Annamacharya Institute of Technology and Sciences::Tirupati
Department of Civil Engineering

Fluid Mechanics

UNIT 1

Introduction to Fluid Mechanics Part-1

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Definition

- **Mechanics** is the oldest physical science that deals with both stationary and moving bodies under the influence of forces.
- The branch of mechanics that deals with bodies at rest is called **statics**, while the branch that deals with bodies in motion is called **dynamics**.
- The subcategory **fluid mechanics** is defined as the science that deals with the behavior of fluids at rest (*fluid statics)* or in motion (*fluid dynamics), and the interaction of fluids with solids* or other fluids at the boundaries.
- The study of fluids at rest is called **fluid statics**.

Definition

- The study of f1uids in motion, where pressure forces are not considered, is called **fluid kinematics** and if the pressure forces are also considered for the fluids in motion. that branch of science is called **fluid dynamics**.
- Fluid mechanics itself is also divided into several categories.
- The study of the motion of fluids that are practically incompressible (such as liquids, especially water, and gases at low speeds) is usually referred to as **hydrodynamics.**
- A subcategory of hydrodynamics is **hydraulics,** which deals with liquid flows in pipes and open channels.

Definition

- **Gas dynamics** deals with the flow of fluids that undergo significant density changes, such as the flow of gases through nozzles at high speeds.
- The category **aerodynamics** deals with the flow of gases (especially air) over bodies such as aircraft, rockets, and automobiles at high or low speeds.
- Some other specialized categories such as **meteorology, oceanography, and hydrology** deal with naturally occurring flows.

What is a Fluid?

- A substance exists in three primary phases: solid, liquid, and gas. A substance in the liquid or gas phase is referred to as a **fluid.**
- Distinction between a solid and a fluid is made on the basis of the substance's ability to resist an applied shear (or tangential) stress that tends to change its shape.
- A solid can resist an applied shear stress by deforming, whereas a fluid deforms continuously under the influence of shear stress, no matter how small.
- In solids stress is proportional to *strain, but in fluids stress is proportional* to *strain rate*.

What is a Fluid?

- Differences between liquid and gases

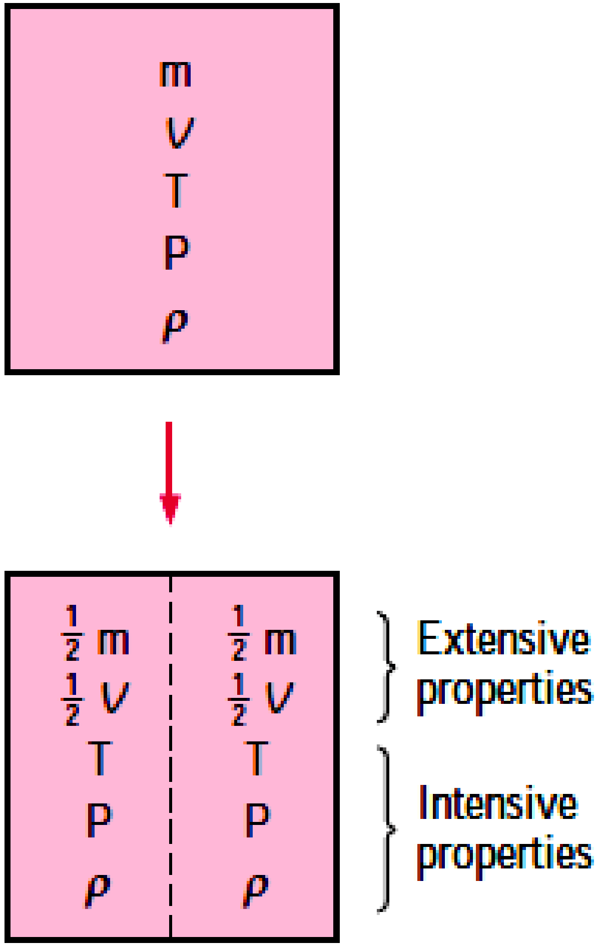
Liquid	Gases
Difficult to compress and often regarded as incompressible	Easily to compress – changes of volume is large, cannot normally be neglected and are related to temperature
Occupies a fixed volume and will take the shape of the container	No fixed volume, it changes volume to expand to fill the containing vessels
A free surface is formed if the volume of container is greater than the liquid.	Completely fill the vessel so that no free surface is formed.

Properties of Fluids

- Any characteristic of a system is called a **property.**
- Some familiar properties are pressure *P*, temperature T, volume V, and mass m.
- Other less familiar properties include viscosity, thermal conductivity, modulus of elasticity, thermal expansion coefficient, electric resistivity, and even velocity and elevation.
- Properties are considered to be either ***intensive or extensive***.
- ***Intensive*** properties are those that are independent of the mass of a system, such as temperature, pressure, and density.
- **Extensive** properties are those whose values depend on the size—or extent—of the system. Total mass, total volume *V*, and total momentum are some examples of extensive properties.

Properties of Fluids

- An easy way to determine whether a property is intensive or extensive is to divide the system into two equal parts with an imaginary partition.
- Each part will have the same value of intensive properties as the original system, but half the value of the extensive properties.



Properties of Fluids

Density or Mass Density

- Density or mass density of a fluid is defined as the ratio of the mass of a f1uid to its volume. Thus mass per unit volume of a fluid is called density. It is denoted the symbol ρ (rho). The unit of mass density in SI unit is kg per cubic meter, i.e ., kg/m^3.
- The density of liquids may be considered as constant while that of gases changes with the variation of pressure and temperature.
- Mathematically mass density is written as.

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}$$

- The value of density of water is 1 gm/cm^3 or 1000 kg/m^3.

Properties of Fluids

Density or Mass Density

- The density of a substance, in general, depends on temperature and pressure.
- The density of most gases is proportional to pressure and inversely proportional to temperature.
- Liquids and solids, on the other hand, are essentially incompressible substances, and the variation of their density with pressure is usually negligible.

Properties of Fluids

Specific weight or Weight Density

- Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume.
- Thus weight per unit volume of a fluid is called weight density and it is denoted by the symbol w.
- Mathematically,

$$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{(\text{Mass of fluid}) \text{ x Acceleration due to gravity}}{\text{Volume of fluid}}$$

$$= \frac{\text{Mass of fluid x g}}{\text{Volume of fluid}}$$

$$= \rho \text{ x g}$$

$$w = \rho g$$

Properties of Fluids

Specific Volume

- Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume.
- Mathematically, it is expressed as

$$\text{Specific volume} = \frac{\text{Volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\dfrac{\text{Mass of fluid}}{\text{Volume}}} = \frac{1}{\rho}$$

- Thus specific volume is the reciprocal of mass density. It is expressed as m^3/kg.
- It is commonly applied to gases.

Properties of Fluids

Specific Gravity.

- Specific gravity is defined as the ratio of the weight density (or density) of a fluid to the weight density (or density) of a standard fluid.
- For liquids, the standard fluid is taken water and for gases, the standard fluid is taken air. Specific gravity is also called relative density. It is dimensionless quantity and is denoted by the symbol *S*.

$$S(\text{for liquids}) = \frac{\text{Weight density (density) of liquid}}{\text{Weight density (density) of water}}$$

$$S(\text{for gases}) = \frac{\text{Weight density (density) of gas}}{\text{Weight density (density) of air}}$$

$$\text{Thus weight density of a liquid} = S \text{ x Weight density of water}$$

$$= S \text{ x } 1000 \text{ x } 9.81 \text{N/m}^3$$

$$\text{Thus density of a liquid} = S \text{ x Density of water}$$

$$= S \text{ x } 1000 \text{kg/m}^3$$

Properties of Fluids

Specific Gravity.

- If the specific gravity of a fluid is known, then the density of the fluid will be equal to specific gravity of fluid multiplied by the density of water.
- For example the specific gravity of mercury is 13.6, hence density of mercury = 13.6 x 1000 = 13600 kg/m^3.

Specific gravities of some substances at 0°C

Substance	SG
Water	1.0
Blood	1.05
Seawater	1.025
Gasoline	0.7
Ethyl alcohol	0.79
Mercury	13.6
Wood	0.3–0.9
Gold	19.2
Bones	1.7–2.0
Ice	0.92
Air (at 1 atm)	0.0013

Properties of Fluids

Example 1.

Calculate the specific weight, density and specific gravity of one liter of a liquid which weighs 7 N.

Solution. Given :

$$\text{Volume} = 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \quad \left(\because\ 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \text{ or } 1 \text{ litre} = 1000 \text{ cm}^3\right)$$

$$\text{Weight} = 7 \text{ N}$$

(*i*) Specific weight (w)

$$= \frac{\text{Weight}}{\text{Volume}} = \frac{7 \text{ N}}{\left(\frac{1}{1000}\right)\text{m}^3} = \mathbf{7000\ N/m^3.\ Ans.}$$

(*ii*) Density (ρ)

$$= \frac{w}{g} = \frac{7000}{9.81} \text{ kg/m}^3 = \mathbf{713.5\ kg/m^3.\ Ans.}$$

(*iii*) Specific gravity

$$= \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{713.5}{1000} \quad \{\because\ \text{Density of water} = 1000 \text{ kg/m}^3\}$$

$$= \mathbf{0.7135.\ Ans.}$$

Example 2. *Calculate the density, specific weight and weight of one liter of petrol of specific gravity = 0.7*

Solution. Given : Volume = 1 litre = $1 \times 1000\ cm^3 = \frac{1000}{10^6}\ m^3 = 0.001\ m^3$

Sp. gravity $S = 0.7$

(*i*) *Density* (ρ)

Density (ρ) $= S \times 1000\ kg/m^3 = 0.7 \times 1000 =$ **700 kg/m³. Ans.**

(*ii*) *Specific weight* (*w*)

$w = \rho \times g = 700 \times 9.81\ N/m^3 =$ **6867 N/m³. Ans.**

(*iii*) *Weight* (*W*)

We know that specific weight $= \frac{\text{Weight}}{\text{Volume}}$

or $w = \frac{W}{0.001}$ or $6867 = \frac{W}{0.001}$

∴ $W = 6867 \times 0.001 =$ **6.867 N. Ans.**

Properties of Fluids

Viscosity

- Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.
- When two layers of a fluid, a distance *'dy' apart move one* over the other at different velocities say u and u+ *du as shown in Fig. 1.1 , the viscosity together with* relative velocity causes a shear stress acting between the fluid layers:

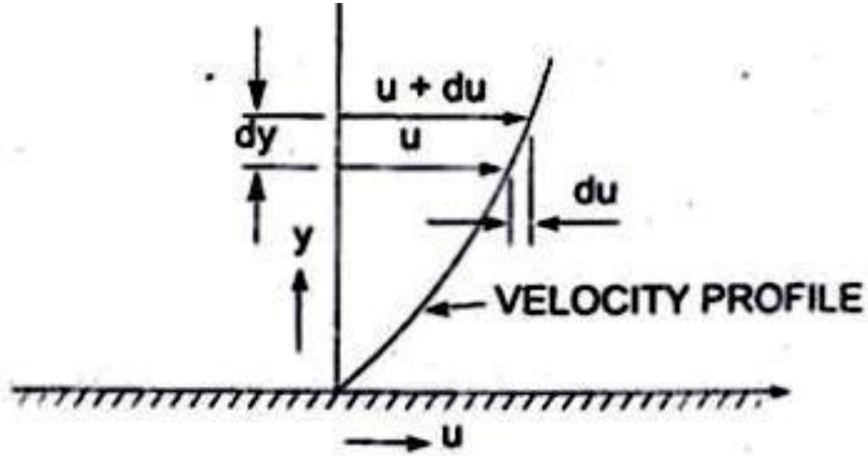


Fig. 1.1 *Velocity variation near a solid boundary.*

Properties of Fluids

Viscosity

- The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer.
- This shear stress is proportional to the rate of change of velocity with respect to *y. It is denoted by symbol* τ called Tau.
- Mathematically,

$$\tau \propto \frac{du}{dy}$$

- or

$$\boxed{\tau = \mu \frac{du}{dy} \qquad (1.2)}$$

Properties of Fluids

- where μ (called mu) *is the constant of proportionality and is known as the coefficient of dynamic viscosity* or only viscosity.
- $\frac{du}{dy}$ *represents the rate of shear strain or rate of shear deformation or velocity gradient.*
- *From equation (1.2) we have*

$$\boxed{\mu = \frac{\tau}{\frac{du}{dy}} \qquad (1.3)}$$

- Thus viscosity is also defined as the shear stress required to produce unit rate of shear strain.

Properties of Fluids

Unit of Viscosity.

- The unit of viscosity is obtained by putting the dimension of the quantities in equation (1.3)

$$\mu = \frac{\text{Shear stress}}{\dfrac{\text{Change of velocity}}{\text{Change of distance}}} = \frac{\text{Force/Area}}{\left(\dfrac{\text{Length}}{\text{Time}}\right) \times \dfrac{1}{\text{Length}}}$$

$$= \frac{\text{Force/(length)}^2}{\dfrac{1}{\text{Time}}} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2}$$

$$\text{SI unit of viscosity} = \frac{\text{Newton second}}{\text{m}^2} = \frac{\text{Ns}}{\text{m}^2}$$

Properties of Fluids

Kinematic Viscosity.

- It is defined as the ratio between the dynamic viscosity and density of fluid.lt is denoted by the Greek symbol (ν) called 'nu' . Thus, mathematically,

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho}$$

- The SI unit of kinematic viscosity is m^2/s.

Newton's Law of Viscosity.

- It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient viscosity. Mathematically, it is expressed as given by equation (1 . 2).

Properties of Fluids

- Fluids which obey the above relation are known as Newtonian fluids and the fluids which do not obey the above relation are called Non-newtonian fluids.

Variation of Viscosity with Temperature

- Temperature affects the viscosity.
- The viscosity of liquids decreases with the increase of temperature while the viscosity of gases increases with increase of temperature. This is due to reason that the viscous forces in a fluid are due to cohesive forces and *molecular momentum transfer.*
- *In liquids the cohesive forces predominates the molecular momentum* transfer due to closely packed molecules and with the increase in temperature, the cohesive forces decreases with the result of decreasing viscosity.

Types of Fluids

1. Ideal Fluid. A fluid, which is incompressible and is having no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity.
2. Real fluid. A fluid, which possesses viscosity, is known as real fluid. All the fluids: in actual practice, are real fluids.
3. Newtonian Fluid. A real fluid, in which the shear stress is directly, proportional to the rate of shear strain (or velocity gradient), is known as a Newtonian fluid.
4. Non-Newtonian fluid. A real fluid, in which shear stress is not proportional to the rate of shear strain (or velocity gradient), known as a Non-Newtonian fluid.

Types of Fluids

5. Ideal Plastic Fluid.

A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain (or velocity gradient), is known as ideal plastic fluid.

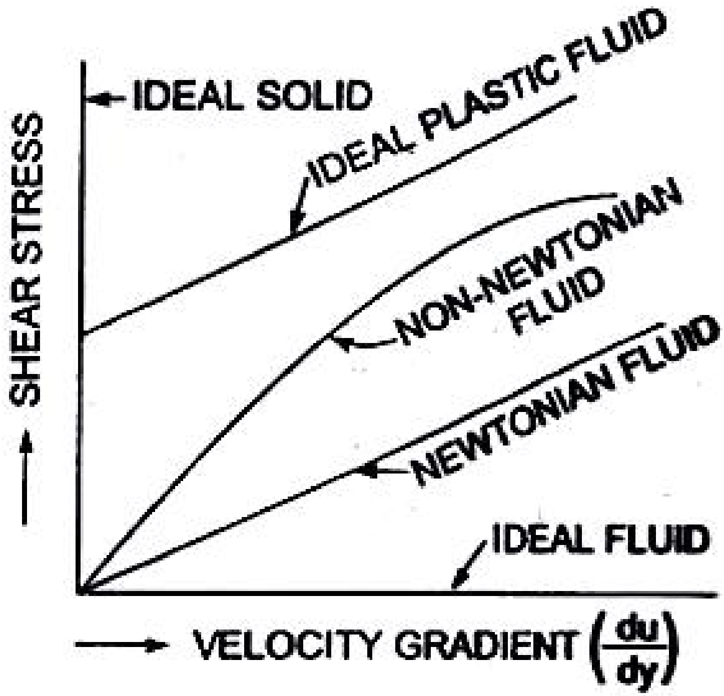


Fig. 1.2 *Types of fluids.*

Example 3

If the velocity distribution over a plate is given by

$$u = \frac{2}{3}y - y^2$$

in which u is velocity in metre per second at a distance y metre above the plate, determine the shear stress at y = 0 and y= 0.15 m. Take dynamic viscosity of fluid as 8.63 poises.

Solution. Given : $u = \frac{2}{3}y - y^2 \quad \therefore \quad \frac{du}{dy} = \frac{2}{3} - 2y$

$$\left(\frac{du}{dy}\right)_{\text{at } y=0} \text{ or } \left(\frac{du}{dy}\right)_{y=0} = \frac{2}{3} - 2(0) = \frac{2}{3} = 0.667$$

Also $$\left(\frac{du}{dy}\right)_{\text{at } y=0.15} \text{ or } \left(\frac{du}{dy}\right)_{y=0.15} = \frac{2}{3} - 2 \times .15 = .667 - .30 = 0.367$$

Value of $\mu = 8.63$ poise $= \frac{8.63}{10}$ SI units $= 0.863$ N s/m^2

Now shear stress is given by equation (1.2) as $\tau = \mu \frac{du}{dy}$.

(*i*) Shear stress at $y = 0$ is given by

$$\tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0} = 0.863 \times 0.667 = \mathbf{0.5756\ N/m^2}. \textbf{ Ans.}$$

(*ii*) Shear stress at $y = 0.15$ m is given by

$$(\tau)_{y=0.15} = \mu \left(\frac{du}{dy}\right)_{y=0.15} = 0.863 \times 0.367 = \mathbf{0.3167\ N/m^2}. \textbf{ Ans.}$$

Example 4

Calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size 0.8 m x 0.8 m and an inclined plane with angle of inclination 30° as shown in Fig. 1.4. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of oil film is 1.5 mm.

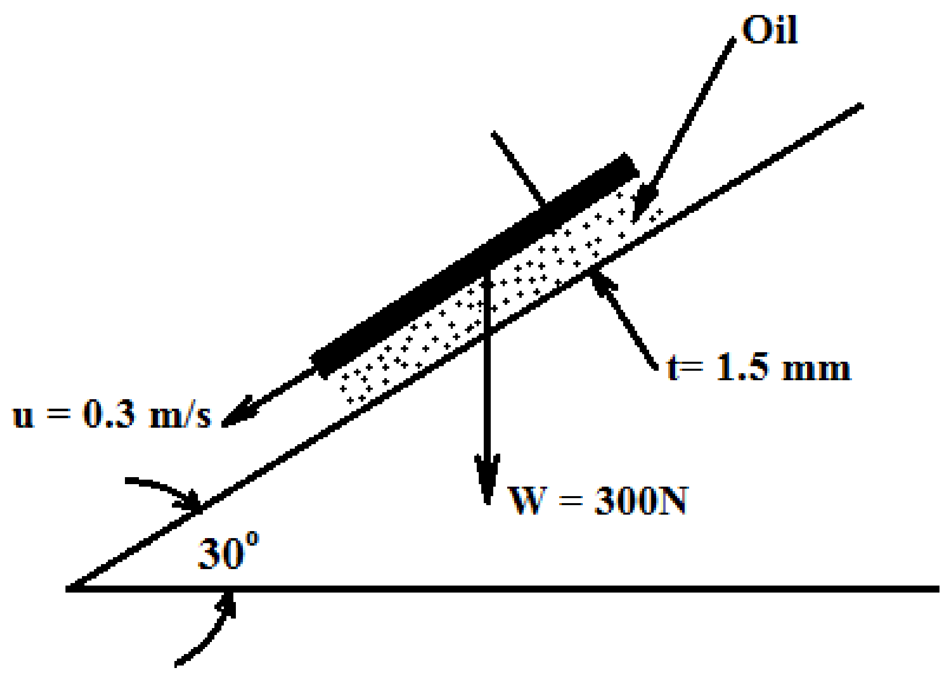


Fig.1.4

Solution. Given :

Area of plate, $A = 0.8 \times 0.8 = 0.64 \text{ m}^2$

Angle of plane, $\theta = 30°$

Weight of plate, $W = 300 \text{ N}$

Velocity of plate, $u = 0.3 \text{ m/s}$

Thickness of oil film, $t = dy = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$

Let the viscosity of fluid between plate and inclined plane is μ.

Component of weight W, along the plane $= W \cos 60° = 300 \cos 60° = 150 \text{ N}$

Thus the shear force, F, on the bottom surface of the plate $= 150 \text{ N}$

and shear stress, $\tau = \dfrac{F}{\text{Area}} = \dfrac{150}{0.64} \text{ N/m}^2$

Now using equation (1.2), we have

$$\tau = \mu \frac{du}{dy}$$

where du = change of velocity $= u - 0 = u = 0.3 \text{ m/s}$

$$dy = t = 1.5 \times 10^{-3} \text{ m}$$

$$\therefore \quad \frac{150}{0.64} = \mu \frac{0.3}{1.5 \times 10^{-3}}$$

$$\therefore \quad \mu = \frac{150 \times 1.5 \times 10^{-3}}{0.64 \times 0.3} = 1.17 \text{ N s/m}^2 = 1.17 \times 10 = \mathbf{11.7 \text{ poise. Ans.}}$$

Example 5

The space between two square flat parallel plates is filled with oil. Each side of the plate is 60 cm. The thickness of the oil film is 12.5 mm. The upper plate, which moves at 2.5 metre per sec requires a force of 98.1 N to maintain the speed. Determine : ·

i. the dynamic viscosity of the oil, and

ii. the kinematic viscosity of the oil if the specific gravity of the oil is 0.95.

Solution. Given:

Each side of a square plate = 60 cm = 0.6 m

Area $A = 0.6 \times 0.6 = 0.36\ m^2$

Thickness of oil film $dy = 12.5\ mm = 12.5 \times 10^{-3}\ m$

Velocity of upper plate $u = 2.5\ m/s$

$\therefore$ Change of velocity between plates, $du = 2.5$ m/sec

Force required on upper plate, $F = 98.1$ N

$\therefore$ Shear stress, $$\tau = \frac{\text{Force}}{\text{Area}} = \frac{F}{A} = \frac{98.1\text{ N}}{0.36\text{ m}^2}$$

(*i*) Let μ = Dynamic viscosity of oil

Using equation (1.2), $$\tau = \mu \frac{du}{dy} \text{ or } \frac{98.1}{0.36} = \mu \times \frac{2.5}{12.5 \times 10^{-3}}$$

$$\therefore \quad \mu = \frac{98.1}{0.36} \times \frac{12.5 \times 10^{-3}}{2.5} = 1.3635 \frac{\text{Ns}}{\text{m}^2} \quad \textbf{Ans.}$$

(*ii*) Sp. gr. of oil, S = 0.95

Let ν = kinematic viscosity of oil

Using equation (1.1 A),

Mass density of oil, $$\rho = S \times 1000 = 0.95 \times 1000 = 950 \text{ kg/m}^3$$

Using the relation, $\nu = \frac{\mu}{\rho}$, we get $$\nu = \frac{1.3635\left(\frac{\text{Ns}}{\text{m}^2}\right)}{950} = .001435 \text{ m}^2\text{/sec} \quad \textbf{Ans.}$$

Compressibility and Bulk Modulus

- Compressibility is the reciprocal of the bulk modulus of elasticity, K which is defined as the ratio of compressive stress to volumetric strain.
- Consider a cylinder fitted with a piston as shown in the Fig.
- Let V= *Volume of a gas enclosed in the cylinder*

  p =*Pressure of gas when volume is* V
- Let the pressure is increased *to* $p+dp$, *the volume of gas* decreases from V *to* $V - dV$.
- Then increase in pressure = dp
- Decrease in volume = dV
- Volumetric strain = - dV/V

End of Chapter 1

Thank you

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Fluid Mechanics

UNIT 1

Introduction to Fluid Mechanics Part-2

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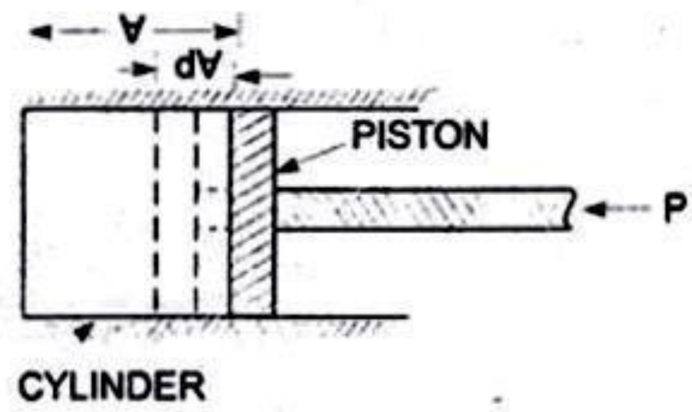
Compressibility and Bulk Modulus

- Compressibility is the reciprocal of the bulk modulus of elasticity, K which is defined as the ratio of compressive stress to volumetric strain.
- Consider a cylinder fitted with a piston as shown in the Fig.
- Let *V= Volume of a gas enclosed in the cylinder*

  p =Pressure of gas when volume is V
- Let the pressure is increased *to p+ dp, the volume of gas* decreases from *V to V – dV.*
- Then increase in pressure = dp
- Decrease in volume = dV
- Volumetric strain = - dV/V

Compressibility and Bulk Modulus

- - ve sign means the volume decreases with increase of pressure.



$$\therefore \text{Bulk modules} \quad K = \frac{\text{Increase of pressure}}{\text{Volumetric strain}}$$

$$= \frac{dp}{\frac{-dV}{V}} = -\frac{dp}{dV}V$$

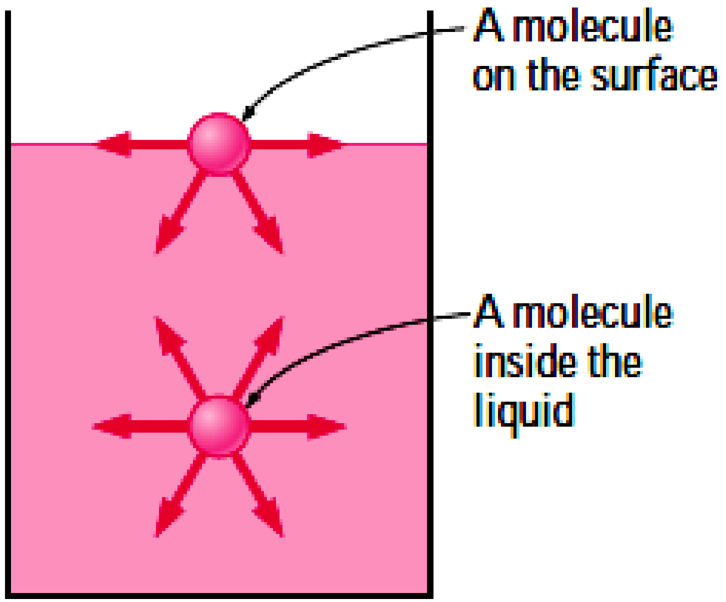
- Compressibility is given by = 1/K

Surface Tension and Capillarity

- Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.
- Surface tension is created due to the unbalanced cohesive forces acting on the liquid molecules at the fluid surface.
- Molecules in the interior of the fluid mass are surrounded by molecules that are attracted to each other equally.
- However, molecules along the surface are subjected to a net force toward the interior.
- The apparent physical consequence of this unbalanced force along the surface is to create the hypothetical skin or membrane.

Surface Tension and Capillarity

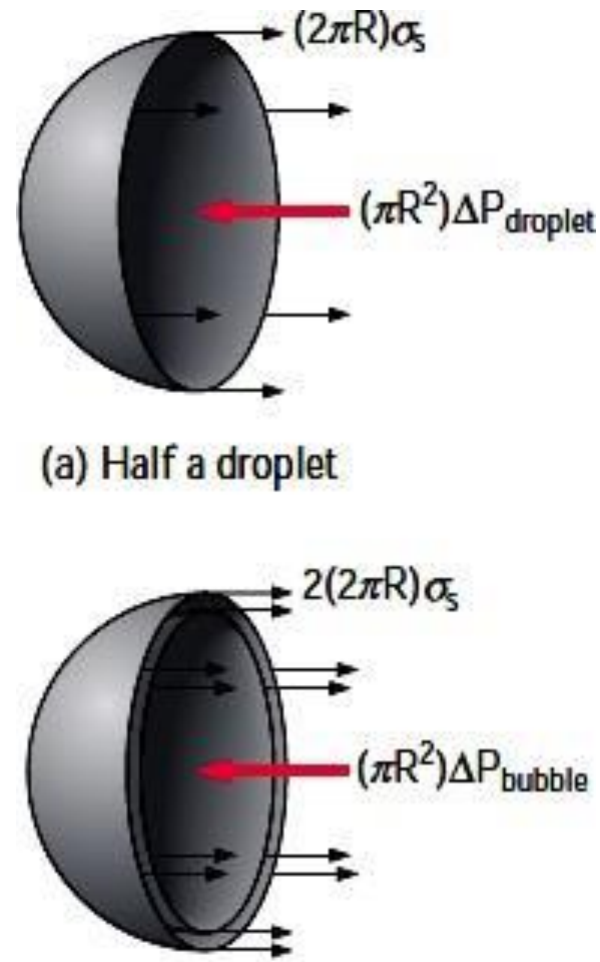
- A tensile force may be considered to be acting in the plane of the surface along any line in the surface.
- The intensity of the molecular attraction per unit length along any line in the surface is called the ***surface tension.***
- It is denoted by Greek letter σ (called sigma).
- The SI unit is N/m.



Surface Tension and Capillarity

Surface Tension on liquid Droplet and Bubble

- Consider a small spherical droplet of a liquid of radius *'R'. On the entire surface of the droplet, the tensile force due to surface tension will be acting.*
- Let σ = surface tension of the liquid
- ΔP= Pressure intensity inside the droplet (in excess of the outside pressure intensity)
- *R= Radius of droplet.*
- Let the droplet is cut into two halves. The forces acting on one half (say left half) will be



(a) Half a droplet

(b) Half a bubble

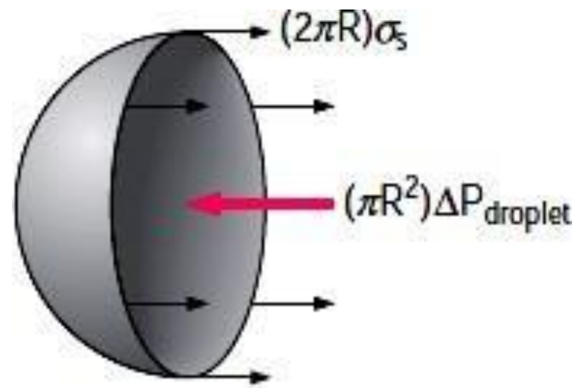
Surface Tension and Capillarity

- (i) tensile force due to surface tension acting around the circumference of the cut portion as shown and this is equal to

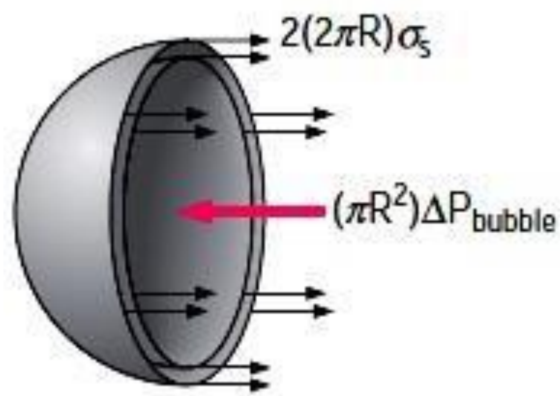
  $= \sigma$ x Circumference

  $= \sigma$ x $2\pi R$

- (ii) pressure force on the area $(\pi/4)d^2$ and
- $= \Delta P \; x \; \pi R^2$ *as shown*



(a) Half a droplet



(b) Half a bubble

Surface Tension and Capillarity

- These two forces will be equal and opposite under equilibrium conditions, *i.e.*,

Droplet: $(2\pi R)\sigma_s = (\pi R^2)\Delta P_{droplet} \rightarrow \Delta P_{droplet} = P_i - P_o = \dfrac{2\sigma_s}{R}$

Bubble: $2(2\pi R)\sigma_s = (\pi R^2)\Delta P_{bubble} \rightarrow \Delta P_{bubble} = P_i - P_o = \dfrac{4\sigma_s}{R}$

- A hollow bubble like a soap bubble in air has two surfaces in contact with air, one inside and other outside. Thus two surfaces are subjected surface tension.

Surface Tension……. Example 1

- Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5 N/m² above atmospheric pressure.

Solution. Given :

Dia. of bubble, $d = 40\text{ mm} = 40 \times 10^{-3}\text{ m}$

Pressure in excess of outside, $p = 2.5\text{ N/m}^2$

For a soap bubble, using equation (1.15), we get

$$p = \frac{8\sigma}{d} \quad \text{or} \quad 2.5 = \frac{8 \times \sigma}{40 \times 10^{-3}}$$

$$\sigma = \frac{2.5 \times 40 \times 10^{-3}}{8}\text{ N/m} = \mathbf{0.0125\text{ N/m. Ans.}}$$

Surface Tension……. Example 2

- The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm² (atmospheric pressure). Calculate the pressure within the droplet if surface tension is given as 0.0725 N/m of water.

Solution. Given :

Dia. of droplet, $d = 0.04 \text{ mm} = .04 \times 10^{-3} \text{ m}$

Pressure outside the droplet $= 10.32 \text{ N/cm}^2 = 10.32 \times 10^4 \text{ N/m}^2$

Surface tension, $\sigma = 0.0725 \text{ N/m}$

The pressure inside the droplet, in excess of outside pressure is given by

or $$p = \frac{4\sigma}{d} = \frac{4 \times 0.0725}{.04 \times 10^{-3}} = 7250 \text{ N/m}^2 = \frac{7250 \text{ N}}{10^4 \text{ cm}^2} = 0.725 \text{ N/cm}^2$$

$\therefore$ Pressure inside the droplet $= p +$ Pressure outside the droplet

$= 0.725 + 10.32 =$ **11.045 N/cm². Ans.**

Surface Tension and Capillarity

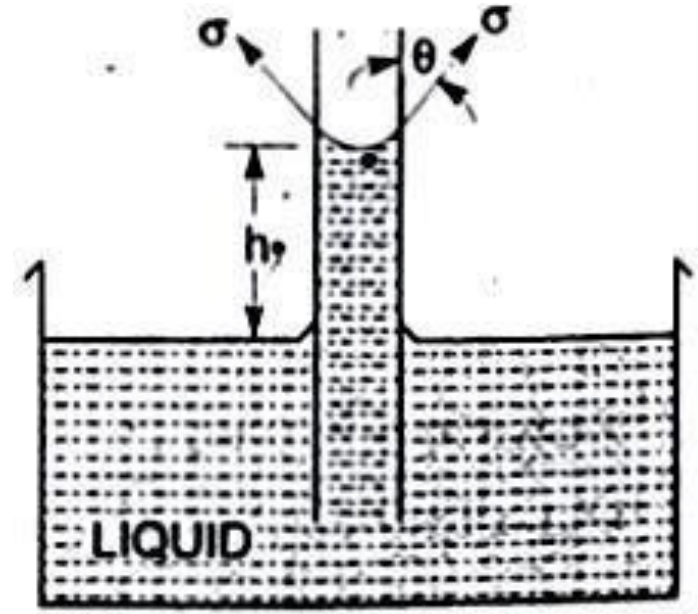
Capillarity

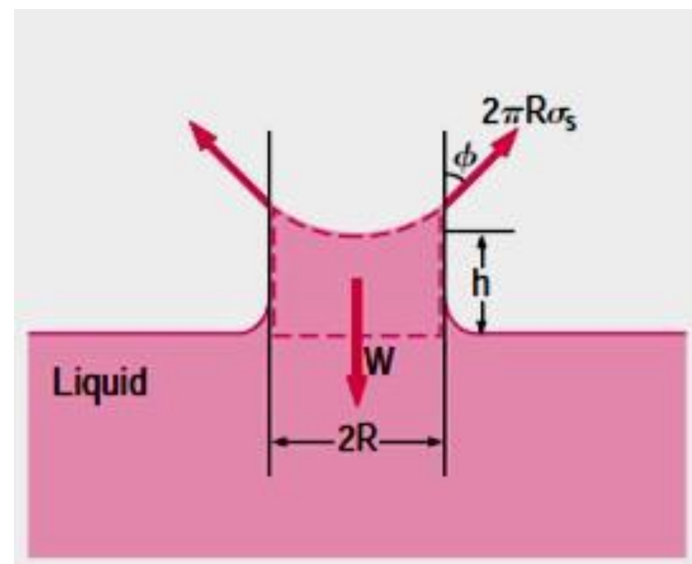
- Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid.
- The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression.
- The attraction (adhesion) between the wall of the tube and liquid molecules is strong enough to overcome the mutual attraction (cohesion) of the molecules and pull them up the wall. Hence, the liquid is said to *wet the solid surface*.
- It is expressed in terms of cm or mm of liquid. Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

Surface Tension and Capillarity

Expression for Capillary Rise

- Consider a glass tube of small diameter ‘d’ opened at both ends and is inserted in a liquid, say water.
- The liquid will rise in the tube above the level of the liquid.
- Let h = the height of the liquid in the tube . Under a state of equilibrium, the weight of the liquid of height h is balanced by the force at the surface of the liquid in the tube. But the force at the surface of the liquid in the tube is due to surface tension.





Expression for Capillary Rise…

- Let σ = Surface tension of liquid

  θ = Angle of contact between the liquid and glass tube

- The weight of the liquid of height h in the tube

  = (Area of the tube x h) x ρ x g

$$= \frac{\pi}{4} d^2 \times h \times \rho \times g$$

where ρ = Density of liquid

Vertical component of the surface tensile force

$$= (\sigma \times \text{Circumference}) \times \cos\theta$$

$$= \sigma \times \pi d \times \cos\theta$$

For equilibrium, equating (1.17) and (1.18), we get

$$\frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos\theta$$

or

$$h = \frac{\sigma \times \pi d \times \cos\theta}{\frac{\pi}{4} d^2 \times \rho \times g} = \frac{4\sigma\cos\theta}{\rho \times g \times d}$$

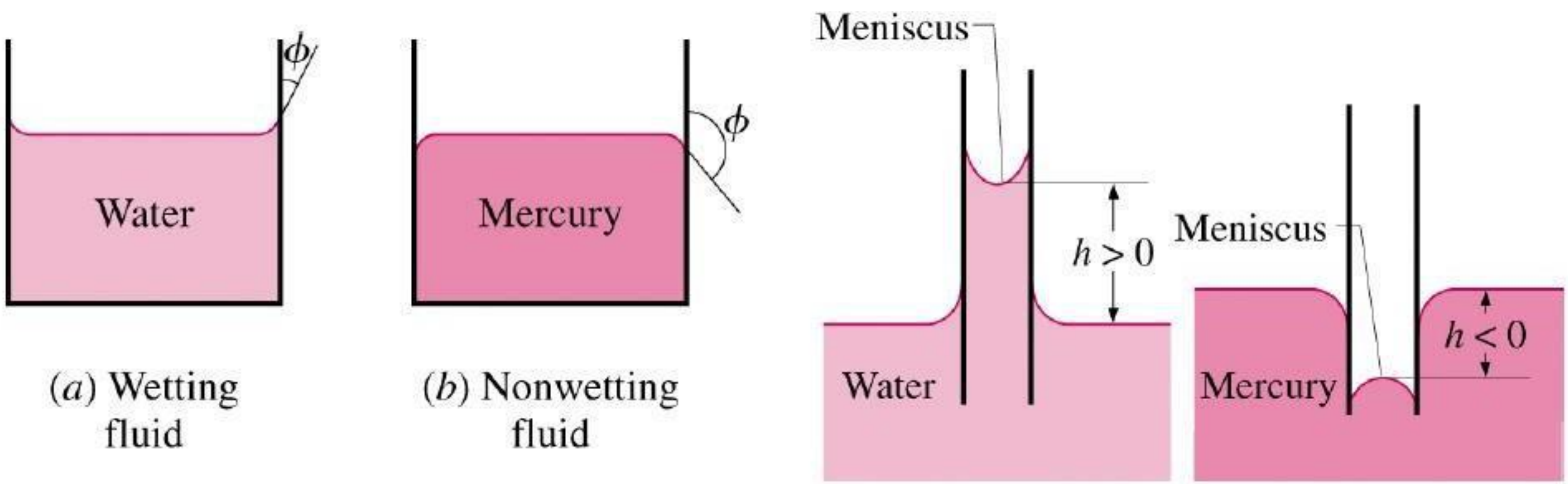
Expression for Capillary Rise…

- The value of θ between water and clean glass tube is approximately equal to zero and hence cos θ is equal to unity. Then rise of water is given by

$$\boxed{h = \frac{4\sigma}{\rho \times g \times d}}$$

- Contact angle depends on both the liquid and the solid.
- If θ is less than 90^o, the liquid is said to "wet" the solid. However, if θ is greater than 90^o, the liquid is repelled by the solid, and tries not to "wet" it.
- For example, water wets glass, but not wax. Mercury on the other hand does not wet glass.

Capillarity



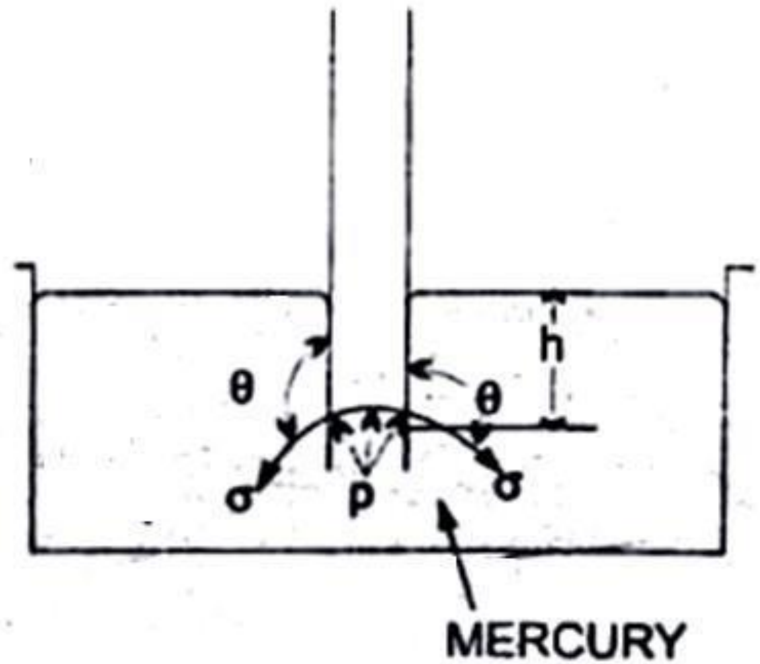
Expression for Capillary Fall

- If the glass tube is dipped in mercury, the revel of mercury in the tube will be lower than the general level of the outside liquid as shown above.

Capillarity

Expression for Capillary Fall

- Let *h = Height of depression in tube.*
- Then in equilibrium, two forces arc acting on the mercury inside the tube.
- First one is due to surface tension acting in the downward direction and is equal to σ x *πd x cos θ.*
- Second force is due to hydrostatic force acting upward and is equal to intensity of pressure at a depth *'h' x Area*



Capillarity

Expression for Capillary Fall

$$= p \times \frac{\pi}{4} d^2 = \rho g \times h \times \frac{\pi}{4} d^2 \; \{\because p = \rho g h\}$$

Equating the two, we get

$$\sigma \times \pi d \times \cos\theta = \rho g h \times \frac{\pi}{4} d^2$$

$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

Value of θ for mercury and glass tube is 128^o

Capillarity…Example 1

- Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in (a) water and (b) mercury. Take surface tensions σ = 0.0725 N/m for water and σ = 0.52 N/m for mercury in contact with air. The specific gravity for mercury is given as 13.6 and angle of contact = 130^o.

Solution. Given :

Dia. of tube,	$d = 2.5$ mm $= 2.5 \times 10^{-3}$ m
Surface tenstion, σ for water	= 0.0725 N/m
σ for mercury	= 0.52 N/m
Sp. gr. of mercury	= 13.6

Capillarity…Example 1

$\therefore$ Density $= 13.6 \times 1000 \text{ kg/m}^3$.

(*a*) **Capillary rise for water ($\theta = 0$)**

Using equation (1.20), we get $h = \dfrac{4\sigma}{\rho \times g \times d} = \dfrac{4 \times 0.0725}{1000 \times 9.81 \times 2.5 \times 10^{-3}}$

$= .0118 \text{ m} = $ **1.18 cm. Ans.**

(*b*) **For mercury**

Angle of constant between mercury and glass tube, $\theta = 130°$

Using equation (1.21), we get $h = \dfrac{4\sigma \cos\theta}{\rho \times g \times d} = \dfrac{4 \times 0.52 \times \cos 130°}{13.6 \times 1000 \times 9.81 \times 2.5 \times 10^{-3}}$

$= -.004 \text{ m} = $ **– 0.4 cm. Ans.**

The negative sign indicates the capillary depression.

Capillarity…Example 2

- Find out the minimum size of glass tube that can be used to measure water level if the capillary rise in the tube is to be restricted to 2 mm. Consider surface tension of water in contact with air as 0.073575 N/m.

Solution. Given :

Capillary rise, $h = 2.0 \text{ mm} = 2.0 \times 10^{-3}$ m

Surface tension, $\sigma = 0.073575$ N/m

Let dia. of tube $= d$

The angle θ for water $= 0$

The density for water, $\rho = 1000 \text{ kg/m}^3$

$$h = \frac{4\sigma}{\rho \times g \times d} \quad \text{or} \quad 2.0 \times 10^{-3} = \frac{4 \times 0.073575}{1000 \times 9.81 \times d}$$

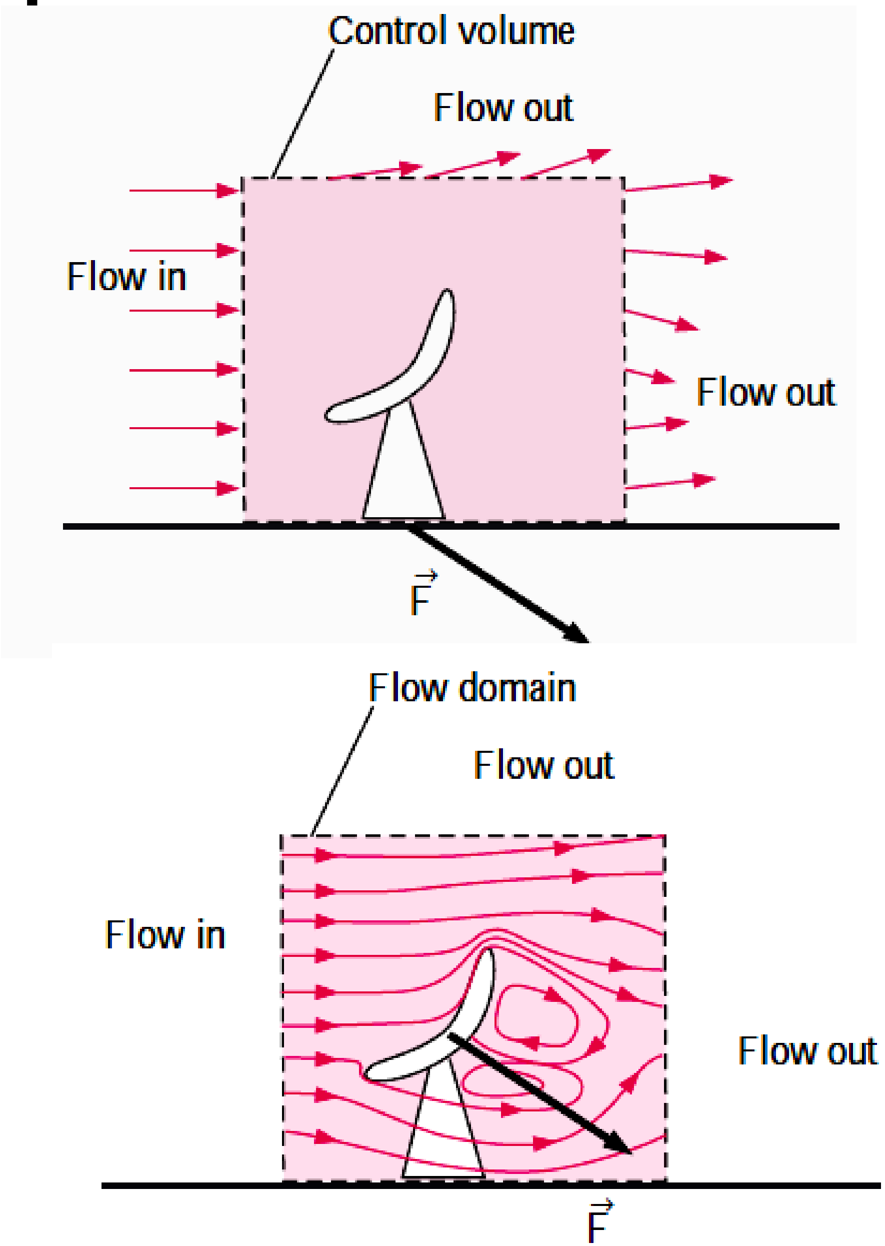
$$\therefore \qquad d = \frac{4 \times 0.073575}{1000 \times 9.81 \times 2 \times 10^{-3}} = 0.015 \text{ m} = \mathbf{1.5 \text{ cm. Ans.}}$$

Thus minimum diameter of the tube should be 1.5 cm.

Flow Analysis Techniques

- In analyzing fluid motion, we might take one of two paths:

1. Seeking an estimate of gross effects (mass flow, induced force, energy change) over a finite region or control volume or
2. Seeking the point-by-point details of a flow pattern by analyzing an infinitesimal region of the flow.



Flow Patterns

- Fluid mechanics is a highly visual subject. The patterns of flow can be visualized in a dozen different ways, and you can view these sketches or photographs and learn a great deal qualitatively and often quantitatively about the flow.
- Four basic types of line patterns are used to visualize flows:

1. A ***streamline*** is a line everywhere tangent to the velocity vector at a given instant.
2. A ***pathline*** is the actual path traversed by a given fluid particle.
3. A ***streakline*** is the locus of particles that have earlier passed through a prescribed point.
4. A ***timeline*** is a set of fluid particles that form a line at a given instant.

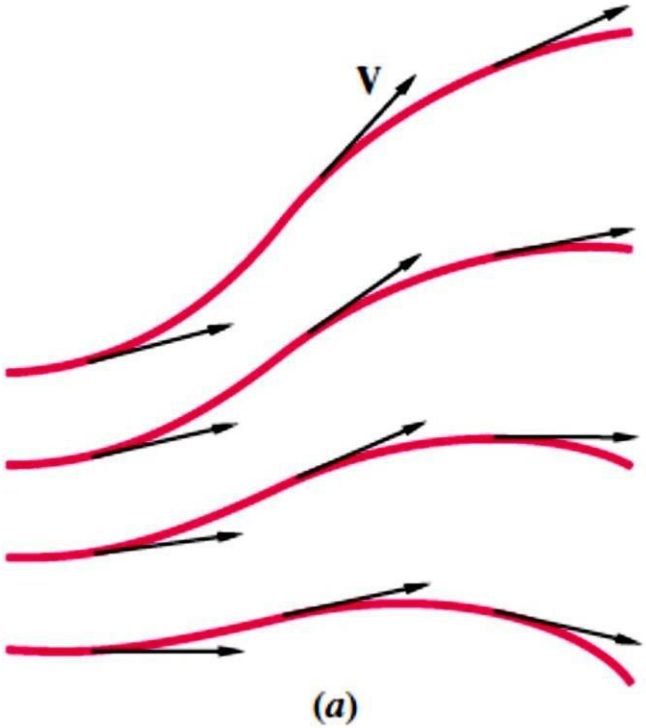
Flow Patterns

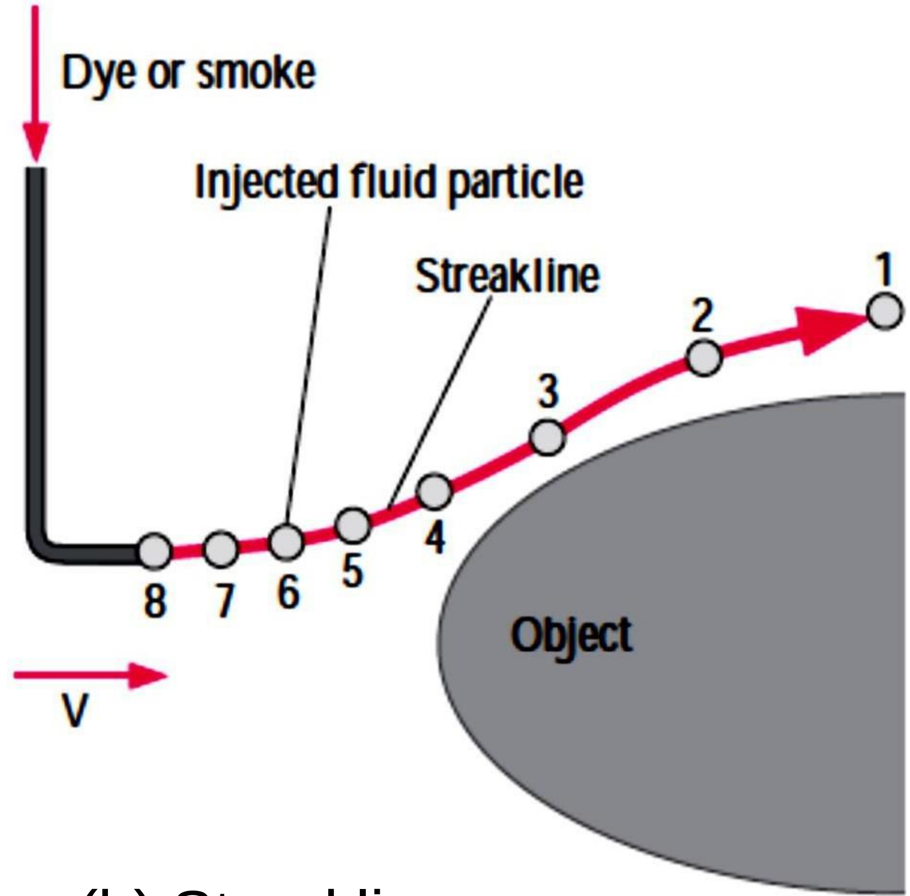
- The streamline is convenient to calculate mathematically, while the other three are easier to generate experimentally.
- Note that a streamline and a timeline are instantaneous lines, while the pathline and the streakline are generated by the passage of time.
- A ***streamline*** is a line that is everywhere tangent to the velocity field. If the flow is steady, nothing at a fixed point (including the velocity direction) changes with time, so the streamlines are fixed lines in space.
- For unsteady flows the streamlines may change shape with time.
- A pathline is the line traced out by a given particle as it flows from one point to another.

Flow Patterns

- A ***streakline*** consists of all particles in a flow that have previously passed through a common point. Streaklines are more of a laboratory tool than an analytical tool.
- They can be obtained by taking instantaneous photographs of marked particles that all passed through a given location in the flow field at some earlier time.
- Such a line can be produced by continuously injecting marked fluid (neutrally buoyant smoke in air, or dye in water) at a given location.
- If the flow is steady, each successively injected particle follows precisely behind the previous one forming a steady streakline that is exactly the same as the streamline through the injection point.

Flow Patterns





(a) Streamlines

(b) Streaklines

Boiling Point

- A liquid boils at the temperature when its vapor pressure equals the surrounding atmospheric pressure.

Vapour pressure

Vapour pressure can be defined as **pressure** formed by the vapor of the **liquid** (or solid) over the surface of the **liquid**. This **pressure** is formed in a thermodynamic equilibrium state in a closed container at a certain temperature.

End of Chapter 1

Thank you

Subject: Fluid Mechanics

Topic: Pressure and Pressure Head

Pressure

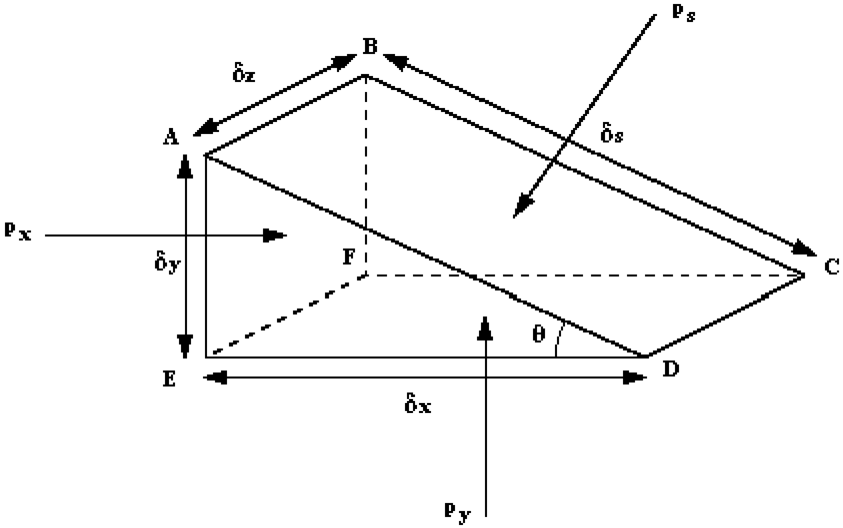
- **Pressure** may be defined as the force exerted on a unit area.

$$P = \frac{F}{A}$$

- **Unit** kgf/m^2 or kgf/cm^2 in **MKS**
- N/m^2 or N/mm^2 in **SI**
- 1 N/m^2 =1 Pascal
- 1 bar=100 kPa=10^5 N/m^2

Pascal's Law

- *"The intensity of pressure at any point in a liquid at rest, is the same in all directions."*



P_x= Force on a face AE

P_y= Force on a face DE

P_z= Force on a face AD

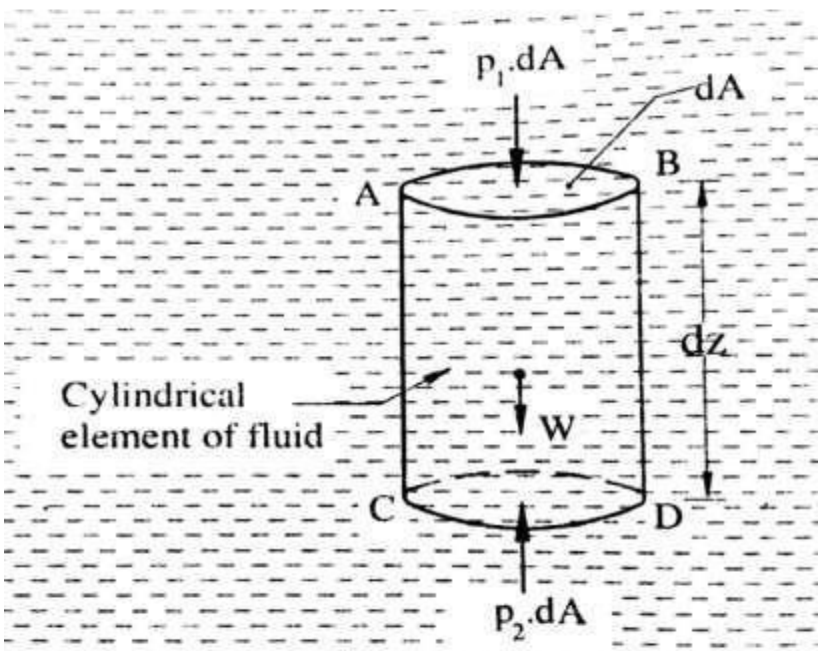
$$P_x = P_y = P_z$$

Application of Pascal’s Law

- In measurement system like manometer pressure gauge etc.
- In construction of machines such as hydraulic press,hydraulic jack, hydraulic lift, hydraulic crane, hydraulic riveter etc.

Hydrostatic Law

- *"The rate of increase of pressure in a vertically downward direction is equal to the weight density of fluid at that point."*



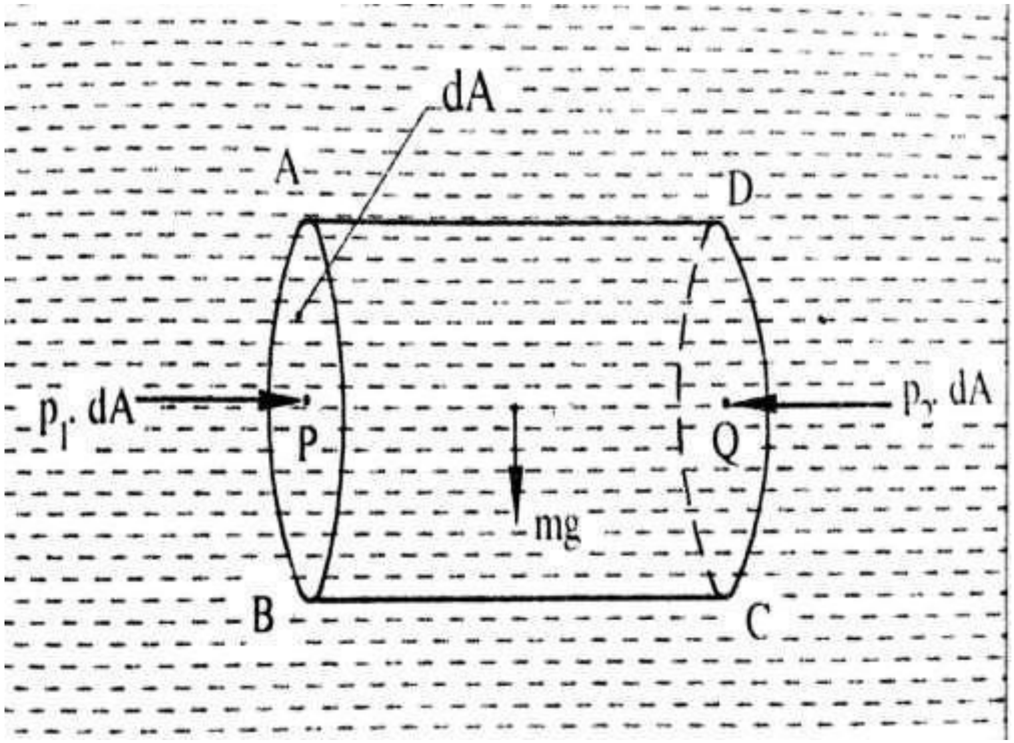
$$dp/dz = \rho g = w$$

$$p = \rho g z$$

Equality of pressure at the same level in a static fluid

- p_1=pressure on face AB
- p_2 =pressure on face CD

❖ $\mathbf{p_1=p_2}$



- Pressure at any two points at the same level in a body of fluid at rest will be the same.

Pressure and Head

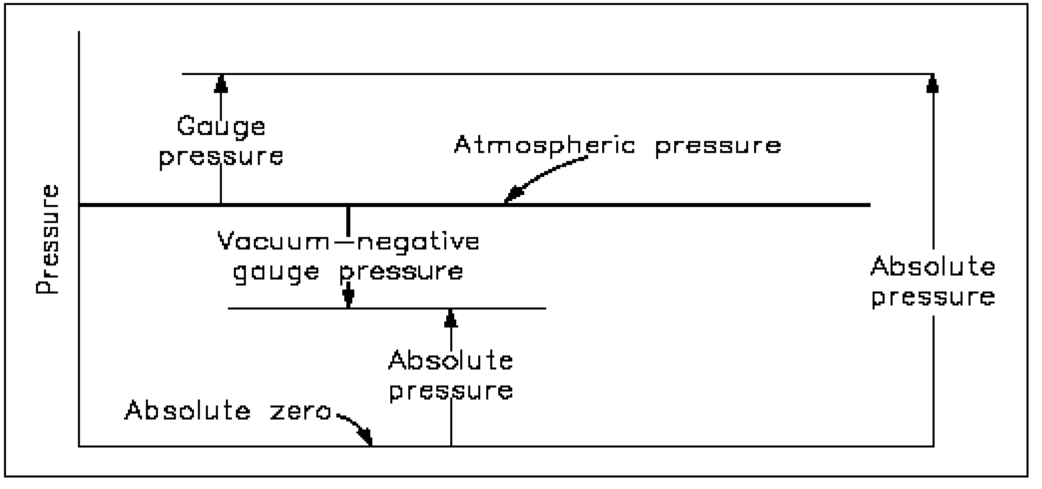
- **Atmosperic Pressure:** It is pressure exerted by the air on the surface of earth.
- Atmospheric pressure is not constant because density of air vary from time to time due to changes in its temperature.

- **Gauge Pressure:** It is measured with the help of pressure gauge.
- In this pressure,atmosperic pressure is considered zero and this pressure is above atmospheric pressure.

- **Vacuum Pressure:** When pressure is below the atmospheric pressure is called vacuum pressure.
- It is also known as negative gauge pressure and is measured by vacuum gauge.

Pressure and Head

- **Absolute Pressure:** It is pressure measured with reference to absolute vacuum pessure.
- It is independent of the changes in atmospheric pressure. It is measured above the absolute zero of pressure.



Manometers

- Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid.
- **Classification of Manometers:**

  (1) Simple Manometer

  - Piezometer
  - U-tube Manometer
  - Single column Manometer

    Vertical single column Manometer

    Inclined single column Manometer

  (2) Differential Manometer

    U-tube differential Manometer

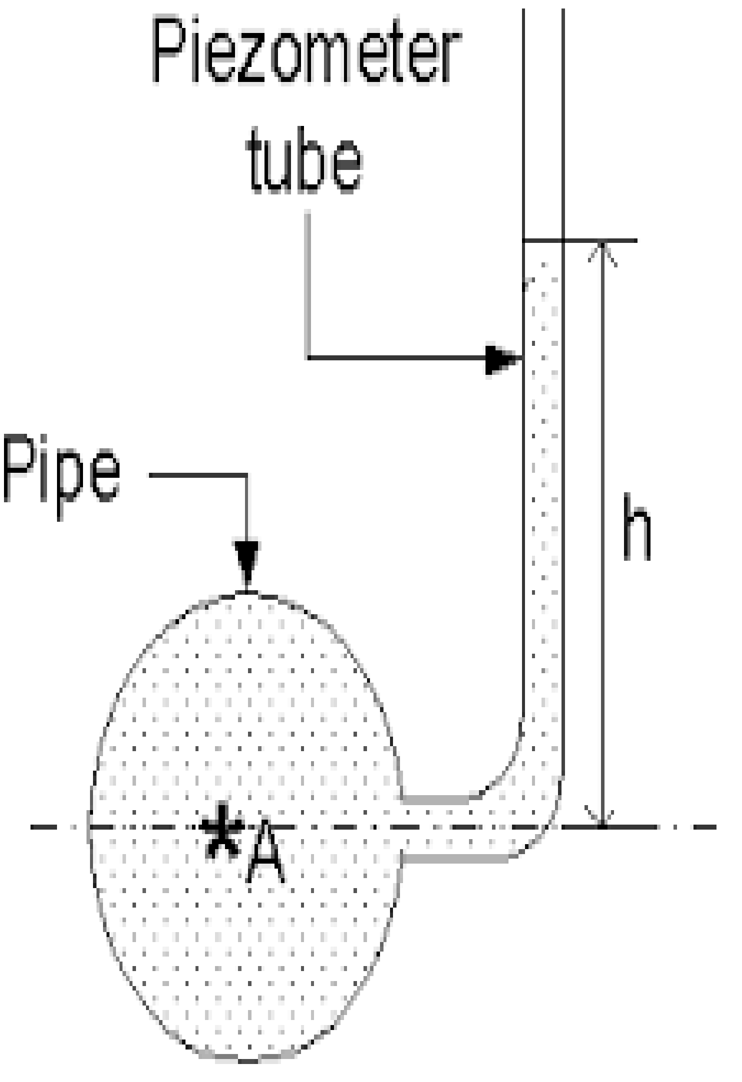
    Inverted U-tube differential Manometer

Piezometer

- It is the simplest form of the Manometer, it measures gauge pressure only.
- The pressure at any point in the fluid is indicated by the height of the liquid in the tube above point A,which can read on the calibrated scale on glass tube.
- Let,p=pressure of fluid

  h=height of fluid in tube

  ρ=density of fluid



h=p/ρg

Piezometer

- **Limitation of Piezometer:**
- One end of tube is open to atm, therefore it measures only gauge pressure.
- It is not suitable for vacuum pressure.
- To measure large pressure with lighter liquid would require very long tube.
- It cannot measure pressure of gas.

U-tube Manometer

- It can be measure vacuum pressure and gas pressure.
- It consists of glass tube bent in U-shape.
- One end is connected to a point at which pressure is to be measured and other end open to the atm.

- Let, h_1=height of light liquid in a left column above datumline X-X

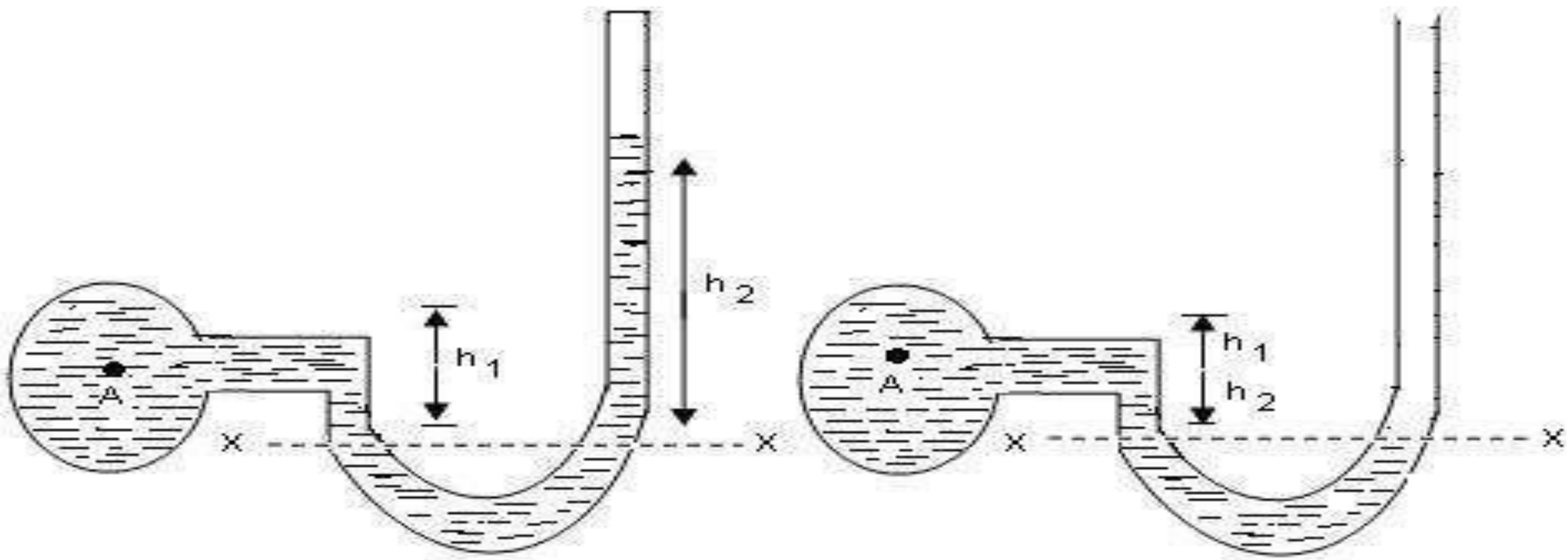
  h_2=height of heavy liquid in a right column above datumline X-X

  ρ_1=density of light liquid

  ρ_2=density of heavy liquid

  p=pressure of fluid in pipe to be measured

U-tube Manometer



(a)Pressure higher than p_{atm}

$$p=\rho_2 g h_2 - \rho_1 g h_1$$

(b)Pressure lower than p_{atm}

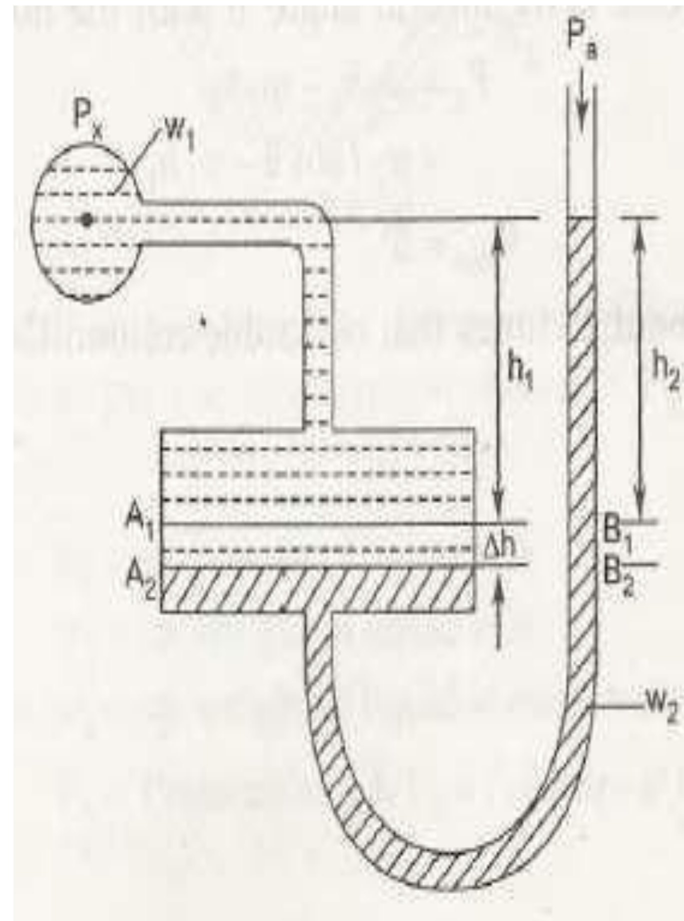
$$p=-(\rho_2 g h_2 + \rho_1 g h_1)$$

Vertical single column manometer

- The A_1 –B_1 is datum line in reservoir and right column when it is not connected to the pipe.
- The A_2 -B_2 is datum line in reservoir and right column when it is connected to the pipe in which fluid pressure is to be measured.
- The Δh is fall of heavy liquid in reservoir.

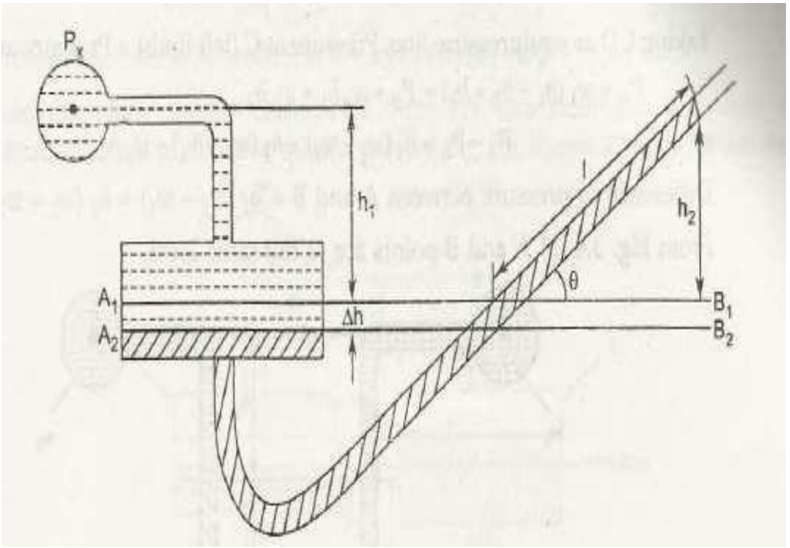
$$\Delta h = a*h_2/A$$

$$h = \frac{ah_2}{A}(S_2 - S_1) + (h_2 S_2 - h_1 S_1)$$



Inclined Single Column Manometer

- It is modified of vertical column manometer.
- The distance moved by heavy liquid in right column increases by providing the inclination of right column therefore the sensitivity can be increased.



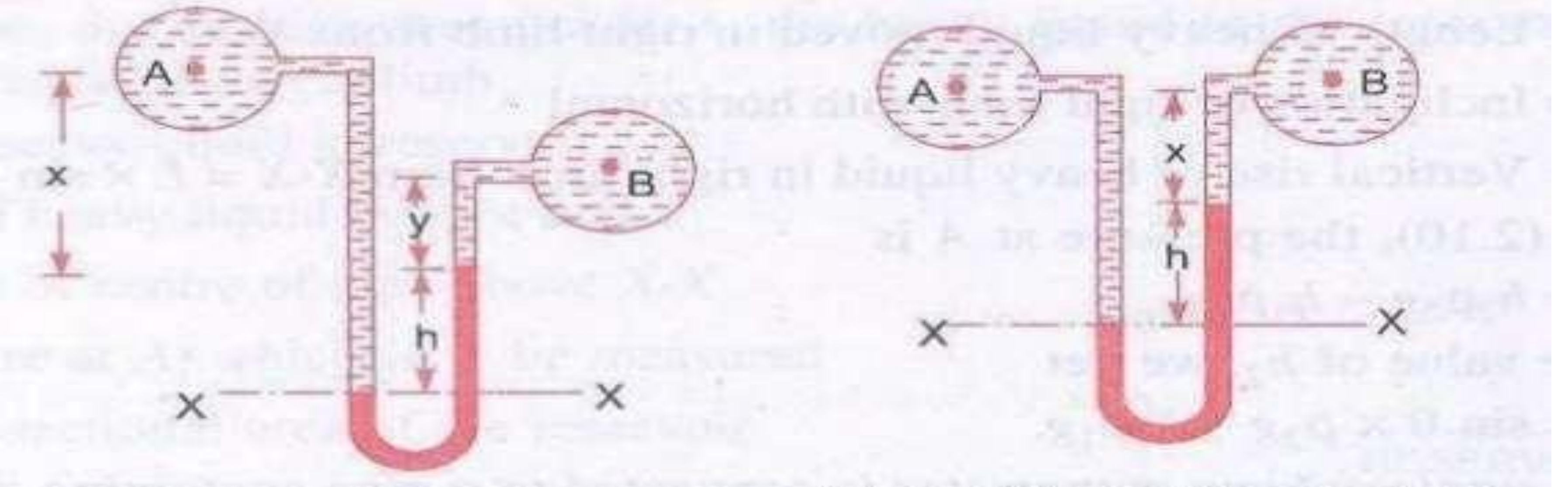
$$p=\rho_2 g l \sin\Theta - \rho_1 g h_1$$

$$h=S_2 l \sin\Theta - S_1 h_1$$

Where, Θ=angle between inclined right narrow tube and horizontal axis

Differential Manometer

- Differential Manometer is used to measure difference between any two points in a pipeline.
- **U-tube Differential manometer:** In U-tube differential manometer, there are two cases mentioned below:



(a)Two pipes at different levels

(b) A and B are at the same level

$p_a - p_b = gh(\rho_g - \rho_a) + \rho_b gy - \rho_a gx$

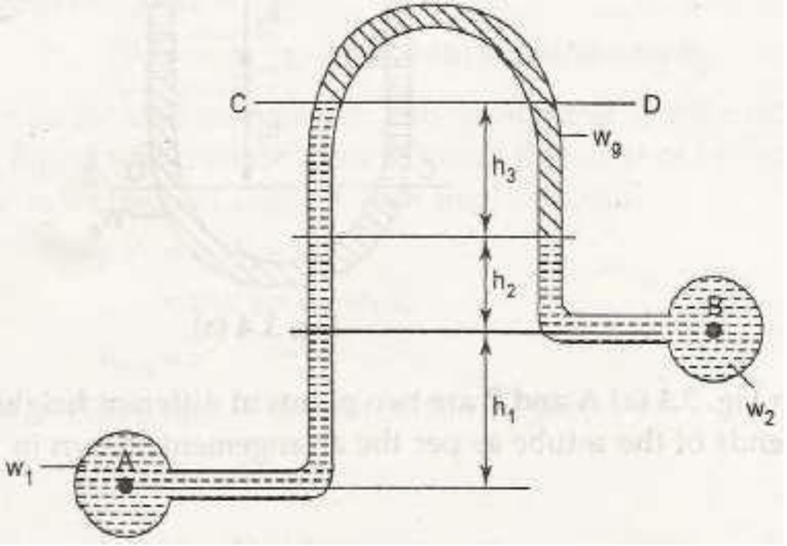
$p_a - p_b = gh(\rho_g - \rho_a)$

Differential Manometer

- **Inverted U-tube differential manometer**:It is normally used for measuring low pressure difference.

•Pressure below C-D in left column=Pressure below C-D in right column

$$p_a - p_b = -\rho_g gh - \rho_b g h_2 + \rho_a g h_1$$



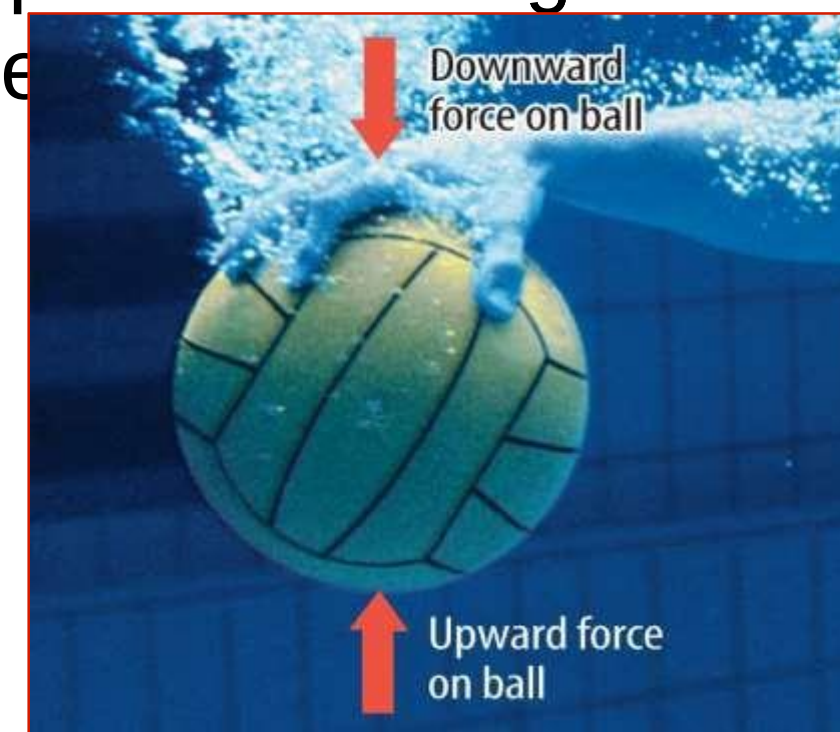
Thank you...

Buoyancy And Floatation

Ups and Downs:
Buoyancy

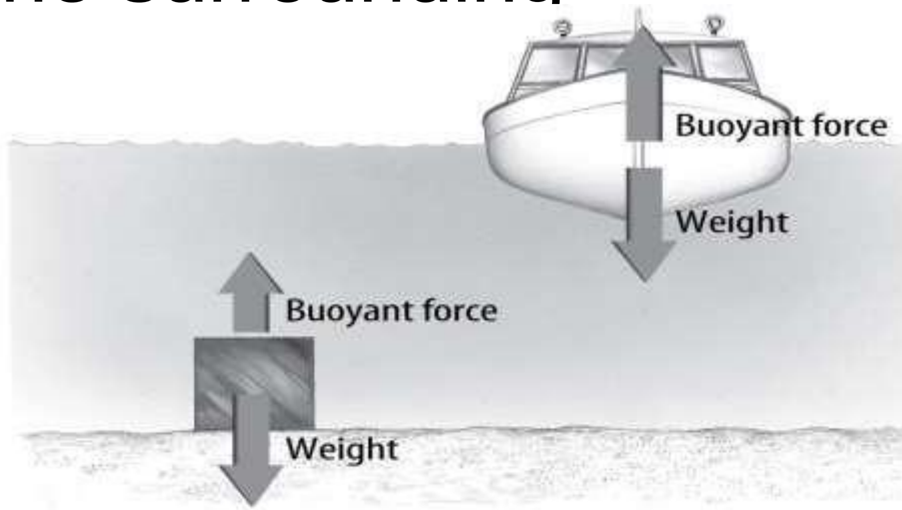
What Is Buoyancy Force

- ➢When a body is immersed in fluid, an upward force is exerted by the fluid on the body.
- ➢This upward force is equal to the weight of the fluid displaced by the
the force of buoyancy.



What causes buoyant force?

- **Buoyant force** is the force on an object exerted by the surrounding fluid.
- When an object pu water pushes back force as it can.
- If the water can push back as hard, the object floats (boat). If not, it sinks (steel).



Forces Acting on Buoyancy

- The **buoyant force** is caused by the difference between the pressure at the top of the object (gravitational force), which pushes it downward, and the pressure at the bottom (buoyant force), which pushes it upward
- Since the pressure at the bottom is always greater than at the top, every object submerged in a fluid feels an upward buoyant force.

Keep It Simple

- **Buoyancy=** ***"the floating force"***
- Water is "heavier" than the object…so the object floats
- Low density-more likely to float
- Buoyant force is measured in Newtons (N)

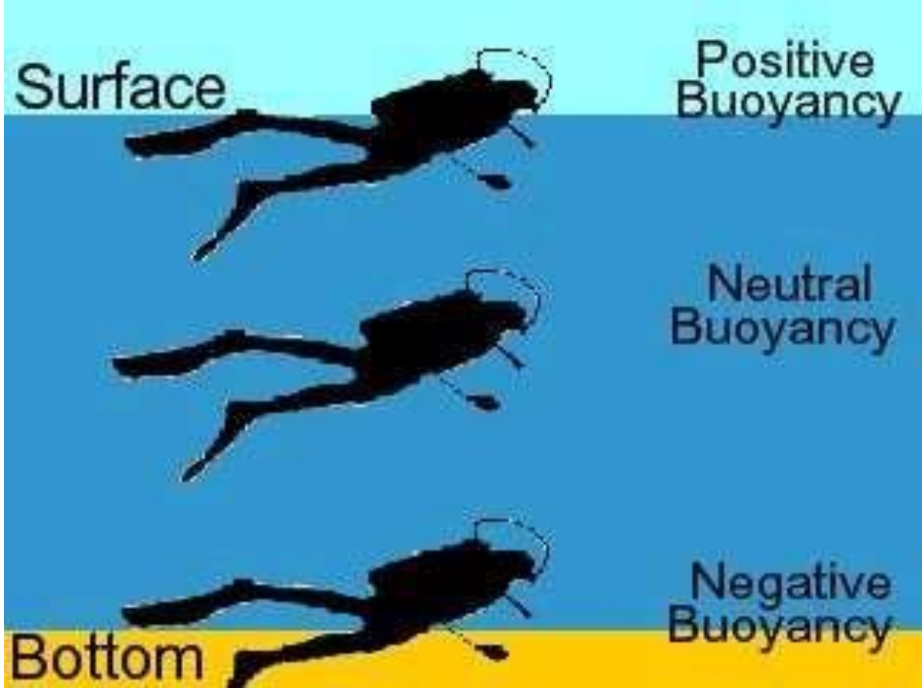
Condition of equilibrium of a floating and sub-merged bodies

- **Positive buoyancy:**

  Buoyant force is greater than weight so the object floats.

- **Neutral buoyancy:**

  Buoyant force is equal to weight so the object is suspended in the fluid.



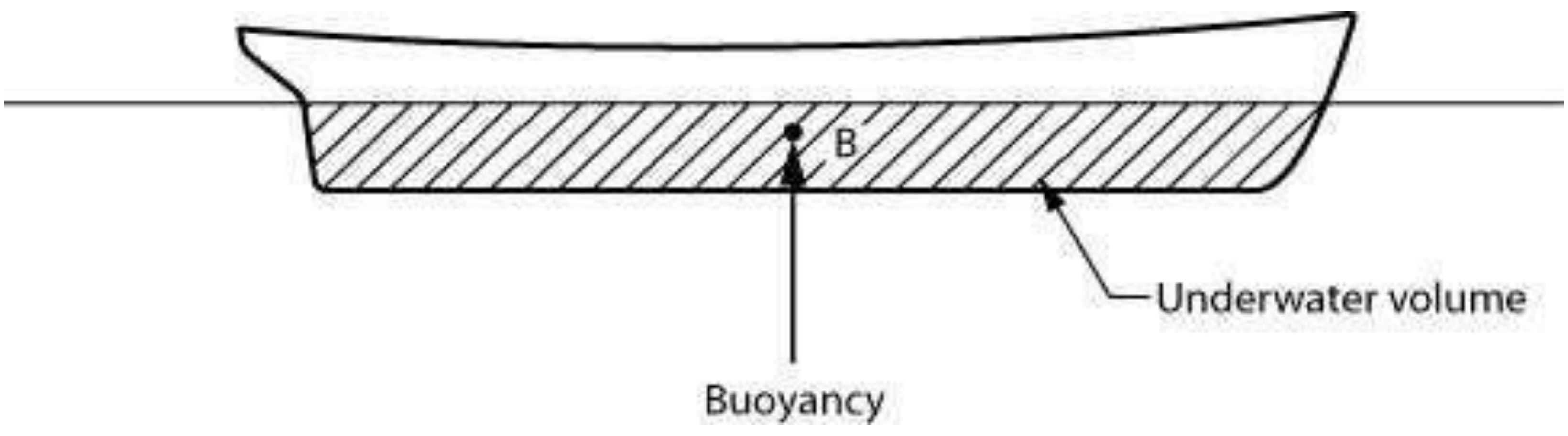
- **Negative buoyancy:**

  Buoyant force is less

Centre of Buoyancy

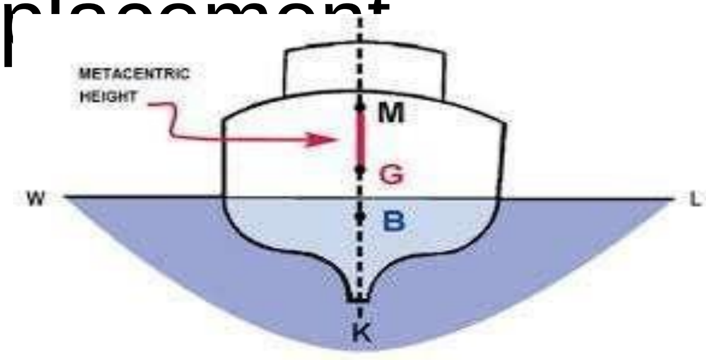
Definition:-

- The point through which the force of buoyancy is supposed to act is known as Centre of Buoyancy.



META-CENTRE

- It is defined as the point about which a body starts oscillating when the body is tilted by a small angle.
- It is the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given small angular displacement.



Meta-centric Height

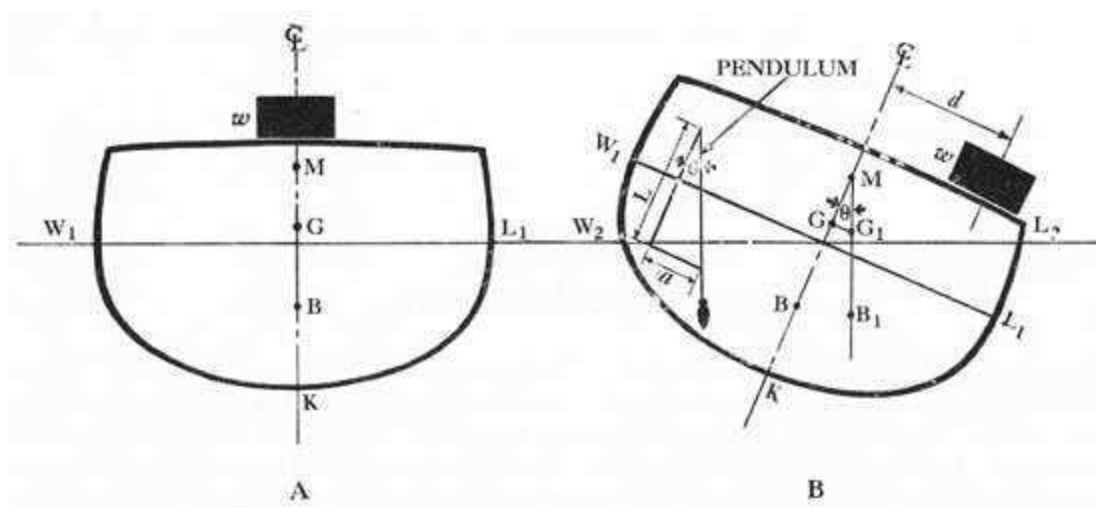
- It is the distance between the meta-centre of floating body and centre of gravity.
- We can find this height by two methods:-

1. Analytical Method

$$GM = \frac{I}{\forall} - BG$$

Here I=M.O.I m^4

$\forall$ =Volume of sub-merged body



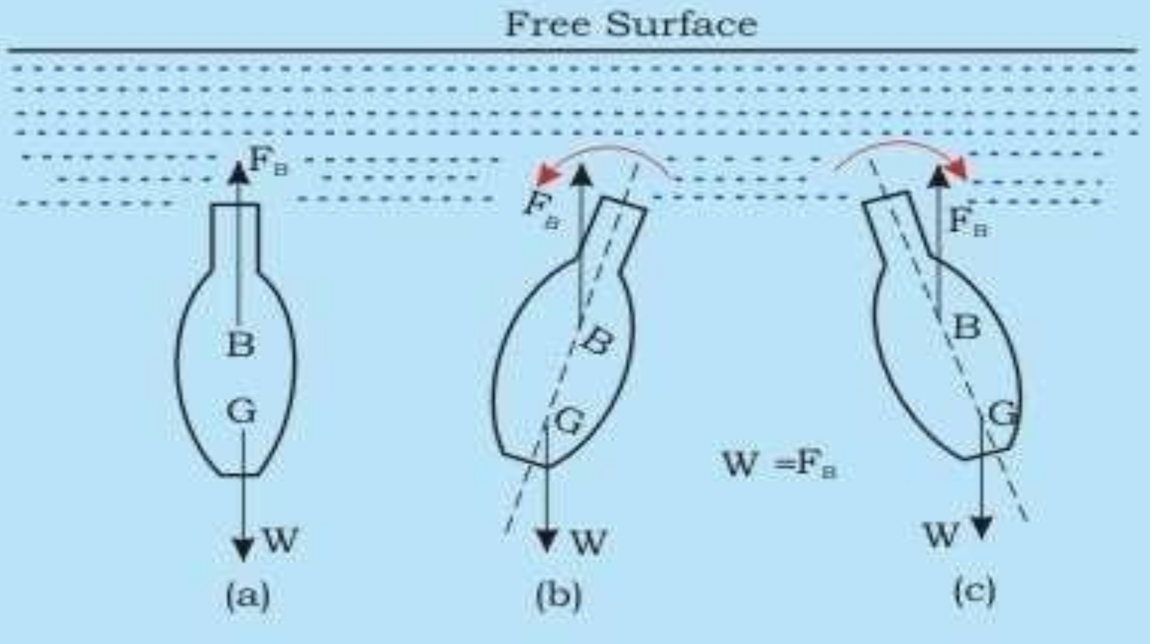
Condition of equilibrium of a floating and sub-merged bodies

Stability of Sub-merged Body:-

a) Stable Equilibrium:-When W= F_b and point B is above G .

b) Unstable Equilibrium:- When W= F_b but B is below G.

c) Neutral Equilibrium:-When W= F_b and B & G are the same point

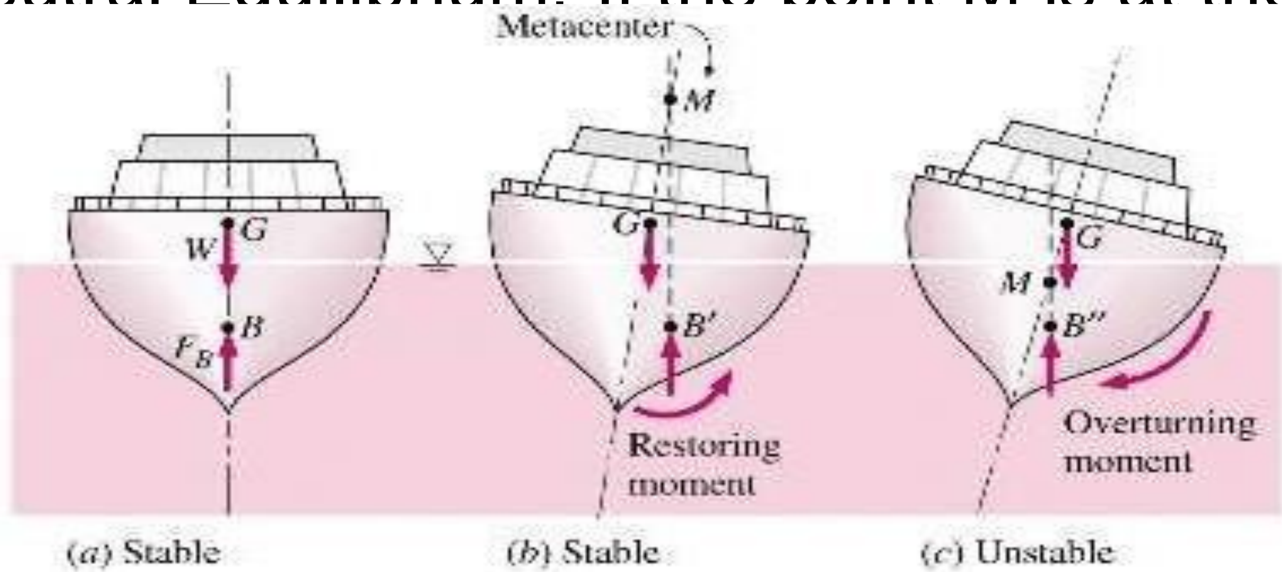


Stability of Floating Body

a) Stable Equilibrium:-If the point M is above G.

b) Unstable Equilibrium:-If the point M is Below G.

c) Neutral Equilibrium:-If the point M is at the G.



THANK

YOU

UNIT- IV

Fluid Dynamics

Equation of motion:-

According to Newton's second law of motion, the net force F_x acting on a fluid element in the direction of x is equal to mass 'm' of the fluid element multiplied by the accelaration a_x in the x-direction. Thus mathematically

$$F_x = m a_x$$

In the fluid flow, the following forces are present

i) F_g, gravity force

ii) F_p, the pressure force

iii) F_v, Force due to viscosity

iv) F_t, force due to turbulance

v) F_c, Force due to compressibility

Net force $F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x$

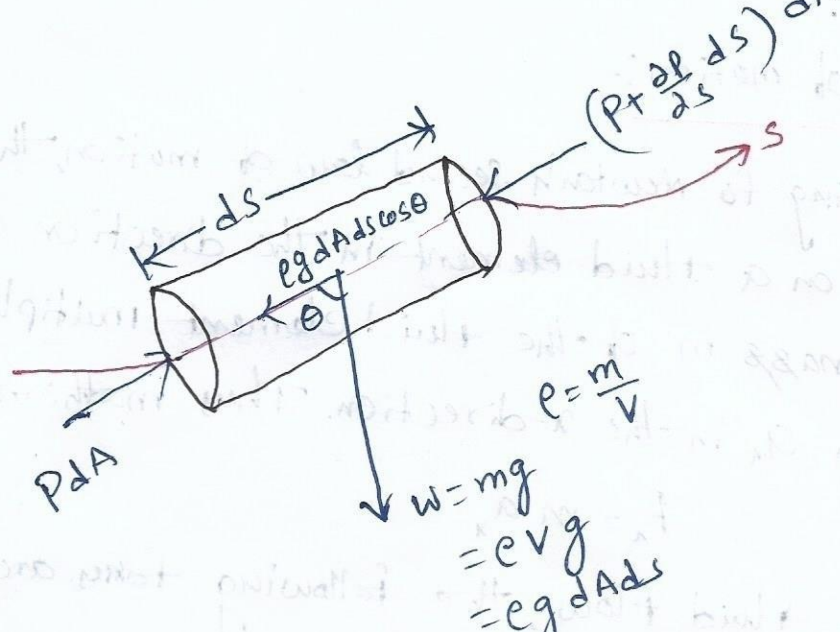
i) If the force due to compressibility, F_c is negligible, the resulting net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x$$ and equation of motions are called Reynold's equations of motion.

ii) For flow, where (F_t) is negligible, the resulting equations of motion are known as Navier stokes equation.

iii) If the flow is assumed to be ideal, viscous force (F_v) is zero and equation of motions are known as Euler's equation of motion.

* Euler's equation of motion :-



$P = F/A$

$F = pA$

Euler's equation of motion

↓

i) Forces due to pressure → pressure force pdA → Direction of flow
pressure force $(p+\frac{\partial p}{\partial s}ds)dA$ → opposite

ii) Force due to gravity

↓

$W = mg = \rho V g = \rho g dA ds$

→ Resultant force = mass of the fluid element × accelaration in the direction.

$$pdA - (p+\frac{\partial p}{\partial s}ds)dA - \rho g dA ds \cos\theta = \rho dA ds \times a_s$$

$$\cancel{pdA} - \cancel{p}dA - \frac{\partial p}{\partial s}ds dA - \rho g dA ds \cos\theta = \rho dA ds a_s$$

$$-\frac{\partial p}{\partial s} ds dA - \rho g dA ds \cos\theta = \rho dA ds a_s \rightarrow (1)$$

$a_s = \frac{dv}{dt}$, $v = \frac{ds}{dt}$ i.e function of s & t

$$a_s = \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t}\frac{dt}{dt}$$

$$= \frac{\partial v}{\partial s} v + \frac{\partial v}{\partial t}$$ ⇒ If the flow is steady then $\frac{dv}{dt} = 0$

a_s value in equation (1)

$$-\frac{\partial p}{\partial s} ds\,dA - \rho g\,dA\,ds\cos\theta = \rho\,dA\,ds \times V\frac{\partial v}{\partial s}$$

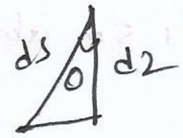
Dividing by $\rho\,dA\,ds$

$$-\frac{\partial p}{\rho\,\partial s} - g\cos\theta = V\frac{\partial v}{\partial s}$$

$$\frac{\partial p}{\rho\,\partial s} + g\cos\theta + V\frac{\partial v}{\partial s} = 0$$

$$\frac{\partial p}{\rho\,\partial s} + g\cos\theta + V\frac{dv}{\partial s} = 0$$

But



$$\cos\theta = \frac{dz}{ds}$$

$$\frac{\partial p}{\rho\,\partial s} + g\frac{dz}{ds} + V\frac{\partial v}{\partial s} = 0$$

$$\boxed{\frac{\partial p}{\rho} + g\,dz + V\,dv = 0} \rightarrow \text{Euler's equation}$$

↓

Integrating

$$\int\frac{\partial p}{\rho} + \int g\,dz + \int v\,dv = \text{constant}$$

→ If flow is compressible, ρ = constant

$$\frac{p}{\rho} + gz + \frac{v^2}{2} = \text{Constant}$$

$$\boxed{\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{Constant}}$$

$\frac{p}{\rho g}$ = pressure head

$\frac{v^2}{2g}$ = Kinetic head

z = potential head

Statement of Bernoulli's theorem:

It states that in a steady, ideal flow of incompressible fluid, the total energy any point of the fluid is always constant

∴ Total energy = constant

So, Assumptions made in Bernoulli's equation

→ The fluid is ideal

→ The flow is steady

→ The flow is incompressible

→ The flow is irrotational

Problem ①:

Water is flowing through a pipe of 5cm diameter under a pressure of 29.43 N/cm^2 (gauge) and with mean velocity of 2 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5m above the datum line.

Given:

Diameter of pipe = 5cm = 0.5m

pressure, $p = 29.43\ N/cm^2 = 29.43 \times 10^4\ N/m^2$

velocity, $v = 2\ m/s$

Datum head, $Z = 5m$

Total head = pressure head + kinetic head + datum head

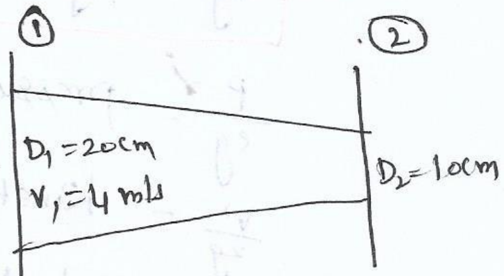
$$\text{pressure head} = \frac{p}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30m$$

$$\text{Kinetic head} = \frac{v^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204m$$

$$\therefore \text{Total head} = \frac{p}{\rho g} + \frac{v^2}{2g} + z = 30 + 0.204 + 5 = \underline{35.204m}$$

Problem ②:

A pipe, through which water is flowing, is having diameters, 20cm and 10cm at the cross sections 1 and 2 respectively. The velocity of water at section 1 is given 4 m/s. Find the velocity head at sections 1 and 2 and also rate of discharge.



Given: $D_1 = 20cm = 0.2m$

$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times 0.2^2 = 0.0314\ m^2$

$V_1 = 4\ m/s$

$D_2 = 10cm = 0.1m$

$A_2 = \frac{\pi}{4}(0.1)^2 = 0.00785\, m^2$

i) velocity head at Section 1 $= \frac{v^2}{2g}$

$= \frac{4 \times 4}{2 \times 9.81} = 0.815\, m.$

ii) velocity head at Section 2 $= \frac{V_2^2}{2g}$

$A_1 V_1 = A_2 V_2$

$V_2 = \frac{A_1 V_1}{A_2} = \frac{0.0314 \times 4}{0.00785} = 16\, m/s.$

$\therefore$ velocity head at Section 2 $= \frac{V_2^2}{2g} = \frac{16^2}{2 \times 9.81} = 83.047\, m.$

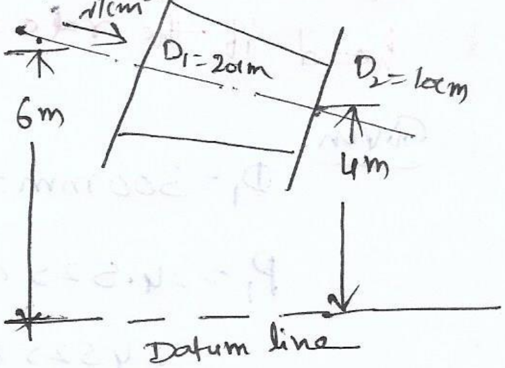
iii) Rate of Discharge $= A_1 V_1$ or $A_2 V_2$

$= 0.0314 \times 4 = \underline{0.125\, m^3/sec}$

problem (3):

The water is flowing through a pipe having diameters 20cm and 10cm at Sections 1 and 2 respectively. The rate of flow through the pipe is 35 lit/sec. The Section 1 is 6m above the datum and Section 2 is 4m above datum. If the pressure at Section 1 is 39.24 N/cm², find the intensity of pressure at Section 2.

Given:



At Section 1

$D_1 = 20cm = 0.2m$

$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} 0.2^2 = 0.0314\, m^2$

$P_1 = 39.24\, N/cm^2$

$= 39.24 \times 10^4\, N/m^2$

$Z_1 = 6m.$

$Z_2 = 4m$

At Section 2 $\quad D_2 = 0.1m.$

$A_2 = \frac{\pi}{4}(0.1)^2 = 0.00785\, m^2 \quad P_2 = ?$

Rate of flow, $Q = 35 \text{ lit/sec} = \frac{35}{1000} = 0.035 \text{ m}^3/\text{sec}$.

$$Q = A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{Q}{A_1} = \frac{0.035}{0.0314} = 1.114 \text{ m/sec}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.035}{0.00785} = 4.456 \text{ m/sec}$$

Applying Bernoulli's equation at sections ① & ②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6 = \frac{P_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4$$

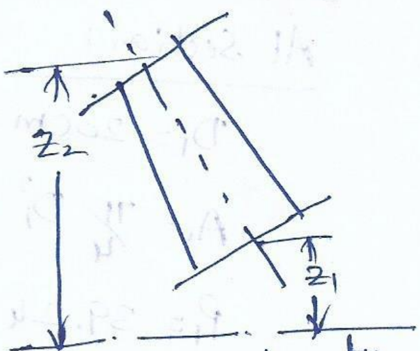
$$46.063 = \frac{P_2}{9810} + 5.012$$

$$P_2 = 40.27 \text{ N/cm}^2$$

Problem ④:

water is flowing through a pipe having diameter 300 mm and 200 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525 N/cm² and the pressure at the upper end is 9.81 N/cm². Determine the difference in datum head if the rate of flow through pipe is 40 lit/sec.

Given,



$D_1 = 300 \text{ mm} = 0.3 \text{ m}$

$P_1 = 24.525 \text{ N/cm}^2$

$= 24.525 \times 10^4 \text{ N/m}^2$

$D_2 = 200 \text{ mm} = 0.2 \text{ m}$

$P_2 = 9.81 \text{ N/cm}^2$

$= 9.81 \times 10^4 \text{ N/m}^2$

* Orifice meter:

⇒ It is a device used for measuring the rate of flow of a fluid through a pipe

⇒ It is a cheaper device as compared to Venturi meter. It works on the same principle as that of venturimeter.

⇒ It consists of a flat circular plate which has a circular sharp edged hole called orifice.

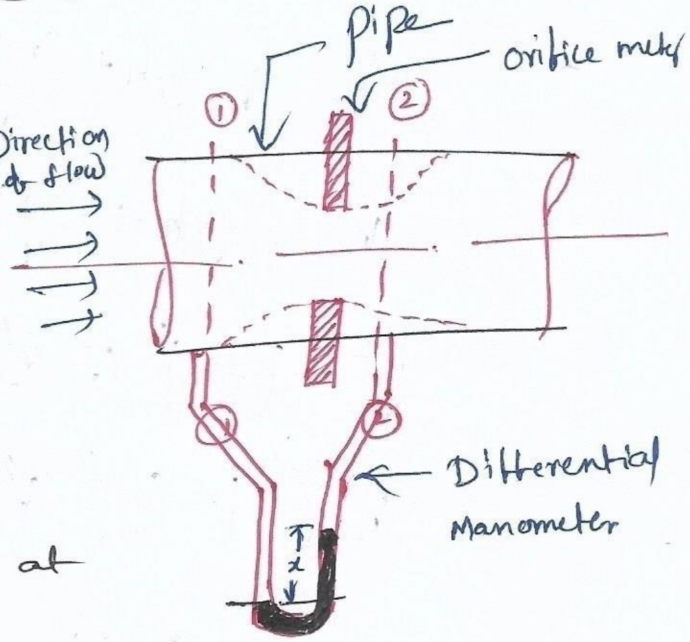
Let

P_1 = pressure at Section ①

V_1 = velocity at section ①

a_1 = area of pipe at section ①

P_2, V_2, a_2 are the corresponding values of at Section ②



Applying Bernoulli's equation at Sections ① & ②

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

$$\frac{P_1}{\rho g}-\frac{P_2}{\rho g}=h$$

$$h=\frac{V_2^2}{2g}-\frac{V_1^2}{2g} \quad \text{or} \quad 2gh=V_2^2-V_1^2$$

$$V_2=\sqrt{2gh+V_1^2} \quad \text{—— ①}$$

Now Section ② is at the Vena-Contracta and a_2 represents the area at the vena-contract. If a_0 is the area of orifice then, we have $C_c=\frac{a_2}{a_0}$, C_c = Coefficient of contraction

$$\therefore a_2 = a_0 \times C_c \quad \text{—— (2)}$$

By continuity equation.

$$a_1 V_1 = a_2 V_2$$

$$V_1 = \frac{a_2}{a_1} V_2 = \frac{a_0 C_c}{a_1} V_2$$

Substituting the value of V_1 in eqn (1)

$$V_2 = \sqrt{2gh + \frac{a_0^2 C_c^2 V_2^2}{a_1^2}}$$

$$V_2^2 = 2gh + \left(\frac{a_0}{a_1}\right)^2 C_c^2 V_2^2$$

$$V_2^2 - \left(\frac{a_0}{a_1}\right)^2 C_c^2 V_2^2 = 2gh$$

$$V_2^2 \left(1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2\right) = 2gh$$

$$V_2 = \frac{\sqrt{2gh}}{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}$$

$$Q = a_2 V_2 = a_0 C_c V_2$$

$$= \frac{a_0 C_c \sqrt{2gh}}{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}$$

The above expression is simplified by using

$$C_d = \frac{C_c \sqrt{1 - \left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$$\boxed{\therefore Q = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}}$$

Problem:

An orifice meter with orifice diameter 10cm is inserted in a pipe of 20cm diameter. The pressure gauges fitted upstream and downstream of the orifice meter gives readings of 19.62 N/cm^2 and 9.81 N/cm^2 respectively. Coefficient of discharge for the orifice meter is given as 0.6. Find the discharge of water through pipe.

Sol:- Given:

Dia. of orifice, $d_0 = 10cm$

$$\text{Area, } a_0 = \frac{\pi}{4}(10)^2 = 78.54\ cm^2$$

Dia. of pipe, $d_1 = 20cm$.

$$a_1 = \frac{\pi}{4}(20)^2 = 314.16\ cm^2$$

$$P_1 = 19.62\ N/cm^2 = 19.62 \times 10^4\ N/m^2$$

$$\frac{P_1}{\rho g} = \frac{19.62 \times 10^4}{1000 \times 9.81} = 20\ m \text{ of water}$$

$$\frac{P_2}{\rho g} = \frac{9.81 \times 10^4}{1000 \times 9.81} = 10\ m \text{ of water}$$

$$h = \frac{P_1}{\rho g} - \frac{P_2}{\rho g} = 20 - 10 = 10\ m \text{ of water}$$

$$C_d = 0.6$$

$$\therefore Q = C_d \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$$= 0.6\ \frac{78.54 \times 314.16}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 9.81 \times 1000}$$

$$Q = \underline{68213.28\ cm^3/s}$$

② An Orifice meter with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter. The pressure difference measured by a mercury oil differential manometer on the two sides of the orifice meter gives a readings of 50 cm of mercury. Find the rate of flow oil of sp. gr. 0.9. when the coefficient of discharge of the Orifice meter 0.64.

Sol:- Given

Dia. of orifice, $d_0 = 15$ cm.

$$\text{Area, } a_0 = \frac{\pi}{4} \times 15^2 = 176.7 \text{ cm}^2$$

Dia. of pipe, $d_1 = 30$ cm

$$\text{Area, } a_1 = \frac{\pi}{4} \times 30^2 = 706.85 \text{ cm}^2$$

Sp. gr. of oil, $S_0 = 0.9$.

Reading of diff. manometer, $x = 50$ cm of mercury

$\therefore$ Differential head, $h = x\left[\frac{S_g}{S_0} - 1\right]$

$$= 50\left[\frac{13.6}{0.9} - 1\right] \text{ cm of oil}$$

$$= 50 \times 14.11 = 705.5 \text{ cm of oil}$$

$\therefore$ The rate of the flow, Q is given by

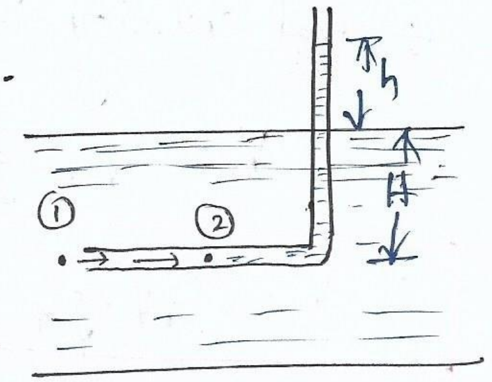
$$Q = C_d \cdot \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$$= 0.64 \times \frac{176.7 \times 706.85}{\sqrt{706.85^2 - 176.7^2}} \times \sqrt{2 \times 9.81 \times 705.5}$$

$$Q = 137414.25 \text{ cm}^3/\text{s}.$$

$$\therefore Q = 137.414 \text{ lit/sec}$$

* pitot- tube:-



⇒ It is a device used for measuring the velocity of flow at any point in a pipe or a channel.

⇒ It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the coversion of the kinetic energy in to pressure energy.

⇒ The pitot-tube consists of a glass tube, bent at right angles as shown in fig.

Consider two points ① & ② at the same level in such a way that point ② is just as the inlet of the pitot-tube and point ① is far away from the tube.

Applying Bernoulli's equation at point ① & ②

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + Z_2$$

But $Z_1 = Z_2$, $V_2 = 0$

$$\frac{P_1}{\rho g} = H \quad , \quad \frac{P_2}{\rho g} = (h+H)$$

$$\therefore \quad H + \frac{v_1^2}{2g} = (h+H) \Rightarrow \frac{v_1^2}{2g} = h + \cancel{H} - \cancel{H}$$

$$\frac{v_1^2}{2g} = h$$

$$v_1^2 = 2gh \Rightarrow \boxed{v_1 = \sqrt{2gh}}$$

this is theoretical velocity

Actual velocity is given by

$$V_1 = C_v \sqrt{2gh}$$

$$\boxed{\therefore V = C_v \sqrt{2gh}}$$

Problem ①

Find the velocity of flow of an oil through a pipe, when the difference of mercury level in a differential U-tube manometer connected to the two tappings of the pitot tube is 100mm. Take coefficient of pitot tube 0.98 and sp.gr of oil 0.8.

Sol:- Difference of mercury level, $x = 100\,mm = 0.1\,m$

sp.gr of oil, $S_o = 0.8$

$S_g = 13.6$

$C_v = 0.98$

$$h = x\left[\frac{S_g}{S_o} - 1\right] = 0.1\left[\frac{13.6}{0.8} - 1\right] = 1.6\,m \text{ of oil.}$$

$$\therefore V = C_v\sqrt{2gh} = 0.98 \times \sqrt{2 \times 9.81 \times 1.6} = \underline{5.49\ m/s}$$

Problem ②

A pitot tube is used to measure the velocity of water in a pipe. The stagnation pressure head is 6m and static pressure head is 5m. Calculate the velocity of flow assuming the coefficient of tube equal to 0.98

Sol:- Stagnation pressure head, $h_s = 6\,m$

Static pressure head, $h_1 = 5\,m$

$h = 6 - 5 = 1\,m.$

$\therefore$ Velocity of flow, $V = C_v\sqrt{2gh}$

$= 0.98 \times \sqrt{2 \times 9.81 \times 1}$

$V = \underline{4.34\ m/s}$

* velocity potential function:-

It is a mathematical function in 2D flow which exists in such a way that it's partial derivative in any direction gives the velocity in that direction.

$$u = \frac{\partial \phi}{\partial x}, \quad v = \frac{\partial \phi}{\partial y}$$

Stream function:-

It is defined as a mathematical function in 2D fluid flow which exists in such a way that it's partial derivative w.r.t y direction gives the +ve velocity in x-direction & it's partial derivative w.r.t x-direction gives the -ve velocity in y-direction.

$$u = \frac{\partial \psi}{\partial y}, \quad v = \frac{-\partial \psi}{\partial x}$$

① stream function $\psi = 5x - 6y$, calculate velocity components and also magnitude & direction of the resultant velocity at any point.

Sol: $\psi = 5x - 6y$

$\frac{\partial \psi}{\partial x} = 5$

$\frac{\partial \psi}{\partial y} = -6$

$u = \frac{\partial \psi}{\partial y} = -6$ units

$v = \frac{\partial \psi}{\partial x} = 5$ units

$V = \sqrt{u^2 + v^2} = \sqrt{6^2 + 5^2} = 7.81$ units/sec

$\tan \phi = v/u = 5/6 = 0.833$

$\phi = \tan^{-1} 0.833$

$\phi = 39°48'$

(2) if $\phi = x(2y-1)$. Determine the velocity at point $p(4,5)$ also determine the value of stream function ψ at the point p

Sol:

$\phi = x(2y-1)$

$\frac{\partial \phi}{\partial x} = 2y-1 \quad , \quad \frac{\partial \phi}{\partial y} = 2x$

$u = \frac{-\partial \phi}{\partial x} = 1-2y$

$v = \frac{-\partial \phi}{\partial y} = -2x$

at point $p(4,5)$, $u = 1-2\times5 = -9$

$v = -2\times4 = -8$

Resultant velocity at p, $v = \sqrt{u^2+v^2} = \sqrt{9^2+8^2} = 12.04$ units/sec

value of stream function ψ at p

$\frac{\partial \psi}{\partial y} = -u = -(1-2y)$

$= 2y-1$

$\frac{\partial \psi}{\partial x} = v = -2x$

$\partial \psi = (2y-1)\,\partial y$

$\psi = y^2 - y + K$

$\frac{\partial \psi}{\partial x} = 0 - 0 + \frac{\partial K}{\partial x}$

$\frac{\partial \psi}{\partial x} = \frac{\partial K}{\partial x}$

$\int \frac{\partial K}{\partial x} = -2x$

$K = -x^2$

Now $\psi = y^2 - y - x^2$

$p(4,5)$

$\psi = 5^2 - 5 - 4^2$

$\psi = 4$ units

1) $\phi = 3xy^2 - x^3$

$\psi = ?$

$u = \frac{d\phi}{dx} = \frac{d}{dx}(3xy^2 - x^3) = 3y^2 - 3x^2$

$v = \frac{\partial \phi}{\partial y} = \partial/\partial y\,(3xy^2 - x^3) = 6xy$

$\frac{\partial \psi}{\partial y} = u = 3y^2 - 3x^2$

$\int \partial\psi = \int (3y^2 - 3x^2)\,dy$

$\psi = y^3 - 3x^2 y + f(x)$

$-\frac{\partial \psi}{\partial x} = v = 6xy$

$\int \partial\psi = \int 6xy\,dx$

$\psi = -3x^2 y + f(y)$

1). if $\phi = \frac{-xy^3}{3} - x^2 + \frac{x^3y}{3} + y^2$, then find

1) velocity component in x & y direction

2) Can ϕ represents the possible case of flow

Sol:

$$\frac{\partial \phi}{\partial x} = \frac{-y^3}{3} - 2x + \frac{3x^2y}{3}$$

$$= \frac{-y^3}{3} - 2x + x^2y$$

$$\frac{\partial \phi}{\partial y} = -xy^3 + \frac{x^3}{3} + 2y$$

The velocity component u & v are

$$u = \frac{-\partial \phi}{\partial x} = \frac{y^3}{3} + 2x - x^2y$$

$$v = \frac{-\partial \phi}{\partial y} = xy^3 - \frac{x^3}{3} - 2y$$

2) if $\phi = 5(x^2 - y^2)$, at point (4,5) Calculate velocity u & v.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} = -2 + 2xy \quad \frac{\partial^2 \phi}{\partial y^2} = -2xy + 2$$

$$-2 + 2xy - 2xy + 2 = 0$$

∴ It is possible case of flow

$\phi = 5(x^2 - y^2)$

$$\frac{\partial \phi}{\partial x} = 10x$$

$$\frac{\partial \phi}{\partial y} = -10y$$

But velocity u & v are

$$u = -\frac{\partial \phi}{\partial x} = -10x$$

$$v = -\frac{\partial \phi}{\partial y} = 10y$$

point (4, 5)

$u = -10 \times 4 = -40$ units

$v = 10 \times 5 = 50$ units

$$Z_B = 4.0 \text{ m}$$

$$V_B = \frac{Q}{\text{Area}} = \frac{0.2}{.1963} = 1.018 \text{ m/s}$$

Total energy at A $$= E_A = \frac{p_A}{\rho g} + \frac{V_A^2}{2g} + Z_A$$

$$= \frac{9.81 \times 10^4}{870 \times 9.81} + \frac{(6.369)^2}{2 \times 9.81} + 0 = 11.49 + 2.067 = 13.557 \text{ m}$$

Total energy at B $$= E_B = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + Z_B$$

$$= \frac{5.886 \times 10^4}{870 \times 9.81} + \frac{(1.018)^2}{2 \times 9.81} + 4.0 = 6.896 + 0.052 + 4.0 = 10.948 \text{ m}$$

(*i*) **Direction of flow.** As E_A is more than E_B and hence flow is taking place from A to B. **Ans.**

(*ii*) Loss of head $= h_L = E_A - E_B = 13.557 - 10.948 =$ **2.609 m. Ans.**

▶ 6.7 PRACTICAL APPLICATIONS OF BERNOULLI'S EQUATION

Bernoulli's equation is applied in all problems of incompressible fluid flow where energy considerations are involved. But we shall consider its application to the following measuring devices :

1. Venturimeter.
2. Orifice meter.
3. Pitot-tube.

6.7.1 Venturimeter. A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of three parts :

(*i*) A short converging part, (*ii*) Throat, and (*iii*) Diverging part. It is based on the Principle of Bernoulli's equation.

Expression for Rate of Flow Through Venturimeter

Consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing (say water), as shown in Fig. 6.9.

Let d_1 = diameter at inlet or at section (1),

p_1 = pressure at section (1)

v_1 = velocity of fluid at section (1),

a = area at section (1) $= \frac{\pi}{4} d_1^2$

and d_2, p_2, v_2, a_2 are corresponding values at section (2).

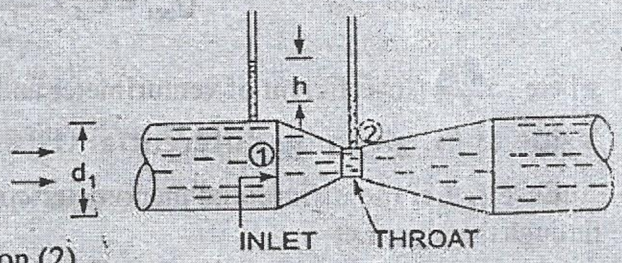


Fig. 6.9 *Venturimeter.*

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As pipe is horizontal, hence $z_1 = z_2$

$$\therefore \quad \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \quad \text{or} \quad \frac{p_1 - p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But $\frac{p_1 - p_2}{\rho g}$ is the difference of pressure heads at sections 1 and 2 and it is equal to h or $\frac{p_1 - p_2}{\rho g} = h$

Substituting this value of $\frac{p_1 - p_2}{\rho g}$ in the above equation, we get

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \qquad \text{...(6.6)}$$

Now applying continuity equation at sections 1 and 2

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad v_1 = \frac{a_2 v_2}{a_1}$$

Substituting this value of v_1 in equation (6.6)

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right] = \frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

or

$$v_2^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$$

$\therefore$

$$v_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$\therefore$ Discharge,

$$Q = a_2 v_2$$

$$= a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \qquad \text{...(6.7)}$$

Equation (6.7) gives the discharge under ideal conditions and is called, theoretical discharge. Actual discharge will be less than theoretical discharge.

$\therefore$

$$Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \qquad \text{...(6.8)}$$

where C_d = Co-efficient of venturimeter and its value is less than 1.

Value of 'h' given by differential U-tube manometer

Case I. Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe. Let

S_h = Sp. gravity of the heavier liquid

S_o = Sp. gravity of the liquid flowing through pipe

x = Difference of the heavier liquid column in U-tube

Then

$$h = x\left[\frac{S_h}{S_o} - 1\right] \qquad \text{...(6.9)}$$

Case II. If the differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given by

$$h = x\left[1 - \frac{S_l}{S_o}\right] \quad \text{...(6.10)}$$

where S_1 = Sp. gr. of lighter liquid in U-tube

S_0 = Sp. gr. of fluid flowing through pipe

x = Difference of the lighter liquid columns in U-tube.

Case III. Inclined Venturimeter with Differential U-tube manometer. The above two cases are given for a horizontal venturimeter. This case is related to inclined venturimeter having differential U-tube manometer. Let the differential manometer contains heavier liquid then h is given as

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[\frac{S_h}{S_o} - 1\right] \quad \text{...(6.11)}$$

Case IV. Similarly, for inclined venturimeter in which differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given as

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[1 - \frac{S_l}{S_o}\right] \quad \text{...(6.12)}$$

Problem 6.10 *A horizontal venturimeter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and the throat is 20 cm of mercury. Determine the rate of flow. Take $C_d = 0.98$.*

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

$\therefore$ Area at inlet, $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

$\therefore$ $a_2 = \frac{\pi}{4} \times 15^2 = 176.7 \text{ cm}^2$

$C_d = 0.98$

Reading of differential manometer $= x = 20$ cm of mercury.

$\therefore$ Difference of pressure head is given by (6.9)

or $$h = x\left[\frac{S_h}{S_o} - 1\right]$$

where S_h = Sp. gravity of mercury = 13.6, S_0 = Sp. gravity of water = 1

$$= 20\left[\frac{13.6}{1} - 1\right] = 20 \times 12.6 \text{ cm} = 252.0 \text{ cm of water.}$$

The discharge through venturimeter is given by eqn. (6.8)

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.9 - 31222.9}} = \frac{86067593.36}{684.4}$$

$$= 125756 \text{ cm}^3\text{/s} = \frac{125756}{1000} \text{ lit/s} = \mathbf{125.756 \text{ lit/s. Ans.}}$$

Problem 6.11 *An oil of sp. gr. 0.8 is flowing through a venturimeter having inlet diameter 20 cm and throat diameter 10 cm. The oil-mercury differential manometer shows a reading of 25 cm. Calculate the discharge of oil through the horizontal venturimeter. Take $C_d = 0.98$.*

Solution. Given :

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_h = 13.6$

Reading of differential manometer, $x = 25$ cm

$\therefore$ Difference of pressure head, $h = x\left[\frac{S_h}{S_o} - 1\right]$

$$= 25\left[\frac{13.6}{0.8} - 1\right] \text{ cm of oil} = 25\,[17 - 1] = 400 \text{ cm of oil.}$$

Dia. at inlet, $d_1 = 20$ cm

$\therefore$ $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times 20^2 = 314.16 \text{ cm}^2$

$d_2 = 10$ cm

$\therefore$ $a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$

$C_d = 0.98$

$\therefore$ The discharge Q is given by equation (6.8)

or $$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - 7a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 400}$$

$$= \frac{21421375.68}{\sqrt{98696 - 6168}} = \frac{21421375.68}{304} \text{ cm}^3\text{/s}$$

$$= 70465 \text{ cm}^3\text{/s} = \mathbf{70.465 \text{ litres/s. Ans.}}$$

Problem 6.12 *A horizontal venturimeter with inlet diameter 20 cm and throat diameter 10 cm is used to measure the flow of oil of sp. gr. 0.8. The discharge of oil through venturimeter is 60 litres/s. Find the reading of the oil-mercury differential manometer. Take $C_d = 0.98$.*

Solution. Given : $d_1 = 20$ cm

$\therefore$ $a_1 = \frac{\pi}{4} 20^2 = 314.16 \text{ cm}^2$

$d_2 = 10$ cm

tube is 0.981 N/cm². Calculate the rate of flow of water through pipe, if the mean velocity of flow is 0.85 times the central velocity. Take $C_v = 0.98$. (Converted to S.I. Units, A.M.I.E., Summer, 1987)

Solution. Given :

Dia. of pipe, $d = 300 \text{ mm} = 0.30 \text{ m}$

∴ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.3)^2 = 0.07068 \text{ m}^2$

Static pressure head $= 100$ mm of mercury (vacuum)

$$= -\frac{100}{1000} \times 13.6 = -1.36 \text{ m of water}$$

Stagnation pressure $= .981 \text{ N/cm}^2 = .981 \times 10^4 \text{ N/m}^2$

∴ Stagnation pressure head $= \frac{.981 \times 10^4}{\rho g} = \frac{.981 \times 10^4}{1000 \times 9.81} = 1 \text{ m}$

∴ h = Stagnation pressure head – Static pressure head

$= 1.0 - (-1.36) = 1.0 + 1.36 = 2.36$ m of water

∴ Velocity at centre $= C_v \sqrt{2gh}$

$$= 0.98 \times \sqrt{2 \times 9.81 \times 2.36} = 6.668 \text{ m/s}$$

Mean velocity, $\overline{V} = 0.85 \times 6.668 = 5.6678 \text{ m/s}$

∴ Rate of flow of water $= \overline{V} \times$ area of pipe

$= 5.6678 \times 0.07068 \text{ m}^3/\text{s} = \mathbf{0.4006 \text{ m}^3/\text{s}}$. **Ans.**

▶ 6.8 THE MOMENTUM EQUATION

It is based on the law of conservation of momentum or on the momentum principle, which states that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction. The force acting on a fluid mass 'm' is given by the Newton's second law of motion,

$$F = m \times a$$

where a is the acceleration acting in the same direction as force F.

But $a = \frac{dv}{dt}$

∴ $F = m \frac{dv}{dt}$

$= \frac{d(mv)}{dt}$ {m is constant and can be taken inside the differential}

∴ $$F = \frac{d(mv)}{dt} \quad \text{...(6.15)}$$

Equation (6.15) is known as the momentum principle.

Equation (6.15) can be written as $F.dt = d(mv)$ ...(6.16)

which is known as the *impulse-momentum equation* and states that the impulse of a force F acting on a fluid of mass m in a short interval of time dt is equal to the change of momentum $d(mv)$ in the direction of force.

Force exerted by a flowing fluid on a Pipe-Bend

The impulse-momentum equation (6.16) is used to determine the resultant force exerted by a flowing fluid on a pipe bend.

Consider two sections (1) and (2), as shown in Fig. 6.18.

Let v_1 = velocity of flow at section (1),

p_1 = pressure intensity at section (1),

A_1 = area of cross-section of pipe at section (1) and

v_2, p_2, A_2 = corresponding values of velocity, pressure and area at section (2).

Let F_x and F_y be the components of the forces exerted by the flowing fluid on the bend in x-and y-directions respectively. Then the force exerted by the bend on the fluid in the directions of x and y will be equal to F_x and F_y but in the opposite directions. Hence component of the force exerted by bend on the fluid in the x-direction $= -F_x$ and in the direction of $y = -F_y$. The other external forces acting on the fluid are p_1A_1 and p_2A_2 on the sections (1) and (2) respectively. Then momentum equation in x-direction is given by

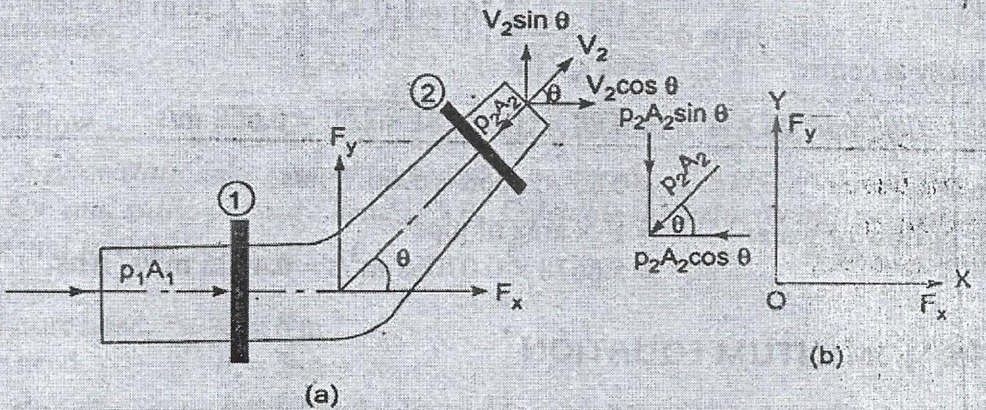


Fig. 6.18 *Forces on bend.*

Net force acting on fluid in the direction of x = Rate of change of momentum in x-direction

$\therefore$ $p_1A_1 - p_2A_2 \cos\theta - F_x$ = (Mass per sec) (change of velocity)

$= \rho Q$ (Final velocity in the direction of x – Initial velocity in the direction of x)

$$= \rho Q (V_2 \cos\theta - V_1) \quad ...(6.17)$$

$$\therefore \quad F_x = \rho Q (V_1 - V_2 \cos\theta) + p_1A_1 - p_2A_2 \cos\theta \quad ...(6.18)$$

Similarly the momentum equation in y-direction gives

$$0 - p_2A_2 \sin\theta - F_y = \rho Q (V_2 \sin\theta - 0) \quad ...(6.19)$$

$$\therefore \quad F_y = \rho Q (- V_2 \sin\theta) - p_2A_2 \sin\theta \quad ...(6.20)$$

Now the resultant force (F_R) acting on the bend

$$= \sqrt{F_x^2 + F_y^2} \quad ...(6.21)$$

And the angle made by the resultant force with horizontal direction is given by

$$\tan\theta = \frac{F_y}{F_x} \quad ...(6.22)$$

Problem 6.29 *A 45° reducing bend is connected in a pipe line, the diameters at the inlet and outlet of the bend being 600 mm and 300 mm respectively. Find the force exerted by water on the bend if the intensity of pressure at inlet to bend is 8.829 N/cm² and rate of flow of water is 600 litres/s.*

Solution. Given :

Angle of bend, $\theta = 45°$

Dia. at inlet, $D_1 = 600$ mm $= 0.6$ m

$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.6)^2 = 0.2827 \text{ m}^2$

Dia. at outlet, $D_2 = 300$ mm $= 0.30$ m

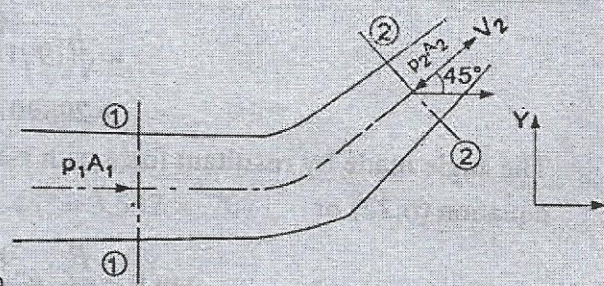


Fig. 6.19

$\therefore$ Area, $A_2 = \frac{\pi}{4} (3)^2 = 0.07068 \text{ m}^2$

Pressure at inlet, $p_1 = 8.829 \text{ N/cm}^2 = 8.829 \times 10^4 \text{ N/m}^2$

$Q = 600 \text{ lit/s} = 0.6 \text{ m}^3\text{/s}$

$$V_1 = \frac{Q}{A_1} = \frac{0.6}{.2827} = 2.122 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.6}{.07068} = 8.488 \text{ m/s.}$$

Applying Bernoulli's equation at section (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

But $z_1 = z_2$

$$\therefore \quad \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} \quad \text{or} \quad \frac{8.829 \times 10^4}{1000 \times 9.81} + \frac{2.122^2}{2 \times 9.81} = \frac{p_2}{\rho g} + \frac{8.488^2}{2 \times 9.81}$$

$$9 + .2295 = p_2/\rho g + 3.672$$

$$\therefore \quad \frac{p_2}{\rho g} = 9.2295 - 3.672 = 5.5575 \text{ m of water}$$

$$\therefore \quad p_2 = 5.5575 \times 1000 \times 9.81 \text{ N/m}^2 = 5.45 \times 10^4 \text{ N/m}^2$$

Forces on the bend in x- and y-directions are given by equations (6.18) and (6.20) as

$$F_x = \rho Q [v_1 - v_2 \cos \theta] + p_1 A_1 - p_2 A_2 \cos \theta$$
$$= 1000 \times 0.6 [2.122 - 8.488 \cos 45°]$$
$$+ 8.829 \times 10^4 \times .2827 - 5.45 \times 10^4 \times .07068 \times \cos 45°$$
$$= -2327.9 + 24959.6 - 2720.3 = 24959.6 - 5048.2$$
$$= 19911.4 \text{ N}$$

and

$$F_y = \rho Q [-V_2 \sin \theta] - p_2 A_2 \sin \theta$$
$$= 1000 \times 0.6 [-8.488 \sin 45°[- 5.45 \times 10^4 \times .07068 \times \sin 45°$$
$$= -3601.1 - 2721.1 = -6322.2 \text{ N}$$

–ve sign means F_y is acting in the downward direction

$\therefore$ Resultant force, $F_R = \sqrt{F_x^2 + F_y^2}$

$$= \sqrt{(19911.4)^2 + (-6322.2)^2}$$

$$= \mathbf{20890.9\ N.\ Ans.}$$

The angle made by resultant force with x-axis is given by equation (6.22) or

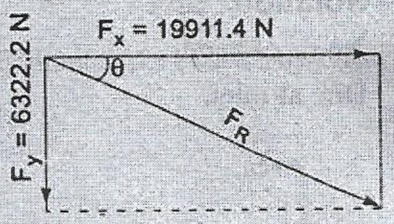


Fig. 6.20

$$\tan\theta = \frac{F_y}{F_x} = \frac{6322.2}{19911.4} = 0.3175$$

$\therefore$ $\theta = \tan^{-1} .3175 = \mathbf{17° 36'. Ans.}$

Problem 6.30 *250 litres/s of water is flowing in a pipe having a diameter of 300 mm. If the pipe is bent by 135° (that is change from initial to final direction is 135°), find the magnitude and direction of the resultant force on the bend. The pressure of water flowing is 39.24 N/cm².*

(A.M.I.E., Winter, 1974)

Solution. Given :

Pressure, $p_1 = p_2 = 39.24$ N/cm^2 $= 39.24 \times 10^4$ N/m^2

Discharge, $Q = 250$ litres/s $= 0.25$ m^3/s

Dia. of bend at inlet and outlet, $D_1 = D_2 = 300$ mm $= 0.3$ m

$\therefore$ Area, $A_1 = A_2 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times .3^2 = 0.07068$ m^2

Velocity of water at (1) and (2), $V = V_1 = V_2 = \frac{Q}{\text{Area}} = \frac{0.25}{.07068} = 3.537$ m/s.

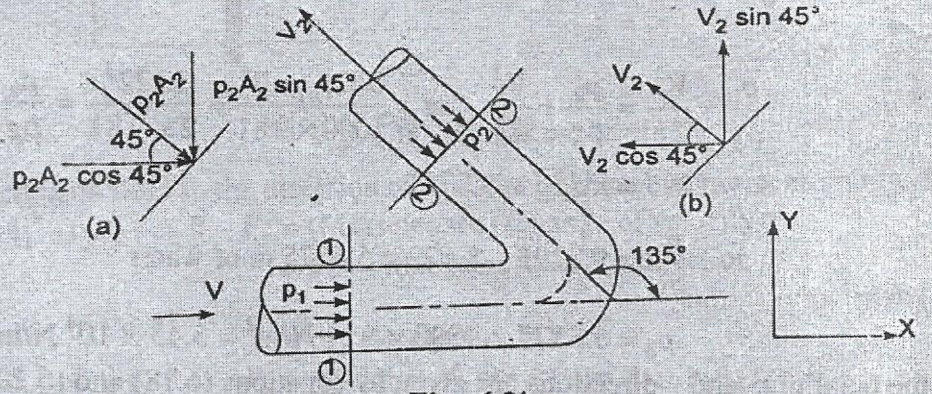


Fig. 6.21

Force along x-axis

$= F_x = \rho Q[V_{1x} - V_{2x}] + p_{1x}A_1 + p_{2x}A_2$

where V_{1x} = initial velocity in the direction of $x = 3.537$ m/s

V_{2x} = final velocity in the direction of $x = -V_2 \cos 45° = -3.537 \times .7071$

p_{1x} = pressure at (1) in x-direction

$= 39.24$ N/cm^2 $= 39.24 \times 10^4$ N/m^2

p_{2x} = pressure at (2) in x-direction

$= p_2 \cos 45° = 39.24 \times 10^4 \times .7071$

$\therefore$ $F_x = 1000 \times .25[3.537 - (-3.537 \times .7071)] + 39.24 \times 10^4 \times .07068 + 39.24 \times 10^4 \times .07068 \times .7071$

$= 1000 \times .25[3.537 + 3.537 \times .7071] + 39.24 \times 10^4 \times .07068\,[1 + .7071]$

1.REYNOLDS NUMBER

- It is the ratio of inertia force to the viscous force.

$$\mathrm{Re} = \frac{\rho V L}{\mu}$$

Where,

ρ is density

V is velocity

L is linear dimension

μ is viscosity

Significance-It is used to identify the nature of flow (Laminar or Turbulent)

2.FROUDE NUMBER

- It is the ratio of inertia force to the gravity force

$$Fr = \frac{V}{\sqrt{gL}}$$

- Where ,

  V is the velocity

  g is the acceleration due to gravity

  L is the characteristics length

Significance-It is used to analyze nature of flow where free surface is present. e.g. weirs ,channels ,spillways

3.WEBERS NUMBER

- It is the ratio of inertia force to surface tension force.

$$We = \frac{V}{\sqrt{\sigma / \rho L}}$$

- Where

  V is the velocity

  σ is the surface tension

  ρ is the density

  L is the characteristics length

Significance-It is used in analyzing formation of droplet and in capillary studies.

4.EULER NUMBER

- It is the ratio of inertia force to pressure force.

$$Eu = \frac{V}{\sqrt{\frac{P}{\rho}}}$$

- Where

  P is the pressure

  ρ is the density

  V is the velocity

Significance-It is used to characterize energy losses in a flow.

5.MACH NUMBER

- It is the ratio of inertia force to elastic force.

$$M = \frac{V}{C}$$

- Where

  V is the velocity of object

  C is the speed of sound

Significance-It is used to analyze fluid flow where compressibility is a important factor.

UNIT-5

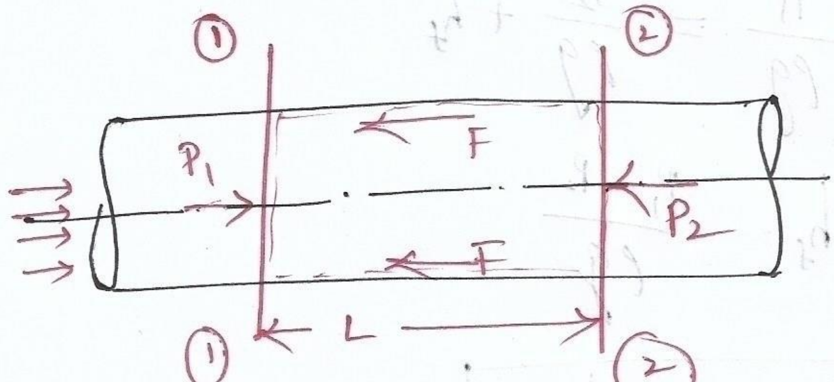
Flow through pipes

* Energy losses in pipelines:-

Energy losses

- Major Energy losses
  - This is due to friction and is calculated by
    1) Darcy - Weisbach equation
    2) Chezy's formula
- Minor Energy losses
  - This is due to
    i) Sudden expansion of pipe
    ii) Sudden contraction of pipe
    iii) Bend in pipe
    iv) An obstruction in pipe
    v) At entrance
    vi) At exit of pipe
    vii) Due to pipe fittings

* Darcy - Weisbach equation for friction loss: (or) Loss of head due to friction:-



→ Consider a uniform horizontal pipe having steady flow. sections 1-1 & Section 2-2.

* Step 1:- Define terms:

d = diameter of pipes

A = Area of cross section of pipe

P_1 = pressure at section 1-1

P_2 = pressure at section 2-2

V_1 = velocity at section 1-1

V_2 = velocity at section 2-2

L = Length of pipe, h_f = head loss due to friction

* Step 2:- Applying Bernoulli's equation:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_f$$

But pipe is horizontal

$$Z_1 = Z_2$$

and the pipe is of Uniform diameter

$$V_1 = V_2$$

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + h_f$$

$$h_f = \frac{P_1 - P_2}{\rho g}$$

$$\boxed{P_1 - P_2 = \rho g h_f} \longrightarrow (1)$$

* **Step 3:** Find frictional resistance

Let f' = frictional resistance per unit wetted area per unit velocity

∴ Frictional resistance

$$= f' \times \text{wetted area} \times (\text{velocity})^2$$

$$= f' \times \pi D L \times V^2$$

$$F = f' \times p \times L \times V^2 \quad \boxed{\because \pi D = p}$$

* **Step 4:** find net force acting the fluid between Sections ①-① and ②-② are

i) At Section ①-①, $P_1 = \frac{F_1}{A} \Rightarrow F_1 = P_1 A \ (+)$

ii) At Section ②-②, $P_2 = \frac{F_2}{A} \Rightarrow F_2 = P_2 A \ (-)$

iii) Frictional resistance $F \ (-)$

∴ Resolving all force the horizontal direction

$$P_1 A - P_2 A - F = 0$$

$$(P_1 - P_2) A = F$$

$$(P_1 - P_2) = F/A$$

$$(P_1 - P_2) = \frac{f' \times p \times L \times V^2}{A}$$

→ But from equation (1)

$$P_1 - P_2 = \rho g h_f$$

$$\rho g h_f = \frac{f' \times P \times L \times V^2}{A}$$

$$h_f = \frac{f'}{\rho g} \times L \times V^2 \times \frac{P}{A}$$

But $\frac{P}{A} = \frac{\text{Wetted perimeter}}{\text{Area}} = \frac{\pi d}{\frac{\pi}{4} d^2} = \frac{4}{d}$

$$\therefore h_f = \frac{f'}{\rho g} \times L \times V^2 \times \frac{4}{d}$$

$$h_f = \frac{f'}{\rho g} \frac{4LV^2}{d}$$

putting $\frac{f'}{\rho} = \frac{f}{2}$

$$h_f = \frac{f}{2g} \frac{4LV^2}{d}$$

$$\boxed{\therefore h_f = \frac{4fLV^2}{2gd}}$$ → Darcy's weisbach equation

Where f = friction factor

Use :- Finding loss of head due to friction in pipes.

* Minor losses in pipe lines :-

Definition:

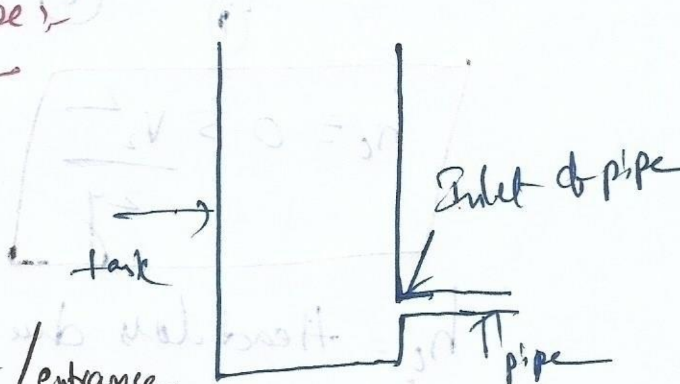
Minor losses are the losses in energy due to the change of velocity (either in magnitude or in direction) of the flowing fluid.

Types of Minor losses:-

1) At the entrance of a pipe
2) Due to sudden enlargement of pipe
3) Due to Sudden Contraction of pipe
4) Due to bend in pipe
5) Due to an obstruction in the pipe
6) In various pipe fittings
7) At the exit / out let of the pipe.

1) At the entrance of pipe :-

$$\boxed{h_i = 0.5 \frac{v^2}{2g}}$$

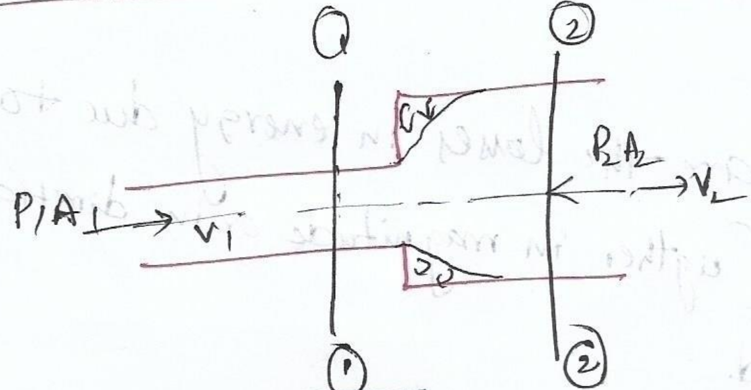


h_i = head loss at inlet / entrance of pipe

v = velocity of liquid in pipe

g = accelaration due gravity

2. Minor losses due to Sudden enlargement :-



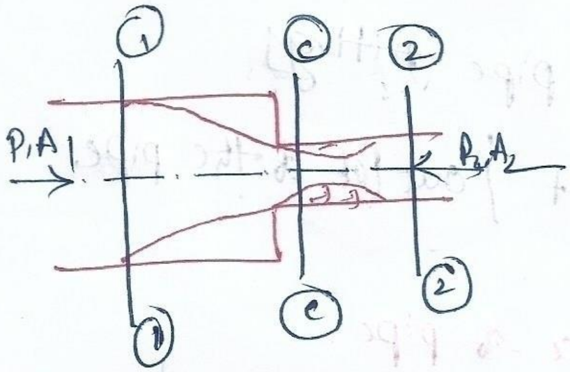
$$\boxed{h_e = \frac{(V_1 - V_2)^2}{2g}}$$

h_e = head loss due to Sudden expansion

V_1 = velocity before expansion

V_2 = velocity after expansion.

3. Minor losses due to Sudden contraction of pipe :-



$$\boxed{h_c = 0.5 \frac{V_2^2}{2g}}$$

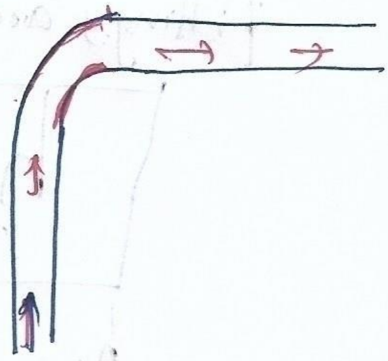
h_c = Head loss due to sudden contraction

V_2 = Velocity after contraction

g = accelaration due to gravity

4. Minor losses due to bend in a pipe:-

$$\boxed{h_b = K_b \frac{V^2}{2g}}$$

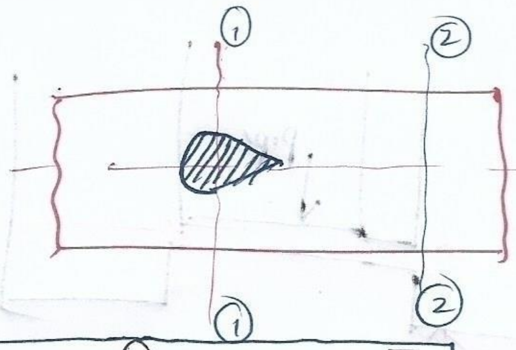


h_b = Head loss due to bend

K_b = Coefficient of bend

V = velocity of liquid

5. Minor losses due to an obstruction in the pipe:-



$$\boxed{h_{ob} = \frac{V^2}{2g}\left[\frac{A}{C_c(A-a)} - 1\right]}$$

h_{ob} = Head loss due to obstruction

A = Cross-section area of pipe

a = Maximum area of obstruction

C_c = Coefficient of contraction

V = velocity of liquid in pipe.

6. Minor losses in Various pipe fittings:-

Fittings are valves, couplings etc..

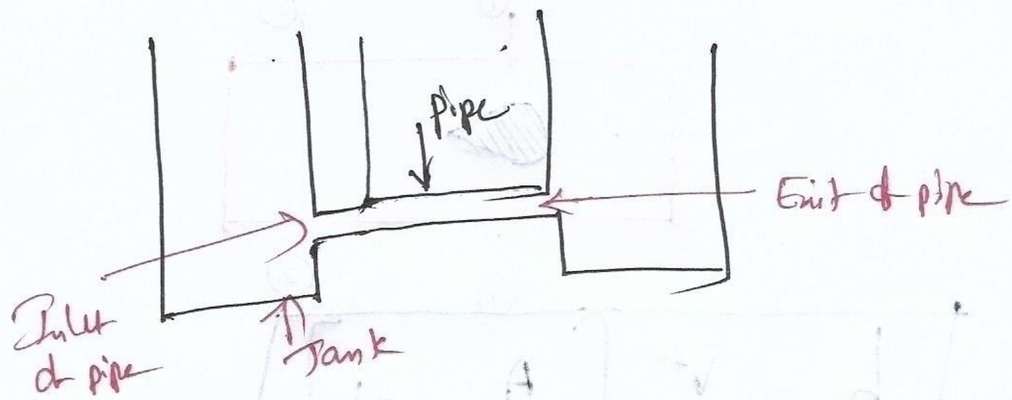
$$\boxed{h_{fi} = K_{fi} \frac{v^2}{2g}}$$

h_{fi} = Head loss due to fittings

K_{fi} = coefficient of fitting

v = velocity of liquid

7. Minor losses at the exit/outlet of pipe:



$$\boxed{h_o = \frac{v^2}{2g}}$$

h_o = Head loss at outlet of pipe

V = Velocity of liquid in pipe

g = accelaration due to gravity

* Hydraulic gradient line: (HGL)

⇒ It is the line joining all the liquid levels in piezometers.

⇒ It is the sum of pressure head & datum head

$$\boxed{\frac{p}{\rho g} + z.}$$

⇒ slope of HGL is called Hydraulic gradient

* Total Energy line :-

⇒ It is graphical representation of total head at various points along pipe

⇒ It is the sum of pressure head, velocity head and datum head $\left[\frac{p}{\rho g} + \frac{v^2}{2g} + z\right]$

⇒ If datum is 0 then energy grade line is TEL.

* Equivalent pipe :-

It is defined as the pipe of uniform diameter having loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters.

L_1 = Length of pipe-1

d_1 = Diameter of pipe-1

L_2 = Length of pipe-2

d_2 = Diameter of pipe 2

L_3 = Length of pipe-3

d_3 = Diameter of pipe-3

H = Total head loss

L = Length of the equivalent pipe

d = Diameter of the equivalent pipe

$$L = L_1 + L_2 + L_3$$

Total head loss in the compound pipe,

Neglecting minor losses.

$$H = \frac{4 f_1 L_1 V_1^2}{2g d_1} + \frac{4 f_2 L_2 V_2^2}{2g d_2} + \frac{4 f_3 L_3 V_3^2}{2g d_3} \longrightarrow (1)$$

$$f = f_1 = f_2 = f_3$$

$$Q = A_1 V_1 = A_2 V_2 = A_3 V_3$$

$$Q = \frac{\pi}{4} d_1^2 V_1 = \frac{\pi}{4} d_2^2 V_2 = \frac{\pi}{4} d_3^2 V_3$$

$$Q = \frac{\pi}{4} d_1^2 V_1$$

$$V_1 = \frac{4Q}{\pi d_1^2}$$

$$V_2 = \frac{4Q}{\pi d_2^2}$$

$$V_3 = \frac{4Q}{\pi d_3^2}$$

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V_1, V_2, V_3 are values substitute in equ (1)

$$H = \frac{4 f_1 L_1 \left(\frac{4Q}{\pi d_1^2}\right)^2}{2g d_1} + \frac{4 f_2 L_2 \left(\frac{4Q}{\pi d_2^2}\right)^2}{2g d_2} + \frac{4 f_3 L_3 \left(\frac{4Q}{\pi d_3^2}\right)^2}{2g d_3}$$

$$H = \frac{4f\, 4^2 Q^2}{2g\, \pi^2}\left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right]$$

$$H = \frac{64 f Q^2}{2g\pi^2}\left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right]$$

$$\frac{4fLV^2}{2gd} = \frac{64 f Q^2}{2g\pi^2}\left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right]$$

$$\because H = \frac{4fLV^2}{2gd}, \quad Q = AV$$

$$Q = \frac{\pi}{4} d^2 V$$

$$H = \frac{4fL\left(\frac{4Q}{\pi d^2}\right)^2}{2gd} \quad V = \frac{4Q}{\pi d^2}$$

$$H = \frac{64 f Q^2 L}{2g\pi^2 d^5}$$

$$\frac{\cancel{64}\cancel{f}\cancel{Q^2} L}{\cancel{2g\pi^2}\, d^5} = \frac{\cancel{64}\cancel{f}\cancel{Q^2}}{\cancel{2g\pi^2}}\left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right]$$

$$\therefore \boxed{\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5} + \cdots \frac{L_n}{d_n^5}}$$

Problem

Three pipes of lengths when 800m, 500m. & 400 m and of diameters 500mm, 400mm, and 300mm respectively. all connected in series. These pipes are to be replaced by a single pipe of length 1700m. Find diameter of single pipe.

Sol:- Given data

$L_1 = 800m$	$D_1 = 500mm = 500 \times 10^{-3} m$
$L_2 = 500m$	$D_2 = 400mm = 400 \times 10^{-3} m$
$L_3 = 400m$	$D_3 = 300mm = 300 \times 10^{-3} m$

$L = 1700m$

$$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

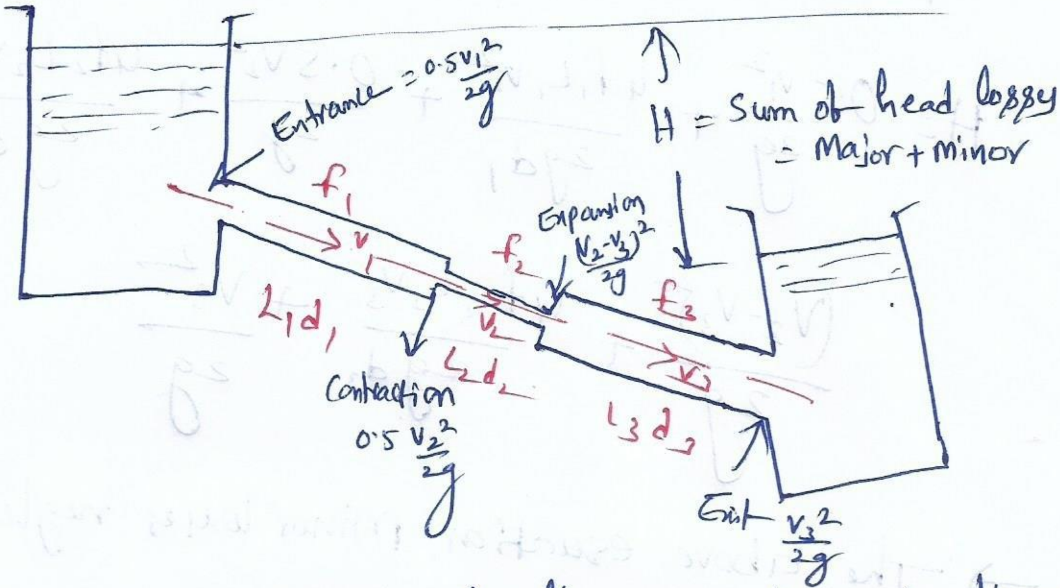
$$\frac{1700}{d^5} = \frac{800}{(500 \times 10^{-3})^5} + \frac{500}{(400 \times 10^{-3})^5} + \frac{400}{(300 \times 10^{-3})^5}$$

$$\frac{1700}{d^5} = 25600 + 48828 + 164609$$

$$d^5 = 7.1118 \times 10^{-3}$$

$$\boxed{d = 0.3718 m}$$

* Pipes in Series (or) Compound Series:-



when the pipes of different lengths and different diameters are connected to each other to form a pipe line.

→ Minor losses

= Entrance loss + Contraction loss + Expansion loss + Exit loss

$$= 0.5\frac{V_1^2}{2g} + 0.5\frac{V_2^2}{2g} + \frac{(V_2-V_3)^2}{2g} + \frac{V_3^2}{2g}$$

→ Major losses

= Head loss due to friction. It is determine by Darcy's weisbach formula

$$h_f = \frac{4fLV^2}{2gd}$$

$$= \frac{4f_1L_1V_1^2}{2gd_1} + \frac{4f_2L_2V_2^2}{2gd_2} + \frac{4f_3L_3V_3^2}{2gd_3}$$

→ Total loss of head

H = Major losses + minor losses

$$H = \frac{0.5 V_1^2}{2g} + \frac{4 f_1 L_1 V_1^2}{2 g d_1} + \frac{0.5 V_2^2}{2g} + \frac{4 f_2 L_2 V_2^2}{2 g d_2} +$$

$$\frac{(V_2 - V_3)^2}{2g} + \frac{4 f_3 L_3 V_3^2}{2 g d_3} + \frac{V_3^2}{2g}$$

→ The above equation, Minor losses neglected

$$H = \frac{4 f_1 L_1 V_1^2}{2 g d_1} + \frac{4 f_2 L_2 V_2^2}{2 g d_2} + \frac{4 f_3 L_3 V_3^2}{2 g d_3}$$

Problem:

Three pipes of diameters 300mm, 200mm and 400mm and lengths 450m, 255m, and 315m respectively, are connected in series. The difference in water surface levels in two tanks is 18m. Determine the rate of flow of water if coefficient of friction are 0.0075, 0.0078 and 0.0072 respectively. Considering i) minor losses ii) Neglecting minor losses.

Sol:- Given data:

$d_1 = 300mm = 0.3m$

$d_2 = 200mm = 0.2m$

$d_3 = 400mm = 0.4m$

$L_1 = 450m$

$L_2 = 255m$

$L_3 = 315m$

$H = 18m$

$f_1 = 0.0075$

$f_2 = 0.0078$

$f_3 = 0.0072$

i) considering minor losses

$$H = 0.5\frac{V_1^2}{2g} + \frac{4f_1L_1V_1^2}{2gd_1} + \frac{0.5V_2^2}{2g}$$

$$+ \frac{4f_2l_2V_2^2}{2gd_2} + \frac{(V_2-V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{2gd_3}$$

$$+ \frac{V_3^2}{2g}$$

$$H = \frac{0.5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{2gd_1} + \frac{0.5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{2gd_2} + \frac{(V_2-V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{2gd_3} + \frac{V_3^2}{2g}$$

from continuity equation

$$A_1V_1 = A_2V_2 = A_3V_3$$

$$\Rightarrow A_1V_1 = A_2V_2 \Rightarrow V_2 = \frac{A_1V_1}{A_2}$$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{\cancel{\frac{\pi}{4}} d_1^2 V_1}{\cancel{\frac{\pi}{4}} d_2^2} = \frac{d_1^2 V_1}{d_2^2} = \frac{(0.3)^2 V_1}{(0.2)^2} = 2.25 V_1$$

$$\Rightarrow A_1 V_1 = A_3 V_3$$

$$V_3 = \frac{A_1 V_1}{A_3} = \frac{\cancel{\frac{\pi}{4}} d_1^2 V_1}{\cancel{\frac{\pi}{4}} d_3^2} = \frac{d_1^2 V_1}{d_3^2} = \frac{(0.3)^2 V_1}{(0.4)^2} = 0.5625 V_1$$

$$\therefore H = \frac{0.5 V_1^2}{2 \times 9.81} + \frac{4 \times 0.0075 \times 450 \times V_1^2}{2 \times 9.81 \times 0.3} + \frac{0.5 (2.25 V_1)^2}{2 \times 9.81} + \frac{4 \times 0.0078 \times 255 \times (2.225 V_1)^2}{2 \times 9.81 \times 0.2}$$

$$+ \frac{(2.25 V_1 - 0.5625 V_1)^2}{2 \times 9.81} + \frac{4 \times 0.0072 \times 315 \times (0.5625 V_1)^2}{2 \times 9.81 \times 0.4} + \frac{(0.5625 V_1)^2}{2 \times 9.81}$$

$$H = 13.23793 V_1^2$$

$$18 = 13.23793 V_1^2$$

$$\boxed{V_1 = 1.16 \text{ m/sec}}$$

$$V_2 = 2.25 V_1$$
$$= 2.25 \times 1.16$$
$$\boxed{V_2 = 2.623 \text{ m/sec}}$$

$$V_3 = 0.5625 V_1$$
$$= 0.5625 \times 1.16$$
$$\boxed{V_3 = 0.655 \text{ m/sec}}$$

ii) Neglecting minor losses :-

$$H = \frac{4 f_1 L_1 V_1^2}{2 g d_1} + \frac{4 f_2 L_2 V_2^2}{2 g d_2} + \frac{4 f_3 L_3 V_3^2}{2 g d_3}$$

$$H = \frac{4 \times 0.0075 \times 450 \times V_1^2}{2 \times 9.81 \times 0.3} + \frac{4 \times 0.0078 \times 255 \times (2.25 V_1)^2}{2 \times 9.81 \times 0.2} + \frac{4 \times 0.0072 \times 315 \times (0.5625 V_1)^2}{2 \times 9.81 \times 0.4}$$

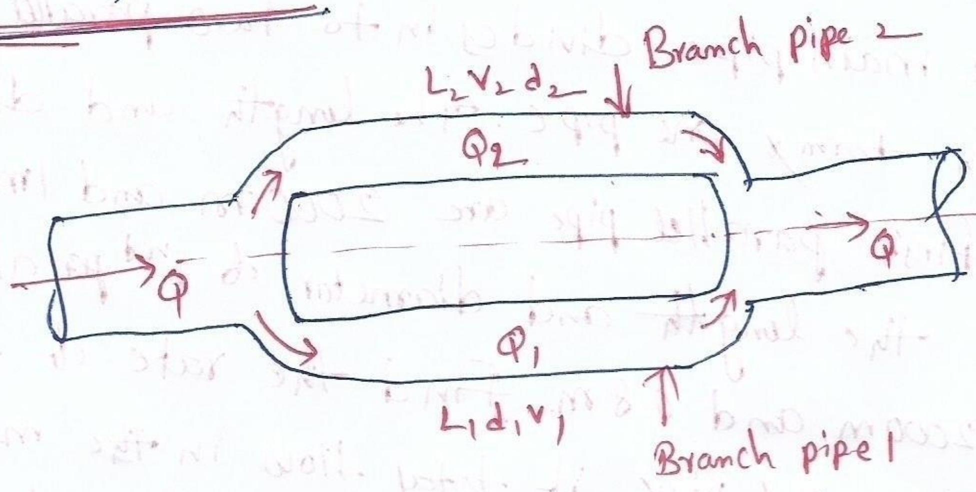
$$18 = 2.2935 V_1^2 + 10.2432 V_1^2 + 0.36523 V_1^2$$

$$V_1 = 1.18 \text{ m/sec}$$

$$V_2 = 2.25 V_1$$
$$= 2.25 \times 1.18$$
$$V_2 = 2.65 \text{ m/sec}$$

$$V_3 = 0.5625 \times 1.18$$
$$V_3 = 0.663 \text{ m/sec}$$

* pipes in parallel :-



* Consider a main pipe which divides into two or more branches as shown in fig, and join again together downstream to form a single pipe, then the branch pipes are said to be connected in parallel.
* The discharge through the main is increased by connecting pipes in parallel.
* The rate of flow in the main pipe is equal to the sum of rate of flow through branch pipes.

Hence above fig. $Q = Q_1 + Q_2 \longrightarrow (1)$

In this arrangement, the loss of head for each branch pipe is same.

$\therefore$ Loss of head in pipe 1 = Loss of head in pipe 2

$$\frac{4 f_1 L_1 V_1^2}{2 g d_1} = \frac{4 f_2 L_2 V_2^2}{2 g d_2} \longrightarrow (2)$$

If $f_1 = f_2$

$$\frac{L_1 V_1^2}{d_1} = \frac{L_2 V_2^2}{d_2} \longrightarrow (3)$$

Problem:

The main pipe divides in to two parallel pipes which again forms one pipe. The length and diameter of the first parallel pipe are 2000 m and 1 m respectively, while the length and diameter of 2nd parallel pipe are 2000m and 0.8 m. Find the rate of flow in each parallel pipe, if total flow in the main is 3 m³/sec. The coefficient of friction for each parallel pipe is same and equal to 0.005.

Sol:- Length of pipe 1, $L_1 = 2000\,m$

Diameter of pipe 1, $d_1 = 1\,m$

Length of pipe 2, $L_2 = 2000\,m$

Diameter of pipe 2, $d_2 = 0.8\,m$

Total flow, $Q = 3\,m^3/sec$

$f_1 = f_2 = 0.005$

Let Q_1 = Discharge in pipe 1

Q_2 = Discharge in pipe 2

$\therefore Q = Q_1 + Q_2$ —— (1)

$$\frac{L_1 V_1^2}{d_1} = \frac{L_2 V_2^2}{d_2}$$

$$\frac{2000 \times V_1^2}{1} = \frac{2000 \times V_2^2}{0.8}$$

$$V_1^2 = \frac{V_2^2}{0.8} \Rightarrow V_1 = \sqrt{\frac{V_2^2}{0.8}} = \frac{V_2}{0.894} \rightarrow (2)$$

Now $Q_1 = A_1 V_1$

$= \frac{\pi}{4} d_1^2 V_1$

$Q_1 = \frac{\pi}{4} \times 1^2 \times \frac{V_2}{0.894} = 0.878 V_2$

$Q_2 = \frac{\pi}{4} d_2^2 \times V_2 = \frac{\pi}{4} \times 0.8^2 \times V_2 = 0.502 V_2$

$\therefore Q = Q_1 + Q_2$

$3 = 0.878 V_2 + 0.502 V_2$

$3 = 1.38 V_2$

$V_2 = \frac{3}{1.38} = 2.17$ m/sec.

$V_1 = \frac{V_2}{0.894} = \frac{2.17}{0.894} = 2.42$ m/s.

$\therefore Q_1 = 0.878 \times 2.17$

$= 1.905$ m³/sec.

$Q_2 = Q - Q_1$

$= 3 - 1.905$

$Q_2 = \underline{1.094 \text{ m}^3/\text{s}}$

Problems:

(1) Find Head loss due to friction in a pipe of diameter 30cm, and length 50m, through which water is flowing at a velocity of 3m/sec using Darcy's formula take kinematic viscosity $\nu = 0.01$ Stoke.

Sol:- Given:

$d = 30cm = 0.3m$

$L = 50m$

$V = 3m/sec$

$\nu = 0.01 \text{ stoke} = 0.01 \times 10^{-4} \, m^2/sec$

$\because f = \frac{16}{Re}$ → used for $Re < 2000$

$\because f = \frac{0.079}{Re^{1/4}}$ → used for $Re > 4000$

$Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu} = \frac{3 \times 0.3}{0.01 \times 10^{-4}} = 9 \times 10^{5}$.
($Re > 4000$)

$f = \frac{0.079}{Re^{1/4}}$

$f = \frac{0.079}{(9 \times 10^{5})^{1/4}}$

$f = 2.56 \times 10^{-3}$

$h_f = \frac{4fLV^2}{2gd} = \frac{4 \times (2.56 \times 10^{-3}) \times 50 \times 3^2}{2 \times 9.81 \times 0.3}$

$\boxed{h_f = 0.784 m}$

(3) A Crude oil of kinematic viscosity 0.4 stoke is flowing through a pipe of diameter is 30cm at the rate of 300lit/sec. Find the head loss due to friction for a length of 50m of the pipe.

Sol:- Given:

$\nu = 0.4$ Stoke

$\nu = 0.4 \times 10^{-4}\ m^2/sec$

$L = 50m$, $Q = 300\ lit/sec = \frac{300}{1000} = 0.3\ m^3/sec$

$d = 30cm = 30 \times 10^{-2}\ m = 0.3m$

$$V = \frac{Q}{A} = \frac{0.3}{\frac{\pi}{4} \times 0.3^2} = 4.24\ m/sec.$$

$$R_e = \frac{VD}{\nu} = \frac{4.24 \times 0.3}{0.4 \times 10^{-4}} = 31800$$

$$f = \frac{0.079}{(31800)^{1/4}} = 0.00591 \qquad \because f = \frac{0.079}{R_e^{1/4}}$$

$$h_f = \frac{4fLV^2}{2gd}$$

$$= \frac{4 \times 0.0591 \times 50 \times 4.24^2}{2 \times 9.81 \times \cancel{0.3}}$$

$$\boxed{h_f = 3.62m}$$

(2) A horizontal pipe of 50mm diameter and 750m long maintains water flow rate of 0.03 m^3/min. Calculate head losses due to friction and the power required to maintain the flow. If viscosity is $1.14 \times 10^{-3} \frac{NS}{m^2}$ and 'f' value is 0.008.

Sol: Given:

$d = 50mm = 50 \times 10^{-3} m$

$L = 750m$

$Q = 0.03 \, m^3/min = \frac{0.03}{60} \, m^3/sec$

$\mu = 1.14 \times 10^{-3} \frac{N\text{-}s}{m^2}$

$f = 0.008$

$h_f = ?$

$Q = AV$

$$V = Q/A = \frac{\frac{0.03}{60}}{\frac{\pi}{4} \times (50 \times 10^{-3})^2} = 0.2546 \, m/s.$$

Reynold's number $R_e = \frac{\rho V D}{\mu} = \frac{1000 \times 0.2546 \times (50 \times 10^{-3})}{1.14 \times 10^{-3}}$

$R_e = 11.16 \times 10^{-3}$ ($R_e > 4000$)

$$\therefore h_f = \frac{4fLV^2}{2gd} = \frac{4 \times 0.008 \times 750 \times 0.2546^2}{2 \times 9.81 \times 50 \times 10^{-3}} = 1.58m$$

$\therefore$ power, $P = \rho g Q h_f = 1000 \times 9.81 \times (5 \times 10^{-4}) \times 1.58$

$= 7.77$ Watts

(4) An oil of specific gravity 0.7 is flowing through a pipe of diameter 30cm at the rate of 500lit/sec. find the head loss due to friction. Length of the flow is 1000m. Take $\nu = 0.29$ Stokes.

Sol:-

$S = 0.7$

$d = 30cm = 0.3m$

$Q = 500 lit/sec = \frac{500}{1000} = 0.5 m^3/sec$

$\nu = 0.29$ Stokes $= 0.29 \times 10^{-4} m^2/sec$

$L = 1000m$

$$V = \frac{Q}{A} = \frac{0.5}{\frac{\pi}{4} \times 0.3^2} = 7.0735 m/sec$$

$$R_e = \frac{VD}{\nu}$$

$$= \frac{7.0735 \times 0.3}{0.29 \times 10^{-4}} = 73211$$

$$f = \frac{0.079}{Re^{1/4}} = \frac{0.079}{(73211)^{1/4}} = 0.0048$$

$$\therefore h_f = \frac{4fLV^2}{2gd}$$

$$= \frac{4 \times 0.0048 \times 1000 \times \cancel{70.0}\,7.0735^2}{2 \times 9.81 \times 0.3}$$

$$\boxed{h_f = 163.3 m}$$

(5) Calculate the discharge through a pipe of diameter 20cm when the difference of pressure head between the two ends of a pipe 500m. and head lost due to friction 4m of water. Take the value of $f = 0.009$.

Sol:- Given: $d = 20\,cm = 20 \times 10^{-2} = 0.2\,m$

$L = 500\,m$

$h_f = 4\,m$

$f = 0.009$

$$h_f = \frac{4fLv^2}{2gd}$$

$$4 = \frac{4 \times 0.009 \times 500 \times v^2}{2 \times 9.81 \times 0.2}$$

$v^2 = 0.872$

$V = 0.933\ m/sec$

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 0.2^2 = 0.0314\,m^2$

$\therefore Q = AV$

$= 0.0314 \times 0.933$

$$\boxed{Q = 0.029\ m^3/sec}$$