Unit - I

Engineering Mechanics : It is a branch of science which deals with the action of force acting on a body which are at rest (or) under uniform motion.

It is a branch of science which deals under the action of a ~~body~~ force on a body it is in rest (or) uniform motion.

It is classified into

1) Statics
2) Dynamics
3) kinematics
4) kinetics.

Statics :- It is a branch of mechanics which deals with the action of forces that are acting on a body which is at rest.

Dynamics : It is a branch of mechanics which deals with the action of forces that are acting on a body which is in uniform motion. body is in uniform motion and considering the force.

Kinematics : It is a branch of dynamics, study of relationship between velocity, displacement and acceleration without considering the forces which cause motion.

kinetics :

It is a branch of dynamics which deals with the motion of bodies considering forces.

Basic Concepts :-

It is used to represent the position of the point in relation to the reference point is called origin

Time:

It is used to define an event.

Mass:

It is used to characteristics and compare the bodies It is used to measure the resistance to change the state of rest (or) motion called inertia.

Force:

It is the effort required to change the state of rest (or) uniform motion of a body.

Particle:

It is defined as a body whose shape and dimension are negligible the mass is concentrated at a point

Ex:- a ball is kept on the table.

Rigid body: A body never undergoes any deformation due to externalsed forces is called rigid bodies.

Ex:- Road roller of radius 'r' and weight 'w' rolls on a road

Fluid:

It is a substance which undergoes deformation with minimum force.

Ex: Liquids and gases.

1. Newton's 1st law :

Every body continuous in its state of rest or of uniform motion, unless an external forces acts on it.

2. Newton's 2nd Law :

The rate of change of momentum of a body is directly proportional to the applied force in the direction of that forces.

$F = ma$ kgf. $(kg \times m/s^2 = \text{Newton})$

3. Newton's 3rd Law :

For every action there will be an equal & opposite reaction.

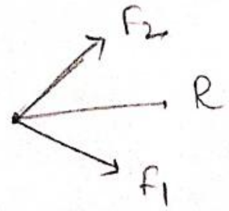
4. Newton's law of gravitation :

The force of attraction between any two forces bodies is directly proportional to their masses and inversly proportional to square of the distance between them.

$$F = G \frac{m_1 m_2}{r^2} \Rightarrow 6.67 \times 10^{-11}\ Nm^2/kg^2.$$

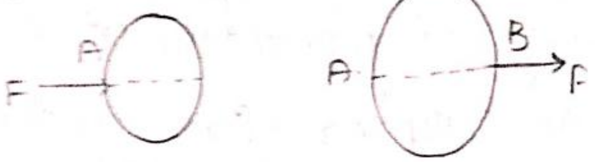
5. parallelogram law of forces :

If two forces acting at a point they represented in magnitude and direction by the two adjacent sides of a parallelogram then their resultant will be represented magnitude and direction by the diagonal of the parallelogram passing through the point



6. Law of Transmissibility of forces :

If a force acting at a point on a rigid body is shifted to any other point in its line of action then external effect of the force on the rigid body remains unchanged.



7. Lami's Theorem :

It states that if three forces acting on a particle in Equilibrium then each force is proportional to the sine of the angle between the other two forces and the constant of proportionality is the same.

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}.$$

8. Laws of Triangle:

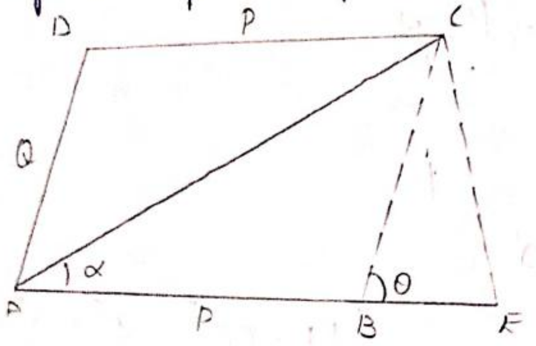
In a, b, c are the sides of a triangles and α, β, γ are the angle between the sides, then the sine law states that

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\sin\theta = \frac{O.S}{H}, \quad \cos\theta = \frac{Ad}{H}.$$

Parallelogram law of Vector :

If the two forces acting at a point their represented in magnitude and direction by the two adjacent sides of parallelogram then their resultant will be represented magnitude and direction by the diagonal of parallelogram passing through the same point.



from Δ^{le} ACE.

$$AC^2 = AE^2 + CE^2$$

$$AC^2 = (AB+BE)^2 + CE^2$$

$$AC^2 = (P^2 + BE)^2 + CE^2$$

$\left\{ \begin{array}{l} AC = R \\ AC^2 = R^2 \end{array} \right\}$

from Δ^{le} BCE.

$$\sin\theta = \frac{CE}{BC} \Rightarrow CE = BC\sin\theta$$

$$\cos\theta = \frac{BE}{BC} \Rightarrow BE = BC\cos\theta$$

$$AC^2 = (P^2 + BC\cos\theta)^2 + (BC\sin\theta)^2$$

$$R^2 = P^2 + BC^2\cos^2\theta + 2PBC\cos\theta + BC^2\sin^2\theta.$$

$$R^2 = P^2 + BC^2(\cos^2\theta + \sin^2\theta) + 2P.BC\cos\theta$$

$$R^2 = P^2 + BC^2 + 2PBC\cos\theta.$$

$$R = \sqrt{P^2 + BC^2 + 2PBC\cos\theta}$$

Δ^{le} ACE

$$\tan\alpha = \frac{CE}{AE} = \frac{CE}{AB+BE} = \frac{BC\sin\theta}{AB + BC\cos\theta}$$

$$\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta} \Rightarrow \alpha = \tan^{-1}\frac{Q\sin\theta}{P+Q\cos\theta}$$

Problem: Find the magnitude of two forces such if they are act at right angle then their resultant is $\sqrt{10}$ newtons & but if they are act at 60° then their resultant is $\sqrt{13}$ Newtons.

Sol Given

$\theta = 90°$, $R_1 = \sqrt{10}$ N

$\theta = 60°$, $R_2 = \sqrt{13}$ N

$R_1 = \sqrt{P^2+Q^2+2PQ\cos\theta}$

$\sqrt{10} = \sqrt{P^2+Q^2+2PQ\cos 90°}$

$\sqrt{10} = \sqrt{P^2+Q^2}$

S.O.B.S

$10 = P^2+Q^2$ — (1)

$R_2 = \sqrt{P^2+Q^2+2PQ\cos\theta}$

$\sqrt{13}\ \cancel{R_2} = \sqrt{P^2+Q^2+2PQ\cos 60°}$

$\sqrt{13}\ \cancel{R_2} = \sqrt{P^2+Q^2+\cancel{2}PQ\cdot\frac{1}{\cancel{2}}}$

$\sqrt{13} = \sqrt{P^2+Q^2+PQ}$

S.O.B.S

$13 = P^2+Q^2+PQ$ — (2)

$13 = P^2+Q^2+PQ$

Sub equ (1) in eq (2)

$13 = 10 + PQ$

$\boxed{PQ = 3}$

$(P+Q)^2 = P^2+Q^2+2PQ$

$(P+Q)^2 = 10+6$

$(P+Q)^2 = 16 \Rightarrow P+Q = 4$

$(P-Q)^2 = P^2+Q^2-2PQ$

$= 10 - 2\times 3$

$= 10-6 = 4$

$(P-Q)^2 = 4 \Rightarrow P-Q = 2$

$P + \cancel{Q} = 4$

$P - \cancel{Q} = 2$

$2P = 6$

$\boxed{P = 3}$

$3 + Q = 4$

$Q = 4-3$

$\boxed{Q = 1}$

$3 - Q = 2$

$-Q = -1$

$\boxed{Q = 1}$

(2) Two equal forces acting at a point whith an angle of 60° the resultant is $20\sqrt{3}$ N. find the magnitude of that force.

Given

$\theta = 60°$, $R = 20\sqrt{3}$

$\boxed{P = Q}$

~~P = Qt~~

$R = \sqrt{P^2+Q^2+2PQ\cos\theta}$

$R = \sqrt{P^2+Q^2+2PQ\cos 60°}$

$20\sqrt{3} = \sqrt{P^2+Q^2+PQ}$

$20\sqrt{3} = \sqrt{P^2+P^2+PQ}$

$20\sqrt{3} = \sqrt{P^2+P^2+P^2}$

$20\sqrt{3} = \sqrt{3P^2}$

$20\cancel{\sqrt{3}} = P\cancel{\sqrt{3}}$

$\boxed{P = 20N}$, $\boxed{Q = 20N}$

(3) The resultant of two forces is 1500 N and angle b/w the forces is 90° the resultant makes an angle 36° with one of the force. find the magnitude of each oth force.

$R = 1500$ N

$\theta = 90°$

$\alpha = 36°$

$\tan\alpha = \dfrac{Q\sin\theta}{P+Q\cos\theta}$

$\tan 36° = \dfrac{Q \times \sin 90°}{P+Q\cos 90°}$

$0.726 = \dfrac{Q}{P}$

$Q = P \times 0.726$

$R = \sqrt{P^2+Q^2+2PQ\cos\theta}$

$1500 = \sqrt{P^2+P^2 \times 0.527}$

$P^2 = .2$

Types of force systems.

* co-planear force system :

All the line of action of forces are lying on a single plane. [Two or more no of forces acting on a same plane

* Non-co planer (or) space force system :

All the lines of action of forces are not lying on a single plane.

* Co-linear force :

line of action of two forces are respresenting a Same line. then the force system is called co-linear. Also if they are lying in the Same plane then they are called co-planer co-linear force system.

* Concurrent forces :

Line of action of all forces pass through a single point [All the forces acting on the single point]

* parallel forces :

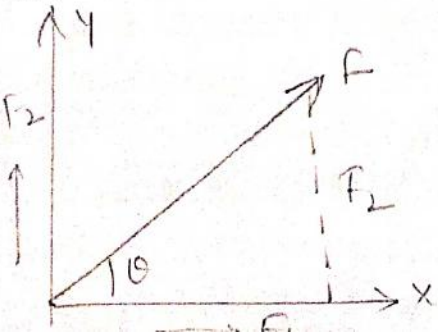
All forces in a system parallel to each other.

* Non-parallel forces :

All forces in a system not parallel to each other.

Resolution of forces :

As we know that single force can be resolved into two mutually perpendicular force components is called as Resolution of forces.

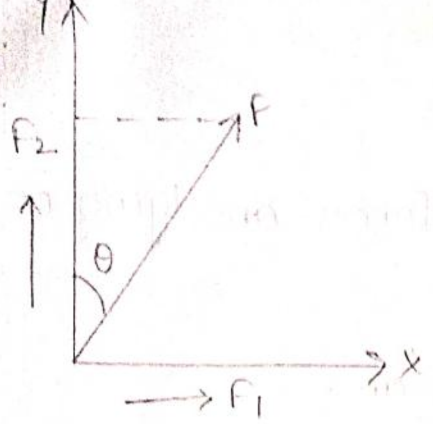


$$\cos\theta = \frac{F_1}{F}$$

$$F_1 = F\cos\theta.$$

$$\sin\theta = \frac{F_2}{F}$$

$$F_2 = F\sin\theta.$$



$\cos\theta = \frac{F_2}{F}$

$F_2 = F\cos\theta$

$\sin\theta = \frac{F_1}{F}$

$F_1 = F\sin\theta.$

Method of Resolution:

When many forces meeting at a point then resultant can be found by method of resolution.

* Each force resolved into two mutually perpendicular Component.
* Find the algebraic Sum of horizontal forces.

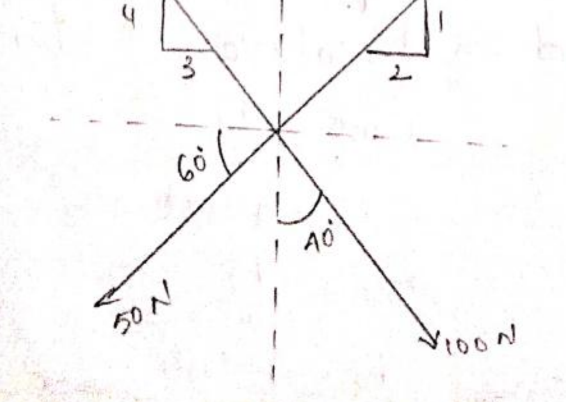
$\Sigma H = 0$ (or) $\Sigma f_x = 0$ $\xrightarrow{} +$ $- \xleftarrow{}$

* Find the algebraic Sum of Vertical component.

$\Sigma V = 0$ (or) $\Sigma f_y = 0$ $\downarrow (-)$ $\uparrow (+)$

* Resultant $R = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2}$
* Inclination $\alpha = \tan^{-1}\left(\frac{\Sigma f_y}{\Sigma f_x}\right)$

(1) The System of 4 forces acting on a body as shown in fig below determine the resultant?

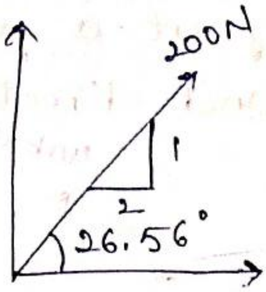


1st form :-

$\tan\theta = \frac{1}{2}$

$\theta = \tan^{-1} \frac{1}{2}$

$= 26.56$



$F_1 = F\cos\theta = 200\cos(26.56^\circ)$

$= 178.89\,N$

$F_2 = F\sin\theta = 200\sin(26.56^\circ)$

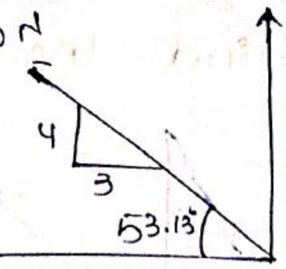
$= 89.42\,N$

2nd form :-

$\tan\theta = \frac{4}{3}$

$\theta = \tan^{-1}\frac{4}{3}$

$= 53.13^\circ$



$F_1 = F\cos\theta$

$= 120\cos(53.13^\circ)$

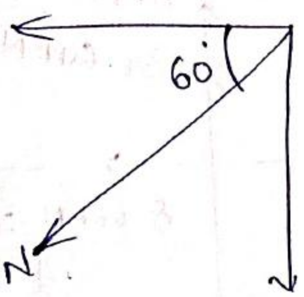
$= 720\,N$

$F_2 = F\sin\theta$

$= 120\sin(53.13^\circ)$

$= 95.99\,N$

3rd Form :-



50N

$F_1 = F\cos\theta$

$= 50\cos 60^\circ$

$= 25\,N$

$F_2 = F\sin\theta$

$= 50\sin 60^\circ$

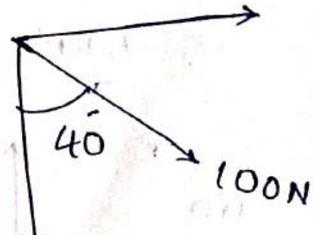
$= 43.30\,N$

4th form

$F_1 = F\sin\theta$

$= 100\sin 40^\circ$

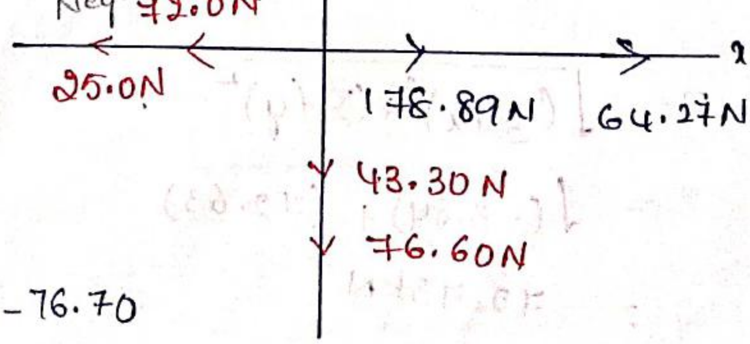
$= 64.27\,N$



$F_2 = F\cos\theta$

$= 100\cos 40^\circ$

$= 76.6\,N$



$\Sigma H = -25.0 - 72.0 + 178.89 + 64.27$

$\Sigma H = 146.16\,N$

ΣV (or) $\Sigma f_y = 95.99 + 89.42 - 43.30 - 76.70$

$= 65.51\,N$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{(146.16)^2 + (65.51)^2}$

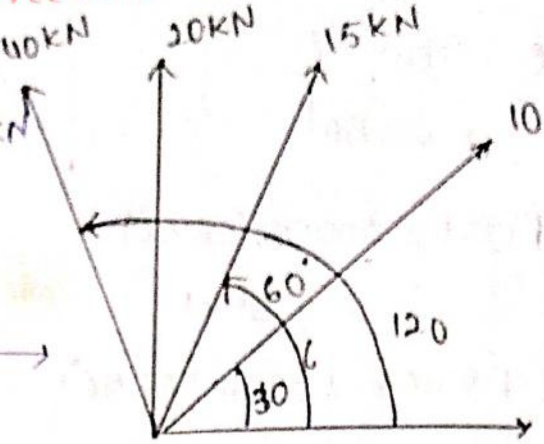
$R = 160.16\,N$

$\alpha = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$

$\alpha = \tan^{-1}\left(\frac{65.51}{146.16}\right)$

$\alpha = 24.14^\circ$

2) Four forces are acting at a point as shown in fig find the magnitude and direction of the resultant



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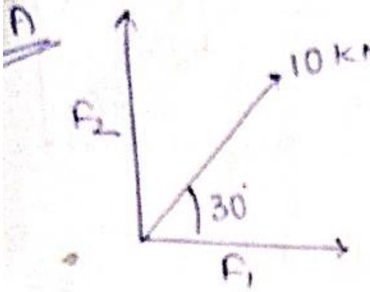
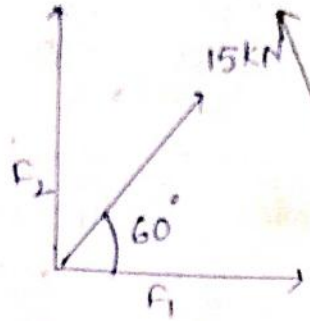




$F_1 = 10 \cos 30^\circ$

$= 8.66 \text{ KN}$

$F_2 = 10 \sin 30^\circ$

$= 5 \text{ KN}$

$F_1 = F_1 \cos 60^\circ$

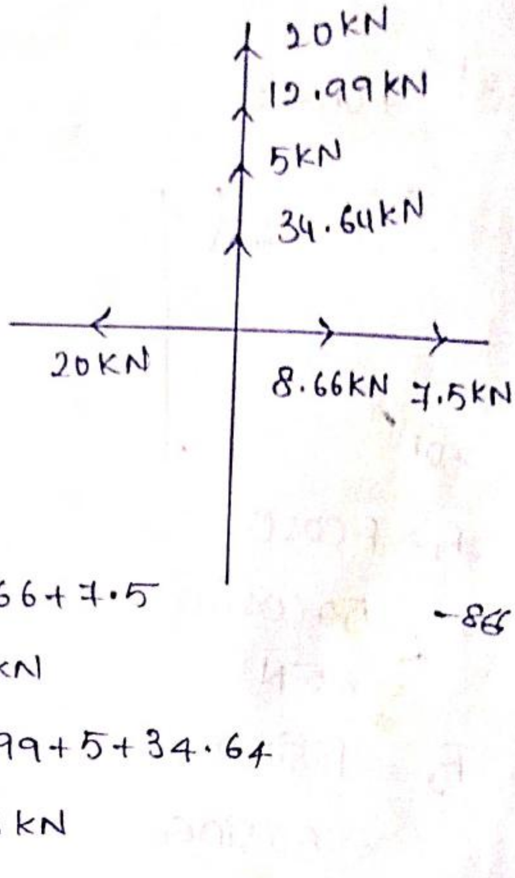
$= 15 \cos 60^\circ$

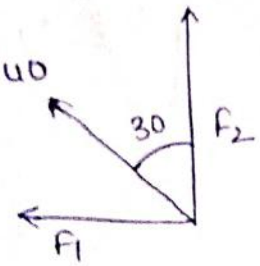
$= 7.5 \text{ KN}$

$F_2 = F \sin\theta$

$= 15 \sin 60^\circ$

$= 12.99 \text{ KN}$





$F_1 = F \cos 30^\circ$

$= 40 \cos 30^\circ$

$= 34.64 \text{ KN}$

$F_2 = F \sin\theta$

$= 40 \sin 30^\circ$

$= 20 \text{KN}$

ΣH

$= -20 + 8.66 + 7.5$

$= -3.84 \text{ KN}$

$\Sigma V = 20 + 12.99 + 5 + 34.64$

$= 72.63 \text{ KN}$

$R = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2}$

$= \sqrt{(-3.84)^2 + (72.63)^2}$

$= 72.73 \text{KN}$

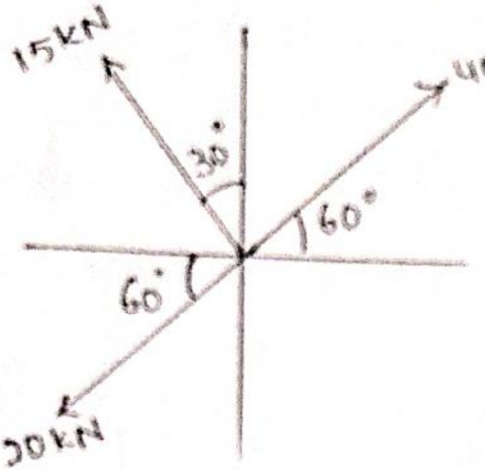
$\tan\theta = \frac{\Sigma f_y}{\Sigma f_x}$

$\alpha = \tan^{-1} \frac{\Sigma f_y}{\Sigma f_x}$

$\alpha = \tan^{-1} \frac{72.63}{-3.84} \Rightarrow \alpha = -86.97$

-86

1



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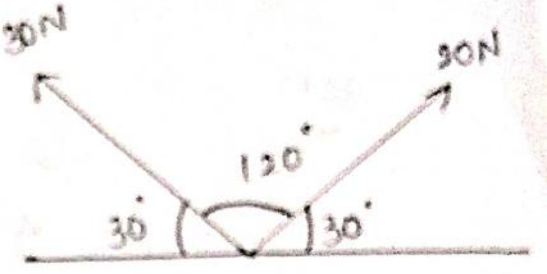
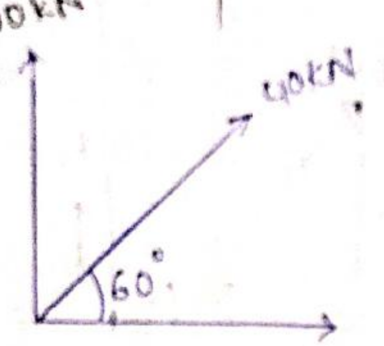
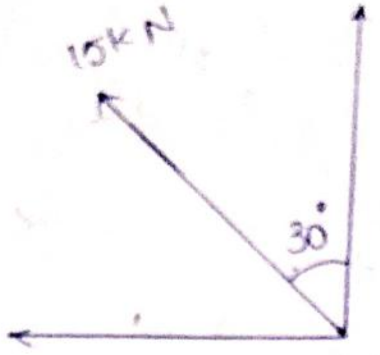
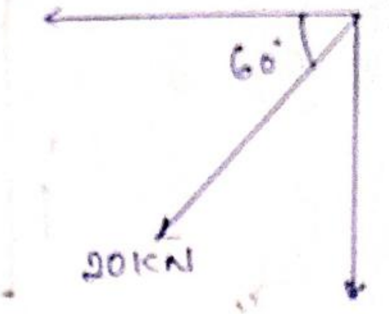








$F_1 = F\cos 60°$

$= 40\cos 60°$

$= 20\ kN$

$F_2 = F\sin\theta$

$= 40\sin 60°$

$= 34.64\ kN$

$F_1 = F\cos 30°$

$= 15\cos 30°$

$= 12.99\ KN$

$F_2 = F\sin\theta$

$= 15\sin 30°$

$= 7.5\ KN$

$F_1 = F\cos\theta$

$= 20\cos 60°$

$= 10\ KN$

$F_2 = F\sin\theta$

$= 20\sin 60°$

$17.32\ KN$

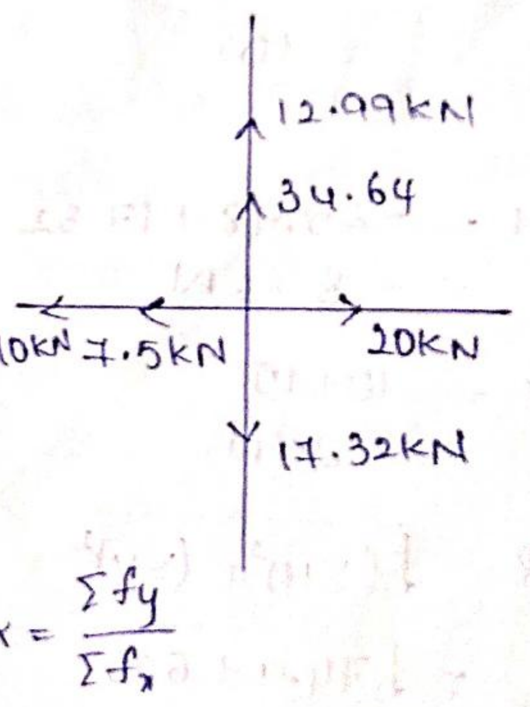
ΣH (or) Σf_x

$-10 - 7.5 + 20 = 2.5\ KN$

ΣV (or) Σf_y.

$34.64 + 12.99 - 17.32\ KN$

$30.31\ KN$



$R = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2}$

$R = \sqrt{(2.5)^2 + (30.31)^2}$

$R = \sqrt{6.25 + 918.69}$

$R = 921.19 = 30.1\ kN$

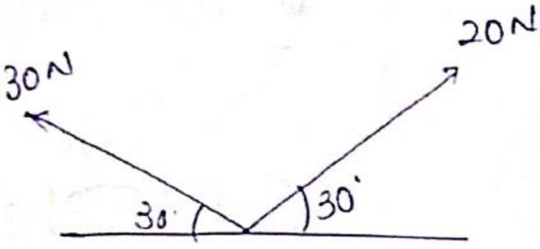
$\tan\alpha = \dfrac{\Sigma f_y}{\Sigma f_x}$

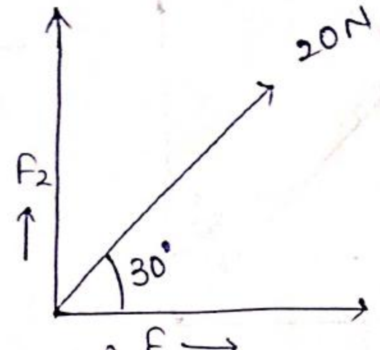
$\alpha = \tan^{-1} \dfrac{30.31}{2.5}$

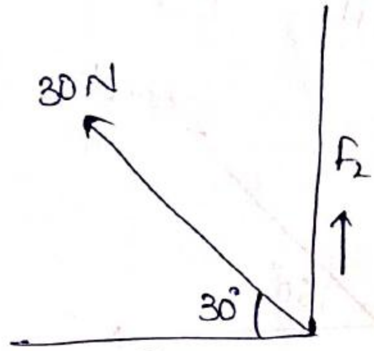
$\alpha = 85.28\ KN$

$\alpha = 85.284\ KN$

(5)







$F_1 = F\cos 30^\circ$

$= 20\cos 30^\circ$

$= 17.32$.

$F_2 = F\sin\theta$

$= 20\sin 30^\circ$

$= 20\sin 30^\circ$

$= 10$

$F_1 = F\cos\theta$

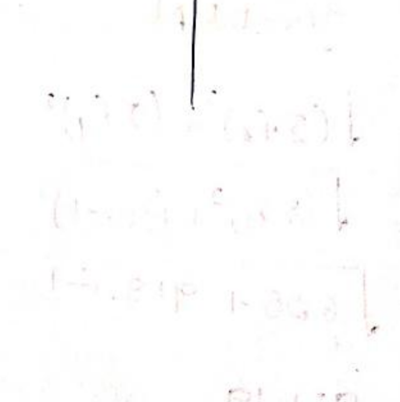
$F_1 = 30\cos 30^\circ$

$F_1 = 25.98$

$F_2 = F\sin\theta$.

$F_2 = 30\sin 30^\circ$

$F_2 = 15$.



$\Sigma H = -25.98 + 17.32$

$= -8.66\,N$.

$\Sigma V = 10 + 15$

$= 25\,N$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{74.9 + 625}$

$= 26.45$

$\tan\alpha = \frac{\Sigma V}{\Sigma H}$

$\tan\alpha = \frac{25}{-8.66} = \alpha = \tan^{-1}\frac{25}{-8.66}$

$= \alpha = -70.8939 = 49.11$

1. Applied force
2. Non- Applied force.

Applied force :

The forces applied externally to a body each force has a point of contact is called applied force.

Non- applied force :

The forces doesn't applied externally to a body of each force has don't have a point of contact is called non- applied force.

Triangular Law of forces :-

If two forces are acting (or) passing at a point they represent sides of a triangle then the resultant of this two forces represents the closing sides of the triangle which are in continuous.

Equilibrium of System of forces :

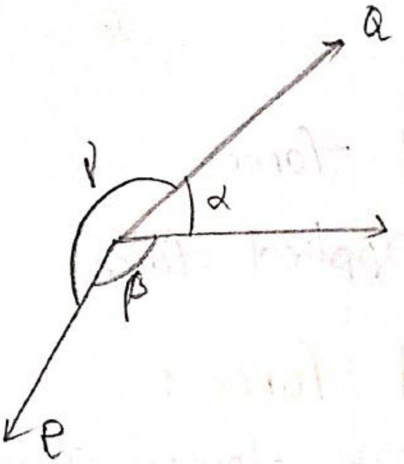
The number of forces acting at a point then the resultant of this forces is must be equals to zero then the body (or) object is in equilibrium condition

Lami's Theorem :

If the three forces are acting at a point which are at equilibrium each force will be proportional to the sine of the angle to the other two forces.

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

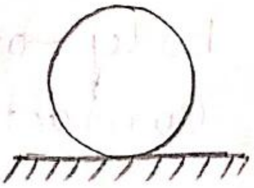


Free Body diagram [FBD] :-

It is very essential to isolate the body from the other body's in contact and draw all forces acting on ~~the~~ a body is known as free body diagram.

Examples :-

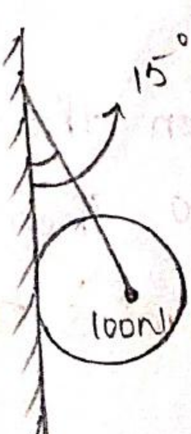
The reaction force is always acts upwards.



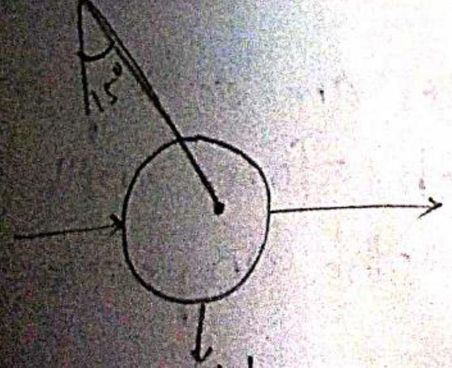
Weight is always acting downwards

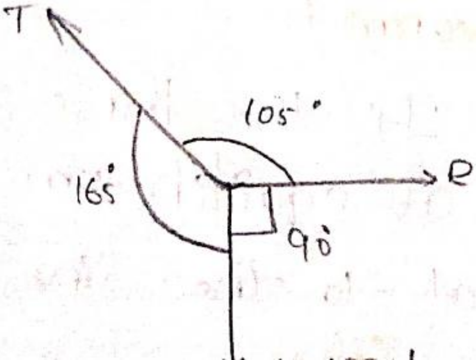
Problems :-

A sphere of weight 100N is tied ~~its~~ to a smooth wall by a string has shown in fig. find the tension in the string and reaction of the wall.



Sol Draw free body diagram from a given diagram.





By applying Lami's Theorem

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\alpha} \quad [\text{formula}]$$

$$\frac{T}{\sin 90^\circ} = \frac{R}{\sin 165^\circ} = \frac{w\,(100N)}{\sin 105^\circ}$$

$$\therefore \frac{R}{\sin 165^\circ} = \frac{100}{\sin 105^\circ}$$

$$R = \frac{100}{\sin 105^\circ} \times \sin 165^\circ$$

$$\boxed{R = 26.79N}$$

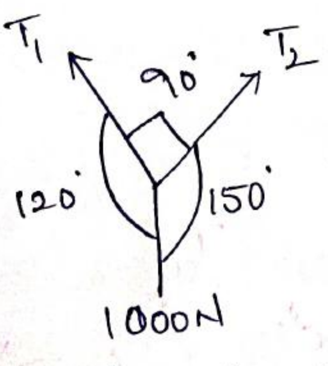
$$\frac{T}{\sin 90^\circ} = \frac{100}{\sin 105^\circ}$$

$$T = \frac{100}{\sin 105^\circ} \times \sin 90^\circ$$

$$\boxed{T = 103.52N}$$

(2) A weight of 1000N is supported by two chains as shown in fig. determine the tension in each chair

A Draw the free body diagram.



By Applying lame's theorem

$$\frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 120^\circ} = \frac{1000}{\sin 90^\circ}$$

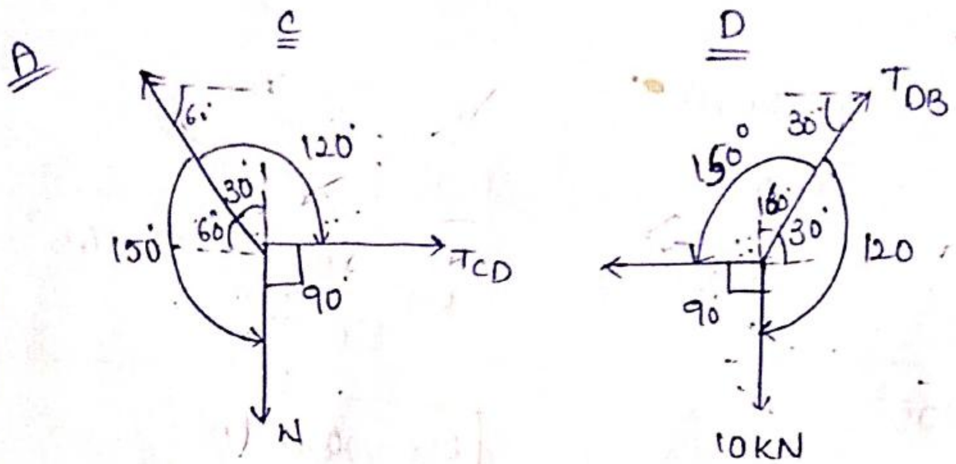
$$T_2 = \frac{1000}{\sin 90^\circ} \times \sin 120^\circ$$

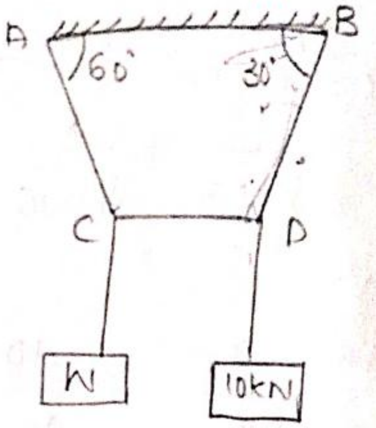
$$\boxed{T_2 = 866.0N}$$

$$T_1 = \frac{1000}{\sin 90^\circ} \times \sin 150^\circ = 500N$$

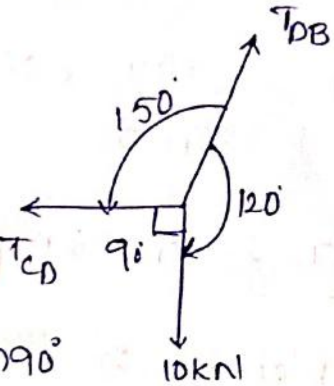
$$\boxed{T_1 = 500N}$$

(3) A Chord is Suspended at A and B caries a load of 10 kN at D. and a load of W at C as shown in fig. Find the value of W so that C, D remains horizontal and also find tension in the strings.





Apply lame's theorem at D:-



$$\frac{T_{DB}}{\sin 90^\circ} = \frac{T_{CD}}{\sin 120^\circ} = \frac{10}{\sin 150^\circ}$$

$$\frac{T_{DB}}{\sin 90^\circ} = \frac{10}{\sin 150^\circ} \Rightarrow T_{DB} = \frac{10}{\sin 150^\circ} \times \sin 90^\circ$$

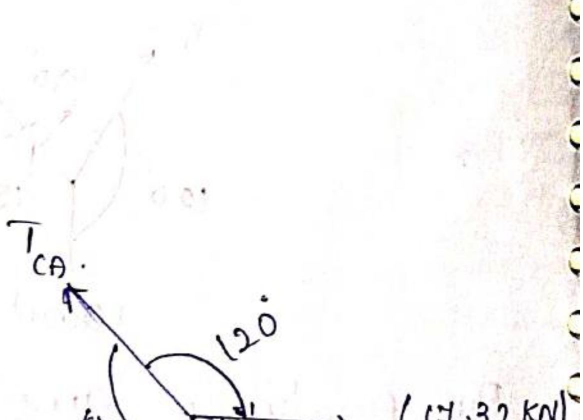
$$\boxed{T_{DB} = 20 \text{ KN}}$$

$$\frac{T_{CD}}{\sin 120^\circ} = \frac{10}{\sin 150^\circ}$$

$$T_{CD} = \frac{10}{\sin 150} \times \sin 120^\circ$$

$$\boxed{T_{CD} = 17.32 \text{ kN}}$$

Apply lami's theorem at 'C':-



$$\frac{T_{CA}}{\sin 90^\circ} = \frac{W}{\sin 120^\circ} = \frac{17.32}{\sin 150^\circ}$$

$$\frac{T_{CA}}{\sin 90^\circ} = \frac{17.32}{\sin 150^\circ}$$

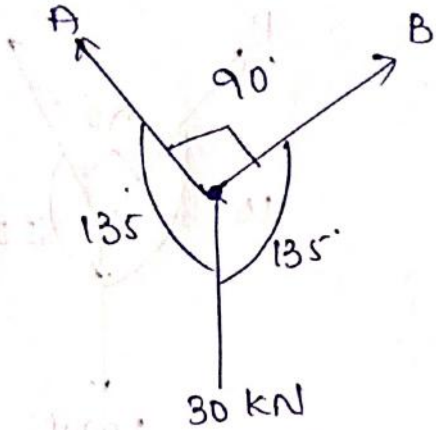
$$T_{CA} = \frac{17.32}{\sin 150^\circ} \times \sin 90^\circ = 34.64 \text{ kN}$$

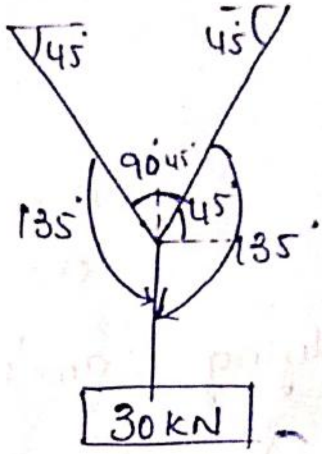
$$\frac{W}{\sin 120^\circ} = \frac{17.32}{\sin 150^\circ}$$

$$W = \frac{17.32}{\sin 150^\circ} \times \sin 120^\circ$$

$$\boxed{W = 30\ kN}$$

(3)





$$\frac{A}{\sin 135^\circ} = \frac{B}{\sin 135^\circ} = \frac{30}{\sin 90^\circ}$$

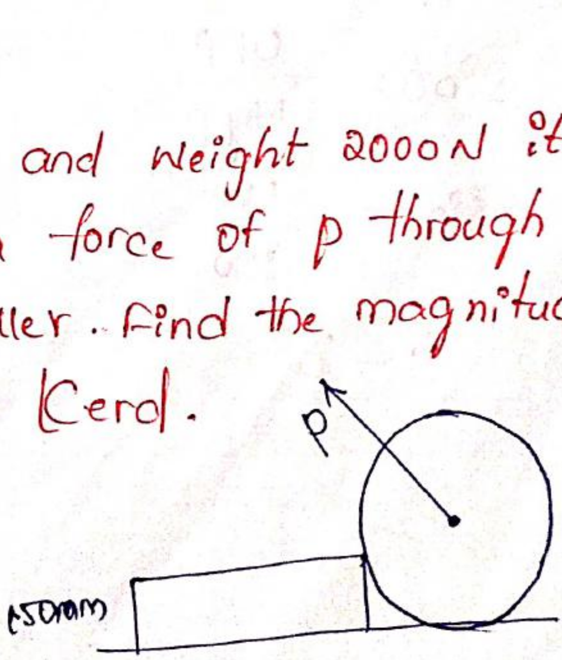
$$A = \frac{30}{\sin 90^\circ} \times \sin 1350$$

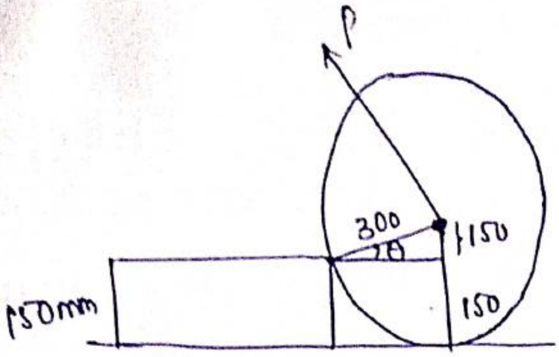
$$= 21.21\ kN$$

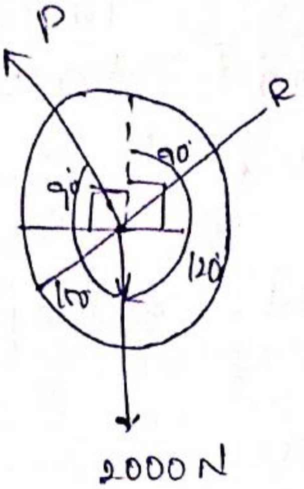
$$B = \frac{30}{\sin 90^\circ} \times \sin 135^\circ$$

$$= 21.21\ kN$$

(4) A roller of radius 300mm and weight 2000N it's pulled of height 150 mm by a force of P through Cure kerds Centre of the roller. Find the magnitude of P required to move over the Cerd.





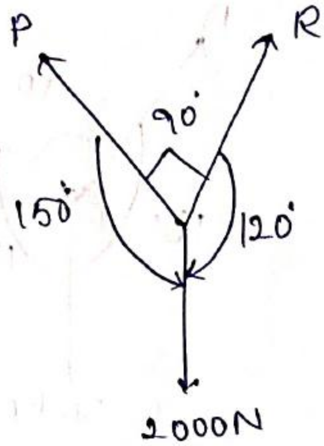


$\sin\theta = \frac{OPP}{Hyp}$

$= \frac{150}{300}$

$\therefore \theta = \sin^{-1}(1/2)$

$\theta = 30^\circ$.



Applying Lame's Theorem:-

$$\frac{P}{\sin 120^\circ} = \frac{R}{\sin 150^\circ} = \frac{W}{\sin 90^\circ}$$

$$\frac{P}{\sin 120^\circ} = \frac{R}{\sin 150^\circ} = \frac{2000}{\sin 90^\circ}$$

$P = \frac{2000}{\sin 90^\circ} \times \sin 120^\circ$

$P = 1732.05N$

$R = \frac{2000}{\sin 90^\circ} \times \sin 150^\circ$

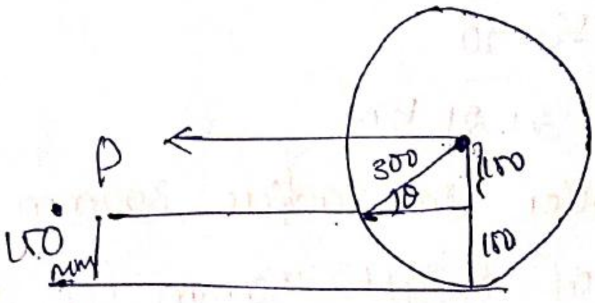
$R = 1000 N$.

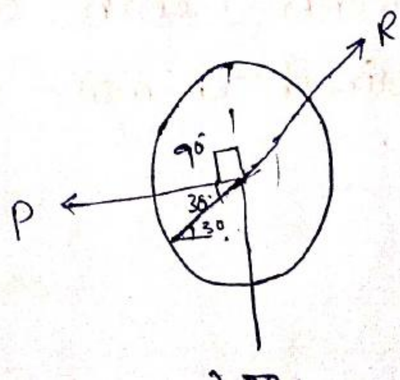
(3)

$\sin\theta = \frac{OPP}{Hyp}$

$\sin\theta = \frac{150}{300}$

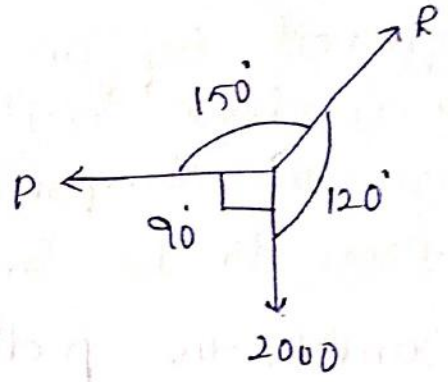
$\theta = 30^\circ$.





By applying Lame's theorem

$$\frac{R}{\sin 90^\circ} = \frac{P}{\sin 120^\circ} = \frac{2000}{\sin 150^\circ}$$



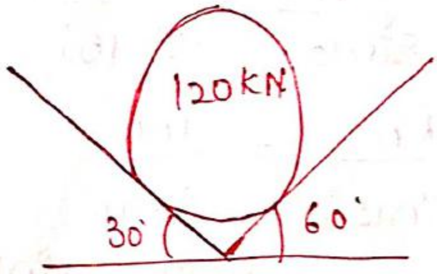
$$R = \frac{2000}{\sin 150^\circ} \times \sin 90^\circ$$

$$R = 4000\ N$$

$$P = \frac{2000}{\sin 150^\circ} \times \sin 120^\circ$$

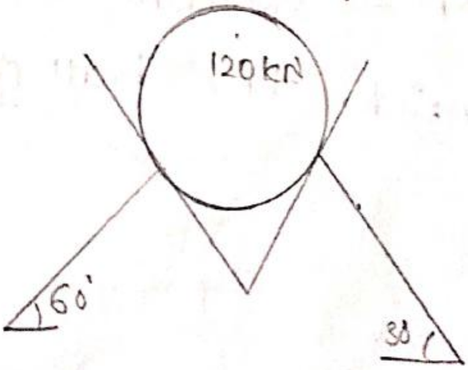
$$P = 3464.10\ N$$

Calculate the reactions at the contact surface of the ball as shown in fig.



(A) By applying Lame's Theorem.

$$\frac{R_1}{\sin 120^\circ} = \frac{R_2}{\sin 150^\circ} = \frac{120}{\sin 90^\circ}$$

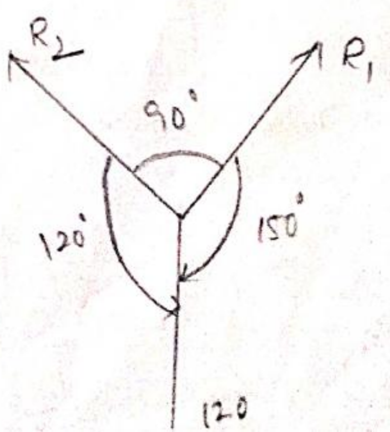


$$R_1 = \frac{120}{\sin 90^\circ} \times \sin 120^\circ$$

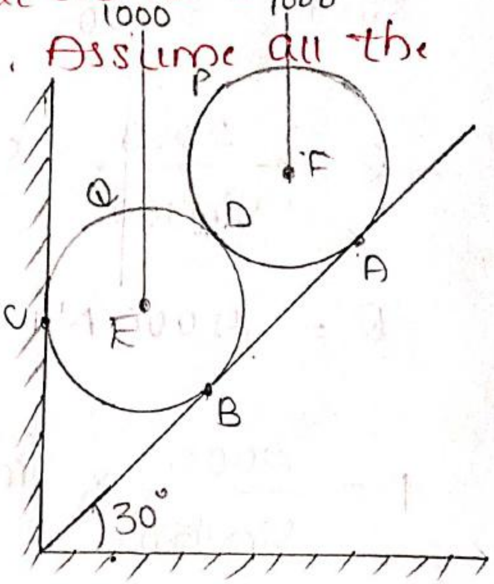
$$R_1 = 103.92\ kN$$

$$R_2 = \frac{120}{\sin 90^\circ} \times \sin 150^\circ$$

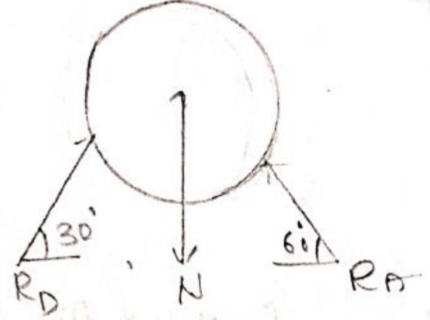
$$R_2 = 60\ kN$$

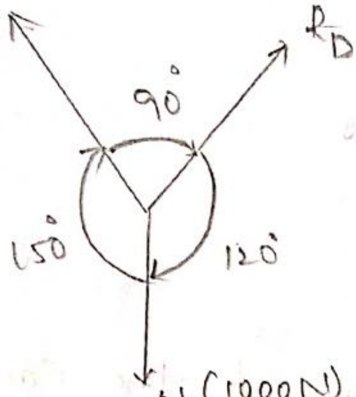


⑦ Two identical rollers each of weight 1000 N are supported by an inclined plane and a vertical wall as shown in fig. find the reactions at the points of supports A, B and C. Assume all the surfaces to be smooth.



A) Consider the p diagram (ball)





$$\frac{R_A}{\sin 120^\circ} = \frac{R_D}{\sin 150^\circ} = \frac{1000}{\sin 90^\circ}$$

$$\frac{R_A}{\sin 120^\circ} = \frac{1000}{\sin 90^\circ}$$

$$R_A = \frac{1000}{\sin 90^\circ} \times \sin 120^\circ$$

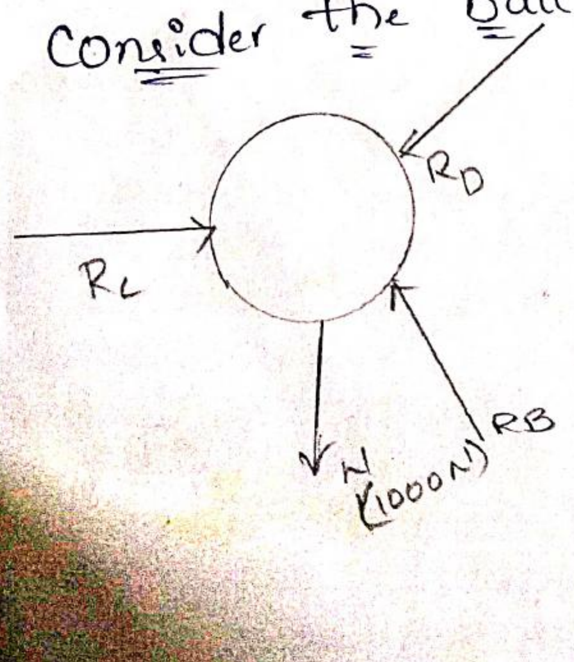
$$R_A = 866.02\,N$$

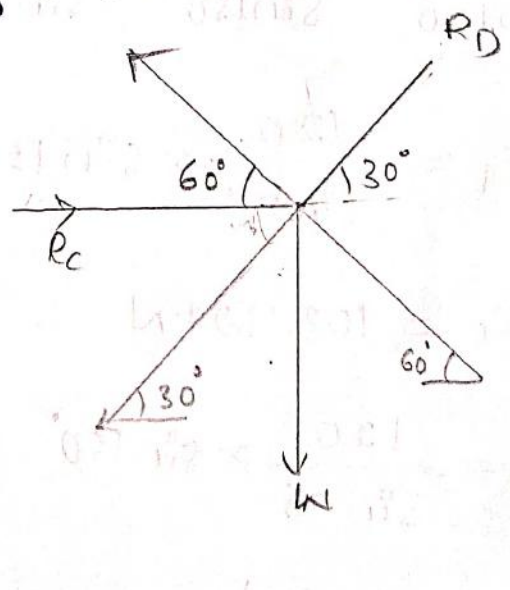
$$\frac{R_D}{\sin 150^\circ} = \frac{1000}{\sin 90^\circ}$$

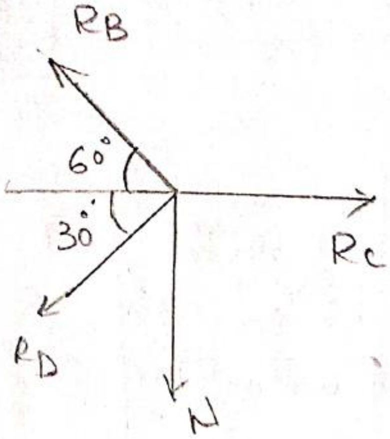
$$R_D = \frac{1000}{\sin 90^\circ} \times \sin 150^\circ$$

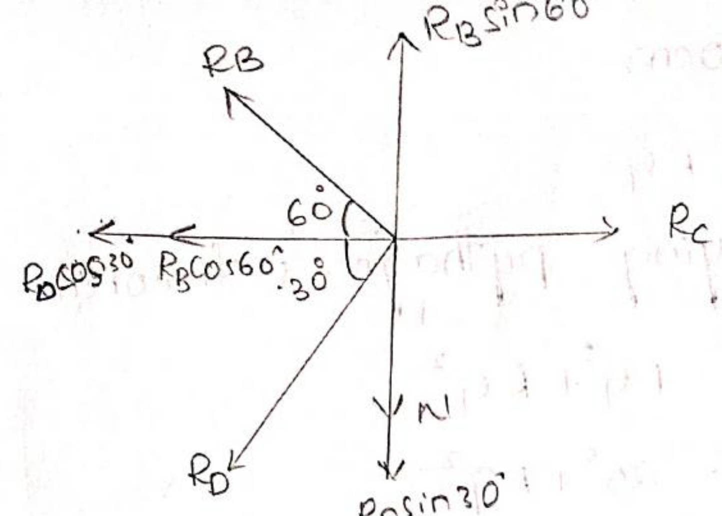
$$R_D = 500\,N$$

Consider the ball Q :-









$\Sigma f_x = 0$

$R_C - R_B \cos 60° - R_D \sin 30° = 0$

$R_C - R_B \cos 60° - 500 \sin 30° = 0$

$R_C - R_B(0.5) - 433.01 = 0$

$R_C - R_B(0.5) = 433.01$ — ①

$\Sigma f_y = 0$

$R_B \sin 60° - 1000 - R_D \sin 30° = 0$

$R_B \sin 60° - 1000 - 250 = 0$

$R_B \sin 60° = 1250$

$R_B = \frac{1250}{\sin 60°}$

$\boxed{R_B = 1443.37\,N}$

R_B value sub in equ ①

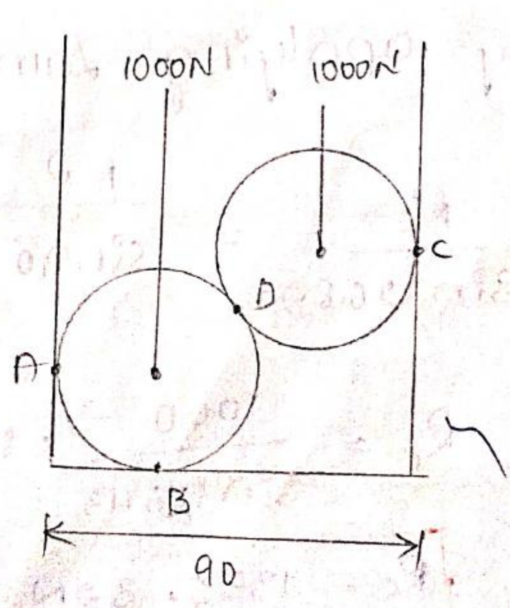
$R_C - 1443.37 \times 0.5 = 433.01$

$R_C = 433.01 + 721.68$

$\boxed{R_C = 1154.69\,N}$

$\boxed{R_D = 500\,N}$

Two sphers each of weight 1000N and of radius 25cm as shown in fig. in horizontal channel width 90 cm. find the reaction of the points of contact.



Weight of each sphere = 1000 N

Radius of each sphere R = 25cm

AF = EC = ED = FD = BF = 25 cm

Width of the channel = 90 cm

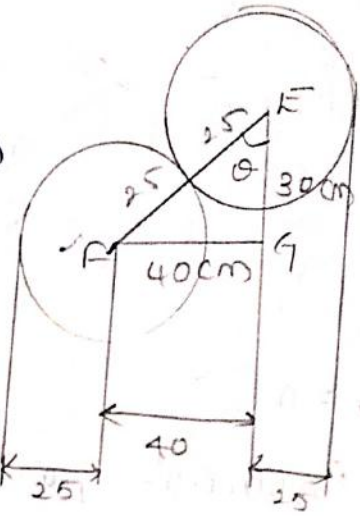
join the centre E to centre F as shown in fig.

EF = 25 + 25 = 50 cm

FG = 40cm.

From Δ^{le} EFG

By applying pythagore's theorem



$EF^2 = FG^2 + EG^2$

$50^2 = 25^2 + EG^2$

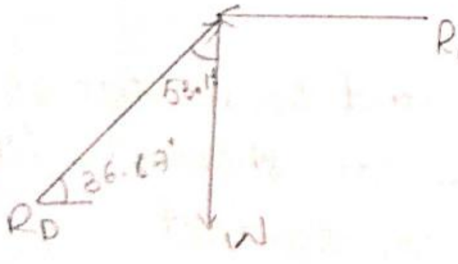
$EG^2 = 900$

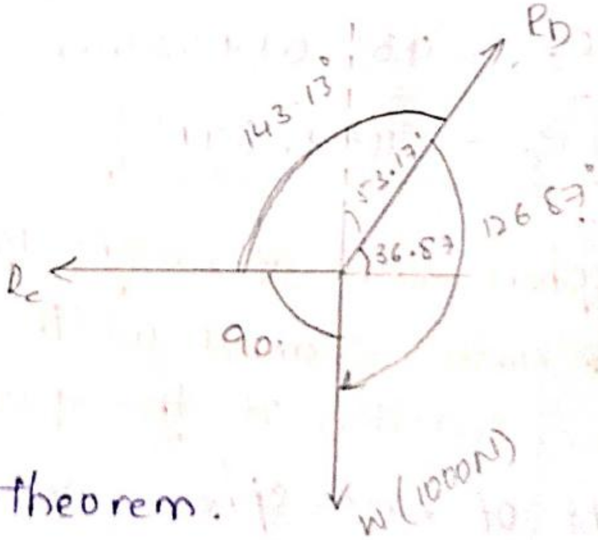
$EG = 30$

$\tan\theta = \frac{40}{30} \Rightarrow \theta = \tan^{-1}\frac{40}{30}$

$\boxed{\theta = 53.13^\circ}$

consider the free body diagram from 2 sphere. and try to calculate the Rc & RD





By applying Lame's theorem.

$$\frac{R_c}{\sin 126.87^\circ} = \frac{RD}{\sin 90^\circ} = \frac{1000}{\sin(143.13^\circ)}$$

$$R_c = \frac{1000}{\sin 143.13} \times \sin 126.87$$

$\boxed{R_c = 1333.32N}$

$R_D \cos 53.13$

1000

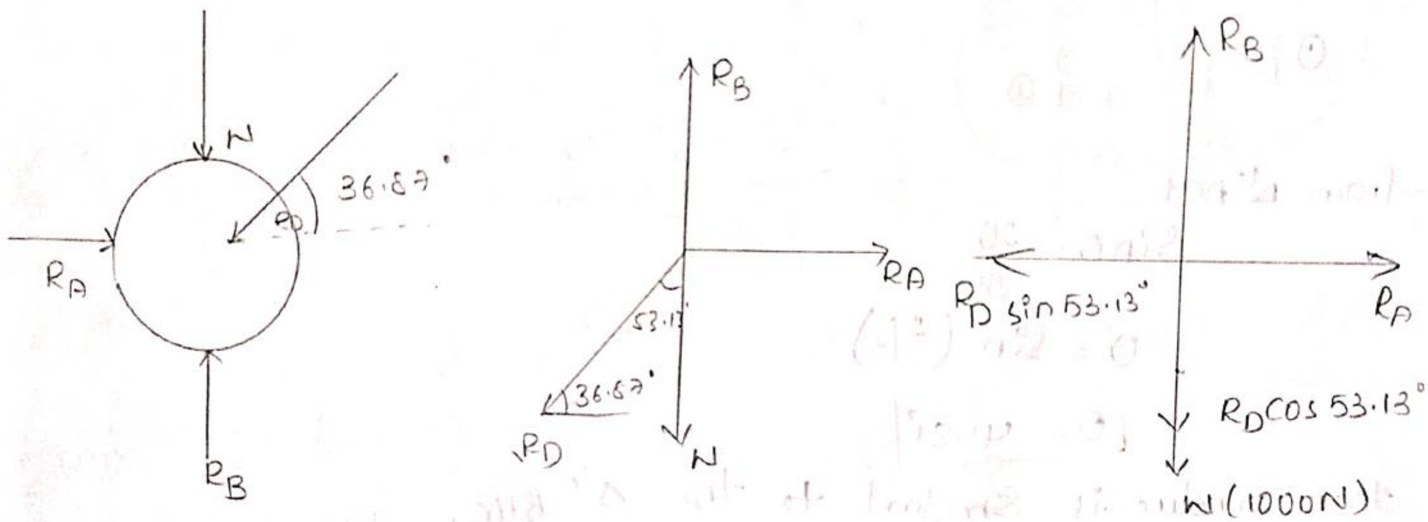
1334

$$\frac{R_D}{\sin 90^\circ} = \frac{1000}{\sin 143.13^\circ}$$

$$R_D = \frac{1000}{\sin 143.13^\circ} \times \sin 90^\circ$$

$$\boxed{R_D = 1666.66\ N}$$

Consider the free body diagram from 1 sphere



$\Sigma f_x = 0$

$R_A - R_D \sin 53.13^\circ = 0$

$R_A = 1666.66 \sin 53.13^\circ$

$R_A = 1333.32^\circ.$

$\Sigma f_y = 0$

$R_B - R_D \cos 53.13^\circ - W = 0$

$R_B = 1666.66 \cos 53.13^\circ + 1000$

$R_B = 1999.99\ N \approx 2000\ N$

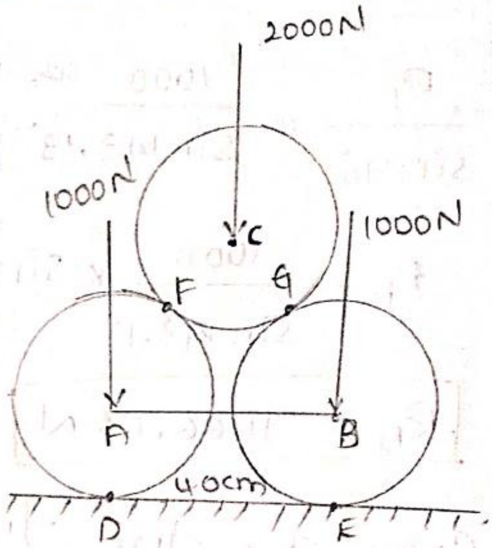
Two Smooth circular cylinders each of weight 1000 N and radius is 15cm are Connected at their centers by a string AB of length 40cm are rest a upon horizantal plane supporting above them third cylinder of weight 2000 N and radius 15cm as shown in figure. Find the force 'S' in the string AB and pressure produced on the floor at the points of Contact D and E.

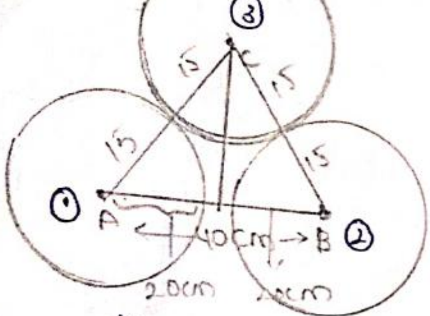
The weight of the cylinder 1 & 2 = 1000N

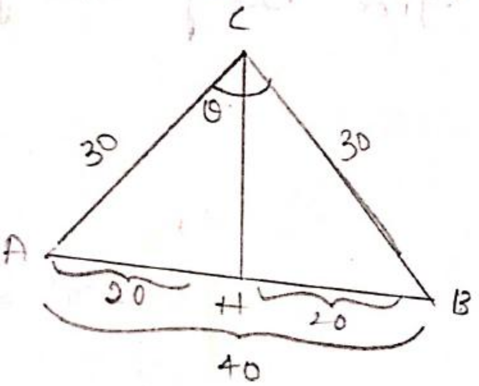
Weight of the cylinder 3 = 2000N

Radius of each cylinder = 15cm.

Length of the string AB = 40cm.







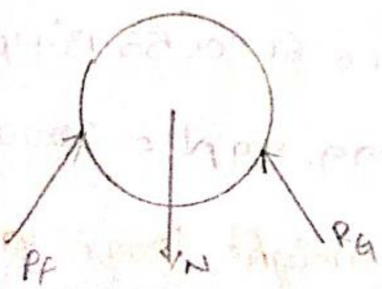
from Δle ACH

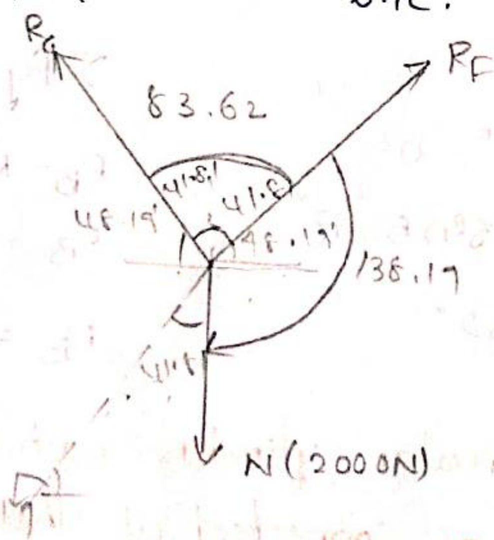
$$\sin\theta = \frac{20}{30}$$

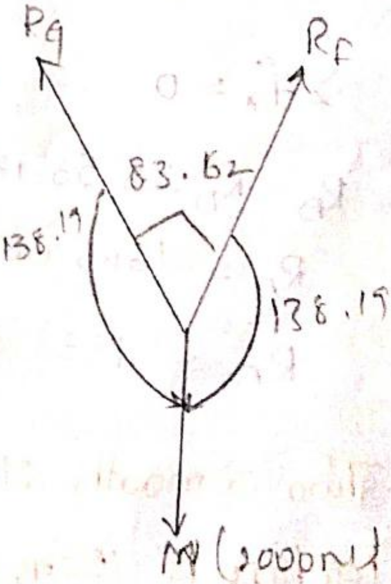
$$\theta = \sin^{-1}(2/3)$$

$$\boxed{\theta = 41.81^\circ}$$

Then θ value is similar to the Δle BHC.







$$\frac{R_G}{\sin 138.19} = \frac{R_f}{\sin 138.19} = \frac{2000}{\sin 83.62}$$

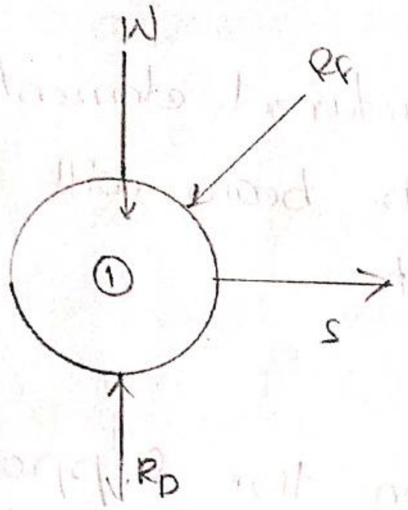
$$\frac{R_f}{\sin 138.19} = \frac{2000}{\sin 83.62}$$

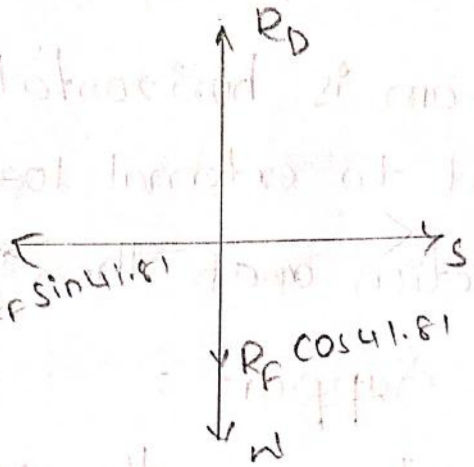
$$R_G = \frac{2000 \times \sin 138.19}{\sin 83.62}$$

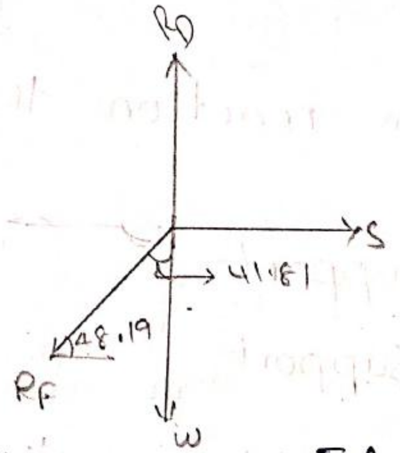
$$R_f = \frac{2000}{\sin 83.62} \times \sin 138.19$$

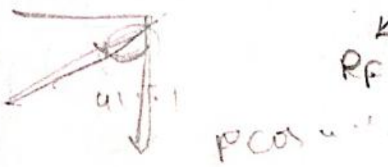
$$R_G = 1341.63N$$

$$R_f = 1341.63N.$$









$\Sigma f_x = 0$

$S - R_F \sin 41.81 = 0$

$S = 1341.63 \times \sin 41.81$

$\boxed{S = 894.41\, N}$

$\Sigma f_y = 0$

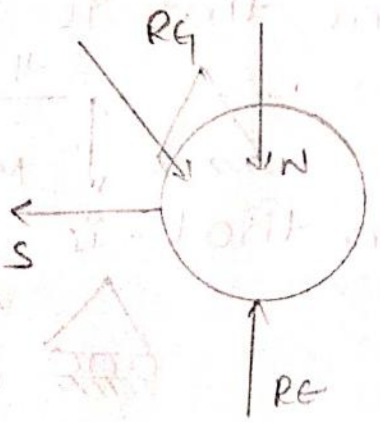
$R_D - R_F \cos 41.81 - W = 0$

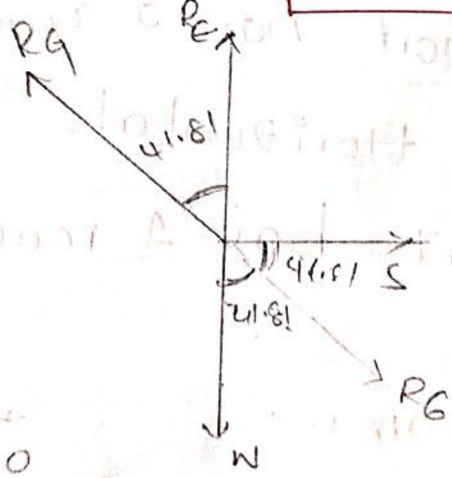
$R_D = W + R_F \cos 41.81$

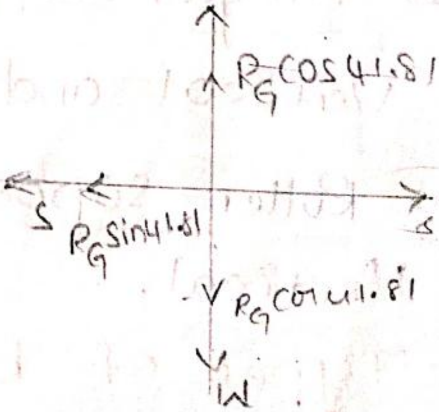
$R_D = 1341.63 \cos 41.81 + 1000$

$\boxed{R_D = 2000 N}$

From cylinder 2







$\Sigma f_x = 0$ $\Sigma f_y = 0$

$-S - R_G \sin 41.81 = 0$

$-R_G \sin 41.81 = S$

$-R_G = \frac{894.41}{\sin 41.81}$

$-R_G = 1341.62$

$R_G = -1341.62$

Beam :-

A beam is horizontal structural element which will subjected to external load, the beam will be having reaction anat the support.

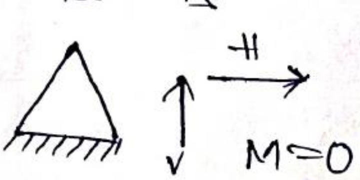
Types of Support :

Based upon the reactions the supports are categorised as

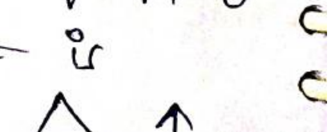
1. Fixed support
2. Hinged support
3. Roller support

⇒ Fixed Support has 3 reactions that is Horizontal, Vertical and moment.

⇒ Hinged Support has 2 reactions that is Vertical and Horizontal.

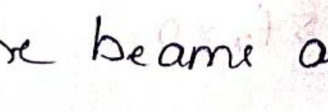
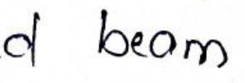
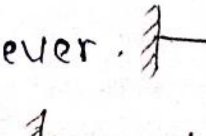


⇒ Roller support has 1 reaction that is Vertical.



Types of beams :

Based upon the supports the beams are categorized as

1. Simply supported beam 
(2) Cantilever beam 
(3) propped cantilever. 
(4) Fixed beam 
(5) Over Hanging beam. 

(6) Continuous beam.

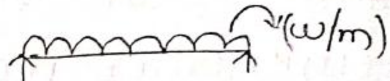
Types of Loads :-

unit (kN (or) N)

1) point load (or) Concentrated load.

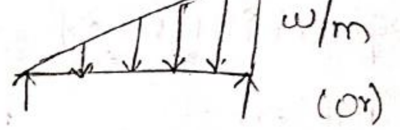
unit kN/m

2) Uniformly distributed load (UDL)

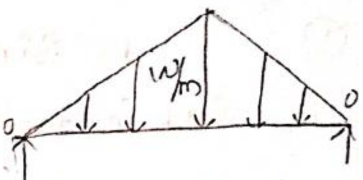


unit w kN/m

3) Uniformly Varying load (UVL)



(or)

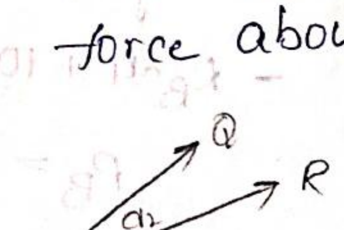


Note :-

* Area of the uniformly distributed load represents a point load acts at the centre of gravity of the load.
* Area of the uniformly varying load represents a point acts at the centre of gravity of load.

Varignon's Theorem :-

The Algebraic sum of moment of system of co-planar forces about a point moment centre is equals to moment of their resultant force about the same point.

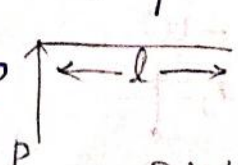


$$P \times d_1 + Q \times d_2 = R \times d$$

Moment :- The product of force × Perpendicular distance

The unit for moment is kN-m

(or) N-mm



If the direction of moment is Clockwise then we can take positive.

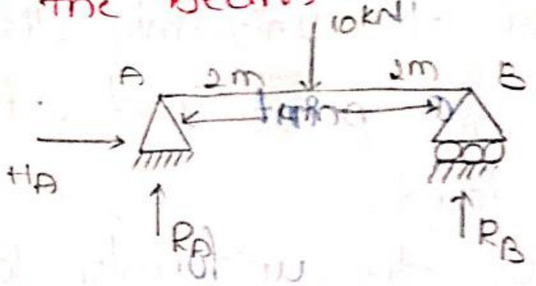
⇒ Anticlockwise means we can take negative.

Analaysis of beam :-

A beam is in equilibrium under the action of external forces and reactions. The unknown reactions can be founde by using 3 equations of equilibrium ($\Sigma H=0$, $\Sigma V=0$, $\Sigma M=0$)

Problems

① A simply supported beam of span (length) 4m is subjected to a concentrated load of 10 kN. acting at the centre of the beam. Find the support reactions.



$\Sigma H=0$

$\boxed{H_A=0}$

$\Sigma V=0$

$R_A+R_B-10=0$

$R_A+R_B=10 \rightarrow ①$

$\Sigma M=0$

$\Sigma M_A=0$

(Taking moments about A = 0)

$-R_B\times 4+10\times 2=0$

$R_B=\frac{20}{4}$

$\boxed{R_B=5}$

R_B sub in equ ①

$R_A+5=10$

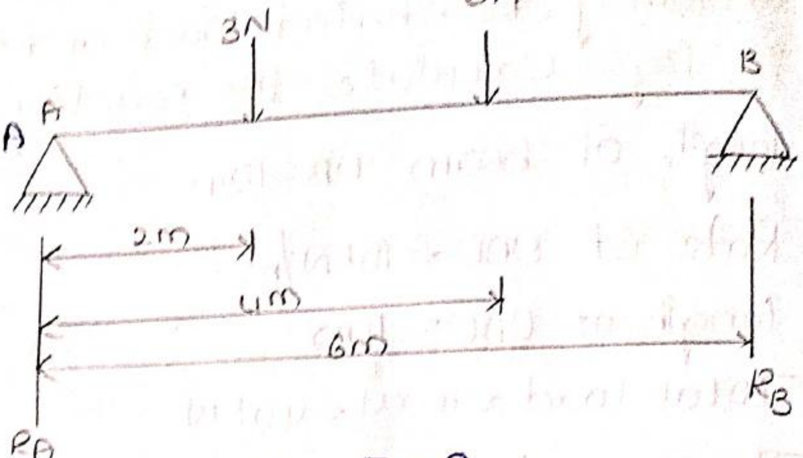
$\boxed{R_A=5}$

② A simply supported beam AB of span 6m carries point load of 3kN and 6kN at a distance of 2m and 4m from left end A as shown in fig. find the reaction

R_A = Reaction @ A

R_B = Reaction @ B

H_A = Horizontal reaction @ A



$\Sigma H = 0 \Rightarrow H_A = 0$

$\Sigma V = 0$

$R_A + R_B - 3 - 6 = 0$

$R_A + R_B = 9$ — ①

$\Sigma m_A = 0$

$R_B \times 6 - (3\times2) - (6\times4) = 0$

$6R_B - 6 - 24 = 0$

$6R_B = 30 \Rightarrow R_B = \frac{\cancel{30}^{5}}{\cancel{6}}$

$\boxed{R_B = 5N}$ → Sub in equ ①

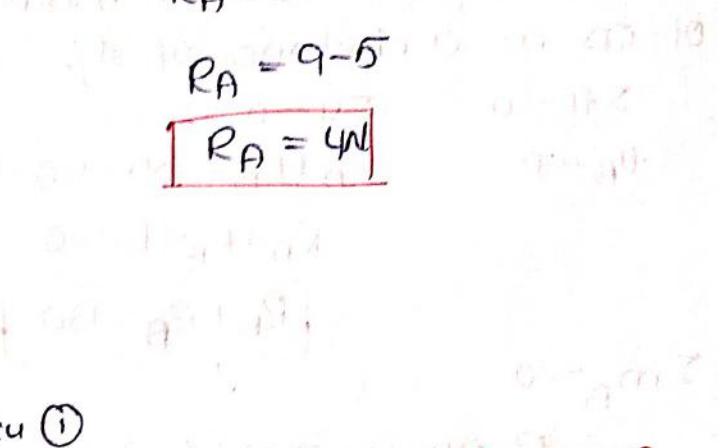
$R_A + 5 = 9$

$R_A = 9 - 5$

$\boxed{R_A = 4N}$

③ A Simply Supported beam AB of length 9m, carries a uniformly distributed load of 10kN/m for a distance of 6m from the left end. Calculate reactions at A & B

$\Sigma H = 0 \Rightarrow H_A = 0$



Rate of UDL = 10kN/m

length of UDL = 6m

Total load = Rate of UDL × length of UDL

= 60kN at middle of AC

= 6/2 = 3m from A.

$\Sigma H = 0 \Rightarrow H_A = 0$

$\Sigma V = 0 \Rightarrow R_A + R_B - 60 = 0$

$R_A + R_B = 60$

$\Sigma m = 0$

$R_B \times 9 - (60\times3) = 0$

$9R_B - 180 = 0$

$R_B = \frac{\cancel{180}^{20}}{\cancel{9}} \Rightarrow \boxed{R_B = 20kN}$

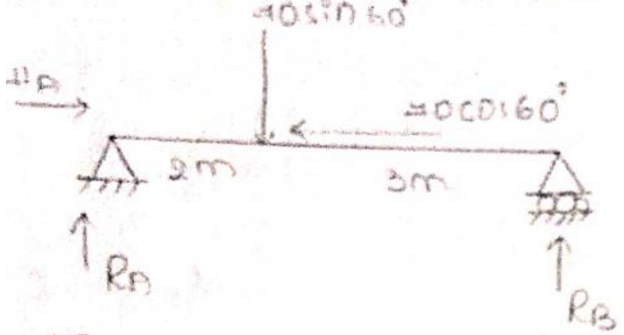
Substitute R_B in ①

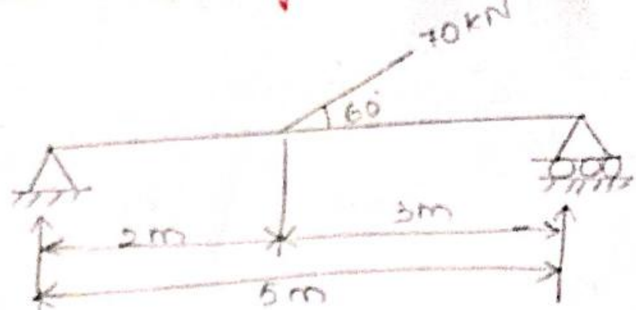
$R_A + 20 = 60$

$R_A = 60 - 20$

$\boxed{R_A = 40kN}$

⑥ Calculate the reactions as shown in fig.





$\Sigma H = 0$

$H_A - 70\cos 60° = 0$

$H_A = 70\cos 60°$

$H_A = 35\,KN$

$\Sigma V = 0$

$R_A + R_B - 70\sin 60° = 0$

$R_A + R_B = 35\sqrt{3}$

$R_A + R_B = 60.62\,KN$

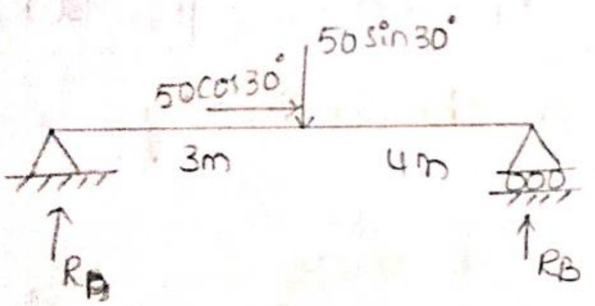
$\Sigma M_A = 0$

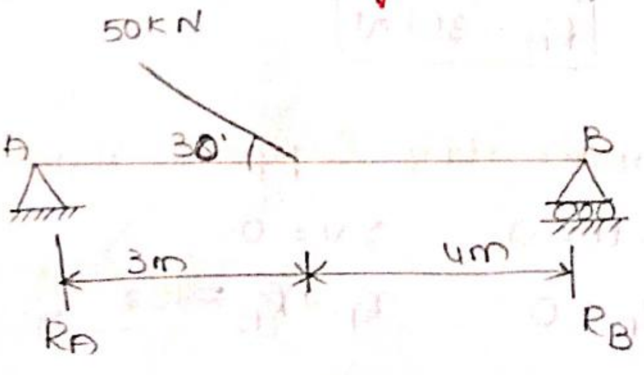
$-R_B \times 5 + 70\sin 60 \times 2 = 0$

$\cancel{-} R_B = \cancel{-}\dfrac{70\sin 60 \times 2}{5}$

$\boxed{R_B = 24.24\,kN}$

⑦ Calculate the reactions as shown in fig.





$\Sigma H = 0$

$R_A + 50\cos 38° = 0$

$R_A = -43.30\,kN$

$\Sigma V = 0$

$R_A + R_B - 50\sin 30° = 0$

$R_A + R_B - 25 = 0 \Rightarrow \boxed{R_A + R_B = 25}$

$\Sigma M_A = 0$

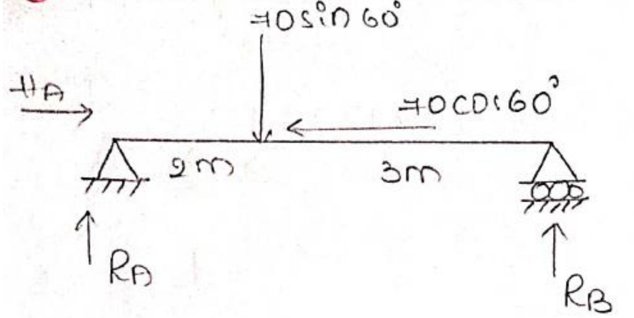
$-R_B$

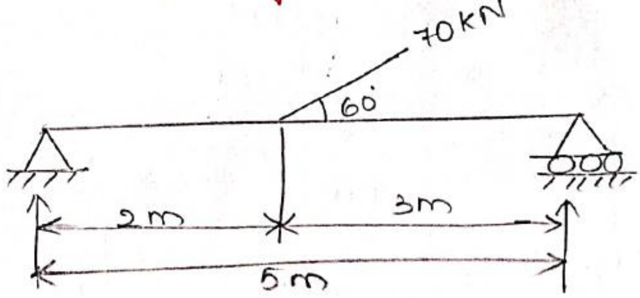
$R_B = 10.71\,kN$

$R_A \cancel{+}\, 10.71 = 25$

$R_A = 14.24\,kN$

⑥ Calculate the reactions as shown in fig.





$\Sigma H = 0$

$H_A - 70\cos 60° = 0$

$H_A = 70\cos 60°$

$H_A = 35\ kN$

$\Sigma V = 0$

$R_A + R_B - 70\sin 60° = 0$

$R_A + R_B = 35\sqrt{3}$

$R_A + R_B = 60.62\ kN$

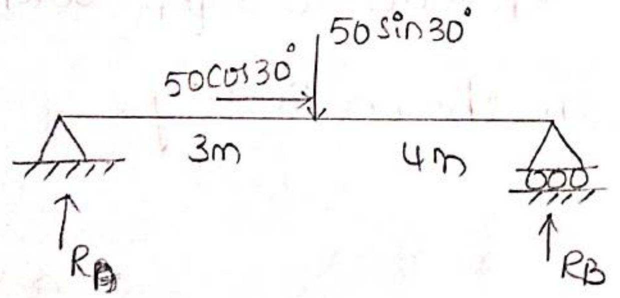
$\Sigma M_A = 0$

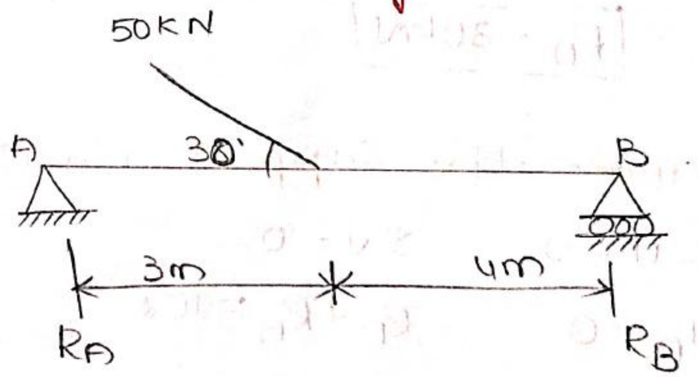
$-R_B \times 5 + 70\sin 60 \times 2 = 0$

$\cancel{-} R_B = \cancel{-}\dfrac{70\sin 60 \times 2}{5}$

$\boxed{R_B = 24.24\ kN}$

⑦ Calculate the reactions as shown in fig.





$\Sigma H = 0$

$R_A + 50\cos 38° = 0$

$R_A = -43.30\ kN$

$\Sigma V = 0$

$R_A + R_B - 50\sin 30° = 0$

$R_A + R_B - 25 = 0 \Rightarrow \boxed{R_A + R_B = 25}$

$\Sigma M_A = 0$

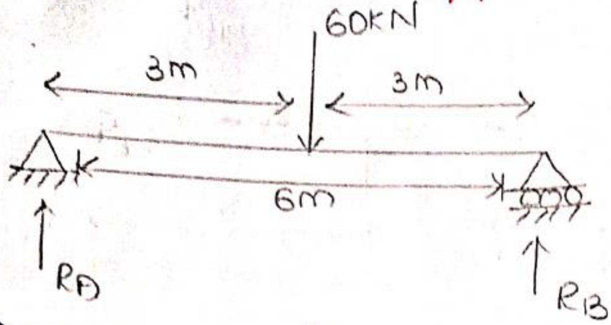
$-R_B$

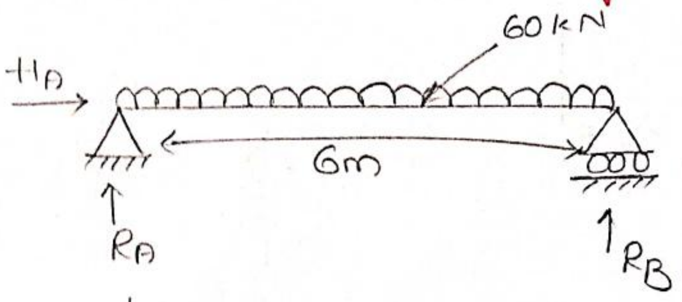
$R_B = 10.71\ kN$

$R_A + 10.71 = 25$

$R_A = 14.24\ kN$

(8) Calculate the Support reactions as shown in fig.





Area of loading = 10×6 = 60KN

$\Sigma H = 0$

$H_A = 0$

$\Sigma V = 0$

$R_A + R_B - 60 = 0$

$R_A + R_B = 60 \longrightarrow (1)$

$\Sigma M_A = 0$

$-R_B \times 6 + 60 \times 3 = 0$

$\cancel{-} 6R_B = \cancel{-}180 \Rightarrow R_B = \frac{\cancel{180}^{30}}{\cancel{6}}$

$\boxed{R_B = 30 kN}$

$R_A + 30 = 60$

$\boxed{R_A = 30 kN}$

$\Sigma H = 0$ $\Sigma V = 0$

$H_A = 0$ $R_A + R_B - 10 \times 6 = 0$

$R_A + R_B = 60$

$\Sigma M_A = 0$

$-R_B \times 6 + 10 \times 6 \times \cancel{6}^{3}/\cancel{2}$

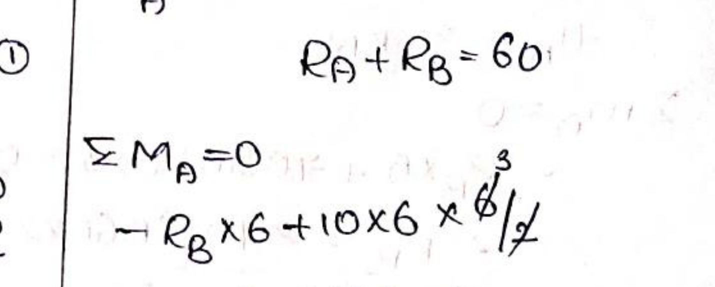
$-6R_B + 180 = 0$

$6R_B = 180$

$\boxed{R_B = 30 kN}$

$R_A = 60 - 30 \Rightarrow \boxed{R_A = 30 kN}$

(9) Calculate the Support reactions as shown in fig.



$\Sigma H = 0$

$H_A = 0$

$\Sigma V = 0$

$R_A + R_B = 10 \longrightarrow (1)$

$\Sigma M_A = 0$

$-R_B \times 4 + 5 \times 2 \times \cancel{2}/\cancel{2}$

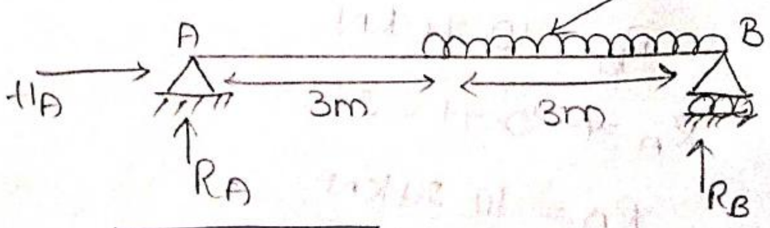
$-4R_B + 10 = 0$

$\boxed{R_B = 2.5 KN}$

$R_A + 2.5 = 10$

$\boxed{R_A = 7.5 KN}$

(10) Calculate the Support reactions as shown in fig.



$\Sigma H = 0 \Rightarrow H_A = 0$

$\Sigma V = 0 \Rightarrow R_A + R_B - 5 \times 3 = 0$

$\boxed{R_A + R_B = 15} \rightarrow (1)$

$\Sigma M_A = 0$

$-R_B \times 6 + 5 \times 3 \times \left(\frac{3}{2} + 3\right) = 0$

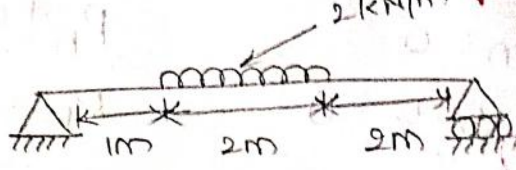
$\cancel{-} R_B \times 6 = \cancel{-} 67.5$

$\boxed{R_B = 11.25 kN}$

$R_A + 11.25 = 15$

$R_A = 15 - 11.25 \Rightarrow \boxed{R_A = 3.75 kN}$

(11) Calculate the Support reactions as shown in fig.



$\Sigma H = 0$ $\quad\quad$ $\Sigma V = 0$

$H_A = 0$ $\quad\quad$ $R_A + R_B - 2\times 2 = 0$

$\boxed{R_A + R_B = 4} \rightarrow (1)$

$\Sigma M_A = 0$

$-R_B \times 5 + 2\times 2\times\left(\frac{2}{2}+1\right)$

$-R_B \times 5 + 6 = 0$

$\cancel{-}5R_B = \cancel{-}6$

$R_B = 6/5$

$\boxed{R_B = 1.2}$

$\Sigma M_B = 0$

$R_A \times 5 - 2\times 2\times\left(\frac{2}{2}+2\right)$

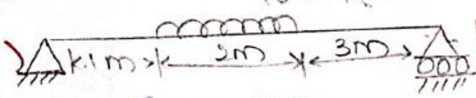
$5R_A - 4\times 3 = 0$

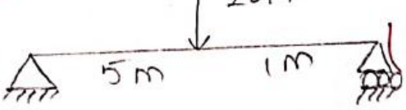
$5R_A = 12$

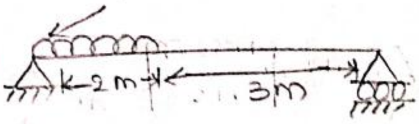
$R_A = \frac{12}{5} \Rightarrow \boxed{R_A = 2.4\,kN}$

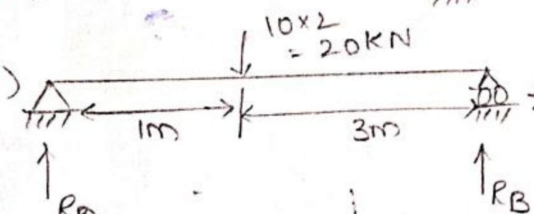
$R_A + R_B = 4$

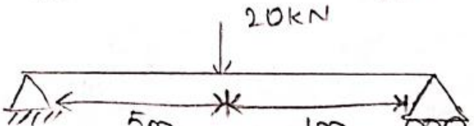
(12) Calculate the Support reactions.

(i) 

(ii) 

(iii) 

(i) 



$\Sigma H = 0$ $\quad\quad$ $\Sigma V = 0$

$H_A = 0$ $\quad\quad$ $R_A + R_B - 20 = 0$

$R_A + R_B = 20 \rightarrow (1)$

$\Sigma M_A = 0$

$-R_B \times 6 + 10\times 2\left(2/2+1\right) = 0$

$-6R_B + 40 = 0$

$\cancel{-}6R_B = \cancel{-}40 \Rightarrow R_B = \frac{40}{6}$ $\quad$ 6.667 kN

$\boxed{R_B = 6.667\,kN}$

$R_A + 6.667 = 20$

$\boxed{R_A = 13.333}$

$\Sigma H = 0$ $\quad\quad$ $\Sigma V = 0$

$H_A = 0$ $\quad\quad$ $R_A + R_B - 20 = 0$

$\boxed{R_A + R_B = 20}$

$\Sigma M_A = 0$

$-R_B \times 6 + 20\times 5 = 0$

$\cancel{-}6R_B = \cancel{-}100$

$R_B = \frac{100}{6} \Rightarrow \boxed{R_B = 16.667\,kN}$

$R_A + 16.667 = 20$

$\boxed{R_A = 3.33\,kN}$

(iii) $\Sigma H = 0$

$H_A = 0$

$R_A + R_B - 5 = 0$

$\boxed{R_A + R_B = 5} \rightarrow (1)$

$\Sigma M_A = 0$

$-R_B \times 5 + 5 \times 1 (1/2) = 0$

$-5R_B + 2.5 = 0$

$\cancel{-}5R_B = \cancel{-}2.5$

$5R_B = 2.5$

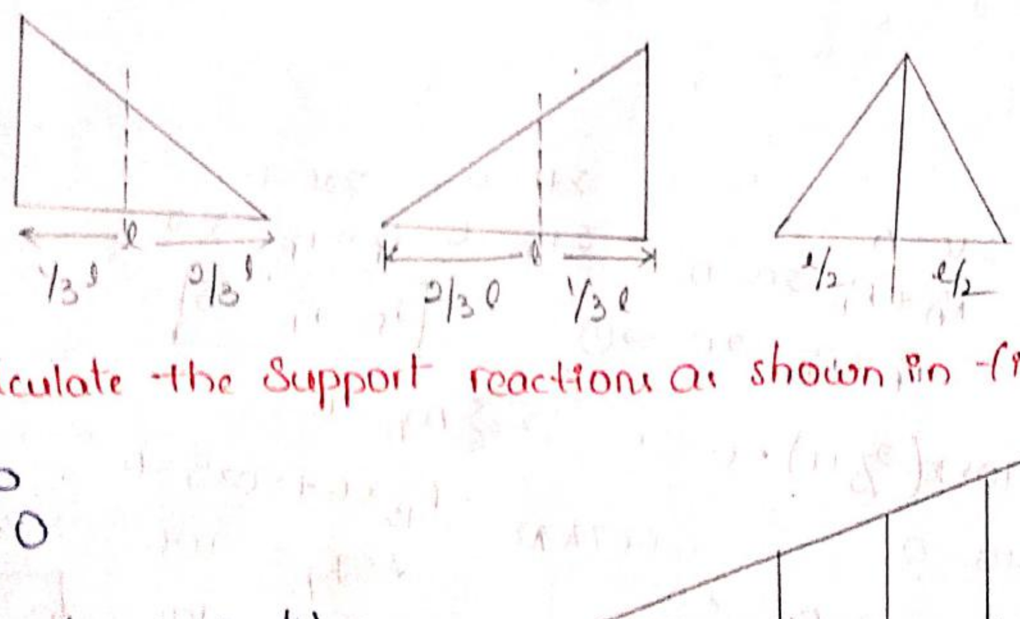
$R_B = \frac{2.5}{5}$

$\boxed{R_B = 0.5 \text{ kN}}$

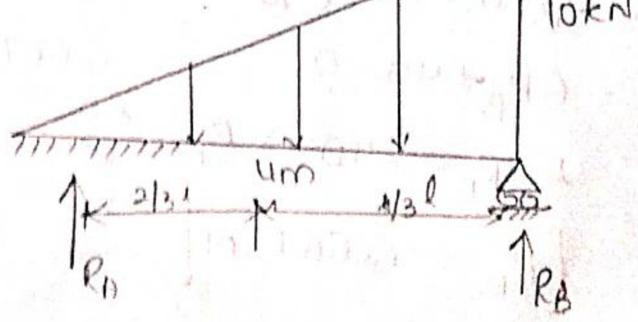
$R_A + 0.5 = 5$

$R_A = 5 - 0.5$

$\boxed{R_A = 4.5 \text{ kN}}$



(15) Calculate the support reactions as shown in fig.



$\Sigma H = 0$

$H_A = 0$

$\Sigma V = 0$

$R_A + R_B - (\text{Area of } \Delta^{le}) = 0$

$R_A + R_B - \left(\frac{1}{2} \times 4 \times 10\right) = 0$

$\boxed{R_A + R_B = 20}$

$\Sigma M_A = 0$

$-R_B \times 4 + (\text{Area of } \Delta^{le}) \times \underbrace{\text{Ler distance}}_{2/3\, l} = 0$

$-R_B \times 4m \left(\frac{1}{2} \times 4 \times 10\right) \times 2/3 \times 4 = 0$

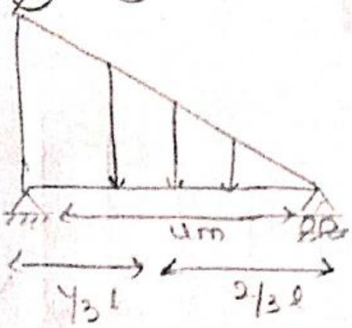
$R_B = 13.33 \text{ kN}$

$R_A + 13.33 = 20$

$R_A = 20 - 13.33$

$\boxed{R_A = 6.67 \text{ kN}}$

(ii) ~~(24)~~ → 21



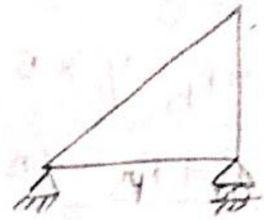
$\Sigma H = 0$

$H_A = 0$

$\Sigma V = 0$

$R_A + R_B - (\frac{1}{2} \times 4 \times 10) = 0$

$\boxed{R_A + R_B = 20}$



$\Sigma M_A = 0$

$-R_B \times 4 + (\text{Area of } \Delta^{le}) \times \perp er \text{ dist} = 0$

$-R_B \times 4 + (\frac{1}{2} \times 4 \times 10) \times \frac{1}{3} \times 4 = 0$

$-R_B \times 4 + (20 \times 4/3) = 0$

$-4R_B = 26.666$

$R_B = \frac{26.666}{4} \Rightarrow \boxed{R_B = 6.666\,kN}$

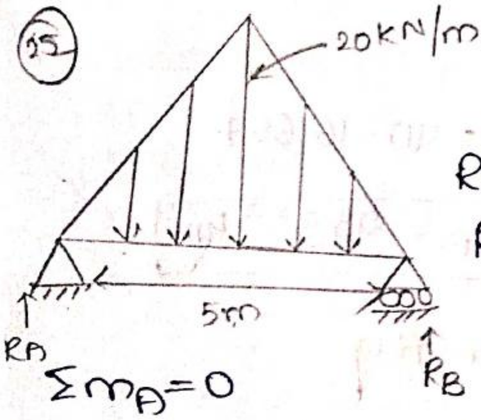
$R_A + 6.6666 = 20$

$R_A = 20 - 6.666\ kN$

$\boxed{R_A = 13.33\,KN}$

(iii)

25



$\Sigma H = 0$

$H_A = 0$

$\Sigma V = 0$

$R_A + R_B - (\text{Area of } \Delta^{le}) = 0$

$R_A + R_B - (\frac{1}{2} \times 5 \times 20) = 0$

$\boxed{R_A + R_B = 50}$

$R_A + R_B = 50$

$R_A + 25 = 50$

$\boxed{R_A = 25\,KN}$

$\Sigma M_A = 0$

$-R_B \times 5 + (\text{Area of } \Delta^{le}) \times \perp er \text{ dist} = 0$

$-R_B \times 5 + (\frac{1}{2} \times 5 \times 20) \times 5/2 = 0$

$-5R_B + 50 \times 5/2 = 0$

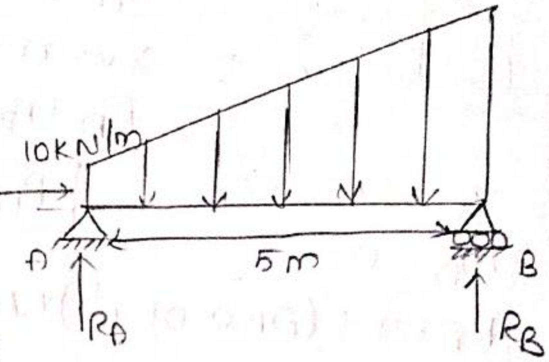
$\cancel{-}5R_B = \cancel{-}125 \Rightarrow R_B = \frac{125}{5} \Rightarrow \boxed{R_B = 25\,kN}$

(14) Calculate the reactions as shown in fig.

$\Sigma H = 0$ $\quad \Sigma V = 0$

$H_A = 0$ $\quad R_A + R_B - \frac{1}{2} \times 5 \times 10 - 5 \times 10 = 0$

$\boxed{R_A + R_B = 75\,KN}$



$\Sigma M_A = 0$

$-R_B \times 5 - w \times l \times l/2 - (\frac{1}{2} \times b \times h) \times 2/3 \times l = 0$

$-R_B \times 5 - 10 \times 5 \times 5/2 - (\frac{1}{2} \times 5 \times 10) \times 2/3 \times 5 = 0$

$-5R_B =$ 41.6667

66.333

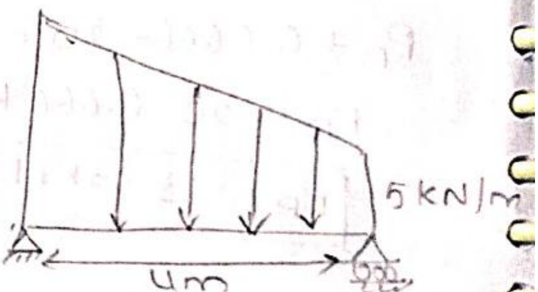
$\Sigma H = 0$ $\quad \Sigma V = 0$

$H_A = 0$ $\quad R_A + R_B - \frac{1}{2} \times b \times h - w \times l$

$R_A + R_B - \frac{1}{2} \times 4 \times 10 - 5 \times 4$

$R_A + R_B - 20 - 20 = 0$

$R_A + R_B = 40$



$\Sigma M_A = 0$

$-R_B \times 4 - 5 \times 4 \times 4/2 - (\frac{1}{2} \times 10 \times 4) \times \frac{1}{3} \times 4$

$-R_B \times 4 = +66.667$

$\boxed{R_B = 16.667\,KN}$

$R_A = 40 - 16.667$

$\boxed{R_A = 23.333\,kN}$

(15) Calculate the Support reaction as shown in fig.

$\Sigma H = 0$ $\quad \Sigma V = 0$

$H_A = 0$ $\quad R_A + R_B - 20 - 10 \times 2 - \frac{1}{2} \times 2 \times 5 = 0$

$\boxed{R_A + R_B = 45}$

$\Sigma M_A = 0$

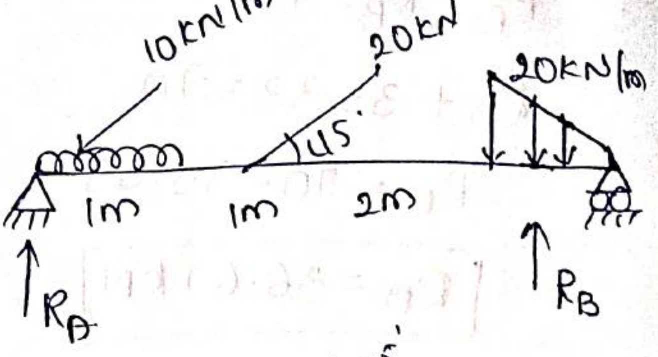
$-R_B \times 7 + 20 \times 1 + (\frac{1}{2} \times 5 \times 2) \times ((2/3 \times 2) + 2 + 1) + (10 \times 2 \times (2/2 + 5))$

$-7R_B + 20 + 5 \times (1.33) + 20 \times 6 = 0$ $\quad R_A + 22.3 = 45$

$+R_B = \frac{+153.33}{7}$

$\boxed{R_B = 22.3\,KN}$

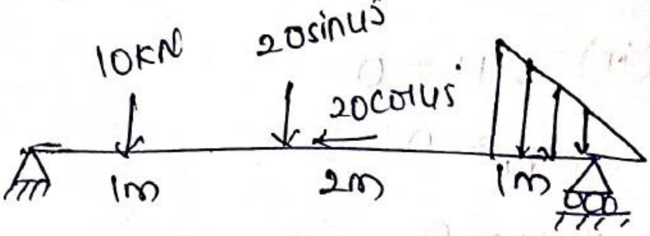
$\boxed{R_A = 22.62\,KN}$



$\Sigma H = 0$

$H_A - 20\cos 45^\circ = 0$

$H_A = 14.14$



$\Sigma V = 0$

$R_A + R_B - 10\times1 - 20\sin45^\circ - \overset{10}{\cancel{20}}\times1\times\frac{1}{\cancel{2}}$

$\boxed{R_A + R_B = 34.14\ kN}$

$\Sigma m_A = 0$

$-R_B\times5 + 10\times1 + \left(\frac{1}{2}\times1\times20\right)\times\left(\frac{2}{3}\times1+2+1+1\right) + (20\sin45\times2)$

$-R_B\times5 + 10 + 48.66 + 28.28$

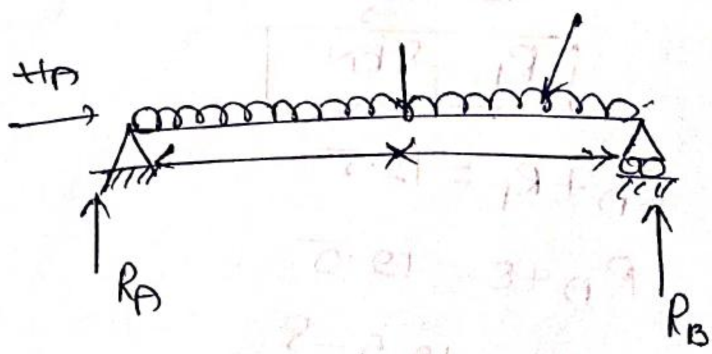
$\cancel{5}R_B = \cancel{17.79}\ N \quad R_B = 16.35\ kN$

$\boxed{R_A = 17.79\ kN}$

(16) Calculate the support reaction as shown in fig.

$\Sigma H = 0$

$H_A = 0$



$\Sigma V = 0$

$R_A + R_B - 10 - 10\times6 = 0$

$\boxed{R_A + R_B = 70}$

$\Sigma m_A = 0$

$-R_B\times6 + 10\times2 + 10\times6\times6/2 = 0$

$-R_B\times6 + 20 + 180 = 0$

$\cancel{-}6R_B = \cancel{+}200$

$R_B = \frac{200}{6} \Rightarrow \boxed{R_B = 33.33\ kN}$

$R_A + R_B = 70$

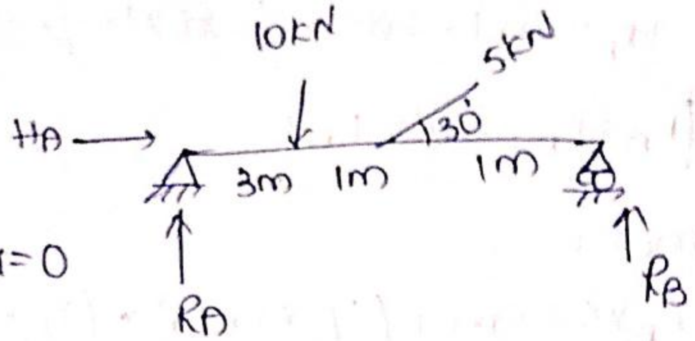
$R_A + 33.33 = 70$

$R_A = 70 - 33.33$

$\boxed{R_A = 36.67\,kN}$

(ii) $\Sigma H = 0$

$H_A = 0$

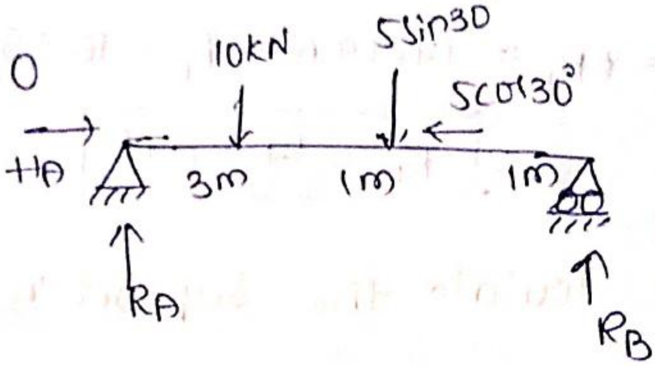


$\Sigma V = 0$

$R_A + R_B + 10\times 3 + 5\sin 30 \times \cancel{4} = 0$

$R_A + R_B = 12.5$

$\Sigma M_A = 0$



$-R_B \times 5 + 10\times 3 + 5\sin 30 \times 4 = 0$

$-R_B \times 5 + 30 + 10 = 0$

$+5R_B = \cancel{-}40$

$5R_B = 40$

$R_B = \frac{\cancel{40}^{8}}{\cancel{5}}$

$\boxed{R_B = 8\,kN}$

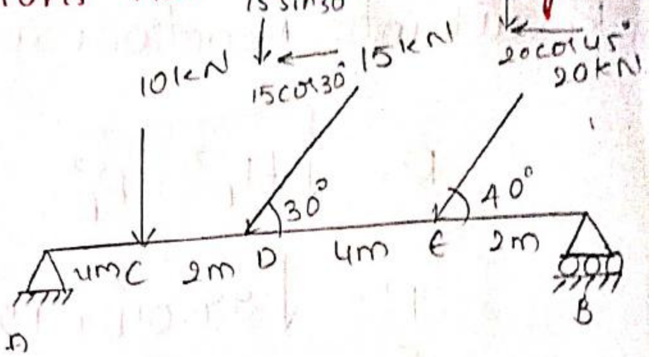
$R_A + R_B = 12.5$

$R_A + 8 = 12.5$

$R_A = 12.5 - 8$

$\boxed{R_A = 4.5\,kN}$

(18) Calculate the Supported reactions has shown in figure



First resolve the Components

(i) Vertical components at load D

$= 15 \sin 30^\circ$

$= 7.5\ kN\ (\downarrow)$

Vertical component at load E

$20 \sin 45^\circ = 14.14\ kN\ (\downarrow)$

(ii) Horizontal component @ D

$= 15 \cos 30^\circ = 12.9\ kN\ (\leftarrow)$

Horizontal component @ E

$20 \cos 45^\circ = 14.14\ kN\ (\leftarrow)$

From equilibrium condition

$\Sigma f_x = 0$

$H_A - 12.9 - 14.14 = 0$

$H_A - 27.04 = 0 \Rightarrow \boxed{H_A = 27.04\ kN}$

$\Sigma f_y = 0$

$R_A + R_B - 10 - 7.5 - 14.14 = 0$

$R_A + R_B - 31.64 = 0 \Rightarrow \boxed{R_A + R_B = 31.64}$

To find R_B taking moments of all forces about $A = 0$

$-R_B \times 12 + 10 \times 4 + 14.14 \times 10 + 7.5 \times 6 = 0$

$\boxed{R_B = 18.86\ kN}$

$R_A + 18.86 = 31.64$

$R_A = 31.64 - 18.86 \Rightarrow \boxed{R_A = 12.78\ kN}$

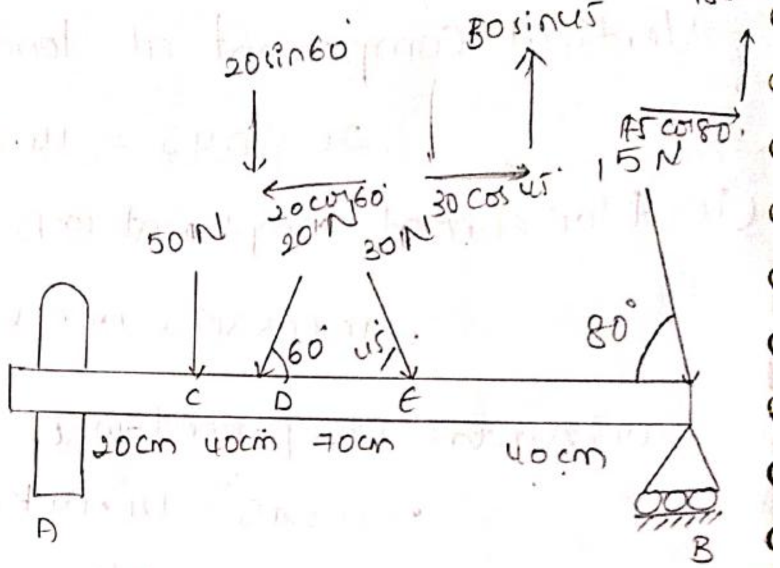
Resultant Reactions @ A.

$$R = \sqrt{H_A^2 + R_A^2}$$

$$R = \sqrt{27.04^2 + 12.78^2}$$

$$\boxed{R = 29.90\ kN}$$

(a)



(i) Vertical component @ D

$= 20 \sin 60°$

$= 17.32\ kN\ (\downarrow)$

Vertical component @ E

$= 30 \sin 45°$

$= 21.21\ kN\ (\downarrow)$

Vertical component at load B

$= 15 \sin 80° = 14.77\ N$ ~~2.60 kN~~ $(\downarrow)$

(ii) Horizontal component at load D $(\longleftarrow)$

$= 20 \cos 60° = 10\ N$

Horizontal component at E $(\longrightarrow)$

$= 30 \cos 45° = 21.21\ N$

Horizontal component at B

$15 \cos 80° = 2.60\ N$ ~~14.77 N~~ $(\longrightarrow)$

From Equilibrium Condition

$\Sigma f_x = 0$

$H_A - 10 + 2.60 + 21.21 = 0$

$H_A = -13.81\ N$

$H_A = -13.81\ N$

$\Sigma f_y = 0$

$R_A + R_B - 50 - 21.21 - 17.32 - 14.81$

$R_A + R_B = 103.34N$

$\Sigma m_A = 0$

$-R_B \times 170 + 50 \times 20 + 17.32 \times 60 + 21.21 \times 130 + 15 \times 170 = 0$

$-R_B \times 170 = 7346.5$

$\cancel{-} R_B = \frac{\cancel{-}7346.5}{170}$

$R_B = 43.214$

$R_A + 43.214 = 103.34$

$R_A = 103.34 - 43.214$

$R_A = 60.126$

Resultant $= \sqrt{(\Sigma H_A)^2 + \Sigma R_A^2}$

$= \sqrt{(-13.81)^2 + (60.126)^2}$

$= 61.69N$

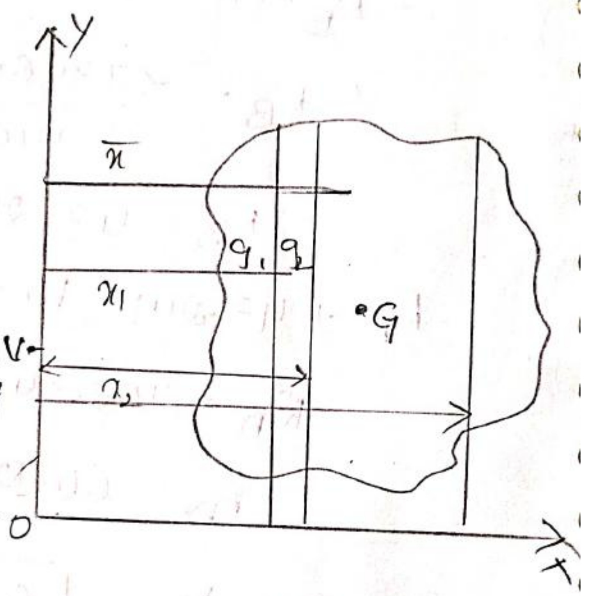
Centre of Gravity & Moment of Inertia

Centre of Gravity:

It is a point at which total area of the Plane. is assumed to the concentrated.

Let G be the centroid of capital area A.

Let $\bar{x}$ and $\bar{y}$ be the distance of Centroid from oy and ox respectively



A = Total Area.

$A_1 + A_2 + A_3 + A_4 + \cdots + A_n$.

$$\therefore \quad \bar{x} = \frac{\text{Total moment of Area}}{\text{Total Area.}}$$

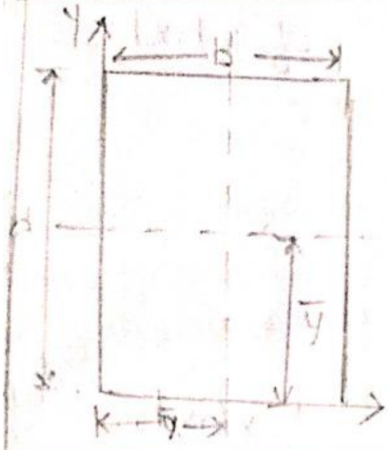
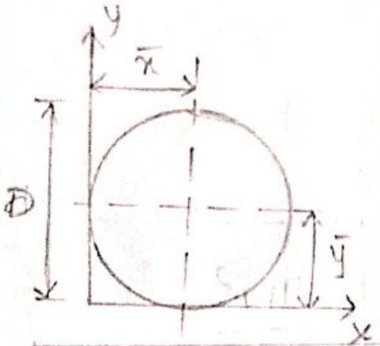
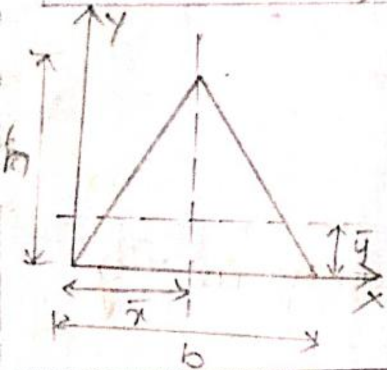
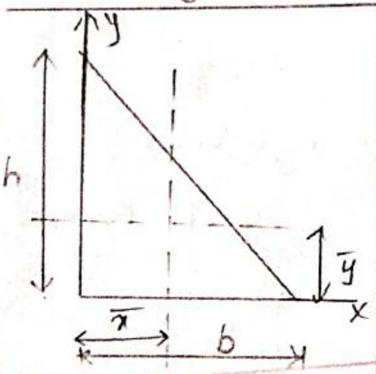
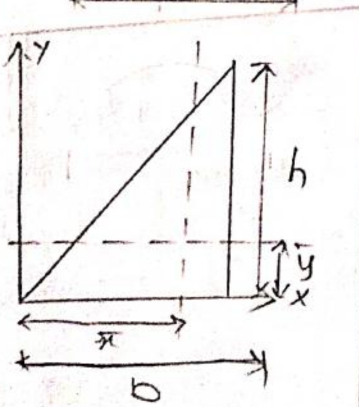
$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + \cdots}{A_1 + A_2 + \cdots A_n}$$

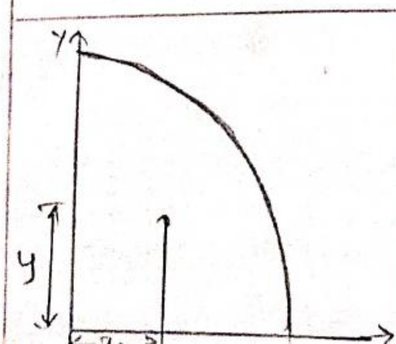
Similarly

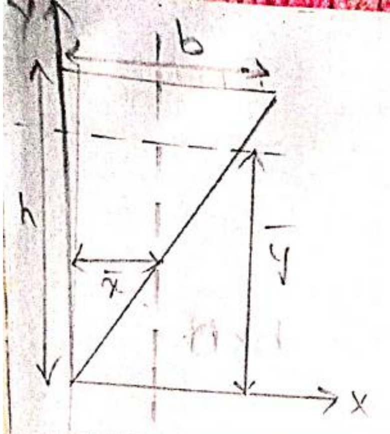
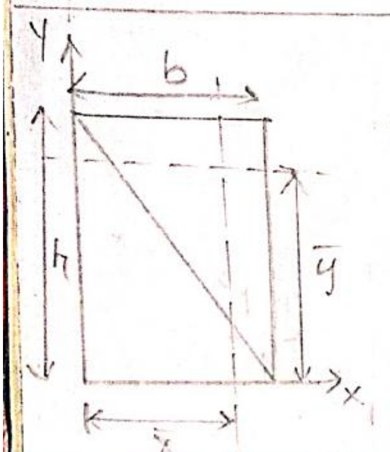
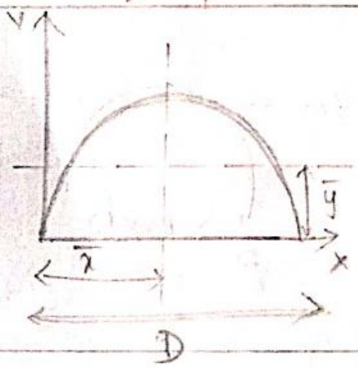
$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + \cdots}{A_1 + A_2 + \cdots A_n}$$

Units for Centroid :

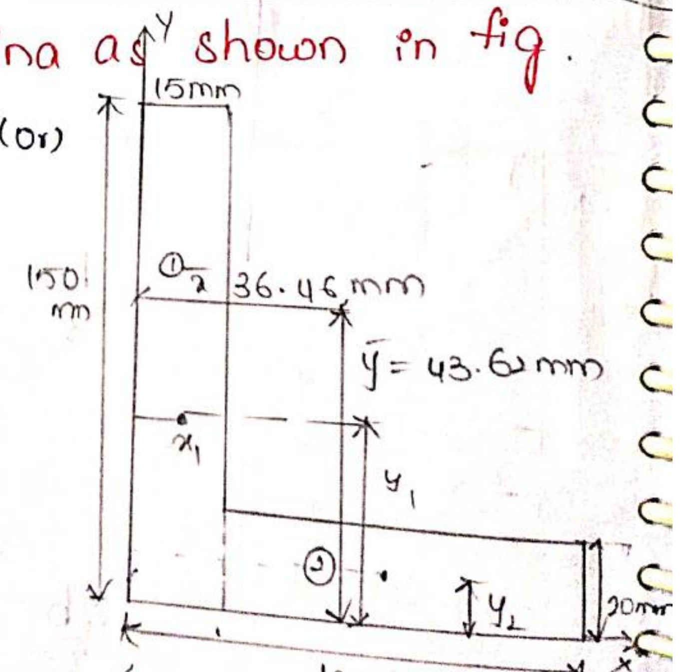
mm (or) cm (or) m.

Shape	$\bar{x}$	$\bar{y}$	Area
	$\frac{b}{2}$	$\frac{d}{2}$	$b \times d$
	$\frac{D}{2}$	$\frac{D}{2}$	$\frac{\pi D^2}{4}$
	$\frac{b}{2}$	$\frac{h}{3}$	$\frac{1}{2} \times b \times h$
	$\frac{b}{3}$	$\frac{h}{3}$	$\frac{1}{2} \times b \times h$
	$\frac{2b}{3}$	$\frac{h}{3}$	$\frac{1}{2} \times b \times h$

	$\bar{x}$	$\bar{y}$	Area
	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$

	$\frac{b}{3}$	$\frac{2h}{3}$	$\frac{1}{2} \times b \times h$
	$\frac{2b}{3}$	$\frac{2h}{3}$	$\frac{1}{2} \times b \times h$
	$\frac{D}{2}$	$\frac{4R}{3\pi}$	$\frac{\pi r^2}{2}$

(1) Locate the centroid of lamina as shown in fig.



This is an L-shaped lamina (or) Angular.

Area :

$A_1 = b_1 \times d_1 = 150 \times 15 = 2250 \text{ mm}^2$

$A_2 = (120-15) \times 20 = 2100 \text{ mm}^2$

$x_1 = \frac{15}{2} = 7.5$ mm

$x_2 = \frac{105}{2} + 15 = 67.5$ mm

$y_1 = \frac{150}{2} = 75$ mm

$y_2 = \frac{20}{2} = 10$ mm

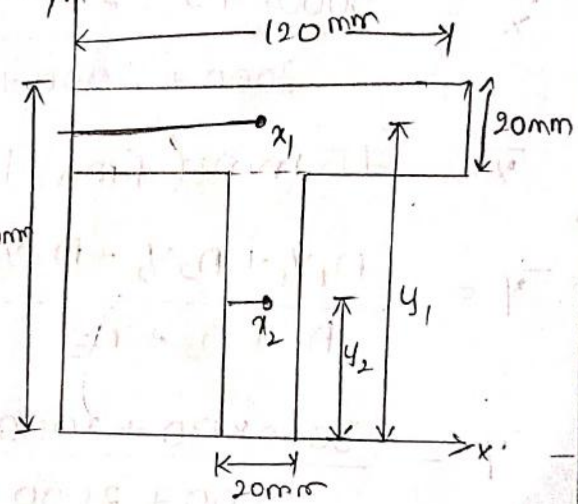
$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2} \qquad \bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2}$$

$$\bar{x} = \frac{2250 \times 7.5 + 2100 \times 67.5}{2250 + 2100} \qquad \bar{y} = \frac{2250 \times 75 + 2100 \times 10}{2250 + 2100}$$

$\bar{x} = 36.46$ mm (Left) $\qquad \bar{y} = 43.62$ mm (bottom)

2) Calculate the centroid of the Section as shown in fig.

$A_1 = 150 \times 2$



Area for 1st rectangle

$A_1 = 120 \times 20$mm

$= 2400 mm^2$

2nd rectangle

$A_2 = (150 - 20) \times 20$

$= 130 \times 20 = 2600 mm^2$

$x_1 = \frac{120}{20} = 60$ mm, $x_2 = \frac{20}{2} + 50$

$x_2 = 60$mm

$y_1 = \frac{20}{2} + 130$

$y_1 = 140$mm

$y_2 = \frac{130}{2} = 65$mm

$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2}$$

$$\bar{x} = \frac{2400 \times 60 + 2600 \times 60}{2400 + 2600}$$

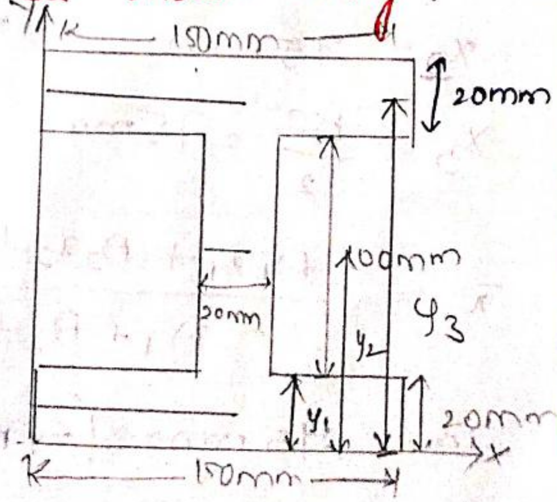
$\bar{x} = 60$mm. (from left)

$$\bar{y} = \frac{A_1y_1 + A_2y_2}{A_1 + A_2}$$

$$\bar{y} = \frac{2400 \times 140 + 2600 \times 65}{2400 + 2600}$$

$\bar{y} = 101$mm (from buttom)

3) Calculate the Centroid of i-section as shown in fig.



$A_1 = 150 \times 20 = 3000 mm^2$

$A_2 = 100 \times 20 = 2000$ mm^2

$A_3 = 150 \times 20 = 3000$ mm^2

$x_1 = \frac{150}{20} = 75$mm, $x_2 = 65 + \frac{20}{2} = 75$mm

$x_3 = 75$mm

$y_1 = 120 + \frac{20}{2}$ | $y_2 = \frac{100}{2} + 20$ | $y_3 = \frac{20}{2} = 10mm$

$= 130mm$ | $y_2 = 70mm$

$\frac{70}{2}$

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3}$$

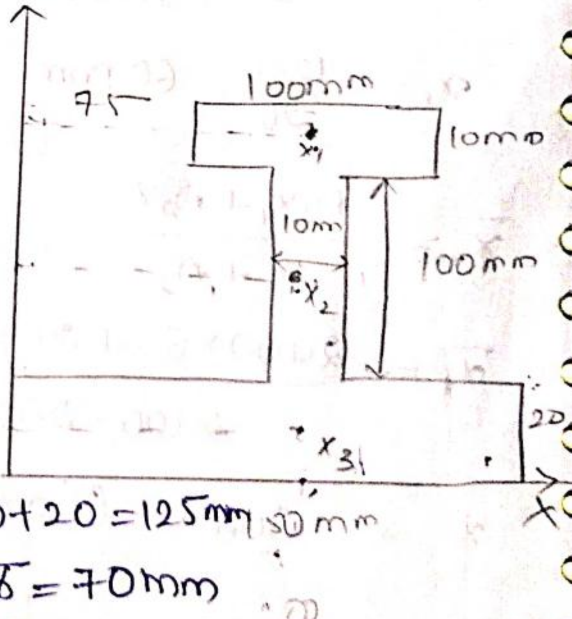
$$\bar{x} = \frac{3000\times75 + 2000\times75 + 3000\times75}{3000 + 2000 + 3000}$$

$\bar{x} = 75mm$ (from left)

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3}$$

$$\bar{y} = \frac{3000\times130 + 2000\times70 + 3000\times10}{3000 + 2000 + 3000}$$

$\bar{y} = 70mm$ (from buttom)



4) $A_1 = 100\times10$

$= 1000mm^2$

$A_2 = 100\times10 = 1000mm^2$

$A_3 = 150\times20 = 3000m^2$

$x_1 = 75mm$ $\qquad y_1 = \frac{120}{2} = 5$

$x_2 = \frac{10}{2} + 75 = \cancel{3/4/8}\ 80$ $\qquad = \cancel{65}\ \frac{10}{2} + 100 + 20 = 125mm$ 50mm

$y_2 = \frac{100}{2} + 20\ \cancel{85} = 70mm$

$x_2 = 75mm$

$x_3 = \frac{150}{2} = 75mm$ $\qquad y_3 = \frac{20}{2} = 10mm$

$\frac{75}{2}$

$\frac{75}{150}$ x

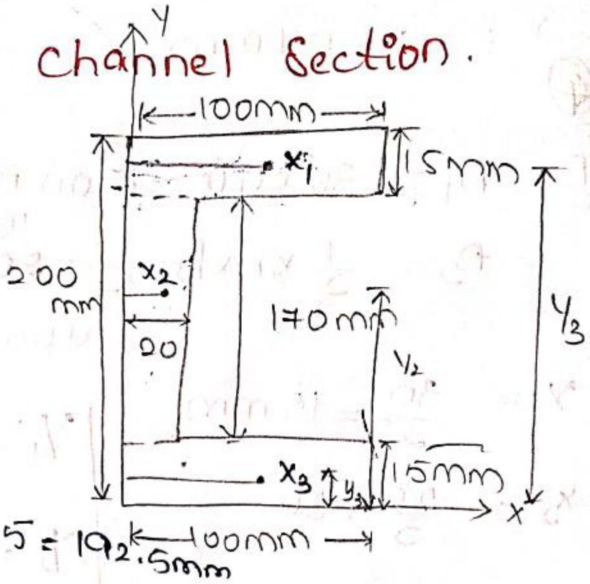
$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3} = \frac{1000\times75 + 1000\times75 + 3000\times75}{1000 + 1000 + 3000}$$

$\bar{x} = 75mm$ (from left)

$$\bar{y} = \frac{1000 \times 125 + 1000 \times 70 + 3000 \times 10}{1000 + 1000 + 3000}$$

$\bar{y} = 45mm$ (from bottom)

(5) Calculate the Centroid for Channel Section.



$A_1 = 100 \times 15$
$= 1500mm^2$

$A_2 = 170 \times 20$
$= 3400 mm^2$

$A_3 = 100 \times 15 = 1500mm^2$.

$x_1 = \frac{100}{2} = 50mm$ | $y_1 = \frac{15}{2} + 170 + 15 = 192.5mm$

$x_2 = \frac{20}{2} = 10mm$ | $y_2 = \frac{170}{2} + 15 = 100mm$

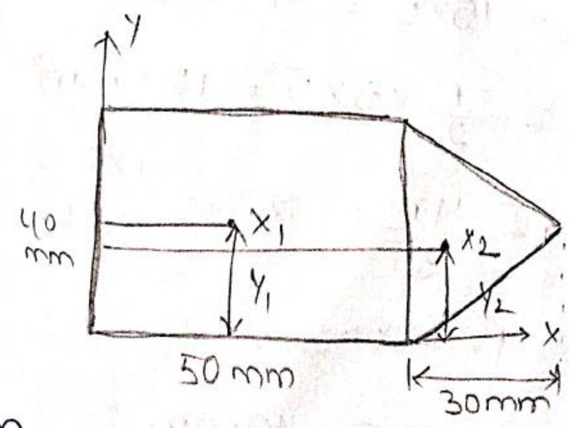
$x_3 = \frac{100}{2} = 50mm$ | $y_3 = \frac{15}{2} = 7.5mm$

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3}, \quad \bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3}$$

$$\bar{x} = \frac{1500 \times 50 + 3400 \times 10 + 1500 \times 50}{1500 + 3400 + 1500} \quad \bar{y} = \frac{1500 \times 192.5 + 3400 \times 100 + 1500 \times 7.5}{1500 + 3400 + 1500}$$

$\boxed{\bar{x} = 28.75mm}$ $\boxed{\bar{y} = 100mm}$

(6) Calculate the Centroid as shown in figure.



Area:-

$A_1 = 50 \times 40 = 2000 mm^2$

$A_2 = \frac{1}{2} \times b \times h = \frac{1}{2} \times 40 \times 30$
$= 600mm^2$

$x_1 = \frac{50}{2} = 25mm$

$x_2 = \frac{b}{3} + 50 = \frac{30}{3} + 50 = 60mm$

$y_1 = \frac{40}{2} = 20mm$

$y_2 = \frac{40}{2} = 20mm$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} \qquad \bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$\bar{x} = \frac{2000 \times 25 + 600 \times 60}{2000 + 600} \qquad \bar{y} = \frac{2000 \times 20 + 600 \times 20}{2000 + 600}$$

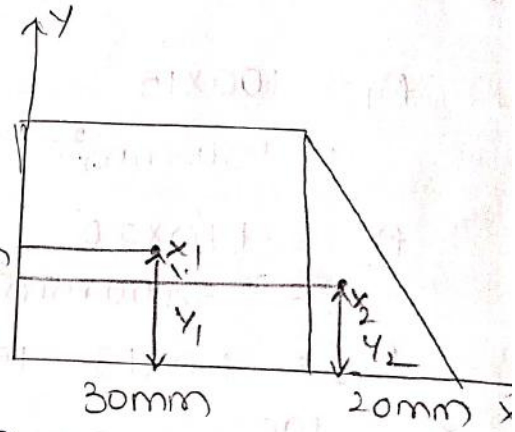
$$\bar{x} = 33.07 \text{mm} \qquad \bar{y} = 20 \text{mm}.$$

7

A $A_1 = 30 \times 40 = 1200 \text{ mm}^2$

$A_2 = \frac{1}{2} \times b \times h = \frac{1}{2} \times 20 \times 40$

$= 400 \text{mm}^2$



$x_1 = \frac{30}{2} = 15 \text{mm}$

$x_2 = \frac{20}{3} + 30$

$= 36.667 \text{ mm}$

$y_1 = \frac{40}{2} = 20 \text{mm}$

$y_2 = \frac{40}{3} = 13.33 \text{mm}$

$$\bar{x} = \frac{1200 \times 15 + 400 \times 36.667}{1200 + 400} \qquad \bar{y} = \frac{1200 \times 20 + 400 \times 13.33}{1200 + 400}$$

$$\boxed{\bar{x} = 20.41 \text{mm}} \qquad \boxed{\bar{y} = 18.33 \text{mm}}$$

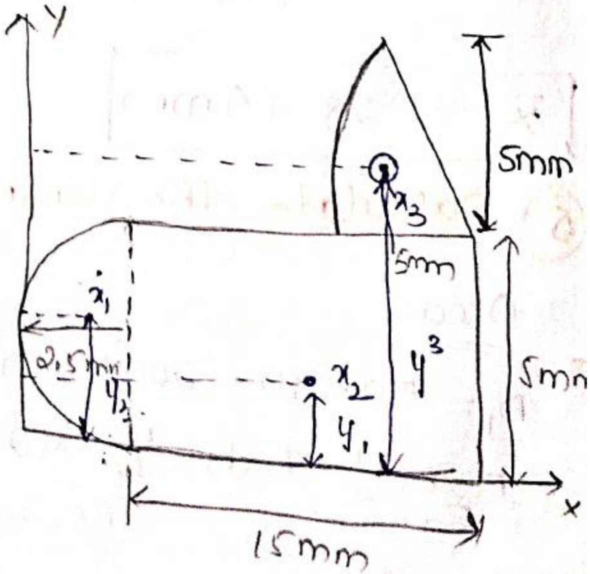
8 Calculate the centroid as shown in fig.

Areas :-

$A_1 = \frac{\pi r^2}{2} = \frac{\pi \times (2.5)^2}{2} = 9.81 \text{mm}^2$

$A_2 = 15 \times 5 = 75 \text{mm}^2$

$A_3 = \frac{1}{2} \times 5 \times 5 = 12.5 \text{mm}^2$



$x_1 = r - \frac{4r}{3\pi} = 2.5 - \frac{4 \times 2.5}{3 \times \pi}$

$x_1 = 1.438 \text{mm}$

$x_2 = \frac{15}{2} + 2.5 = 10 \text{mm}$

$x_3 = \frac{5}{2} + 10 + 2.5 = 15 \text{mm}$

$y_1 = \frac{5}{2} = 2.5mm$

$y_2 = \frac{5}{2} = 2.5mm$

$y_3 = 5 + \frac{5}{3} = 6.67mm$

$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3}$

$\bar{x} = \frac{9.81 \times 1.438 + 75 \times 10 + 12.5 \times 15}{9.81 + 75 + 12.5}$

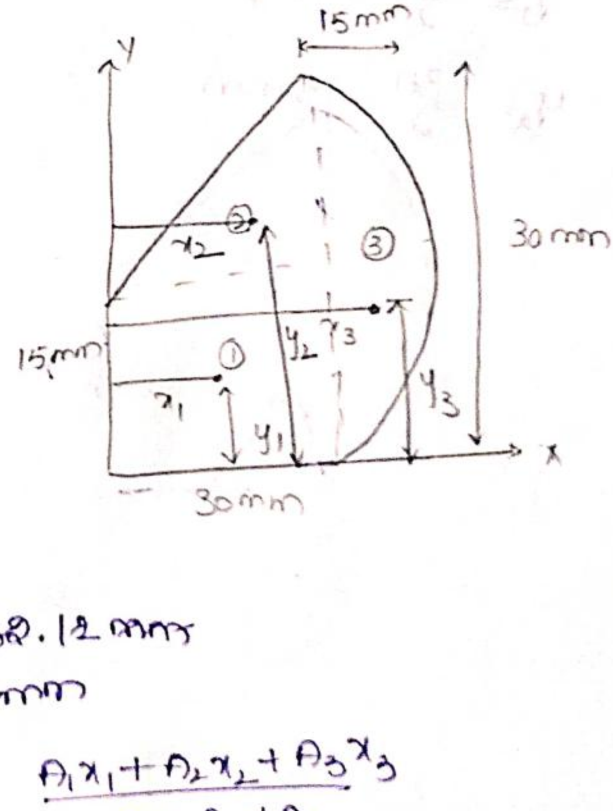
$\bar{x} = 9.779mm$ (from left)

$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3}$

$\bar{y} = \frac{9.81 \times 2.5 + 75 \times 2.5 + 12.5 \times 6.67}{9.81 + 75 + 12.5}$

$\bar{y} = 3.0356mm$ (from bottom)

(4) Calculate the centroid of the following figure.



Areas:-

$A_1 = 15 \times 30 = 450 mm^2$

$A_2 = \frac{1}{2} \times 30 \times 15$

$= 225 mm^2$

$A_3 = \frac{\pi r^2}{2} = \frac{\pi \times 15^2}{2} = 39.26 mm^2$ $353.42 mm^2$

$x_1 = \frac{30}{2} = 15mm$

$x_2 = \frac{2b}{3} = \frac{2 \times 30}{3} = 20mm$

$x_3 = \frac{4r}{3\pi} + 30 = \frac{4 \times 15}{3\pi} + 30 = 31.08\ 32.12mm$

$= 36.36mm$

$y_1 = \frac{30}{2} = 15mm$

$y_2 = \frac{2b}{3} = \frac{2 \times 30}{3} = 20mm$

$y_3 = r$

$y_1 = \frac{15}{2} = 7.5mm$

$y_2 = \frac{h}{3} + 15 = 20mm$

$y_3 = \frac{30}{2} = 15mm$

$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3}$

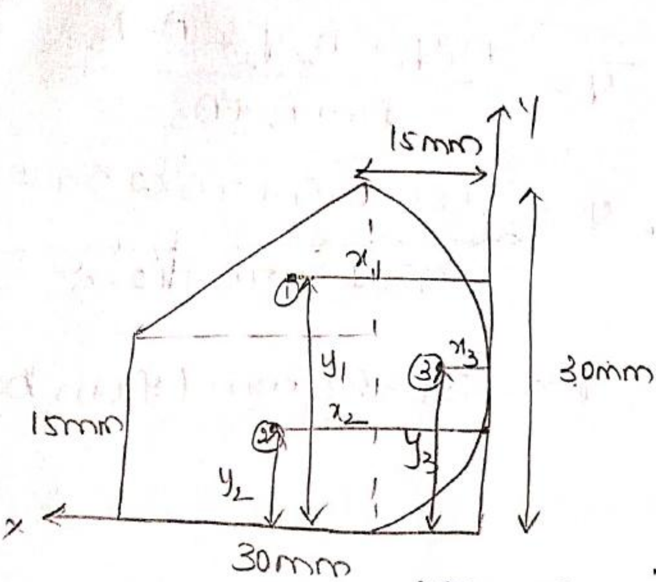
$\bar{x} = \frac{450 \times 15 + 225 \times 20 + 353.42 \times 36.36}{450 + 225 + 353.42}$

$\bar{x} = 23.43 mm$ (from left)

$\bar{y} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3}$

$\bar{y} = \frac{450 \times 7.5 + 225 \times 20 + 353.42 \times 15}{450 + 225 + 353.42}$

$\bar{y} = 12.8216mm$ (from bottom)



$$x_1 = \frac{30}{2} + 15$$

$$x_1 = 15 + 15 = 30\text{mm}$$

$$x_2 = \frac{30}{3} + 15 = \overset{25}{\cancel{10}}\text{mm}$$

$$x_3 = \frac{15}{2} + \frac{4r}{3\pi} = \frac{4(15)}{3\pi} \Rightarrow 15 - \frac{4(15)}{3\pi}$$

$$x_3 = 6.366\text{mm} \quad 8.634\text{mm}$$

$$\bar{x} = \frac{450 \times 30 + 225 \times 25 + 353.42 \times 8.633}{450 + 225 + 353.42}$$

$$y_1 = \frac{h}{3} + 15 = \frac{15}{3} + 15 = 20\text{mm}$$

$$y_2 = \frac{15}{2} = 7.5\text{mm}$$

$$y_3 = \frac{30}{2} = 15\text{mm}$$

Areas

10

$A_1 = \frac{\pi r^2}{2}$

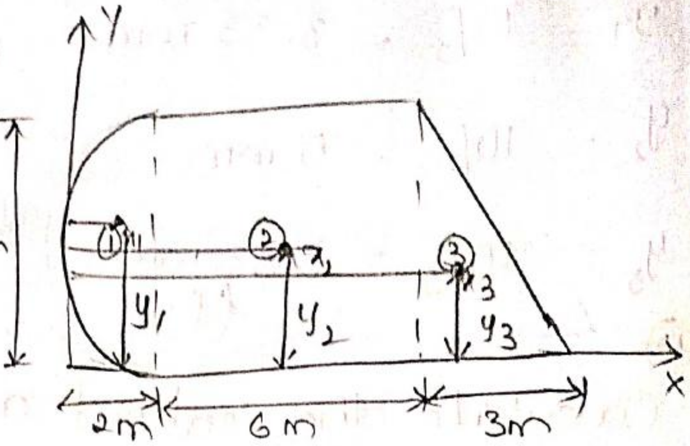
$= \frac{\pi \times 2^2}{2}$

$= 6.28\ mm^2$

$A_2 = 6 \times 4 = 24\ mm^2$

$A_3 = \frac{1}{2} \times 3 \times 4$

$= 6\ mm^2$



$x_1 = r - \frac{4r}{3\pi} = 2 - \frac{4(2)}{3\pi}$

$x_1 = 1.15\ mm$

$x_2 = \frac{6}{2} + 2 = 5\ mm$

$x_3 = \frac{3}{3} + 8 = 9\ mm$

$\bar{x} = \frac{6.28 \times 1.15 + 24 \times 5 + 6 \times 9}{6.28 + 24 + 9}$

$\bar{x} = 4.99\ mm$ (from left)

$y_1 = \frac{4}{2} = 2\ mm$

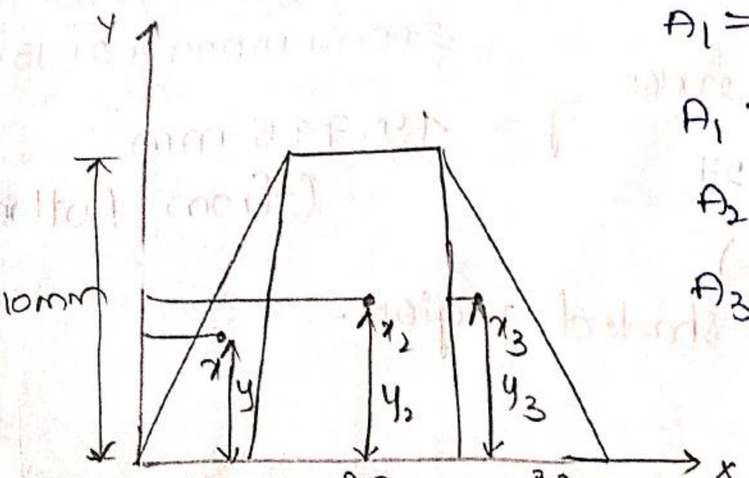
$y_2 = \frac{4}{2} = 2\ mm$

$y_3 = \frac{4}{3} = 1.33\ mm$

$\bar{y} = \frac{6.28 \times 2 + 24 \times 2 + 6 \times 1.33}{6.28 + 24 + 6}$

$\bar{y} = 1.88\ mm$ (from bottom)

11 Calculate the Centroid of the following figure



$A_1 = \frac{1}{2} \times 10 \times 10$

$A_1 = 50\ mm^2$

$A_2 = 10 \times 20 = 200\ mm^2$

$A_3 = \frac{1}{2} \times 30 \times 10$

$= 150\ mm^2$

$x_1 = \frac{2 \times 10}{3} = 6.66\ mm$

$x_2 = \frac{20}{2} + 10 = 20\ mm$

$x_3 = \frac{30}{3} = 33.33$

$\bar{x} = \frac{50 \times 6.66 + 200 \times 20 + 150 \times 10}{50 + 200 + 150}$

$\bar{x} = 19.\cancel{5825}\ mm$ $\bar{x} = 19.99\ m$

$y_1 = 10/3 = 3.33$ mm

$y_2 = 10/2 = 5$ mm

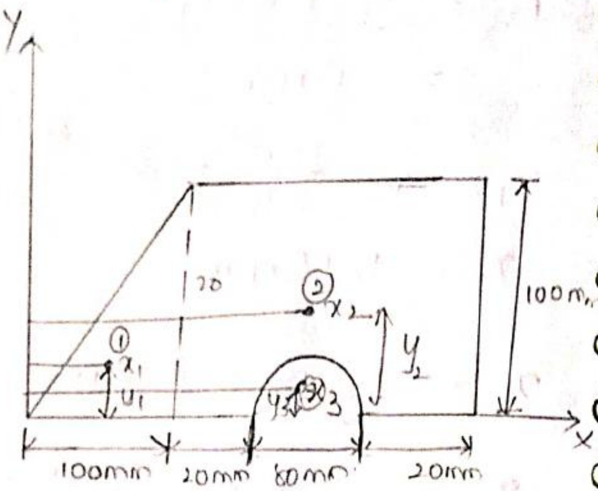
$y_3 = 10/3 = 3.33$ mm

$\bar{y} = \dfrac{50 \times 3.33 + 200 \times 5 + 50 \times 3.33}{50 + 200 + 50}$

$\bar{y} = 4.45$ mm (from bottom)

(12)

Calculate the centroid as shown in fig.



Areas:-

$A_1 = \frac{1}{2} \times 100 \times 100$

$= 5000 mm^2$

$A_2 = 120 \times 100 = 12000 mm^2$

$A_3 = \dfrac{\pi r^2}{2} = \dfrac{\pi \times 40^2}{2} = 2513.27 mm^2$

$x_1 = \dfrac{2b}{3} = \dfrac{2 \times 100}{3} = 66.66$ mm

$x_2 = \dfrac{120}{2} + 100 = 160$ mm

$x_3 = \dfrac{80}{2} + 120 = 160$ mm

$\bar{x} = \dfrac{A_1 x_1 + A_2 x_2 - A_3 x_3}{A_1 + A_2 - A_3}$

$\bar{x} = \dfrac{5000 \times 66.66 + 12000 \times 160 - 2513.27 \times 160}{5000 + 12000 + 2513.27}$

$\bar{x} = 127.78$ mm (from left)

$y_1 = \dfrac{100}{3} = 33.33$ mm

$y_2 = \dfrac{100}{2} = 50$ mm

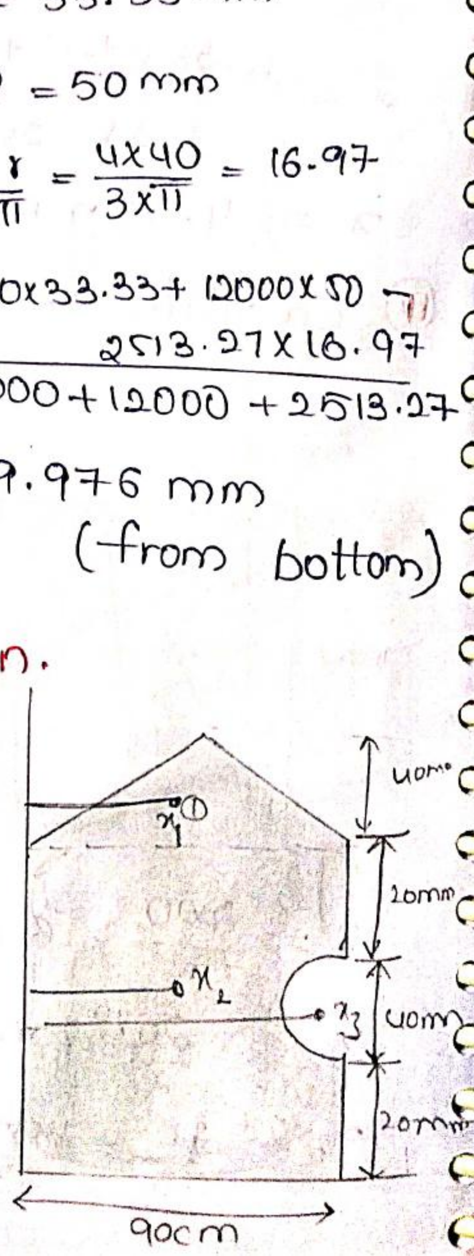
$y_3 = \dfrac{4 \times r}{3\pi} = \dfrac{4 \times 40}{3 \times \pi} = 16.97$

$\bar{y} = \dfrac{5000 \times 33.33 + 12000 \times 50 - 2513.27 \times 16.97}{5000 + 12000 + 2513.27}$

$\bar{y} = 49.976$ mm

(from bottom)

Calculate the centroid for shaded region.



Areas

$A_1 = \frac{1}{2} \times 90 \times 40$

$A_1 = 1800 mm^2$

$A_2 = 90 \times 80 = 7200 mm^2$

$A_3 = \dfrac{4 \times 20^2}{2} = 628.31 mm^2$

$x_1 = \frac{90}{2} = 45mm$

$x_2 = \frac{90}{2} = 45mm$

$x_3 = r - \frac{4r}{3\pi} + 70$

$x_3 = 20 - \frac{4 \times 20}{3 \times \pi} + 70$

$x_3 = 81.51 mm$

$\bar{x} = \frac{1800 \times 45 + 7200 \times 45 - 628.31 \times 81.51}{1800 + 7200 - 628.31}$

$\bar{x} = 42.25 mm$

(from left)

$y_1 = \frac{40}{3} + 80$

$y_1 = 93.33 mm$

$y_2 = \frac{80}{2} = 40 mm$

$y_3 = \frac{40}{2} + 20 = 40mm$

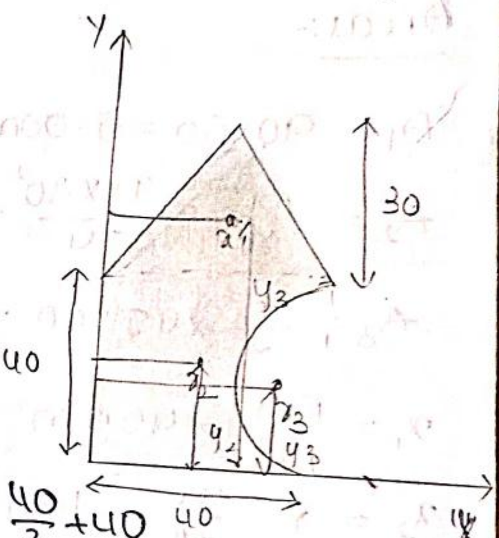
$\bar{y} = \frac{1800 \times 93.33 + 7200 \times 40 - 628.31 \times 40}{1800 + 7200 - 628.31}$

$\bar{y} = 51.46 mm$ (from bottom)

(14) $A_1 = \frac{1}{2} \times 40 \times 30 = 600mm^2$

$A_2 = 40 \times 40 = 1600mm^2$

$A_3 = \frac{\pi \times 20^2}{2} = 628.31mm^2$



$x_1 = \frac{40}{2} = 20 mm$

$x_2 = \frac{40}{2} = 20 mm$

$x_3 = \cancel{20 + 40mm}\ 20 - \frac{4 \times 20}{3 \times \pi} + 20$

$x_3 = 31.51 mm$

$\bar{x} = 15.39 mm$

$y_1 = \frac{40}{3} + 40$

$y_1 = 50mm$

$y_2 = \frac{40}{2} = 20mm$

$y_3 = \frac{40}{2} = 20mm$

$\bar{y} = \frac{600 \times 50 + 1600 \times 20 - 628.31 \times 20}{600 + 1600 - 628.31}$

$\bar{y} = 31.45 mm$

(15) Calculate the centroid of Shaded region as shown in fig.

Area:

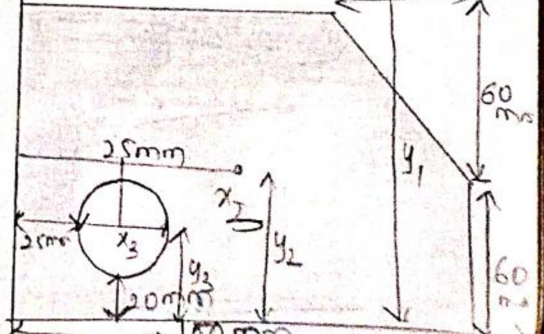
$A_1 = 120 \times 150$
$= 18000 mm^2$

$A_2 = \frac{1}{2} \times 70 \times 60$
$= 2100 mm^2$

$A_3 = \frac{\pi D^2}{4}$

$A_3 = \frac{\pi \times 25^2}{4}$

$A_3 = 490.89 mm^2$



$x_1 = \frac{150}{2} = 75mm$

$x_2 = \frac{2\times70}{3} + 80 = 126.67mm$

$x_3 = \frac{25}{2} + 25 = 37.5mm$

$y_1 = \frac{120}{2} = 60mm$

$y_2 = \frac{2\times60}{3} + 60 = 100mm$

$y_3 = \frac{25}{2} + 20$

$y_3 = 32.5\ mm$

$\bar{x} = \frac{18000\times75 - 2100\times126.67 - 490.87\times37.5}{18000 - 2100 - 490.87}$

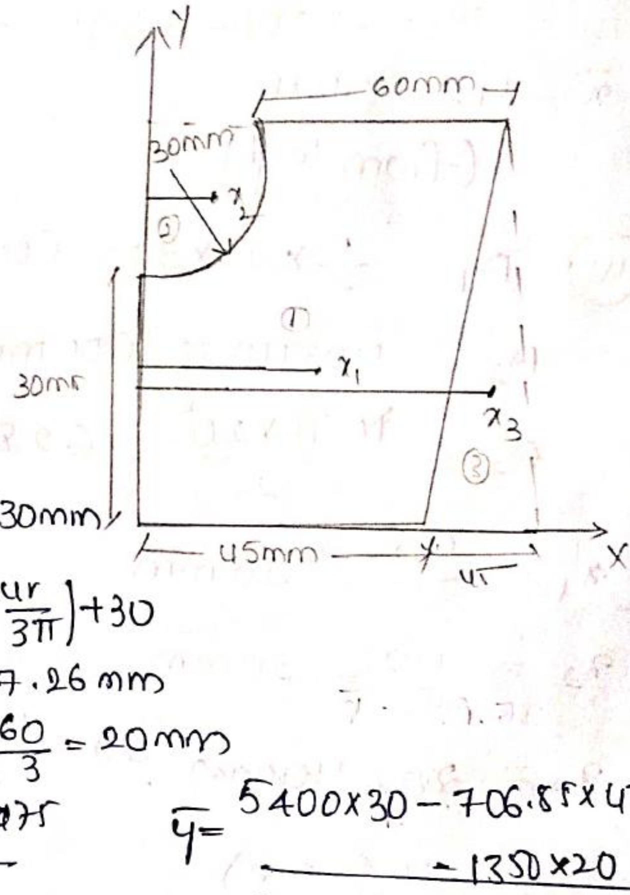
$\bar{x} = 69.15mm$ (from left)

$\bar{y} = \frac{18000\times60 - 2100\times100 - 490.87\times32.5}{18000 - 2100 - 490.87}$

$\bar{y} = 55.42mm$ (from bottom)

(16)

Area:-



$A_1 = 90\times60 = 5400mm^2$

$A_2 = \frac{\pi r^2}{4} = \frac{\pi\times30^2}{4} = 706.85mm^2$

$A_3 = \frac{1}{2}\times45\times60 = 1350\ mm^2$

$x_1 = \frac{90}{2} = 45mm$

$x_2 = r - \frac{4r}{3\pi} = 12.73\ mm$

$x_3 = \frac{2\times45}{3} + 45 = 75mm$

$y_1 = \frac{60}{2} = 30mm$

$y_2 = \left(r - \frac{4r}{3\pi}\right) + 30$

$y_2 = 47.26\ mm$

$y_3 = \frac{60}{3} = 20mm$

$\bar{x} = \frac{5400\times45 - 706.85\times12.73 - 1350\times75}{5400 - 706.85 - 1350}$

$\bar{x} = 39.70\ mm$ (from left)

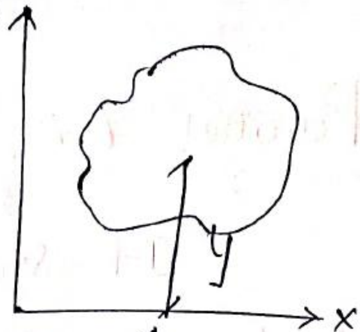
$\bar{y} = \frac{5400\times30 - 706.85\times47.26 - 1350\times20}{5400 - 706.85 - 1350}$

$\bar{y} = 30.38mm$ (from bottom)

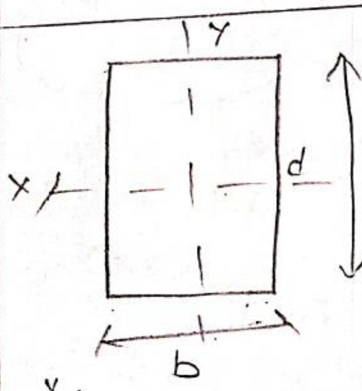
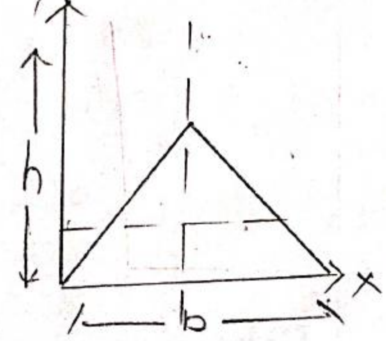
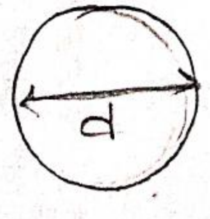
Moment of inertia

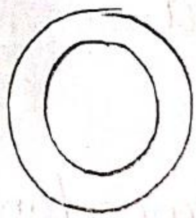
* Moment Of inertia placed very important role in bending strength and stiffness of a member.
* Moment of inertia of an area with respect to any axis line in its plane is defined as Sum of product of elemental areas and its Square of their distance to be axis.
* Moment of inertia denoted by "I".

$$\boxed{I = \Sigma_1^1 dAY^2}$$



* Units for moment of Inertia mm^4 (or) cm^4

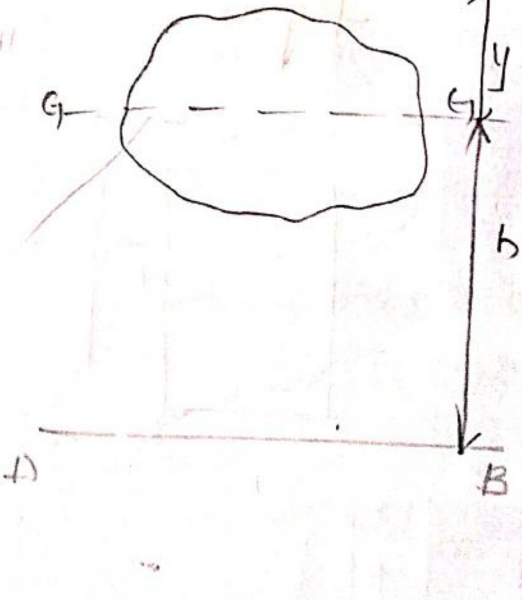
S.NO	figure	Moment of inertia x-axis	Moment of inertia Y-axis
1.		$\frac{bd^3}{12}$	$\frac{db^3}{12}$
2.		$\frac{bh^3}{36}$	$\frac{hb^3}{36}$
(3)		$\frac{\pi D^4}{64}$	$\frac{\pi D^4}{64}$

*		$\frac{\pi}{64}(D^4-d^4)$	$\frac{\pi}{64}(D^4-d^4)$
5.		$0.111r^4$	$\frac{\pi r^4}{8}$

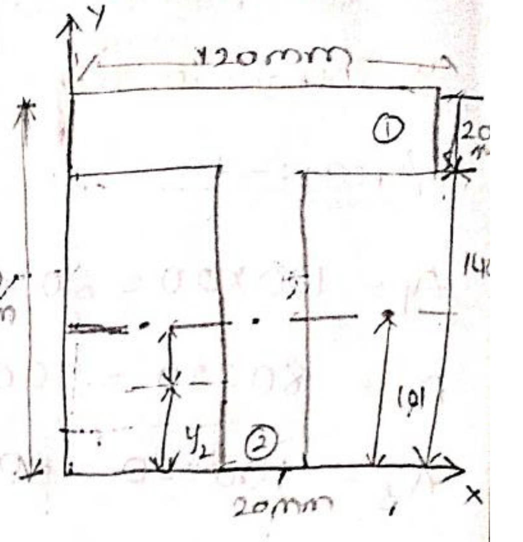
Parallel Axis Theorem:

It states that moment of inertia of an area about any axis is equals to the moment of inertia about parallel axis passing through its centroid + Area multiplied by square of the distance between the axis.

$$I_{AB} = I_G + Ah^2$$



Calculate the centroidal moment of inertia of T-section as shown in fig.



Areas

$A_1 = 120 \times 20 = 2400 mm^2$

$A_2 = 130 \times 20 = 2600 mm^2$

$x_1 = \frac{120}{2} = 60mm$ | $y_1 = \frac{20}{2} + 130 = 140mm$

$x_2 = \frac{20}{2} + 50 = 60mm$ | $y_2 = \frac{130}{2} = 65mm$

$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2}$ $\bar{y} = 101$ mm (from bottom)

$\bar{x} = 60mm$ (from left)

374166

3.741×10^8

Moment of inertia from x-axis.

$I = I_1 + I_2$

$I_1 = I_G + Ah^2$

$I_1 = \frac{bd^3}{12} + Ah^2$

$I_1 = \frac{120 \times 20^3}{12} + 2400 \times (\bar{x} - x_1)^2$ (101-140)

$I_1 = \frac{120 \times 20^3}{12} + 2400 \times (60-60)^2$

$I_1 = 80 \times 10^3 mm^4$ = 3.73×10^6

Calc (ii):-

$I_2 = I_G + A_2h^2$

$= \frac{bd^3}{12} + A_2h^2$

$= \frac{20 \times 130^3}{12} + 2600 \times (\bar{x} - x_2)^2$ (101-65)

$= 3.66 \times 10^6 mm^4$ = 7.05×10^6

$I = I_1 + I_2$

$I_{xx} = 3.74 \times 10^6 mm^4$ = 9.914×10^6

Y-axis

$I = I_1 + I_2$

$I_1 = I_G + Ah^2$

$I_1 = \frac{db^3}{12} + 2400(\bar{y} - y_1)^2$ (0)

$I_1 = \frac{20 \times 120^3}{12} + 2400(101 - 140)^2$

$I_1 = 6.53 \times 10^6 mm^4$

$= 2.88 \times 10^6 mm^4$

Calc ii):-

$I_2 = I_1 + I_2$

$I_2 = \frac{db^3}{12} + A_2h^2$

$I_2 = \frac{130 \times 20^3}{12} + 2600 \times (101-65)^2$ (0)

$I_2 = 3.45 \times 10^6 mm^4$

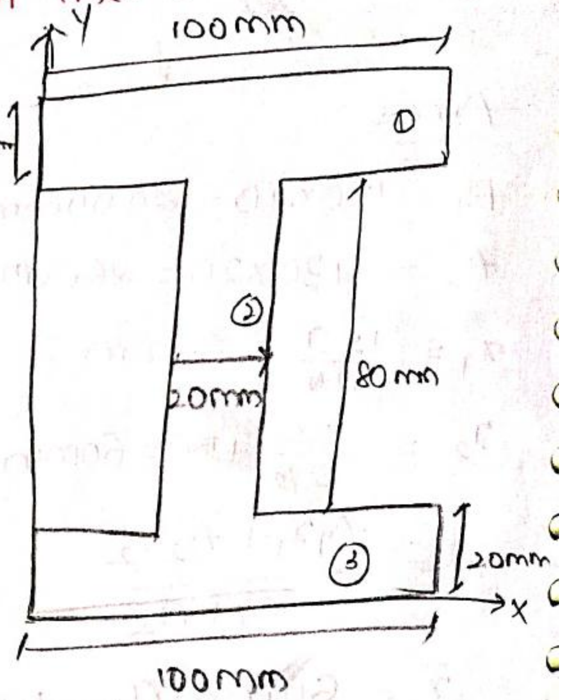
$= 86.66 \times 10^3$

$I_{yy} = I_1 + I_2$

$I_{yy} = 9.98 \times 10^6 mm^4$

$= 2.96 \times 10^6 mm^4$

Calculate the Centroidal moment of Inertia I-section as shown in fig.



Area:-

$A_1 = 100 \times 20 = 2000 mm^2$

$A_2 = 80 \times 20 = 1600 mm^2$

$A_3 = 100 \times 20 = 2000 mm^2$

$x_1 = \frac{100}{2} = 50 mm$ | $y_1 = 110 mm$

$x_2 = 40 + \frac{20}{2} = 50mm$ | $y_2 = 60mm$

$x_3 = \frac{100}{2} = 50mm$ | $y_3 = \frac{20}{2} = 10mm$

$\bar{x} = 50mm$ | $\bar{y} = 60mm$

$I = I_1 + I_2 + I_3$

$I_1 = I_G + A_1 h^2$

$I_1 = \frac{bd^3}{12} + 2000 \times (\bar{x} - x_1)^2$ $(60-110)^2$

$I_1 = \frac{100 \times 20^3}{12} + 2000 \times (50-50)^2$

$I_1 = 66.66 \times 10^3 mm^4 = 5.06 \times 10^6$

$I_2 = I_G + A_2 h^2$

$I_2 = \frac{20 \times 80^3}{12} + 1600 \times (50-10)^2$ $(60-60)$

$I_2 = 853.33 \times 10^3 mm^4$

$I_3 = I_G + A_3 h^2$

$I_3 = \frac{100 \times 20^3}{12} + 2000 \times (50-50)^2$ $(60-10)^2$

$I_3 = 66.66 \times 10^3 mm^4 = 5.06 \times 10^6$

$I = I_1 + I_2 + I_3$

$I = 66.66 \times 10^3 + 853.33 \times 10^3 + 66.66 \times 10^3 = 10.97 \times 10^6$

$\boxed{I = 986.65 \times 10^3 mm^4}$

y-axis:-

$I = I_1 + I_2 + I_3$

$I_1 = I_G + A_1 h^2$

$I_1 = \frac{db^3}{12} + 2000 \times (60-110)$ $50-50$

$I_1 = \frac{20 \times 100^3}{12} + 2000 \times (60-110)$

$I_1 = 1.56 \times 10^6 / 6.66 \times 10^6 mm^4$

1.66×10^6

$I_2 = I_G + A_2 h^2$

$I_2 = \frac{80 \times 20^3}{12} + 1600 \times (0)^2$ $(50-50)^2$

$I_2 = 53.33 \times 10^3 mm^4$

$I_3 = \frac{20 \times 100^3}{12} + 2000 \times (60-10)^2$ $(50-50)^2$

$I_3 = 6.66 \times 10^6 = 1.66 \times 10^6$

$I = I_1 + I_2 + I_3$

$I = 13.379 \times 10^6 mm^4$

Second method :-

$$I_{yy} = \left[\frac{d_1 b_1^3}{12} + A_1 h^2\right] + \left(\frac{d_2 b_2^3}{12} + A_2 h^2\right) + \left[\frac{d_3 b_3^3}{12} + A_3 h^3\right]$$

$\bar{y} - y_1 = 110 - 60 = 50mm$

$\bar{y} - y_2 = 60 - 60 = 0mm$

$\bar{y} - y_3 = 60 - 10 = 50mm$

$I_{yy} = 2\left[6.66\times10^6\right] + \left(53.33\times10^3\right)$

$= 13.37\times10^6\ mm^4$

(A) $A_1 = 150\times15 = 2250\ mm^2$

$A_2 = (120-15)\times20 = 2100\ mm^2$

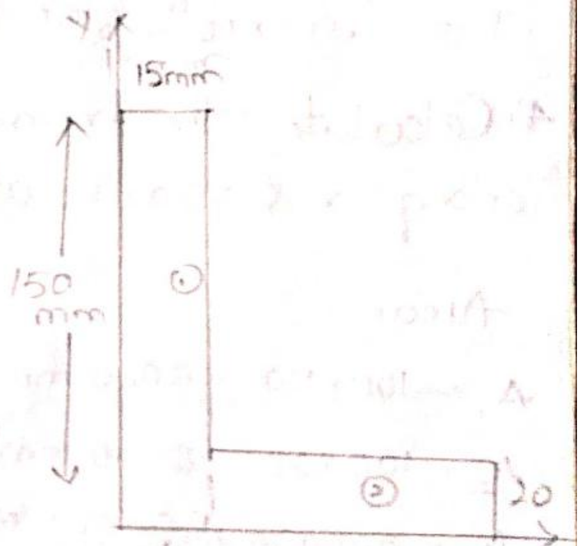
$x_1 = \frac{15}{2} = 7.5mm$

$x_2 = \frac{105}{2} + 15 = 67.5mm$

$y_1 = \frac{150}{2} = 75mm$

$y_2 = \frac{20}{2} = 10mm$

$\bar{x}_1 = 36.46mm$, $\bar{y} = 43.62\ mm$.



x-axis

$I = I_1 + I_2$

$I_1 = I_G + Ah^2$

$I_1 = \frac{bd^3}{12} + 2250(\bar{y} - y_1)^2$

$I_1 = \frac{150\times15^3}{12} + 2250(43.62 - 75)^2$

$I_1 = \frac{15\times150^3}{12} + 2250(43.62 - 75)^2$

$I_1 = 6.4343\times10^6$

$I_2 = I_G + A_2 h^2$

$I_2 = \frac{20\times120^3}{12} + 2100(43.62-10)^2$

$I_2 = 2.44\times10^6$

$I = I_1 + I_2$

$I = 8.87\times10^6\ mm^4$

y-plane

$$I_1 = \frac{db^3}{12} + 2250 \times (\bar{x} - x_1)^2$$

$$I_1 = \frac{150 \times 15^3}{12} + 2250 \times (36.46 - 7.5)^2$$

$$I_1 = 1.92 \times 10^6 \text{ mm}^4$$

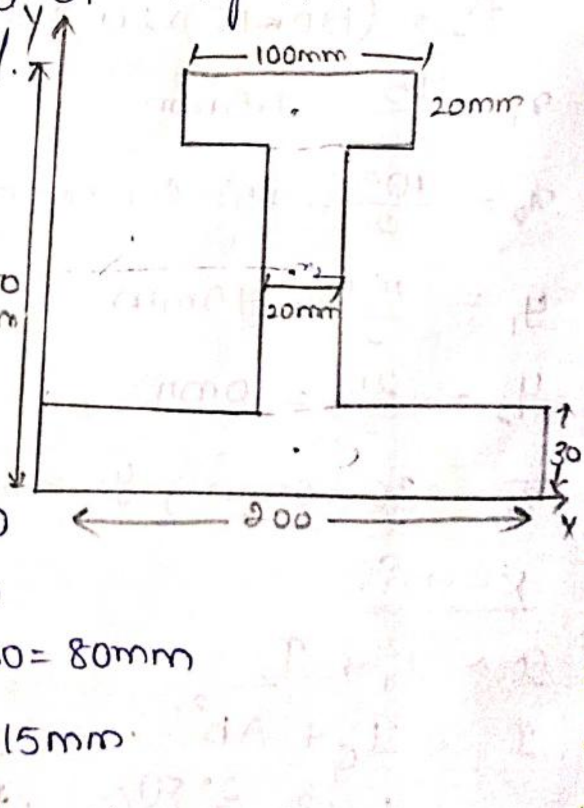
$$I_2 = \frac{20 \times 105^3}{12} + 2100 \times (36.46 - 67.5)^2$$

$$I_2 = 3.95 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2$$

$$I = 1.92 \times 10^6 + 3.95 \times 10^6 = 5.87 \times 10^6 \text{ mm}^4$$

4) Calculate the moment of inertia of unsymmetrical I-section along x & y axis at the centroid.



Area:

$A_1 = 100 \times 20 = 2000 \text{ mm}^2$

$A_2 = 100 \times 20 = 2000 \text{ mm}^2$

$A_3 = 200 \times 30 = 6000 \text{ mm}^2$

$x_1 = \frac{100}{2} + 50 = 100 \text{ mm}$ $\quad y_1 = \frac{20}{2} + 130 = 140 \text{ mm}$

$x_2 = \frac{20}{2} + 90 = 100 \text{ mm}$ $\quad y_2 = \frac{100}{2} + 30 = 80 \text{ mm}$

$x_3 = \frac{200}{20} = 100 \text{ mm}$ $\quad y_3 = \frac{30}{2} = 15 \text{ mm}$

$\bar{x} = 100 \text{ mm}$ $\quad \bar{y} = \cancel{103}\ 53 \text{ mm}$

$$I = I_1 + I_2 + I_3$$

$$I_1 = \frac{bd^3}{12} + 2000 \times (\bar{y} - y_1)^2$$

$$I_1 = \frac{100 \times 20^3}{12} + 2000 \times (53 - 140)^2$$

$$I_1 = 15.20 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{20 \times 100^3}{12} + 2000(53 - 80)^2$$

$$= 3.12 \times 10^6 \text{ mm}^4$$

$$I_3 = 9.14 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3$$

$$I = 27.34 \times 10^6$$

Y - axis

$I = I_1 + I_2 + I_3$

$I_1 = I_G + Ah^2$

$I_1 = \frac{db^3}{12} + 2000 \times (\bar{x} - x_1)^2$

$I_1 = \frac{20 \times 100^3}{12} + 2000 \times (100 - 100)^2$

$= 1.66 \times 10^6 \, mm^4$

$I_2 = \frac{db^3}{12} + 2000 \times (\bar{x} - x_2)^2$

$I_2 = \frac{100 \times 20^3}{12} + 2000 (100 - 100)^2$

$= 66.66 \times 10^3 mm^4$

$I_3 = \frac{30 \times 200^3}{12} + 6000 (100 - 100)^2$

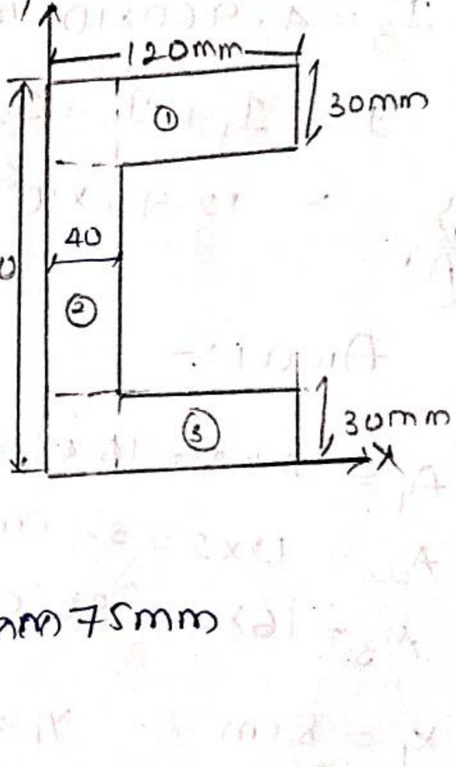
$I_3 = 20 \times 10^6 \, mm^4$

$I_1 + I_2 + I_3$

$I = 1.66 \times 10^6 + 66.66 \times 10^3 + 20 \times 10^6$

$I = 21.72 \times 10^6 \, mm^4$

(5) Area:-



$A_1 = 120 \times 30 = 3600 \, mm^2$

$A_2 = 90 \times 40 = 3600 \, mm^2$

$A_3 = 120 \times 30 = 3600 \, mm^2$

$x_1 = \frac{120}{2} = 60 \, mm$

$x_2 = \frac{40}{2} = 20 \, mm$

$x_3 = \frac{120}{2} = 60 \, mm$

$\bar{x} = 46.6\cancel{7} \, mm$

$y_1 = \frac{30}{2} + 90 + 30$

$y_1 = 135 mm$

$y_2 = \frac{90}{2} + 30 = \cancel{60mm}\ 75 mm$

$y_3 = \frac{30}{2} = 15 mm$

$\bar{y} = \cancel{45}\ 75$

$I = I_1 + I_2 + I_3$

$I_1 = \frac{bd^3}{12} + 3600 \times (75 - 135)^2$

$= \frac{120 \times 30^3}{12} + 3600 \times (75 - 135)^2$

$= 13.23 \times 10^6 \, mm^4$

$I_2 = \frac{40 \times 90^3}{12}$

$I_2 = 2.43 \times 10^6$

$I_3 = \frac{120 \times 30^3}{12} + 3600 (75 - 15)^2$

$I_3 = 13.23 \times 10^6$

$\boxed{I = 28.89 \, mm^4}$

$$I_1 = \frac{db^3}{12} + A_1 h_1^2$$

$$= \frac{30 \times 120^3}{12} + 3600(46.67 - 60)^2$$

$$= 4.960 \times 10^6 \, mm^4$$

$$I_2 = \frac{db^3}{12} + 3600(46.67 - 20)^2$$

$$= \frac{90 \times 40^3}{12} + 3600(46.67 - 20)^2$$

$$I_2 = 3.03 \times 10^6 \, mm^4$$

$$I_3 = 4.960 \times 10^6 \, mm^4$$

$$I = I_1 + I_2 + I_3$$

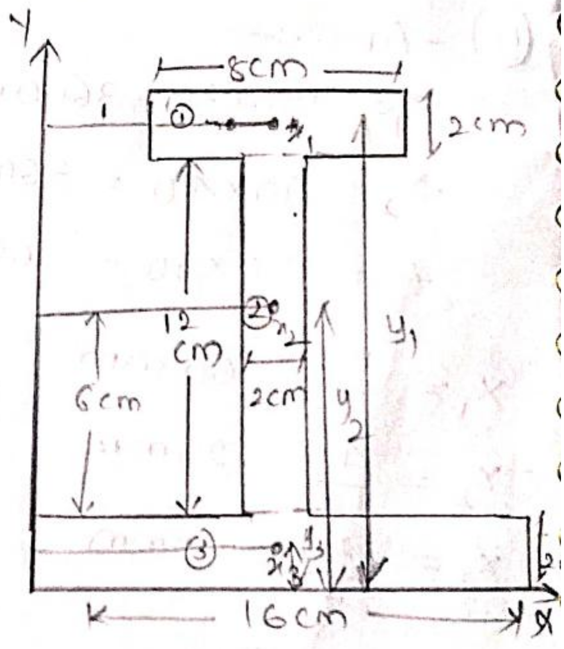
$$= 12.95 \times 10^6$$

Short cut method :-

$$= \frac{120 \times 150^3}{12} - \frac{80 \times 90^3}{12}$$

$$= 28.895 \times 10^6 \, mm^4$$

6) (A)



Area :-

$A_1 = 8 \times 2 = 16 \, cm^2$

$A_2 = 12 \times 2 = 24 \, cm^2$

$A_3 = 16 \times 2 = 32 \, cm^2$

$x_1 = 8 \, cm$ $\quad y_1 = \frac{2}{2} + 14 = 15 \, cm$

$x_2 = 8 \, cm$ $\quad y_2 = \frac{12}{2} + 2 = 8 \, cm$

$x_3 = 8 \, cm$ $\quad y_3 = \frac{2}{2} = 1 \, cm$

$\bar{x} = 8 \, cm$ $\quad \bar{y} = 6.44 \, cm$

$$I = I_1 + I_2 + I_3$$

$$I_1 = \frac{bd^3}{12} + 16 \times (\bar{y} - y_1)^2$$

$$I_1 = \frac{8 \times 2^3}{12} + 16 \times (6.44 - 15)^2$$

$$I_1 = 1.17 \times 10^3 \, cm^4$$

$$I_2 = \frac{bd^3}{12} + 24 \times (\bar{y} - y_2)^2$$

$$I_2 = \frac{2 \times 12^3}{12} + 24 \times (6.44 - 8)^2$$

$$I_2 = 346.40 \times 10^0 \, cm^4$$

$$I_3 = \frac{bd^3}{12} + 32 \times (\bar{y} - y_3)^2$$

$$I_3 = \frac{16 \times 2^3}{12} + 32 \times (6.44 - 1)^2$$

$$I_3 = 957.66 \times 10^0 \, cm^4$$

$$\boxed{I = 2.47 \times 10^3} \, mm^4$$

Y–Y (or) Y–axis :–

$I = I_1 + I_2 + I_3$

$I_1 = \frac{db^3}{12} + 16 \times (\bar{x} - x_1)^2$

$I_1 = \frac{2 \times 8^3}{12} + 16 \times (8-8)^2$

$I_1 = 85.33 \text{ cm}^4$

$I_3 = \frac{db^3}{12} + 32 \times (8-8)^2$

$I_3 = \frac{2 \times 16^3}{12} + 32(0)^2$

$= 682.66 \text{ cm}^4$

$I_2 = \frac{db^3}{12} * 24 \times (\bar{x} - x_2)^2$

$I_2 = \frac{12 \times 2^3}{12} + 24 \times (8-8)^2$

$I_2 = 8 \text{ cm}^4$

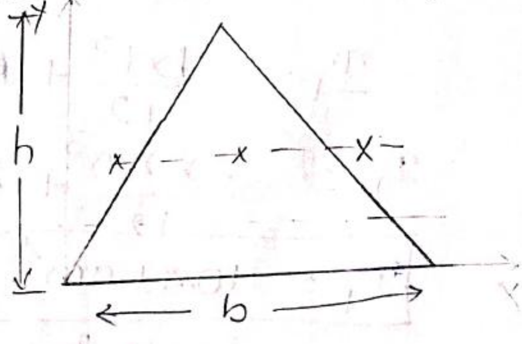
$I = I_1 + I_2 + I_3$

$I = 85.33 + 8 + 682.66$

$I = 775.99$

$\boxed{I = 775.99 \text{ cm}^4}$

(7) Determine the moment of inertia of triangle area about its base.



$I_{AB} = I_G + Ah^2$

$= \frac{bh^3}{36} + \left(\frac{1}{2} \times b \times h\right) \times \left(\frac{h}{3}\right)^2$

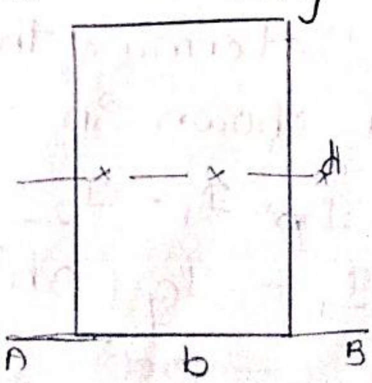
$= \frac{bh^3}{36} + \frac{bh^3}{18}$

$= \frac{bh^3 + 2bh^3}{36}$

$= \frac{bh^3}{12}$

18 | 36, 18
2 | 2, 1
1, 1

(8) Determine the moment of inertia of rectangle about its base :



$I_{AB} = I_G + Ah^2$

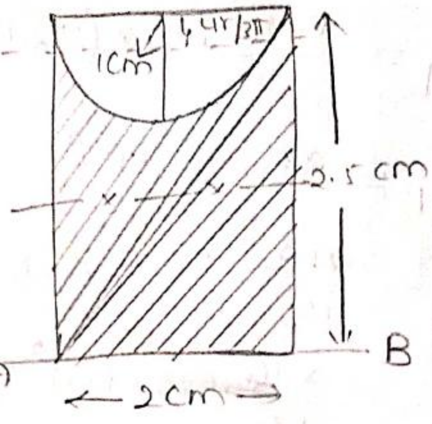
$= \frac{bd^3}{12} + b \times d \times \left(\frac{d}{2}\right)^2$

$= \frac{bd^3}{12} + \frac{bd^3}{4}$

$= \frac{bd^3 + 3bd^3}{12} = \frac{bd^3}{3}$

$= \frac{bd^3}{3}$

Determine the moment of Interia of Area shown by the section line about x-axis coincide with the base.



$I_{AB} = I_G + Ah^2.$

$= I_G +$

$I_{xx} = I_1 + I_2$

$I_1 = I_G + Ah^2$

$I_1 = \frac{bd^3}{12} + A \times h^2$

$I_1 = \frac{bd^3}{12} + b \times d \times h_1^2$

$I_1 = \frac{2 \times 2.5^3}{12} + 2 \times 2.5 \times 1.56$

$\boxed{I_1 = 10.41 \text{ cm}^4}$

$I_2 = I_G + Ah_2^2$

$I_2 = 0.111 r^4 + \frac{\pi r^2}{2} \times h_2^2$

$I_2 = 0.111(1)^4 + \frac{\pi (1)^2}{2} \times \left(2.5 - \frac{4r}{3\pi}\right)^2$

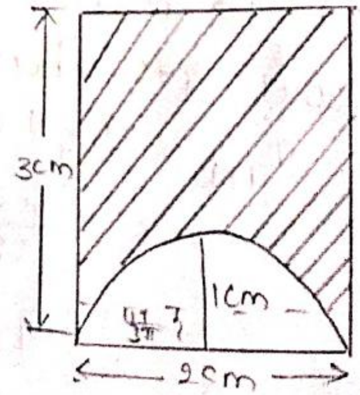
$\boxed{I_2 = 6.8087 \text{ cm}^4}$

$I_{xx} = I_1 - I_2$

$I_{xx} = 10.41 - 6.8087$

$\boxed{I_{xx} = 3.654 \text{ cm}^4}$

Determine the moment of inertia abolut its base as shown in figure.



$I_{xx} = I_1 - I_2$

$I_1 = I_G + Ah^2$

$= \frac{bd^3}{12} + A \times h^2$

$= \frac{bd^3}{12} + b \times d \times h_1^2$

$= \frac{2 \times 3^3}{12} + 2 \times 3 \times \left(\frac{3}{2}\right)^2$

$$\boxed{I_1 = 18\ cm^4}$$

$$I_2 = I_G + Ah^2$$

$$= 0.111\,r^4 + \frac{\pi r^2}{2} \times h_2^2$$

$$= 0.111(1)^4 + \frac{\pi \times (1)^2}{2} \times \left(\frac{4r}{3\pi}\right)^2$$

$$= 0.111 + \frac{\pi}{2} \times \left(\frac{4}{3\pi}\right)^2$$

$$\boxed{I_2 = 0.393 cm^4}$$

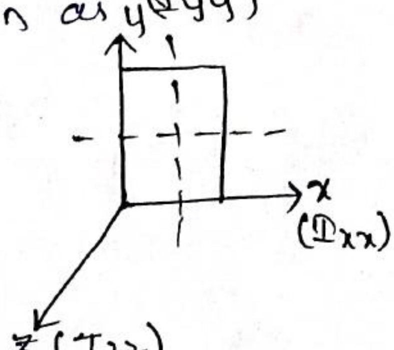
$$I_{xx} = I_1 - I_2 = 18 - 0.393$$

$$\boxed{= 17.607 cm^4}$$

Perpendicular Axis Theorem :-

Moment of Inertia of 2-dimensional object about an axis passing perpendicular from it is equal to sum of moment of Inertia of the object about two mutually perpendicular axis lying in the plane of the object.

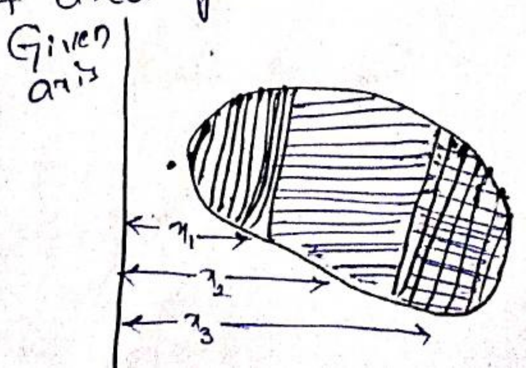
Perpendicular axis theorem can be written as



$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$

Radius of gyration:

Radius of gyration of a body about an axis is a distance such that it's square multiply the area it gives moment of Inertia of area given axis.



$$I = r_1^2 a_1 + r_2^2 a_2 + r_3^2 a_3$$

$$I = r^2 a$$

$$r^2 = I/a \Rightarrow r = \sqrt{\frac{I}{a}}$$

Unit - III :- Simple stresses and strains

Introduction:-

Stress:- The force of resistance per unit area offered by a body against deformation is known as "stress"

$$\sigma = \frac{P}{A}$$

Where, σ = stress

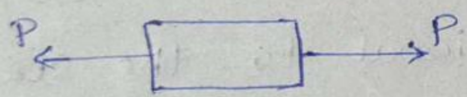
P = load

A = Area

Strain:- The ratio of change in the dimension to the original dimension is known as strain.

Types of stresses

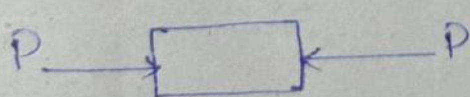
1) Tensile stress (σ_t)



$\sigma = P/A$

The stress induced in a body when subjected to equal and opposite pullings as shown in fig.

2) Compressive stress (σ_c):-



$\sigma_c = P/A$

The stress induced in a body when subjected to equal and opposite pushings as shown in figure.

Types of strains

1) Tensile strain (e_t)

The ratio of increase in the length to the original length is known a tensile strain

$$e = \frac{dL}{L}$$

$dL \rightarrow$ Increase in length

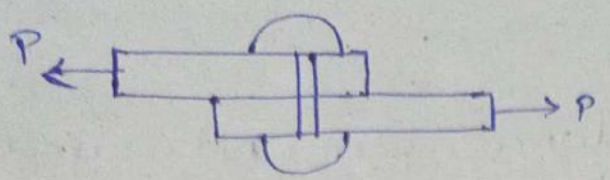
2) Compressive strain (e_c)

The ratio of decrease in length to the original length is known as compressive strain.

$$e = \frac{dL}{L}$$

$dL \rightarrow$ decrease in length.

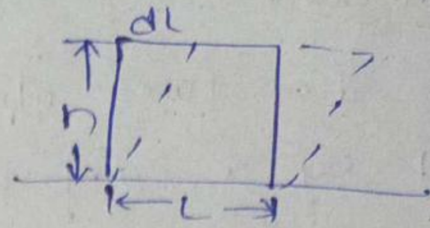
3) Shear Stress (τ)



The stress induced in a body when subjected to equal and opposite forces which are acting tangentially across the resisting section

$$\tau = \frac{\text{Shear Resistance}}{\text{Shear Area}}$$

3) Shear Strain (φ)



The corresponding strain Related to the shear stress is known as shear strain.

$$\phi = \frac{dL}{h}$$

dL = transversal displacement

* Hooke's law:-

When a material is loaded within elastic limit the stress is proportional to the strain.

$$\sigma \propto e$$

* Young's Modulus (or) Modulus of Elasticity:-

The ratio of tensile or compressive stresses ~~to the~~ corresponding to the strain is known as young's modulus.

$$\boxed{E = \frac{\sigma}{e}}$$

Shear Modulus:- (or) Modulus of Rigidity:-

The ratio of shear stress to shear strain is known as shear Modulus.

$$\boxed{G = \frac{\tau}{\phi}}$$

Longitudinal Strain:-

When a body is subjected to a axial tensile load the ratio of axial deformation to the original length of the body is known as longitudinal (or) linear strain $= \frac{\delta l}{L}$ → length

Lateral strain:-

The strain at right angles to the direction of applied load is known as lateral strain.

lateral strain $= \frac{\delta b}{b}$ (or) $\frac{\delta d}{d}$

b ↓ Width

d ↓ depth

poison's Ratio:- (μ)

$$\mu = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

factor of safety:-

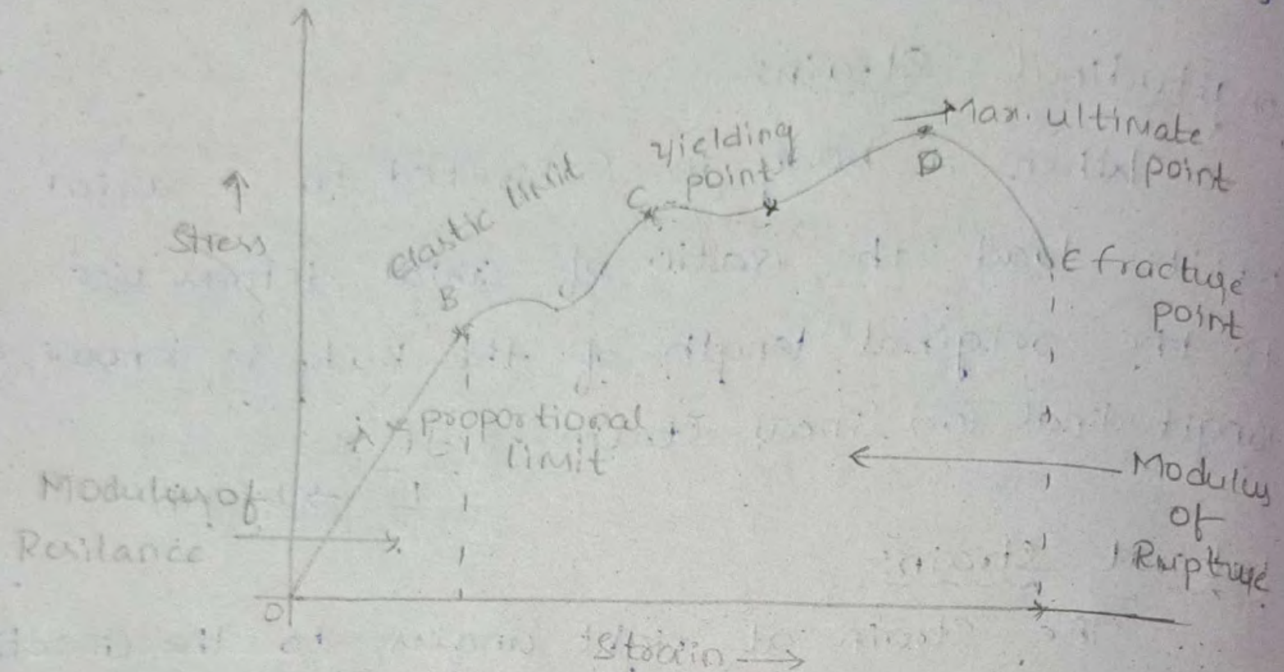
$$\text{factor of safety} = \frac{\text{Ultimate stress}}{\text{Working stress}}$$

Stress - Strain Curve diagram for Mild Steel (fe 415):

fe250 = low carbon steel

fe415 = Mild steel

fe500 = HYSD (High yield strength Deformation)



Stress-Strain Curve

→ proportional limit:-

OA - proportional limit - $\sigma \propto e$

Hooke's law

→ Elastic limit:-

AB - Elastic limit — limiting value reaching elastic point.

→ Yield point:-

C D - yield point - Extension

→ Ultimate point:-

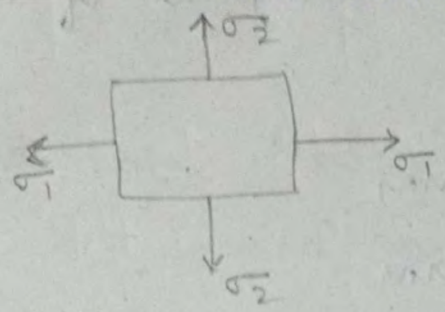
D - Ultimate point - strength of material

→ fracture point:-

E - fracture point - Breaking point.

Relationship between stress and strain:-

Case-I :- Two dimensional



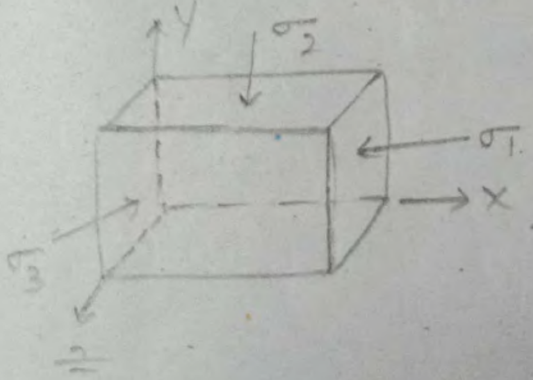
1) Consider a two dimensional plane body as shown in figure.

2) The plane subjected to 2 Mutually perpendicular forces along with the stress σ_1, σ_2

3) The strain e_1 and e_2 will produced by the stress σ_1 and σ_2 in x and y directions.

$$\left.\begin{aligned} e_1 &= \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} \\ e_2 &= \frac{\sigma_2}{E} - \frac{\mu\sigma_1}{E} \end{aligned}\right\} \text{Total strains}$$

Case-II :- Three-dimensional



$$e_1 = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E} - \mu\frac{\sigma_3}{E}$$

$$e_2 = \frac{\sigma_2}{E} - \frac{\mu\sigma_3}{E} - \frac{\mu\sigma_1}{E}$$

$$e_3 = \frac{\sigma_3}{E} - \frac{\mu\sigma_1}{E} - \frac{\mu\sigma_2}{E}$$

problems:-

* A Rod 150cm long and of diameter 2cm is Subjected to an axial pull of 20KN. If the Modulus of elasticity of the material $E = 2\times10^5$ N/mm² find the. 1) stress 2) strain 3) elongation of the rod.

Sol:- Given data,

length $= L = 150$ cm $= 1500$ mm

diameter $(d) = 2$ cm $= 20$ mm

Axial pull $(P) = 20$ KN $= 20\times10^3$ N

Young's Modulus $(E) = 2\times10^5$ N/mm²

1) $\text{Stress} = \dfrac{\text{Load}}{\text{Area}}$

$$\text{Area} = \frac{\pi(d)^2}{4}$$

$$= \frac{\pi(20)^2}{4}$$

$$A = 314.159 \text{ mm}^2$$

$$\text{Stress } (\sigma) = \frac{20\times10^3}{314.159}$$

$$\boxed{\text{Stress } (\sigma) = 63.66 \text{ N/mm}^2}$$

2) Strain

$$e = \frac{\sigma}{E}$$

$$= \frac{63.66}{2\times10^5}$$

$$\boxed{e = 0.000318}$$

3) $\delta L = e \times L$ $\qquad e = \frac{\delta L}{L}$

$= 0.000318 \times 150$

$\boxed{\delta L = 0.0477 \text{ cm}}$

elongation of the rod $(\delta l) = 0.0477$ cm

* find the minimum diameter of steel wire which is used to ~~ro~~ rise a load of 4000 N If the stress in the rod is not to exceed 95 MN/mm².

$K = 10^3$

Sol:- Given data,

$M = 10^6$

$G = 10^9$

$T = 10^{12}$

$P = 4000 \text{ N}$

$\sigma = 95 \text{ MN/m}^2$

$= \frac{95 \times 10^6 \text{ N}}{(10^3)^2 \text{ mm}^2}$

$\sigma = 95 \text{ N/mm}^2$

* $\text{Stress} = \frac{\text{load}}{\text{Area}}$

$\text{Area} = \frac{\text{load}}{\text{Stress}}$

$\frac{\pi (D)^2}{4} = \frac{4000}{95}$

$D^2 = \frac{4000}{95} \times \frac{4}{\pi}$

$D = \sqrt{\frac{4000}{95} \times \frac{4}{\pi}} \Rightarrow \boxed{D = 7.322 \text{ mm}}$

* find the Young's Modulus of a Brass Rod of diameter 25 MM and its length 250 MM which is subjected to a tensile load of 50KN. When the extension of the rod is equal to 0.3 MM

Sol:- Given data,

$d = 25\ MM$

$l = 250\ MM$

$P = 50\ KN$

$\delta l = 0.3\ MM$

1) $Stress = \frac{load}{Area}$

$$= \frac{50 \times 10^3 N}{\frac{\pi (25)^2}{4}}$$

$$\boxed{\sigma = 101.859}$$

2) $Strain = \frac{\delta l}{l}$

$$= \frac{0.3}{250}$$

$$\boxed{e = 0.0012}$$

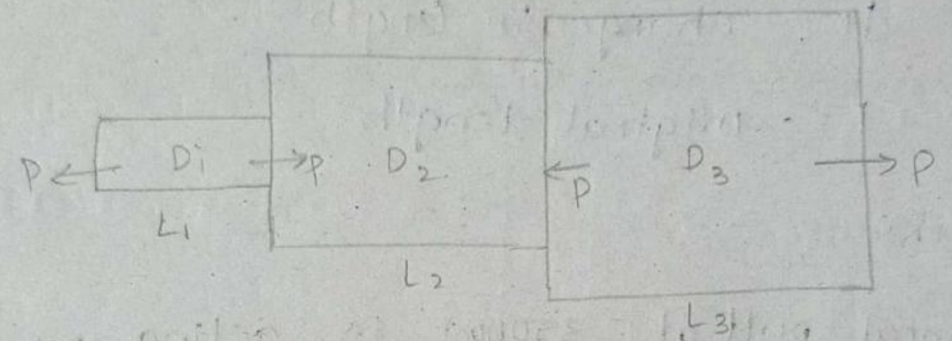
3) $E = \frac{\sigma}{e}$

$$= \frac{101.859}{0.0012}$$

$$\boxed{E = 84882.5\ N/MM^2}$$

Bars of Varying Sections:-

* Consider a bar of different length and different diameters as shown in figure.
* This bar is subjected to an axial load P
* Through each section is subjected to same axial load the stresses and strains will be different.



Total Extension:-

$$dL = P\left[\frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2} + \frac{L_3}{A_3 E_3}\right]$$

$$E = \frac{\sigma}{e}$$

$$E = \frac{P/A}{dL/L}$$

$$E = \frac{P}{A} \times \frac{L}{dL}$$

$$dL = \frac{PL}{AE}$$

$$dL = dL_1 + dL_2 + dL_3$$

① Bar $E_1 = \frac{\sigma_1}{e_1}$

$$= \frac{\frac{P}{A_1}}{\frac{dL_1}{L_1}}$$

$$= \frac{P}{A_1} \times \frac{L_1}{dL_1}$$

$$\boxed{dL_1 = \frac{PL_1}{AE_1}}$$

Similarly for both Remaining 2 bars

Where,

$E =$ Young's Modulus

$\sigma =$ Stress

$e =$ Strain

$P =$ Load

$A_1 =$ Area of cross-section

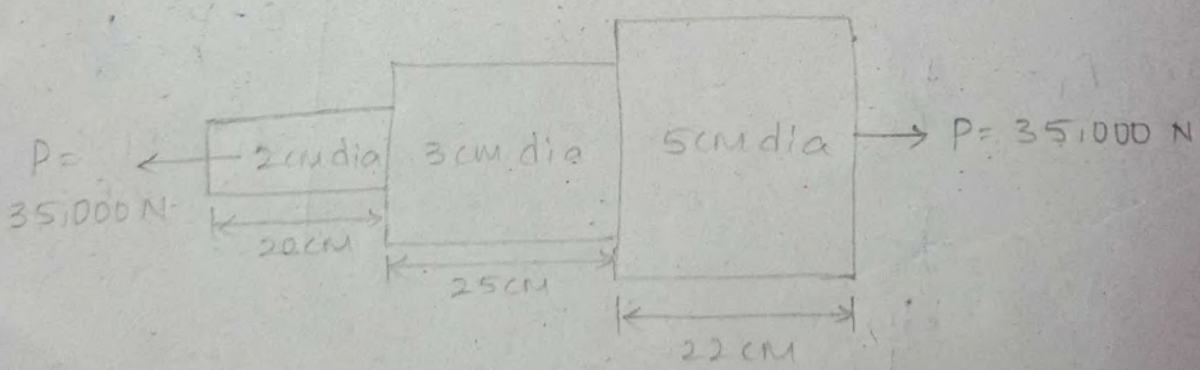
$dL =$ change in length

$L =$ original length

problems:-

* An axial pull of 35000N is acting on a bar consisting of 3 length as shown in figure If the Young's modulus $E = 2.1 \times 10^5 \, N/mm^2$ Determine the 1) stresses in each section

2) Total extension of the bar.



Given data, $P = 35,000 \, N$

$d_1 = 2 \, cm = 20 \, mm$

$d_2 = 3 \, cm = 30 \, mm$

$d_3 = 5 \, cm = 50 \, mm$

$L_1 = 20\,cm = 200\,mm$

$L_2 = 25\,cm = 250\,mm$

$L_3 = 22\,cm = 220\,mm$

Young's Modulus $E = 2.1 \times 10^5\ N/mm^2$

$$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi (20)^2}{4} = 314.159\ mm^2$$

$$A_2 = \frac{\pi (d_2)^2}{4} = \frac{\pi (30)^2}{4} = 706.86\ mm^2$$

$$A_3 = \frac{\pi (d_3)^2}{4} = \frac{\pi (50)^2}{4} = 1963.495\ mm^2$$

1) Stresses

$$\sigma_1 = \frac{P}{A_1} = \frac{35000}{314.159} = 111.41\ N/mm^2$$

$$\sigma_2 = \frac{P}{A_2} = \frac{35000}{706.86} = 49.515\ N/mm^2$$

$$\sigma_3 = \frac{P}{A_3} = \frac{35000}{1963.495} = 17.82\ N/mm^2$$

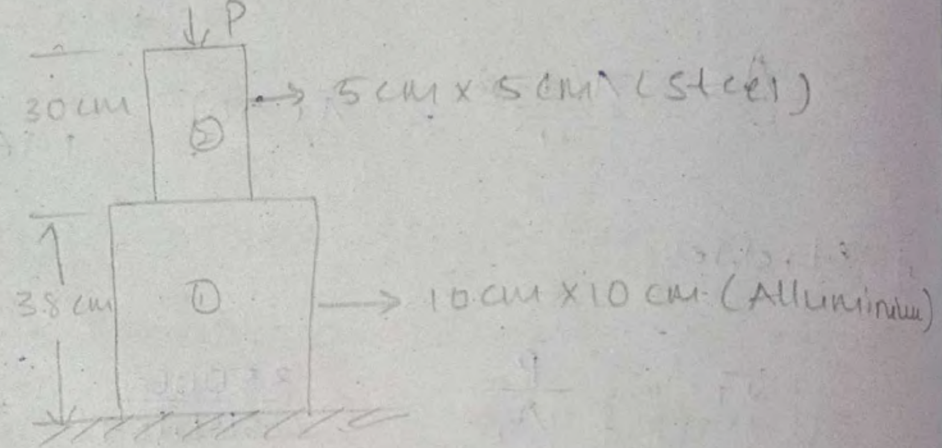
2) Total Extension of the bar

$$dL = P\left[\frac{L_1}{A_1 E_1} + \frac{L_2}{A_1 E_2} + \frac{L_3}{A_3 E_3}\right]$$

$$= \frac{35000}{2.1 \times 10^5}\left[\frac{200}{314.159} + \frac{250}{706.86} + \frac{220}{1963.495}\right]$$

$$\boxed{dL = 0.184\ mm}$$

* A member formed by connecting a steel bar to a Alluminium bar as shown in figure. Assuming that the bars are prevented from buckling on the both the case. Calculate the magnetude of force 'P' that will cause the total length of the member to decrease 0.25 mm. The values of E for steel and Aluminium are $2.1 \times 10^5 N/mm^2$ and $7 \times 10^4 N/mm^2$ respectively.



Sol: Given that.

$dL = 0.25 mm$

$E_s = 2.1 \times 10^5 N/mm^2$

$E_{Al} = 7 \times 10^4 N/mm^2$

$d_2 = 5 cm = 50 mm$

$d_1 = 10 cm = 100 mm$

$L_2 = 30 cm = 300 mm$

$L_1 = 38 cm = 380 mm$

$A_2 = 100 \times 100$
$= 10000 mm^2$

$A_1 = 50 \times 50$
$= 2500 mm^2$

$$dl = P\left[\frac{A L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2}\right]$$

$$0.25 = P\left[\frac{300}{\cancel{1963.4998} \times 2.1\times10^5} + \frac{380}{\cancel{7853.998} \times 7\times10^4}\right]$$

$\cancel{10000}$ 2500 $\cancel{2500}$ 10000

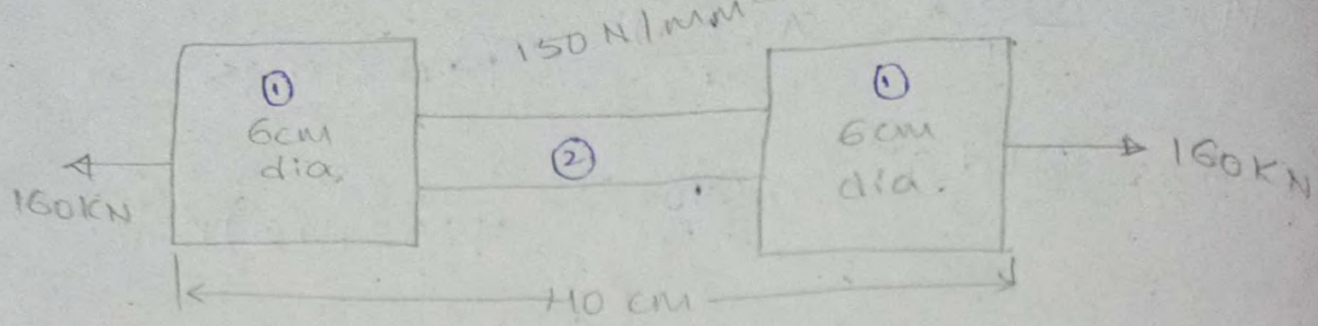
$$0.25 = P\left[1.1143\times10^{-6}\right]$$

$$\cancel{P} = \frac{\cancel{1.4187\times10^{-6}}}{\cancel{0.25}}$$

$$P = \frac{0.25}{1.1143\times10^{-6}}$$

$$\boxed{P = 224.359\times10^{3}\,N}$$

* The bar as shown in fig. is subjected to tensile load of 160 KN if the stress in the middle portion limited to $160 N/mm^2$. Determine the diameter of the middle portion. And the length of the middle portion. If the total elongation of the bar is 0.2mm and $E = 2.1\times10^5 N/mm^2$



Sol:- Given data,

$P = 160KN = 160 \times 10^3 N$

$\sigma_2 = 150\ N/mm^2$

$dL = 0.2mm$

$L_2 = ?$

$d_2 = ?$

$E = 2.1 \times 10^5\ N/mm^2$

$L = 40cm = 400mm$

$d_1 = 6cm = 60mm$

$$A_1 = \frac{\pi (D_1)^2}{4} = \frac{\pi (60)^2}{4} = 2827.43\ mm^2$$

$$\sigma_2 = \frac{P}{A_2} \Rightarrow 150 = \frac{160 \times 10^3}{\frac{\pi (d_2)^2}{4}}$$

$\cancel{150}\ \cancel{160 \times 10^3} = 1$

$$\frac{\pi (d_2)^2}{4} = \frac{160 \times 10^3}{150} \rightarrow \boxed{\frac{\pi (d_2)^2}{4} = 1066.66\ mm^2}$$

$$(d_2)^2 = \frac{160 \times 10^3 \times 4}{150 \times \pi}$$

$$d_2 = \sqrt{\frac{160 \times 10^3 \times 4}{150 \times \pi}}$$

$$\boxed{d_2 = 36.85mm}$$

$L_1 + L_2 = L = 40\,cm = 400\,mm$

$L_1 = 400 - L_2$

$L_2 = 400 - L_1$

$$dL = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2}\right]$$

$$= \frac{160\times10^3}{2.1\times10^5}\left[\frac{400-L_2}{2827.43} + \frac{L_2}{1066.66}\right]$$

$$= 0.762\left[\frac{1066.66(400-L_2) + 2827.43(L_2)}{3.0159\times10^6}\right]$$

$$= 0.762\left[\frac{426664 - 1066.66L_2 + 2827.43L_2}{3.0159\times10^6}\right]$$

$$= 0.762\left[\frac{426664 + 1760.77L_2}{3.0159\times10^6}\right]$$

$$dL = \overset{2.5266\times10^{-7}}{\cancel{\ \ }}\left(426664 + 1760.77L_2\right)$$

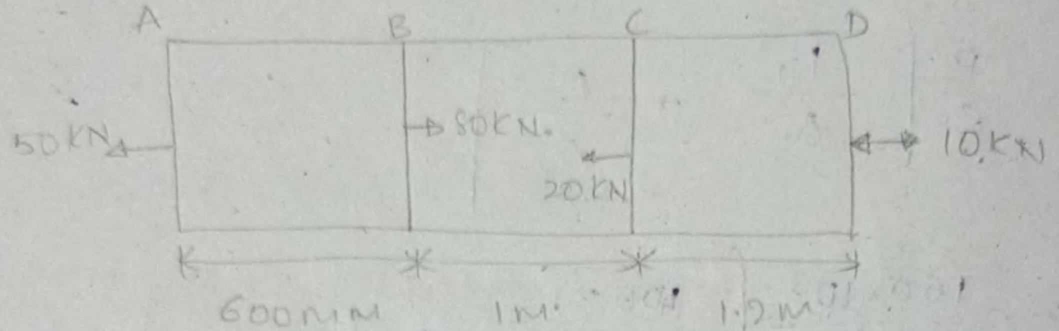
$$\underset{0.2}{dL} = \overset{0.1078}{\cancel{\ \ }} + \overset{4.448\times10^{-4}}{\cancel{\ \ }}L_2$$

$$L_2 = \frac{0.2 - 0.1078}{4.448\times10^{-4}}$$

$$\boxed{L_2 = 207.28\,mm}$$

$\boxed{L_2 = \cancel{492.5022\,mm}}$

* A Gross bar having c/s area of 1000 mm² ~~of 1000 mm²~~ is subjected to axial force as shown in figure. find the total elongation of the bar. Take $E = 1.05 \times 10^5\ N/mm^2$.



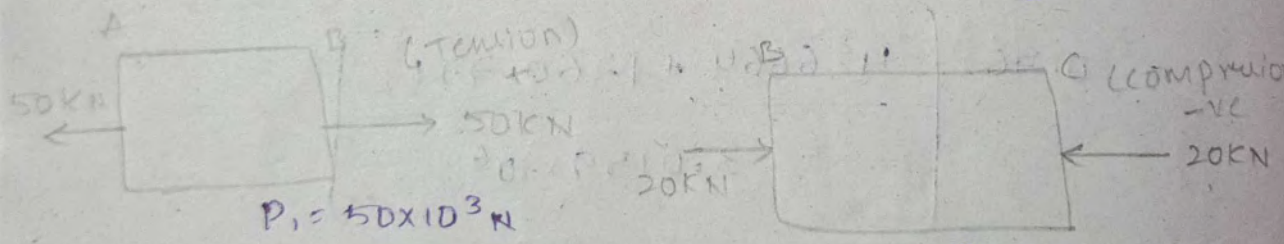
Sol:-

Given data,

$$A = 1000mm^2$$

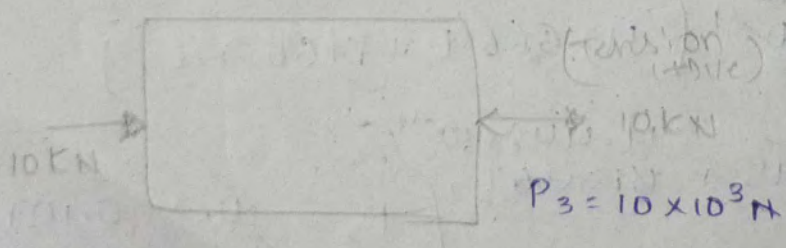
$$E = 1.05 \times 10^5\ N/mm^2$$

$$dL = \frac{1}{AE}\left[P_1L_1 + P_2L_2 + P_3L_3\right]$$



$P_1 = 50 \times 10^3 N$

$P_2 = 20 \times 10^3 N$

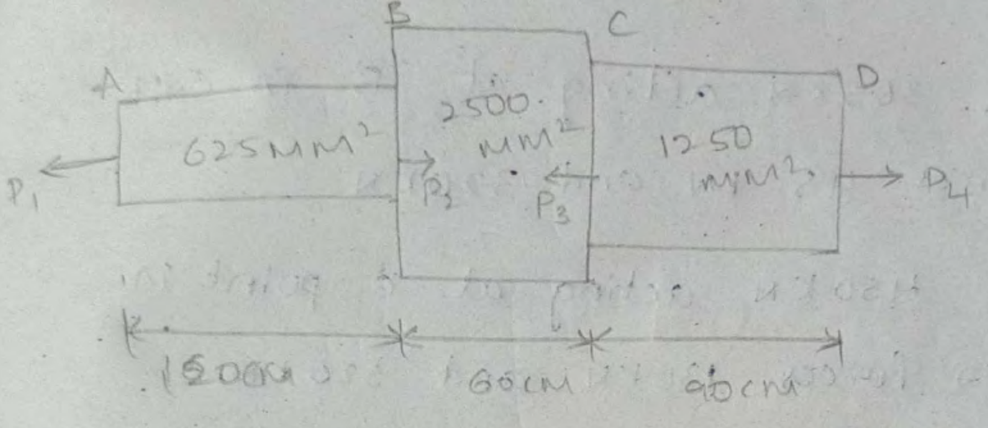


$P_3 = 10 \times 10^3 N$

$$dL = \frac{1}{1000 \times 1.05 \times 10^5}\left[50 \times 10^3 (600) \mp 20 \times 10^3 (1000) \mp 10 \times 10^3 (1200)\right]$$

$$\boxed{dL = -0.019\ mm}$$

* A member ABCD is subjected to point loads P_1, P_2 and P_3 and P_4 as shown in figure. Calculate the force P_2 in equilibrium condition. If $P_1 = 45KN$, $P_3 = 450KN$ and $P_4 = 130KN$. Determine the total elongation of the member. Assuming the $E = 2.1 \times 10^5 N/mm^2$



Soln

Given data,

$P_1 = 45KN = 45 \times 10^3 N$

$P_3 = 450KN = 450 \times 10^3 N$

$P_4 = 130KN = 130 \times 10^3 N$

$E = 2.1 \times 10^5 N/mm^2$

$L_1 = 120 cm = 1200 mm$

$L_2 = 60 cm = 600 mm$

$L_3 = 90 cm = 900 mm$

$A_1 = 625 mm^2$

$A_2 = 2500 mm^2$

$A_3 = 1250 mm^2$

from Equilibrium condition

$P_1 + P_3 = P_2 + P_4$

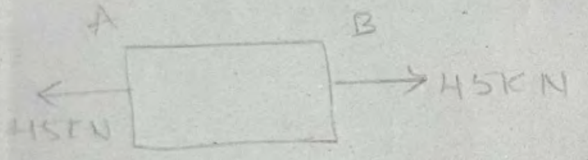
$45 + 450 = P_2 + 130$

$P_2 = 45 + 450 - 130$

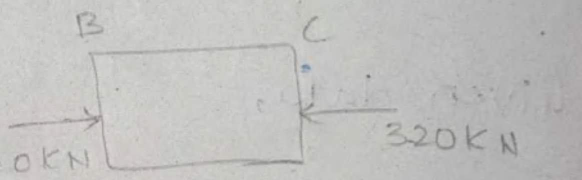
$P_2 = 365\,KN$

The force of 365 KN acting at 'B' is split into two forces 45 KN and 320 KN

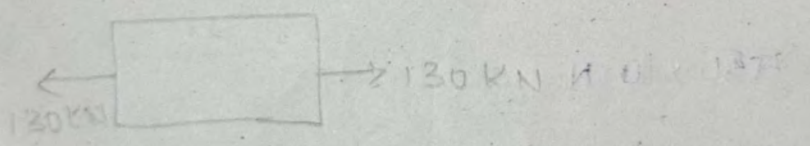
The force of 450 KN acting at C point is split into two forces 130 KN and 320 KN.



$P_1 = 45 \times 10^3\,N$



$P_2 = 320 \times 10^3\,N$



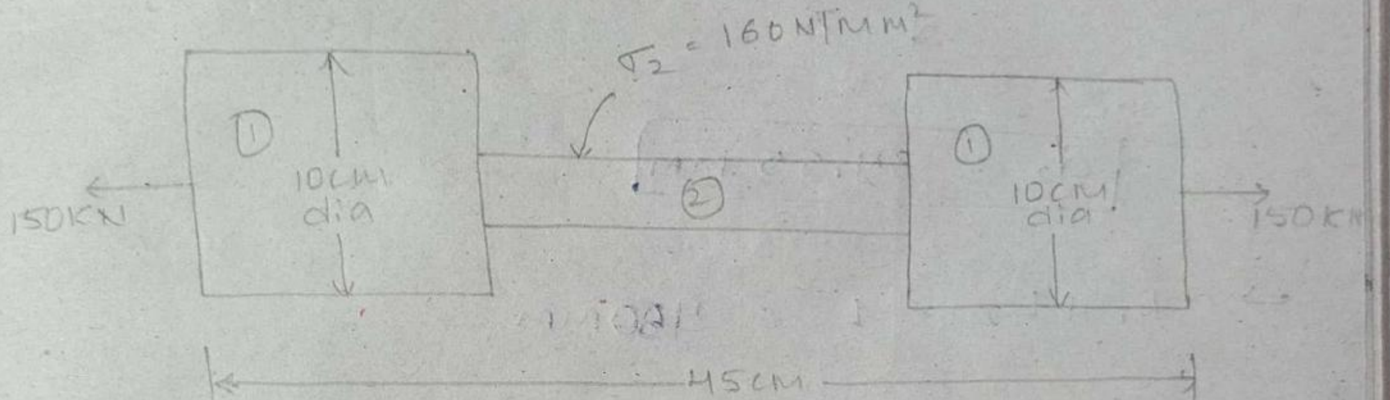
$P_3 = 130 \times 10^3\,N$

$$dL = \frac{1}{E}\left[\frac{P_1 L_1}{A_1} + \frac{P_2 L_2}{A_2} + \frac{P_3 L_3}{A_3}\right]$$

$$dL = \frac{1}{2.1 \times 10^5}\left[\frac{45 \times 10^3 \times (1200)}{625} - \frac{320 \times 10^3 (600)}{2500} + \frac{130 \times 10^3 \times (900)}{1250}\right]$$

$$\boxed{dL = 0.49\,MM}$$

* The bar shown in fig.1.28 is subjected to a tensile load of 150KN. If the stress in the middle portion is limited to 160 N/mm², determine the diameter of the middle portion. find also the length of the middle portion if the total elongation of the bar is to be 0.25cm. Young's modulus is given as equal to 2.0×10^5 N/mm².



Sol:
Given data,

Tensile load (P) = 150KN = 150×10^3 N

Stress in middle portion (σ_2) = 160 N/mm²

Total elongation of the bar (dL) = 0.25cm

= 2.5 mm

Young's modulus $E = 2.0 \times 10^5$ N/mm²

L = 45cm = 450mm

d_1 = 10cm = 100mm

$$A_1 = \frac{\pi (d_1)^2}{4} = \frac{\pi (100)^2}{4} = 7853.98 \text{ mm}^2$$

$$\sigma_2 = \frac{P}{A_2} \Longrightarrow A_2 = \frac{P}{\sigma_2}$$

$$A_2 = \frac{150 \times 10^3}{160}$$

$$A_2 = 937.5\,mm^2$$

$$\therefore \quad A_2 = \frac{\pi (d_2)^2}{4}$$

$$937.5 = \frac{\pi (d_2)^2}{4}$$

$$d_2 = \sqrt{\frac{937.5 \times 4}{\pi}}$$

$$\boxed{d_2 = 34.55\ MM}$$

$$\Rightarrow \quad L_1 + L_2 = L = 450\,MM$$

$$L_1 = 450 - L_2$$

$$L_2 = 450 - L_1$$

$$dL = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2}\right]$$

$$0.25 = \frac{150 \times 10^3}{2.0 \times 10^5}\left[\frac{450 - L_2}{7853.98} + \frac{L_2}{937.5}\right]$$

$$0.25 = 0.75\left[\frac{937.5(450 - L_2) + 7853.98(L_2)}{7853.98 \times 937.5}\right]$$

$$0.25 = 0.75\left[\frac{421875 - 937.5L_2 + 7853.98L_2}{7.363 \times 10^6}\right]$$

$$\frac{\cancel{2.5}}{0.25} = 0.75\left[\frac{421876 + 6916.48 L_2}{7.363 \times 10^6}\right]$$

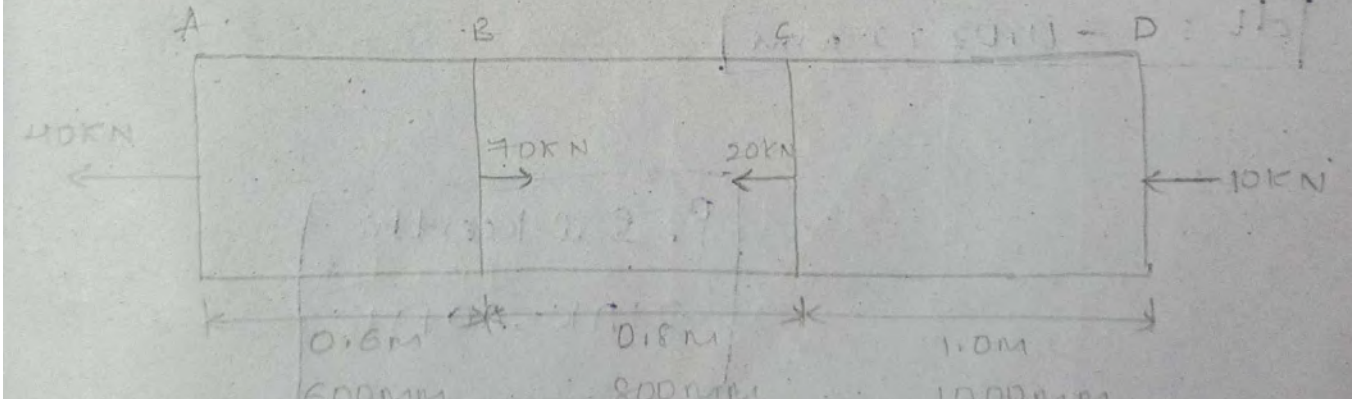
$$\frac{0.25}{\cancel{2.5}} = 1.0186 \times 10^{-7}\left[421876 + 6916.48 L_2\right]$$

$$0.25\cancel{218} = 0.043 + 7.045 \times 10^{-4} L_2$$

$$L_2 = \frac{0.25 - 0.043}{7.045 \times 10^{-4}}$$

$$\boxed{L_2 = 293.83\ mm}$$

* A brass bar, having cross-section area of $900 mm^2$, is subjected to axial forces as shown a fig. 1.29 in which AB = 0.6m, BC = 0.8m and CD = 1.0m.
find the elongation of the bar. Take $E = 1 \times 10^5$ N/mm^2.



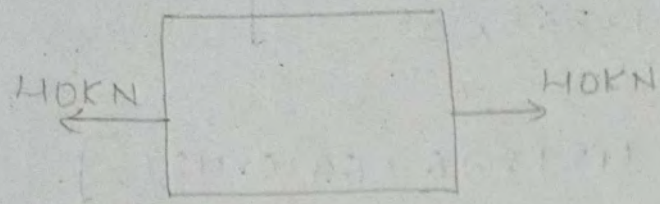
Sol: Given data,

$A = 900 mm^2$

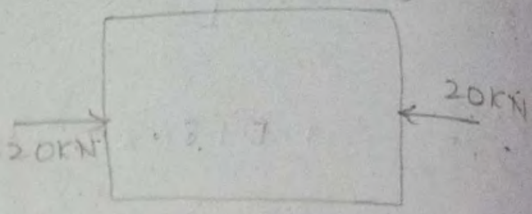
$E = 1 \times 10^5 N/mm^2$

$$dL = \frac{1}{AE}\left[P_1 L_1 + P_2 L_2 + P_3 L_3\right]$$

Tension (+ve)



compression (-ve)



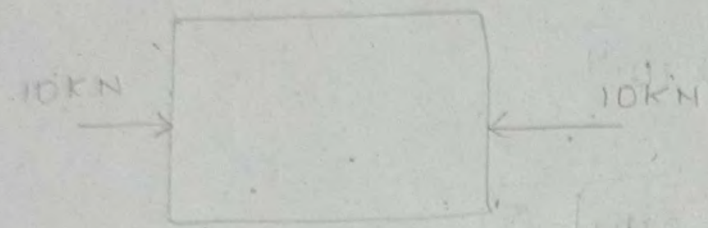
$P_1 = 40\times10^3$ N

$L_1 = 600$ MM

$P_2 = 20\times10^3$ N

$L_2 = 800$ mm

compression (-ve)



$P_3 = 10\times10^3$ N $\quad L_3 = 1000$ MM

$$dL = \frac{1}{900\times1\times10^5}\left[40\times10^3(600) - 20\times10^3(800) - 10\times10^3(1000)\right]$$

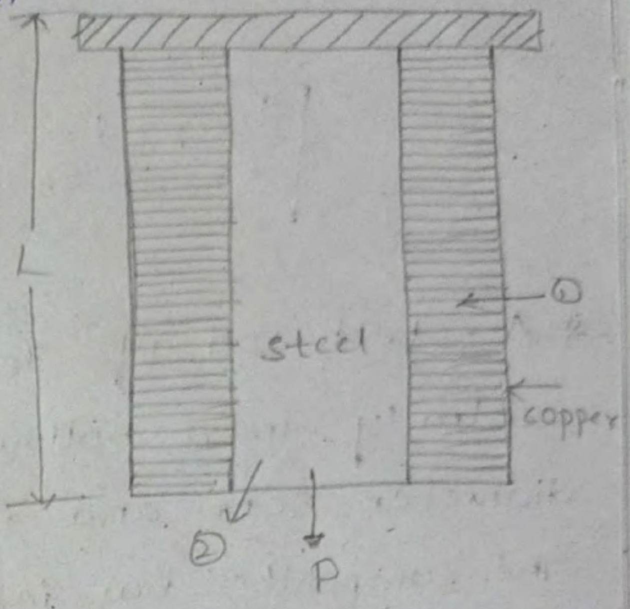
$$dL = 1.111\times10^{-8}\left[-2000000\right]$$

$$\boxed{dL = -0.0222 \text{ MM}}$$

P. Sai keerthi

21AK5A0171

Analysis of Bars of Composite Section



* The extension or compression in each bay is equal hence the deformation per unit length is equal.

* The total external load on the composite bars is equal to the sum of the loads carried by different materials.

E_1 and E_2 are young's modulus

$P \rightarrow$ Total load

A_1, A_2 are Areas

σ_1, σ_2 are stress of bars

$$P = P_1 + P_2 \longrightarrow (1)$$

$$\sigma_1 = \frac{P_1}{A_1} \qquad \sigma_2 = \frac{P_2}{A_2}$$

$$P_1 = \sigma_1 A_1 \qquad P_2 = \sigma_2 A_2$$

Now,

$$\boxed{P = \sigma_1 A_1 + \sigma_2 A_2}$$

Young's Modulus $E = \frac{\sigma}{e}$

$$E_1 = \frac{\sigma_1}{e_1} \; ; \; E_2 = \frac{\sigma_2}{e_2}$$

$$e_1 = \frac{\sigma_1}{E_1} \quad ; \quad e_2 = \frac{\sigma_2}{E_2}$$

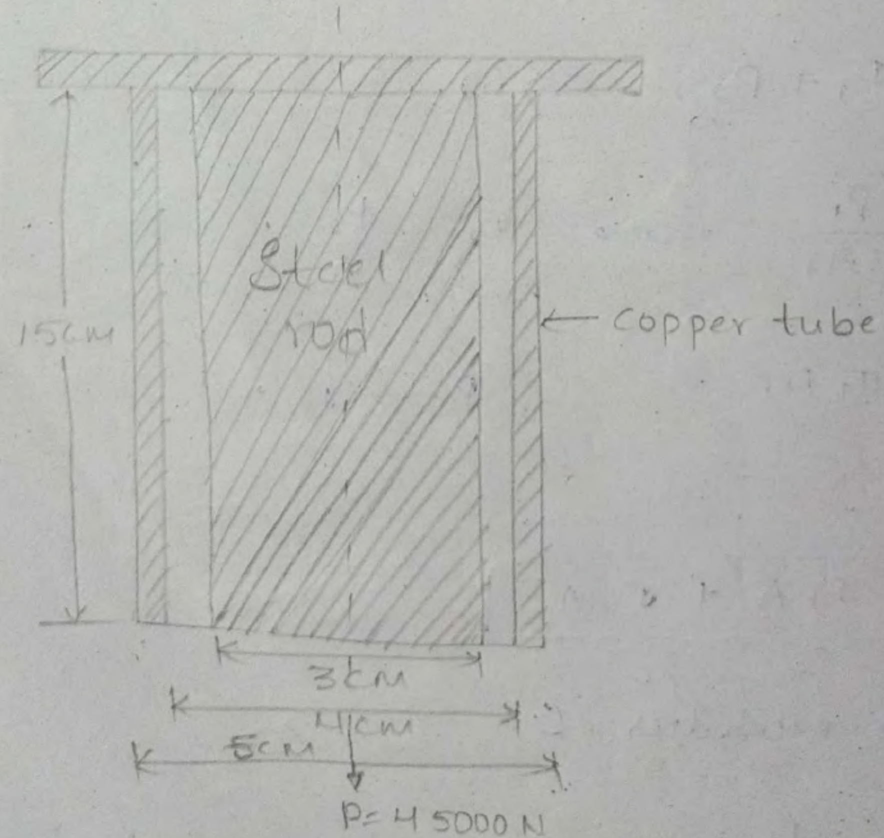
$$\boxed{\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}} \longrightarrow \frac{E_1}{E_2} \text{ Modular ratio}$$

* A steel rod of 3cm diameter is enclosed centrally in a hollow copper tube of external diameter 5cm and internal diameter of 4cm. The composite bar is then subjected to an axial pull of 45000N. If the length of each bar is equal to 15cm, determine.

i) The stresses in the rod and tube, and

ii) Load carried by each bar.

Take E for steel = 2.1×10^5 N/mm^2 and for copper = 1.1×10^5 N/mm^2.



Sol:-

Given data,

Diameter of the steel rod = 3cm = 30mm

External diameter of copper = 5cm = 50mm

Internal diameter of copper = 4cm = 40mm

Area of steel $(A_{st}) = \frac{\pi(30)^2}{4} = 706.85 mm^2$

Area of copper $(A_c) = \frac{\pi}{4}(D_{ex}^2 - D_{in}^2)$

$= \frac{\pi}{4}(50^2 - 40^2)$

$A_c = 706.85 mm^2$

P = 45000 N

L = 15 cm = 150 mm

$E_s = 2.1 \times 10^5 N/mm^2$ $E_c = 1.1 \times 10^5 N/mm^2$

① Stress in each rod and tube:-

$e_c = e_{st}$

$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$ (or) $\frac{\sigma_c}{E_c} = \frac{\sigma_s}{E_s}$

$\sigma_s = \frac{E_s}{E_c} \times \sigma_c$

$\sigma_s = \frac{2.1 \times 10^5}{1.1 \times 10^5} \sigma_c$

$\sigma_s = 1.909 \sigma_c \longrightarrow ①$

$P = P_1 + P_2$

$$45000 = \sigma_1 A_1 + \sigma_2 A_2$$

$$45000 = \sigma_A A_A + \sigma_C \times A_C$$

$$45000 = 1.909\,\sigma_C \times 706.85 + \sigma_C \times 706.85$$

$$45000 = 1349.37\,\sigma_C + 706.85\,\sigma_C$$

$$45000 = 2056.23\,\sigma_C$$

$$\sigma_C = \frac{45000}{2056.23}$$

$$\boxed{\sigma_C = 21.88\ N/mm^2}$$

$$\therefore\ \sigma_A = 1.909\,\sigma_C$$

$$= 1.909 \times 21.88$$

$$\boxed{\sigma_A = 41.778\ N/mm^2}$$

(2) Load carried by each bar:-

$P = \sigma_1 \times A_1$ $\quad$ $P_2 = \sigma_2 \times A_2$

$P_A = \sigma_A \times A_A$ $\quad$ $P_C = \sigma_C \times A_C$

$P_A = 41.77 \times 706.85$ $\quad$ $P_C = 21.88 \times 706.85$

$\boxed{P_A = 29525.125\,N}$ $\quad$ $\boxed{P_C = 15465.878\,N}$

$$P = P_1 + P_2$$

$$= 29525.125 + 15465.878$$

$$45000 = 44991.003\,N \simeq 45000\,N$$

* A compound tube ~~ka~~ consists of a steel tube 140mm Internal diameter and 160mm external diameter and an outer brass tube 160mm Internal diameter and 180 mm Internal diameter. The two tubes are of the same length. The compound tube carries an axial load of 900 kN. find the stresses and the load carried by each tube and the amount it shortens. Length of each tube is 140mm. Take E for steel as 2×10^5 N/mm^2 and for brass as 1×10^5 N/mm^2

Sol: Given data,

Internal diameter of steel tube = 140mm

External diameter of steel tube = 160mm

Internal diameter of brass tube = 160 mm

External diameter of brass tube = 180mm

Load (P) = 900 kN

$P = 900 \times 10^3$ N

Length (L) = 140mm

$E_s = 2 \times 10^5$ N/mm^2

$E_b = 1 \times 10^5$ N/mm^2

⇒ Stresses in each tube:-

$$e_s = e_b$$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_b}{E_b}$$

$$\sigma_s = \frac{E_s}{E_b} \times \sigma_b$$

$$= \frac{2 \times 10^5}{1 \times 10^5} \sigma_b$$

$\sigma_s = 2\sigma_b$

We know that

$$P = P_1 + P_2$$

$$= \sigma_1 A_1 + \sigma_2 A_2$$

$$P = \sigma_s A_s + \sigma_b A_b$$

$$A_s = \frac{\pi}{4}(160^2 - 140^2)$$

$$\boxed{A_s = 4712.38 \, mm^2}$$

$$A_b = \frac{\pi}{4}(180^2 - 160^2)$$

$$\boxed{A_b = 5340.71 \, mm^2}$$

$\Rightarrow$ $900 \times 10^3 = 2\sigma_b \times 4712.38 + \sigma_b \times 5340.71$

$$900 \times 10^3 = 9424.76\,\sigma_b + 5340.71\,\sigma_b$$

$$900 \times 10^3 = 14765.47\,\sigma_b$$

$$\sigma_b = \frac{900 \times 10^3}{14765.47}$$

$$\boxed{\sigma_b = 60.953 \, N/mm^2}$$

$\Rightarrow$ $\sigma_s = 2 \times \sigma_b$

$$= 2 \times 60.953$$

$$\boxed{\sigma_s = 121.91 \, N/mm^2}$$

Load carried by each tube;-

$P_1 = \sigma_1 \times A_1$; $P_2 = \sigma_2 \times A_2$

$P_s = \sigma_s \times A_s$; $P_b = \sigma_b \times A_b$

$P_s = 121.906 \times 4712.38$ $P_b = 60.953 \times 5340.71$

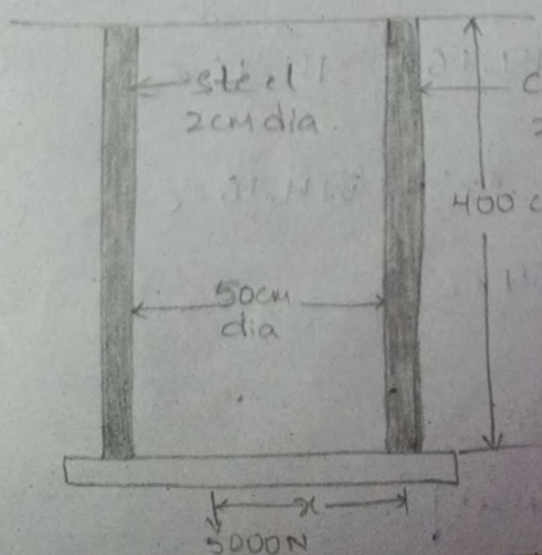
$\boxed{P_s = 574467.39\,N}$ $\boxed{P_b = 325532.29\,N}$

$P = P_s + P_b$

$= 574467.39 + 325532.29$

$900000 = 899999.68\,N \simeq 900000\,N$

* Two vertical rods one of steel and the other of copper are each rigidly fixed at the top & 50cm apart. Diameter and lengths of each rod are 2cm and 4m respectively. A cross bar fixed to the rods at the lower ends carries a load of 5000N such that the cross bar remains horizontal even after loading. find the stresses in each rod and the position of the load on the bar. Take E for steel $= 2 \times 10^5\,N/mm^2$ and E for copper $= 1 \times 10^5\,N/mm^2$.



Sol:-

Given:-

Distance between the rods = 50 cm = 500 mm

Diameter of steel & copper rods = 2 cm = 20 mm

Length of each rod = 4 m = 4000 mm

$$A_s = A_c = \frac{\pi (20)^2}{4} = 314.16 \text{ mm}^2$$

$E_s = 2 \times 10^5 \text{ N/mm}^2$ $E_c = 1 \times 10^5 \text{ N/mm}^2$

$P = 5000 \text{ N}$

① Strain in steel = Strain in copper

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \frac{E_s}{E_c} \times \sigma_c$$

$$= \frac{2 \times 10^5}{1 \times 10^5} \times \sigma_c$$

$$\boxed{\sigma_s = 2\sigma_c}$$

② $P = P_1 + P_2$

$$P = \sigma_s A_s + \sigma_c A_c$$

$$5000 = 2\sigma_c \times 314.16 + 314.16\sigma_c$$

$$5000 = 628.32\sigma_c + 314.16\sigma_c$$

$$5000 = 942.64\sigma_c$$

$$\sigma_c = \frac{5000}{942.64}$$

$$\boxed{\sigma_c = 5.31 \text{ N/mm}^2}$$

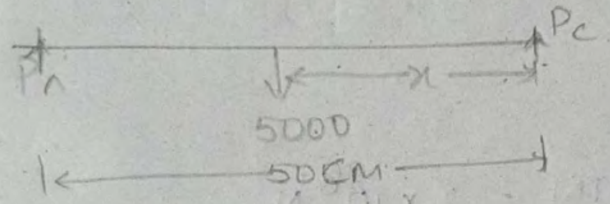
$\sigma_s = 2 \times 5.31$

$$\boxed{\sigma_s = 10.61 \text{ N/MM}^2}$$

(3) $P_s = \sigma_s \times A_s$ $\qquad$ $P_c = \sigma_c \times A_c$

$= 10.61 \times 314.16$ $\qquad$ $= 5.31 \times 314.16$

$$\boxed{P_s = 3333.24 \text{ N}} \qquad \boxed{P_c = 1668.19 \text{ N}}$$



Taking moments

$P_c \times 0 - 5000 \times x + P_s \times 500 = 0$

$-5000x + 3333 \times 500 = 0$

$-5000x + 1666500 = 0$

$x = \dfrac{1666500}{5000}$

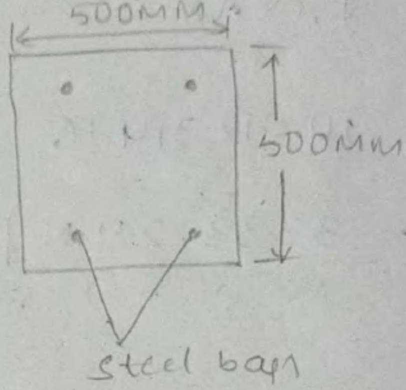
$$\boxed{x = 333.3 \text{ MM}}$$

load 5000 N is acting @ a distance of 333.3MM from copper rod.

* A load of 2 MN is applied on short concrete column. 500MM x 500MM. The column is reinforced with 4 steel bars 10 MM dia, one of each in corner. find the stresses in concrete and steel bars.

$E_s = 2.1 \times 10^5$

$E_c = 1.4 \times 10^5$



Sol:-

Given,

$$\text{load} = 2\ MN = 2 \times 10^6\ N$$

$$\text{column size} = 500\ MM \times 500\ MM$$

$$\text{dia of steel bars} = 10\ MM$$

$$\text{No. of bars} = 4$$

$$E_s = 2.1 \times 10^5\ N/MM^2$$

$$E_c = 1.4 \times 10^5\ N/MM^2$$

$$\text{Area of column} = 500 \times 500$$

$$\boxed{A_{CO} = 250000\ MM^2}$$

$$\text{Area of steel bars} = 4 \times \frac{\pi (d)^2}{4}$$

$$= 4 \times \frac{\pi (10)^2}{4}$$

$$\boxed{A_s = 314.16\ MM^2}$$

Gross Area = $A_{steel} + A_{concrete}$

$A_{concrete}$ = Gross Area − A_{st}

= 250000 − 314.16

$$\boxed{A_c = 249685.84 \, mm^2}$$

① strain in steel = strain in copper

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \frac{E_s}{E_c} \sigma_c$$

$$= \frac{2.1 \times 10^5}{1.4 \times 10^4} \sigma_c$$

$$\boxed{\sigma_s = 15 \sigma_c}$$

② $P = P_s + P_c$

$= \sigma_s \times A_s + \sigma_c \times A_c$

$= 15 \sigma_c \times 314.16 + 249685.84 \sigma_c$

$2 \times 10^6 = 4712.4 \sigma_c + 249685.84 \sigma_c$

$2 \times 10^6 = 254398.24 \sigma_c$

$$\sigma_c = \frac{2 \times 10^6}{254398.24}$$

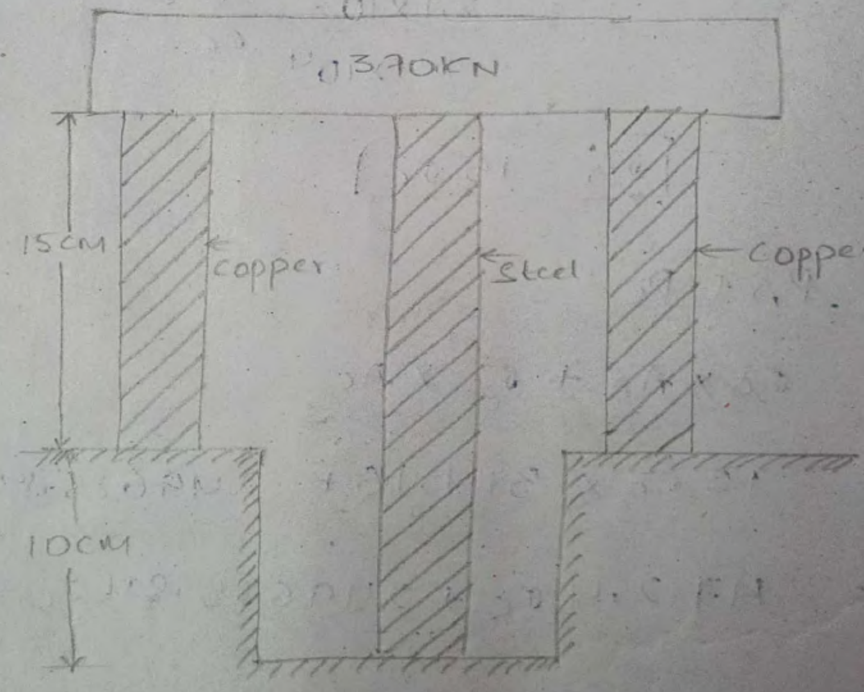
$$\boxed{\sigma_c = 7.86 \, N/mm^2}$$

$\therefore \quad \sigma_s = 15 \times \sigma_c$

$= 15 \times 7.86$

$$\boxed{\sigma_s = 117.9 \ N/mm^2}$$

* A steel rod and two copper rods together support a load of 370 KN as shown in figure. The c/s area of steel rod is 2500 mm² and of each copper rod is 1600 mm². find the stresses in the rods. Take E for steel $= 2 \times 10^5 \ N/mm^2$ and for copper $E = 1 \times 10^5 \ N/mm^2$.



Sol: Given data

$P = 370 \ KN$

$P = 370 \times 10^3 \ N$

Area of steel rod (A_s) $= 2500 \ mm^2$

Area of 2 copper rods $(A_c) = 2 \times 1600$

$= 3200 \ mm^2$

$E_s = 2\times10^5$ N/mm²

$E_c = 1\times10^5$ N/mm²

length of copper rod = 15 cm = 150 mm

length of steel rod = 15 + 10 = 25 cm = 250 mm

(1) strain in steel = strain in copper

$$e_s = e_c$$

$$L_s \times \frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c} \times L_c$$ 'L' is differ for steel and copper

$$\frac{\sigma_s}{2\times10^5} \times 250 = \frac{\sigma_c}{1\times10^5} \times 150$$

$$\sigma_s = \frac{2\times10^5}{1\times10^5} \times \frac{150}{250}\sigma_c$$

$$\boxed{\sigma_s = 1.2\sigma_c} \longrightarrow (1)$$

(2) $P = P_s + P_c$

$= \sigma_s A_s + \sigma_c A_c$

$370\times10^3 = 1.2\sigma_c \times 2500 + \sigma_c \times 3200$

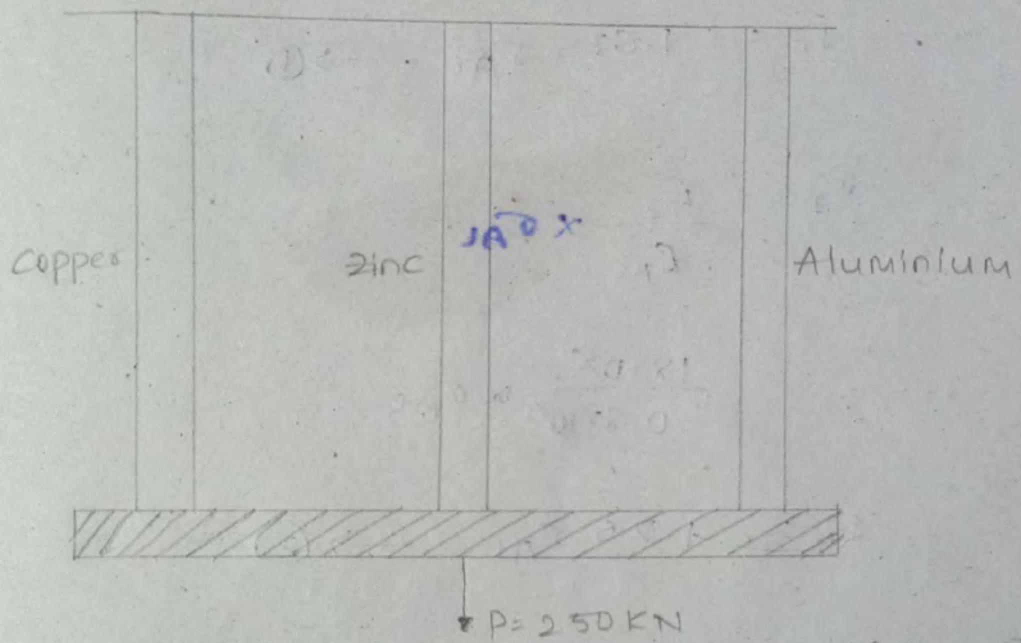
$370\times10^3 = 3000\sigma_c + 3200\sigma_c$

$370\times10^3 = 6200\sigma_c$

$$\sigma_c = \frac{370\times10^3}{6200}$$

$$\boxed{\sigma_c = 59.67 \text{ N/mm}^2}$$

* Three bars made of copper, zinc and aluminium are of equal length and have c/s 500, 750 and 1000 square mm Respectively. They are rigidly connected at their ends. If this compound member is subjected to a longitudinal pull of 250KN. estimate the proportional of the load carried on each rod and the induced stresses. Take the value of E for copper = 1.3×10^5 N/mm², for zinc = 1.0×10^5 N/mm² and for aluminium = 0.8×10^5 N/mm².



Sol: Given data,

$P = 250\ KN = 250 \times 10^3\ N$

$L_C = L_Z = L_{Al}$

$A_C = 500\ mm^2$

$A_Z = 750\ mm^2$

$A_{Al} = 1000\ mm^2$

$E_C = 1.3 \times 10^5\ N/mm^2$

$E_Z = 1 \times 10^5\ N/mm^2$

$E_{Al} = 0.8 \times 10^5\ N/mm^2$

(1) strain in copper = strain in zinc = strain in Aluminium

(1) (2) (3)

$$\frac{\sigma_c}{E_c} = \frac{\sigma_z}{E_z} = \frac{\sigma_{AL}}{E_{AL}}$$

$$\sigma_c = \frac{E_c}{E_{AL}} \times \sigma_{AL}$$

$$= \frac{1.3\times10^{\cancel{8}}}{0.8\times10^{\cancel{5}}}\,\sigma_{AL}$$

$$\sigma_c = 1.625\,\sigma_{AL} \longrightarrow (1)$$

$$\sigma_z = \frac{E_z}{E_{AL}} \times \sigma_{AL}$$

$$= \frac{1\times10^{\cancel{8}}}{0.8\times10^{\cancel{5}}} \times \sigma_{AL}$$

$$\sigma_z = 1.25\,\sigma_{AL} \longrightarrow (2)$$

(2) $P = P_c + P_z + P_{AL}$

$$250\times10^3 = \sigma_c \times A_c + \sigma_z \times A_z + \sigma_{AL} \times A_{AL}$$

$$= 1.625\,\sigma_{AL} \times 500 + 1.25 \times \sigma_{AL} \times 750 + \sigma_{AL} \times 1000$$

$$= 812.5\,\sigma_{AL} + 937.5\,\sigma_{AL} + 1000\,\sigma_{AL}$$

$$250\times10^3 = 2750\,\sigma_{AL}$$

$$\sigma_{AL} = \frac{250\times10^3}{2750}$$

$$\simeq \cancel{90.18}$$

$$\boxed{\sigma_{AL} = 90.91\ N/mm^2}$$

$\Rightarrow \sigma_C = 1.625 \times \sigma_{AL} = 1.625 \times 90.91 = 147.73$ N/mm²

$\Rightarrow \sigma_Z = 1.25 \times \sigma_{AL} = 1.25 \times 90.91 = 113.64$ N/mm²

$$\sigma = \frac{P}{A}$$

$$P = \sigma \times A$$

$\therefore \Rightarrow P_C = \sigma_C \times A_C = 147.73 \times 500 = 73865$ N

$P_Z = \sigma_Z \times A_Z = 113.64 \times 750 = 85230$ N

$P_{AL} = \sigma_{AL} \times A_{AL} = 90.91 \times 1000 = 90910$ N

1) The resistance per unit area, offered by a body against deformation is known as "Stress."

The stress is given by

$$\sigma = \frac{P}{A}$$

Where,

P = External force or load

A = Cross-sectional area.

→ Stress is expressed as kgf/m^2, kgf/cm^2, N/m^2 and N/mm^2.

→ $1\ N/m^2 = 10^{-4}\ N/cm^2$ or $10^{-6}\ N/mm^2$.

2) The ratio of change of dimension of the body to the original dimensions is known as "Strain."

3) The stress induced in a body, which is subjected to two equal and opposite pulls is known as "tensile stress."

4) The stress induced in a body, which is subjected to two equal and opposite pushes is known as "compressive stress."

5) "Elasticity" is the property by virtue of which certain materials return back to their original position after the removal of the external forces

6) "Hooke's law" states that stress is proportional to the strain within elastic limit.

7) The ratio of tensile stress (or compressive stress) to the corresponding strain is known as "Young's Modulus" (or) "Modulus of elasticity" and is denoted by 'E'

$$E = \frac{\text{Tenisle or Compressive Stress}}{\text{Corresponding strain}}$$

8) The ratio of shear stress to the corresponding shear strain within the elastic limit, is known as "Modulus of Rigidity" or "Shear Modulus". It is denoted by 'C' [or 'G' or 'N']

9) Total change in the length of a bar of different lengths and of different diameters when subjected to an axial load P, is given by

$$dL = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right]$$ When E is same

$$dL = P\left[\frac{L_1}{E_1A_1} + \frac{L_2}{E_2A_2} + \frac{L_3}{E_3A_3}\right]$$ When E is different

10) The total extension of a uniformly tapering circular rod of diameters D_1 and D_2, when the rod is subjected to an axial load P is given by

$$dL = \frac{4PL}{\pi E D_1 D_2}$$ Where,

L = Total length of the rod.

11) A composite bar is made up of two or more bars of equal lengths but the different materials rigidly fixed with each other and behaving as one

unit for extension or compression.

12) In case of a composite bar having equal length

(i) Total load on the composite bar is equal to the sum of loads carried by each different materials.

(ii) strain in ~~eq~~ each bar is equal.

13) The stresses induced in a body due to change in temperature are known as <u>thermal stresses.</u>

14) Thermal strain and thermal stress is given by thermal strain, $e = \alpha . T$ and thermal stress $p = \alpha . T . E$

Where, α = Co-efficient of linear expansion,

T = Rise or fall of temperature.

E = Young's modulus.

15) Total elongation of a uniformly tapering Rectangular bar when subjected to an axial load P is given by

$$dL = \frac{PL}{Et(a-b)} \log_e \frac{a}{b}.$$

Where,

L = Total length of bar

α = Width at bigger end

E = Young's modulus

t = thickness of bar

b = Width at smaller end

16) In case of a composite bar having two or more bars of different lengths, the extension or compression in each bar will be equal. And the total load will be equal to the sum of the loads carried by each member.

17) In case of nut and bolt used on a tube with washers, the tensile load on the bolts is equal to the compressive load on the tube.

18) Elongation of a bar due to its own weight is given by

$$\delta L = \frac{W}{E} \times \frac{L^2}{2} \quad \text{or} \quad \frac{WL}{2E}$$

Where,

W = Weight per unit volume of the bar material

L = length of bar.

* A steel rod 20mm in diameter passes centrally through a steel tube of 25 mm internal diameter and 30mm diameter. The tube is 800 mm long and is closed by rigid washers of negligible thickness which are fastened by nuts threaded on the rod. The nuts are tightened until the compressive load on the tube is 20 KN. Calculate the stresses in the tube and the rod.
find the increase in these stresses when one nut is tightened by one-quarter of a turn relative to the other. There are 4 threads per 10mm. Take $E = 2\times10^5 N/mm^2$.

Sol: Given,

Dia. of rod = 20mm

Internal dia of steel tube = 25mm

External dia of steel tube = 30mm

length of tube, L = 800mm

Load $P = 20KN = 20\times10^3 N$

$E = 2\times10^5 N/mm^2$

Area of Rod $(A_r) = \frac{\pi(20)^2}{4} = \cancel{100}\ 314.16 mm^2$

Area of tube $(A_t) = \frac{\pi}{4}(30^2 - 25^2) = \cancel{88.45}$

$215.98 mm^2$

When the nuts are tightened, the tube will be compressed and the rod will be elongated. This means that the tube will be under compression and Rod will be under tension. Since no external forces have been applied, the compressive load on the tube must be equal to the tensile load on the rod.

Let, σ_s = stress in the tube,

σ_r = stress in rod.

Now,

Tensile ~~stress~~ load on the rod = compressive load on the tube.

$\therefore \sigma_r \times A_r = \sigma_t \times A_t$

$$\cancel{\sigma_r = \frac{A_t}{A_r} \times \sigma_t}$$

$$\cancel{= \frac{314.16}{215.98} \sigma_t}$$

$$\sigma_r = \frac{A_t}{A_r} \times \sigma_t$$

$$= \frac{215.98}{314.16} \sigma_t$$

$$\sigma_r = 0.6875\,\sigma_t \longrightarrow (1)$$

(i) When the compressive load on the tube is 20×10^3 N.

$$\sigma_t = \frac{\text{load}}{\text{Area of tube}}$$

$$= \frac{20 \times 10^3}{215.98}$$

$$\boxed{\sigma_t = 92.60\ N/mm^2}$$ (compressive)

(ii) Substituting this value in equation ①

$\sigma_r = 0.6875\ \sigma_t$

$= 0.6875 \times 92.6$

$\boxed{\sigma_r = 63.66\ N/mm^2}$ (tensile)

(iii) Stresses in the rod and tube, when one nut is tightened by one quarter of a turn.

σ_r^* = stress in the rod

σ_t^* = Stress in the tube due to tightening of the nut by one-quarter of a turn.

As the stress in the tube is compressive and stress in the rod in tensile. hence there will be decrease in the length of tube but there will be increase in the length of the rod.

∴ Decrease in the length of tube. = Strain×L

$\Rightarrow \dfrac{\text{Stress in tube}}{E} \times L$

$\dfrac{\sigma_t^*}{2\times10^5} \times 800 = 0.004 \times \sigma_t^*$

Increas in the length of the rod = $\dfrac{\text{Stress in rod}}{E} \times L$

$= \dfrac{\sigma_r^*}{E} \times L$

$= \dfrac{\sigma_r^*}{2\times10^5} \times 800$

$= \dfrac{0.6875\ \sigma_t^*}{2\times10^5} \times 800$

$= 0.00275 \times \sigma_t^*$

Axial advancement of the nut = one-quarter of turn

= $\frac{1}{4}$ of a turn.

But in one turn, the advancement of the nut is

$\frac{1}{4}$ th of 10 mm.

$\therefore$ Axial advancement of the nut = $\frac{1}{4} \times \frac{1}{4} \times 10 = 0.625$ mm

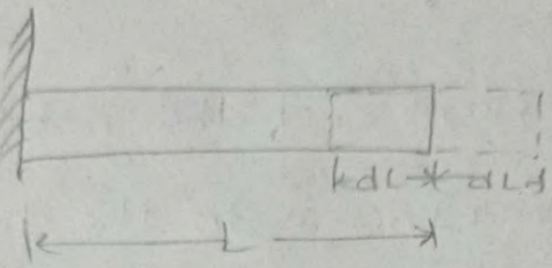
But axial advancement of the nut = Decrease in

length of tube + Increas in the length of rod.

Thermal (or) Temperature Stresses:-

When temperature of a body raised (or) lowered the body is not allowed to expand (or) contrast freely.

Consider a body which is heated to a certain temperature



Where, L = length of the body (original)

dL = Change in length

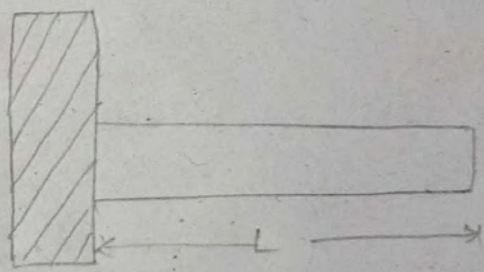
T = Temperature

α = Coefficient of thermal Expansion (or) contraction

Strain $e = \frac{dL}{L}$

$= \frac{\cancel{L} \times \alpha \times T}{\cancel{L}}$

$\boxed{\text{Thermal } e = \alpha \times T}$



$dL = L \times \alpha \times T$

~~Strain~~ Young's modulus $E = \frac{\text{Stress}}{\text{Strain}}$

$E = \frac{\sigma}{e}$

$E = \frac{\sigma}{\alpha \times T}$

$\boxed{\text{Thermal Stress } \sigma = \alpha . T . E}$

case-ii If support yields by certain Amount (δ)

strain $e = \frac{dL}{L}$

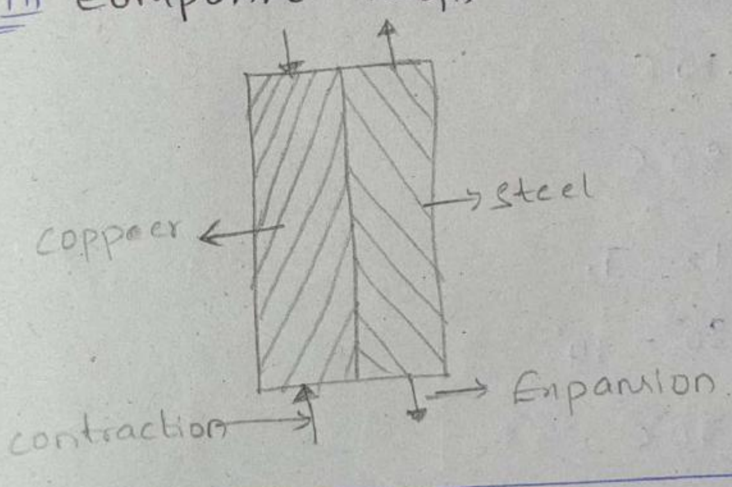
$$e = \frac{L \times \alpha \times T - \delta}{L}$$

Young's modulus $E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{e}$

$$E = \frac{\sigma}{\frac{L \times \alpha \times T - \delta}{L}}$$

$$\boxed{\sigma = \left[\frac{L\alpha T - \delta}{L}\right] \times E}$$

case-iii Composite bars



$$\boxed{\alpha_s T + \frac{\sigma_s}{E_s} = \alpha_c T \mp \frac{\sigma_c}{E_c}}$$

Where,

T = Temperature.

σ_s , σ_c = Thermal stress of steel & copper

α_s , α_c = coefficient of Thermal Expansion (or) contraction of steel and copper

E_s , E_c = Young's modulus of steel and copper for the equilibrium of the system compression in copper should be equal to the tension in the steel.

$$P_c = P_s$$

$$\boxed{\sigma_c \times A_c = \sigma_s \times A_s}$$

* A rod is 2M long at a temperature of 10°c. find the expansion of the rod. When the temperature is raised to 80°c. If this expansion is prevented. find the stress in the material of the rod. Take $E = 1 \times 10^5$ MN/MM², $\alpha = 0.000012$°C.

Sol: Given, data,

length = 2M

$T_1 = 10°C$

$T_2 = 80°C$

$T = T_2 - T_1$

$= 80° - 10°$

$T = 70°C$

$E = 1 \times 10^5 \times 10^6$ N/M²

$\alpha = 0.000012°C$

① Expansion $dL = \alpha \times T \times L$

$= 0.000012 \times 70 \times 2$

$dL = 1.682 \times 10^{-3}$ M

$$\boxed{dL = 0.168 \text{ CM}}$$

(2) stress $\sigma = \alpha \times T \times E$

$= 0.000012 \times 70 \times 1 \times 10^5 \times 10^6$

$\sigma = 84 \times 10^6 \, N/m^2$

$84 \times \frac{\cancel{10^6}}{\cancel{10^6}} \frac{N}{mm^2}$

$\boxed{\sigma = 84 \, N/mm^2}$

* A steel rod of 3cm diameter and 5m long is connected to grips and the rod is maintain at a temperature of 95°C. Determine the stress and pull when the temperature falls to 30°C

If 1) The end, without any yield

2) The end yields by 0.12cm

Take $E = 2 \times 10^5 \, MN/m^2$ $\alpha = 12 \times 10^{-6} \, °/c$

Sol: Given data,

dia = 3 cm = 30 mm

$A = \frac{\pi (30^2)}{4} = 706.86 \, mm^2$

$T_1 = 95°C$ $T_2 = 30°C$

$T = 95 - 30$

$T = 65°C$

$L = 5m = 5000 \, mm$

$E = 2 \times 10^{11} \, N/m^2$

$\alpha = 12 \times 10^{-6} \, °/c$

(1) stress $\sigma = \alpha \times T \times E$

$= 12 \times 10^{-6} \times 65 \times 2 \times 10^{11}$

$\boxed{\sigma = 156 \times 10^6 \, N/m^2}$ $\boxed{\sigma = 156 \, N/mm^2}$

load $P = \sigma \times A$

$= 156 \times \cancel{10^6} \times 706.86$

$$\boxed{P = 110270.16\ N}$$

(2) With yield

$\sigma = \dfrac{L \alpha T - \delta}{L} \times E$

$\delta = 0.12\ cm = 1.2\ mm = 0.012\ m$

$$\sigma = \left[\frac{(5000 \times 12 \times 10^{-6} \times 65) - 1.2}{5000}\right] \times 2 \times \frac{10^{11}}{10^6}\ N/mm^2$$

$$\boxed{\sigma = 108\ N/mm^2}$$

pull $P = \sigma \times A$

$= 108 \times 706.86$

$$\boxed{P = 76340.88\ N}$$

* A steel rod of 20 mm in diameter passes through a copper tube 50 external diameter and 40 mm internal diameter. The tube is closed at each end by rigid plates of negligible thickness. The nuts are tightened lightly home on the projecting parts of the rod. If the temp. of the assembly is raised by 50°C, Calculate the stresses developed in copper & steel. Take E for steel and copper are 200 GN/m² and 100 GN/m² and α for steel and copper is $12 \times 10^{-6}/°C$ & $18 \times 10^{-6}/°C$.

Sol: Given data,

Dia of steel rod = 20 MM = d_s

External dia of copper tube = 50 MM

Internal dia of copper tube = 40 M

Area of steel rod = $\frac{\pi(20)^2}{4}$ = 314.159 MM^2

Area of copper tube = $\frac{\pi}{4}(50^2 - 40^2)$ = 706.86 MM^2

$E_s = 200\ GN/M^2 = 200 \times \frac{10^9}{10^6}\ N/MM^2$

$E_s = 200 \times 10^3\ N/MM^2$

$E_c = 100\ GN/MM^2 = 100 \times \frac{10^9}{10^6}\ N/MM^2$

$E_c = 100 \times 10^3\ N/MM^2$

$\alpha_s = 12 \times 10^{-6} /°C$ $\qquad T = 50°C$

$\alpha_c = 18 \times 10^{-6} /°C$

(i) $P_c = P_s$

$\sigma_c \times A_c = \sigma_s \times A_s$

$\sigma_c \times 706.86 = \sigma_s \times 314.159$

$\cancel{\sigma_c = \sigma_s \times \frac{314.159}{706.86}}$ $\qquad \sigma_s = \frac{706.86}{314.159}\sigma_c$

$\boxed{\sigma_s = 2.25\ \sigma_c} \longrightarrow (1)$

(2) $\alpha_s T + \frac{\sigma_s}{E_s} = \alpha_c T - \frac{\sigma_c}{E_c}$

$$12\times10^{-6}\times50 + \frac{2.25\,\sigma_c}{200\times10^3} = 18\times10^{-6}\times50 - \frac{\sigma_c}{100\times10^3}$$

$$\cancel{6.1125\times10^{-4}\sigma_c = 8.9\times10^{-4}}$$

$$6\times10^{-4} + 1.125\times10^{-5}\sigma_c = 9\times10^{-4} - 1\times10^{-5}\sigma_c$$

$$1.125\times10^{-5}\sigma_c + 1\times10^{-5}\sigma_c = 9\times10^{-4} - 6\times10^{-4}$$

$$2.125\times10^{-5}\sigma_c = 3\times10^{-4}$$

$$\sigma_c = \frac{3\times10^{-4}}{2.125\times10^{-5}}$$

$$\therefore \boxed{\sigma_c = 14.117\ N/mm^2}$$

Substitute σ_c value in equ (1)

$$\sigma_s = 2.25\,\sigma_c$$

$$= 2.25\times14.117$$

$$\boxed{\sigma_s = 31.765\ N/mm^2}$$

* A steel tube of 30MM external diameter and 20MM internal diameter encloses a copper rod of 15MM diameter to which it is rigidly joined at each end. If, at a temperature of 10°C there is no longitudinal stress, calculate the stresses in the rod and tube when the temperature 200°C, Take

the $E_s = 2.1 \times 10^5 \, N/mm^2$ $E_c = 1 \times 10^5 \, N/mm^2$

$\alpha_s = 11 \times 10^{-6} /°C$ $\alpha_c = 18 \times 10^{-6} /°C$.

Sol: Given data,

External diameter of steel tube = 30 mm

Internal diameter of steel tube = 20 mm

Dia of copper rod = 15 mm

$A_s = \frac{\pi (30^2 - 20^2)}{4}$

$\boxed{A_s = 392.69 \, mm^2}$

$A_c = \frac{\pi (15)^2}{4}$

$\boxed{A_c = 176.71 \, mm^2}$

$E_s = 2.1 \times 10^5 \, N/mm^2$

$E_c = 1 \times 10^5 \, N/mm^2$

$\alpha_s = 11 \times 10^{-6} /°C$

$\alpha_c = 18 \times 10^{-6} /°C$

$T_1 = 200°C$

$T_2 = 10°C$

$T = 200° - 10°$

$\boxed{T = 190°}$

(1) $P_s = P_c$

$\sigma_s A_s = \sigma_c A_c$

$\sigma_s \times 392.69 = \sigma_c \times 176.71$

$\sigma_c = \frac{392.69}{176.71} \sigma_s$

$\boxed{\sigma_c = 2.6 \sigma_s}$ $\rightarrow$ (1)

$$\alpha_s T + \frac{\sigma_s}{E_s} = \alpha_c T - \frac{\sigma_c}{E_c}$$

$$11\times10^{-6} \times 190^\circ + \frac{\sigma_s}{2\times10^5} = 18\times10^{-6} \times 190^\circ - \frac{2.2\,\sigma_s}{1\times10^5}$$

$$2.09\times10^{-3} + 5\times10^{-6}\,\sigma_s = 3.42\times10^{-3} - 2.2\times10^{-5}\,\sigma_s$$

$$5\times10^{-6}\,\sigma_s + 2.2\times10^{-5}\,\sigma_s = 3.42\times10^{-3} - 2.09\times10^{-3}$$

$$2.67\times10^{-5}\,\sigma_s = 1.33\times10^{-3}$$

$$\sigma_s = \frac{1.33\times10^{-3}}{2.67\times10^{-5}}$$

$$\sigma_s = 49.81\ N/mm^2$$

Substitute σ_s value in equ (1)

$$\sigma_c = 2.2\times\sigma_s$$

$$= 2.2\times 49.81$$

$$\boxed{\sigma_c = 109.58\ N/mm^2}$$

1) A steel tube of 30 mm ex.dia and 25 mm internal dia encloses a gun metal rod of 20 mm diameter which is rigidly joined up each end. The temperature of the whole assembly raised to 140°C and the nuts on the rod lightly fixed at the ends of the tube. find the intensity of the stress in the rod when temperature fallen to 30°c. E_s=

Sol: Given data,

Ex.dia of steel tube $(D_s) = 30\,mm$

In. dia of steel tube $(d_s) = 25\,mm$

dia of metal rod = 20 mm

$$A_s = \frac{\pi(30^2 - 25^2)}{4}$$

$$A_s = 215.98\,mm^2$$

$$E_s = 2.1 \times 10^5\,N/mm^2$$

$$A_g = \frac{\pi(20)^2}{4}$$

$$E_c = 1 \times 10^5\,N/mm^2$$

$$A_g = 314.159\,mm^2$$

$$\alpha_s = 12 \times 10^{-6}/°C$$

$$\alpha_g = 20 \times 10^{-6}/°C$$

$$T_1 = 140°C$$

$$T_2 = 30°C$$

$$T = 140° - 30°$$

$$\boxed{T = 110°C}$$

① $P_s = P_g$

$$\sigma_s A_s = \sigma_g A_g$$

$$\sigma_s \times 215.98 = \sigma_g \times 314.159$$

$$\sigma_s = \frac{314.159}{215.98}\sigma_g$$

$$\boxed{\sigma_s = 1.45\,\sigma_g} \longrightarrow (1)$$

(2) $\alpha_s T + \frac{\sigma_s}{E_s} = \alpha_g T - \frac{\sigma_g}{E_g}$

$$12\times10^{-6}\times110 + \frac{1.45\sigma_g}{2.1\times10^5} = 20\times10^{-6}\times110 - \frac{\sigma_g}{1\times10^5}$$

$$1.32\times10^{-3} + 6.9\times10^{-6}\sigma_g = 2.2\times10^{-3} - 1\times10^{-5}\sigma_g$$

$$6.9\times10^{-6}\sigma_g + 1\times10^{-5}\sigma_g = 2.2\times10^{-3} - 1.32\times10^{-3}$$

$$1.69\times10^{-5}\sigma_g = 8.8\times10^{-4}$$

$$\sigma_g = \frac{8.8\times10^{-4}}{1.69\times10^{-5}}$$

$$\boxed{\sigma_g = 52.07\ N/mm^2}$$

Substitute σ_g value in equ (1)

$$\sigma_s = 1.45\sigma_g$$

$$= 1.45\times52.07$$

$$\boxed{\sigma_s = 75.5\ N/mm^2}$$

Elastic Constants:-

Longitudinal strain = $\frac{\delta L}{L}$

Lateral strain = $\frac{\delta b}{b}$ (or) $\frac{\delta d}{d}$

volumetric strain = L, b, d $\quad \frac{\delta v}{V}$

Young's Modulus :- $E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{e}$

poission's ratio (μ) = $\frac{\text{Lateral strain}}{\text{longitudinal strain}}$

* Determine the changes in length, width, thickness of a steel bar which is 4 m long and 30 mm width, 20 mm thick, is subjected to an axial pull of 30 KN in the friction of the length. $\mu = 0.3$ $E = 2 \times 10^5$ N/mm^2.

Sol:- Given data,

Length = 4 M = 4000 MM

Width = 30 MM

Thick (d) = 20 MM

pull (P) = 30 KN = 30×10^3 N

$E = 2 \times 10^5$ N/mm^2

$\mu = 0.3$

$\delta L = ?$, $\delta b = ?$, $\delta t = ?$

$E = \frac{\text{stress}}{\text{strain}}$

$\text{stress} = \frac{P}{A} = \frac{30 \times 10^3}{30 \times 20} = 50$ N/mm^2

$e = \frac{\sigma}{E} \Rightarrow e = \frac{50}{2 \times 10^5}$

$\boxed{e = 2.5 \times 10^{-4}}$

$\boxed{\sigma = 50 N/mm^2}$

$\boxed{e = 2.5 \times 10^{-4}}$

$e = \frac{\delta L}{L}$

$\delta L = 2.5 \times 10^{-4} \times 4000$

$\boxed{\delta L = 1 mm}$

$\mu = \frac{\text{Lateral strain}}{\text{longitudinal strain}}$

lateral strain = $\mu \times$ longitudinal strain

$= 0.3 \times 2.5 \times 10^{-4}$

$\boxed{\text{lateral strain} = 7.5 \times 10^{-5}}$

$\frac{\delta b}{b} = 7.5 \times 10^{-5}$

$\delta b = 7.5 \times 10^{-5} \times 30$

$\delta b = 2.25 \times 10^{-3}$

$\boxed{\delta b = 0.00225 mm}$

$\frac{\delta t}{t} = 7.5 \times 10^{-5}$

$\delta t = 7.5 \times 10^{-5} \times 20$

$\boxed{\delta t = 0.0015 mm}$

* Determine the value of young's modulus and poissons ratio of a metallic bar of length 30cm, width 4 cm and depth 4cm. When the bar is subjected to compressive load of 400KN. The decrease in length is given as 0.075 cm and increase in the width is 0.003cm.

Sol:- Given data,

$L = 30\ cm = 300\ mm$

$b = 4\ cm = 40\ mm$

$d = 4\ cm = 40\ mm$

$P = 400\ KN = 400 \times 10^3\ N$

$\delta L = 0.075\ cm = 0.75\ mm$

$\delta b = 0.003 = 0.03\ mm$

$$E = \frac{\text{Stress}}{\text{strain}}$$

$$\text{Stress} = \frac{P}{A} = \frac{400 \times 10^3}{40 \times 40} = 250\ N/mm^2$$

$$\text{Strain} = \frac{\delta L}{L} = \frac{0.75}{300} = 0.0025$$

$$\text{lateral strain} = \frac{\delta b}{b} = \frac{0.03}{40} = 0.00075$$

$$E = \frac{\sigma}{e}$$

$$E = \frac{250}{0.0025}$$

$$\boxed{E = 1 \times 10^5\ N/mm^2}$$

$$\text{poissions Ratio} = \frac{\text{Lateral strain}}{\text{longitudinal strain}}$$

$$\mu = \frac{0.00075}{0.0025}$$

$$\boxed{\mu = 0.3}$$

* Volumetric Strain:-

$$e_v = \frac{\delta V}{V}$$

$$\frac{\delta V}{V} = \frac{\delta l}{l} + \frac{\delta b}{b} + \frac{\delta t}{t}$$

$$\frac{\delta V}{V} = \text{longitudinal} + 2(\text{lateral strain})$$

$\mu = \frac{\text{la. strain}}{\text{long strain}}$

~~lat~~ = $\mu \times$ longitu direct

$$\frac{\delta V}{V} = \text{longitudinal} - 2\mu \times \text{longitudinal}$$

$$\frac{\delta V}{V} = \text{longitudinal}\ (1-2\mu)$$

$$\boxed{\frac{\delta V}{V} = \frac{\delta L}{L}[1-2\mu]}$$

$$\boxed{\frac{\delta V}{V} = \frac{(\sigma_x + \sigma_y + \sigma_z)}{E}(1-2\mu)}$$

*10M

* The Relationship between Young's Modulus and Bulk modulus and Rigidity modulus.

$$\text{Young's Modulus} = \frac{\sigma}{e}$$

$$\text{Bulk Modulus } (K) = \frac{E}{3(1-2\mu)}$$

$$\text{Rigidity Modulus } [C \text{ (or) } G] = \frac{E}{2(1+\mu)}$$

Problem:-

* for a material Young's Modulus $E = 1.2 \times 10^5 \, N/mm^2$, poissons ratio $\mu = 1/4$, Calculate the bulk Modulus.

Sol:-

$E = 1.2 \times 10^5 \, N/mm^2$

$\mu = 1/4 = 0.25$

Bulk Modulus $K = \dfrac{E}{3(1-2\mu)}$

$= \dfrac{1.2 \times 10^5}{3(1-2 \times 0.25)}$

$$\boxed{K = 0.8 \times 10^5 \, N/mm^2}$$

* A bar of 30 mm diameter is subjected a pull of 60 kN. The extension on the gauge length of 200 mm is 0.1 mm. And change in diameter is 0.004 mm. Calculate young's Modulus, poisons ratio, Bulk Modulus.

Sol:- Given data,

$d = 30 \, mm$

$P = 60 \, kN = 60 \times 10^3 \, N$

$L = 200 \, mm$

$\delta d = 0.004 \, mm$

$\delta L = 0.1 \, mm$

i) Young's Modulus $E = \dfrac{\sigma}{e}$

$\sigma = \dfrac{P}{A} = \dfrac{60 \times 10^3}{\frac{\pi (30)^2}{4}}$

$\sigma = 84.88 \, N/mm^2$

$e = \dfrac{\delta L}{L} = \dfrac{0.1}{200}$

$e = 5 \times 10^{-4}$

$$E = \frac{84.88}{5 \times 10^{-4}}$$

$$\boxed{E = 1.697 \times 10^5 \ N/mm^2}$$

2) Poisons Ratiom $(\mu) = \frac{\text{lateral strain}}{\text{longitudinal strain}}$

longitudinal ~~lateral~~ strain $= \frac{\delta l}{L}$

$$= \frac{0.1}{200}$$

$$= 0.0005$$

~~longi.~~ lateral strain $= \frac{\delta d}{d}$

$$= \frac{0.004}{30}$$

$$= 0.00013$$

$$\mu = \frac{0.00013}{0.0005}$$

$$\boxed{\mu = 0.26}$$

3) Bulk Modulus $K = \frac{E}{3(1-2\mu)}$

$$= \frac{1.697 \times 10^5}{3(1 - 2 \times 0.26)}$$

$$\boxed{K = 1.178 \times 10^5 \ N/mm^2}$$

* Determine the poison's ratio and Bulk Modulus of a material for the young's Modulus $1.2\times10^5\ N/mm^2$ and Modulus of regidity $4.8\times10^4\ N/mm^2$.

Sol:- Given that,

$$E = 1.2\times10^5\ N/mm^2$$

$$C = 4.8\times10^4\ N/mm^2$$

Modulus of rigidity $C = \dfrac{E}{2(1+\mu)}$

$$4.8\times10^4 = \frac{1.2\times10^5}{2(1+\mu)}$$

$$1+\mu = \frac{1.2\times10^5}{2\times4.8\times10^4}$$

$$1+\mu = 1.25$$

$$\mu = 1.25-1$$

$$\boxed{\mu = 0.25}$$

Bulk Modulus $K = \dfrac{E}{3(1-2\mu)}$

$$= \frac{1.2\times10^5}{3(1-2(0.25))}$$

$$\boxed{K = 0.8\times10^5\ N/mm^2}$$

04/3/22

* Calculate the Modulus of rigidity and Bulk modulus of a cylindrical bar with diameter 30 mm and length 1.5 m and the longitudinal strain is 4 times of lateral strain. Find the change in volume when the bar is subjected to an hydrostatic pressure 100 N/mm². Take $E = 1\times10^5$ N/mm²

Sol: Given data,

$d = 30$ mm

$l = 1.5$ m $= 1500$ mm

$P = 100$ N/mm²

$E = 1\times10^5$ N/mm²

$$\text{poison's ration} = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

longitudinal strain = 4 × lateral strain

$$\mu = \frac{\text{lateral strain}}{4\times\text{lateral strain}}$$

$$= \frac{1}{4}$$

$$\boxed{\mu = 0.25}$$

① Bulk Modulus $K = \dfrac{E}{3(1-2\mu)}$

$$= \frac{1\times10^5}{3(1-2\times0.25)}$$

$$\boxed{K = 0.66\times10^5 \text{ N/mm}^2}$$

② Modulus of Rigidity C or $G = \dfrac{E}{2(1+\mu)} = \dfrac{1\times10^5}{2(1+0.25)}$

$$\boxed{G = 0.4\times10^5 \text{ N/mm}^2}$$

Bulk Modulus $K = \frac{\text{stress}}{\left(\frac{\delta v}{v}\right)}$

stress = $100 \, N/mm^2$

volume = $A \times L$

$= \frac{\pi (30)^2}{4} \times 1500$

volume = 1.06×10^6 (or) $1060287.52 \, mm^3$

$\Rightarrow \quad 0.66 \times 10^5 = \frac{100}{\frac{\delta v}{1060287.52}}$

$\delta v = \frac{100 \times 1060287.52}{0.66 \times 10^5}$ (or) $\frac{100 \times 1060287.52}{0.667 \times 10^5}$

$\boxed{\delta v = 1606.496 \, mm^3}$ $\boxed{\delta v = 1589.64 \, mm^3}$

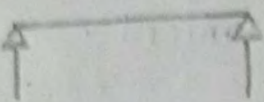
5/3/22

4. Shear force and Bending Moment

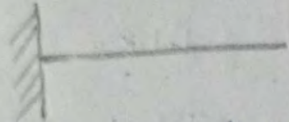
Beam:- It is a Structural element that resist lateral loads with respect to axis of the beam

Types of beam:

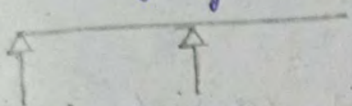
1) Simply supported beam



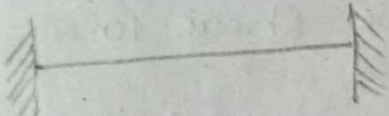
2) Cantilever beam



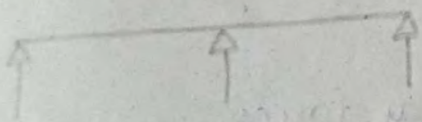
3) Over hanging beam



4) fixed beam

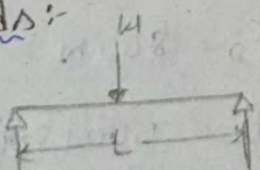


5) Continuous beam

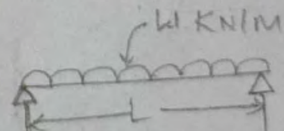


Types of loads:-

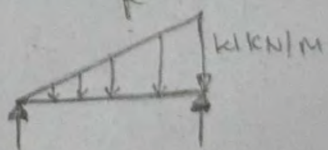
1) Point load



2) Uniformly Distributed load (udl)

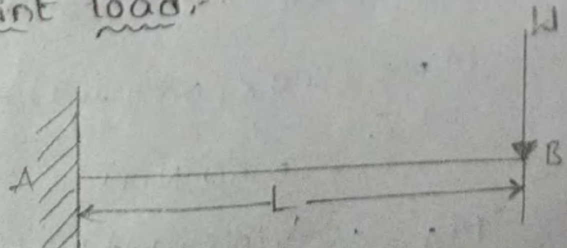


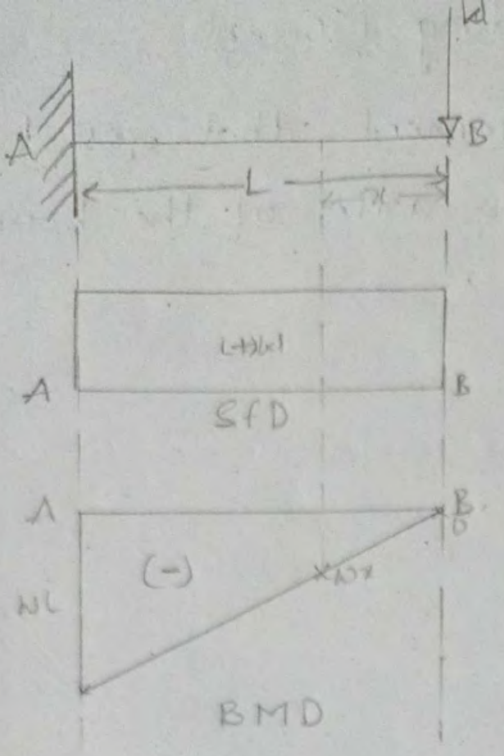
3) uniformly varying load (uvl)



I) Cantilever beam:-

① Point load:-





Shear force (F) = +W

Bending Moment.

$M_B = 0$

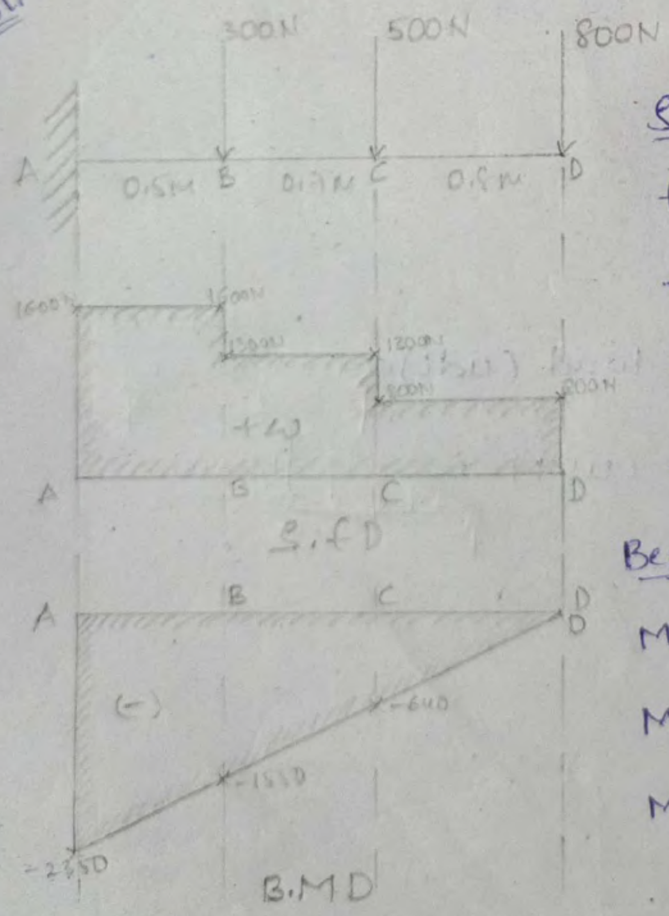
$M_x = -W \times x$

$M_A = -W \times L$

Problems:

* Cantilever beam of length 2m carries point load as shown in figure. Draw the shear force and Bending moment diagram.

Sol:



Shear force:

$F_D = 800\,N$

$F_C = 500 + 800 = 1300\,N$

$F_B = 300 + 500 + 800 = 1600\,N$

$F_A = 1600\,N$

Bending Moment $M_x = -W \times x$

$M_D = 0$

$M_C = -800 \times 0.8 = -640\,N.M$

$M_B = -800 \times (0.8 + 0.7) - 500(0.7)$

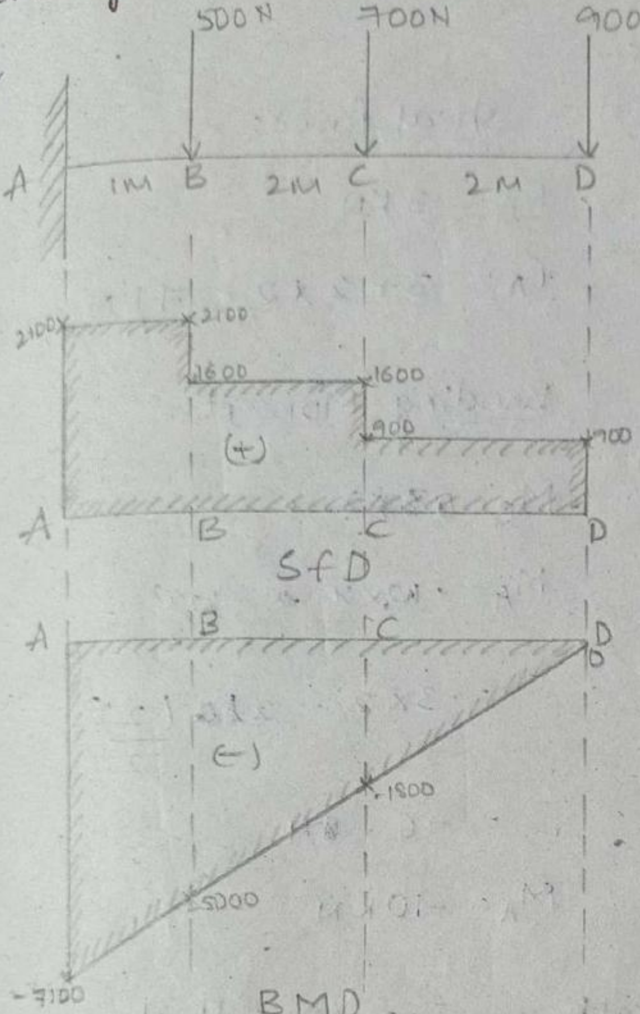
$= -1550\,N.M$

$M_A = -800 \times (0.8 + 0.7 + 0.5) - 500(0.7 + 0.5) - 300(0.5)$

$M_A = -2350\,N.M$

* Cantilever beam of length 5m carries point load as shown in figure. Draw the shear force and Bending Moment Diagram.

Sol:



Shear force:

$f_D = 900\,N$

$f_C = 900 + 700 = 1600\,N$

$f_B = 900 + 700 + 500 = 2100\,N$

$f_A = 2100\,N$

Bending Moment: $M_x = -W \times x$

$M_D = 0$

$M_C = -900(2) = -1800\,N.M$

$M_B = -900(2+2) - 700(2)$

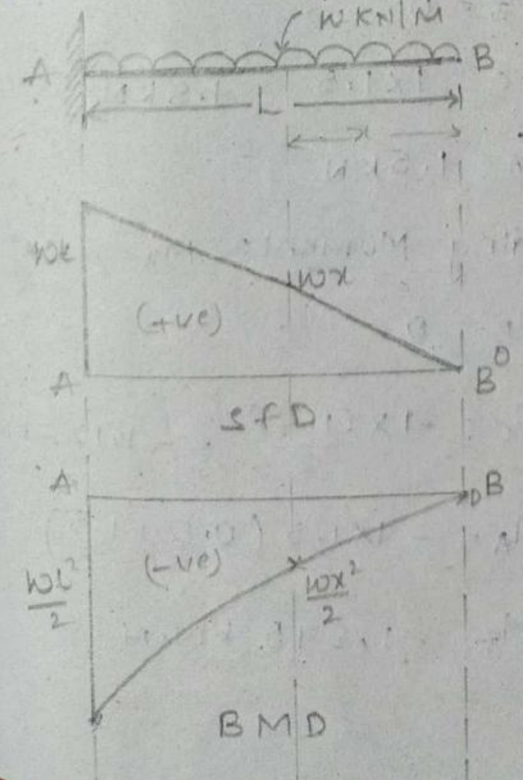
$= -3600 - 1400$

$= -5000\,N.M$

$M_C = -900(2+2+1) - 700(2+1) - 500(1)$

$= -7100\,N.M$

② UDL [uniformly distributed load]:-



Shear force:

$f_B = 0$

$f_x = +W \times x$

$f_A = +W \times L$

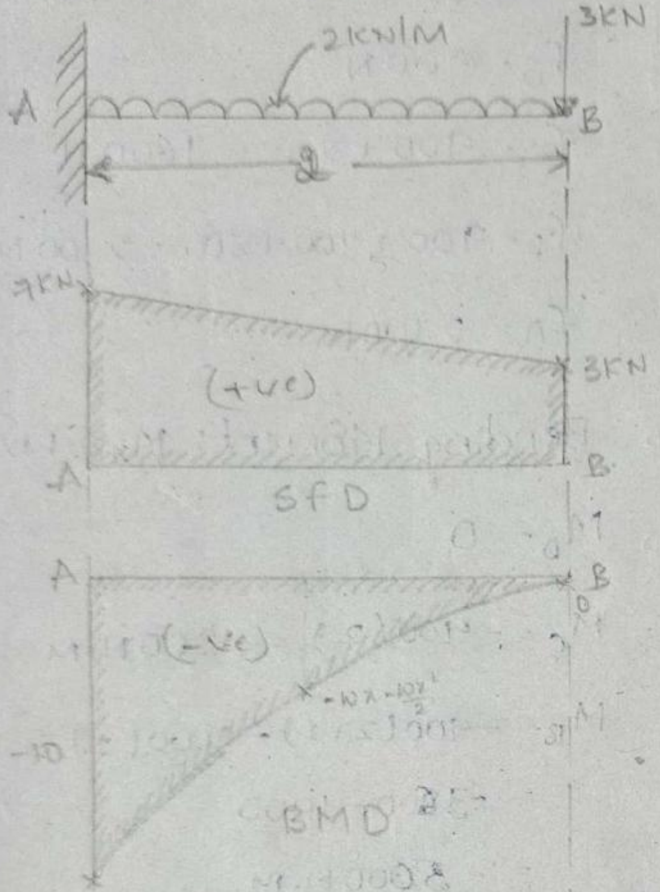
Bending Moment:

$M_B = 0$

$M_x = -W \times x \times \frac{x}{2} = -\frac{Wx^2}{2}$

$M_A = -W \times L \times \frac{L}{2} = -\frac{WL^2}{2}$

* A cantilever beam of length 2m carries a udl of 2 KN/m over the whole length and point load 3 KN at the free end. Draw the shear force and bending moment diameter.



<u>Shear force:</u>

$f_B = 3\,KN$

$f_A = 3 + 2\times2 = 7\,KN$

<u>Bending Moment:</u>

$M_B = 0$

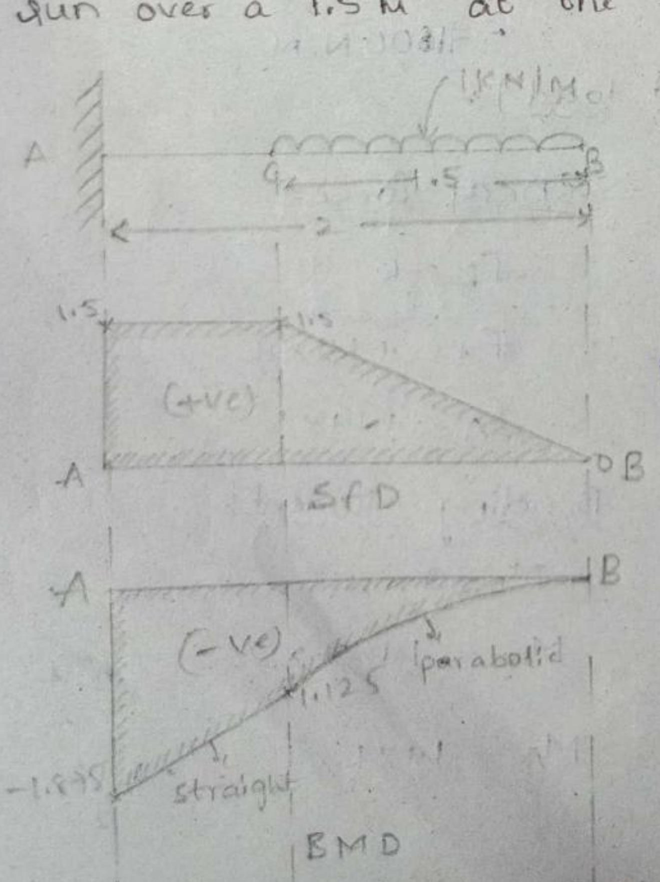
$M_A = -W_1 \times x - \frac{W_2 x^2}{2}$

$= -3\times2 - 2\frac{(2)^2}{2}$

$= -6 - 4$

$M_A = -10\,KN$

* A cantilever beam of length 2m carries udl of 1 KN/m run over a 1.5 m at the free end. Draw SFD and BMD.



<u>Shear force:</u>

$f_B = 0$

$f_C = 1\times1.5 = 1.5\,KN$

$f_A = 1.5\,KN$

<u>Bending Moment:</u> $M_x = -\frac{wx^2}{2}$

$M_B = 0$

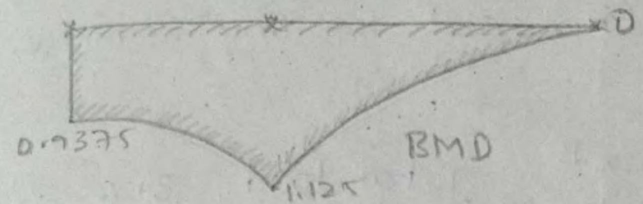
$M_C = -1\times\frac{(1.5)^2}{2} = -1.125\,KN.m$

$M_A = -1\times1.5\left(0.5 + \frac{1.5}{2}\right)$

$M_A = -1.875\,KN.m$

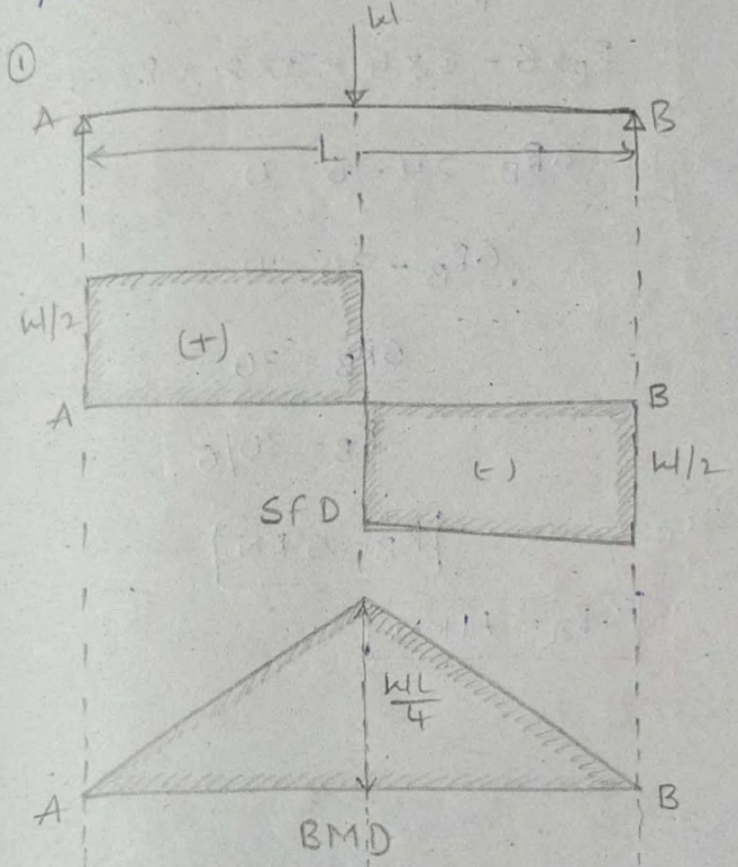
A.S.A

$M_A = -\frac{1.5}{2}\left(0.5 + \frac{1.5}{2}\right) = -0.9375$



Simply Supported beam:-

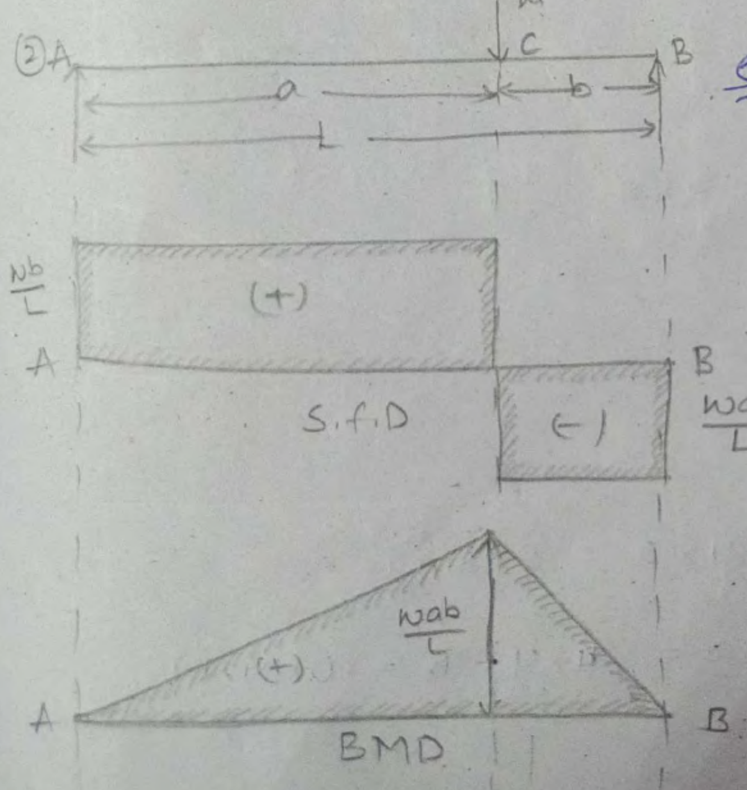
① Point load:-



Shear force:-

$+\frac{W}{2}$ to $-\frac{W}{2}$

Bending Moment:- $\frac{WL}{4}$



Shear force:-

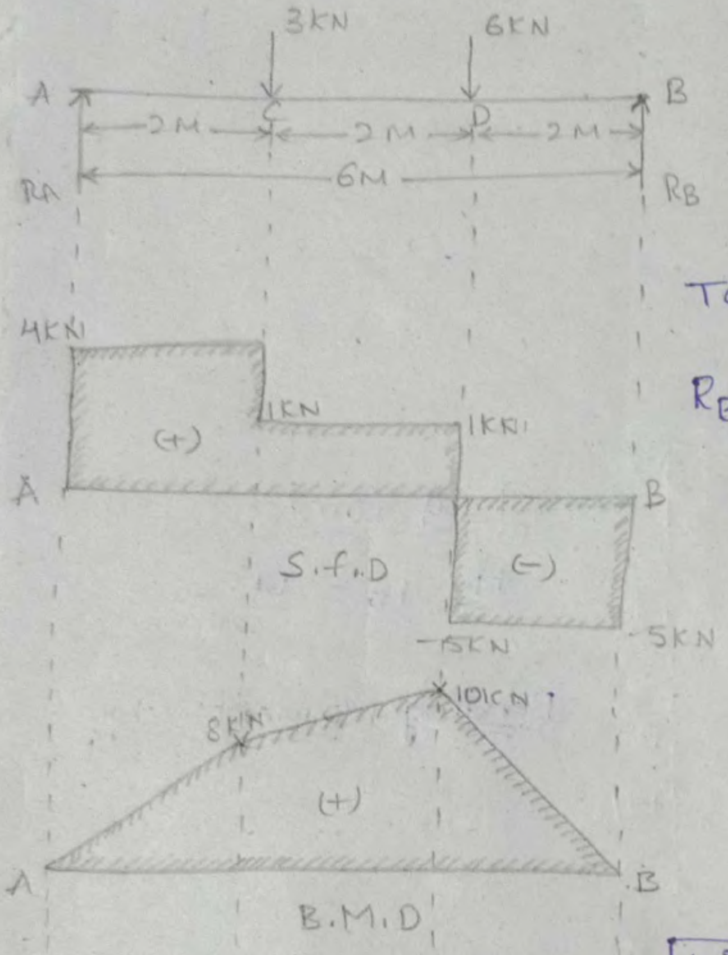
$R_A = \frac{Wb}{L}$

$R_B = \frac{Wa}{L}$

Bending Moment:-

$M_C = \frac{Wab}{L}$

* A simply supported beam of length 6M carries a point load 3KN and 6KN at the distance of 2M, 4M from the left end. Draw shear force and bending moment.



Total load = $3+6 = R_A + R_B$

$R_A + R_B = 9\,KN \longrightarrow (1)$

Taking moments @ A = 0

$R_B \times 6 - 6\times4 - 3\times2 + R_A\times 0 = 0$

$6R_B - 24 - 6 = 0$

$6R_B - 30 = 0$

$6R_B = 30$

$R_B = 30/6$

$\boxed{R_B = 5\,KN}$

$\boxed{\therefore R_A = 4\,KN}$

Shear force:-

$f_A = 4\,KN$

$f_C = +4-3 = 1\,KN$

$f_D = +4-3-6 = -5\,KN$

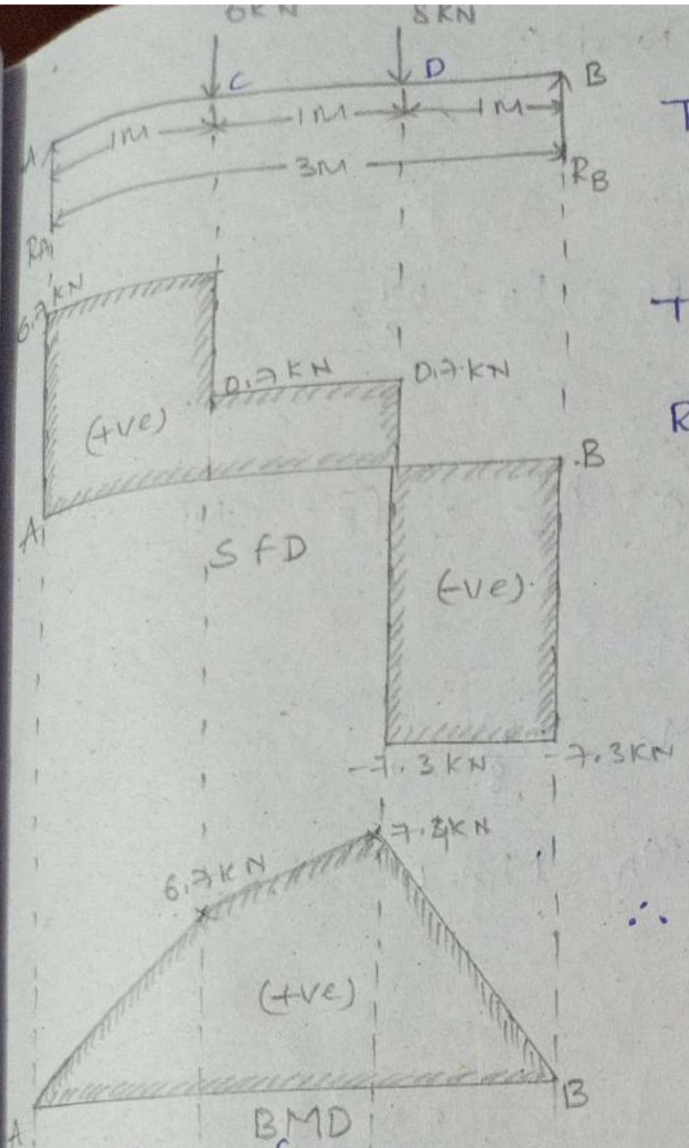
$f_B = -5\,KN$

Bending Moment:-

$M_A = M_B = 0$

$M_C = R_A \times 2 = 4\times2 = 8\,KN.M$

$M_D = R_A \times 4 - 3\times2 = 4\times4 - 6 = 10\,KN.M$



Total load $= 6+8 = R_A + R_B$

$R_A + R_B = 14$ KN

Taking moments @ A = 0

$R_B \times 3 - 8 \times 2 - 6 \times 1 + R_A \times 0 = 0$

$3R_B - 16 - 6 = 0$

$R_B = \frac{22}{3}$

$R_B = 7.3$ KN

$\therefore R_A = 14 - 7.3$

$R_A = 6.7$ KN

Shear force:-

$f_A = 6.7$ KN

$f_C = 6.7 - 6 = 0.7$ KN

$f_D = 6.7 - 6 - 8 = -7.3$ KN

$f_B = -7.3$ KN

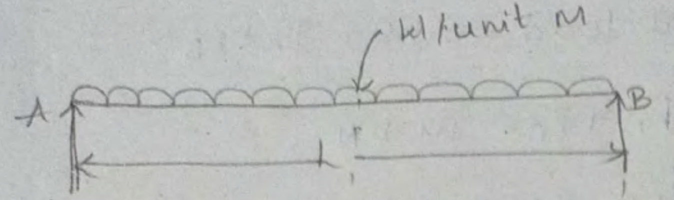
Bending Moment:-

$M_A = 0 \quad M_B = 0$

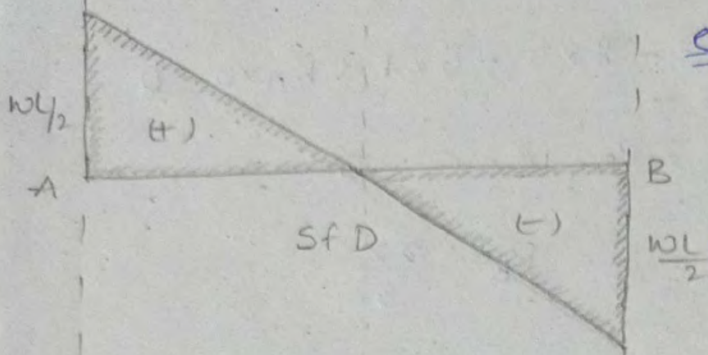
$M_C = R_A \times 1 = 6.7 \times 1 = 6.7$ KN.m

$M_D = R_A \times 2 - 6 \times 1 = 6.7 \times 2 - 6 = 7.4$ KN.M

② Udt (uniformly distributed load):-



$$R_A = R_B = \frac{wL}{2}$$



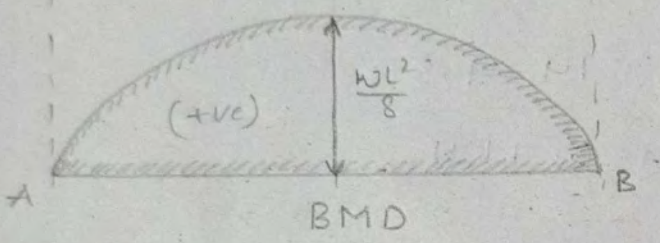
S.f:-

$$f_A = \frac{wL}{2}, \quad f_B = -\frac{wL}{2}$$

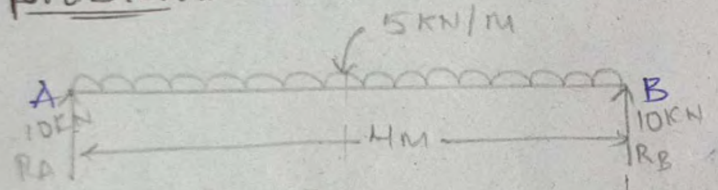
B.M

$$M_C = \frac{wL^2}{8},$$

$$M_A = M_B = 0$$



problem:-

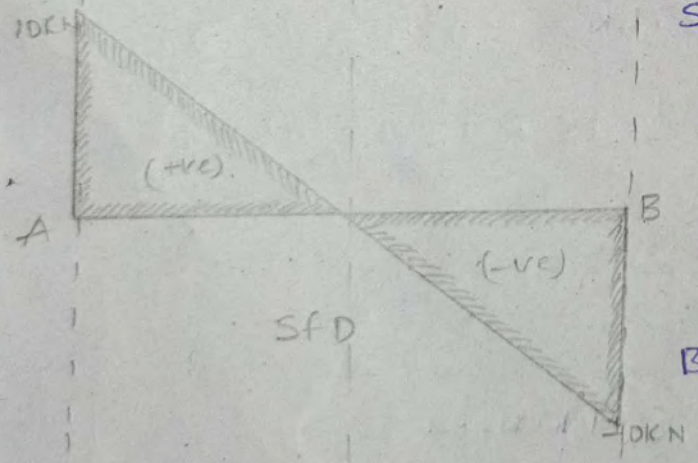


$$R_A = R_B = \frac{5 \times 4}{2} = 10\,KN$$

Shear force:-

$$f_A = \frac{wL}{2} = \frac{5 \times 4}{2} = 10\,KN$$

$$f_B = -\frac{wL}{2} = -\frac{5 \times 4}{2} = -10\,KN$$



Bending Moment:-

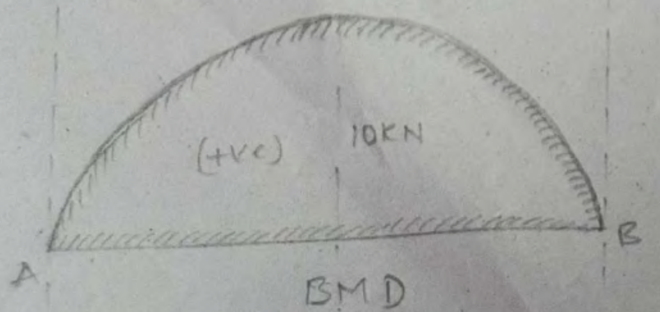
$$M_C = \frac{wL^2}{8}$$

$$= \frac{5(4)^2}{8}$$

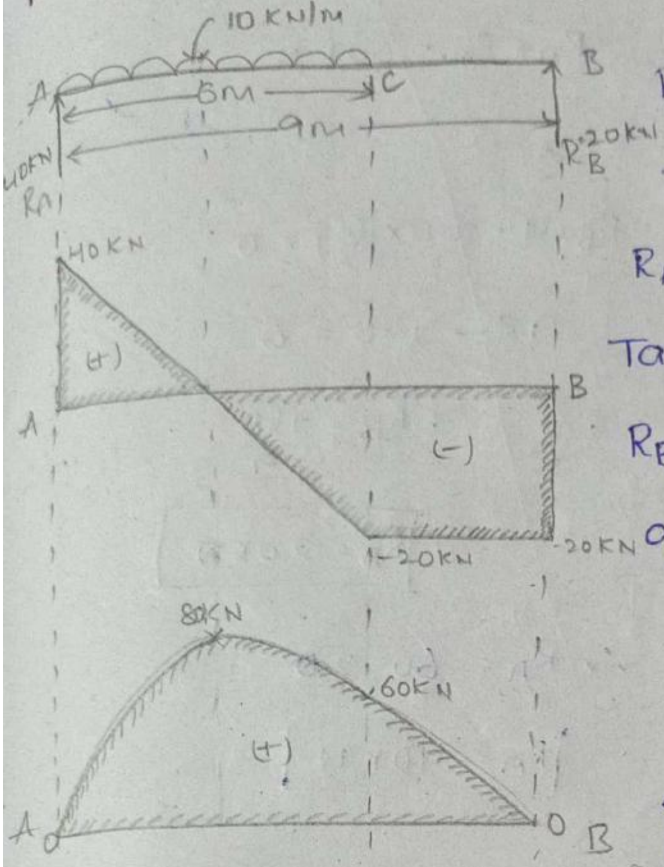
$$M_C = 10\,KN$$

$$M_A = 0$$

$$M_B = 0$$



Draw the Shear force and Bending Moment diagram for simply supported beam length 9M. carring udl 10 KN/M upto a distance of 6M from the left end.



Reactions:

Total load = $10 \times 6 = 60KN$

$R_A + R_B = 60KN$

Taking Moments @ A = 0

$R_B \times 9 - 10 \times 6 \left(\frac{6}{2}\right) + R_A \times 0 = 0$

$9R_B - 180 = 0$

$R_B = \frac{180}{9}$

$\boxed{R_B = 20KN}$

$\boxed{R_A = 40KN}$

Shear force:

$f_A = 40KN$

$f_C = 40KN - 10 \times 6 = -20KN$

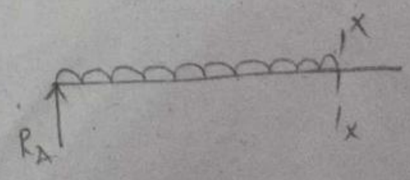
$f_B =$ ~~40 - 10x6~~ $= -20KN$

Bending Moment:

$M_A = M_B = 0$

$M_C = 6 \times 10 = 60KN.M$

Max. BM Shear force $_{x-x} = 0$



$R_A - 10 \times x = 0$

$40 - 10 \times x = 0$

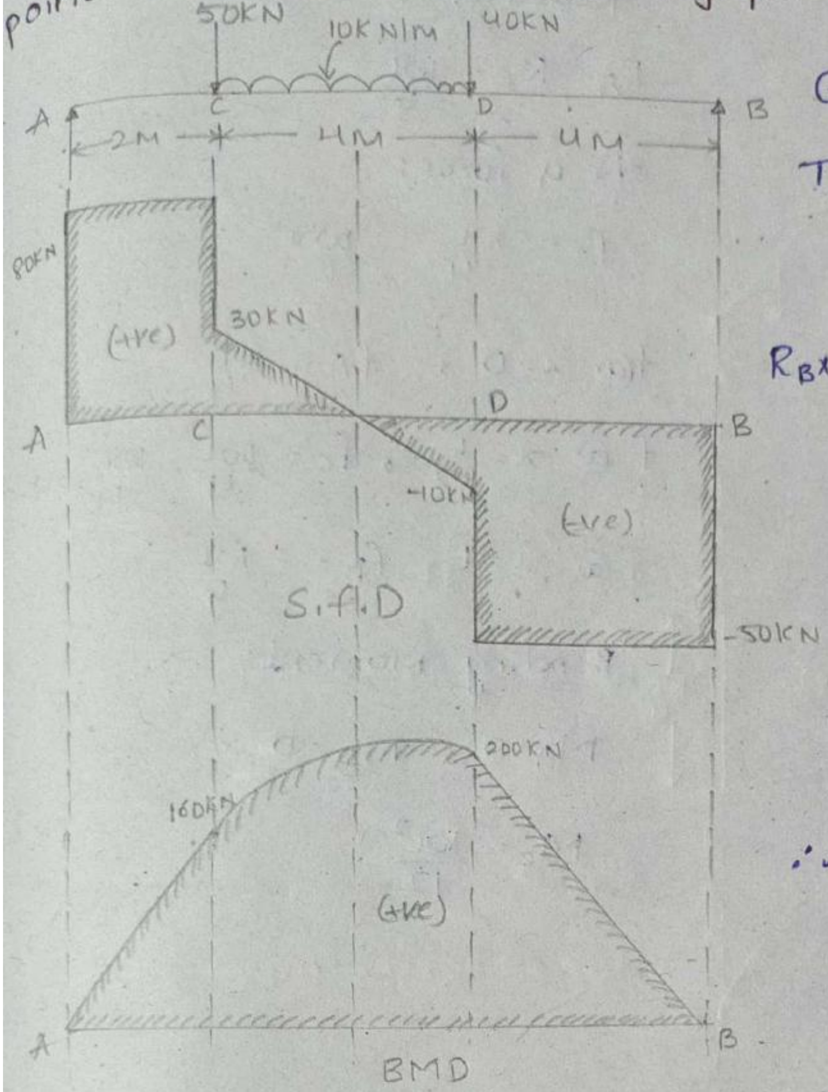
$\boxed{x = 4M}$

$M_x = R_A \times 4 - 10 \times 4 \times \frac{4}{2}$

$M_x = 40 \times 4 - 80$

$\boxed{M_x = 80KN.M}$

*A simply supported beam L = 10 m carries udl and point load as shown in figure.



Given data,

Total load = 40 + (10×4) + 50

$R_A + R_B = 130$ KN

$R_B \times 10 - 40\times6 - 10\times4\left(2+\frac{4}{2}\right) - 50\times2 = 0$

$R_B \times 10 - 500 = 0$

$R_B = \frac{500}{10}$

$\boxed{R_B = 50 KN}$

$\therefore R_A = 130 - 50$

$\boxed{R_A = 80 KN}$

Shear force:-

$f_A = 80$ KN

$f_C = 80 - 50 = 30$ KN

$f_D = 80 - 50 - 10\times4 = -10$ KN

$f_B = 80 - 50 - (10\times4) - 40 = -50$ KN

Bending Moment:-

$M_A = 0$ $\quad M_B = 0$

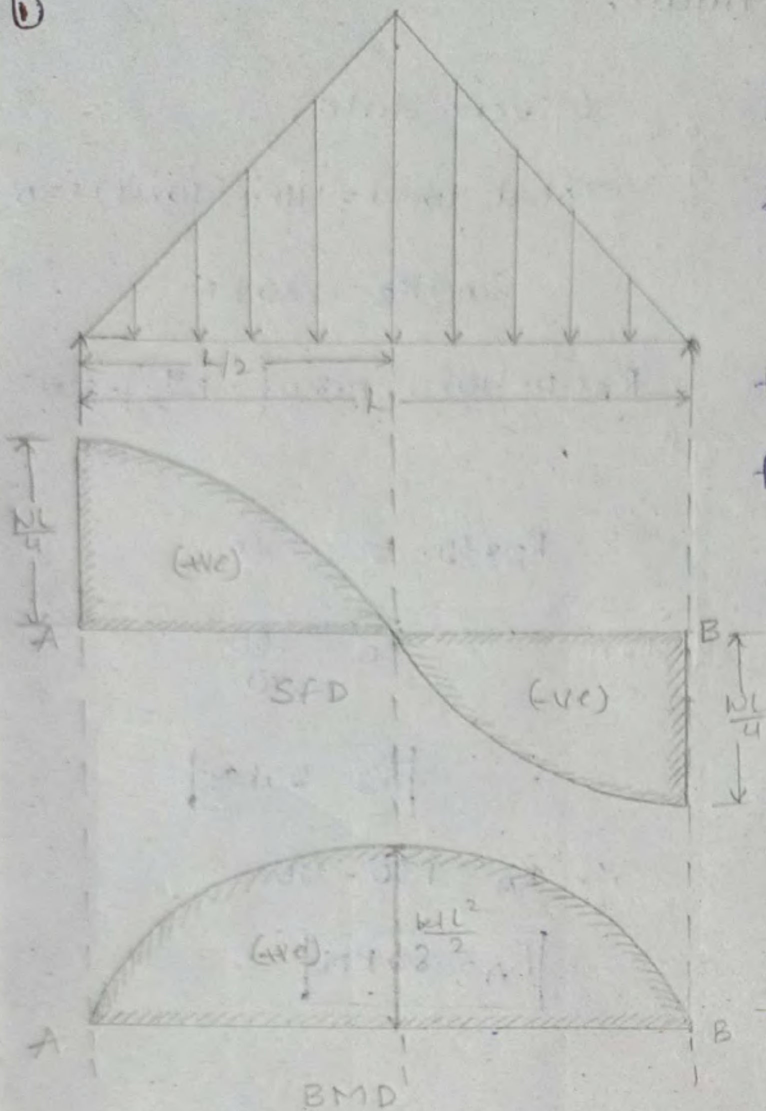
$M_C = 80\times2 = 160$ KN.m

$M_D = 80\times6 - 50\times4 - (10\times4)\frac{4}{2} - 40\times0$

$M_D = 200$ KN.m

UVL

①



Reactions:-

$R_A = R_B = \frac{wL}{4}$

Shear force:-

$f_x = \frac{wL}{4} - \frac{wx^2}{L}$

$f @ x=0, \quad f_A = \frac{wL}{4}$

$f @ x=\frac{L}{2}, \quad f_C = \frac{wL}{4} - \frac{wL}{4} = 0$

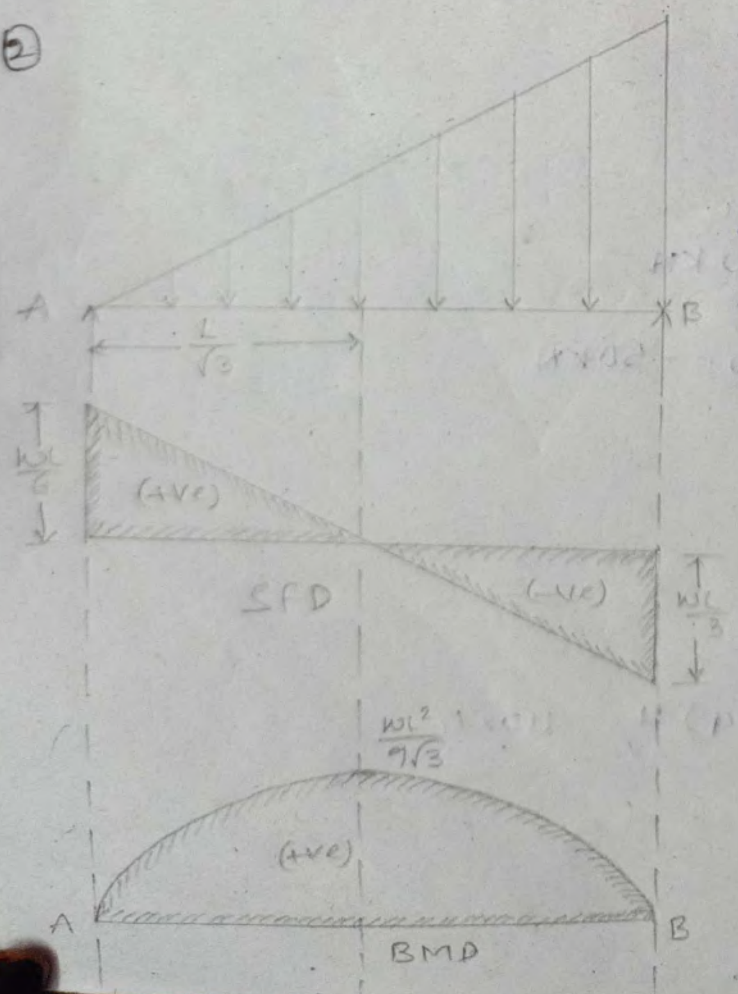
$f @ x=L, \quad f_B = -\frac{wL}{4}$

Bending moment:-

$M_A = 0, \quad M_B = 0$

$M_C = \frac{wL^2}{12}$

②



Reactions:-

$R_A = \frac{wL}{6}; \quad R_B = \frac{wL}{3}$

Shear force:-

$f @ x=0,$

$f_A = \frac{wL}{2}$

$f_B = -\frac{wL}{3}$

$f_C = 0$

Bending moment:-

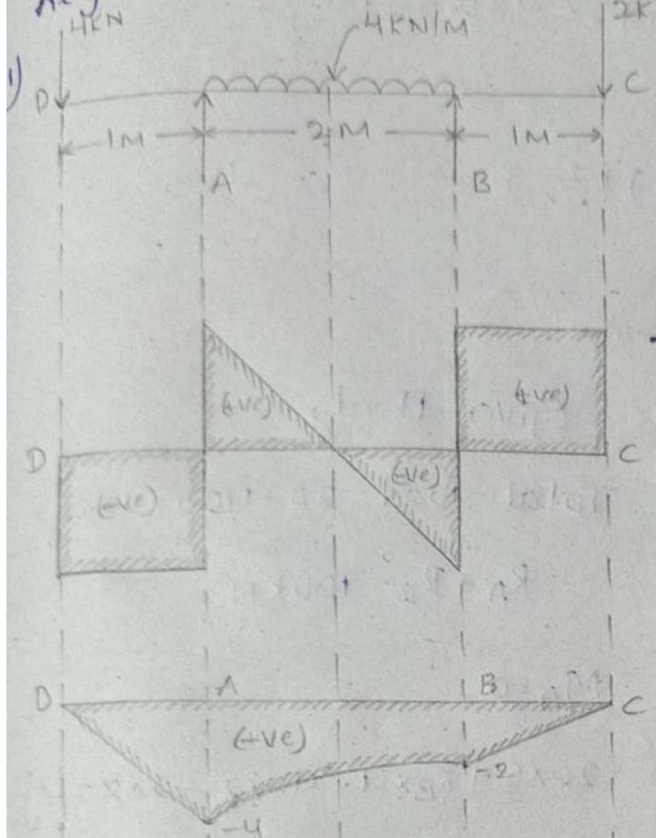
$M_A = 0 \quad M_B = 0$

$B.M = \frac{wL^2}{9\sqrt{3}}$

Over Hanging beam

point of Contraflexure: It is the point when the B.M is zero. After changing from sign positive to negative (or) negative to positive.

1)



Given that,

Total load = $4+(4\times2)+2$

$R_A+R_B = 14\text{ KN} \longrightarrow (1)$

Taking moments @ D = 0

$-2\times4 + R_B\times3 - (4\times2)\left(1+\frac{2}{2}\right) + R_A\times1 = 0$

$-8+3R_B-16+R_A=0$

$3R_B+R_A=24 \longrightarrow (2)$

Solving equation (1) and (2)

$R_A+R_B=14\text{KN}$

$3R_A+R_B=24\text{KN}$

$2R_B=10\text{KN}$

$R_B=\frac{10}{2}=5\text{KN}$

$\boxed{R_B=5\text{KN}}$

$R_A=14-R_B$

$=14-5$

$\boxed{R_A=9\text{KN}}$

Shear force:

$f_C=2\text{KN}$

$f_B=2-5=-3\text{KN}$

(L.H.S) $f_A=2-5+4(2)=5\text{KN}$

$f_D=-4\text{KN}$

$f_A=2-5+4(2)-9=-4\text{KN}$

$2-R_B+4x=0$

$2-5+4x=0$

$x=\frac{3}{4}$

$\boxed{x=0.75\text{M}}$

Bending Moment:-

$M_C = 0$

$M_B = -2\times1 = -2\ KN.M$

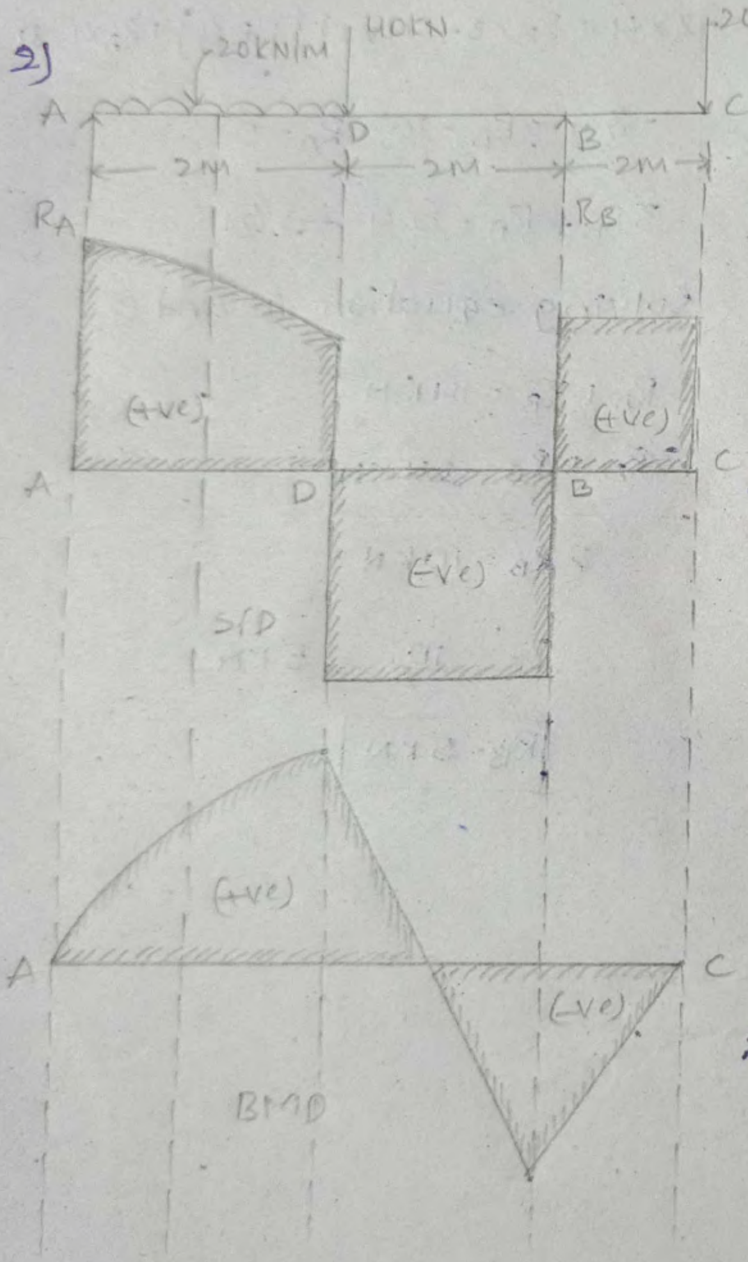
$M_x = -2\times(1+0.75)+5\times0.75-4\times0.75\times\frac{0.75}{2}$

$M_x = -0.875\ KN.M$

$M_A = -2\times3+R_B\times2\,(-4\times2)\left(\frac{2}{2}\right)$

$M_A = -4\ KN.M.$

2)



Given that,

Total load $= 20+40+(20\times2)$

$R_A + R_B = 100\ KN$

$M_A = 0$

$-20\times5+R_B\times4-40\times2-20\times2\left[\frac{2}{2}\right] = 0$

$-100+R_B\times4-80-40=0$

$R_B = \frac{220}{4}$

$\boxed{R_B = 55\ KN}$

$\therefore\ R_A+R_B = 100$

$R_A = 100 - R_B$

$= 100-55$

$\boxed{R_A = 45\ KN}$

Shear force:-

$f_C = 20 KN$

$f_B = 20 - 55 = -35 KN$

$f_D = 20 - 55 + 20 \times 2 = 5 KN$

$f_A = 20 - 55 + 40 + 20 \times 2 = 45 KN$

$\boxed{f_A = 45 KN}$

Bending Moment:-

$M_C = 0 \;, \; M_A = 0$

$M_B = -20 \times 1 = -20 KN.M$

$M_D = -20 \times 3 + (55 \times 2) = 50 KN.M$

point of Contraflexure:-

$M_x = 0$

$-20(1+x) + 55x = 0$

$-20 - 20x + 55x = 0$

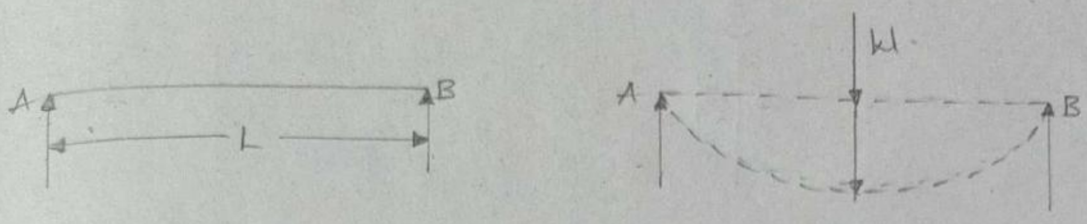
$35x = 20$

$x = \frac{20}{35}$

$\boxed{x = 0.571 M}$

UNIT-V Deflection of Beams

Uniform bending - slope, deflection and Radius of curvature.



Before Loading After loading

If a beam carries udl (or) point load the beam will be deflected from its original position.

The Radius of curvature:

$$\frac{M}{I} = \frac{E}{R}$$

Where,

M = Moment [Bending Moment]

I = Moment of Inertia

E = Young's Modulus

R = Radius of curvature.

*

Design Parameters:-

Deflection = y

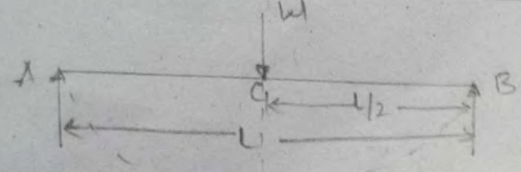
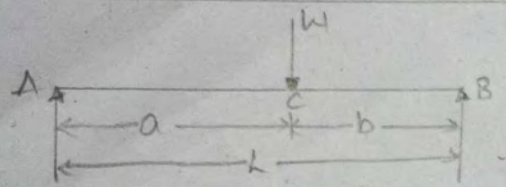
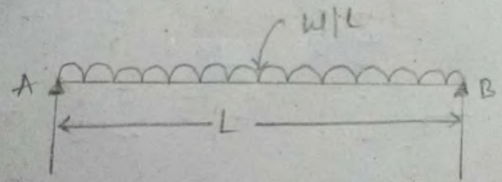
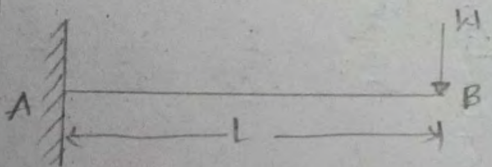
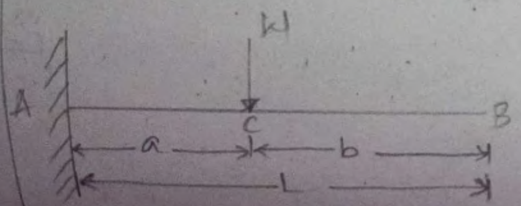
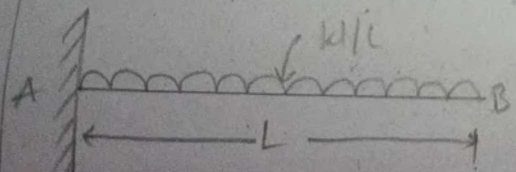
Slope = $\theta = \frac{dy}{dx}$

B.M = $\frac{d^2y}{dx^2} \times EI$

S.F = $\frac{d^3y}{dx^3} \times EI$

Rate of loading = $\frac{d^4y}{dx^4} \times EI$

Deflection and slope formulaes:-

Beam + load	Deflection	Slopes θ_A	θ_B	θ_C
	$y_C = \dfrac{-WL^3}{48EI}$	$-\dfrac{WL^2}{16EI}$	$-\dfrac{WL^2}{16EI}$	–
	$y_C = \dfrac{-Wa^2b^2}{3EIL}$	$\dfrac{-Wab}{6EIL}(a+2b)$	–	$\dfrac{Wab}{3EIL}(a-b)$
	$y_C = \dfrac{-5}{384} \cdot \dfrac{WL^3}{EI}$	$\dfrac{-WL^3}{24EI}$	$-\dfrac{WL^3}{24EI}$	–
	$y_B = \dfrac{-WL^3}{3EI}$	–	$-\dfrac{WL^2}{2EI}$	–
	$y_C = -\dfrac{Wa^3}{3EI}$	–	$-\dfrac{Wa^2}{2EI}$	$-\dfrac{Wa^2}{2EI}$
	$y_B = \dfrac{WL^3}{8EI} = \dfrac{wL^4}{8EI}$	–	$-\dfrac{WL^2}{6EI}$	–

Methods of determining Slope & deflection:-

1) Double Integration Method:-

$$M = EI \frac{d^2y}{dx^2}$$

$$\int \frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\int \frac{dy}{dx} = \text{slope}$$

y = deflection

2) Macauly's Method:-

$$M_x = \frac{M}{EI} \frac{d^2y}{dx^2}$$

y_x = ~~Slope~~ deflection

3) Mohr's theorem and Moment Area method:-

$$\theta = \frac{\text{Area of B.M.D}}{EI} = \frac{A}{EI}$$

$$y = \frac{A\bar{x}}{EI}$$

* Where θ = slope

y = deflection

A = Area of B.M.D

$\bar{x}$ = distance of C.G of area

E = Young's Modulus

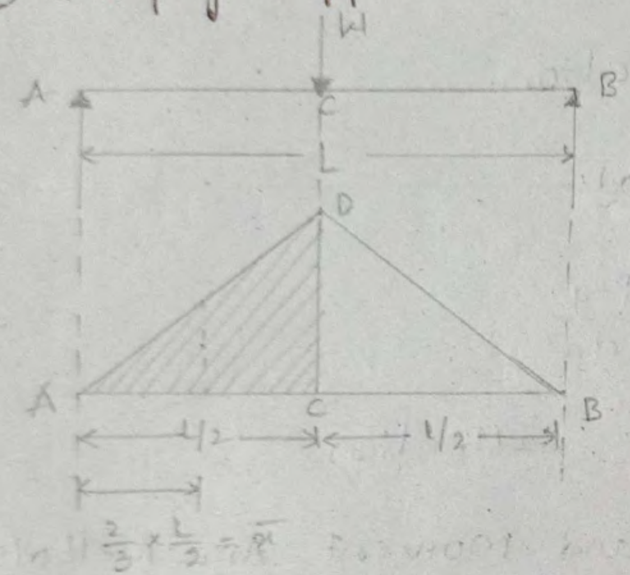
I = Moment of Inertia

EI = Flexural Rigidity.

1) The change of slope between any two points is equal to the net area of the bending moment diagram. (BMD) between the points divided by EI.

2) The total deflection between any two points is equal to the moment of Area of bending moment diagram divided by EI

① Simply Supported beam - [Point load]:-



BMD

$$\text{Slope} = \theta = \frac{A}{EI} \text{ (from A to C)}$$

$$A = \frac{1}{2} \times AC \times CD$$

$$A = \frac{1}{2} \times \frac{L}{2} \times \frac{WL}{4}$$

$$A = \frac{WL^2}{16}$$

$$\boxed{\text{Slope } \theta = \frac{WL^2}{16EI}}$$

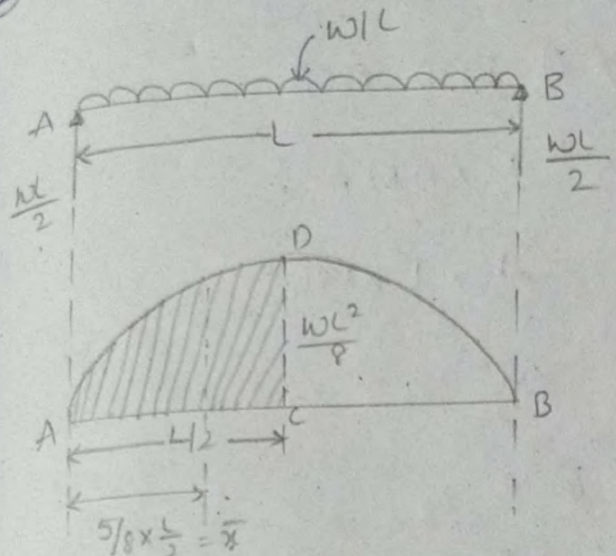
Deflection $y = \frac{A\bar{x}}{EI}$

$$= \frac{WL^2}{16EI} \times \bar{x}$$

$$= \frac{WL^2}{16EI} \times \frac{\cancel{2}}{3} \times \frac{L}{\cancel{2}}$$

$$\boxed{y = \frac{WL^3}{48EI}}$$

② Simply supported beam - [Udl]:-



Slope $= \theta = \frac{A}{EI}$ (B.M.D from A to C)

ACD Δ

$A = \frac{2}{3} \times AC \times CD$

$= \frac{2}{3} \times \frac{L}{2} \times \frac{wL^2}{8}$

$A = \frac{wL^3}{24}$

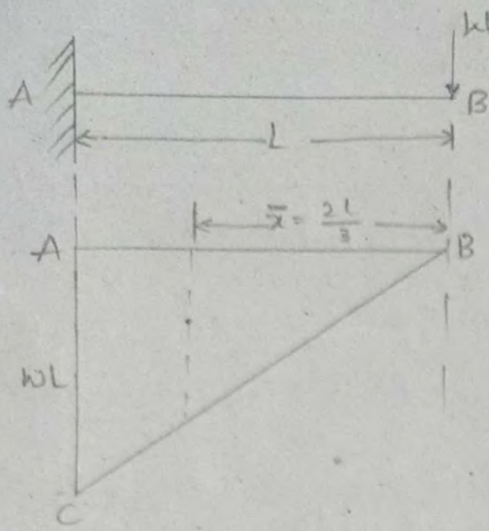
$$\boxed{\text{Slope } \theta = \frac{wL^3}{24EI}}$$

deflection $y = \frac{A\bar{x}}{EI}$

$= \frac{wL^3}{24EI} \times \frac{5}{8} \times \frac{L}{2}$

$$\boxed{y = \frac{5wL^4}{384EI} \text{ (or) } \frac{wL^4}{EI} \frac{5}{384}}$$

(3) Cantilever beam - [point load @ free end] :-



Area of BMD ACB

$A = \frac{1}{2} \times AB \times AC$

$= \frac{1}{2} \times L \times WL$

$A = \frac{WL^2}{2}$

Slope @ B

$\theta_B = \frac{A}{EI} = \frac{WL^2}{2EI}$

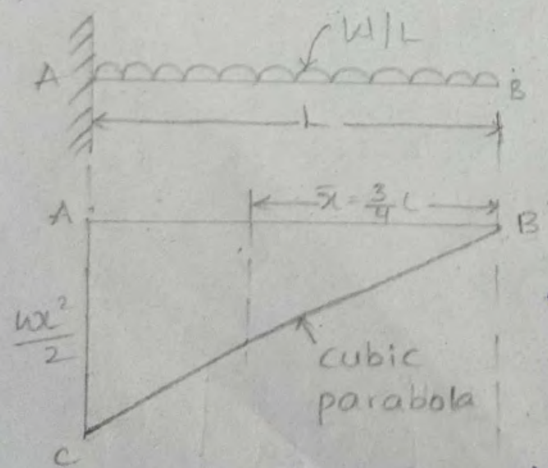
deflection $y = \frac{A\bar{x}}{EI}$

$\bar{x}$ = distance of C.G of area B.M.D from B = $\frac{2L}{3}$

$$y = \frac{\frac{WL^2}{\cancel{2}}}{EI} \times \frac{\cancel{2}L}{3}$$

$$\boxed{y = \frac{WL^3}{3EI}}$$

(4) Cantilever beam - [Udl] :-



Area of BMD ABC

$A = \frac{1}{3} \times L \times \frac{WL^2}{2}$

$A = \frac{WL^3}{6}$

$$\boxed{\text{Slope } \theta_B = \frac{A}{EI} = \frac{WL^3}{6EI}}$$

Deflection $y = \frac{A\bar{x}}{EI}$ where $\bar{x}$ = distance of C.G of BMD from B

$\bar{x} = \frac{3L}{4}$

$y = \frac{WL^3}{\cancel{6}_2 EI} \times \frac{\cancel{3}L}{4}$

$$\boxed{y_B = \frac{WL^4}{8EI}}$$

* Conjugate beam:-

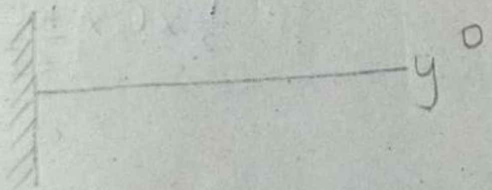
1) Conjugate beam its an imaginary beam of length equal to the original beam for the load diagram is equal to bending moment at the point is divided by EI.

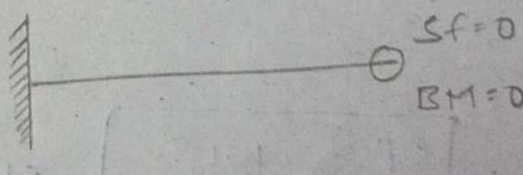
2) The slope at any section of the given beam is equal to the shear force of the conjugate beam.

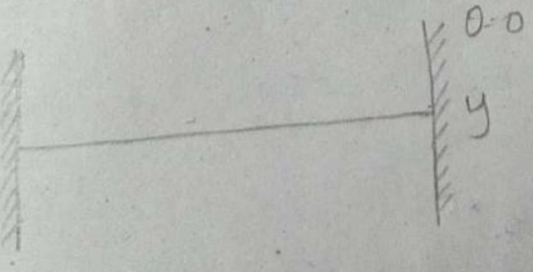
3) The deflection at any section for the given beam is equal to the bending moment of the conjugate beam.

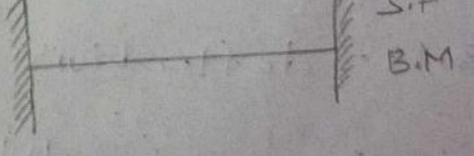
Relation b/w conjugate beam and Actual beam:-

Actual beam	conjugate beam.
* simply supported or rollers support at the end deflection	* simply supported at the end bending moment = '0' but shear force exist.
* At the free end slope and deflection exist.	* At the free end shear force and BM are zero.
* At the fixed end slope and deflection are zero.	* At the fixed end S.f and BM exist.





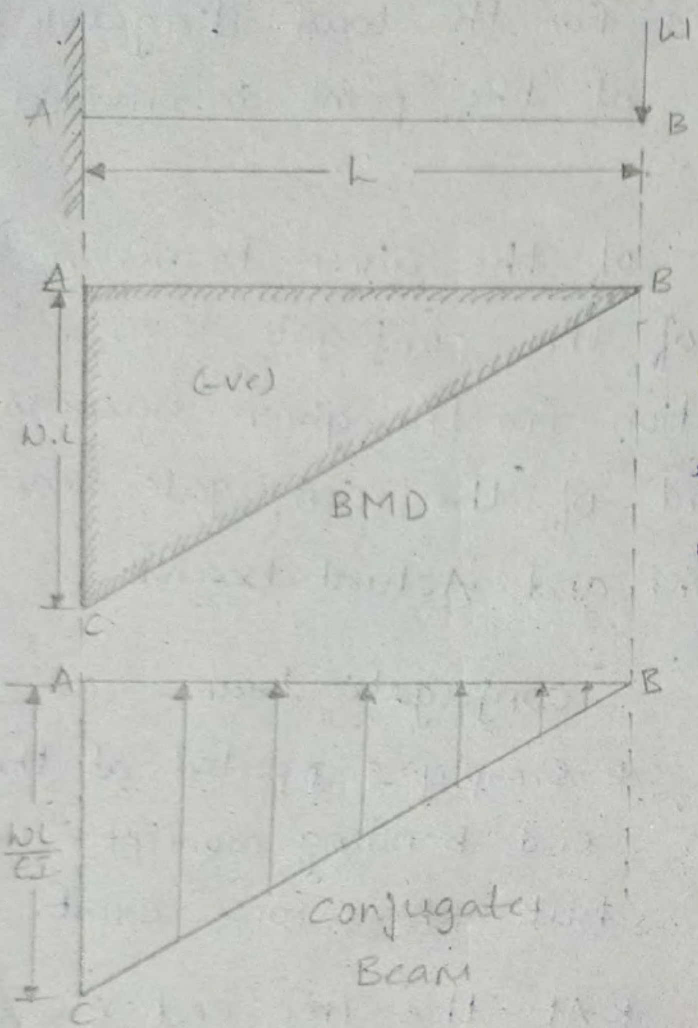




* B.M diagram positive

* $\frac{M}{EI}$ will be positive.

Slope and deflection of a cantilever with a point load at the free end:-



A cantilever beam of length and carrying point load 'W' at the free end.

The bending moment is zero at free end and B.M at A = WL

* The conjugate beam when we draw by B.M divided by EI as shown in figure.

Total load on conjugate beam = SF

$$= \frac{1}{2} AB \times AC$$

$$= \frac{1}{2} \times L \times \frac{WL}{EI}$$

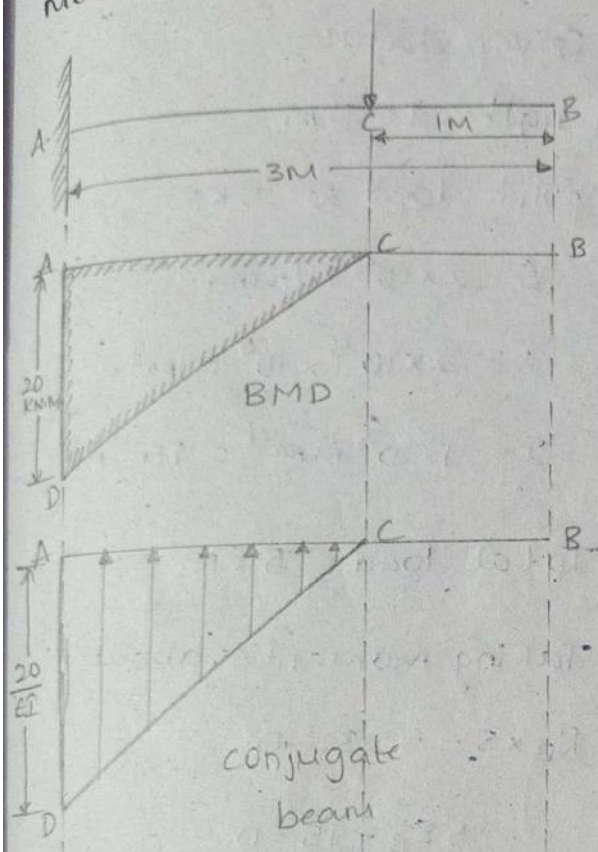
$$\boxed{SF = \frac{WL^2}{2EI}} \rightarrow \text{Slope}$$

B.M = deflection

$$= \frac{1}{2} \times L \times \frac{WL}{EI} \times \frac{2}{3} \times L$$

$$\boxed{y = \frac{WL^3}{3EI}}$$

* Cantilever beam of length 3M carries a point load 10KN at a distance of 2M from the fix end. If $E = 2\times10^5 N/MM^2$ and $I = 10^8 mm^4$. find the slope and deflection at the free end using conjugate beam method.



Given data,

$L = 3M = 3000 MM$

$E = 2\times10^5 N/MM^2$

$I = 10^8 MM^4$

$BM @ A = -20 KN.M$

$= -20\times10^3 N.M$

$= -20\times10^3\times10^3 MM$

$= -20\times10^6 MM$

$\theta = \frac{1}{\cancel{2}} \times \cancel{2} \times \frac{20}{EI}$

$\theta = \frac{20\times10^6}{2\times10^5\times10^8}$

$\theta = 1\times10^{-6}$ rad.

Bending moment = load x centroidal distance

$= 1 + \frac{2}{3}(2)$

centroidal distance = 2.33M

$BM = \left(\frac{1}{2}\times2\times10^3\times\frac{20\times10^6}{EI}\right)\left[1+\frac{2}{3}\times2\right]$

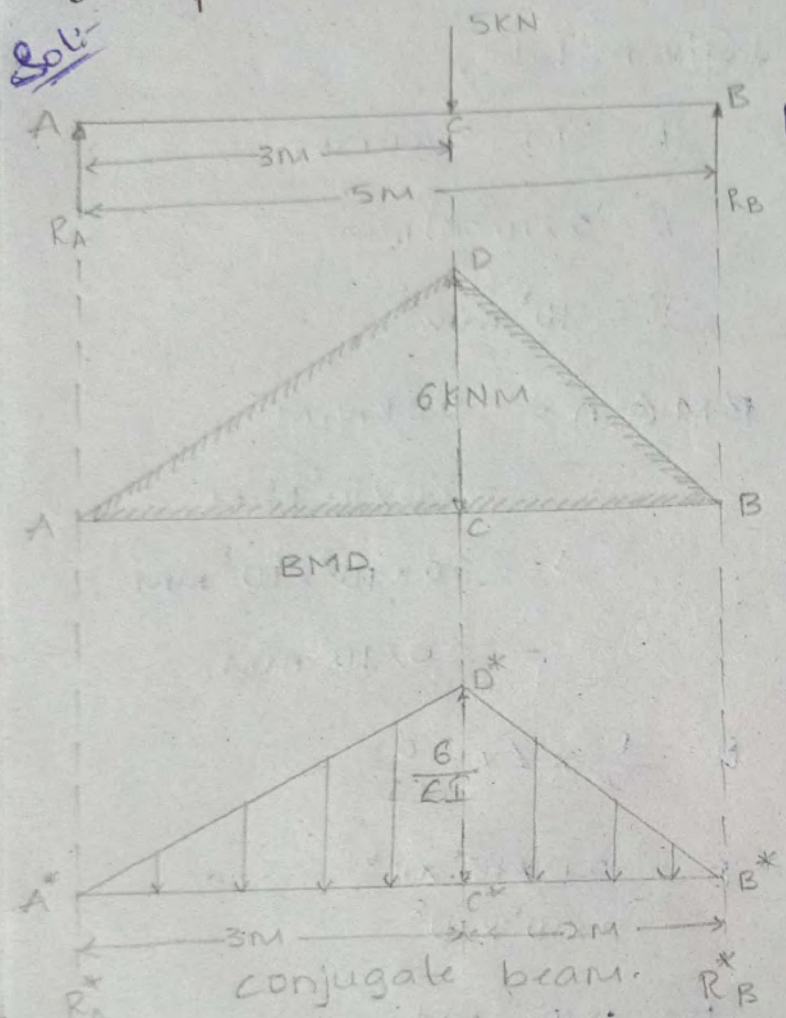
$= \left(\frac{1}{2}\times2\times10^3\times\frac{20\times10^6}{2\times10^5\times10^8}\right)\left(1+\frac{2}{3}\times2\right]$

$BM = y = 0.00233 MM$

$\boxed{y = 2.33MM}$

* A simply supported beam of length 5m carries a point load of 5KN at a distance of 3m from the left end. If $E = 2\times10^5 \, N/mm^2$. and $I = 10^8 \, mm^4$, determine the slope at the left support and deflection under the point load using conjugate beam method.

Sol:-



Given data:-

length $l = 5M$

point load $w = 5KN$

$E = 2\times10^5 \, N/mm^2$

$= 2\times10^5 \times 10^6 \, N/m^2$

$I = 1\times10^8 \, mm^4 = 10^{-4} m^4$

Total load = 5 KN

Taking moments about A,

$R_B \times 5 - 5\times3 = 0$

$5R_B - 15 = 0$

$R_B = \frac{15}{5}$

$\boxed{R_B = 3KN}$

$R_A = 5 - R_B$

$R_A = 5 - 3 \Rightarrow \boxed{R_A = 2KN}$

Bending Moment:-

$M_A = 0 \qquad M_B = 0$

$M_C = R \times 3 = 2\times3$

$M_C = 6KN.M$

Now construct the conjugate beam. The vertical load at C^* on conjugate beam $= \frac{BM@C}{EI}$

$$= \frac{6 KN.M}{EI}$$

$R_A^* = ?$, $R_B^* = ?$

R_A^* = Reaction at A^* for conjugate beam

R_B^* = Reaction at B^* for conjugate beam

Taking moments about A^*, we get

$R_B^* \times 5$ = load on $A^*C^*D^*$ x distance of C.G of $A^*C^*D^*$ from A^* + load on $B^*C^*D^*$ x distance of C.G of $B^*C^*D^*$ from A^*

$$= \left[\frac{1}{2} \times 3 \times \frac{6}{EI}\right] \times \left[\frac{2}{3} \times 3\right] + \left(\frac{1}{2} \times 2 \times \frac{6}{EI}\right) \times \left[3 + \frac{1}{3} \times 2\right]$$

$$= \frac{18}{EI} + \frac{6}{EI} \times \frac{11}{3}$$

$$= \frac{18}{EI} + \frac{22}{EI}$$

$$= \frac{40}{EI}$$

$$R_B^* = \frac{40}{EI} \times \frac{1}{5}$$

$$\boxed{R_B^* = \frac{8}{EI}}$$

R_A^* = Total load (i.e., load $A^*B^*D^*$) $- R_B^*$

$$= \left[\frac{1}{2} \times 5 \times \frac{6}{EI}\right] - \frac{8}{EI}$$

$$= \frac{15}{EI} - \frac{8}{EI}$$

$$\boxed{R_A^* = \frac{7}{EI}}$$

let,

θ_A = Slope at A for the given beam i.e., $\left(\frac{dy}{dx}\right)$ at A

y_c = Deflection at c for the given beam.

Then According to conjugate beam method,

θ_A = shear force at A^* for conjugate beam = R_A^*

$$= \frac{7}{EI}$$

$$= \frac{7}{2\times10^8\times10^{-4}}$$

$$\boxed{\theta_A = 0.00035 \text{ radians}}$$

y_c = BM at c^* for conjugate beam

$= R_A^* \times 3$ - load $A^*C^*D^*$ x distance of C.G of $A^*C^*D^*$ from C^*

$$= \frac{7}{EI}\times3 - \left[\frac{1}{2}\times3\times\frac{6}{EI}\right]\times\left[\frac{1}{3}\times3\right]$$

$$= \frac{21}{EI} - \frac{9}{EI}$$

$$= \frac{12}{EI}$$

$$y_c = \frac{12}{2\times10^8\times10^{-4}}$$

$$\boxed{y_c = 0.6\text{ mm}}$$

* A simply supported beam of length 4m carries a point of 3KN at a distance of 1m from each end. If $E = 2\times10^5$ N/mm^2 and $I = 10^8$ mm^4 for the beam, then using conjugate beam method determine:

(i) Slope at each end and under each load
(ii) deflection under each load and at the centre.

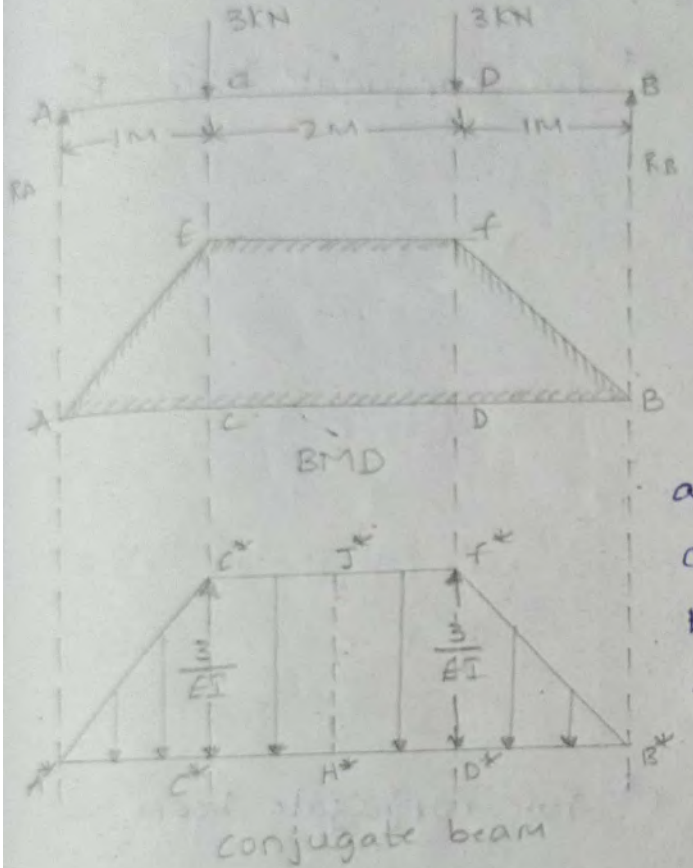
Sol:- Given data.

length $L = 4\,M$

$E = 2\times10^5\,N/MM^2 = 2\times10^5\times10^6\,N/M^2$

$= 2\times10^5\times10^3\,KN/m^2 = 2\times10^8\,KN/M^2$

$I = 10^8\,MM^4 = \frac{10^8}{10^{12}}\,M^4 = 10^{-4}\,M^4.$



Bending Moment

$M_C = R_A\times1 = 3\times1 = 3\,KNM$

$M_D = R_B\times1 = 3\times1 = 3\,kNM$

Now by dividing the BM at any section by EI, we can construct the conjugate beam as shown in figure. The loading are shown on the conjugate beam.

let, $R_A^* =$ Reaction at A^* for the conjugate beam

$R_B^* =$ Reaction at B^* for conjugate beam

The loading on the conjugate beam is Symmetrical

$R_A^* = R_B^* =$ Half of total load on conjugate beam

$= \frac{1}{2}[$Area of trapezoidal $A^*B^*f^*E^*]$

$= \frac{1}{2}\left[\frac{(E^*f^* + A^*B^*)}{2}\times E^*C^*\right]$

$= \frac{1}{2}\left(\frac{(2+4)}{2}\times\frac{3}{EI}\right.$

$$\boxed{R_A^* = \frac{4.5}{EI}}$$

i) Slope at each end and under each load,

Let, θ_A = Slope at A for the given beam i.e., $\left(\frac{dy}{dx}\right)$ at A.

θ_B = Slope at B for the given beam

θ_C = Slope at C for the given beam

θ_D = Slope at D for the given beam

Then According to conjugate beam method,

θ_A = Shear force at A^* for conjugate beam $= R_A^*$

$$= \frac{4.5}{EI}$$

$$= \frac{4.5}{2\times10^8\times10^{-4}}$$

$$\boxed{\theta_A = 0.000225 \text{ rad}}$$

$$\theta_B = R_B^* = \frac{4.5}{EI}$$

$$\boxed{\theta_B = 0.000225 \text{ rad}}$$

θ_C = Shear force at C^* for conjugate beam

$= R_A^*$ - Total load $A^*C^*D^*$

$$= \frac{4.5}{EI} - \frac{1}{2}\times1\times\frac{3}{EI}$$

$$= \frac{3}{EI}$$

$$= \frac{3}{2\times10^8\times10^{-4}}$$

$$\boxed{\theta_C = 0.00015 \text{ rad}}$$

Similarly, $\boxed{\theta_D = 0.00015 \text{ rad}}$ [By symmetry]

ii) Deflection under each load

Due to symmetry; the deflection under each load

let, y_C = deflection at C for the given beam and

y_D = deflection at D for the given beam

Now according to conjugate beam method,

y_C = B.M at C^* for conjugate method

$= R_A^* \times 1.0 -$ (load $A^* C^* E^*$) $\times$ distance of C.G of C.G of $A^* C^* E^*$ from C^*.

$$= \frac{4.5}{EI} \times 1 - \left[\frac{1}{2} \times 1 \times \frac{3}{EI}\right] \times \frac{1}{3}$$

$$= \frac{4.5}{EI} - \frac{0.5}{EI}$$

$$= \frac{4.0}{EI}$$

$$= \frac{4}{2 \times 10^8 \times 10^{-4}}$$

$$\boxed{y_C = 0.2 \text{ mm}}$$

and also, $\boxed{y_D = 0.2 \text{ mm}}$

Deflection at centre of beam.

= B.M at centre of the conjugate beam

$= R_A^* \times 2.0 -$ load $A^* C^* E^* \times$ distance of $A^* C^* E^*$ from the centre of beam $-$ load $C^* H^* J^* E^* \times$ distance of C.G of $C^* H^* J^* E^*$ from the centre of beam.

$$= \frac{4.5}{E.I} \times 2.0 - \left[\frac{1}{2} \times 1 \times \frac{3}{EI}\right] \times \left[1 + \frac{1}{3}\right) - \left[1 \times \frac{3}{EI}\right] \times \frac{1}{2}$$

$$= \frac{9}{EI} - \frac{2}{EI} - \frac{1.5}{EI}$$

$$= \frac{5.5}{EI}$$

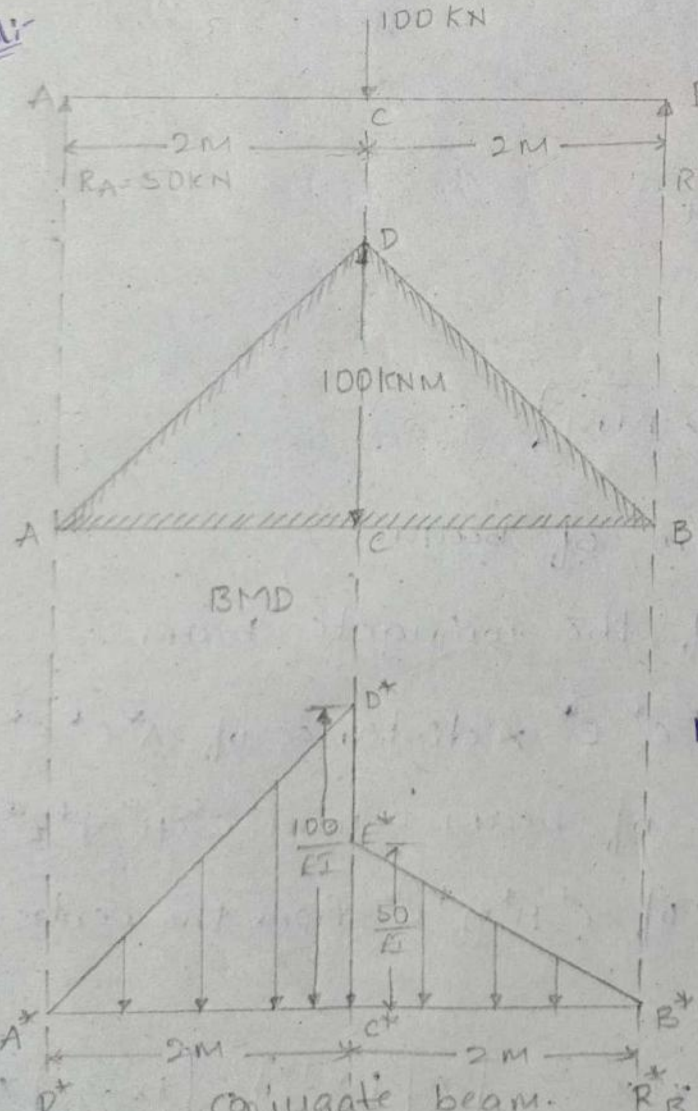
$$= \frac{5.5}{2\times10^{8}\times10^{-4}}$$

$$= 0.000275 \text{ m}$$

deflection @ center of beam. $= 0.275$ mm

* A simply supported beam AB of span 4 m carries a point of 100 kN at its centre. The value of I for the left half is 1×10^{8} mm^4 and for the right half portion I is 2×10^{8} mm^4. Find the slopes at the two supports and deflection under the load.
Take $E = 200$ GN/m^2.

Soln:-



Given data,

$l = 4$ m

length AC = length of BC = 2 m

$W = 100$ kN

Moment of Inertia for AC

$I = 1\times10^{8}$ mm^4

$I = 10^{-4}$ m^4

Moment of Inertia for BC

$I = 2\times10^{8}$ mm^4

$= 2\times10^{-4} = 2I$ $[10^{-4} = I]$

$E = 200$ GN/m^2

$= 200\times10^{9}$ N/m^2

$= 200\times10^{6}$ kN/m^2

The reactions at A and B will be equal, as point load is acting at the centre.

$$R_A = R_B = \frac{100}{2} = 50\,KN$$

Now, BM at A and B are zero

$$BM @ C = R_A \times 2 = 50 \times 2$$

$$= 100\,KNM$$

Now we can construct the conjugate beam by dividing B.M at any section by the product of E and M.O.I.

The conjugate beam is shown in diagram. The loading are shown on the conjugate beam. The loading on the length A^*C^* will be $A^*C^*D^*$ whereas the loading on length B^*C^* will be $B^*C^*E^*$.

The ordinates $C^*D^* = \dfrac{BM @ C}{E \times MI \text{ for } AC} = \dfrac{100}{EI}$

The ordinates $C^*E^* = \dfrac{BM @ C}{E \times MI \text{ for } BC} = \dfrac{100}{E \times 2I} = \dfrac{50}{EI}$

let, R_A^* = Reactions at A^* for conjugate beam

R_B^* = Reactions at B^* for conjugate beam

$R_A^* = ?$ $R_B^* = ?$

Taking moments of all forces about A^*, we get

$R_B^* \times 4$ = load $A^*C^*D^*$ x distance of C.G of $A^*C^*D^*$ from A + load $B^*C^*E^*$ x distance of C.G of $B^*C^*E^*$ from A^*

$$4R_B^* = \left(\frac{1}{2} \times 2 \times \frac{100}{EI}\right) \times \left[\frac{2}{3} \times 2\right] + \left[\frac{1}{2} \times 2 \times \frac{50}{EI}\right] \times \left[2 + \frac{1}{3} \times 2\right]$$

$$= \frac{400}{3EI} + \frac{400}{3EI}$$

$$4 R_B^* = \frac{800}{3EI}$$

$$R_B^* = \frac{200}{3EI}$$

and R_A^* = Total load on conjugate beam $- R_B^*$

$$= \left[\frac{1}{2} \times 2 \times \frac{100}{EI} + \frac{1}{2} \times 2 \times \frac{50}{EI} \right] - \frac{200}{3EI}$$

$$= \frac{150}{EI} - \frac{200}{3EI}$$

$$R_A^* = \frac{250}{3EI}$$

(i) Slopes at the supports

let,

θ_A = Slope at A i.e., $\left(\frac{dy}{dx}\right)$ at A for the given beam

θ_B = Slope at B i.e., $\left(\frac{dy}{dx}\right)$ at B for the given beam

The according to the conjugate beam method,

θ_A = Shear force at A^* for conjugate beam $= R_A^*$

$$= \frac{250}{3EI}$$

$$= \frac{250}{3 \times 200 \times 10^6 \times 10^4}$$

$$\boxed{\theta_A = 0.004166 \text{ rad}}$$

θ_B = Shear force at B^* for conjugate beam $= R_B^*$

$$= \frac{200}{3EI}$$

$$= \frac{200}{3 \times 200 \times 10^{6} \times 10^{-4}}$$

$$\boxed{\theta_B = 0.003333 \text{ rad.}}$$

ii) Deflection under the load

Let, y_c = Deflection at c for the given beam.

Then according to the conjugate beam method,

y_c = B.M at point c^* of the conjugate beam.

= $R_A^* \times 2$ - (load $A^* c^* D^*$) x distance of C.G. of $A^* c^* D^*$ from c^*

$$= \frac{250}{3EI} \times 2 - \left[\frac{1}{2} \times 2 \times \frac{100}{EI}\right] \times \left[\frac{1}{3} \times 2\right]$$

$$= \frac{500}{3EI} - \frac{200}{3EI}$$

$$= \frac{100}{EI}$$

$$y_c = \frac{100}{200 \times 10^{6} \times 10^{-4}} \text{ m}$$

$$= \frac{1}{200} \text{ m}$$

$$= \frac{1}{200} \times 1000$$

$$\boxed{y_c = 5 \text{ mm}}$$