(c).**

- The sum pattern is used for transmission, while both the sum pattern and the difference pattern are used on reception.

- The signal received with the difference pattern provides the magnitude of the angle error.

- The sum signal provides the range measurement and is also used as a reference to extract the sign of the error signal.

- Signals received from the sum and the difference patterns are amplified separately and combined in a phase-sensitive detector to produce the error-signal characteristic shown in **Fig (d).**

Phase-sensitive detector

- The system contains a phase sensitive detector that compares two signals of the same frequency.

- It is a nonlinear device

- The output indicates the direction of the angle error relative to the boresight.

- Though phase comparison is done, the magnitude of the angle error signal is determined by comparison of amplitude signals.

Amplitude-comparison monopulse (One angular coordinate)

- A block diagram of the amplitude-comparison-monopulse tracking radar for a single angular coordinate is shown in below **Fig.**
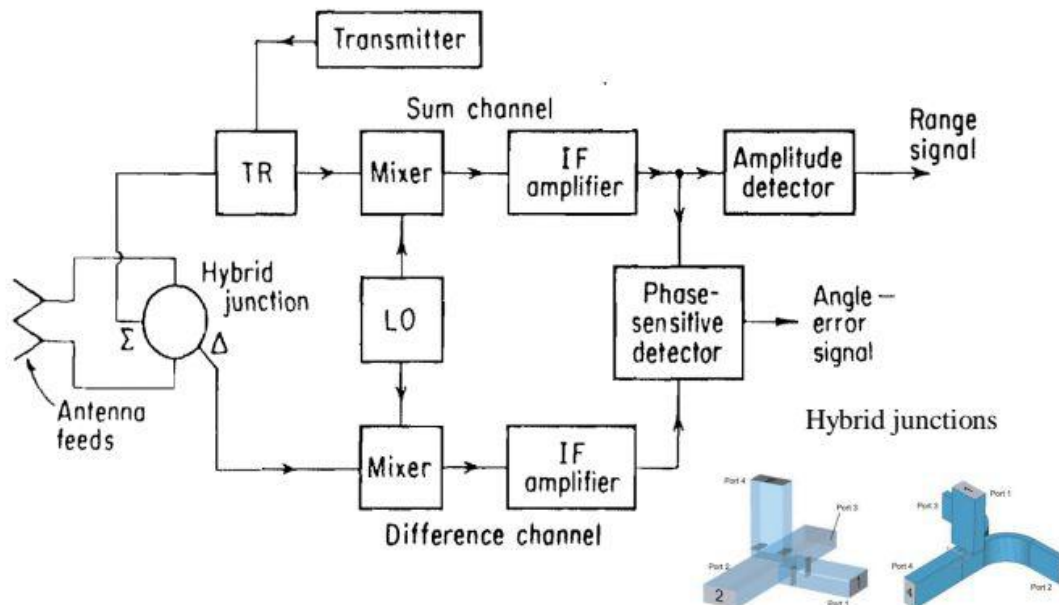
- The two adjacent antenna feeds are connected to the two arms of a *hybrid junction* such as a *"magic T,* a *"rat race,"* or a *"short-slot coupler"*.

- The sum and difference signals appear at the two other arms of the *hybrid*. On reception, the outputs of the sum arm and the difference arm are each heterodyned to an intermediate frequency and amplified as, in any super heterodyne receiver.

- The transmitter is connected to the sum arm. Range information is also extracted from the sum channel.

- A duplexer is included in the sum arm for the protection of the receiver.

- The output of the phase-sensitive detector is an error signal whose magnitude is proportional to the angular error and whose sign is proportional to the direction.

- The output of the monopulse radar is used to perform automatic tracking.

- The angular error signal actuates a servo-control system to position the antenna, and the range output from the sum *channel feeds* into an automatic-range-tracking unit.

- The sign of the difference signal (and the direction of the angular error) is determined by comparing the phase of the difference signal with the phase of the sum signal.

- If the sum signal in the IF portion of the receiver were $A_S \cos (\omega_{IF}t)$ the difference signal would be either $A_d \cos (\omega_{IF}t)$ or $-A_d \cos (\omega_{IF}t)$ ($A_s>0$, $A_d>0$), depending on which side of center is the target.

- Since $-A_d \cos (\omega_{IF}t) = A_d \cos \omega_{IF}(t+\pi)$, the sign of the difference signal may be measured by determining whether the difference signal is in phase with the sum or 180^0 out of phase.

- The purpose of the phase-sensitive detector is **only** to conveniently furnish the *sign* of the error signal.

Amplitude-comparison monopulse
(Two angular Coordinate)
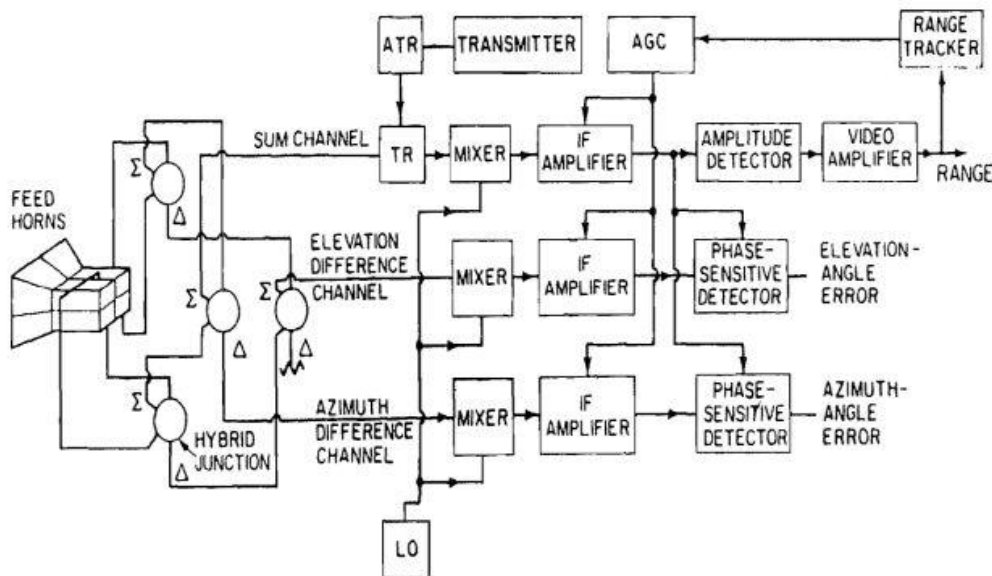
- A block diagram of a monopulse radar with provision for extracting error signals in both elevation and azimuth is shown in above **Fig**

- The cluster of four feeds generates four partial overlapping antenna beams. The feeds might be used with a parabolic reflector, Cassegrain antenna, or a lens.

- All four feeds generate the sum pattern. The difference pattern in one plane is formed by taking the sum of two adjacent feeds and subtracting this from the sum of the other two adjacent feeds.

- The difference pattern in the orthogonal plane is obtained by adding the differences of the orthogonal adjacent pairs.

- A total of four hybrid junctions generate the sum channel, the azimuth difference channel, and the elevation difference channel.

- Three separate mixers and IF amplifiers are shown, one for each channel.

- All three mixers operate from a single local oscillator in order to maintain the phase relationships between the three channels.

- Two phase-sensitive detectors extract the angle-error information, one for azimuth, the other for elevation.

- Range information is extracted from the output of the sum channel after amplitude detection.

- An alternative approach to using three identical amplifiers in the monopulse receiver is to use only one IF channel which amplifies the sum signal and the two difference signals on a time-shared basis.

- The sum signal is passed through the single IF amplifier followed by the two difference signals delayed in time by a suitable amount.

- Automatic gain control (AGC) is required in order to maintain a stable closed-loop servo system for angle tracking and to insure that angle error signal is not affected by changes in the received signal amplitude.

- The AGC results in a constant angle sensitivity independent of target size and range.

- With AGC the output of the angle-error detector is proportional to the difference signal normalized (divided) by the sum signal. The output of the sum channel is constant.

[SUM PATTERN : A+B +C +D
AZIMUTH DIFF. PATTERN : (A+B) – (C +D)
ELEVATION DIFF. PATTERN : (B+D) – (A+C)]

Important Requirements of Amplitude-comparison monopulse

- The monopulse antenna must generate a sum pattern with high efficiency (maximum boresight gain), and a difference pattern with a large value of slope at the crossover of the offset beams.

- *The greater the signal-to-noise ratio and the steeper the slope of the error signal in the vicinity of zero angular error, the more accurate is the measurement of angle.*

- Furthermore, the side lobes of both the sum and the difference patterns must be low.

- The antenna must be capable of the desired bandwidth, and the patterns must have the desired polarization characteristics.

Phase comparison Monopulse

- **The** tracking techniques discussed thus far in this chapter are based on the comparison of the amplitude of echo signals received from two or more antenna positions.

- The sequential-lobing and conical-scan techniques used a single, time-shared antenna beam while the Monopulse technique used two or more simultaneous beams.

- The difference in amplitudes in the several antenna positions is proportional to the angular error.

- A tracking radar which operates with phase information is similar to an **active interferometer** and is also called *interferometer radar.* It has also been called *simultaneous phase comparison radar* or *phase-comparison Monopulse*.

- Phase comparison monopulse uses two antenna beams to obtain an angle measurement in one coordinate. But the two beams cover the same region of space.

- Here the two beams look in the same direction whereas in earlier case they looked at slightly different directions

Amplitude comparison monopulse

Phase comparison monopulse
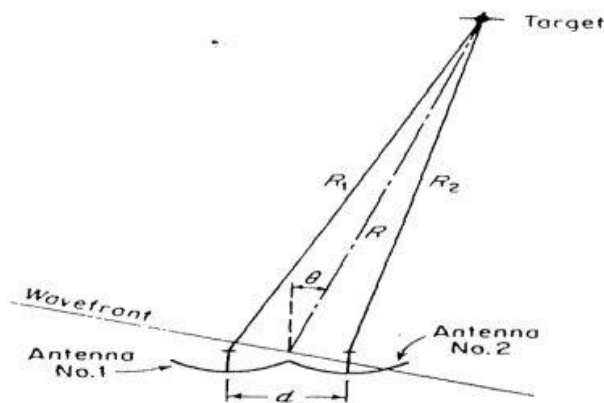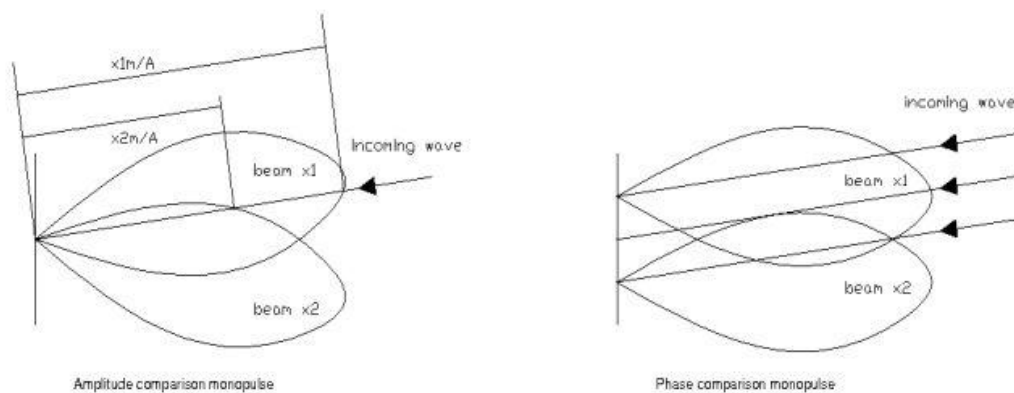
Wave front phase relationships in phase comparison Monopulse Radar

- In above **Fig** two antennas are shown separated by a distance **d**.

- The distance to the target is **R** and is assumed large compared with the antenna separation **d**.

- The line of sight to the target makes an angle θ to the perpendicular bisector of the line joining the two antennas.

- The distance from antenna 1 to the target is:

$$R_1 = R + (d/2)\sin\theta$$

and the distance from antenna 2 to the target is:

$$R_2 = R - (d/2)\sin\theta$$

- The phase difference between the echo signals in the two antennas is approximately:

$$\Delta\emptyset = (2\pi/\lambda).d.\sin\theta$$

For small angles where $\sin\theta \approx \theta$, the phase difference is a linear function of the angular error and may be used to position the antenna via a servo-control loop.
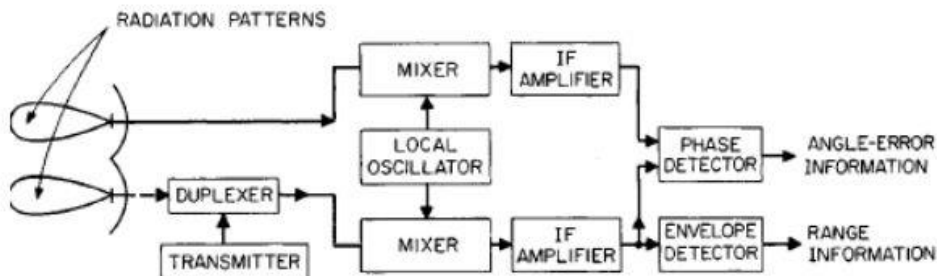
Block diagram of a phase comparison monopulse radar (one angle coordinate).

Limitations

•.Effect of grating lobes due to wide separation between the antennas

•Ambiguities in the angle measurement

•To overcome these problems, a portion of the parabolic reflectors is sliced off to achieve this.

Important points to remember in amplitude and phase comparison monopulse

- A total of four hybrid junctions generate sum, azimuth and elevation difference channel
- Range information is extracted form the output of the sum channel after amplitude detection
- The angular error signal is obtained by comparing echo amplitudes which actuates a servo mechanism to position the antenna.
- The angle of arrival is determined by comparing the phase difference between signals from two separate antennas
- Antennas of phase comparison are not offset from the axis

Limitations to Tracking Accuracy

Major effects that determine the accuracy of a tracking radar:

- Glint or angle noise or angular scintillation: which affects all tracking radars especially at short range. The greater the target in angle, the greater is the glint error.
- Receiver noise: affects all radars and mainly determines tracking accuracy at long range.
- RCS or Amplitude fluctuations of the target echo that bother conical scan and sequential lobing trackers but not monopulse.
- Servo noise
- Antenna Beamwidth
- Atmospheric effects

Tracking in range

- In the early days o radar, tracking of target in range was usually done manually by an operator who watched an A-scope or similar presentation and positioned a handwheel to maintain a marker on the display over desired target pip.

- The setting of the handwheel was a measure of the target range and was converted to an electrical signal and supplied to a data processor.

- Manually tracking has many limitations and it cannot be used in systems such as missiles where there is no operator present.

- It was soon replaced by closed loop automatic tracking, such as *split gate tracker.*

- Split gate tracker uses two split range gates called early gate and late gate.

- The echo pulse is shown in Fig.a, the relative position of the gates at a particular instant shown in Fig.b, and the error signal shown in Fig c.

- The portion of the signal energy contained in the early gate is less than that in the late gate.

- If the outputs of the two gates are subtracted, an error signal **shown in fig c** will result which is used to reposition the center of the gates.

Figure : Split-range-gate tracking.(a)Echo pulse (b)early-late range gates(c) difference signal between early and late range gates
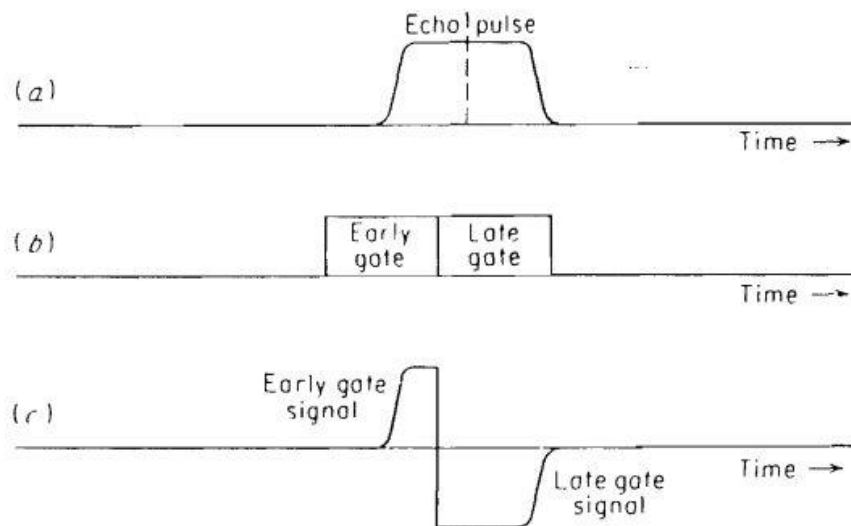
- The magnitude of the error signal is a measure of the difference between the center of the pulse and the center of the gates.

- The sign of the error signal determines the direction in which the gates must be repositioned by a feedback-control system.

- When the error signal is zero the range gates are centered on the pulse.

- The range gating necessary to perform automatic tracking in Range offers several advantages as by products.

- It isolates one target, excluding targets at other ranges. This permits the boxcar generator to be employed.

- Also, range gating improves the signal-to-noise ratio since it eliminates the noise from the other range intervals. Hence the width of the gate should be sufficiently narrow to minimize extraneous noise.

- A reasonable compromise is to make the gate width two to five times of the pulse width.

Acquisition and Scanning Patterns

- A tracking radar must first find and acquire its target before it can operate as a tracker.

- Most tracking radars employ a narrow pencil beam for accurate tracking in angle; but it can be difficult to search a large volume targets when using a narrow antenna beamwidth.

- Search must be done with care to cover the entire volume uniformly and efficiently.

- Some other radar, therefore must first find the target to be tracked and then designated the target's coordinates to the tracker. These radars have been called acquisition radars or designation radars that search a large volume.

Types of Scanning Patterns

The purpose of using scanning antenna is to find the direction of the target with respect to the transmitter. The direction of the antenna at the instance when echo is received, gives the direction of location of the target.

Examples of acquisition search patterns: (a) Trace of helical scanning beam (b) Palmer scan (c) spiral scan(d) raster, or TV, scan (e) nodding scan. The raster scan is sometimes called an n-bar scan, where *n* is the number of horizontal rows.

(a)   (b)   (c)

(d)   (e)
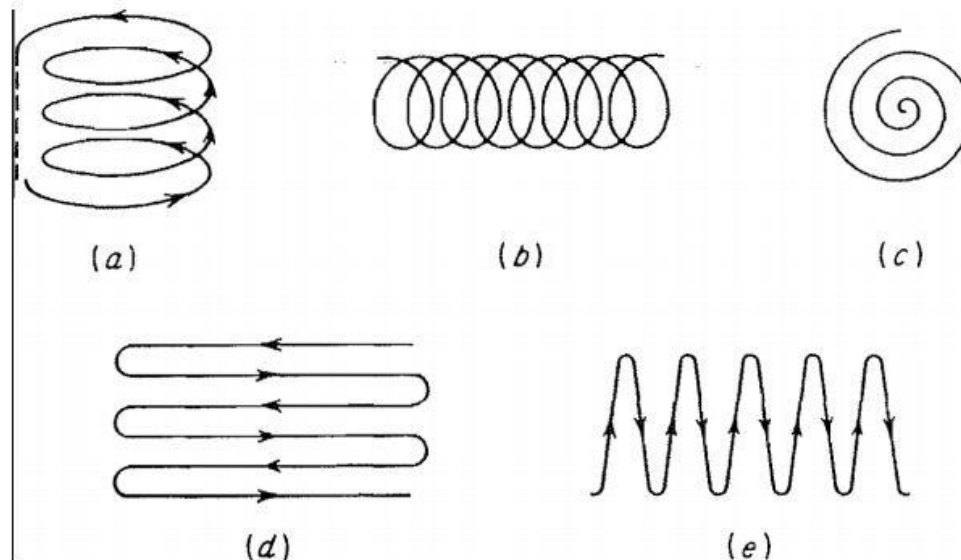
a) **Helical Scanning**

*   Helical scanning covers a hemisphere.

*   In the *helical scan*, the antenna is continuously rotated in azimuth   while it is simultaneously raised or lowered in elevation.

*   It's typical speed of rotation is 6rpm along with a rise of 20% and was utilized in world war II for anti-aircraft gun batteries as fire controlled radar.

b) **Palmer Scan**

*   The *Palmer* scan consists of a rapid circular scan (conical scan) about   the axis of the antenna, combined with a linear movement of the axis of rotation.

*   When the axis of rotation is held stationary the Palmer scan reduces to the *conical* scan.

*   Because of this feature the palmer scan is used with conical scan tracking radars which must operate in both search and track mode.

c) Spiral Scan

- The spiral scan covers an angular search volume with circular symmetry.
- Both the spiral scan and the Palmer scan suffer from the disadvantage that all parts of the scan volume do not receive the same energy unless the scanning speed is varied during the scan cycle.
- As a consequence, the number of hits returned from a target when searching with a constant scanning rate depends upon the position of the target within the search area.

d) Raster Scan

- The raster or TV, scan, unlike the Palmer or the spiral scan, scans the search area in a uniform manner.
- The raster scan is a simple and convenient means for searching a limited sector, rectangular in shape.

e) Nodding Scan

- The antenna is moved rapidly in elevation while it rotates slowly in azimuth thus scanning in both planes.
- The pattern covers the complete hemisphere i.e. elevation angle extending to $90^0$and the azimuth scan angle to 360^0
- Used in height finding radars

Comparison of Trackers

- Of the four continuous-tracking-radar techniques that have been discussed (sequential lobing, conical scan, amplitude-comparison monopulse, and phase-comparison monopulse), conical scan and amplitude-comparison monopulse have seen more application than the other two.
- In phase comparison four antennas are placed in awkward direction and its side lobe levels are higher than desired.
- Sequential lobing suffers more losses with complex antenna and feed system
- Amplitude comparison has high SNR
- It has higher precision in target tracking due to the absence of target amplitude fluctuations
- Angle error in two coordinates can be obtained by a single pulse
- Conical scan integrates no of pulses and then extracts angle measurement but vice versa in monopulse.

SNR

- The SNR from a monopulse radar is greater than that from a conical scan since it views target at the peak of sum pattern.
- SNR is 2 to 4 db greater.

Accuracy

- Due to high SNR, the range accuracy is also high in monopulse.
- The accuracy is not affected by fluctuations in the amplitude of the echo signal.
- Both systems are degraded by the wandering of the apparent position of the a target caused by glint.

Complexity

- Monopulse is more complex of the two.
- Conical scan has to rotate or nutate the beam at high speed.
- The cassegrain is a popular choice for monopulse
- A space fed phased array can implement monopulse by using a multiple feed similar to cassegrain.

Min No. of Pulses

- A monopulse can perform on the basis of a single pulse. For a phased array one pulse is sufficient
- The conical scan tracker requires a minimum no. of four pulses per revolution of beam to extract an angle measurement in two coordinates.
- The monopulse first makes its angle measurement and then integrates a no. of measurements to obtain the required SNR.
- The conical scan integrates a no. of pulses first and then extracts the angle measurement.

Susceptibility to ECM

- Conical scan tracker is more vulnerable to spoofing that takes advantage of its conical scan frequency
- It can also suffer from deliberate amplitude fluctuations.
- A well designed monopulse is hard to deceive.

Application

- Monopulse trackers should be used when good angle accuracy is needed.
- When high performance tracking is not necessary, the conical scan tracker might be used for its low cost.

Comparison of Monopulse Tracking and Conical Scan Tracking

Monopulse Tracking	Conical Scan Tracking
Multiple beams are used to determine the angle of arrival of the echo signal	A single antenna beam on a time shared basis is used
Signle pulse is required to derive angle error information	Multiple pulses are required
High SNR	Low SNR
More acuurate tracking	Less accurate tracking
Complex Design	Simple Design
Cassegrain antenna is used	Horn antenna is used
High cost	Low cost

The **Search radar** is usually less precise and only distinguishes between targets that are hundreds of yards or even miles apart. Radar resolution is usually divided into two categories viz. range resolution and angular resolution (i.e. bearing resolution).

- Distance coverage: Long, medium, short ranges (20 km to 2000 km)
- High power density on the target: high peak power, long pulses, long pulse trains, high antenna gain
- Low PRFs
  Search options: rapid search rate with narrow beams or slower search rate with wide beams

Tracking Radar

The **Tracking radar** continuously emits the EM waves in the air and detects the targetted object when it comes in the path of the waves.

- ˇ Accurate angle and range measurement required

  ˇ Minimize time on target for rapid processing

  ˇ Special tracking techniques: monopulse, conical scan, beam switching

DETECTION OF RADAR SIGNALS
IN NOISE

Introduction

Matched filter receiver

Response characteristics and derivation

Correlation function and cross correlation receiver

Efficiency of Non-matched filters

Matched filter with non-white noise

- The two basic operations performed by radar are (1) *detection* of the presence of reflecting objects, and (2) *extraction* of information from the received waveform to obtain such target data as position, velocity, and perhaps size.

- In this chapter some aspects of the problem of detecting radar signals in the presence of noise will be considered. Noise ultimately limits the capability of any radar.

Matched-Filter Receiver

- A network whose frequency-response function maximizes the output peak-signal-to- mean-noise (power) ratio is called a matched filter. This criterion, or its equivalent, is used for the design of almost all radar receivers.

- The frequency-response function, denoted H(f), expresses the relative amplitude and phase of the output of a network with respect to the input when the input is a pure sinusoid.

- The magnitude |H(f)|of the frequency-response function is the receiver amplitude pass band characteristic.

- If the bandwidth of the receiver pass band is wide compared with that occupied by the signal energy, extraneous noise is introduced by the excess bandwidth which lowers the output signal-to-noise ratio. On the other hand, if the receiver bandwidth is narrower than the bandwidth occupied by the signal, the noise energy is reduced along with a considerable part of the signal energy.

- The net result is again a lowered signal-to-noise ratio. Thus there is an optimum bandwidth at which the signal-to-noise ratio is a maximum. This is well known to the radar receiver designer.

- The rule of thumb quoted in pulse radar practice is that the receiver bandwidth B should be approximately equal to the reciprocal of the pulse width τ. This is a reasonable approximation for pulse radars with conventional superheterodyne receivers. It is not generally valid for other waveforms.

- The exact specification of the optimum receiver characteristic involves the frequency- response function and the shape of the received waveform.

- The receiver frequency-response function, is assumed to apply from the antenna terminals to the output of the IF amplifier.

- The second detector and video portion of the well designed radar superheterodyne receiver will have negligible effect on the output signal-to- noise ratio if the receiver is designed as a matched filter. Narrow banding is most conveniently accomplished in the IF.

- The bandwidths of the RF and mixer stages of the normal superheterodyne receiver are usually large compared with the IF bandwidth. Therefore the frequency-response function of the portion of the receiver included between the antenna terminals to the output of the IF amplifier is taken to be that of the IF amplifier alone.

- Thus we need only obtain the frequency-response function that maximizes the signal-to-noise ratio at the output of the IF. The IF amplifier may be considered as a filter with gain.

- For a received waveform s(t) with a given ratio of signal energy E to noise energy No (or noise power per hertz of bandwidth), North showed that the frequency-response function of the linear, time-invariant filter which maximizes the output peak-signal-to-mean- noise (power) ratio for a fixed input signal-to-noise (energy) ratio is

$$H(f) = G_a S^*(f) \exp\left(-j2\pi f t_1\right)$$

where $S(f) = \int_{-\infty}^{\infty} s(t) \exp\left(-j2\pi ft\right) dt =$ voltage spectrum (Fourier transform) of input signal

$S^*(f) =$ complex conjugate of $S(f)$

$t_1 =$ fixed value of time at which signal is observed to be maximum

$G_a =$ constant equal to maximum filter gain (generally taken to be unity)

- The noise that accompanies the signal is assumed to be stationary and to have a uniform spectrum (white noise). It need not be Gaussian.

- The filter whose frequency-response function is given by Eq. above has been called the North filter, the conjugate filter, or more usually the matched filter. It has also been called the Fourier transform criterion.

- The frequency-response function of the matched filter is the conjugate of the spectrum of the received waveform except for the phase shift exp (- j2Πft1). This phase shift varies uniformly with frequency. Its effect is to cause a constant time delay.

- The frequency spectrum of the received signal may be written as an amplitude spectrum |S(f)|and a phase spectrum exp [- jφs (f)].

- The matched- filter frequency-response function may similarly be written in terms of its amplitude and phase spectra |H(f)| and exp [- jφm(f)]. Ignoring the constant Ga, Eq. above for the matched filter may then be written as

$$|H(f)| \exp\left[-j\phi_m(f)\right] = |S(f)| \exp\left\{j[\phi_s(f) - 2\pi f t_1]\right\}$$

or
$$|H(f)| = |S(f)|$$

and
$$\phi_m(f) = -\phi_s(f) + 2\pi f t_1$$

- Thus the amplitude spectrum of the matched filter is the same as the amplitude spectrum of the signal, but the phase spectrum of the matched filter is the negative of the phase spectrum of the signal plus a phase shift proportional to frequency.

- The matched filter may also be specified by its impulse response h(t), which is the inverse Fourier transform of the frequency-response function.

$$h(t) = \int_{-\infty}^{\infty} H(f) \exp(j2\pi ft)\, df$$

- Physically, the impulse response is the output of the filter as a function of time when the input is an impulse (delta function).

Since $S^*(f) = S(-f)$, we have

$$h(t) = G_a \int_{-\infty}^{\infty} S(f) \exp[j2\pi f(t_1 - t)]\, df = G_a s(t_1 - t)$$

- A rather interesting result is that the impulse response of the matched filter is the image of the received waveform; that is, it is the same as the received signal run backward in time starting from the fixed time t1.

- Figure 1 shows a received waveform s(t) and the impulse response h(t) of its matched filter. The impulse response of the filter, if it is to be realizable, is not defined for t< 0. (One cannot have any response before the impulse is applied.) Therefore we must always have t < t1.

- This is equivalent to the condition placed on the transfer function H(f) that there be a phase shift exp (-j2ΠIft1). However, for the sake of convenience, the impulse response of the matched filter is sometimes written simply as s(- t).
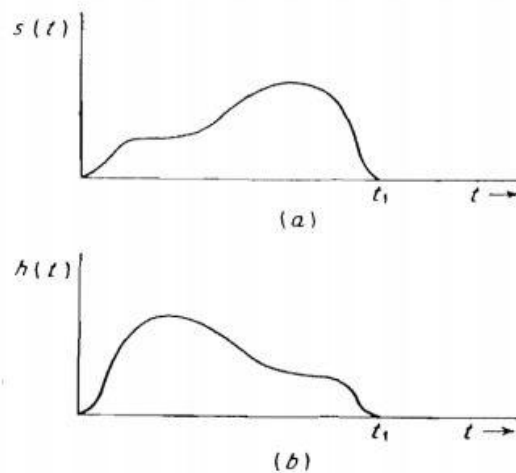

Fig.1 (a) Received waveform s(t); (b) impulse response h(t) of the matched filter.

Derivation of the matched-filter characteristic

- The frequency-response function of the matched filter has been derived by a number of authors using either the calculus of variations or the Schwartz inequality. We shall derive the matched-filter frequency-response function using the Schwartz inequality.

- We wish to show that the frequency-response function of the linear, time-invariant filter which maximizes the output peak-signal-to-mean-noise ratio is

$$H(f) = G_a S^*(f) \exp(-j2\pi f t_1)$$

- When the input noise is stationary and white (uniform spectral density). The ratio we wish to maximize is

$$R_f = \frac{|s_o(t)|^2_{max}}{N}$$

- Where $|s_o(t)|$ max = maximum value of output signal voltage and N = mean noise power at receiver output. The ratio R_f is not quite the same as the signal-to-noise ratio which has been considered in the radar equation.

- The output voltage of a filter with frequency-response function H(f) is

$$|s_o(t)| = \left| \int_{-\infty}^{\infty} S(f) H(f) \exp(j2\pi f t) \, df \right|$$

- Where S(f) is the Fourier transform of the input (received) signal. The mean output noise power is

$$N = \frac{N_o}{2} \int_{-\infty}^{\infty} |H(f)|^2 \, df$$

- Where No is the input noise power per unit bandwidth. The factor appears before the integral because the limits extend from - ∞ to +∞, whereas No is defined as the noise power per cycle of bandwidth over positive values only. Assuming that the maximum value of $|s_o(t)2|$ occurs at time t = t1, the ratio Rf becomes

$$R_f = \frac{\left| \int_{-\infty}^{\infty} S(f) H(f) \exp(j2\pi f t_1) \, df \right|^2}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 \, df}$$

Schwartz's inequality states that if P and Q are two complex functions, then

$$\int P^* P \, dx \int Q^* Q \, dx \geq \left| \int P^* Q \, dx \right|^2$$

- The equality sign applies when $P = kQ$, where k is a constant. Letting

$$P^\bullet = S(f)\exp(j2\pi ft_1) \quad \text{and} \quad Q = H(f)$$

and recalling that

$$\int P^\bullet P\, dx = \int |P|^2\, dx$$

- We get, on applying the Schwartz inequality to the numerator of Eq. earlier, we get

$$R_f \leq \frac{\int_{-\infty}^{\infty} |H(f)|^2\, df \int_{-\infty}^{\infty} |S(f)|^2\, df}{\dfrac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\, df} = \frac{\int_{-\infty}^{\infty} |S(f)|^2\, df}{\dfrac{N_0}{2}}$$

From Parseval's theorem,

$$\int_{-\infty}^{\infty} |S(f)|^2\, df = \int_{-\infty}^{\infty} s^2(t)\, dt = \text{signal energy} = E$$

Therefore we have

$$R_f \leq \frac{2E}{N_0}$$

- The frequency-response function which maximizes the peak-signal-to-mean-noise ratio Rf may be obtained by noting that the equality sign in Eq. applies when $P = kQ$, or

$$H(f) = G_a S^*(f)\exp(-j2\pi ft_1)$$

Where the constant k has been set equal to $1/G_a$.

6

The matched filter and the correlation function

- The output of the matched filter may be shown to be proportional to the input signal cross-correlated with a replica of the transmitted signal, except for the time delay t1.

- The cross correlation function R(t) of two signals $y(\lambda)$ and $s(\lambda)$, each of finite duration, is defined as

$$R(t) = \int_{-\infty}^{\infty} y(\lambda)s(\lambda - t) \, d\lambda$$

- The output yo(t) of a filter with impulse response h(t) when the input is yin(t) = s(t) + n(t) is

$$y_0(t) = \int_{-\infty}^{\infty} y_{in}(\lambda)h(t - \lambda) \, d\lambda$$

- If the filter is a matched filter, then $h(\lambda) = s(t1 - \lambda)$ and Eq. above becomes

$$y_0(t) = \int_{-\infty}^{\infty} y_{in}(\lambda)s(t_1 - t + \lambda) \, d\lambda = R(t - t_1)$$

- Thus the matched filter forms the cross correlation between the received signal corrupted by noise and a replica of the transmitted signal.

- The replica of the transmitted signal is "built in" to the matched filter via the frequency-response function.

- If the input signal yin (t) were the same as the signal s(t) for which the matched filter was designed (that is, the noise is assumed negligible), the output would be the autocorrelation function.

Cross correlation receiver(correlation detection)

$$y_0(t) = \int_{-\infty}^{\infty} y_{in}(\lambda)s(t_1 - t + \lambda)\, d\lambda = R(t - t_1)$$

- Equation above describes the output of the matched filter as the cross correlation between the input signal and a delayed replica of the transmitted signal.

- This implies that the matched-filter receiver can be replaced by a cross-correlation receiver that performs the same mathematical operation as shown in Fig.
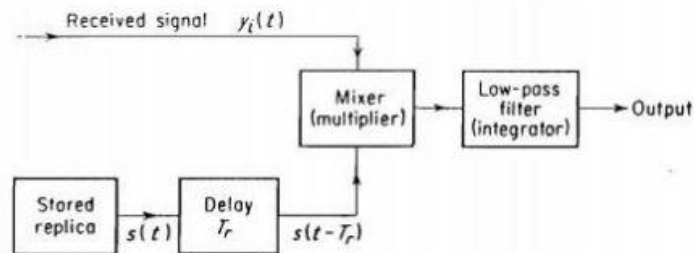

Fig: Block diagram of a cross-correlation receiver

- The input signal y (t) is multiplied by a delayed replica of the transmitted signal s(t - Tr), and the product is passed through a low-pass filter to perform the integration.

- The cross-correlation receiver of above Fig tests for the presence of a target at only a single time delay Tr. Targets at other time delays, or ranges, might be found by varying Tr. However, this requires a longer search time.

- The cross-correlation receiver and the matched-filter receiver are equivalent mathematically, the choice as to which one to use in a particular radar application is determined by which is more practical to implement.

- The matched-filter receiver, or an approximation, has been generally preferred in the vast majority of applications.

Efficiency of non-matched filters

- In practice the matched filter cannot always be obtained exactly. It is appropriate, therefore, to examine the efficiency of non matched filters compared with the ideal matched filter.

- The measure of efficiency is taken as the peak signal-to-noise ratio from the non matched filter divided by the peak signal-to-noise ratio (2E/No) from the matched filter.

- Figure. Plots the efficiency for a single-tuned (RLC) resonant filter and a rectangular-shaped filter of half-power bandwidth B_τ when the input is a rectangular pulse of width τ

- The maximum efficiency of the single-tuned filter occurs for $B_\tau \approx 0.4$. The corresponding loss in signal-to-noise ratio is 0.88 dB as compared with a matched filter. .
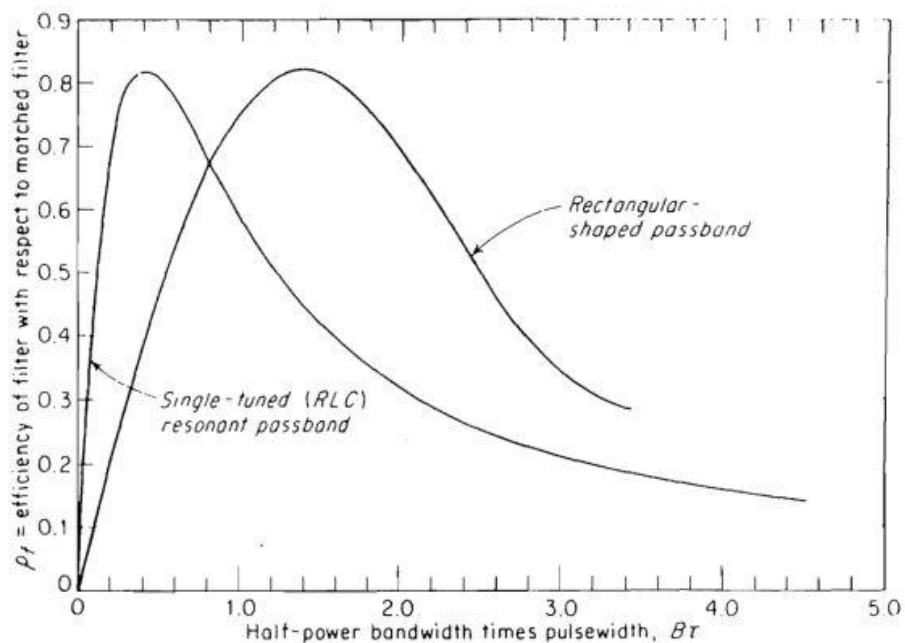
- Table lists the values of Bτ which maximize the signal-to-noise ratio (SNR) for various combinations of filters and pulse shapes. It can be seen that the loss in SNR incurred by use of these non-matched filters is small.

Input signal	Filter	Optimum Bτ	Loss in SNR compared with matched filter, dB
Rectangular pulse	Rectangular	1.37	0.85
Rectangular pulse	Gaussian	0.72	0.49
Gaussian pulse	Rectangular	0.72	0.49
Gaussian pulse	Gaussian	0.44	0 (matched)
Rectangular pulse	One-stage, single-tuned circuit	0.4	0.88
Rectangular pulse	2 cascaded single-tuned stages	0.613	0.56
Rectangular pulse	5 cascaded single-tuned stages	0.672	0.5

Table: Efficiency of nonmatched filters compared with the matched filter

Matched filter with nonwhite noise

- In the derivation of the matched-filter characteristic, the spectrum of the noise accompanying the signal was assumed to be white; that is, it was independent of frequency.

- If this assumption were not true, the filter which maximizes the output signal-to-noise ratio would not be the same as the matched filter.

- It has been shown that if the input power spectrum of the interfering noise is given by [Ni(f)]2, the frequency-response function of the filter which maximizes the output signal-to-noise ratio is

$$H(f) = \frac{G_a S^*(f) \exp(-j2\pi f t_1)}{[N_i(f)]^2}$$

- When the noise is nonwhite, the filter which maximizes the output signal-to-noise ratio is called the NWN (nonwhite noise) matched filter.

- For white noise $[N_i(f)]2$ = constant and the NWN matched-filler frequency-response function of Eq. above reduces to that of Eq. discussed earlier in white noise. Equation above can be written as

$$H(f) = \frac{1}{N_i(f)} \times G_a \left(\frac{S(f)}{N_i(f)} \right)^* \exp(-j2\pi f t_1)$$

- This indicates that the NWN matched filter can be considered as the cascade of two filters.

- The first filter, with frequency-response function $1/N_i(f)$, acts to make the noise spectrum uniform, or white. It is sometimes called the whitening filter.

- The second is the matched filter when the input is white noise and a signal whose spectrum is $S(f)/N_i(f)$.

RADAR RECEIVERS

Noise figure and Noise Temperature

Displays-Types

Duplexers-Branch type and Balanced Type

Circulators as duplexers

Phased array Antennas Introduction, Basic concepts, Radiation Pattern

Beam steering and Beam width changes

Series vs parallel feeds

Applications, Advantages and Limitations

Radar Receivers

- The receiver section is connected to the antenna by duplexer. The receiver section is super heterodyne type and it consists of mixer, IF amplifier, detector and video amplifier.

- The first stage of the receiver is usually an RF amplifier with very low noise properties. The RF amplifier is followed by a mixer of fairly low noise figure.

- The mixer is a crystal diode which is fed to IF amplifier. The IF amplifier operates at 30 or 60 MHz or at nearby frequency.

- IF stage provides most of the receiver gain. The IF amplifier should be a low noise device to ensure that the overall noise figure of the receiver does not deteriorate to great extent.

- The down conversion from microwave frequency to IF frequency may be done in number of stages to ensure adequate image frequency suppression.

- The detector is crystal diode whose output is amplified by video amplifier having the same bandwidth as the IF amplifier.

- The output signal is then fed to a display unit which may be a CRT.

Characteristics of radar receiver

- The function of the radar receiver is to detect desired echo signals in the presence of noise, interference, or clutter.

- It must separate wanted from unwanted signals, and amplify the wanted signals to a level where target information can be displayed to an operator or used in an automatic data processor.

- The design of the radar receiver will depend not only on the type of waveform to be detected, but on the nature of the noise, interference, and clutter echoes with which the desired echo signals must compete.

- Noise can enter the receiver via the antenna terminals along with the desired signals, or it might be generated within the receiver itself. At the microwave frequencies usually used for radar, the external noise which enters via the antenna is generally quite low. The measure of receiver internal noise is the noise-figure.

- Good receiver design is based on maximizing the output signal-to-noise ratio. To maximize the output signal-to-noise ratio, the receiver must be designed as a matched filter, or its equivalent. The matched filter specifies the frequency response function of the IF part of the radar receiver.

- Obviously, the receiver should be designed to generate as little internal noise as possible, especially in the input stages where the desired signals are the weakest.

- Receiver design also must be concerned with achieving sufficient gain, phase, and amplitude stability, dynamic range, tuning, ruggedness, and simplicity.

- Protection must be provided against overload or saturation, and burnout from nearby interfering transmitters. Timing and reference signals are needed to properly extract target information.

- Timing and reference signals are needed to properly extract target information. Specific applications such as MTI radar, tracking radar, or radars designed to minimize clutter place special demands on the receiver.

- Receivers that must operate with a transmitter whose frequency can drift need some means of automatic frequency control (AFC).

- Radars that encounter hostile counter-measures need receivers that can minimize the effects of such interference.

- Thus there can be many demands placed upon the receiver designer in meeting the requirements of modern high-quality radar systems.

- Although the super regenerative, crystal video and tuned radio frequency (TRF) receivers have been employed in radar systems, the superheterodyne has seen almost exclusive application because of its good sensitivity, high gain, selectivity, and reliability. No other receiver type has been competitive to the superheterodyne.

Receiver Noise Figure

Noise figure:

Noise figure of a receiver is a measure of the noise produced by a practical receiver as compared with the noise of an ideal receiver. The noise figure F_n may be defined as:

$$F_n = \frac{S_{in}/N_{in}}{S_{out}/N_{out}} = \frac{N_{out}}{kT_0\,B_n\,G} \qquad \text{Eq.(1)}$$

Where
S_{in} = available input signal power

N_{in} = available input noise power (equal to kT_oB)

S_{out} = available output signal power

N_{out} = available output noise power

"Available power" refers to the power which would be delivered to a matched load. The available gain G is equal to S_{out}/S_{in}.

k = Boltzmann's constant = 1.38×10^{-23} J/deg, T_o = standard temperature of **290 K** (approximately room temperature) and B_n is the noise bandwidth (defined earlier).

The product $kT_o \approx 4 \times 10^{-21}$ W/Hz. The purpose of defining a standard temperature is to refer any measurements to a common basis of comparison.

- Equation above permits two different but equivalent interpretations of noise figure.

- It may be considered as the degradation of the signal-to-noise ratio caused by the network (receiver), or it may be interpreted as the ratio of the actual available output noise power to the noise power which would be available if the network merely amplified the thermal noise.

- The noise figure may also be written

$$F_n = \frac{kT_0 B_n G + \Delta N}{kT_0 B_n G} = 1 + \frac{\Delta N}{kT_0 B_n G}$$

- where ΔN is the additional noise introduced by the network itself. The noise figure is commonly expressed in decibels, that is, $10 \log F_n$. The term noise factor is also used at times instead of noise figure. The two terms are synonymous.

Noise figure of networks in cascade

- Consider two networks in cascade, each with the same noise bandwidth Bn but with different noise figures and available gain (Fig).

- Let F1, G1 be the noise figure and available gain, respectively, of the first network, and F2, G2 be similar parameters for the second network.

- The problem is to find Fo, the overall noise-figure of the two circuits in cascade.
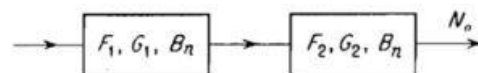
- From the definition of noise figure the output noise No of the two circuits in cascade is

$\mathbf{N_o = F_o G_1 G_2 kT_0 B_n}$ =Noise from network **1 at output of network2**+ **Noise ΔN_2 introduced by network2**

$$N_o = kT_0 B_n F_1 G_1 G_2 + \Delta N_2 = kT_0 B_n F_1 G_1 G_2 + (F_2 - 1)kT_0 B_n G_2$$

$$F_o = F_1 + \frac{F_2 - 1}{G_1}$$

15

- The contribution of the second network to the overall noise-figure may be made negligible if the gain of the first network is large. This is of importance in the design of multistage receivers.

- It is not sufficient that only the first stage of a low-noise receiver have a small noise figure.

- The succeeding stage must also have a small noise figure, or else the gain of the first stage must be high enough to swamp the noise of the succeeding stage if the first network is not an amplifier but is a network with loss (as in a crystal mixer), the gain GI should be interpreted as a number less than unity.

- Extending the same method, the noise figure of N networks in cascade may be shown to be:

$$F_o = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_N - 1}{G_1 G_2 \cdots G_{N-1}}$$

Noise temperature

- The noise introduced by a network may also be expressed as an **effective** Noise **temperature,** T_e defined as that (fictional) temperature at the input of the network which would account for the noise ΔN at the output. Therefore $\Delta N = kT_e B_0 G$ and

$$F_n = 1 + \frac{T_e}{T_0}$$

$$T_e = (F_n - 1)T_0$$

- The system noise temperature Ts is defined as the effective noise temperature of the receiver system including the effects of antenna temperature Ta. (It is also sometimes called the system operating noise temperature) If the receiver effective noise temperature is Te, then

$$T_s = T_a + T_e = T_0 F_s$$

- where Fs is the system noise-figure including the effect of antenna temperature. The effective noise temperature of a receiver consisting of a number of networks in cascade is

$$T_e = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \cdots$$

where Ti and Gi , are the effective noise temperature and gain of the ith network.

- The effective noise temperature and the noise figure both describe the same characteristic of a network. In general, the effective noise temperature has been preferred for describing low-noise devices, and the noise figure is preferred for conventional receivers.

Displays-Types

- The purpose of the display is to visually present the information contained in the radar echo signal in a form suitable for operator interpretation and action.

- When the display is connected directly to the video output of the receiver, the information displayed is called raw video. This is the "traditional" type of radar presentation.

- When the receiver video output is first processed by an automatic detector or automatic detection and tracking processor (ADT), the output displayed is sometimes called synthetic video.

- The cathode-ray tube (CRT) has been almost universally used as the radar display. There are two basic cathode-ray tube displays. One is the dejection-modulated CRT, such as the A-scope, in which a target is indicated by the deflection of the electron beam.

- The other is the intensity modulated CRT, such as the PPI, in which a target is indicated by intensifying the electron beam and presenting a luminous spot on the face of the CRT.

- In general, deflection-modulated displays have the advantage of simpler circuits than those of intensity- modulated displays, and targets may be more readily discerned in the presence of noise or interference.

- On the other hand, intensity-modulated displays have the advantage of presenting data in a convenient and easily interpreted form.

- The deflection of the beam or the appearance of an intensity-modulated spot on a radar display caused by the presence of a target is commonly referred to as a blip.

Types of display presentations: The various types of CRT displays which might be used for surveillance and tracking radars are defined as follows:

- **A-scope:** A deflection-modulated display in which the vertical deflection is proportional to target echo strength and the horizontal coordinate is proportional to range.

- **B-scope:** An intensity-modulated rectangular display with azimuth angle indicated by the horizontal coordinate and range by the vertical coordinate.

- **C-scope:** An intensity-modulated rectangular display with azimuth angle indicated by the horizontal coordinate and elevation angle by the vertical coordinate.

- **D-scope:** A C-scope in which the blips extend vertically to give a rough estimate of distance.

- **E-scope:** An intensity-modulated rectangular display with distance indicated by the horizontal coordinate and elevation angle by the vertical coordinate. Similar to the RHI in which target height or altitude is the vertical coordinate.

- **F-Scope:** A rectangular display in which a target appears as a centralized blip when the radar antenna is aimed at it. Horizontal and vertical aiming errors are respectively indicated by the horizontal and vertical displacement of the blip.

- **G-Scope:** A rectangular display in which a target appears as a laterally centralized blip when the radar antenna is aimed at it in azimuth, and wings appear to grow on the pip as the distance to the target is diminished; horizontal and vertical aiming errors are respectively indicated by horizontal and vertical displacement of the blip.

- **H-scope:** A B-scope modified to include indication of angle of elevation. The target appears as two closely spaced blips which approximate a short bright line, the slope of which is in proportion to the sine of the angle of target elevation.

- **I-scope:** A display in which a target appears as a complete circle when the radar antenna is pointed at it and in which the radius of the circle is proportional to target distance; incorrect aiming of the antenna changes the circle to a segment whose arc length is inversely proportional to the magnitude of the pointing error, and the position of the segment indicates the reciprocal of the pointing direction of the antenna.

- **J-scope:** A modified A-scope in which the time base is a circle and targets appear as radial deflections from the time base.

- **K-scope:** A modified A-scope in which a target appears as a pair of vertical deflections. When the radar antenna is correctly pointed at the target, the two deflections are of equal height, and when not so pointed, the difference in deflection amplitude is an indication of the direction and magnitude of the pointing error.

- **L-scope:** A display in which a target appears as two horizontal blips, one extending to the right from a central vertical time base and the other to the left; both blips are of equal amplitude when the radar is pointed directly at the target, any inequality representing relative pointing error, and distance upward along the baseline representing target distance.

- **M-scope:** A type of A-scope in which the target distance is determined by moving an adjustable pedestal signal along the baseline until it coincides with the horizontal position of the target signal deflections; the control which moves the pedestal is calibrated in distance.

- **N-scope**: A K-scope having an adjustable pedestal signal, as in the M-scope, for the measurement of distance.

- **O-scope:** An A-scope modified by the inclusion of an adjustable notch for measuring distance.

- **PPI or Plan Position Indicator (also called P-scope):** An intensity-modulated circular display on which echo signals produced from reflecting objects are shown in plan position with range and azimuth angle displayed in polar (rho-theta) coordinates, forming a map-like display. An offset, or off center, PPI has the zero position of the time base at a position other than at the center of the display to provide the equivalent of a larger display for a selected portion of the service area. A delayed PPI is one in which the initiation of the time base is delayed.

- **R-scope:** An A-scope with a segment of the time base expanded near the blip for greater accuracy in distance measurement.

- **RHI or Range-Height Indicator:** An intensity modulated display with height (altitude) as the vertical axis and range as the horizontal axis.

- PPI, A-scope, B-scope, and RHI are among the more usual displays employed in radar.

Duplexers

The duplexer is the device that 1) switches the radar antenna to either transmitter or the receiver 2) serves to protect the receiver from burnout or damage during the transmission.

Branch type Duplexer
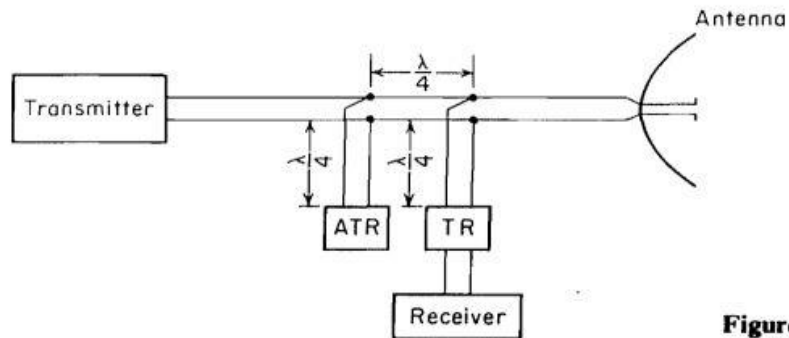
**Figure : Principle of branch-type duplexer**

- The branch-type duplexer, shown in above Fig is one of the earliest duplexer configurations. It consists of a TR (transmit-receive) switch and an ATR (anti-transmit receive) switch, both of which are gas-discharge tubes.

- When the transmitter is turned on, the TR and the ATR tubes ionize; that is, they break down, or fire. The TR in the fired condition acts as a short circuit to prevent transmitter power from entering the receiver.

- Since the TR is located a quarter wavelength from the main transmission line, it appears as a short circuit at the receiver but as an open circuit at the transmission line so that it does not impede the flow of transmitter power.

- Since the ATR is displaced a quarter wavelength from the main transmission line, the short circuit it produces during the fired condition appears as an open circuit on the transmission line and thus has no effect on transmission.

- During reception, the transmitter is off and neither the TR nor the ATR is fired.

- The open circuit of the ATR, being a quarter wave from the transmission line, appears as a short circuit across the line.

- Since this short circuit is located a quarter wave from the receiver branch line, the transmitter is effectively disconnected from the line and the echo signal power is directed to the receiver.

- The branch-type duplexer is of limited bandwidth and power handling capability, and has generally been replaced by the balanced duplexer and other protection devices. It is used, inspite of these limitations, in some low-cost radars.

- **Balanced duplexer**
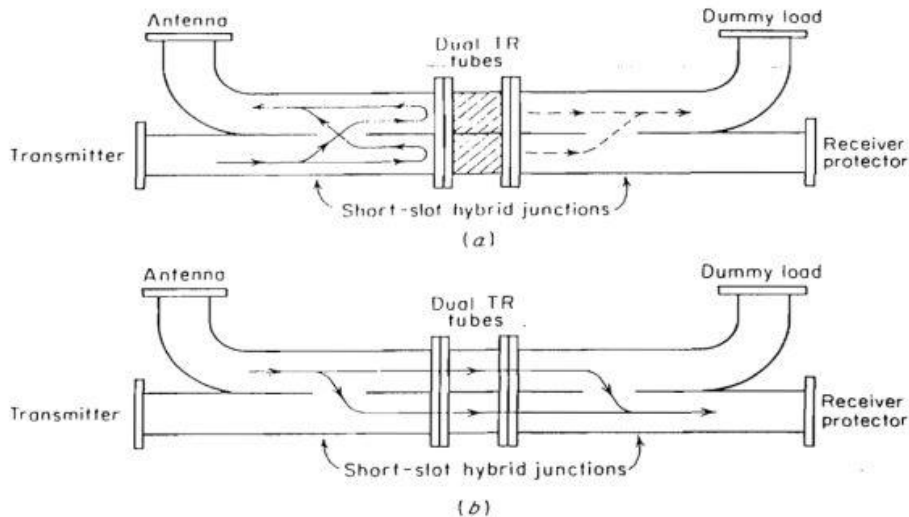
Figure : Balanced duplexer using dual TR tubes and two short-slot hybrid junctions.
(a) Transmit Condition (b) Receive condition.

- The balanced duplexer is based on the short-slot hybrid junction which consists of two sections of waveguides joined along one of their narrow walls with a slot cut in the common narrow wall to provide coupling between the two.

- The short-slot hybrid may be considered as a broadband directional coupler with a coupling ratio of 3 dB.

- In the transmit condition (Figure a) power is divided equally into each waveguide by the first short slot hybrid junction. Both TR tubes break down and reflect the incident power out the antenna arm as shown.

- *The short-slot hybrid has the property that each time the energy passes through the slot in either direction, its phase is advanced 90^0.* Therefore, the energy travels as indicated by the solid lines.

- Any energy which leaks through the TR tubes (shown by the dashed lines) is directed to the arm with the matched dummy load and not to the receiver. In addition to the attenuation provided by the TR tubes, the hybrid junctions provide an additional 20 to 30 dB of isolation.

- On reception the TR tubes are unfired and the echo signals pass through the duplexer and into the receiver as shown in Fig.6b. The power splits equally at the first junction and because of the 90^0 phase advance on passing through the slot, the energy recombines in the receiving arm and not in the dummy-load arm.

- The power-handling capability of the balanced duplexer is inherently greater than that of the branch-type duplexer and it has wide bandwidth, over ten percent with proper design. *A receiver protector,* is usually inserted between the duplexer and the receiver for added protection.
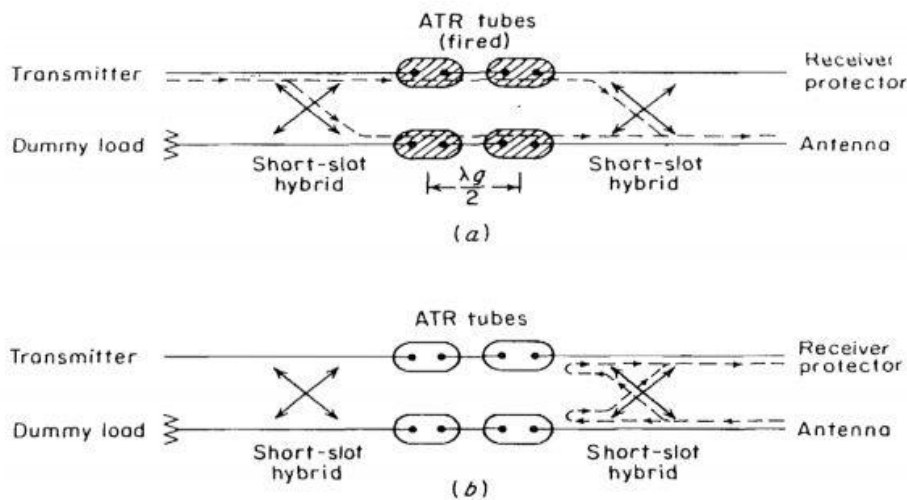
Balanced duplexer using ATR tubes.
(a) Transmit condition (b) receive condition

CIRCULATORS AS DUPLEXERS
CIRCULATOR AND RECEIVER PROTECTOR

- The ferrite circulator is a three or four-port device that can in principle, offer separation of the transmitter and receiver without the need for the conventional duplexer configurations explained earlier.

- The circulator does not provide sufficient protection by itself and requires a receiver protector like duplexers.

- The isolation between the transmitter and receiver ports of a circulator is seldom sufficient to protect the receiver from damage.

- However, it is not the isolation between transmitter and receiver ports that usually determines the amount of transmitter power at the receiver, but the impedance mismatch at the antenna which reflects transmitter power back into the receiver.

- The VSWR is a measure of the amount of power reflected by the antenna. For example, a VSWR of 1.5 means that about 4 percent of the transmitter power will be reflected by the antenna mismatch in the direction of the receiver, which corresponds to an isolation of only 14 dB. About 11 percent of the power is reflected when the VSWR is 2.0, corresponding to less than 10 dB of isolation. Thus, a receiver protector is almost always required.

- It also reduces to safe level radiations from nearby transmitters.

- The receiver protector might use solid-state diodes for an all solid-state configuration, or it might be a passive TR-limiter consisting of a radioactive primed TR-tube followed by a diode limiter.

- The ferrite circulator with receiver protector is attractive for radar applications because of its long life, wide bandwidth, and compact design.
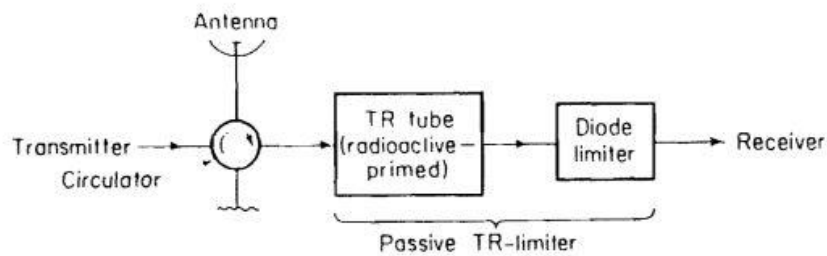

Figure : circulator and receiver protector. A four-port circulator is shown with the fourth port terminated in a matched load to provide greater isolation between the transmitter and the receiver than provided by a three-port circulator

Introduction to phased array antennas

- The phased array is a directive antenna made up of individual radiating antennas, or elements, which generate a radiation pattern whose shape and direction is determined by the relative phases and amplitudes of the currents at the individual elements.

- By properly varying the relative phases it is possible to steer the direction of the radiation.

- The radiating elements might be dipoles open-ended waveguides, slots cut in waveguide, or any other type of antenna.

- Became of interest to Radar due to the inherent flexibility it has offered in steering the beam by means of electronic control rather than by physical movement of the antenna.

- It has been considered in those radar applications where it is necessary to shift the beam rapidly from one position in space to another, or where it is required to obtain information about many targets at a flexible, rapid data rate.

- The full potential of a phased-array antenna requires the use of a computer that can determine in real time, on the basis of the actual operational situation, how best to use the capabilities offered by the array.

- Initially In World War 2, the United States, Great Britain, and Germany used radar with fixed phased-array antennas in which the beam was scanned by mechanically actuated phase shifters.

- A major advance in phased array technology was made in the early 1950s with the replacement of mechanically actuated phase shifters by electronic phase shifters.

- Frequency scanning in one angular coordinate was the first successful electronic scanning technique to be applied.

- The introduction of digitally switched phase shifters employing either ferrites or diodes in the early 1960s made a significant improvement in the practicality of phased arrays that could be electronically steered in two orthogonal angular coordinates.

Basic concept

- An array antenna consists of a number of individual radiating elements suitably spaced with respect to one another.

- Two common geometrical forms of array antennas used in radar are the linear array and the planar array.

Linear array

- A *linear array* consists of elements arranged in a straight line in one dimension.

- The linear array generates a fan beam when the phase relationships are such that the radiation is perpendicular to the array.

- When the radiation is at some angle other than broadside, the radiation pattern is a conical-shaped beam.

- The linear array can also act as a feed for a parabolic-cylinder antenna.

Planar array

- A *planar array* is a two dimensional configuration of elements arranged to lie in a plane. The planar array may be thought of as a linear array of linear arrays.

- The two-dimensional planar array is the most commonly used in radar applications since it is fundamentally the most versatile of all radar antennas.

- A rectangular aperture can produce a fan shaped beam. A square or a circular aperture produces a pencil beam.

- The array can be made to simultaneously generate many search and/or tracking beams with the same aperture.

Broadside array

- A *broadside array* is one in which the direction of maximum radiation is perpendicular, or almost perpendicular to the line (or plane) of the array.

- The broadside linear-array antenna may be used where broad coverage in one plane and narrow beam width in the orthogonal plane are desired.

End fire

- An *end fire* array has its maximum radiation parallel to the array.
- The end fire array is a special case of the linear or the planar array when the beam is directed along the array.
- End fire linear arrays have not been widely used in radar applications.
- They are usually limited to low or medium gains since an end fire linear antenna of high gain requires an excessively long array.
- Small end fire arrays are sometimes used as the radiating elements of a broadside array if directive elements are required.

Other types of array antennas

- An array whose elements are distributed on a non planar surface is called a *conformal array*.
- An array in which the relative phase shift between elements is controlled by electronic devices is called an *electronically scanned array*.
- In an electronically scanned array the antenna elements, the transmitters, the receivers, and the data-processing portions of the radar are often designed as a unit.

Radiation pattern

- Consider a linear array made up of **N** elements equally spaced a distance *d* apart as shown in *Fig.*
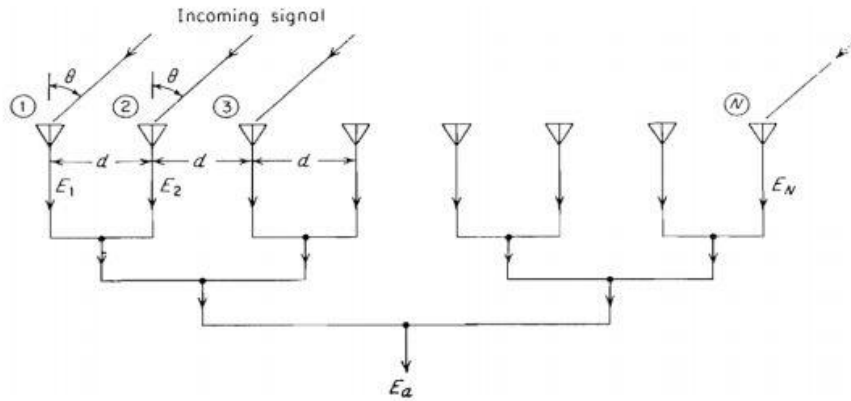

Figure : N-element linear array.

- The elements are assumed to be isotropic point sources radiating uniformly in all directions with equal amplitude and phase.

- The outputs of all the elements are summed via lines of equal length to give a sum output voltage E_a.

- Element **1** will be taken as the reference signal with zero phase. The difference in the phase of the signals in adjacent elements is $\Psi = 2\pi \, (d/\lambda) \, \sin \theta$, where θ is the direction of the incoming radiation.

- It is further assumed that the amplitudes and phases of the signals at each element are weighted uniformly. Therefore the amplitudes of the voltages in each element are the same and, for convenience, will be taken to be unity.

- The sum of all the voltages from the individual elements, when the phase difference between adjacent elements is Ψ, can be written as

$$E_a = \sin \omega t + \sin (\omega t + \psi) + \sin (\omega t + 2\psi) + \cdots + \sin \left\lfloor \omega t + (N-1)\psi \right\rfloor$$

where ω is the angular frequency of the signal. The sum can be written as

$$E_a = \sin \left\lfloor \omega t + (N-1)\frac{\psi}{2} \right\rfloor \frac{\sin (N\psi/2)}{\sin (\psi/2)}$$

- The first factor is a sine wave of frequency ω with a phase shift $(N - 1)\ \psi/2$. The second term represents the amplitude factor of the form **sin ($N\psi/2$)/sin ($\psi/2$)**. The field intensity pattern is the magnitude of the equation **2**, or

$$|E_a(\theta)| = \left| \frac{\sin\left[N\pi(d/\lambda)\sin\theta\right]}{\sin\left[\pi(d/\lambda)\sin\theta\right]} \right|$$

- The pattern has nulls whenever the numerator is zero.

- $N\Pi(d/\lambda)\sin\theta = 0, \pm\Pi, \pm 2\Pi..., \pm n\Pi$, where n = integer. The denominator, however, is zero when $\Pi(d/\lambda)\sin\theta, = 0, \pm\Pi, \pm 2\Pi..., \pm n\Pi$. Note that when the denominator is zero, the numerator is also zero.

- The value of the field intensity pattern is indeterminate when both the denominator and numerator are zero. However, by applying L'Hopital's rule (differentiating numerator and denominator separately) it is found that ǀEa ǀis a maximum whenever $\sin\theta = \pm n\lambda/d$.

- These maxima all have the same value and are equal to N. The maximum at $\sin\theta = 0$ defines the main beam. The other maxima are called grating lobes. They are generally undesirable and are to be avoided.

- The radiation pattern is equal to the normalized square of the amplitude, or

$$G_a(\theta) = \frac{|E_a|^2}{N^2} = \frac{\sin^2\left[N\pi(d/\lambda)\sin\theta\right]}{N^2\sin^2\left[\pi(d/\lambda)\sin\theta\right]}$$

- When directive elements are used, the resultant array antenna radiation pattern is

$$G(\theta) = G_e(\theta)\frac{\sin^2\left[N\pi(d/\lambda)\sin\theta\right]}{N^2\sin^2\left[\pi(d/\lambda)\sin\theta\right]} = G_e(\theta)G_a(\theta)$$

- where Ge (θ) is the radiation pattern of an individual element. The resultant radiation pattern is the product of the element factor Ge(θ) and the array factor Ga(θ).

- Grating lobes caused by a widely spaced array may therefore be eliminated with directive elements which radiate little or no energy in the directions of the undesired lobes.

- For example, when the element spacing $d = 2\lambda$, grating lobes occur at $\theta = \pm30°$ and $\pm90°$ in addition to the main beam at $\theta = 0°$. If the individual elements have a beamwidth somewhat less than 60°, the grating lobes of the array factor will be suppressed.

- In a two-dimensional, rectangular planar array, the radiation pattern may sometimes be written as the product of the radiation patterns in the two planes which contain the principal axes of the antenna.

- If the radiation patterns in the two principal planes are $G_1(\theta_e)$ and $G_2(\theta_a)$ the two-dimensional antenna pattern is

$$G(\theta_e, \theta_a) = G_1(\theta_e)G_2(\theta_a)$$

- Thus, the normalized radiation pattern of a uniformly illuminated rectangular array is

$$G(\theta_e, \theta_a) = \frac{\sin^2\left[N\pi(d/\lambda)\sin\theta_a\right]}{N^2\sin^2\left[\pi(d/\lambda)\sin\theta_a\right]} \frac{\sin^2\left[M\pi(d/\lambda)\sin\theta_e\right]}{M^2\sin^2\left[\pi(d/\lambda)\sin\theta_e\right]}$$

- Where N = number of radiating elements in θ_a dimension with spacing d and M the number in θ_e dimension.

Beam steering phased array antennas

- The beam of an array antenna may be steered rapidly in space without moving large mechanical masses by properly varying the phase of the signals applied to each element.

- Consider an array of equally spaced elements. The spacing between adjacent elements is d, and the signals at each element are assumed to be of equal amplitude.

- If the same phase is applied to all elements, the relative phase difference between adjacent elements is zero and the position of the main beam will be broadside to the array at an angle $\theta = 0$.

- The main beam will point in a direction other than broadside if the relative phase difference between elements is other than zero.

- The direction of the main beam is at an angle θ_0, when the phase difference is $\emptyset = 2\pi (d/\lambda) \sin \theta_0$. The phase at each element is therefore $(\emptyset_c + m \emptyset)$ where $m = 0, 1, 2 \ldots (N - I)$ and $\emptyset_c$ is any constant phase applied to all elements.

- The normalized radiation pattern of the array when the phase difference between adjacent elements is **Ø** is given by:

$$G(\theta) = \frac{\sin^2\left[N\pi(d/\lambda)(\sin\theta - \sin\theta_0)\right]}{N^2 \sin^2\left[\pi(d/\lambda)(\sin\theta - \sin\theta_0)\right]}$$

- The maximum of the radiation pattern occurs when   **$\sin\theta = \sin\theta_0$**
- The above Equation states that the main beam of the antenna pattern may be positioned to an angle **θ_0** by the insertion of the proper phase shift **Ø** at each element of the array. If variable, rather than fixed, phase shifters are used, the beam may be steered as the relative phase between elements is changed
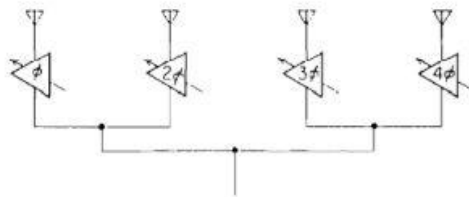

FIG: Steering of an antenna beam with variable phase shifters (parallel-fed array).

- Using an argument similar to the non scanning array described previously, grating lobes appear at an angle **θ_g** whenever the denominator is zero, or when

$$\pi\frac{d}{\lambda}\left(\sin\theta_g - \sin\theta_0\right) = \pm n\pi$$

or

$$\left|\sin\theta_g - \sin\theta_0\right| = \pm n\frac{\lambda}{d}$$

- If a grating lobe is permitted to appear at -90^0 when the main beam is steered to $+90°$, it is found from the above that **$d = \lambda/2$.**

- Thus the element spacing must not be larger than a half wavelength if the beam is to be steered over a wide angle without having undesirable *grating* lobes appear.

- Practical array antennas do not scan +/- 90°. If the scan is limited to +/- 60^0 the element spacing should be less than **0.54λ.**

Change of beam width with steering angle

- The half-power beam width in the plane of scan increases as the beam is scanned off the broadside direction. The beam width is approximately inversely proportional to **cos θ_0**, where θ_0 is the angle measured from the normal to the antenna.

- This may be proved by assuming that the sine in the denominator of **G(θ)** discussed earlier can be replaced by its argument, so that the radiation pattern is of the form $(\sin^2 u)/u^2$, where $u = N\Pi (d/\lambda)(\sin\theta - \sin\theta o)$.

- The $(\sin^2 u)/u^2$ antenna pattern is reduced to half its maximum value when $u = \pm 0.443\Pi$. Denote by $\theta+$ the angle corresponding to the half-power point when $\theta > \theta o$, and $\theta-$, the angle corresponding to the half-power point when $\theta < \theta o$; that is, $\theta+$ corresponds to $u = +0.443\Pi$ and $\theta-$ to $u = -0.443\Pi$.

- The $\sin\theta - \sin\theta o$, term in the expression for u can be written

$$\sin \theta - \sin \theta_0 = \sin (\theta - \theta_0) \cos \theta_0 - [1 - \cos (\theta - \theta_0)] \sin \theta_0$$

- The second term on the right-hand side of Eq. above can be neglected when θo is small (beam is near broadside), so that

$$\sin \theta - \sin \theta_0 \approx \sin (\theta - \theta_0) \cos \theta_0$$

- Using the above approximation, the two angles corresponding to the 3-dB points of the antenna pattern are

$$\theta_+ - \theta_0 = \sin^{-1} \frac{0.443\lambda}{Nd \cos \theta_0} \approx \frac{0.443\lambda}{Nd \cos \theta_0}$$

$$\theta_- - \theta_0 = \sin^{-1} \frac{-0.443\lambda}{Nd \cos \theta_0} \approx \frac{-0.443\lambda}{Nd \cos \theta_0}$$

- The half-power beamwidth is

$$\theta_B = \theta_+ - \theta_- \approx \frac{0.886\lambda}{Nd \cos \theta_0}$$

- Therefore, when the beam is positioned an angle θo off broadside, the beamwidth in the plane of scan increases as $(\cos \theta o)^{-1}$.

- The variation of the beam shape with scan angle is graphically shown in **Fig below**.

- The antenna radiation pattern is plotted in spherical coordinates as a function of the two direction cosines, **cos α_x** and **cos α_y** of the radius vector specifying the point of observation.

- The angle **Ø** is measured from the **cos α_x** axis, and **θ** is measured from the axis perpendicular to the **cos α_x** and **cos α_y** axes.

- In **Fig.** **Ø** is taken to be a constant value of **90^0** and the beam is scanned in the **θ** coordinate.

- At **θ= 0** (beam broad side to the array) a symmetrical pencil beam of half-power width **B_0** is assumed.

- The shape of the beam at the other angular positions is the projection of the circular beam shape on the surface of the unit sphere.

- It can be seen that as the beam is scanned in the **θ** direction, it broadens in that direction, but is constant in the **Ø** direction.

- For **θ≠0,** the beam shape is not symmetrical about the center of the beam, but is eccentric.
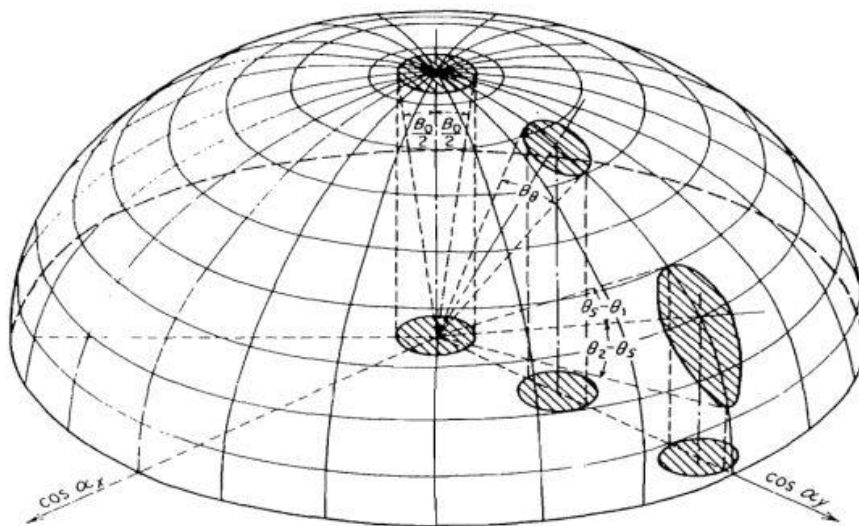
Beam width and eccentricity of the scanned beam

Applications of phased array antennas

The phased array antenna has seen application in radar for a wide variety of purposes:

- Aircraft surveillance from on board ship
- Satellite surveillance
- Ballistic missile defense
- Air defense
- Aircraft landing systems
- Mortar and artillery location
- Tracking of ballistic missiles and Airborne bomber radar (EAR).
- Many developmental array radars have been developed and built in USA. Although much effort and funds have been spent on this activity, *except for limited-scan arrays* there has not been any large serial production of such radars compared to the serial production of radars with mechanically rotating reflector antennas.

Advantages phased array antennas.

- **Inertialess rapid beam-steering:** The beam from an array can be scanned, or switched from one position to another, in a time limited only by the switching speed of the phase shifters. Typically, the beam can be switched in several microseconds, but it can be considerably shorter if desired.

- **Multiple, independent beams:** A single aperture can generate many simultaneous independent beams. Alternatively, the same effect can be obtained by rapidly switching a single beam through a sequence of positions.

- **Potential for large peak and / or average power:** If necessary, each element of the array can be fed by a separate high-power transmitter with the combining of the outputs made in space to obtain a total power greater than can be obtained from a single transmitter.

- **Control of the radiation pattern**. A particular radiation pattern may be more readily obtained with the array than with other microwave antennas since the amplitude and phase of each array element may be individually controlled. Thus, radiation patterns with extremely low side lobes or with a shaped main beam may be achieved conveniently. Separate monopulse sum and difference patterns, each with it's own optimum shape, can also be generated.

- **Graceful degradation**. The distributed nature of the array means that it can fail only gradually and not l at once (catastrophically).

- **Convenient aperture shape**. The shape of the array permits flush mounting and it can be strengthened to resist blast.

- **Electronic beam stabilization**. The ability to steer the beam electronically can be used to stabilize the beam direction when the radar is on an unstable platform, such as a ship or aircraft that is subject to roll, pitch, and yaw disturbances.

Limitations of phased array antennas

1. The major limitation that has limited the widespread use of the conventional phased array in radar is its high cost, which is due in large part to its complexity.

2. When graceful degradation has gone too far a seperate maintenance is needed.

3. When a planar array is electronically scanned, the change of mutual coupling that accompanies a change in beam position makes the maintenance of low sidelobes more difficult.

4. Although the array has the potential for radiating large power, it is seldom that an array is required to radiate more power than can be radiated by other antenna types or to utilize a total power which cannot possibly be generated by current high-power microwave tube technology that feeds a single transmission line.

Series Vs Parallel feeds

- The phase relationship between the adjacent elements of the array can be obtained with either series fed or parallel fed arrangement.
- In series fed arrangement, the energy may be transmitted from one end to the line or it may be fed from the center out to each end.
- The adjacent elements are connected by a phase shifter with phase ϕ.
- All the phase shifters are identical and introduce the same amount of phase shift which is less than 2Π radians.
- In parallel fed the energy to be radiated is divided between the elements by a power splitter.
- When a series of power splitters are used to create a tree like structure is called a corporate feed.
- Equal lengths of line transmit the energy to each element so that no unwanted phase difference are introduced by the lines themselves.
- The maximum phase change required of each phase shifter in the parallel-fed array is many times 2Π *radians*.

- In a series-fed array containing N phase shifters, the signal suffers the insertion loss of a single phase shifter N times. In a parallel-fed array the insertion loss of the p h s e shifter is introduced effectively but once.

- Since each phase shifter in the series-fed linear array of below Fig has **the same value of** phase shift, only a single control signal is needed to steer the beam. The N-element parallel-fed linear array similar to that of below Fig requires a separate control signal for each phase shifter or N - **1 total.**

Parallel Feed
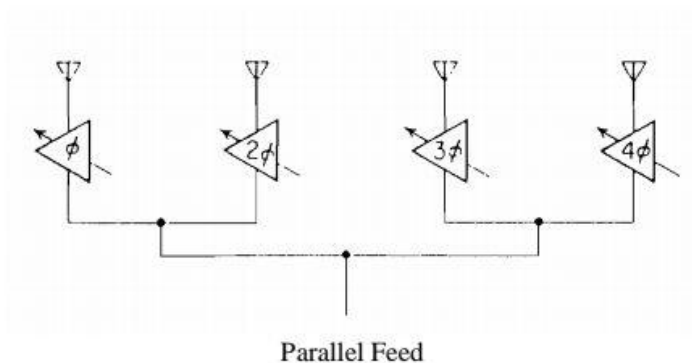
Series arrangements for applying phase relationships in an array.
(a) fed from one end; *(b) center-fed.*

(a)

(b)