ANNAMACHARYA INSTITUTE OF TECHNOLOGY AND SCIENCES, TIRUPATI
(AUTONOMOUS)

Department of Electrical and Electronics Engineering

Year/Sem: I/II

Branch of Study: EEE

Subject Name ELECTRICAL CIRCUIT ANALYSIS-I

Subject Code:23APC0201

SYLLABUS

UNIT-I : INTRODUCTION TO ELECTRICAL CIRCUITS

Basic Concepts of passive elements of R, L, C and their V-I relations, Sources (dependent and independent), Kirchoff's laws, Network reduction techniques (series, parallel, series - parallel, star-to delta and delta-to-star transformation), source transformation technique, nodal analysis and mesh analysis to DC networks with dependent and independent voltage and current sources.

UNIT-II : NETWORK THEOREMS (DC & AC EXCITATIONS

Superposition theorem, Thevenin's theorem, Norton's theorem, Maximum Power Transfer theorem, Reciprocity theorem, Millman's theorem and compensation theorem.

UNIT-III : MAGNETIC CIRCUITS

Basic definition of MMF, flux and reluctance, analogy between electrical and magnetic circuits, Faraday's laws of electromagnetic induction - concept of self and mutual inductance, Dot convention coefficient of coupling and composite magnetic circuit, analysis of series and parallel magnetic circuits.

UNIT-IV : SINGLE PHASE CIRCUITS

Characteristics of periodic functions, Average value, R.M.S. value, form factor, representation of a sine function, concept of phasor and phasor diagrams. Steady state analysis of R, L and C circuits to sinusoidal excitations-response of pure resistance, inductance, capacitance, series RL circuit, series RC circuit, series RLC circuit, parallel RL circuit, parallel RC circuit.

UNIT-V : RESONANCE AND LOCUS DIAGRAMS

Series Resonance: Characteristics of a series resonant circuit, Q-factor, selectivity and bandwidth, expression for half power frequencies; Parallel resonance: Q-factor, selectivity and bandwidth; Locus diagram: RL, RC, RLC with R, L and C variables.

Basic Electrical circuits

Unit - I

[redacted]

Basics

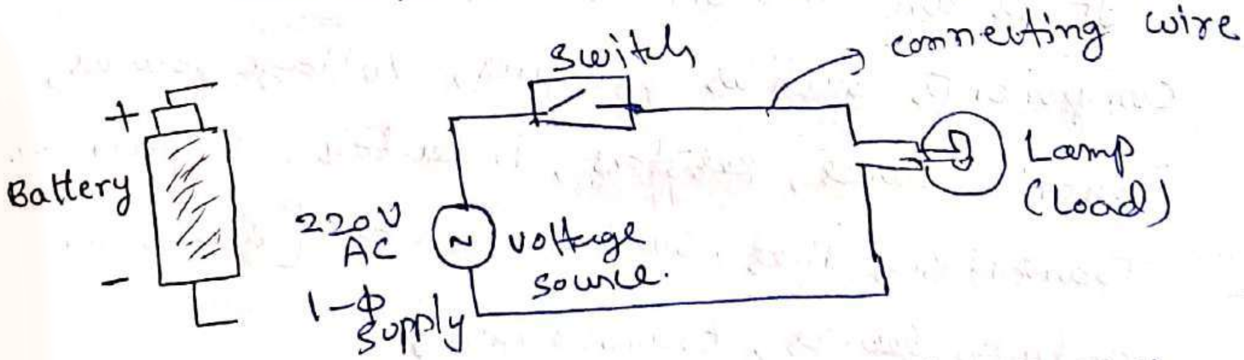
Concept of Electrical circuit

→ The inter connection of electrical elements is called as electrical circuit.

- There electrical elements are
  1. Active elements [Eg: Voltage source (V) & current source (I)
  2. passive elements [Eg: Resistor inductor capacitor]

- The main purpose of electrical circuit is to transfer energy from source to load.

Eg: Simple electrical circuit



- In the above ckt, it consists of voltage source, switch, connecting wire & electrical lamp.

When ever the switch is on electrical current is flowing through Lamp it emits light.

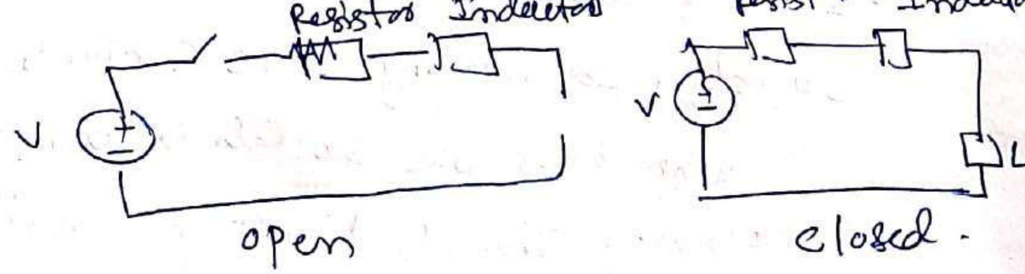
- During switch is ON, The current starts from Source & flowing through switch, and Lamp Then return back to Source.
- Here The current has a complete path of flow is called closed circuit.
- During switch OFF, The current is break in switch, so that current can not flow. Then The circuit is called open circuit.
- If a network contains at least one closed path, Then it is called an electrical circuit.

Basic definitions

(1) Electrical network

It is an interconnection of various electrical components such as Batteries, voltage sources, current sources, Resistors, inductors, capacitors, Transmission lines, switches & etc. [also semi-conductor devices, transformers]

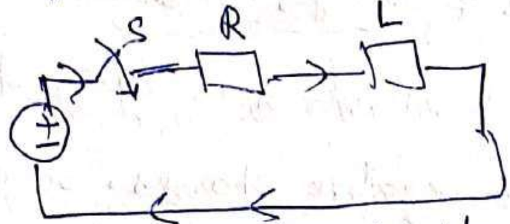
Eg:



open closed.

(2) Electrical circuit

It is a electrical network that has a closed loop giving a return path for the current.



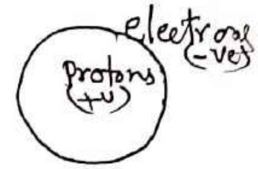
closed ckt.

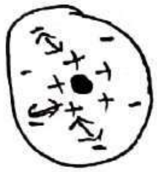
Difference b/w electrical network & circuit

Electrical Network	Electrical circuit
(1) It can be either closed or open path	(1) It has always closed path for current.
(2) It is not necessary have both active & passive elements	(2) It must have active & passive elements.
(3) All the networks are not circuit	(3) All the circuits are networks.
<u>Eg:</u> In a building, building is a network	<u>Eg</u>: In a building, room is a circuit

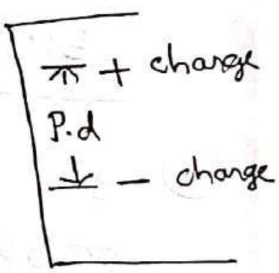
③ voltage:

(i) → According to structure of an atom, There are two types of charges. These are positive & negative. A force of attraction is exists b/w these positive & negative charges. so certain amount of energy is required to overcome the force and move the charges through a specific distance.



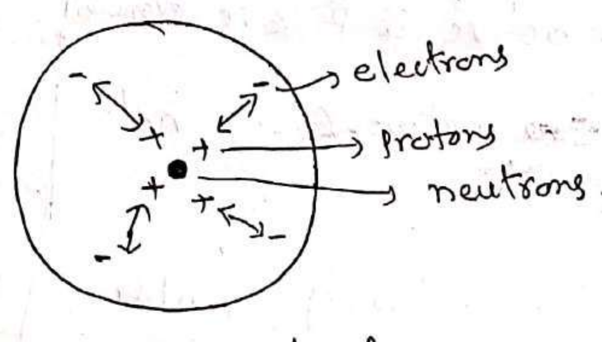


- The difference in potential energy of the charges is called the potential difference.

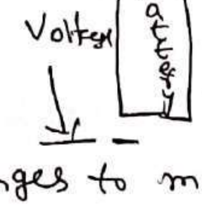


- Potential difference in electrical terminology is called voltage.
- It is denoted by Symbol V. unit is volts.

→

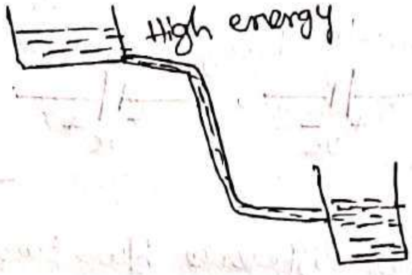


(ii) → The difference in energy level from one end of the battery to the other end of battery.

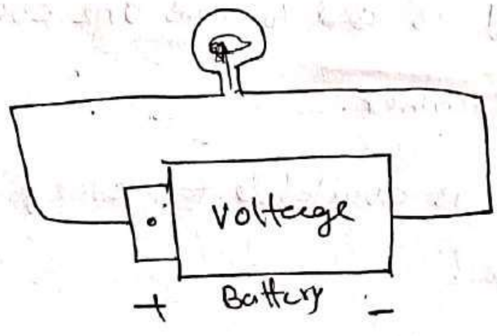


→ The energy differe causes the charges to move from a higher to a lower voltage in a closed circuit.

Eg:

1. 

water flows from high energy to low energy.

2. 

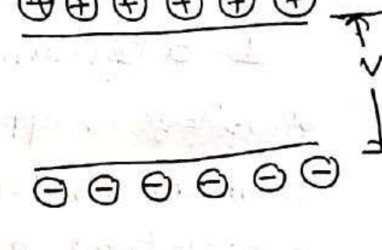
(iii) Voltage is the pressure (force) in the electrical circuit that ~~pushes~~ pushes the charged electrons (current) through the conducting loop, then illuminating a light.

voltage = pressure.

Def of voltage

The potential difference between two charges or two conductors or two points is called as voltage.

- It is denoted by 'v' or 'v'



- units are volts.
- mathematically, voltage can be expressed as work done per unit charge

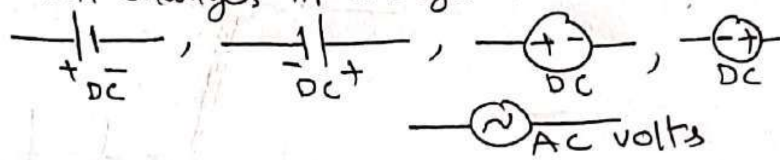
$$V = \frac{W}{Q} \qquad \frac{\text{Joules}}{\text{coulombs}} = \text{volts.}$$

Energy (W)

Q → charge.

- $\vartheta = \frac{dw}{dq}$ → Small changes in energy or work; dq → small changes in charge.

→ It is denoted by



one volt (1 V)

one volt is the potential difference b/w two points when one joule of energy is used to pass one coulomb of charge from one point to the other.

(Q) If 70 J of energy is available for every 30 C of charge, what is the voltage?

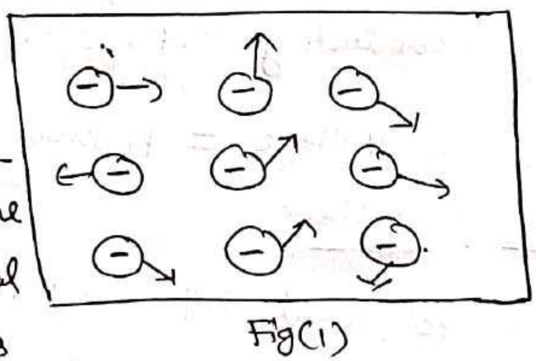
Sol

$$V = \frac{W}{Q} = \frac{70}{30} = 2.33 \text{ volts.}$$

Current

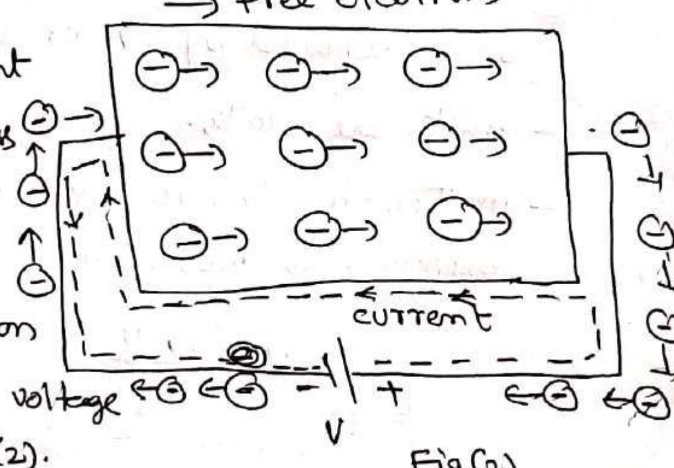
There are free electrons available in all semiconductive & conductive materials.

These free electrons move at random in all the directions within the structure in the absence of external pressure or voltage, which is shown in fig (1).



Fig(1)

If a certain amount of voltage is applied across the material, then all the free electrons move in one direction depending on the polarity of applied voltage which is shown in fig (2).



Fig(2)

This movement of free electrons from one end of the material to the other end constitutes an electric current.

Def.

~~The flow of of free~~

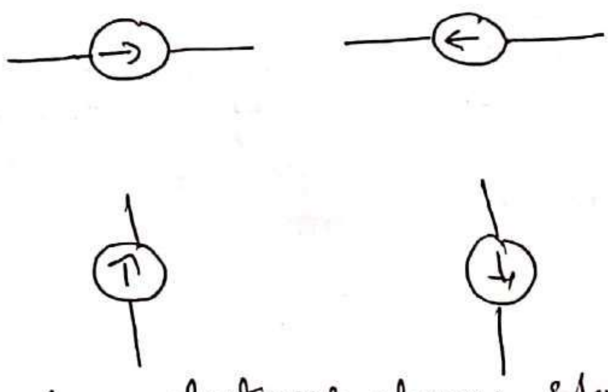
The rate of flow of free electrons in a conductive or Semi conductive materials is called as electric current or simply current.

- It is denoted by I or i. Unit is amperes or amp.
- In mathematically, current is expressed as

$$I = \frac{Q}{t} = \frac{\text{charge}}{\text{time}} = \frac{\text{coulomb}}{\text{seconds}} = \text{Amperes.}$$

$$i = \frac{dq}{dt} \quad \begin{array}{l} \to \text{small changes in charge} \\ \to \text{Small changes in time.} \end{array}$$

- It is denoted as symbollically as

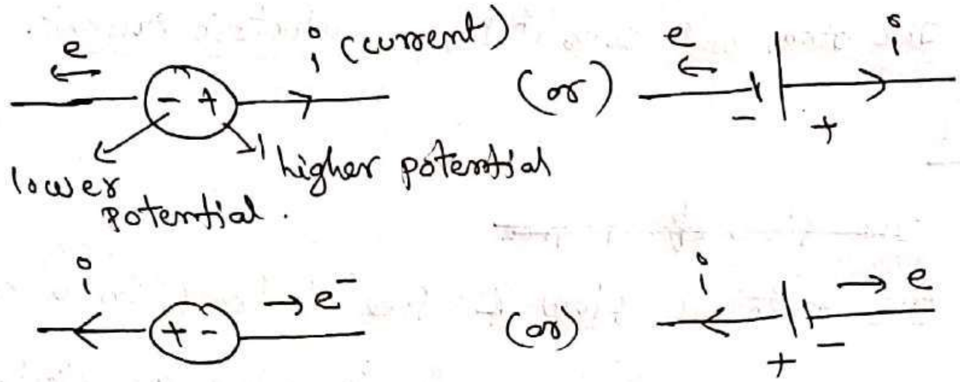


Note: 1. The free electrons always flow from −ve to +ve.

2. The current is always flows from +ve to −ve

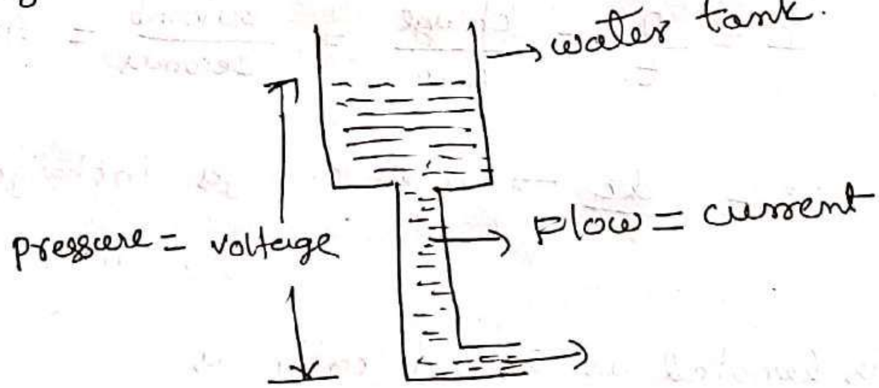
3. The conventional direction of current is always flows in the opposite to direction of electrons.

→ directions of current



→ current is always flow from (+ve) terminal or higher potential to −ve terminal or lower potential.

→ Eg.



Power

Def:

The rate of change of energy is called as power.

- It is denoted by symbol 'P' or p
- units are watts or KW or MW
- In mathematically, it is expressed as

$$P = \frac{w}{t} = \frac{\text{Energy}}{\text{time}} = \frac{\text{Joule}}{\text{second}} = \text{watts.}$$

$$p = \frac{dw}{dt} = \frac{\text{small changes in energy}}{\text{Small changes in time}}$$

$$p = \frac{dw}{dt} \times \frac{dq}{dq}$$

$$= \frac{dw}{dq} \times \frac{dq}{dt}$$

$$\boxed{p = v \times i} \text{ watts.}$$

→ Instantaneous power.

Scanned with CamScanner

Energy

Def:

Capacity to do work is called Energy

(or)

It is the capacity of doing work is called as energy.

- Energy is nothing but stored energy.
- It is denoted by symbol W
- Units are Joules.

→ In mathematically,

we know, $P = \frac{dw}{dt}$

$$dw = P\,dt$$

Integrating both sides

$$\int 1 \cdot dw = \int P \cdot dt$$

$$\boxed{w = \int P \cdot dt} \text{ Joules}$$

Passive Elements

An element is capable only of receiving power is called as passive elements

Eg: Resistor, Inductor, capacitor

Note: ① Some passive elements like Inductors & capacitors are capable of storing a finite amount of energy and return it later to an external element.

② Passive elements cannot supply average power greater than '0'.

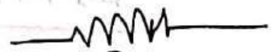
① Resistor & Resistance

Resistor

It is the material which is having property of resistance is called Resistor.

(or)

It is the material with a predetermined electrical resistance like 1Ω, 10Ω, 100Ω & 1000Ω etc.

— symbol 

R → Resistance.

→ Eg: Resistor of 100Ω. or resistance of 100Ω.

R = 100Ω.

Resistance

Def: It is the property of a material which opposes the flow of free electrons.

- It is denoted by 'R'
- Unit is ohms or Ω
- mathematically,

$R \propto$ length of the wire (l)

$\propto \frac{1}{a}$ [area of cross section of area]

$$R \propto \frac{l}{a}$$

$R = \frac{\rho l}{a} \Omega$, $\rho \rightarrow$ Resistivity in ohm-metre

$l \rightarrow$ length in metres

$a \rightarrow$ cross sectional area.

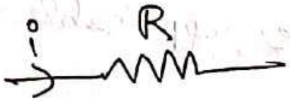
- Resistance of a given material depends on the physical properties (Resistivity).

	material	Resistivity (ρ) Ω-m (20°c)
conductive materials	Silver	1.59×10^{-8}
	Copper	1.68×10^{-8}
	Gold	2.44×10^{-8}
	Aluminium	2.8×10^{-8}
Semi conductive materials	Germanium	4.6×10^{-1}
	Sillicon	6.4×10^{2}
Insulator	Quartz	7×10^{17}

Voltage drop :

→ when an electrical current flows through any resistor, heat is generates due to collision of free-electronics.

So Resistor is always heat dissipates i.e voltage is always dropped). and is called voltage drop.

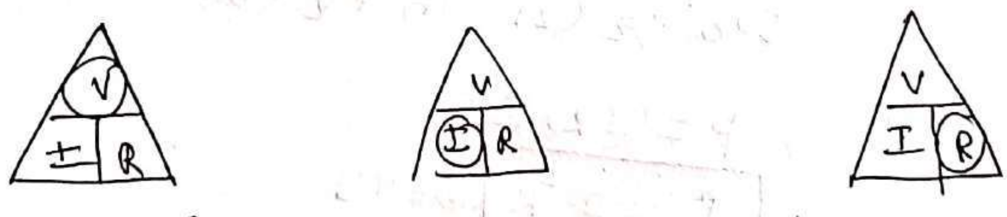


voltage drop of particular resistor

$\boxed{V_R = iR}$ volts.

ohm's Law

According to this, current through the conductor is directly proportional to the voltage across b/w two points. here R → resistance.



$V = I \times R$ $\qquad I = \frac{V}{R}$ Amps $\qquad R = \frac{V}{I}$

$I \propto V$ —(1)

$I \propto \frac{1}{R}$ —(2)

voltage drop.

$I = \frac{V}{R} \Rightarrow \boxed{V = IR}$ volts.

Scanned with CamScanner

Limitations of ohm's Law

There are some limitations using ohm's Law

1. It can not be applicable to temparature varying cases
2. It can not be applied to semiconductor materials. (Ge, Si)
3. It can not be applicable to unilateral elements (Diodes)
4. It is not suitable for non linear elements.

→ voltage across resistor $V = iR$ volts.

→ power absorbed by the resistor

$$P = VI \quad -(1)$$

we know $V = IR \quad -(2)$

sub eq (2) in eq (1).

$$P = (IR)\, I$$

$$\boxed{P = I^2 R} \text{ watts}$$

also $P = V \times \frac{V}{R}$ (from eq (2))

$$\boxed{P = \frac{V^2}{R}} \text{ watts.}$$

$$\boxed{P = VI = I^2 R = \frac{V^2}{R}} \text{ watts}$$

Energy lost in a resistor

we know

$$\omega = \int_0^t p \cdot dt$$

$$= p \int_0^t 1 \cdot dt$$

$$\boxed{\omega = pt}$$

$$\boxed{\omega = I^2 R t}$$ joules $[P = I^2 R]$

Inductor

Def: It is a material which posses the property of inductance.

- It stores energy in the form of electromagnetic field.
- A wire of certain length, when twisted into a coil becomes a basic inductor.

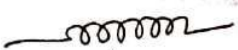
Inductor.

- Inductor never dissipates energy which only stores energy.

Inductance

Def: It is the property of the material which doesnot allow sudden change in current is called inductance.

- It is denoted by symbol 'L'
- It's unit is Henry (H).
- It is represented by



L

- whenever the current flowing through it an emf is induced in it.
- For constant values of current, voltage across inductor is zero.
- Inductor allows only Linear current $\frac{di}{dt}$

voltage across inductor

voltage across inductor is proportional to the rate of change of current passing through it.

$$V \propto \frac{di}{dt}$$

$$\boxed{v = L\frac{di}{dt}} \text{ volts} \quad (1)$$

current passing through inductor

$$v = L\frac{di}{dt}$$

$$\frac{v}{L} = \frac{di}{dt}$$

$$di = \frac{v}{L}dt$$

Integrating on both sides

$$\int_0^t di = \frac{1}{L}\int_0^t v\,dt$$

$$\underbrace{i(t) - i(0)}_{0} = \frac{1}{L}\int_0^t v\,dt$$

$$\boxed{i(t) = \frac{1}{L}\int_0^t v\,dt} \text{ Amps.}$$

Power absorbed by inductor

$$p = vi$$

$$= L\frac{di}{dt} \times i$$

$$\boxed{P = Li\frac{di}{dt}} \text{ watts}$$

Energy stored by inductor

$$w \text{ (or) } E = \int P.\,dt$$

$$= \int Li\frac{di}{\cancel{dt}}\cancel{dt}$$

$$\boxed{E = \frac{1}{2}Li^2} \text{ Joules}$$

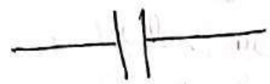
Capacitor :

Def : It is the material which posses the property of capacitance.

- It stores energy in electrostatic field.

capacitance

Def : Two conducting surfaces separated by an insulating medium exhibit the property of a capacitance.



$C \rightarrow$ capacitance.

- It will be denoted by symbol 'C'
- unit is Farad (F).

Def 2 : It is the Property of the material which does not allow sudden change in voltage.

- mathematically, charge of capacitor is proportional to voltage.

$$Q \propto V$$

$$Q = CV$$

$$\boxed{C = \frac{Q}{V}} = \frac{\text{charge}}{\text{voltage}}$$

also $e = \frac{dq}{dV}$ $\rightarrow$ small changes in charge
$\rightarrow$ small changes in voltage.

current through capacitor

$$i = \frac{dq}{dV}$$

$$= \frac{dq}{dV} \times \frac{dt}{dt}$$

$$C = i \cdot \frac{dt}{dv}$$

$$\boxed{i = C\frac{dv}{dt}} \text{ amps}$$

(or)

$$i = \frac{dq}{dt}$$

$$= \frac{d}{dt}(CV)$$

$$\boxed{i = C\frac{dv}{dt}}$$

→ current passing through capacitor is proportional to rate of change of voltage across capacitor voltage in it.

we know $i = C\frac{dv}{dt}$

$$\frac{i}{C} = \frac{dv}{dt}$$

$$dv = \frac{1}{C}\, i\, dt$$

Integrating both sides

$$\int_0^t dv = \frac{1}{C}\int_0^t i\, dt$$

$$v(t) - \underbrace{v(0)}_{0} = \frac{1}{C}\int_0^t i \cdot dt$$

$$\boxed{v(t) = \frac{1}{C}\int_0^t i\, dt} \text{ volts.}$$

Power absorbed by capacitor

$$P = Vi$$

$$= V \cdot C\frac{dv}{dt}$$

$$\boxed{P = VC\frac{dv}{dt}} \text{ watts}$$

Energy stored by capacitor

$$W \text{ (or) } E = \int P \cdot dt$$

$$= \int V \cdot C \cdot \frac{dv}{\cancel{dt}}\, \cancel{dt}$$

$$\boxed{E = \frac{1}{2}CV^2} \text{ Joules.}$$

V – I Relationships for passive elements

Element	Voltage (volts)	Current (Amps)	Power (watts)	Energy (Joules)
Resistance (R) (Ω)	$V = IR$	$I = \frac{V}{R}$	$P = I^2 R$	$E = I^2 R t$
Inductance (L) (H)	$v = L\frac{di}{dt}$	$i = \frac{1}{L}\int v\,dt$	$P = Li\frac{di}{dt}$	$E = \frac{1}{2}Li^2$
Capacitance (C) (F)	$v = \frac{1}{c}\int i\,dt$	$i = c\frac{dv}{dt}$	$P = cv\frac{dv}{dt}$	$E = \frac{1}{2}cv^2$

Problem: A 4Ω resistor has a current $i = 2.5A$. Find voltage, power & energy by resistor. $t = 2$ sec

Sol Given

$R = 4\Omega$, $i = 2.5A$.

① Voltage across resistor

$$V = iR$$
$$= 2.5 \times 4 = 10 \text{ volts}$$

② power absorbed by resister

$$P = i^2 R$$
$$= 2.5^2 \times 4 =$$

③ Energy stored by resistor

$$E = I^2 R t$$
$$=$$

Classification of Elements in The circuit/Network

Network/circuit elements are classified into 5 types. These are.

1. Active & passive elements
2. Unilateral & Bilateral elements
3. Linear & non linear elements
4. Lumped & distributed elements
5. Time variant & Time invariant elements.

(1) Active & passive elements

Active elements

- Active element which delivers energy to The other elements.
- Active element are also called as energy donaes.

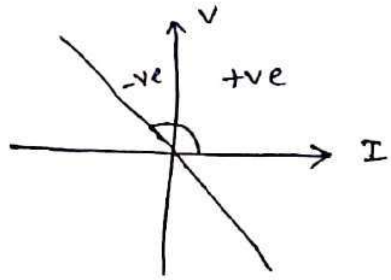
  Eg: voltege source, current source, Battery etc.

- Active element requires external source to Their operation

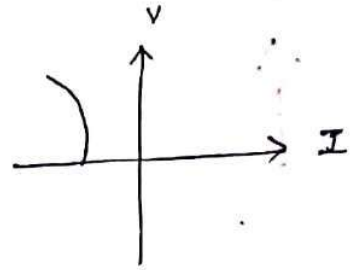
  Eg: Ge, Si, diodes.

* Active elements are non-linear category.

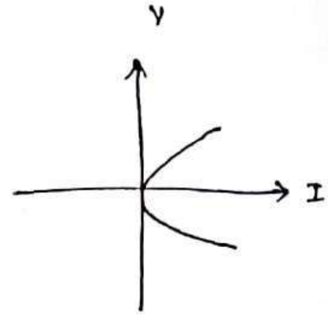
V-I characteristics:-



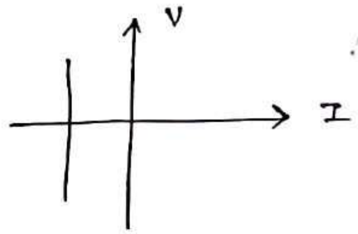
Active elements



Active elements



Active elements



Active elements

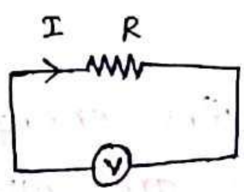
NOTE :- Except $1^{st}, 3^{rd}$ quadrant, all the quadrants becomes active elements

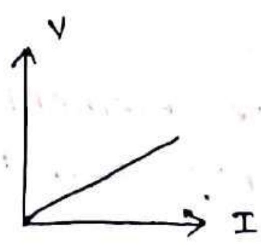
passive elements :-

* passive elements which receives energy from the other elements i.e., These elements are called energy acceptors.

Eg :- Resistor, Inductor, Capacitor.

* passive elements are linear category.

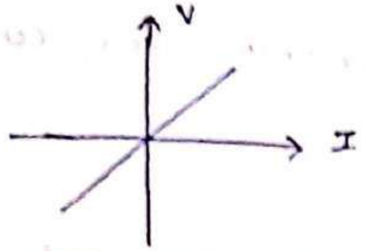




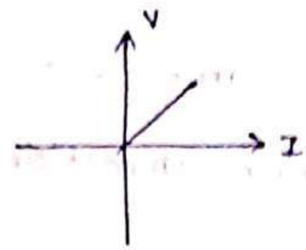
$V \propto I$

$$\boxed{V = IR}$$

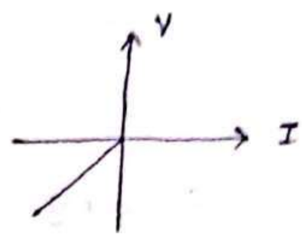
V-I characteristics:-



passive elements



passive elements



passive elements

2. unilateral and Bilateral elements:-

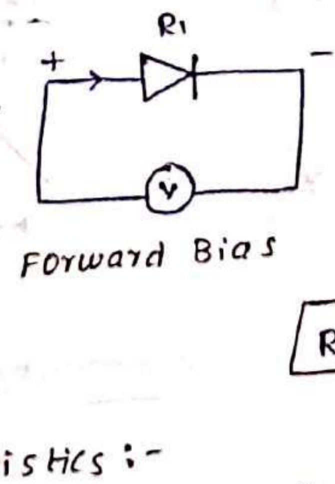
→ unilateral elements:-

It does not satisfies V-I relation for current flowing in either direction

(OR)

It has different V-I relation for current flowing in either direction

Eg:- Diodes, Rectifiers, Transistors

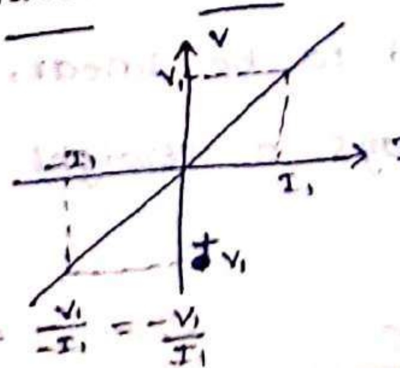


Forward Bias

Reverse Bias

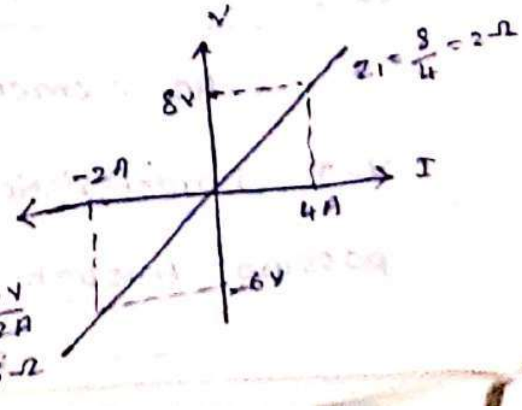
$$\boxed{R_1 \neq R_2}$$

V-I characteristics:-



$Z_1 = \frac{V_1}{I_1}$

$Z_1 = \frac{V_1}{-I_1} = \frac{-V_1}{I_1}$



$Z_1 = \frac{8}{4} = 2\Omega$

$R_2 = \frac{-6V}{-2A}$

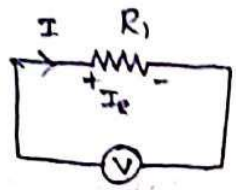
$R_2 = 3\Omega$

Scanned with CamScanner

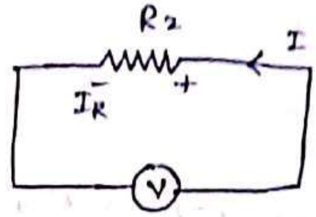
→ Bilateral elements :-

* It has same V-I relationship for current flowing in either direction.

Eg:- R, L, C, Transmission lines, Incandecent lamp filaments



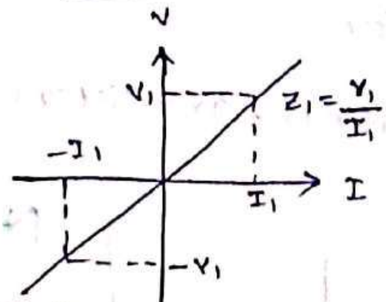
$R_1 = \frac{V}{I}$



$R_2 = \frac{V}{I}$

$\boxed{R_1 = R_2}$

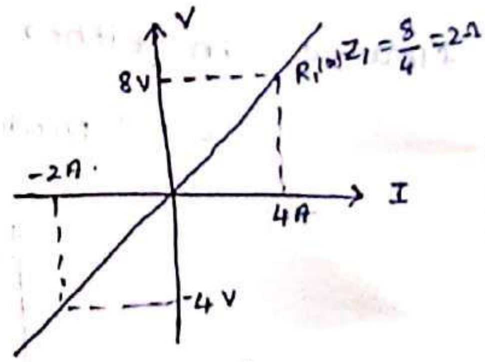
→ V-I characteristics :-



$Z_2 = \frac{-V_1}{-I_1}$

$Z_2 = \frac{V_1}{I_1}$

$\boxed{Z_1 = Z_2}$



R_2 (or) $Z_2 = \frac{-4}{-2} = 2\Omega$

$\boxed{R_1 = R_2}$

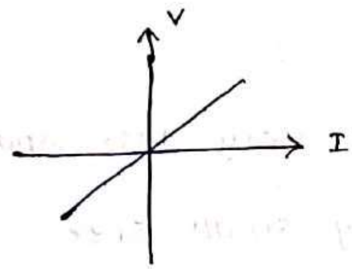
3. Linear and non-linear elements :-

→ Linear elements :-

* An element is said to be linear, the V-I characteristics is always a straight line passing through origin.

Eg:- R, L and C

→ V-I characteristics :-



$V \propto I$

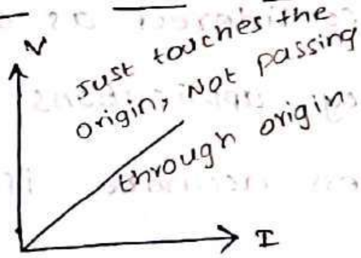
$\boxed{V = IR}$

→ Non-linear elements :-

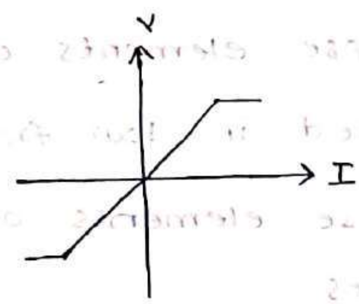
* An element is said to be non-linear, then the V-I characteristics is not a straight line passing through origin.

Eg :- Diodes, Rectifiers, Transistors, SCR's etc.

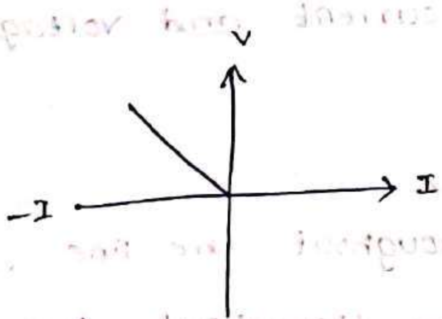
→ V-I characteristics :-



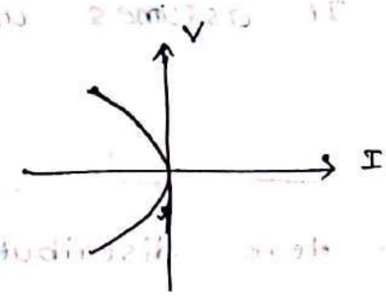
Non-linear



Non-linear



Non-linear



Non-linear

4. Lumped and Distributed elements:-

→ Lumped Elements:-

* Which occupies very less space
* which are very small size.
* It has π Networks and T-Networks

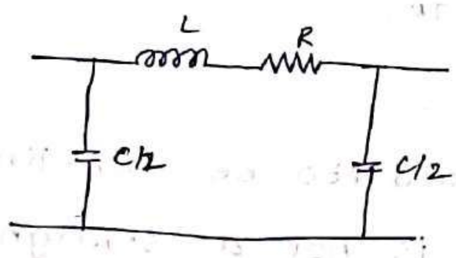


fig:- π-Network

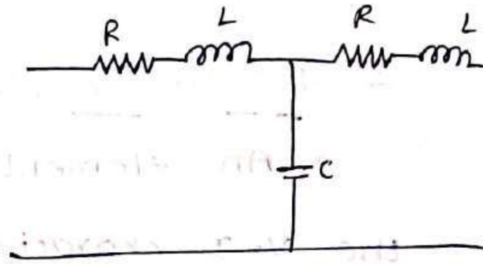
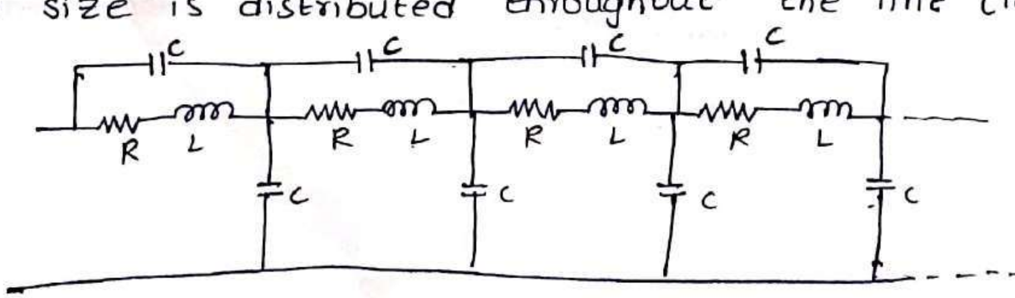


fig:- T-Network.

* These elements are visible and physically present like R, L & C.
* These elements are considered as separte elements
* used in low frequency applications.
* These elements are less accurate if frequency increases.
* It assumes uniform current and voltage.

→ Distributed Elements:-

* Here distributed throughout the line.
* The size is distributed throughout the line (large size)
*



* These elements are invisible and does not physically present.

* These are not physically separated.

* Used at high frequency applications.

* Less accurate if frequency decreases

* It does not having uniform I & V

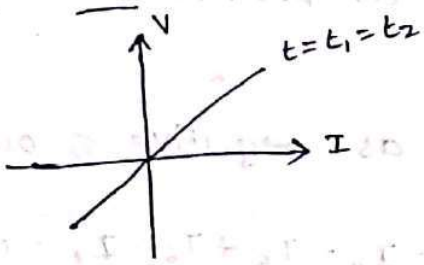
5. Time invariant and time variant elements :-

→ Time invariant elements :-

* An element is said to be time invariant when its V-I characteristics does not change with time.

Eg :- Fourier series, Laplace transform.

→ V-I characteristics :-

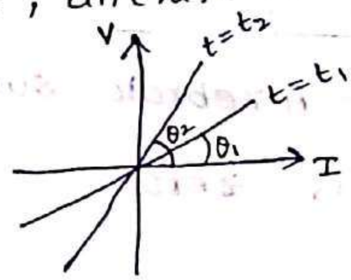


→ Time variant elements :-

* An element is said to be time variant when its V-I characteristics change with time

Eg :- Human vocal craft, aircraft

→ V-I characteristics :-



$$P = CV\frac{dv}{dt} \quad (or) \quad P = Q\frac{dv}{dt}$$

$\therefore$ power a/c capacitor is $\boxed{P = CV\frac{dv}{dt}}$ (or)

$$\boxed{P = Q \cdot \frac{dv}{dt}}$$

Energy across the capacitor :-

$$E = \int P \cdot dt$$

$$E = \int CV \cdot \frac{dv}{dt} \times dt$$

$$E = \int CV \cdot dv$$

$$E = \frac{CV^2}{2}$$

$$E = \frac{1}{2} CV^2 \quad (or)$$

$$E = \frac{1}{2} QV$$

$\therefore$ Energy across the capacitor is $\boxed{E = \frac{1}{2} QV}$

*** 5M Types of Energy sources :- 5

Energy sources

- Independent sources
  - Ideal
    - Ideal voltage source
    - Ideal current source
  - practical
    - voltage source
    - current source
- Dependent sources
  - VCVS
  - VCIS
  - ICIS
  - ICVS

VCVS – Voltage controlled voltage source

VCIS – Voltage controlled current source

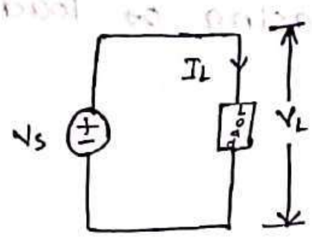
ICIS – current controlled current source

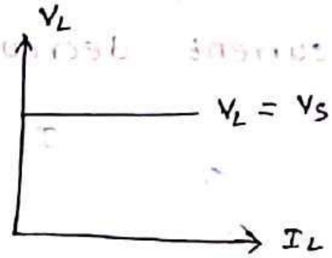
ICVS – current controlled voltage source

Independent sources :-

Ideal voltage source : It does not depends on any element.

IVS

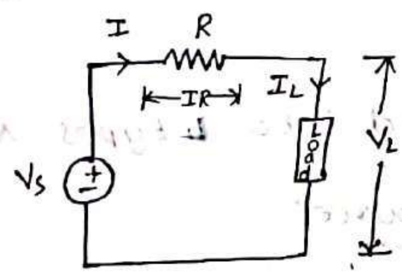




* It has Internal resistance = 0

fig : V–I characteristics

practical voltage source :-



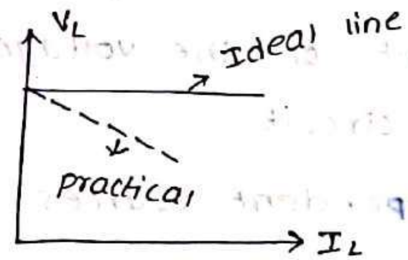
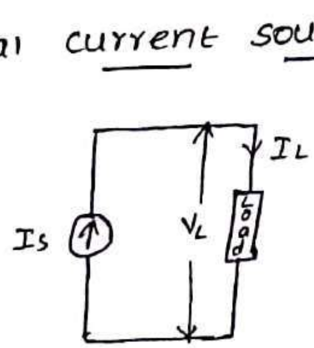


fig :- V–I characteristics

$V_S = V_1 + V_L$

$V_L = V_S - V_1$

Ideal current source :-



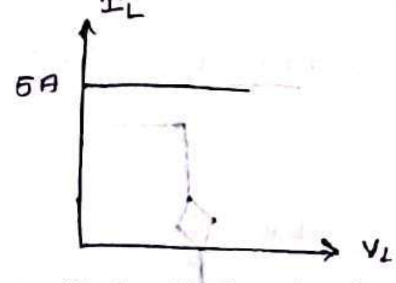


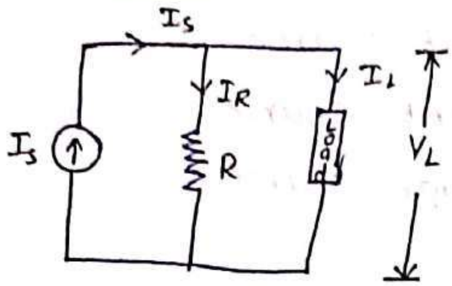
Fig :- V–I characteristics

* Load current is independent of load voltage.

It has Internal resistance = Infinity (α)

ICS

Practical current source :-



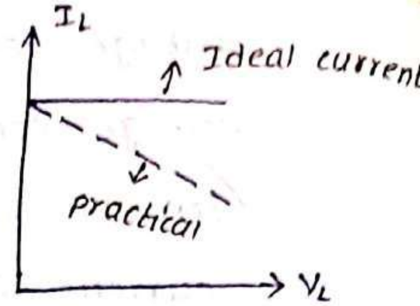


fig:- V-I characteristics

$$I_s = I_R + I_L$$

$$I_L = I_s - I_R$$

* Load current decreases with increasing of load voltage

$$I_L = I_s - \frac{V_L}{R}$$

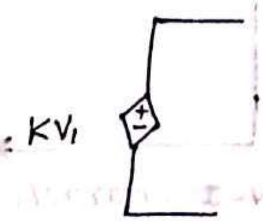
Dependent Sources :-

→ Value of quantity supplied by source is dependent on the voltage (or) current somewhere else in the circuit.

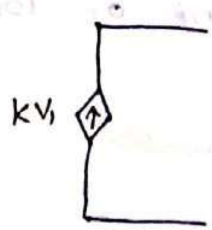
* Dependent sources are classified into 4 types. Namely

1. Voltage controlled voltage source
2. Voltage controlled current source
3. current controlled current source
4. current controlled voltage source.

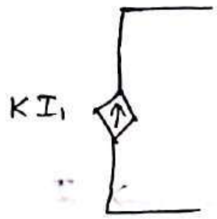
1. voltage controlled voltage source :-



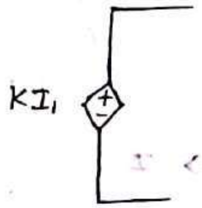
2. voltage controlled current source :-



3. current controlled current source :-



4. current controlled voltage source :-



** 5M/10M Network elements :- S

* Network elements are classified into 5 types. Namely

1. Active and passive elements
2. unilateral and Bilateral elements
3. Linear and non-linear elements
4. Lumped and distributed elements
5. Time variant and Time invariant elements

1. Active and passive elements :-

Active elements :-

* Active elements which delivers energy to the other elements i.e., these elements are called as energy donors.

Eg:- voltage source, current source, Battery etc.

(OR)

* Active elements recquires external source to their operation. Eg :- Ge, Si diodes

Kirchhoff's Laws

In 1847, a German physicist, Kirchhoff, formulated two fundamental laws of electricity.

1. Kirchhoff's voltage ~~divis~~ Law's (KVL)
2. Kirchhoff's current Law's (KCL).

(1) Kirchhoff's voltage law (KVL)

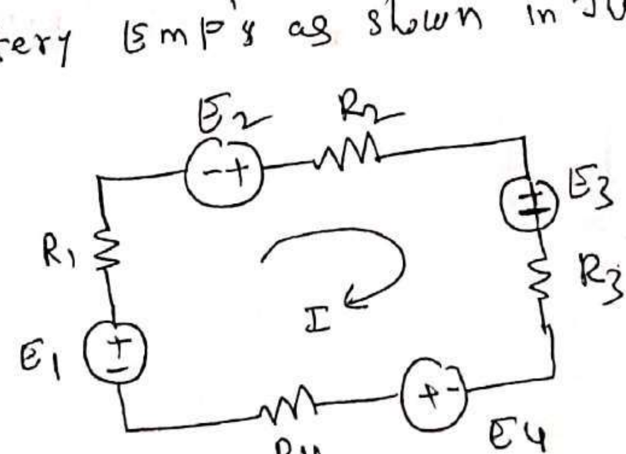
It states that, in any closed loop or mesh, the algebraic sum of EMF's of voltage sources plus the voltage drops across the network elements is zero.

$$\therefore \boxed{\Sigma \text{ EMF's} + \Sigma \text{ voltage drops} = 0}$$

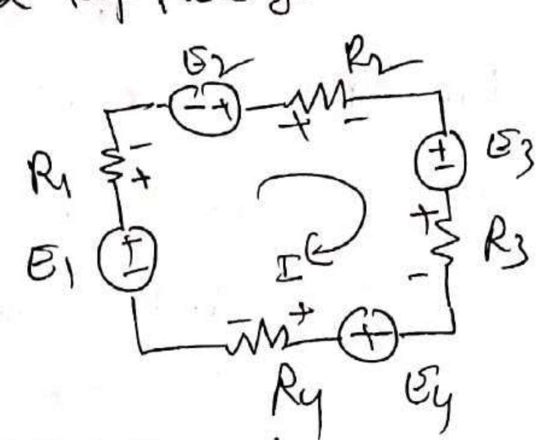
Explanation

If ABCDA is a closed loop or mesh as shown in fig. & E_1, E_2, E_3 & E_4 are the source EMF's.

The network elements R_1, R_2, R_3 & R_4 are connected to the battery EMF's as shown in the fig.



Apply KVL to the closed loop, we get



$\cancel{E_1 + IR_1}$

$$-E_1 + IR_1 - E_2 + IR_2 + E_3 + IR_3 - E_4 + IR_4 = 0$$

$$\left(-E_1 - E_2 + E_3 - E_4\right) + I\left(R_1 + R_2 + R_3 + R_4\right) = 0$$

$$\boxed{\Sigma \text{ EMF's} + \Sigma \text{ Voltage drops} = 0}$$

Note:

(1) KVL depends on the law of conservation of energy.

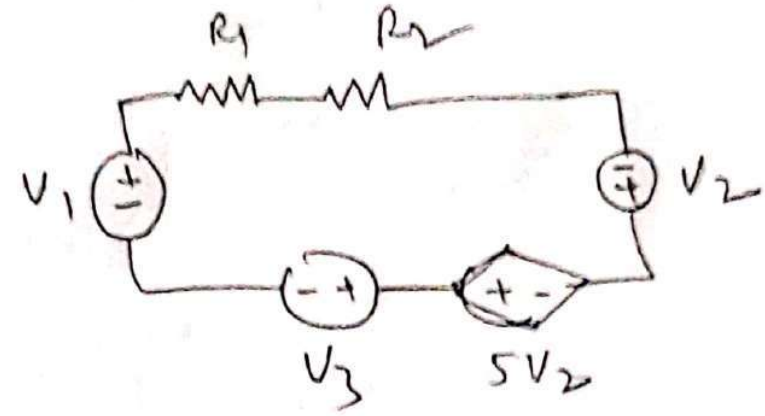
(2) KVL is not applicable for distributed parameters like transmission lines.

(3) Loop analysis = KVL + ohm's law

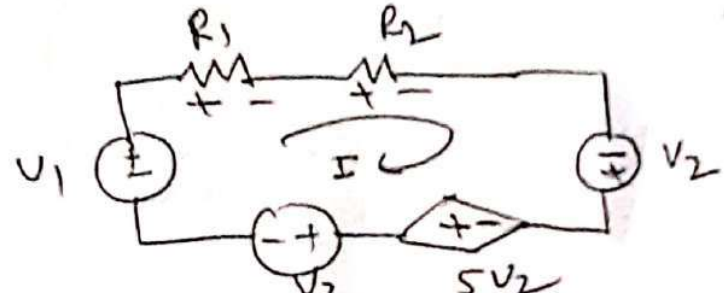
(4) No. of loop equations $= b - n + 1$

where n = no. of nodes
b = no. of branches

Eg 2



Sol



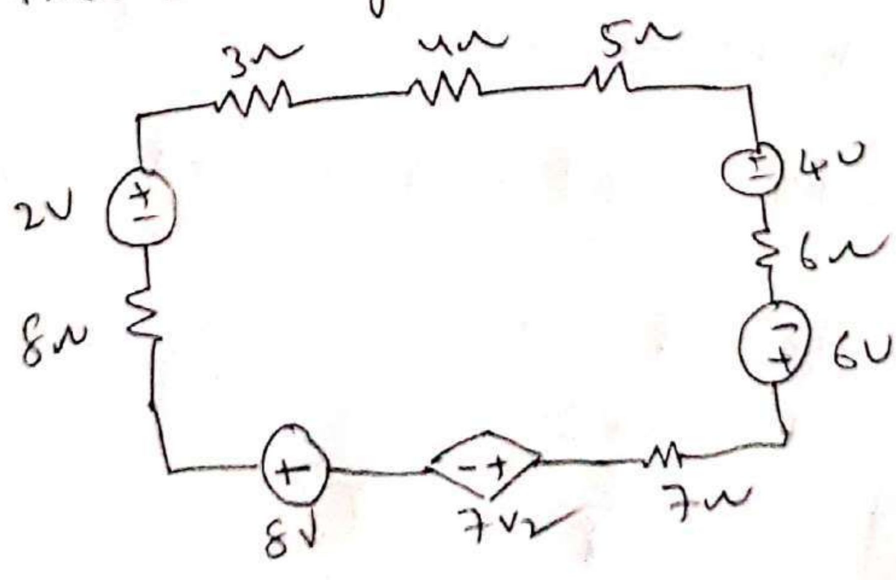
apply KVL

$$-V_1 + IR_1 + IR_2 - V_2 - 5V_2 + V_3 = 0$$

$$(-V_1 - V_2 + V_3) + (IR_1 + IR_2) = 0$$

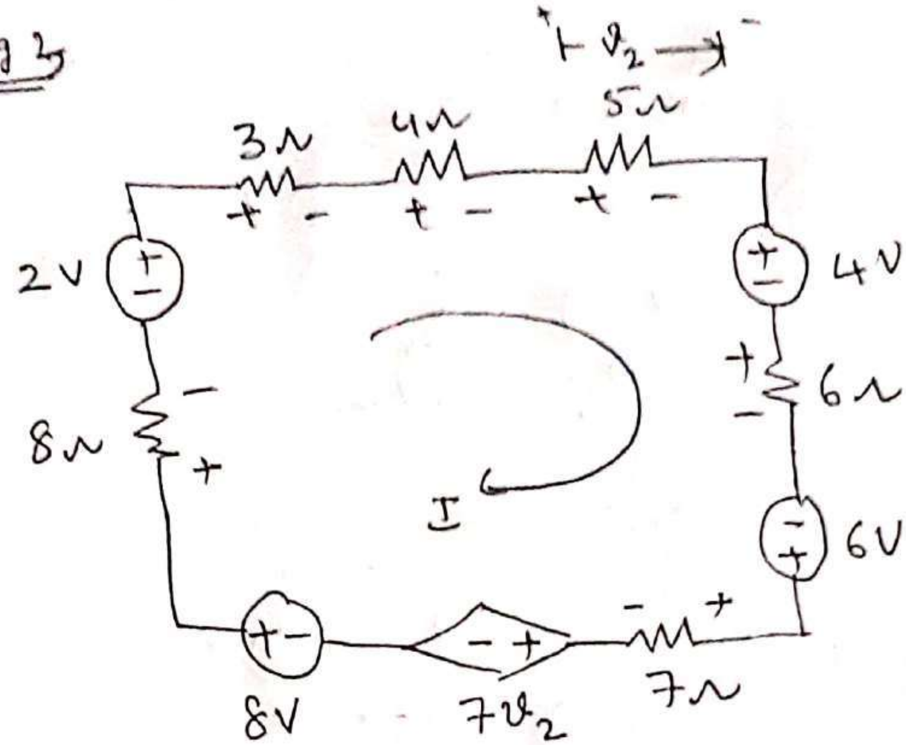
$$\Sigma \text{Emf's} + \Sigma \text{vol-drops} = 0$$

Eg 3 Find 'I' using KVL



10)

→ Eg 3



Sol

Apply KVL for the above loop.

$$-2+3I+4I+5I+4+6I-6+7I+7v_2-8+8I=0$$

here $v_2 = 5I$

$$-2+12I+4+6I-6+7I+7(5I)-8+8I=0$$

$$(-2+4-6-8)+(12I+6I+7I+35I+8I)=0$$

$$-12+68I=0$$

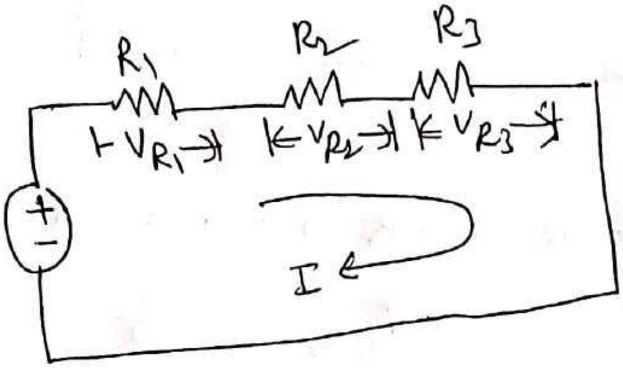
$$12 = 68I$$

$$I = \frac{12}{68} = 0.18\text{ A}.$$

Voltage division

Voltage division is possible only in Series circuits

Eg:



From the fig.

voltage drop across $R_1 = V_{R_1}$

" " $R_2 = V_{R_2}$

" " $R_3 = V_{R_3}$

$$\boxed{V_S = V_{R_1} + V_{R_2} + V_{R_3}}$$

According to vol. division

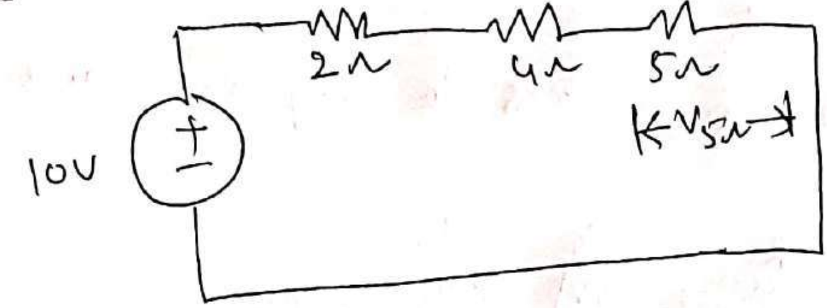
voltage drop across R_1 (V_{R_1}) = Total vol × $\frac{\text{Same Resis}}{\text{Total res}}$ ~~voltage~~ branch

$$= V_S \times \frac{R_1}{R_1 + R_2 + R_3}$$

lly $$V_{R_2} = V_S \times \frac{R_2}{R_1 + R_2 + R_3}$$

lly $$V_{R_3} = V_S \times \frac{R_3}{R_1 + R_2 + R_3}$$

Eg: Find voltage across 5Ω resistor using vol. div



So)

Voltage across 5Ω resistor is

$$V_{5\Omega} = 10 \times \frac{5}{2+4+5}$$

$$= 10 \times \frac{5}{11}$$

$$\boxed{V_{5\Omega} = \frac{50}{11}} \text{ volts.}$$

(2) Kirchhoff's current Law (KCL)

It states that, at any node in any electrical circuit, the sum of incoming currents is equal to sum of outgoing currents.

(or)

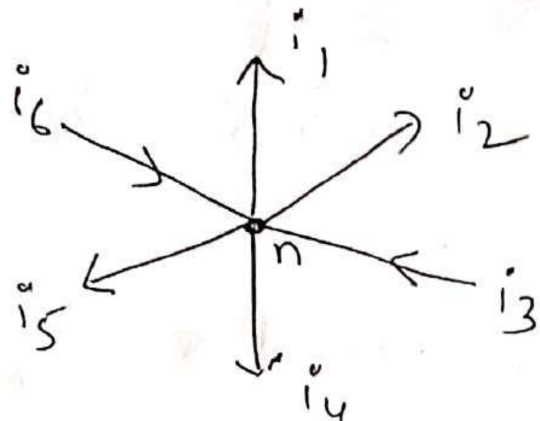
It states that, the algebraic sum of currents meeting at a node is equal to zero.

$$\Sigma I = 0$$

Note: It is also called Point's Law.

Explanation

Let $i_1, i_2, i_3, i_4, i_5, i_6$ are the currents meeting at node 'n' as shown in fig.



Here i_3 & i_6 are incoming current
i_1, i_2, i_4, i_5 are outgoing currents.

According to KCL

$$i_3 + i_6 = i_1 + i_2 + i_4 + i_5$$

Incoming currents = outgoing currents

(or)

$$i_3 + i_6 - i_1 - i_2 - i_4 - i_5 = 0$$

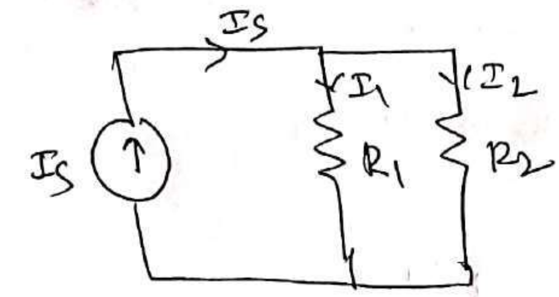
$$\Sigma i = 0$$

Note:
(1) KCL works on the principle of law of conservation of charge.
(2) KCL is not applicable for distributed parameter like transmission lines.

current divission

current divission is possible only in parallel circuits

Eg



According to current divission rule.

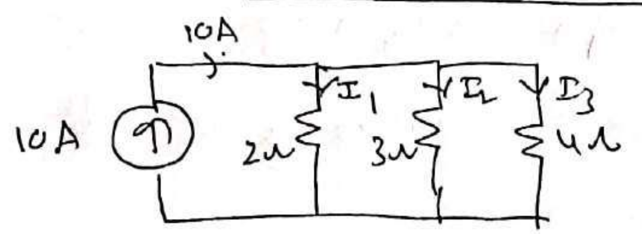
$$I_s = I_1 + I_2$$

$$I_1 = \text{Total current} \times \frac{\text{opposite resistance}}{\text{Total resistance}}$$

$$\boxed{I_1 = I_S \times \frac{R_2}{R_1+R_2}} \text{ Amps}$$

$$\text{IIIy} \quad \boxed{I_2 = I_S \times \frac{R_1}{R_1+R_2}} \text{ Amps.}$$

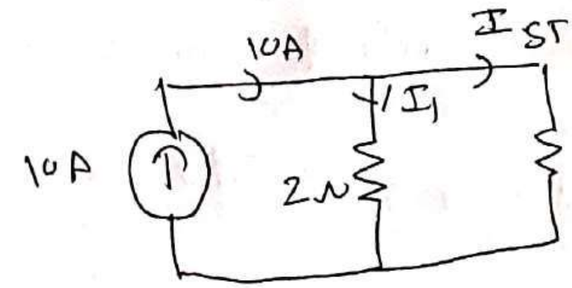
Eg



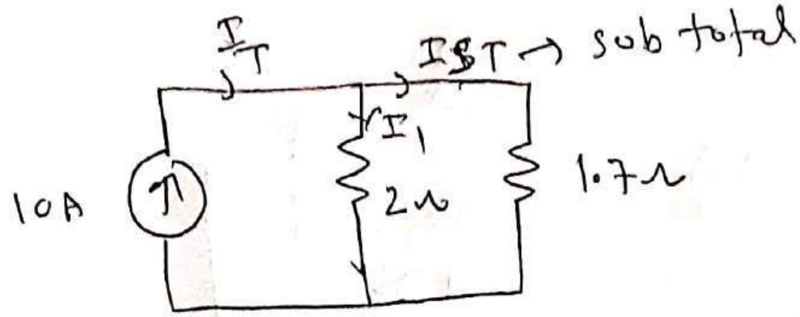
Find I_1, I_2, I_3 currents

Sol)

here Three resistance are in parallel. So the simple procedure is like This



$$\frac{3\times4}{3+4} = \frac{12}{7} = 1.7\,\Omega$$



According to current division rule

$$I_1 = 10 \times \frac{1.7}{2+1.7}$$

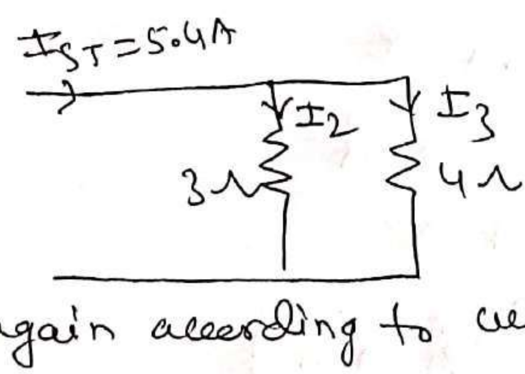
$$\boxed{I_1 = 4.61\,A}$$

$$I_{ST} = 10 \times \frac{2}{2+1.7}$$

$$= 10 \times \frac{2}{3.7}$$

$$= 5.4\,A.$$

Now the subtal circuit is



Now, again according to current division rule

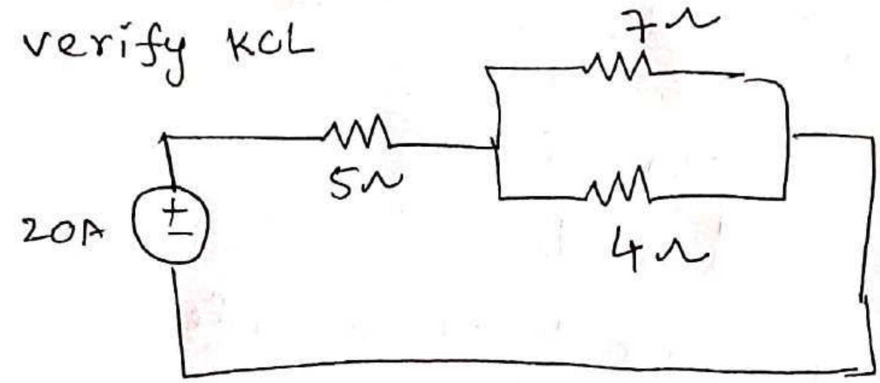
$$I_2 = 5.4 \times \frac{4}{3+4} = 3.09\,A.$$

$$I_3 = 5.4 \times \frac{3}{3+4} = 2.31\,A.$$

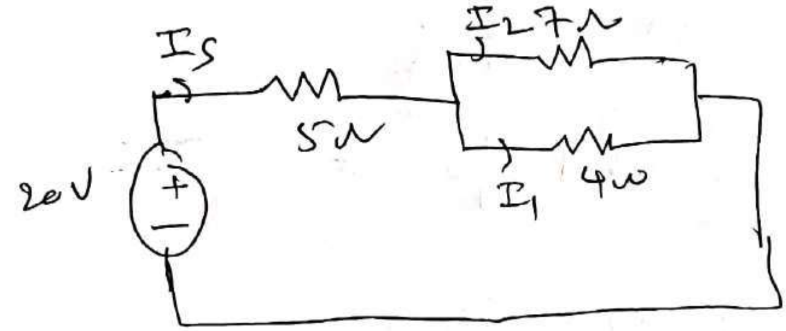
$\therefore$ $I_1 = 4.61\,A$, $I_2 = 3.09\,A$, $I_3 = 2.31\,A$.

$\therefore$ $I_S(10) = I_1(4.61) + I_2(3.09) + I_3(2.31)\,A.$

→ verify KCL



<u>sol</u>



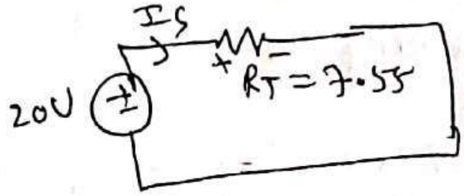
First of we want to find out total current I_S. for That simplify the resistance using series parallel resistances.

$$R_T = 5 + \frac{7 \times 4}{7+4}$$

$$= 5 + \frac{28}{11}$$

$$= 5 + 2.55$$

$$R_T = 7.55$$

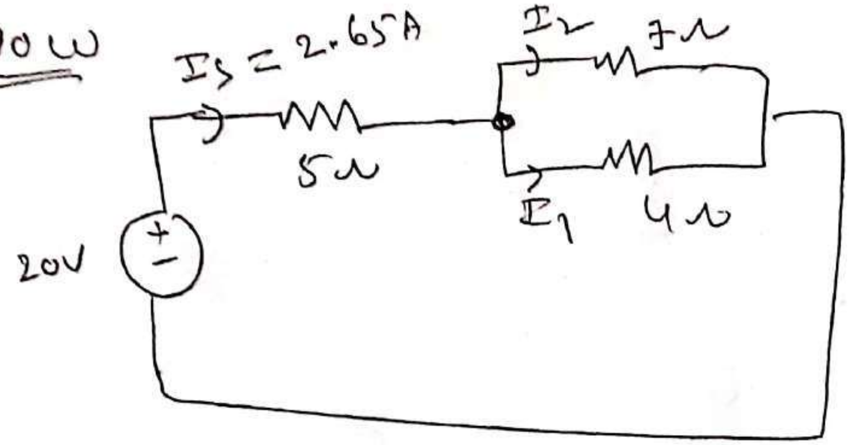


According to ohm's law (or) KVL

$$-20 + I_S \times 7.55 = 0$$

$$\boxed{I_S = \frac{20}{7.55} = 2.65 A.}$$

NOW



According to current division rule.

$$I_1 = 2.65 \times \frac{7}{7+4} = 1.69A$$

$$I_2 = 2.65 \times \frac{4}{4+7} = 0.96$$

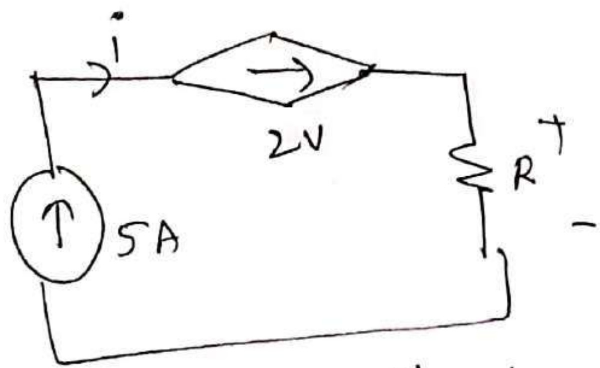
$$I_S(2.65) = I_1(1.69) + I_2(0.96)$$

$$2.65A = 2.65A.$$

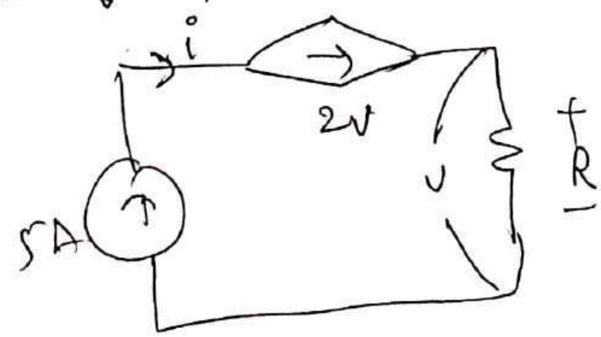
Incoming currents = outgoing currents

Hence KCL is verified.

→ obtain the value of 'R' in the ckt using kcl rule



Sol Let V is shown in following fig.



we know for series circuit, currents are equal or same.

$$i = 5 = 2v$$

$$\boxed{v = \frac{5}{2} = 2.5\,V}$$

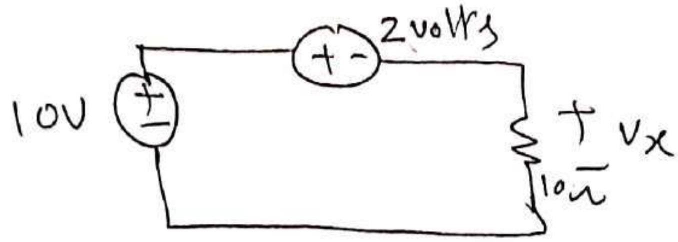
However, From ohm's law

$$v = iR$$

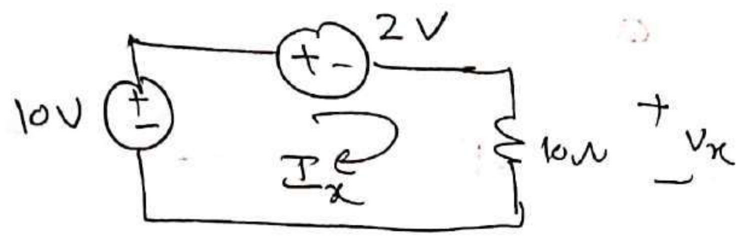
$$2.5 = 5 \times R$$

$$\boxed{R = \frac{2.5}{5} = 0.5\,\Omega}$$

(Q) Find V_x & I_x using KVL



Sol



Apply KVL to the above ckt

$$-10 + 2 + V_x = 0$$

$$\boxed{V_x = 8V}$$

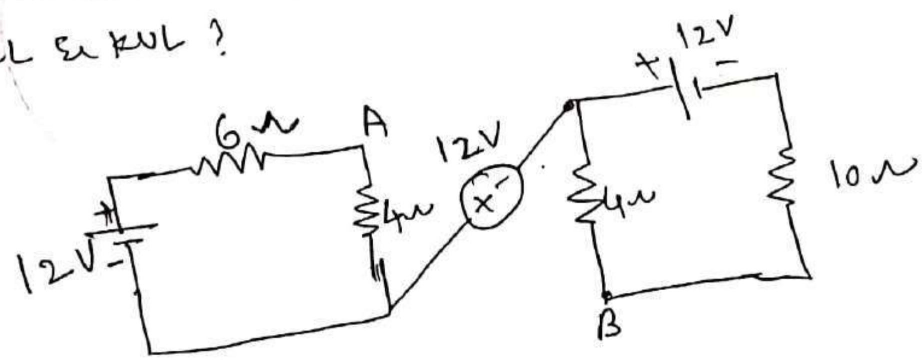
To find V_x

$$V_x = I_x \times R$$

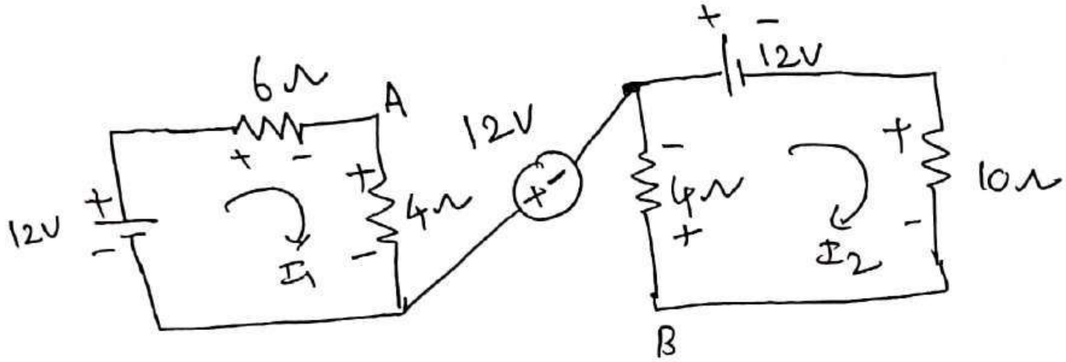
$$8 = I_x \times 10$$

$$\boxed{I_x = 0.8A.}$$

(Q) Find the voltage b/w A & B for the following ckt using KCL & KVL ?

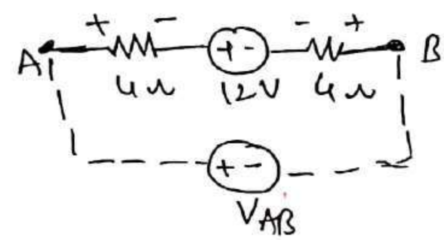


Sol



From A to B, voltage across A & B is

$$V_{AB} = V_{4\Omega} + 12 - V_{4\Omega}$$



$-V_{AB} + V_{4\Omega} + 12 - V_{4\Omega} = 0$

To find $V_{4\Omega}$ (First loop)

apply KVL for loop1

$$-12 + I_1 6 + 4I_1 = 0$$

$$10 I_1 = 12$$

$$I_1 = 1.2A.$$

$$V_{4\Omega} = 1.2 \times 4 = 4.8\ V.$$

To find $V_{4\Omega}$ (Second loop)

apply KVL for loop2

$$+12 + 10I_2 + 4I_2 = 0$$

$$14I_2 = -12$$

$$I_2 = -12/14 = -0.86$$

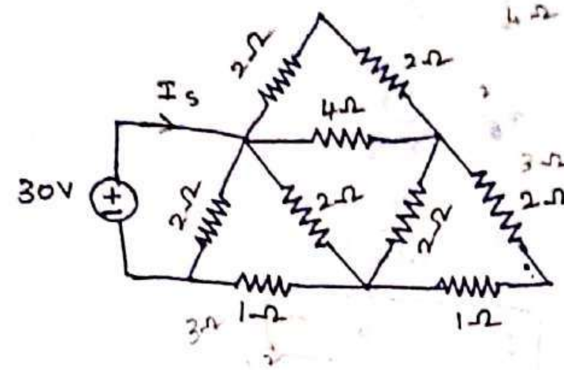
$$V_{4\Omega} = -0.86 \times 4 = -3.44\ V.$$

V_{AB}

$$V_{AB} = V_{4\Omega} + 12 - V_{4\Omega}$$

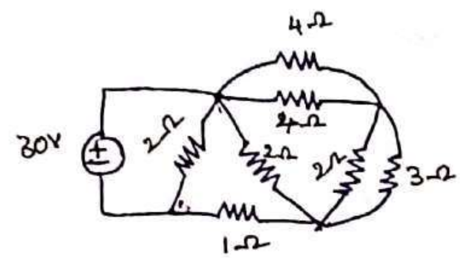
$$= 4.8 + 12 - (-3.44) = 20.24\ V.$$

Find the source current for the following circuit using Equivalent Resistance method.



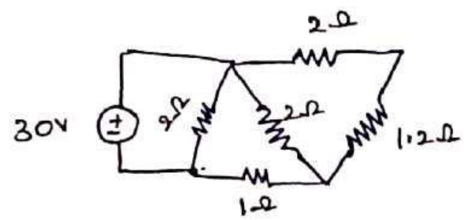
$$2+2 = 4\,\Omega$$

$$2+1 = 3\,\Omega$$



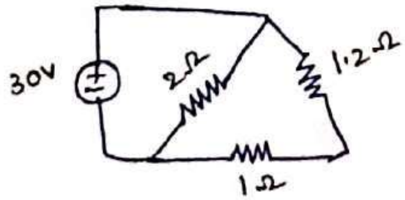
$$\frac{4\times4}{4+4} = 2\,\Omega$$

$$\frac{3\times2}{3+2} = 1.2\,\Omega$$

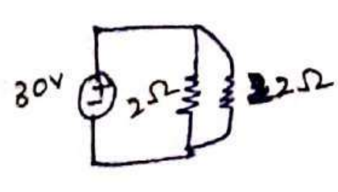


$$2+1.2 = 3.2\,\Omega$$

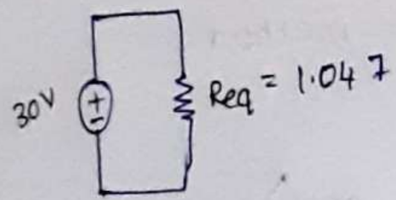
$$\frac{3.2\times2}{3.2+2} = 1.2\,\Omega$$



$$1.2+1 = 2.2\,\Omega$$



$$\frac{2.2\times2}{2+2.2} = 1.047$$



$$I_s = \frac{V_s}{R_{eq}}$$

$$I_s = \frac{30}{1.047}$$

$$\boxed{I_s = 28.45\ A}$$

Network Reduction Techniques

The main purpose of Network Reduction Techniques are to simplify the complex network into simple network for finding the network parameters.

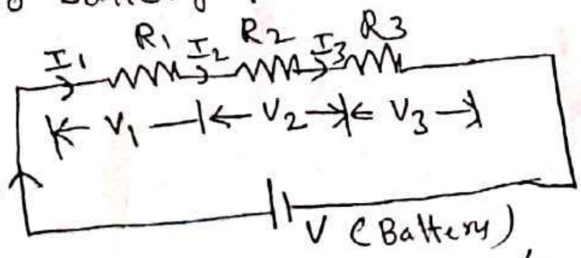
There are several techniques

1. series, parallel, series – parallel
2. star-to-delta or delta-to-star transformations

1. Series, parallel & series - parallel connections

(A) Resistances in series connection:

(i) Let R_1, R_2, R_3 are the 3 resistances connected in series to battery of 'v' volts as shown in fig.



(ii) In series connection, the current flowing through all resistances is same.

$$I = I_1 = I_2 = I_3 \quad -(1)$$

(iii) But in series connection, the voltage is dropped across each resistances

$$V = V_1 + V_2 + V_3 \quad -(2)$$

(iv) But from ohm's Law,

$$\left.\begin{aligned} V &= IR_{eq} \\ V_1 &= I_1R_1 = IR_1 \\ V_2 &= I_2R_2 = IR_2 \\ V_3 &= I_3R_3 = IR_3 \end{aligned}\right\} \text{eq-(3)}$$

(v) Sobsticuting eq (3) in eq (2)

$$IR_{eq} = IR_1 + IR_2 + IR_3$$

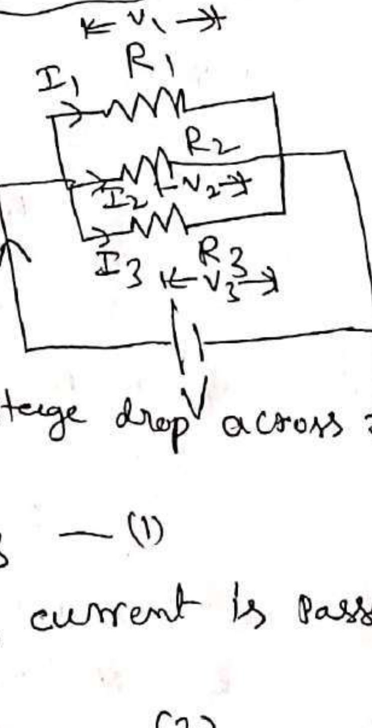
$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

where $R_{eq} \rightarrow$ equivalent Resistance of R_1, R_2, R_3 when They are connected in series.

~~(2) Resi~~

~~For Eg~~:

(2) Resistances are in parallel connection



(i) Let R_1, R_2, R_3 be The 3 resistances are connected in parallel to voltage of 'v' volts as shown in fig.

(ii) In parallel connection, The voltage drop across The all The resistances are Same.

$$V = V_1 = V_2 = V_3 \quad — (1)$$

(iii) But, In parallel connection, current is passes Through each resistance.

$$I = I_1 + I_2 + I_3 \quad — (2)$$

(iv) But from ohms law

$$\left.\begin{aligned} I &= \frac{V}{R_{eq}} \\ I_1 &= \frac{V_1}{R_1} = \frac{V}{R_1} \\ I_2 &= \frac{V_2}{R_2} = \frac{V}{R_2} \\ I_3 &= \frac{V_3}{R_3} = \frac{V}{R_3} \end{aligned}\right\} \quad -(3)$$

Note:
If two Resistors in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{R_2 + R_1}{R_1 R_2}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

(v) By substicuting eq (3) in eq (2)

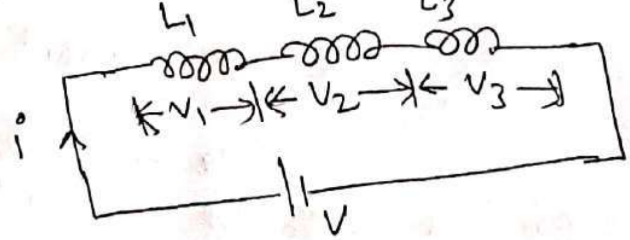
$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

where $R_{eq} \rightarrow$ Total resistance or resultant of $R_1 R_2 R_3$ when They are connected in parallel.

Inductors

(1) Inductors in series



Like Resistances in Series.

$$V = V_1 + V_2 + V_3$$

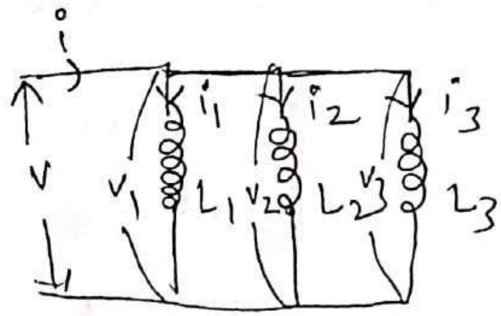
$$L_{eq}\frac{di}{dt} = L_1\frac{di_1}{dt} + L_2\frac{di_2}{dt} + L_3\frac{di_3}{dt}$$

$$= L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt}$$

$$L\frac{di}{dt} = (L_1 + L_2 + L_3)\frac{di}{dt} \Rightarrow \boxed{L_{eq} = L_1 + L_2 + L_3}$$

Scanned with CamScanner

Inductors in parallel



$$i = i_1 + i_2 + i_3$$

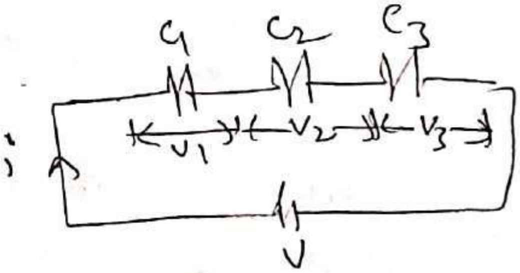
$$= \frac{1}{L_1}\int v\,dt + \frac{1}{L_2}\int (v\,dt) + \frac{1}{L_3}\int v\,dt$$

$$V = V_1 = V_2 = V_3$$

$$\frac{1}{L_{eq}}\int v\,dt = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)\int v\,dt$$

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$$

capacitors in Series



$$V = V_1 + V_2 + V_3$$

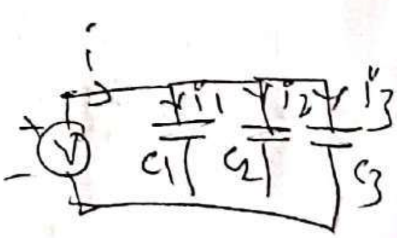
$$\frac{1}{C_{eq}}\int i\,dt = \frac{1}{C_1}\int i\,dt + \frac{1}{C_2}\int i\,dt + \frac{1}{C_3}\int i\,dt$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Note: If two capacitors are in Series.

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \qquad \left[\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}\right]$$

capacitors in parallel



$$i = i_1 + i_2 + i_3$$

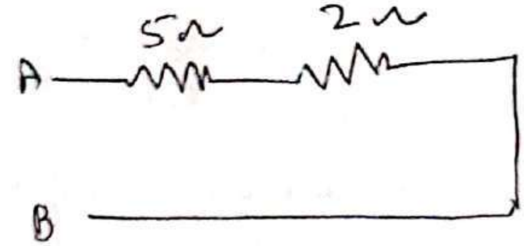
$$C_{eq}\frac{dv}{dt} = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dv}{dt}$$

$$\boxed{C_{eq} = C_1 + C_2 + C_3}$$

Problems on series, parallel & series-parallel resistance

(1) Find equivalent resistance b/w point A & B.

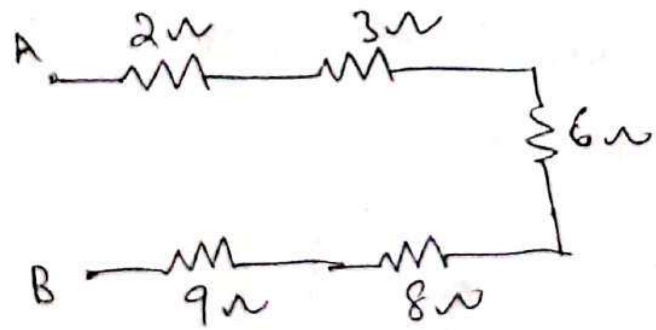
a)



Sol

$$\boxed{R_{AB} = R_{eq} = 5 + 2 = 7\,\Omega}$$

b)

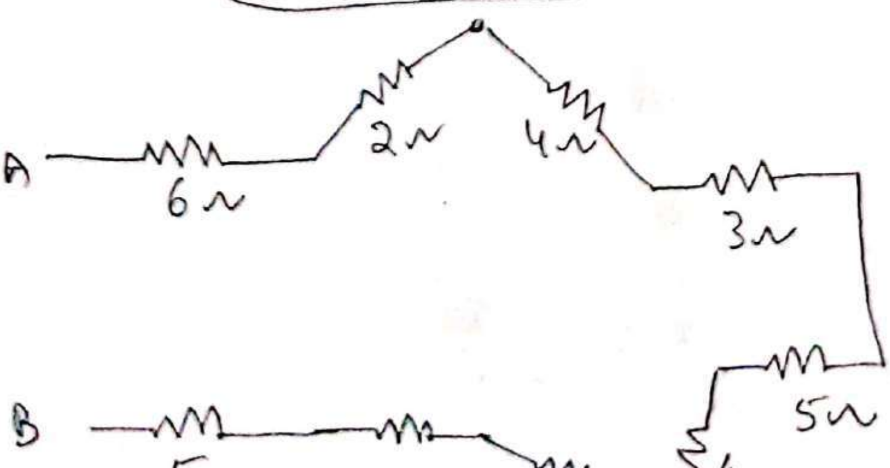


Sol

$$R_{AB} = R_{eq} = 2 + 3 + 6 + 8 + 9$$

$$\boxed{R_{AB} = 28\,\Omega}$$

c)

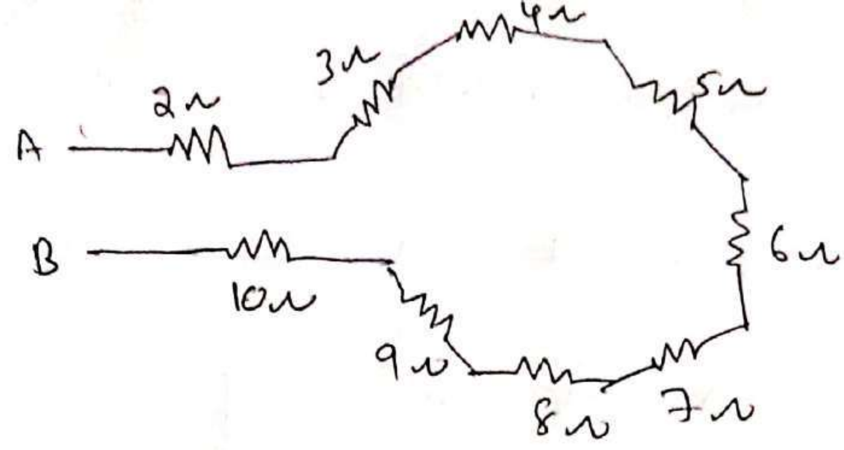


Sol

$$R_{AB} = R_{eq} = 6 + 2 + 4 + 3 + 5 + 4 + 8 + 2 + 5$$

$$\boxed{R_{AB} = 39\,\Omega}$$

d)

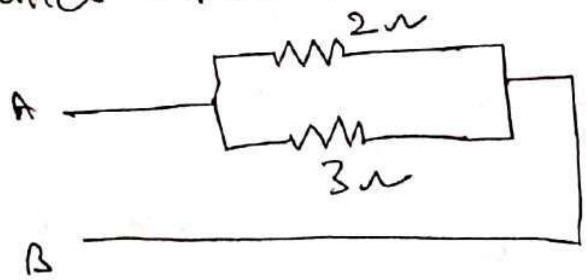


$R_{AB} = R_{eq} = 2+3+4+5\Omega+6\Omega+7\Omega+8+9+10$

$$\boxed{R_{AB} = 54\,\Omega}$$

(2) Find the equivalent resistance b/w following Parallel circuits?

A)



~~$R_{AB} = \frac{2\times3}{}$~~

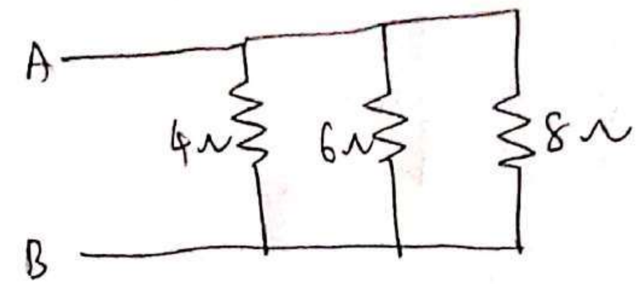
$\frac{1}{R_{AB}} = \frac{1}{2}+\frac{1}{3}$

$\frac{1}{R_{AB}} = \frac{3+2}{3\times2}$

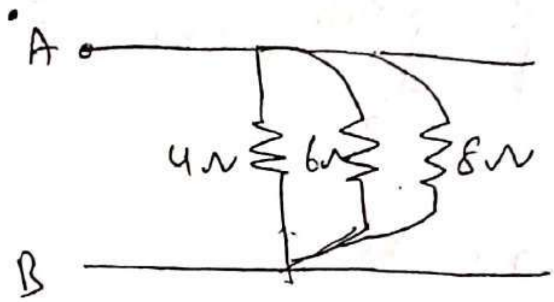
$$\boxed{R_{AB} = \frac{2\times3}{2+3}}$$

$$\boxed{R_{AB} = \frac{6}{5}\,\Omega}$$

(B)



<u>Sol</u>



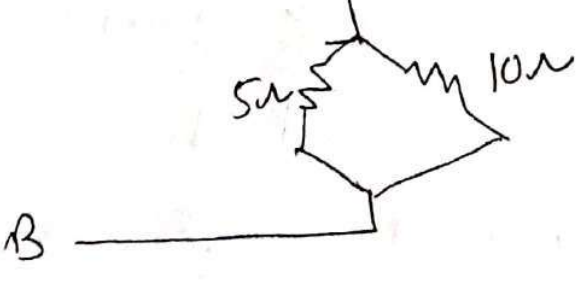
$$\frac{1}{R_{AB}} = \frac{1}{4} + \frac{1}{6} + \frac{1}{8}$$

$$\frac{1}{R_{AB}} = \frac{12+8+6}{48}$$

$$\frac{1}{R_{AB}} = \frac{26}{48}$$

$$\boxed{R_{AB} = \frac{48}{26}\,\Omega}$$

(C)

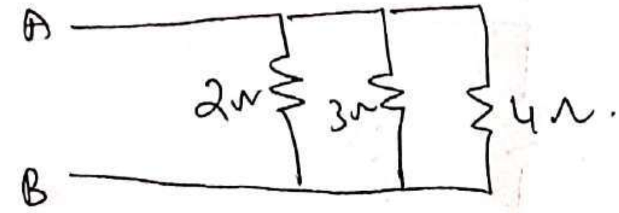


$R_{AB} = ?$

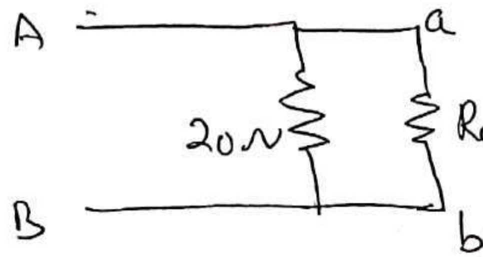
<u>Sol</u> $\frac{1}{R_{AB}} = \underbrace{\frac{1}{5} + \frac{1}{10}}$ $\Rightarrow R_{AB} = \frac{5\times10}{5+10} = \frac{50}{25} = 2\Omega$

Scanned with CamScanner

d)



Sol)



$$R_{ab} = \frac{3 \times 4}{3+4} = \frac{12}{7} = 1.71\,\Omega$$

$$R_{AB} = \frac{20 \times 1.71}{20 + 1.71}$$

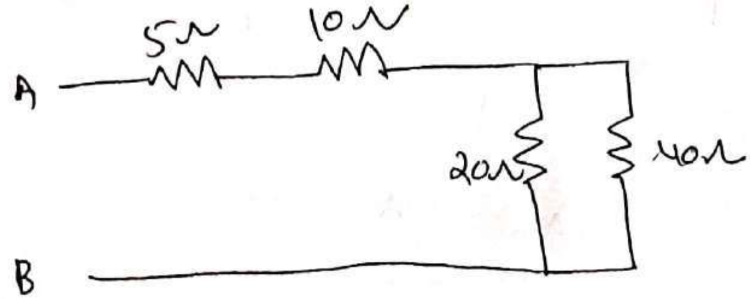
$$= \frac{34.2}{21.71}$$

$$\boxed{R_{AB} = 1.58\,\Omega}$$

Series - Parallel

(3) Find equivalent resistance between A & B.

(A)

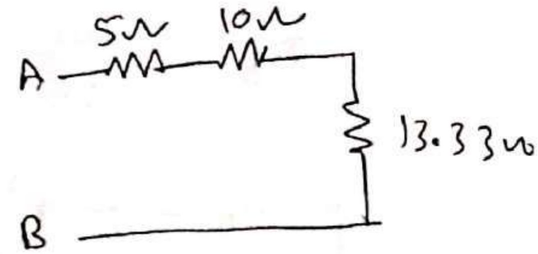


Sol) **step 1:** 20Ω & 40Ω are connected in parallel.

$$= \frac{20 \times 40}{20+40} = \frac{800}{60} = 13.33\,\Omega$$

Scanned with CamScanner

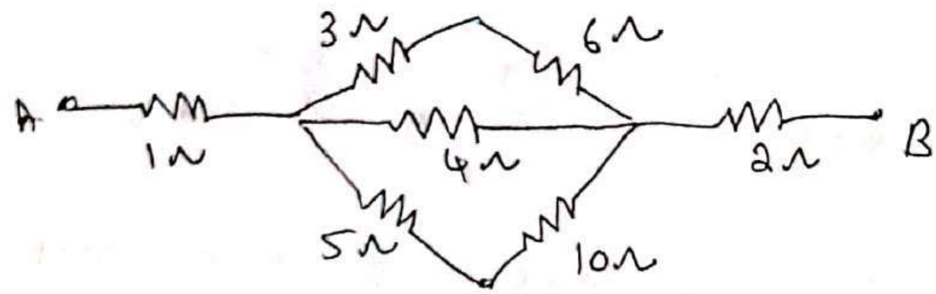
step 2: 5 Ω, 10 Ω & 13.33 Ω are in series.



$R_{AB} = 5+10+13.33$

$$\boxed{R_{AB} = 28.33\ \Omega}$$

(B)



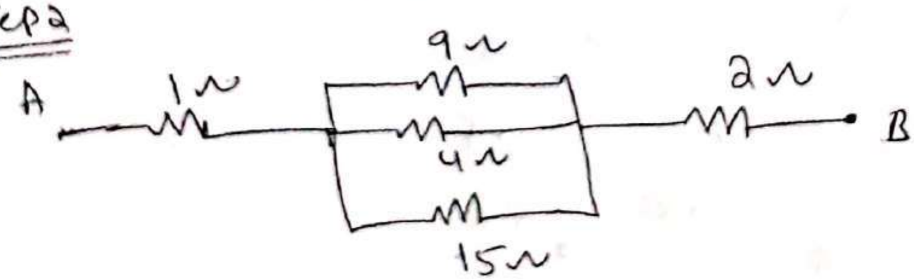
Sol

step 1: 3 Ω & 6 Ω are series and 5 Ω & 10 Ω are in Series

$\Rightarrow\ 3+6 = 9\ \Omega$

$5+10\ \Omega = 15\ \Omega$

step 2



here 9 Ω, 4 Ω & 15 Ω resistances are in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{9} + \frac{1}{4} + \frac{1}{15}$$

$$\frac{1}{R_{eq}} = \cancel{\frac{15+9+4}{9\times4\times15}} \quad \frac{4\times15 + 15\times9 + 9\times4}{9\times4\times15}$$

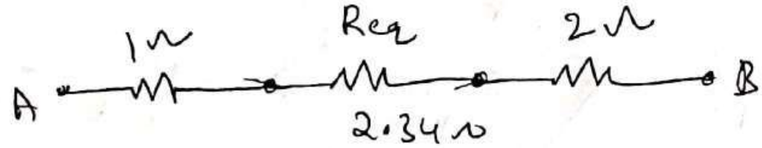
$$= \frac{60+135+36}{9\times4\times15} = \frac{231}{540}$$

$$\frac{1}{R_{eq}} = \frac{231}{540}$$

$$R_{eq} = \frac{540}{231}$$

$$\boxed{R_{eq} = 2.34}$$

step3:

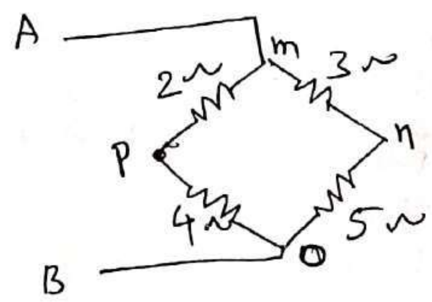


here Three resistances are in series

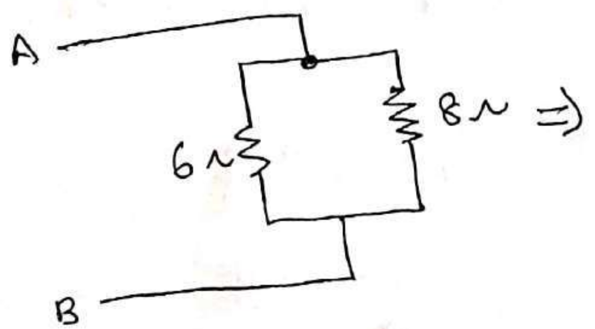
$$R_{AB} = 1 + 2.34 + 2$$

$$\boxed{R_{AB} = 5.34\,\Omega}$$

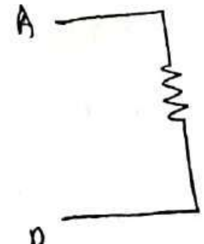
(c) → Determine the equalent resistance.



sol



$\Rightarrow$

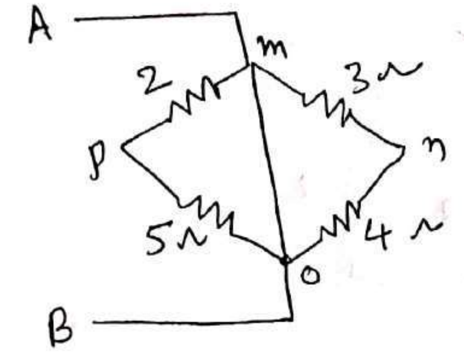


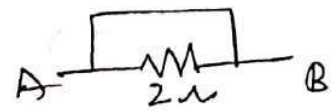
$$\frac{6\times 8}{6+8} = \frac{\cancel{48}^{24}}{\cancel{14}_{7}}$$

$$= 3.4\,\Omega$$

$$\boxed{R_{AB} = 3.4\,\Omega}$$

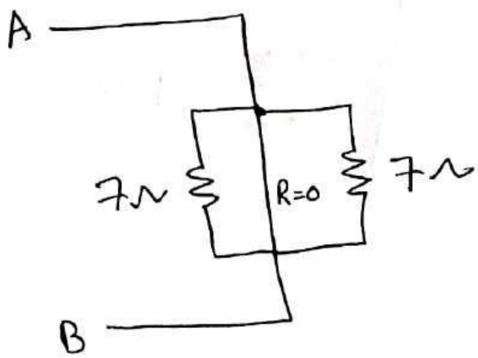
→





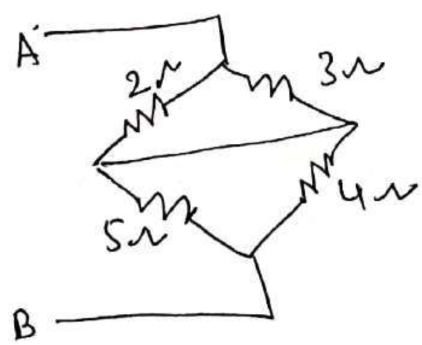
<u>Sol</u> In the above ckt

$$R_{AB} = \frac{0\times2}{0+2} = 0\Omega$$

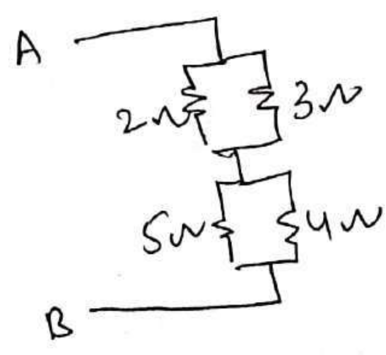


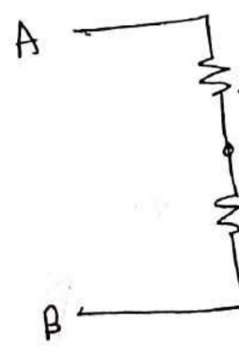
$$\boxed{R_{AB} = 0\Omega}$$

→



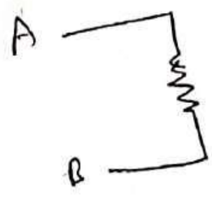
<u>Sol</u>





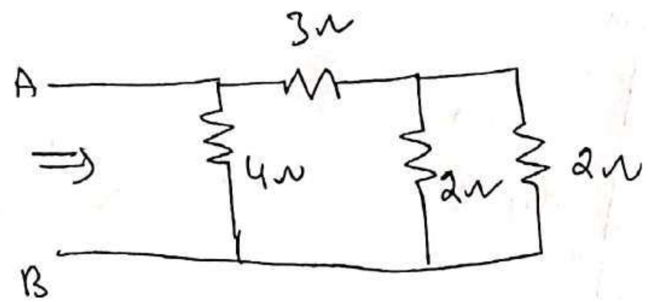
$$\frac{2\times3}{2+3} = \frac{6}{5} = 1.2\Omega$$

$$\frac{5\times4}{5+4} = \frac{20}{9} = 2.2\Omega$$

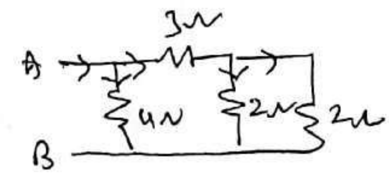


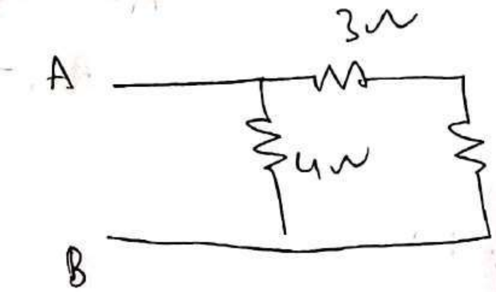
$$1.2 + 2.2 = 3.4\Omega$$

→ Find equivalent resistance b/w A & B

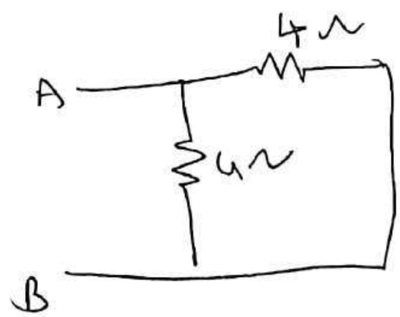


Sol



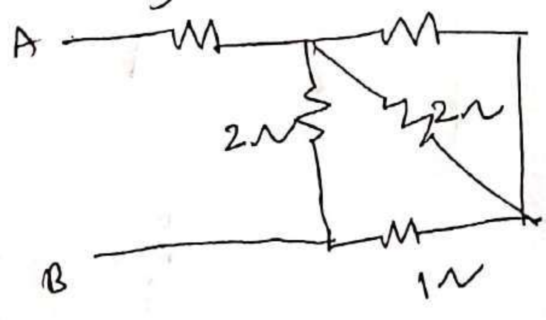


$\frac{2\times2}{2+2} = 1\Omega$

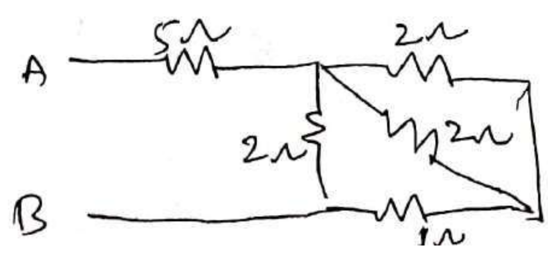


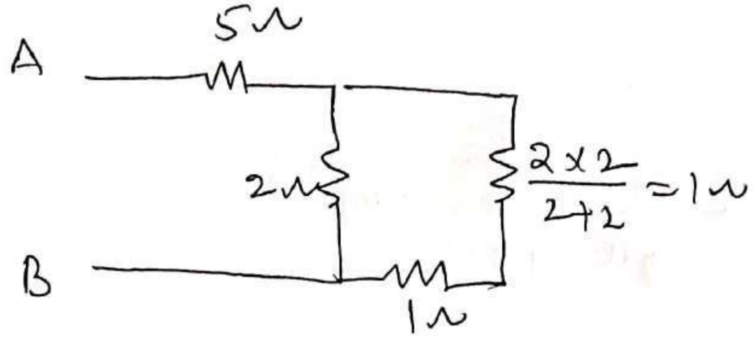
$$R_{AB} = \frac{4\times4}{4+4} = \frac{16}{8} = 2\Omega$$

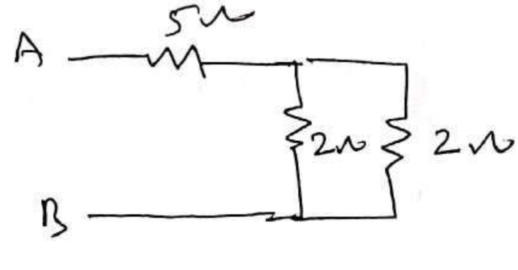
→

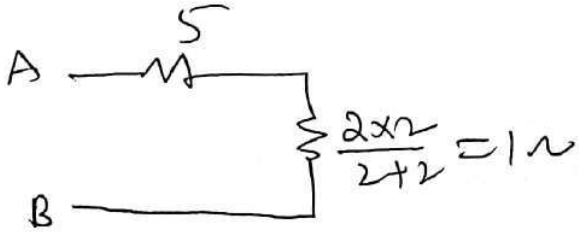


Sol



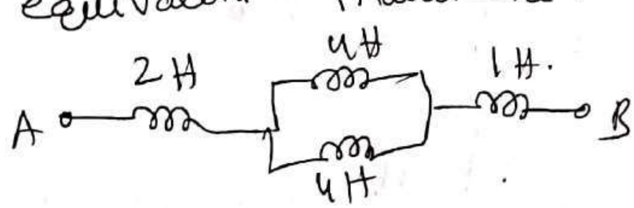




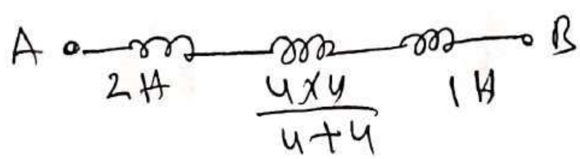


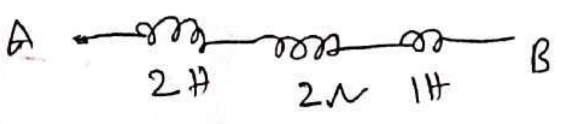
$$\boxed{R_{AB} = 5+1 = 6\,\Omega}$$

→ Find equivalent inductance.

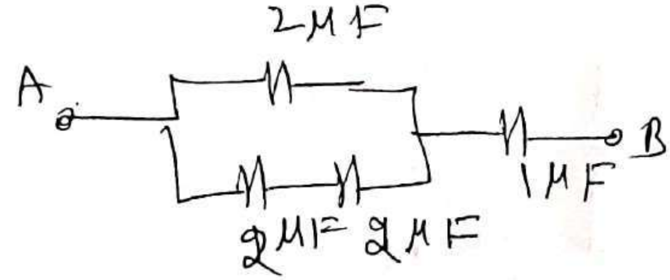


Sol It is like Resistance are connected in parallel.



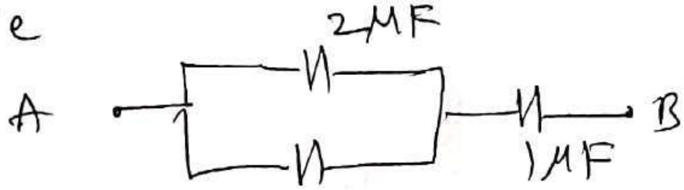


→ Find equivalent capacitance.

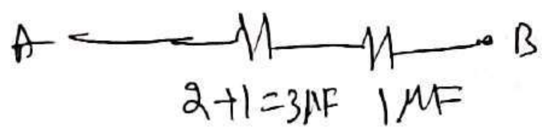


Sol

here

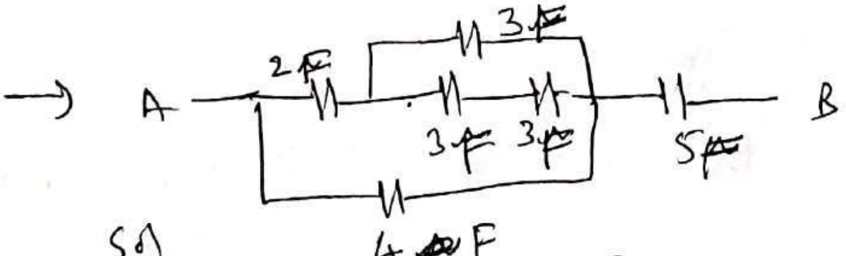


$$\frac{2\times 2}{2+2} = 1\,\mu F$$



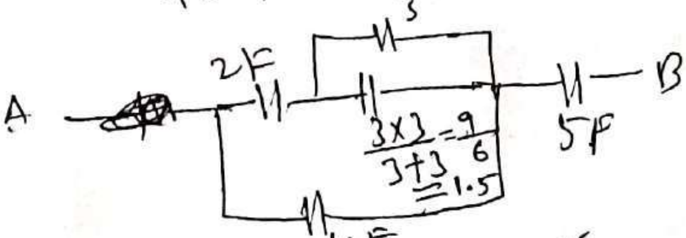
$$\frac{3\times 1}{3+1} = \frac{3}{4} = 0.75\,F$$

→



Sol

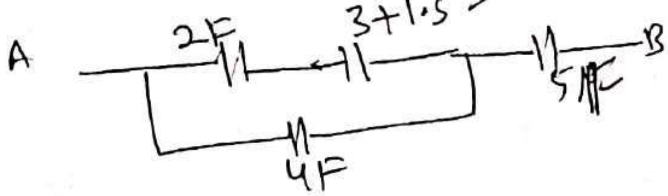
⇒



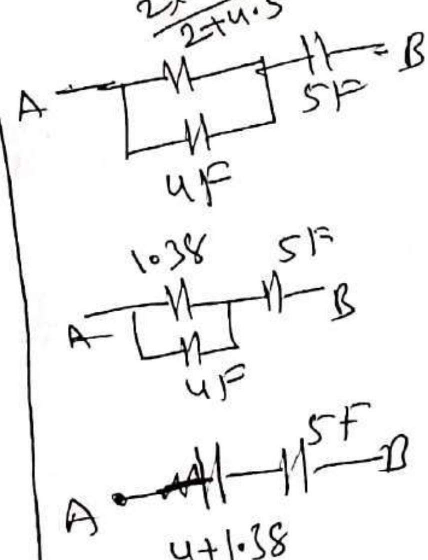
$$\frac{3\times 3}{3+3} = \frac{9}{6} = 1.5$$

$3+1.5=4.5$

⇒



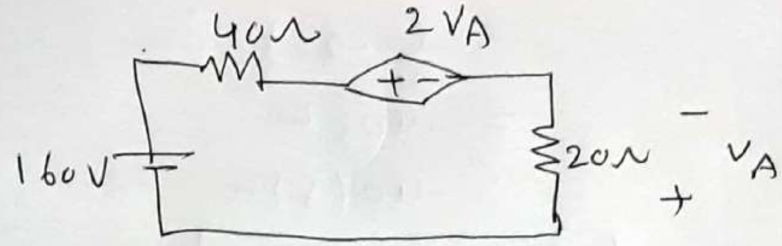
$$\frac{2\times 4.5}{2+4.5} = \frac{9}{6.5}$$



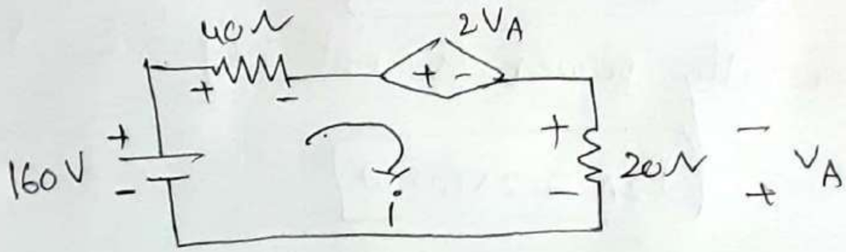
$4+1.38$
$=5.38$

$$C_{AB} = \frac{5.38\times 5}{5.38+5} = 2.59F$$

Q) Find the current, power absorbed by each resistor, Power by dependent source using KVL ?



Sol



(i) Apply KVL to the above ckt.

$$-160 + 40i + 2V_A - V_A = 0$$

where $V_A = -20i$

$$-160 + 40i + V_A = 0$$

$$-160 + 40i - 20i = 0$$

$$20i = 160$$

$$\boxed{i = 8A}$$

(ii) Power absorbed by each resistance.

$$P_{40\Omega} = i^2R = 8^2 \times 40 = 2560 \text{ watts.}$$

$$P_{20\Omega} = i^2R = 8^2 \times 20 = 1280 \text{ W.}$$

(iii) power by dependent source is

$$P = VI$$

$$= 2V_A \times i$$

$$= 2(-20i) \times i$$

$$= -40i^2$$

$$= -40(8)^2$$

$$\boxed{P_{2A} = -2560\,W}$$

Note: Generally power is not negative. so take

$$\boxed{P_{2A} = 2560\,W}$$

Scanned with CamScanner

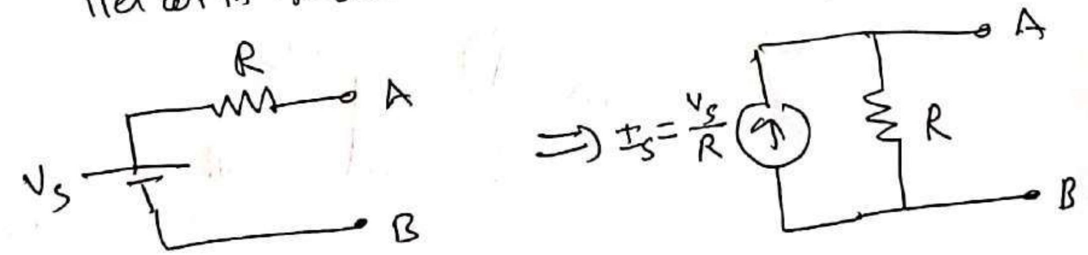
Source Transformation

Source transformation is a technique which is used to solving the networks for finding the solution.

- Basically sources are either voltage source or current source and sometimes it is necessary to convert voltage source to current source and vice versa in the network analysis.

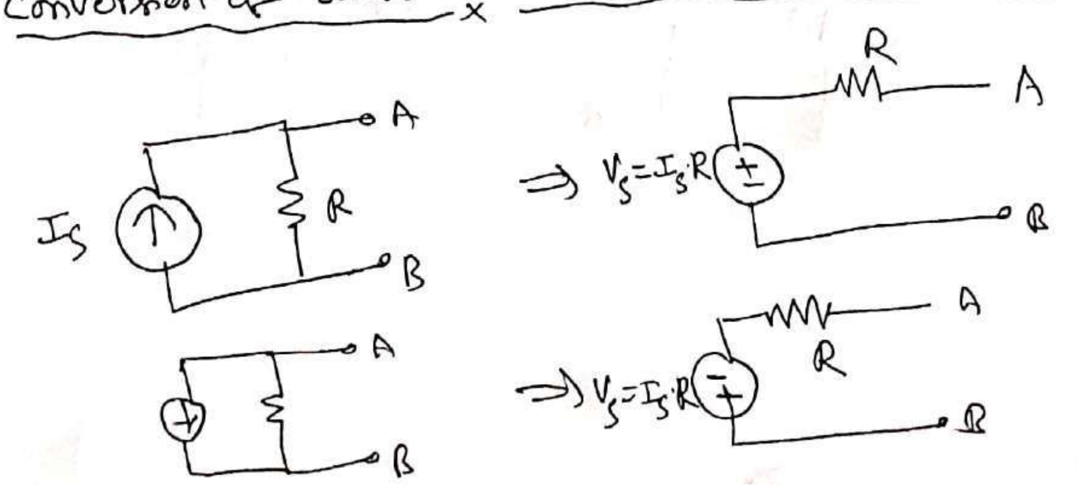
(i) Conversion of voltage source to current source.

Voltage source represent voltage V_s in series with resistance 'R'. it is shown in fig 1. current source represent current (I_s) in parallel with resistance 'R' it is shown in fig (2).



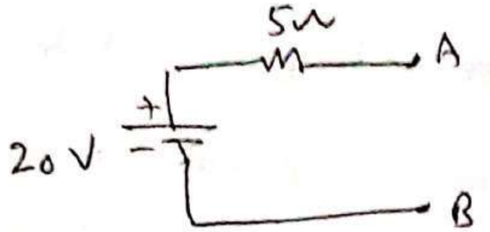
Volteege soure

(ii) Conversion of current source to voltage source

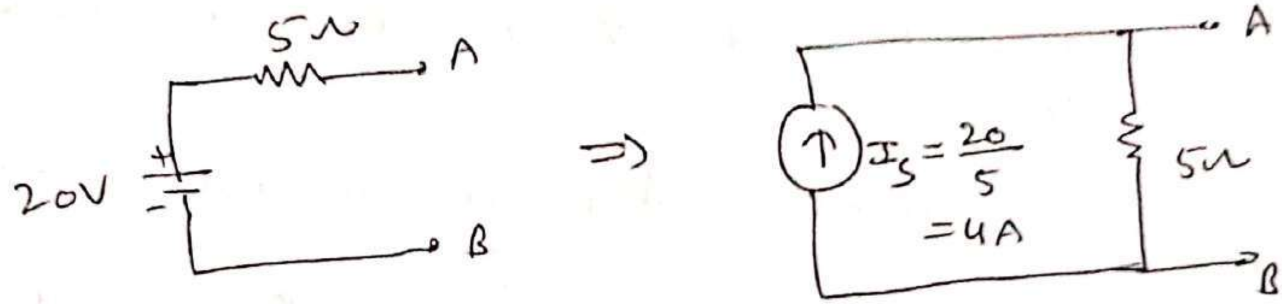


Problems on source transformation

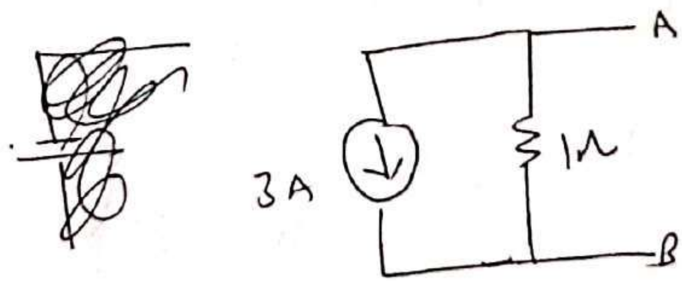
① convert voltage source to equivalent current source



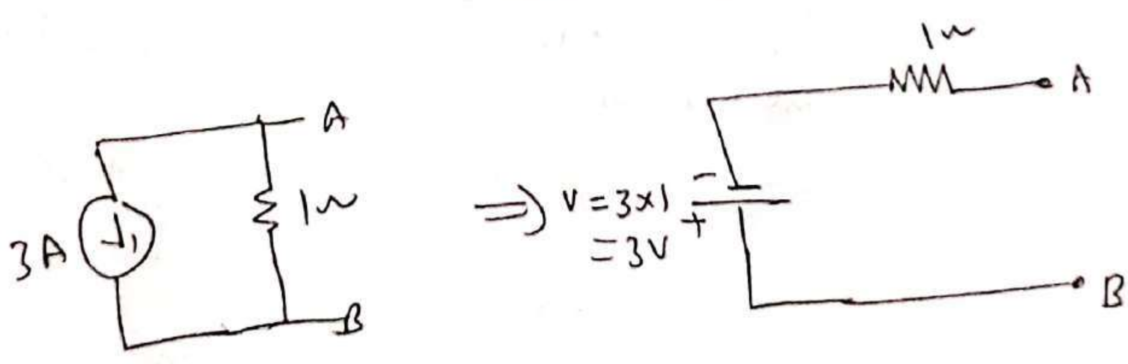
sol



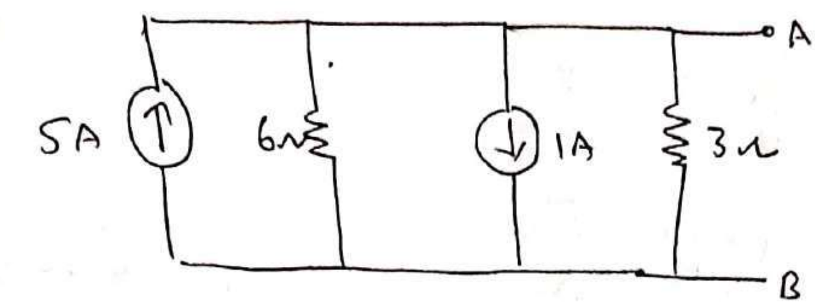
② convert to equivalent voltage source.



sol

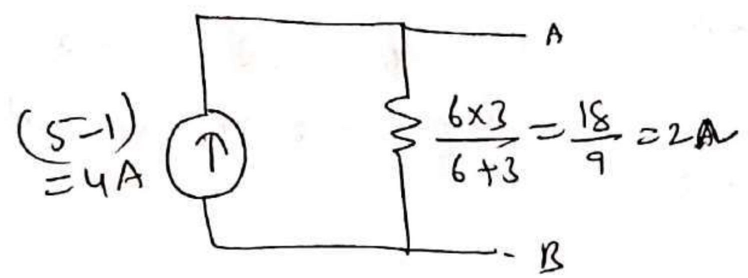


(3) Find single voltage source for the following ckt

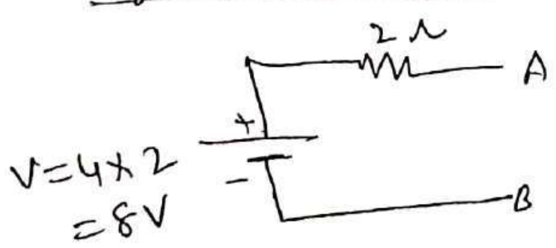


Sol

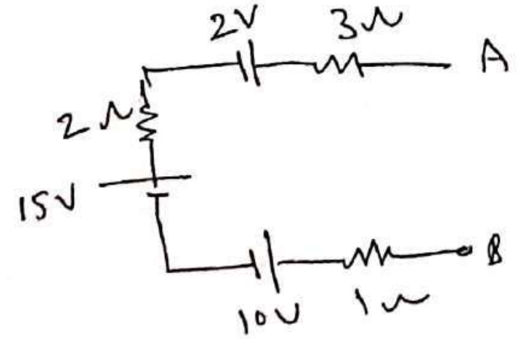
First convert current source into single source



Single voltage source is

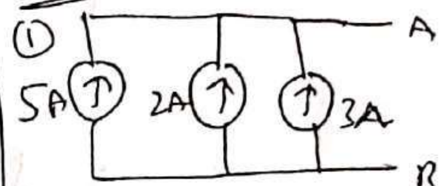


(4) Find single source

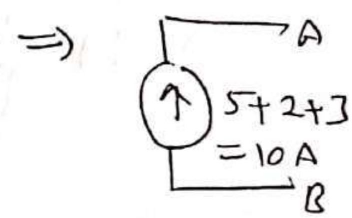


Note

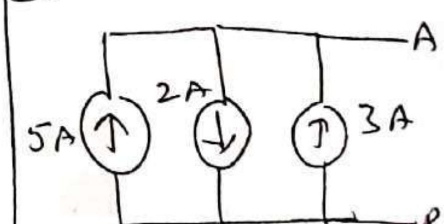
(1)



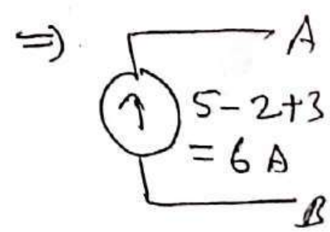
⟹



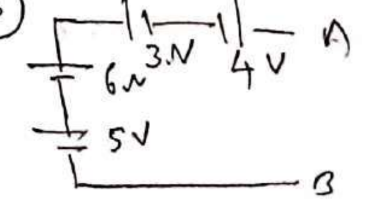
(2)

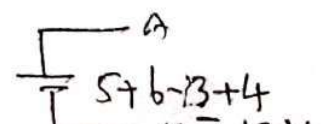


⟹

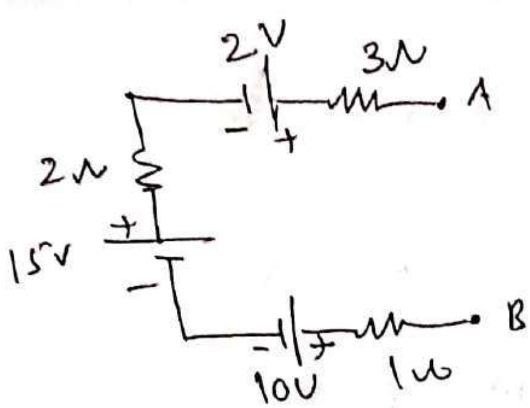


(3)

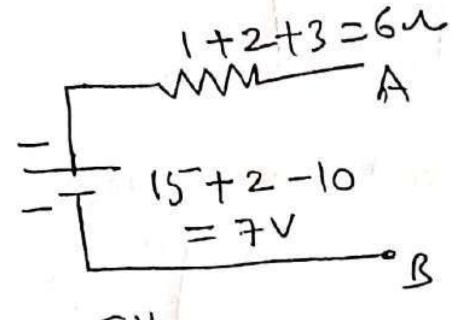




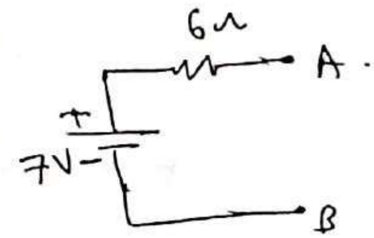
Sol



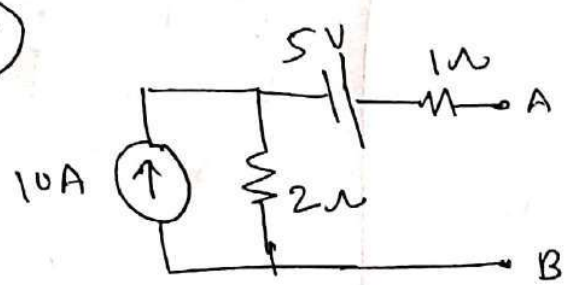
Sol



⇒



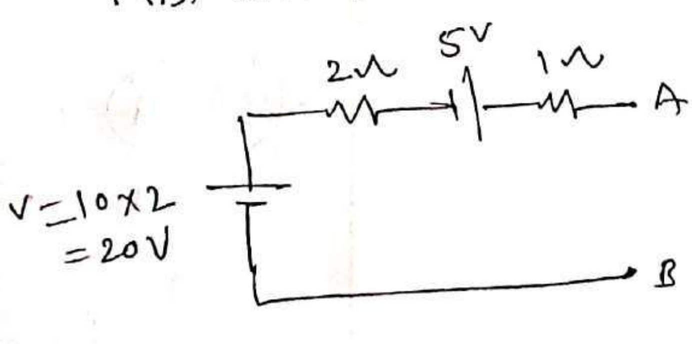
(5)

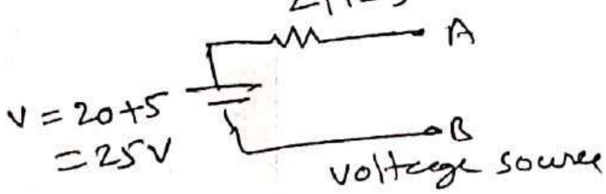


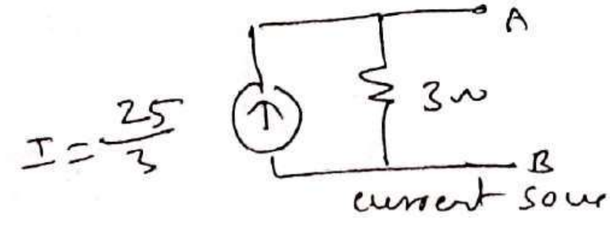
Find ~~a~~ single source using Source Transformation?

Sol **Sol**

First convert ~~voltage~~ current source into ~~current~~ voltage source.







Star-delta and delta-star transformation

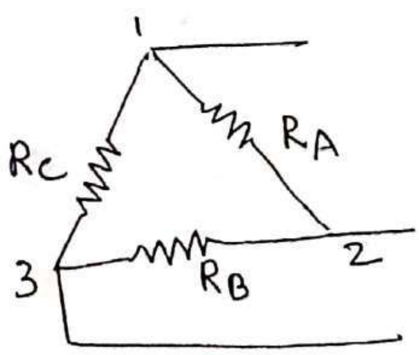
By solving networks, by the application of Kirchhoff's laws, Sometimes experiences great difficulty due to a large no. of Simultaneous equations that have to be solved complicated.

In order avoid the difficulties, Delta-star & star-delta are very useful for reduction of complex n/ws. In this n/w's are simplified by replacing delta by equivalent star & vice versa.

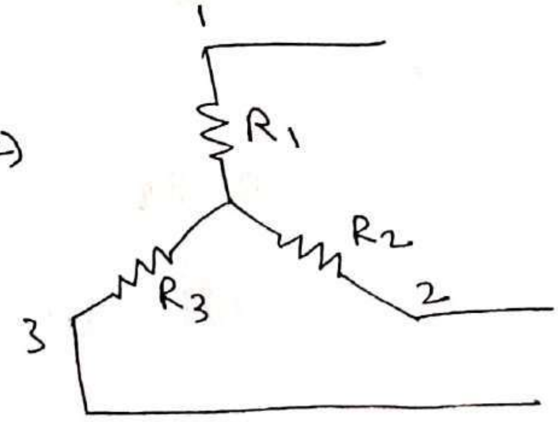
Delta-star transformation (or) [$\Delta - Y(\lambda)$]

Consider R_A, R_B, R_C are the three resistances are connected in delta connection b/w terminals 1, 2 & 3 as shown in fig 1 and these resistances can be replaced by equivalent resistan and these resistances can be replaced by equivalent resistances connected in star as shown in fig 2.

R_1, R_2 & R_3 are connected in star as shown in fig 2.



Delta.



star

Step 1: In delta connection, Terminals b/w 1 & 2, the resistance R_A is parallel with $R_B + R_C$. So equalent resistance is

$$\frac{R_A \times (R_C + R_B)}{R_A + R_C + R_B} \quad -(1)$$

Similarly, In star connection, The resistance b/w same terminals 1 & 2 is

$$R_1 + R_2 \quad -(2)$$

eq (1) & (2) are equating,

$$R_1 + R_2 = \frac{R_A \cdot (R_B + R_C)}{R_A + R_B + R_C} \quad -(3)$$

||ly for 2 & 3, 3 & 1 terminals.

$$R_2 + R_3 = \frac{R_B \cdot (R_C + R_A)}{R_A + R_B + R_C} \quad -(4)$$

$$R_3 + R_1 = \frac{R_C \cdot (R_A + R_B)}{R_A + R_B + R_C} \quad -(5)$$

Subtracting eq (3) - eq (4)

$$R_1 + R_2 - R_2 - R_3 = \frac{R_A \cancel{R_B} + R_A R_C - R_B R_C - R_{\cancel{A}} R_B}{R_A + R_B + R_C}$$

$$R_1 - R_3 = \frac{R_A R_C - R_B R_C}{R_A + R_B + R_C} \quad -(6)$$

Now add eq (5) & (6)

$$\cancel{R_3} + R_1 + R_1 - \cancel{R_3} = \frac{R_A R_C + R_{\cancel{B}} R_C + R_A R_C - R_{\cancel{B}} R_C}{R_A + R_B + R_C}$$

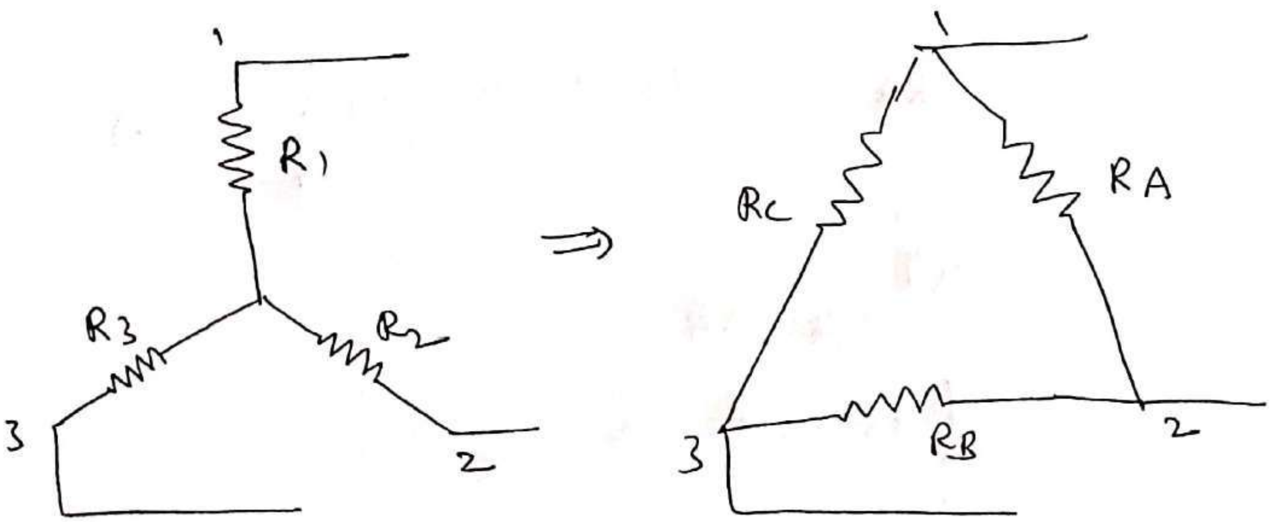
$$2R_1 = \frac{2 R_A R_C}{R_A + R_B + R_C}$$

star $\boxed{R_1 = \frac{R_A R_C}{R_A + R_B + R_C}}$ delta $\quad -(7)$

IIly
$$\boxed{R_2 = \frac{R_A R_B}{R_A + R_B + R_C}} \quad \text{— (8)}$$

$$\boxed{R_3 = \frac{R_B R_C}{R_A + R_B + R_C}} \quad \text{— (9)}$$

(2) Star to delta ~~conn~~ Transformation (or) [λ to Δ]



From eq (7), (8) & (9),

$$R_1 R_2 = \frac{R_A^2 R_B R_C}{(R_A + R_B + R_C)^2} \quad \text{— (10)}$$

$$R_2 R_3 = \frac{R_B^2 R_C R_A}{(R_A + R_B + R_C)^2} \quad \text{— (11)}$$

$$R_3 R_1 = \frac{R_C^2 R_A R_B}{(R_A + R_B + R_C)^2} \quad \text{— (12)}$$

Add (10), (11) & (12) eq

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_A R_B R_C (R_A + R_B + R_C)}{(R_A + R_B + R_C)^{2}}$$

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_A R_B R_C}{R_A + R_B + R_C} \quad -(13)$$

But $$R_1 = \frac{R_A R_C}{R_A + R_B + R_C} \quad -(14)$$

sub ~~RA~~ eq (14) in eq (13)

$$R_1R_2 + R_2R_3 + R_3R_1 = R_1 \times R_B$$

$$\boxed{R_B = R_2 + R_3 + \frac{R_2R_3}{R_1}} \quad -(15)$$

ly.

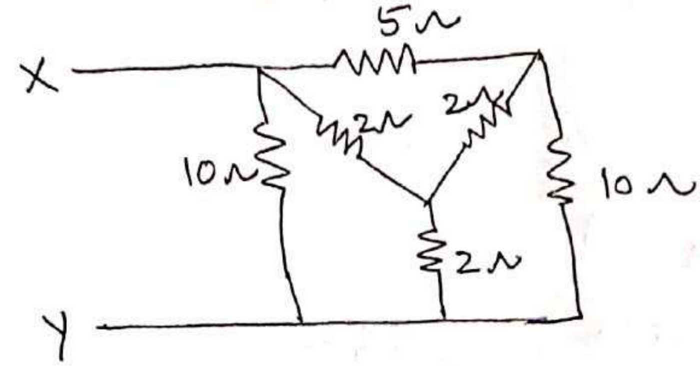
~~$R_A = R_A$~~

$$\boxed{R_A = R_1 + R_2 + \frac{R_1R_2}{R_3}} \quad -(16)$$

$$\boxed{R_C = R_1 + R_3 + \frac{R_1R_3}{R_2}} \quad -(17)$$

Problems on Y-Δ or Δ-Y

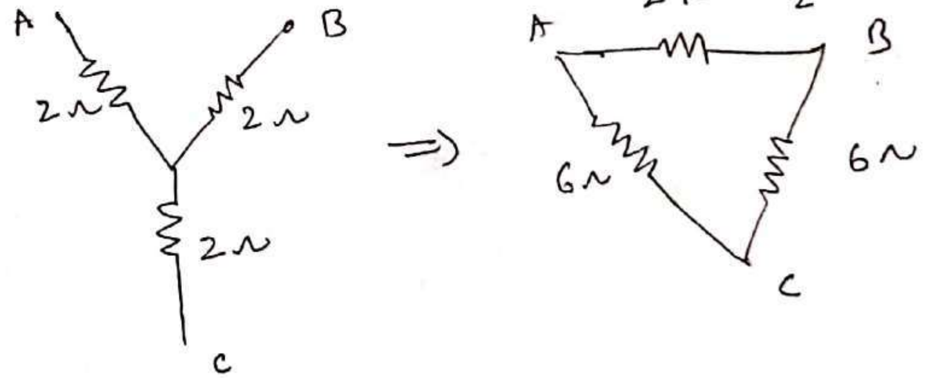
(1) Find equivalent resistance b/w x & y using star-delta transformation



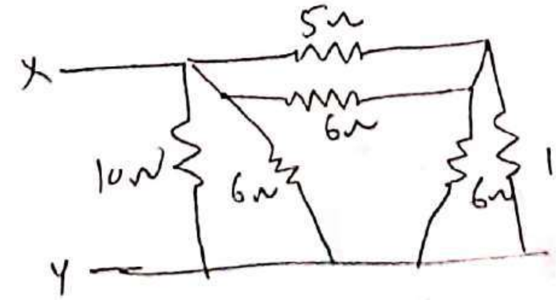
Sol

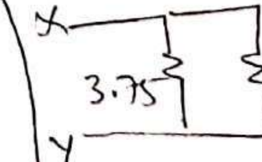
First convert star n/w into delta n/w

$2+2+\frac{2\times 2}{2}=6\Omega$



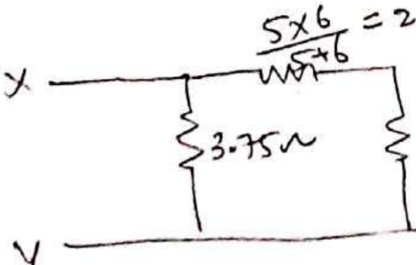
Then



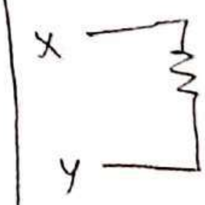


$2.72+3.75=6.47\Omega$

$\frac{5\times 6}{5+6}=2.72\Omega$



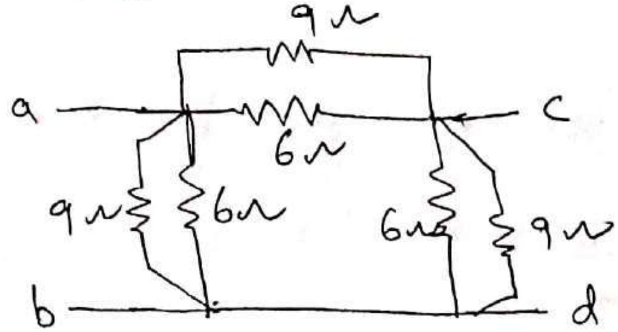
$\frac{6\times 10}{6+10}=3.75\Omega$



$\frac{3.75\times 6.47}{3.75+6.47}=2.37\Omega$

$\boxed{R_{xy}=2.37\Omega}$

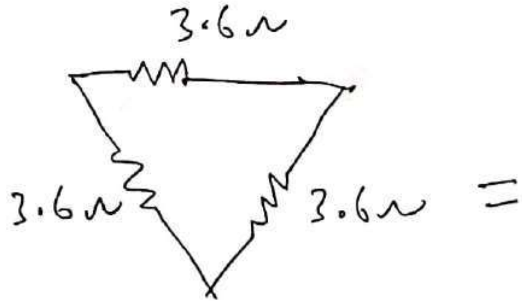
(2) Find The equivalent ~~resistance b/w~~ star



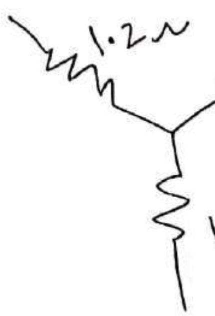
Sol

$$\frac{9 \times 6}{9+6} = 3.6$$

Redraw

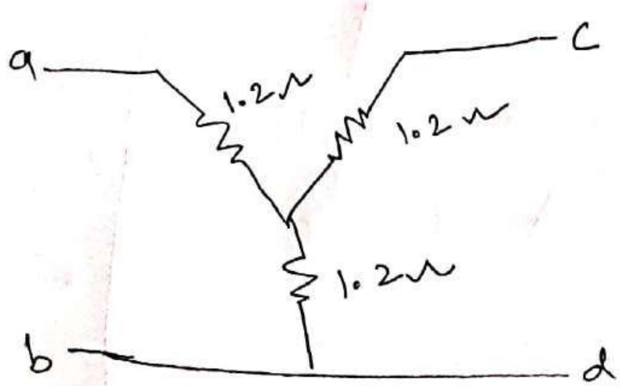


=



$$\frac{3.6 \times 3.6}{3.6+3.6+3.6} = 1.2\,\Omega.$$

Equivalent star



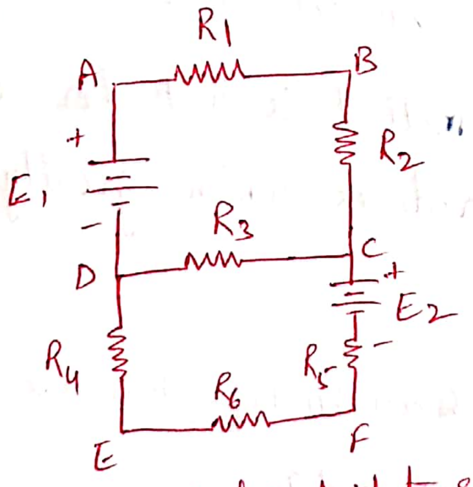
Network Analysis:

UNIT-I

[redacted]

Network Theorems (DC & AC), Mesh and Nodal Analysis

Mesh:- Mesh (or loop) is a set of branches forming a closed path in a network in such a way that if one branch is removed then remaining branches do not form a closed path. A loop also can be defined as a closed path which originates from a particular node, terminating at the same node, travelling through various other nodes without travelling through any node twice. In the fig. paths A-B-C-D-A, A-B-C-F-E-D-A, D-C-F-E-D etc are loops of the network.



An Electrical Network

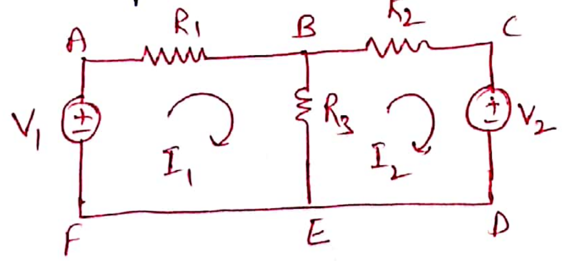
Node:- A point at which two or more elements are joined together is called a node. The junction points are also the nodes of the network. In the network shown in the Fig. A, B, C, D, E and F are the nodes of the network.

Loop Analysis & Mesh Analysis:-

This method of analysis is specially useful for the circuits that have many nodes and loops. The difference between application of Kirchoff's laws and loop analysis is, in loop analysis instead of branch currents, the loop currents are considered for writing the equations. The another difference is each branch of the network may carry more than one current. The total branch current must be decided by the algebraic sum of all currents through that branch. While in analysis using Kirchoff's laws, each branch current carries only one current.

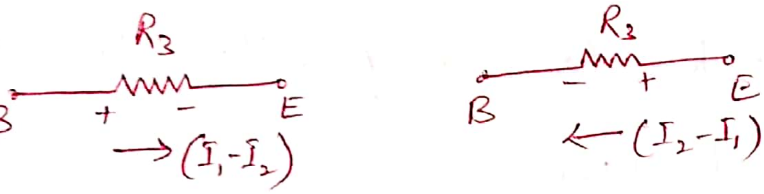
The advantage of this method is that for complex network the number of unknowns reduces which greatly simplifies calculation work.

Consider following network shown in Fig. There are two loops. So assuming two loop currents as I_1 and I_2



While assuming loop currents, Consider the loops such that each element of the network will be included atleast once in any of the loops.

Now Branch B-E Carries two currents I_1 from B to E and I_2 from E to B. So net current through branch B-E will $(I_1 - I_2)$ and Corresponding drop across R_3 must be as shown below in Fig.

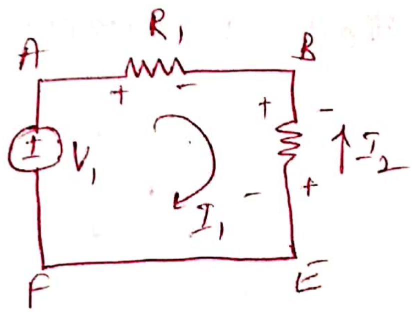


Consider loop A-B-E-F-A

For branch B-E, polarities of voltage drops will be B +ve, E -ve for current I_1 while E +ve, B -ve for current I_2 flowing through R_3.

Now while writing loop equations assume main loop current as positive and remaining loop current must be treated as negative for common branches.

Writing loop equations for the network shown in the Fig



For loop A-B-E-F-A

$$-I_1 R_1 - I_1 R_3 + I_2 R_3 + V_1 = 0$$

For loop B-C-D-E-B

$$-I_2 R_2 - V_2 - I_2 R_3 + I_1 R_3 = 0$$

By solving above simultaneous equations any unknown branch current can be determined

Scanned by CamScanner

1. While assuming loop currents make sure that atleast one loop current links with every element.

2. No two loops should be identical.

3. Choose minimum number of loop currents.

4. If current in a particular branch is required, then try to choose loop current in such a way that only one loop current links with that branch.

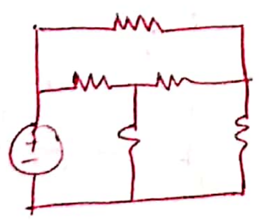
→ If a network has large no. of voltage sources it is useful to use Mesh analysis.

$\boxed{\text{KVL + Ohm's law = Mesh analysis}}$

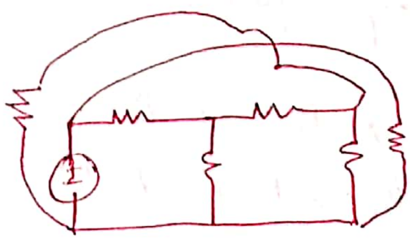
→ Mesh analysis is only applicable for planar network. For non planar circuits mesh analysis is not applicable.

** → A Circuit is said to be planar if it can be drawn on a planar surface without crossovers.

** → A non planar circuit can't be drawn on a plane surface without a crossover.



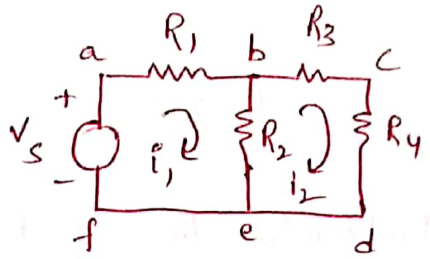
Planar



Non planar

Eg:-

Check:-



→ Check whether the circuit is planar or not.

→ Select mesh currents

→ Write KVL for every loop & solve it

for mesh ① $V_s = i_1 R_1 + R_2(i_1 - i_2)$

② $R_3 i_2 + R_4 i_2 + R_2(i_2 - i_1) = 0$

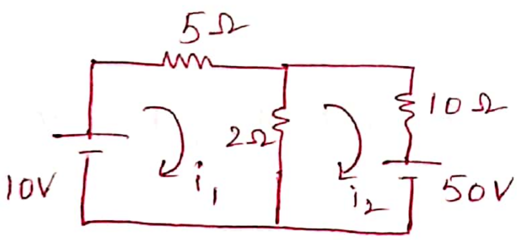
No. of Mesh equations = No. of branches - (No. of nodes - 1)

$$\boxed{M = B - (N-1)}$$

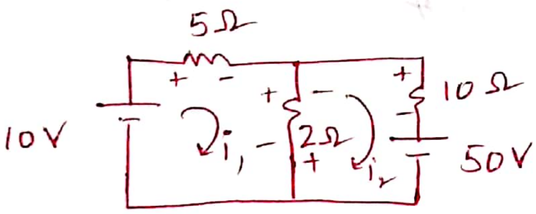
In the above ckt

$$M = 5 - (4-1) = 2$$

Problem 1:- Write mesh currents equations in the circuit shown and determine the currents.



Sol:-



Apply KVL to mesh ①

$$10 - 5i_1 - 2(i_1 - i_2) = 0 \Rightarrow -7i_1 + 2i_2 = -10 \rightarrow ①$$

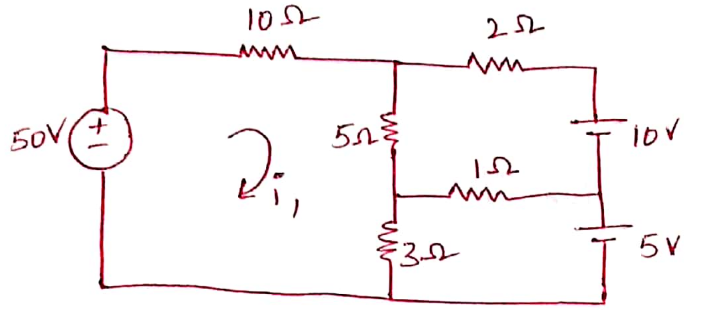
Apply KVL to mesh ②

$$-10i_2 - 50 - 2(i_2 - i_1) \Rightarrow 2i_1 - 12i_2 = 50 \rightarrow ②$$

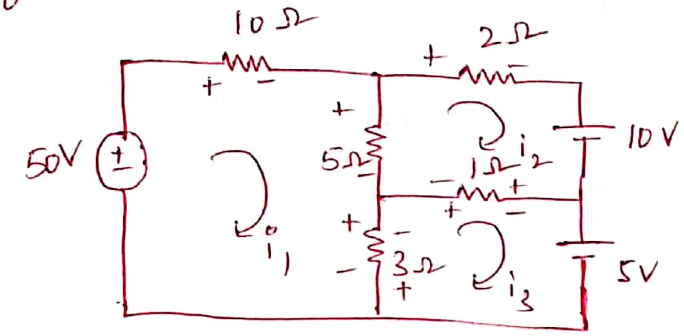
$i_1 = 0.25A$

$i_2 = -4.125A$

Problem 2:- Determine the mesh currents I_1 in the circuit shown



Sol:-



Apply KVL to loop ①

~~$50 - 10i_1 - 5i_2 - 3$~~

$$50 - 10i_1 - 5(i_1 - i_2) - 3(i_1 - i_3) = 0 \Rightarrow -18i_1 + 5i_2 + 3i_3 = -50 \rightarrow ①$$

Apply KVL to loop ②

$$-2i_2 - 10 - 1(i_2 - i_3) - 5(i_2 - i_1) = 0 \Rightarrow 5i_1 - 8i_2 + i_3 = 10 \rightarrow ②$$

Apply KVL to loop ③

$$-3(i_3 - i_1) - 1(i_3 - i_2) - 5 = 0 \Rightarrow 3i_1 + i_2 - 4i_3 = 5 \rightarrow ③$$

$i_1 = 3.3A$

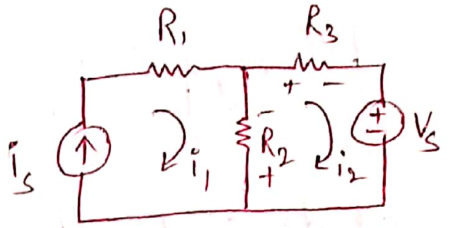
$i_2 = 0.99A$

$i_3 = 1.47A$

Mesh Current Analysis with Current Sources:-

→ Mesh current i_1 is equal to i_s

i.e., $i_1 = i_s$



→ Write KVL for second loop

$$-R_3 i_2 - V_s - R_2(i_2 - i_1) = 0$$

$$V_s = i_1 R_2 - i_2(R_2 + R_3)$$

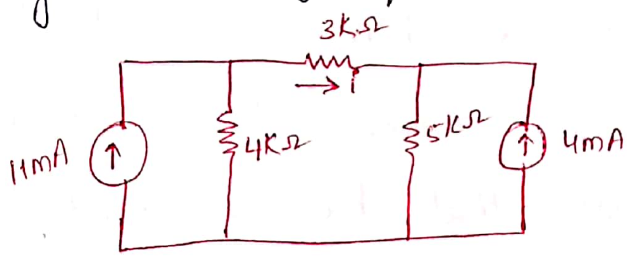
since $i_1 = i_s$

$$\frac{V_s - i_s R_2}{R_2 + R_3} = -i_2 \Rightarrow i_2 = \frac{-V_s + i_s R_2}{R_2 + R_3}$$

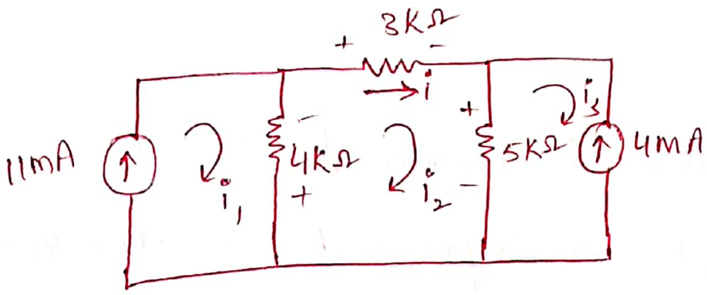
** → Presence of current sources reduces the no of mesh equations in mesh analysis.

Problem 1:-

Using mesh analysis find 'i' in the circuit shown in fig.



Sol:



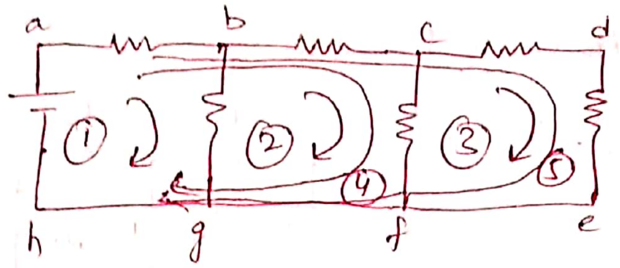
From the circuit $i_1 = 11mA$, $i_3 = -4mA$

Apply KVL to mesh ②

$$-3000 i_2 - 5000(i_2 - i_3) - 4000(i_2 - i_1) = 0$$

$$4000 i_1 - 12000 i_2 + 5000 i_3 = 0 \Rightarrow 4000 \times 11 \times 10^{-3} - 12000 i_2 + 5000 \times -4 \times 10^{-3} = 0$$

$\therefore i_2 = 2mA \Rightarrow \boxed{i = 2mA}$



ab, bc, cd, de, cf, bg, ah are branches

①, ②, ③ are mesh equations → a, b, c, d (e/f/g/h) are nodes /junctions.

Common Node

Reference node

→ Mesh is defined as a loop which doesnot contain any other loop.

→ loops ⟶ ①, ②, ③, ④, ⑤ are loops

→ Mesh is always a loop

But every loop not a mesh

$$\text{Mesh equations} = M = [B-(N-1)] = 7-(5-1) = 7-4 = 3$$

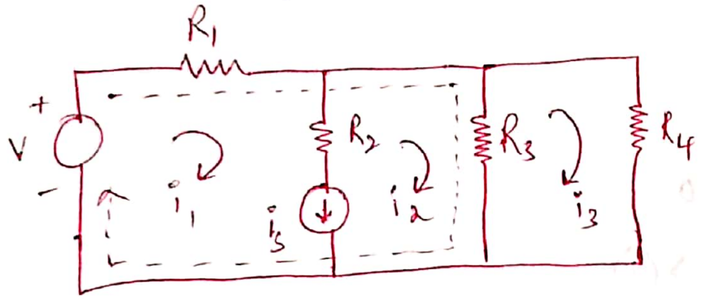
Super Mesh:- A Supermesh occurs when a current source is contained between two essential meshes. It is a larger mesh created from two meshes that has an independent or dependent current source as a common element.

Super Mesh Analysis:-

When a current source is common to two meshes then we use the concept of super mesh to analyze the circuit using mesh current method.

A Supermesh encloses more than one mesh. for each common current source between two meshes, the no. of meshes reduces by one, thus reducing the no. of Independent mesh equations by one.

Eg: In the fig.. i_s is the current source common to mesh ① & ②. Now we can create supermesh shown in dotted line as in fig that consists of the interior of mesh ① & ②.



Now we can apply KVL for super mesh

$$R_1 i_1 + R_3(i_2 - i_3) = V \Rightarrow R_1 i_1 + R_3 i_2 - R_3 i_3 = V \rightarrow ①$$

Consider mesh ③

$$-R_3(i_3 - i_2) - R_4 i_3 = 0 \Rightarrow R_3(i_3 - i_2) + R_4 i_3 = 0 \rightarrow ②$$

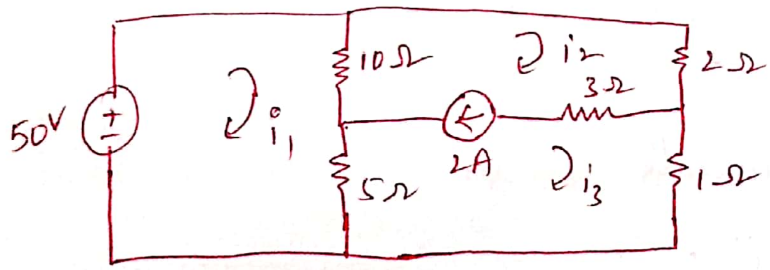
Finally the current i_s from current source is equal to difference between two meshes currents i.e.,

$$i_1 - i_2 = i_s \rightarrow ③$$

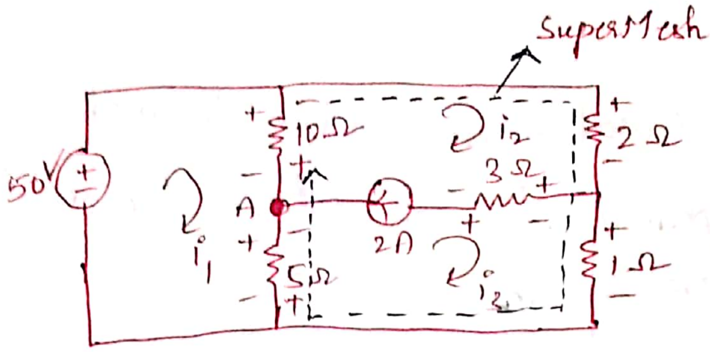
from ①, ② & ③ we calculate i_s.

$$\boxed{\text{Super Mesh analysis} = \text{Ohm's Law} + \text{KVL} + \text{KCL}}$$

Problem:- Determine current in 5Ω resistor in the network given in figure.



Sol:



Apply KVL to loop ①

$$50 - 10(i_1 - i_2) - 5(i_1 - i_3) = 0$$

$$-15i_1 + 10i_2 + 5i_3 = -50 \longrightarrow ①$$

Apply KVL to supermesh (1 & 2 meshes)

$$-10(i_2 - i_1) - 2i_2 - i_3 - 5(i_3 - i_1) = 0$$

$$15i_1 - 12i_2 - 6i_3 = 0 \longrightarrow ②$$

Apply KCL at node A

$$i_2 - i_3 = 2 \longrightarrow ③$$

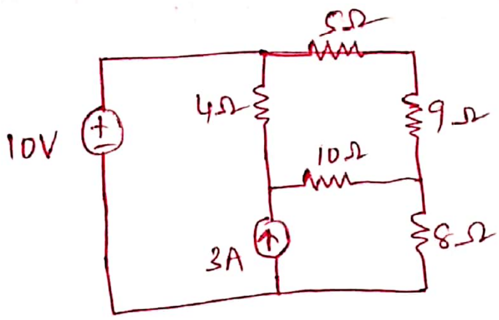
$i_1 = 20A$

$i_2 = 17.33A$

$i_3 = 15.33A$

Current in 5Ω resistor $= (i_1 - i_3) = (20 - 15.33)A = 4.67A \downarrow$

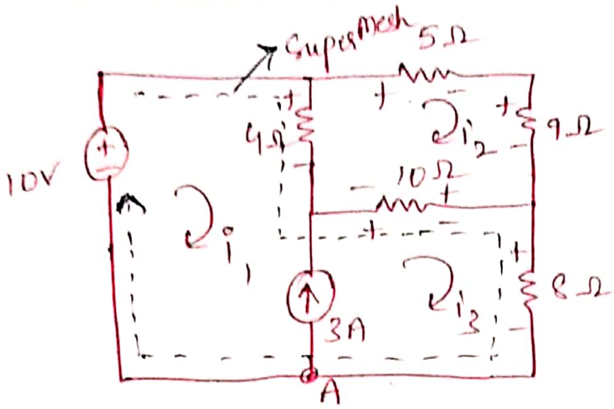
Problem 2:- Find mesh current i_1 in the circuit shown.



Sol:

Assume loop currents

Apply KVL to mesh ① & ③



$$10 - 4(i_1 - i_2) - 10(i_3 - i_2) - 8i_3 = 0$$

$$-4i_1 + 14i_2 - 18i_3 = -10 \rightarrow (1)$$

Apply KVL to loop (2)

$$-5i_2 - 9i_2 - 10(i_2 - i_3) - 4(i_2 - i_1) = 0$$

$$4i_1 - 28i_2 + 10i_3 = 0 \rightarrow (2)$$

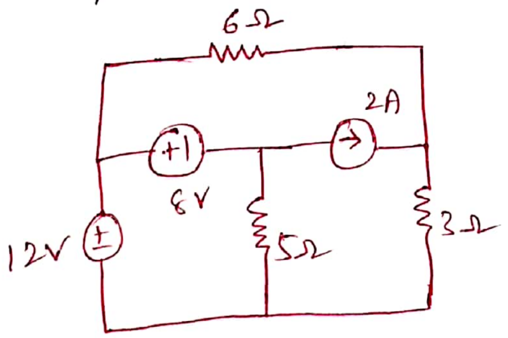
Apply KCL to ~~loop (3)~~ node A

$$i_3 - i_1 = 3 \rightarrow (3)$$

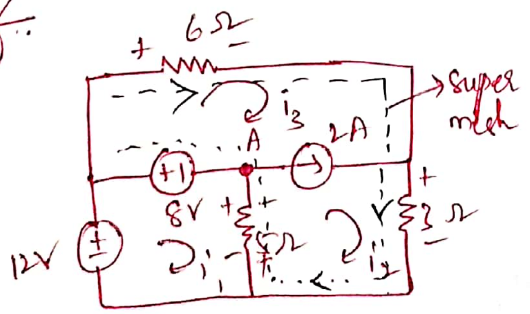
Solve (1), (2) and (3)

$$i_1 = -1.933A,\ i_2 = 0.104A,\ i_3 = 1.06A$$

Problem 3:- Find voltage across 3Ω resistor by using mesh analysis (supermesh problem)



Sol:



Apply KVL to loop (1)

$$12 - 8 - 5(i_1 - i_2) = 0$$

$$-5i_1 + 5i_2 = -4 \rightarrow (1)$$

Apply KVL to super mesh

$$-6i_3 - 3i_2 - 5(i_2 - i_1) = 0$$

$$5i_1 - 8i_2 - 6i_3 = 0 \rightarrow (2)$$

$+8$

$$5i_1 - 8i_2 - 6i_3 = -8 \rightarrow (2)$$

Scanned by CamScanner

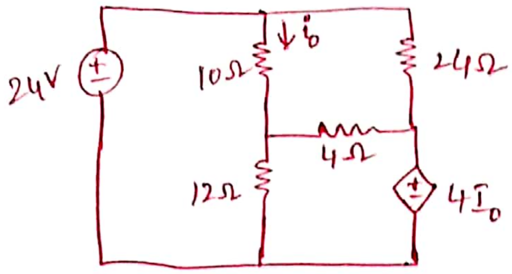
Apply KCL at node A

$$i_2 - i_3 = 2A \rightarrow (3)$$

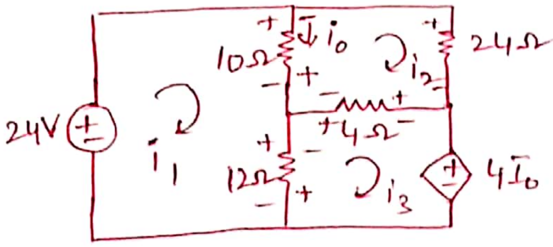
$$i_1 = 3.46A, \quad i_2 = 2.66A, \quad i_3 = 0.66A$$

Dependent Sources Mesh Method Problems:-

Problem 1:- Find the current i_0 for the circuit shown in the figure.



Sol:-



$$i_0 = (i_1 - i_2)$$

Apply KVL to loop (1)

$$24 - 10(i_1 - i_2) - 12(i_1 - i_3) = 0$$

$$-22i_1 + 10i_2 + 12i_3 = -24 \rightarrow (1)$$

Apply KVL to loop (2)

$$-24i_2 - 4(i_2 - i_3) - 10(i_2 - i_1) = 0$$

$$10i_1 - 38i_2 + 4i_3 = 0 \rightarrow (2)$$

Apply KVL to loop (3)

$$-4(i_3 - i_2) - 4i_0 - 12(i_3 - i_1) = 0$$

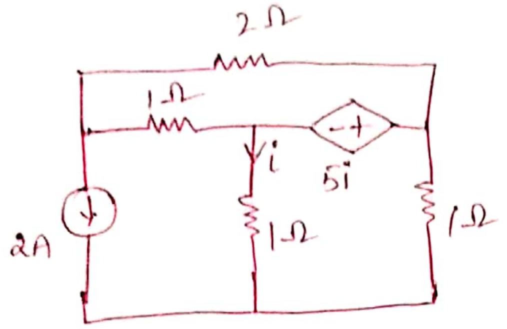
$$-4i_3 + 4i_2 - 4(i_1 - i_2) - 12i_3 + 12i_1 = 0$$

$$8i_1 + 8i_2 - 16i_3 = 0 \rightarrow (3)$$

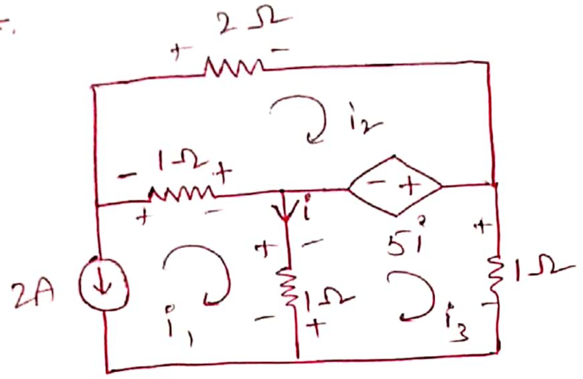
$$i_1 = 2.25A, \quad i_2 = 0.75A, \quad i_3 = 1.5A$$

$$i_0 = i_1 - i_2 = 2.25 - 0.75 = 1.5A$$

Problem 2:- Using Mesh analysis find the magnitude of current 1.12 dependent source and current through 2Ω resistor.



Sol:



Apply KVL to loop ②

$-2i_2 - 5i - 1(i_2 - i_1) = 0$

$-2i_2 - 5(i_1 - i_3) - 1(i_2 - i_1) = 0$

$-4i_1 - 3i_2 + 5i_3 = 0 \rightarrow ①$

$i_1 = -2A$

$i = (i_1 - i_3)$

Apply KVL to loop ③

$5i - i_3 - (i_3 - i_1) = 0$

$5(i_1 - i_3) - 2i_3 + i_1 = 0$

$6i_1 - 7i_3 = 0 \rightarrow ②$

$6 \times -2 - 7i_3 = 0$

$\Rightarrow -7i_3 = 12 \Rightarrow i_3 = \frac{-12}{7} = -1.71A$

Substitute i_3 and i_1 in ①

$-4 \times -2 - 3i_2 + 5 \times -1.71 = 0$

$-3i_2 = -8 + 8.55$

$= 0.55$

$i_2 = \frac{-0.55}{3} = -0.18 A.$

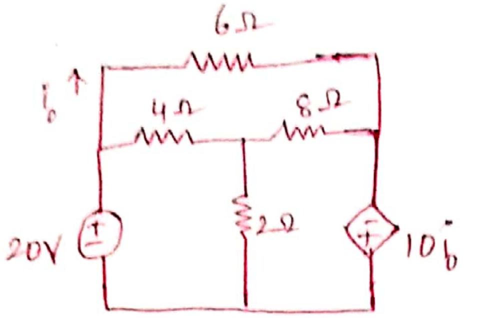
$\therefore i_1 = -2A,\ i_2 = -0.18A,\ i_3 = -1.71A.$

$i = i_1 - i_3 = -2 + 1.71 = -0.29A.$

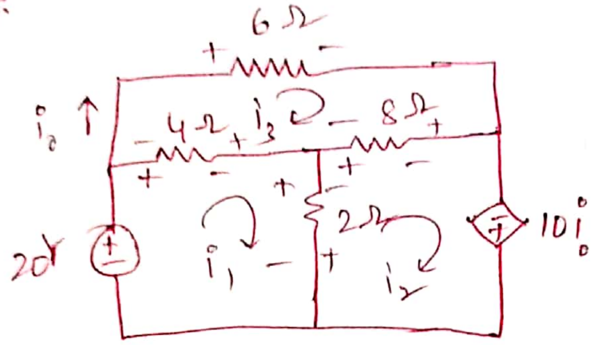
Magnitude of current source $= 5i = 5 \times 0.29 = 1.45V$

Scanned by CamScanner

Problem 3:- Find the current i_o in the circuit shown in fig.



Sol:



$$i_o = i_3$$

Apply KVL to loop ①

$$20 - 4(i_1 - i_3) - 2(i_1 - i_2) = 0$$

$$-6i_1 + 2i_2 + 4i_3 = -20 \rightarrow ①$$

Apply KVL to loop ②

$$-8(i_2 - i_3) + 10i_o - 2(i_2 - i_1) = 0$$

$$2i_1 - 10i_2 + 8i_3 + 10i_3 = 0$$

$$2i_1 - 10i_2 + 18i_3 = 0 \rightarrow ②$$

Apply KVL to loop ③

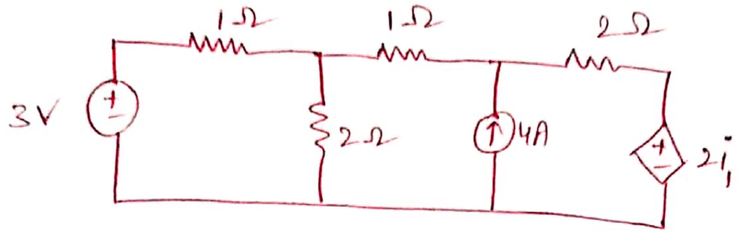
$$-6i_3 - 8(i_3 - i_2) - 4(i_3 - i_1) = 0$$

$$4i_1 + 8i_2 - 18i_3 = 0 \rightarrow ③$$

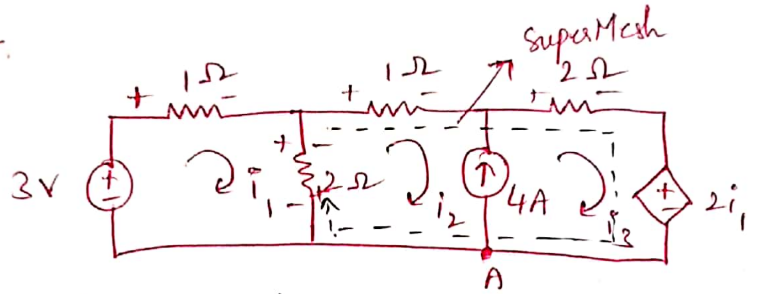
$$\boxed{\begin{aligned} i_1 &= -3.214A \\ i_2 &= -9.64A \\ i_3 &= -5A \end{aligned}}$$

Dependent Source Super Mesh Problems:-

Problem 1:- Find the loop currents i_1, i_2 and i_3 in the network of by mesh analysis.



Sol.



Apply KVL to loop ①

$$3 - 1i_1 - 2(i_1 - i_2) = 0$$

$$-3i_1 + 2i_2 = -3 \longrightarrow ①$$

Loops ② and ③ forms a super mesh.

So Apply KVL to loops ② & ③

$$-i_2 - 2i_3 - 2i_1 - 2(i_2 - i_1) = 0$$

$$-3i_2 - 2i_3 = 0 \longrightarrow ②$$

Apply KCL at node A.

$$i_3 - i_2 = 4 \longrightarrow ③$$

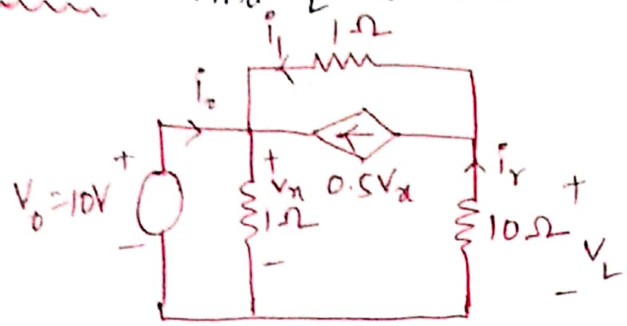
Solve ①, ② & ③

$$i_1 = -0.06A$$

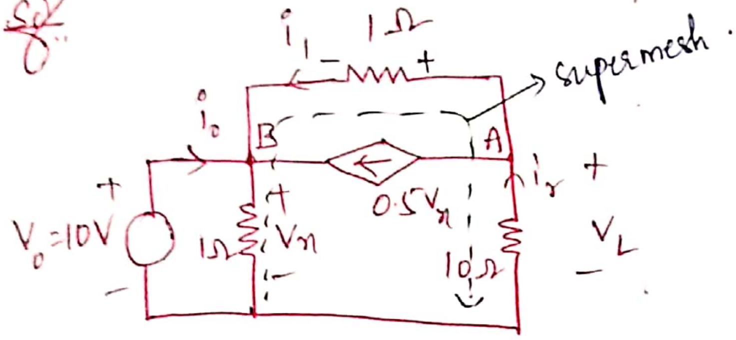
$$i_2 = -1.6A$$

$$i_3 = 2.4A$$

Problem2:- Find V_L using KVL



Sol:-



$V_x = 10V$.

$i_r = \frac{-V_L}{10}$

From fig Apply KCL at node A.

$i_r = 0.5V_x + i_1 \Rightarrow i_1 = i_r - 0.5V_x$

At B apply KCL

$i_o + 0.5V_x + i_1 = \frac{V_x}{1}$

Apply KVL to Super mesh.

$+V_x + i_1 - V_L = 0$

$V_x + (i_r - 0.5V_x) - V_L = 0$

$V_x + \left(\frac{-V_L}{10} - 0.5V_x\right) - V_L = 0$.

$10 - \frac{V_L}{10} - 0.5 \times 10 - V_L = 0$

$5 - 0.1V_L - V_L = 0$

$-1.1V_L = -5$

$\boxed{V_L = 4.54V}$

Nodal Analysis:-

This method is mainly based on Kirchoff's Current Law (KCL). This method uses the analysis of different nodes of the network. Every junction point in a network where two or more branches meet is called a "node".

If network has more current sources we use nodal analysis.

$$\boxed{\text{KCL + ohm's Law = Nodal analysis}}$$

→ In general circuit in a 'N' node circuit, one of the nodes is chosen as reference node or datum node, then it is possible to write (N-1) node equations by assuming (N-1) node voltages.

→ In general circuit reference node we assume at zero potential or ground.

→ The node voltage is the voltage of a given node with respect to one particular node, called reference node which is assumed at zero potential.

→ Select a node as a reference node. Assign voltages to other nodes as $V_1, V_2 \ldots V_{n-1}$ to remaining $(n-1)$ nodes. The voltages are referenced with respect to the reference node.

→ Apply KCL to each of the $(n-1)$ non reference nodes. Use ohm's law to express the branch currents in terms of node voltages

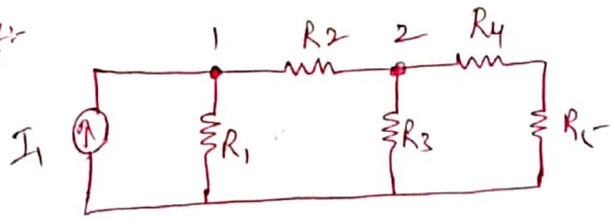
→ Solve the resulting simultaneous equations to obtain the unknown node voltages.

The reference node is commonly called as ground since it is assumed to have zero potential.

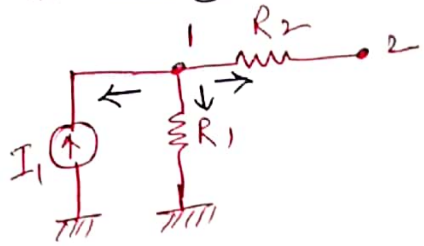
The node voltage is the voltage of a given node with respect to one particular node, called the reference node, which we assume at zero potential 

Current flows from higher potential to lower potential in a resistor.

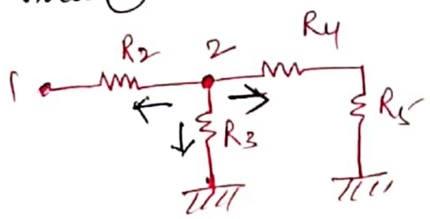
Eg:-



At node ①



At node ②



Apply KCL at node ①

$$\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} - I_1 = 0$$

$$\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} = I_1 \rightarrow ①$$

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_2}{R_4 + R_5} = 0 \rightarrow ②$$

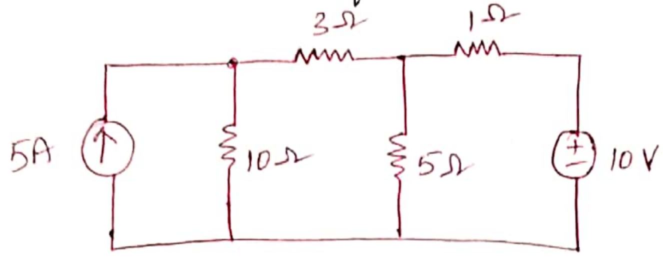
Rearranging the above equations

$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{R_2}\right) = I_1$$

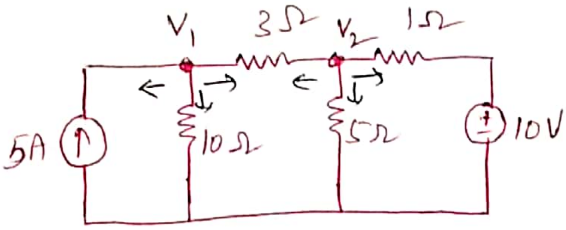
$$V_1\left(\frac{-1}{R_2}\right) + V_2\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4 + R_5}\right) = 0$$

By solving the above equations, we obtain V_1 and V_2 voltages at each node.

Problem :- Write the node equations and determine the current in each branch of the network shown in figure.



Sol :-



Apply KCL at node 1

$$-5 + \frac{V_1}{10} + \frac{V_1 - V_2}{3} = 0$$

$$0.1 V_1 + 0.33 V_1 - 0.33 V_2 = 5$$

$$0.43 V_1 - 0.33 V_2 = 5 \rightarrow (1)$$

Apply KCL to node 2

$$\frac{V_2 - V_1}{3} + \frac{V_2}{5} + \frac{V_2 - 10}{1} = 0$$

$$0.33 V_2 - 0.33 V_1 + 0.2 V_2 + V_2 - 10 = 0$$

$$-0.33 V_1 + 1.53 V_2 = 10 \rightarrow (2)$$

$$V_1 = 19.94\,V \quad , \quad V_2 = 10.83\,V$$

Current flowing through 10Ω = $\frac{V_1}{10} = \frac{19.94}{10} = 1.99$ Amps ↓

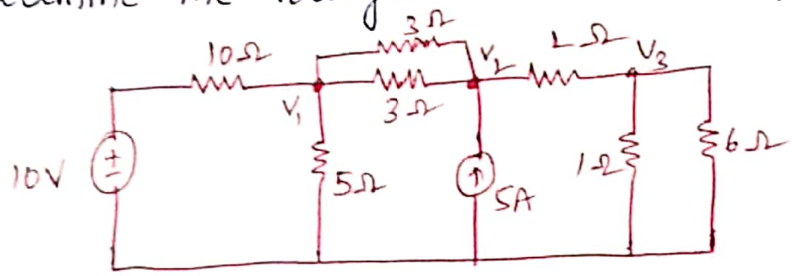
Current flowing through 3Ω = $\frac{V_1 - V_2}{3} = \frac{19.94 - 10.83}{3} = 3.03$ A from node 1 to 2

Current flowing through 5Ω = $\frac{V_2}{5} = \frac{10.83}{5} = 2.16$ A

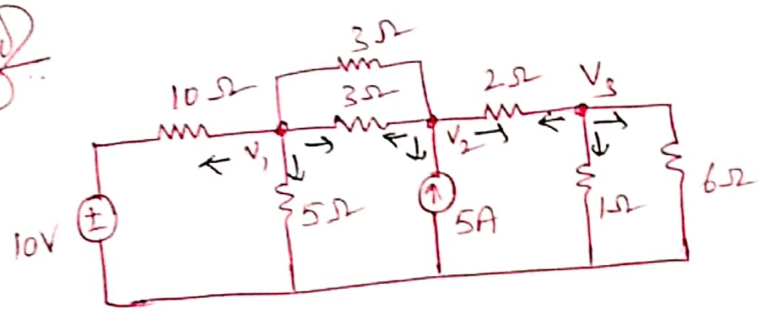
Current flowing through 1Ω = $\frac{V_2 - 10}{1} = \frac{10.83 - 10}{1} = \cancel{9.94A}\ 0.83$ A from node 2 to 10V source.

Problem 2:-

Determine the voltages at each node for the circuit



Sol:



Apply KCL at node ①

$$\frac{V_1-10}{10} + \frac{V_1}{5} + \frac{V_1-V_2}{1.5} = 0$$

$$0.1V_1 + 0.2V_1 + 0.66V_1 - 0.66V_2 = 1$$

$$0.96V_1 - 0.66V_2 = 1 \rightarrow ①$$

Apply KCL at node ②

$$-5 + \frac{V_2-V_1}{1.5} + \frac{V_2-V_3}{2} = 0$$

$$0.66V_2 - 0.66V_1 + 0.5V_2 - 0.5V_3 = 5$$

$$-0.66V_1 + 1.16V_2 - 0.5V_3 = 5 \rightarrow ②$$

Apply KCL at node ③

$$\frac{V_3-V_2}{2} + \frac{V_3+0}{0.85} = 0$$

$$0.5V_3 - 0.5V_2 + 1.17V_3 = 0$$

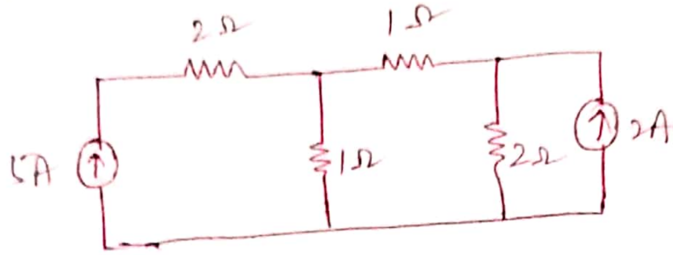
$$-0.5V_2 + 1.67V_3 = 0 \rightarrow ③$$

$V_1 = 8.06V$

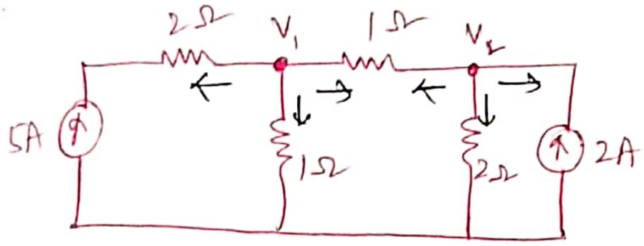
$V_2 = 10.21V$

$V_3 = 3.05V$

Problem 3:- Using nodal analysis, find the current in the resistors.



Sol:



Apply KCL at node ①

$$-5 + \frac{V_1}{1} + \frac{V_1 - V_2}{1} = 0$$

$$2V_1 - V_2 = 5 \rightarrow ①$$

Apply KCL at node ②

$$\frac{V_2 - V_1}{1} + \frac{V_2}{2} - 2 = 0$$

$$-V_1 + 1.5V_2 = 2 \rightarrow ②$$

$$V_1 = 4.75V, \; V_2 = 4.5V$$

Current flowing through 2Ω = 5A from current source 5A to node 1

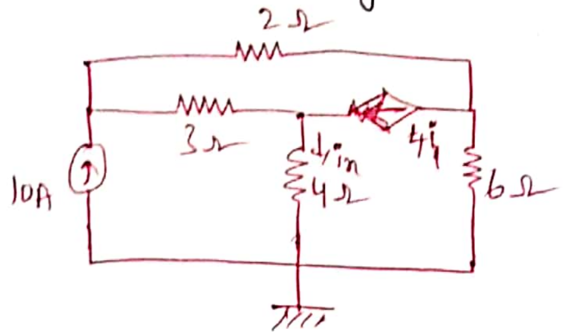
Current flowing through 1Ω = $\frac{V_1}{1} = \frac{4.75}{1} = 4.75A$

Current flowing through 1Ω = $\frac{V_1 - V_2}{1} = \frac{4.75 - 4.5}{1} = 0.25A$ from node 1 to 2

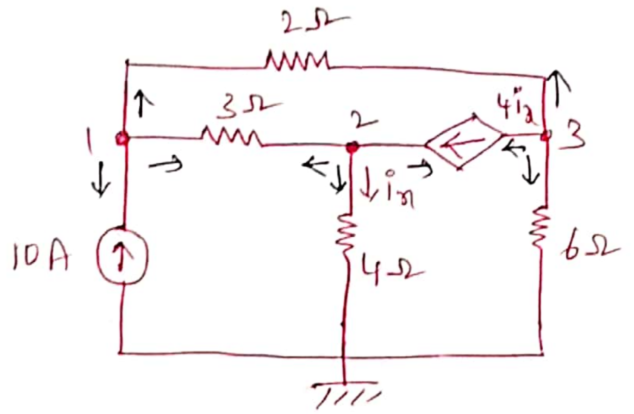
Current flowing through 2Ω = $\frac{V_2}{2} = \frac{4.5}{2} = 2.25A$ from node 2 to reference node

Dependent Source Nodal Analysis Problems:-

Problem:- Find node voltages at three non reference nodes in the circuit.



Sol:



Apply KCL at node ①

$$-10 + \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{2} = 0$$

$$0.333V_1 - 0.333V_2 + 0.5V_1 - 0.5V_3 = 10$$

$$0.833V_1 - 0.333V_2 - 0.5V_3 = 10 \rightarrow ①$$

Apply KCL at node ②

$$\frac{V_2 - V_1}{3} + i_n - 4i_a = 0 \quad , \quad i_n = \frac{V_2}{4}$$

$$\frac{V_2 - V_1}{3} + \frac{V_2}{4} - 4\left(\frac{V_2}{4}\right) = 0$$

$$0.33V_2 - 0.333V_1 + 0.25V_2 - V_2 = 0$$

$$-0.333V_1 - 0.42V_2 = 0 \rightarrow ②$$

Apply KCL at node ③

$$\frac{V_3}{6} + \frac{V_3 - V_1}{2} + 4i_n = 0$$

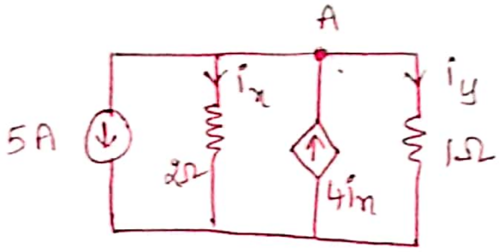
$$0.166V_3 + 0.5V_3 - 0.5V_1 + 4\left(\frac{V_2}{4}\right) = 0$$

$$0.166V_3 + 0.5V_3 - 0.5V_1 + V_2 = 0$$

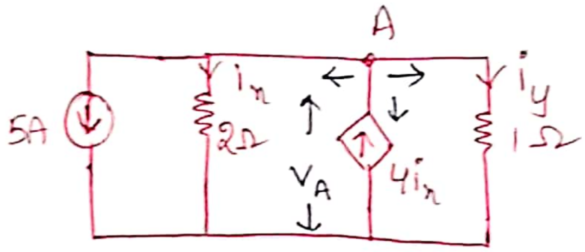
$$-0.5V_1 + V_2 + 0.666V_3 = 0$$

$$V_1 = 80.62V \ , V_2 = -63.92 = -64V \ , V_3 = 156.51V$$

Problem 2:- obtain i_x, i_y and numerical value of current dependent source.



Sol:



Apply KCL at node A.

$$5 + i_x - 4i_x + i_y = 0$$

Assume node voltage at A to be V_A.

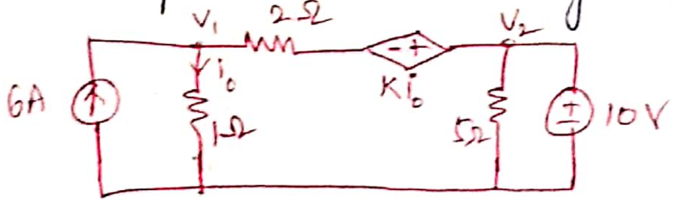
$$5 + \frac{V_A}{2} - \cancel{4}^{2}\left(\frac{V_A}{\cancel{2}}\right) + \frac{V_A}{1} = 0$$

$$-\frac{V_A}{2} = -5 \Rightarrow V_A = 10V$$

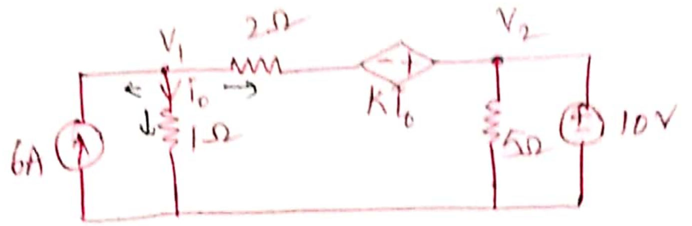
$$i_x = \frac{10}{2} = 5A$$

$\therefore$ Numerical value of current dependent source $= 4i_x = 4 \times 5$
$= 20A$.

Problem 3:- If power loss in 1Ω resistor is 25W, find the value of K in the dependent source using nodal method.



Sol



Apply KCL at node ①, Given $V_2 = 10V$

$$-6 + i_0 + \frac{V_1 + Ki_0 - V_2}{2} = 0$$

$$-6 + \frac{V_1}{1} + \frac{V_1 + Ki_0 - 10}{2} = 0$$

$$\frac{2V_1 + V_1 + Ki_0 - 10}{2} = 6$$

$$3V_1 + Ki_0 - 10 = 12$$

$$3V_1 + K\left(\frac{V_1}{1}\right) - 10 = 12$$

Given

$$(K+3)V_1 - 10 = 12 \longrightarrow ①$$

Power loss in 1Ω resistor $= i_0^2 \times 1 = \left(\frac{V_1}{1}\right)^2 \times 1 = 25$

$$V_1^2 = 25$$

$$V_1 = 5V$$

Substitute V_1 in ①

$$(K+3)5 - 10 = 12$$

$$5K + 15 = 22$$

$$5K = 22 - 15 = 7$$

$$K = 7/5 = 1.4$$

$$\therefore \boxed{K = 1.4}$$

Super Node:- Whenever a voltage source (Independent or Dependent) is connected between the two non reference nodes, then these two nodes form a generalized node called "Super Node". A Super node can be regarded as a surface enclosing the voltage source and its two nodes.

Super Node Analysis:-

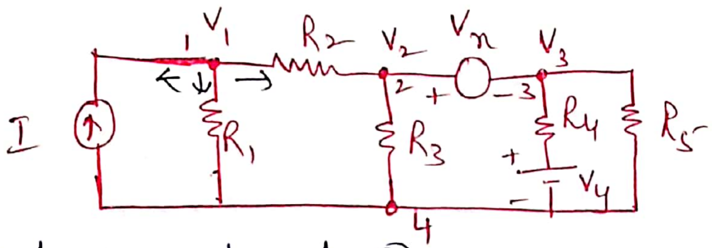
Suppose any of the branches in the network has a voltage source, then it is slightly difficult to apply nodal analysis

→ One way to overcome this difficulty is to apply the supernode technique.

→ In this method, the two adjacent nodes that are connected by a voltage source are reduced to a single node & then the equations are formed by applying KCL.

$$\boxed{\text{Super Node analysis} = \text{Ohm's law} + \text{KVL} + \text{KCL}}$$

Consider the circuit below



Node 1, 2, 3 are non reference node
Node 4 is reference nod

Apply KCL at node ①

$$-I + \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} = 0$$

$$\Rightarrow I = V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{R_2}\right) \rightarrow ①$$

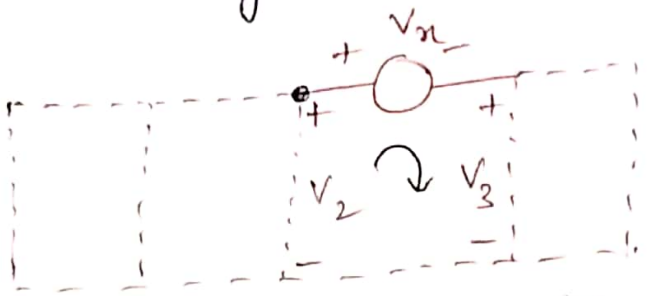
→ V_n is between nodes ② & ③ it is slightly difficult to find out the current. The supernode technique can be conveniently applied in this case.

Accordingly we can write combined equation for nodes ② & ③

$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_3 - V_4}{R_4} + \frac{V_3}{R_5} = 0 \longrightarrow ②$$

Since V_x is in between two non reference nodes we apply KCL & KVL to determine the node voltages.

A Supernode may be regarded as a closed surface enclosing the voltage source and its two nodes.



Apply KVL to path consisting of V_x, V_1 & V_2

$$-V_x - V_3 + V_2 = 0$$

$$V_2 - V_3 = V_x \longrightarrow (3)$$

By solving equations (1), (2) and (3)

V_1, V_2 and V_3 can be obtained.

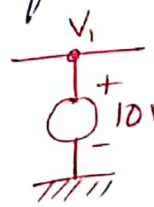
Note the following properties of Super Node.

1. The voltage source inside the supernode provides a constraint equation needed to solve for node voltages.

2. A Supernode has no voltage of its own.

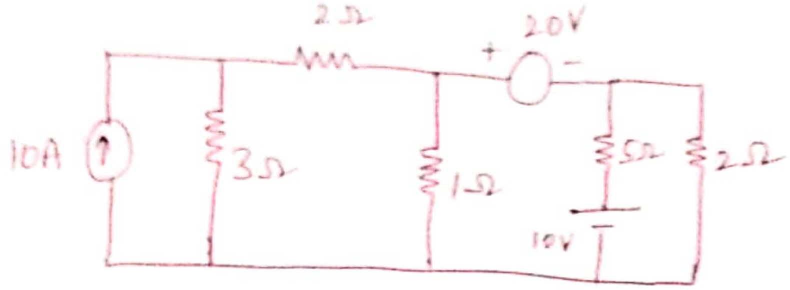
3. A Supernode requires the application of both KVL and KCL.

Note:- If a voltage source is connected between the reference node and non reference node we set the voltage at the non reference node equal to the voltage of the voltage source. For eg... $V_1 = 10V$

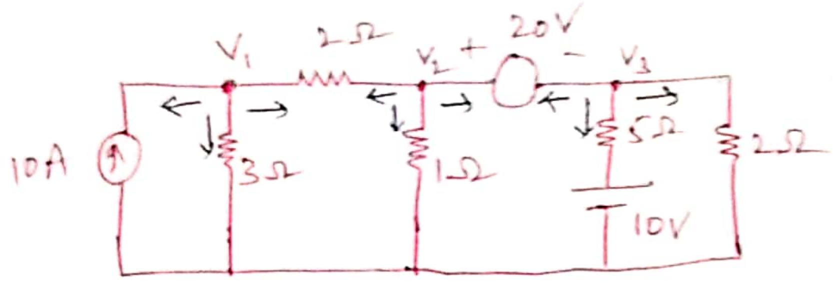


Scanned by CamScanner

Problem:- Determine the current in 5Ω resistor for the circuit 1.26 shown below.



Sol:



Since 20V voltage source is in between two non reference nodes Nodes ② & ③ form a Super Node.

Apply KCL to node ①

$$-10 + \frac{V_1}{3} + \frac{V_1 - V_2}{2} = 0$$

$$V_1\left(\frac{1}{3}+\frac{1}{2}\right) - V_2\left(\frac{1}{2}\right) = 10$$

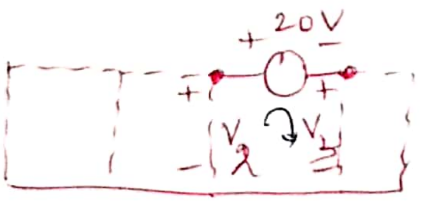
$$0.833V_1 - 0.5V_2 = 10 \longrightarrow ①$$

Apply KCL to Super Node.

$$\frac{V_2 - V_1}{2} + \frac{V_2}{1} + \frac{V_3 - 10}{5} + \frac{V_3}{2} = 0$$

$$V_1\left(\frac{-1}{2}\right) + V_2\left(\frac{1}{2}+1\right) + V_3\left(\frac{1}{5}+\frac{1}{2}\right) = 2$$

$$-0.5V_1 + 1.5V_2 + 0.7V_3 = 2 \longrightarrow ②$$



Apply KVL to the path consisting of super node:

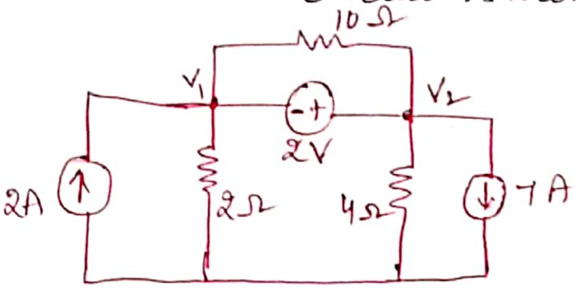
$$V_2 - 20 - V_3 = 0 \Rightarrow V_2 - V_3 = 20 \longrightarrow ③$$

Solve ①, ② & ③ equations

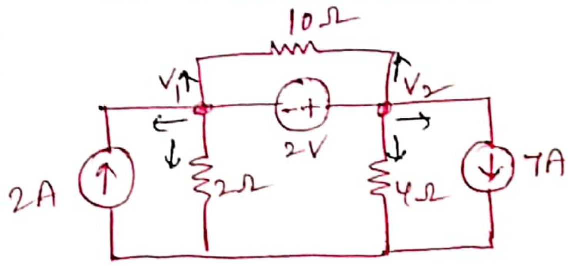
$V_1 = 18.95V$, $V_2 = 11.58V$, $V_3 = -8.41V$

Current in 5Ω resistor $= \frac{V_3 - 10}{5} = \frac{-8.41 - 10}{5} = -3.68$ Amps from node 3 to source 10V.

Problem 2:- For the circuit shown, find node voltages.



Sol:



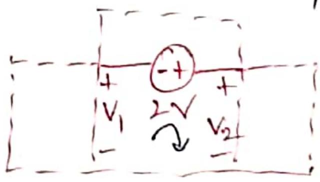
Nodes ① & ② form a supernode since they have a voltage source b/w two non reference nodes.

Apply KCL to ① & ② Nodes

$$-2 + \frac{V_1}{2} + \frac{V_1 - V_2}{10} + \frac{V_2 - V_1}{10} + \frac{V_2}{4} + 7 = 0$$

$$V_1\left(\frac{1}{2} + \frac{1}{10} - \frac{1}{10}\right) + V_2\left(\frac{-1}{10} + \frac{1}{10} + \frac{1}{4}\right) = -5$$

$$0.5V_1 + 0.25V_2 = -5 \rightarrow ①$$



Apply KVL to the path containing voltage source b/w two non reference nodes.

$$V_1 + 2 - V_2 = 0$$

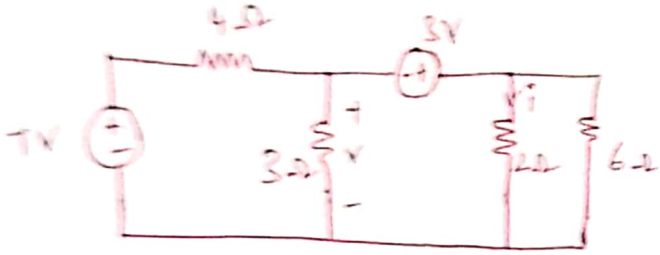
$$V_1 - V_2 = -2 \rightarrow ①$$

Solve ① & ② equations to obtain V_1 & V_2

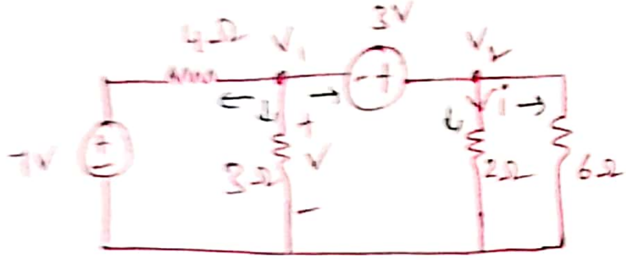
$V_1 = -7.33V$

$V_2 = -5.33V$

Problem: Find V & i in the Circuit.



Sol



Node 1 and 2 form a super node.

Apply KCL to Super Node

$$\frac{V_1 - 7}{4} + \frac{V_1}{3} + \frac{V_2}{2} + \frac{V_2}{6} = 0$$

$$V_1\left(\frac{1}{4}+\frac{1}{3}\right) + V_2\left(\frac{1}{2}+\frac{1}{6}\right) = \frac{7}{4}$$

$$0.58V_1 + 0.66V_2 = 1.75 \rightarrow (1)$$

Apply KVL to path Containing voltage source between nodes

$$V_1 + 3 - V_2 = 0$$

$$V_1 - V_2 = -3 \rightarrow (2)$$

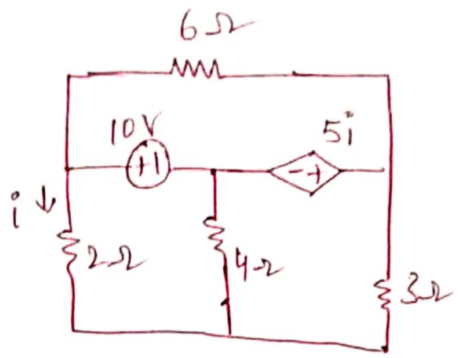
$$V_1 = -0.185V, \quad V_2 = 2.814V$$

$$V = V_1 = -0.185V$$

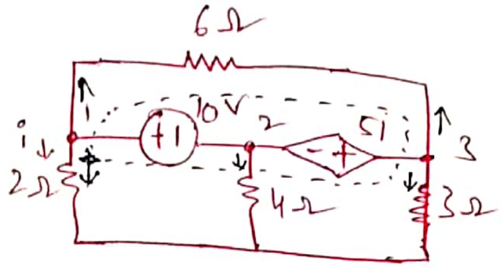
$$i = \frac{V_2}{2} = \frac{2.814}{2} = 1.407 \text{ Amps.}$$

Dependent sources Super Node analysis Problems:-

Problem 1:- Find V_1, V_2 and V_3 in the Circuit shown using nodal analysis.



Sol:-



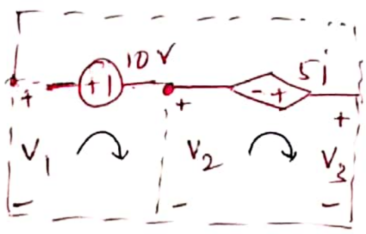
Nodes 1, 2, 3 form super Node.

Apply KCL to Super Node.

$$\frac{V_1}{2} + \frac{V_1 - V_3}{6} + \frac{V_2}{4} + \frac{V_3 - V_1}{6} + \frac{V_3}{3} = 0$$

$$V_1\left(\frac{1}{2} - \frac{1}{6} + \frac{1}{6}\right) + V_2\left(\frac{1}{4}\right) + V_3\left(\frac{-1}{6} + \frac{1}{6} + \frac{1}{3}\right) = 0$$

$$0.5V_1 + 0.25V_2 + 0.333V_3 = 0 \longrightarrow (1)$$



Apply KVL to loop (1)

$$V_1 - 10 - V_2 = 0$$

$$V_1 - V_2 = 10 \longrightarrow (2)$$

Apply KVL to loop (2)

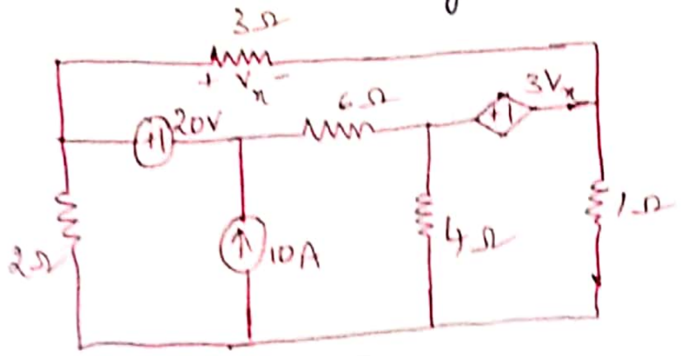
$$V_2 + 5i - V_3 = 0$$

$$V_2 - V_3 = -5i \Rightarrow V_2 - V_3 = 5\left(\frac{-V_1}{2}\right)$$

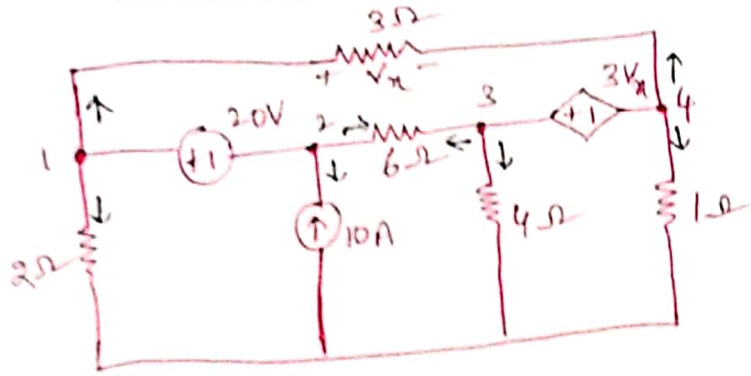
$$V_2 - V_3 + 2.5V_1 = 0 \longrightarrow (3)$$

$V_1 = 3.04V$, $V_2 = -6.95V$, $V_3 = 0.65V$

Problem 2:- Find node voltages in the circuit



Sol:



Nodes ① & ②, ③ & ④ form super nodes

Apply KCL to ① & ② $V_x = V_1 - V_4$

$$\frac{V_1}{2} + \frac{V_1 - V_4}{3} + \frac{V_2 - V_3}{6} - 10 = 0$$

$$V_1\left(\frac{1}{2} + \frac{1}{3}\right) + V_2\left(\frac{1}{6}\right) - V_3\left(\frac{1}{6}\right) - V_4\left(\frac{1}{3}\right) = 10$$

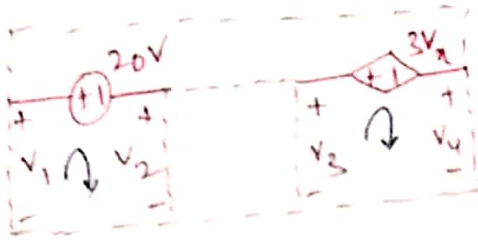
$$0.83V_1 + 0.16V_2 - 0.16V_3 - 0.33V_4 = 10 \rightarrow ①$$

Apply KCL to ③ & ④

$$\frac{V_3 - V_2}{6} + \frac{V_3}{4} + \frac{V_4}{1} + \frac{V_4 - V_1}{3} = 0$$

$$V_1\left(\frac{-1}{3}\right) + V_2\left(\frac{-1}{6}\right) + V_3\left(\frac{1}{6} + \frac{1}{4}\right) + V_4\left(1 + \frac{1}{3}\right) = 0$$

$$-0.33V_1 - 0.16V_2 + 0.41V_3 + 1.33V_4 = 0 \rightarrow ②$$



Apply KVL to loop ①

$V_1 - 20 - V_2 = 0 \Rightarrow V_1 - V_2 = 20 \rightarrow ③$

Apply KVL to loop ③

$V_3 - 3V_x - V_4 = 0 \Rightarrow V_3 - 3(V_1 - V_4) - V_4 = 0$

$$-3V_1 + V_3 + 2V_4 = 0 \rightarrow ④$$

From (4) $V_3 = 3V_1 - 2V_4$

Substitute in (1), (2) & (3).

From (1) $0.83V_1 + 0.16V_2 - 0.16(3V_1 - 2V_4) - 0.33V_4 = 10$

$0.35V_1 + 0.16V_2 - 0.01V_4 = 10 \rightarrow (5)$

From (2) $-0.33V_1 - 0.16V_2 + 0.41(3V_1 - 2V_4) + 1.33V_4 = 0$

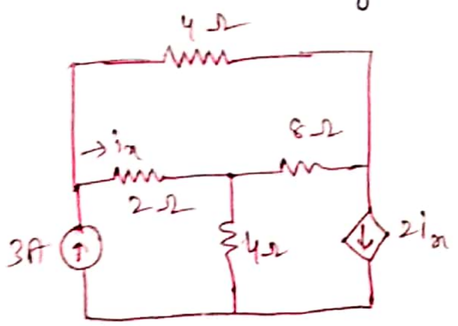
$0.9V_1 - 0.16V_2 + 0.51V_4 = 0 \rightarrow (6)$

From (3) $V_1 - V_2 = 20 \rightarrow (7)$

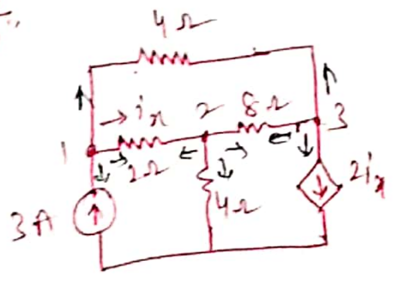
solve (5), (6) & (7) $V_1 = 25.04$, $V_2 = 5.04V$, $V_3 = 3V_1 - 2V_4$

$V_4 = -42.61V$, $V_3 = 160.34V$

Problem 3:- Determine voltages at the nodes. (Nodal analysis problem)



Sol.



Apply KCL at node (1).

$-3 + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} = 0$

$V_1\left(\frac{1}{2} + \frac{1}{4}\right) + V_2\left(\frac{-1}{2}\right) + V_3\left(\frac{-1}{4}\right) = 3$

$0.75V_1 - 0.5V_2 - 0.25V_3 = 3 \rightarrow (1)$

Apply KCL at node (2)

$\frac{V_2 - V_1}{2} + \frac{V_2}{4} + \frac{V_2 - V_3}{8}$

$V_1\left(\frac{-1}{2}\right) + V_2\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right) + V_3\left(\frac{-1}{8}\right) = 0$

$-0.5V_1 + 0.875V_2 - 0.125V_3 = 0 \rightarrow (2)$

$\boxed{V_1 = 4.8V,\ V_2 = 2.4V,\ V_3 - 2.4V}$

Apply KCL at node (3)

$2i_x + \frac{V_3 - V_2}{8} + \frac{V_3 - V_1}{4} = 0$

$2\left(\frac{V_1 - V_2}{2}\right) + \frac{V_3 - V_2}{8} + \frac{V_3 - V_1}{4} = 0$

$V_1\left(1 - \frac{1}{4}\right) + V_2\left(-1 - \frac{1}{8}\right) + V_3\left(\frac{1}{8} + \frac{1}{4}\right) = 0$

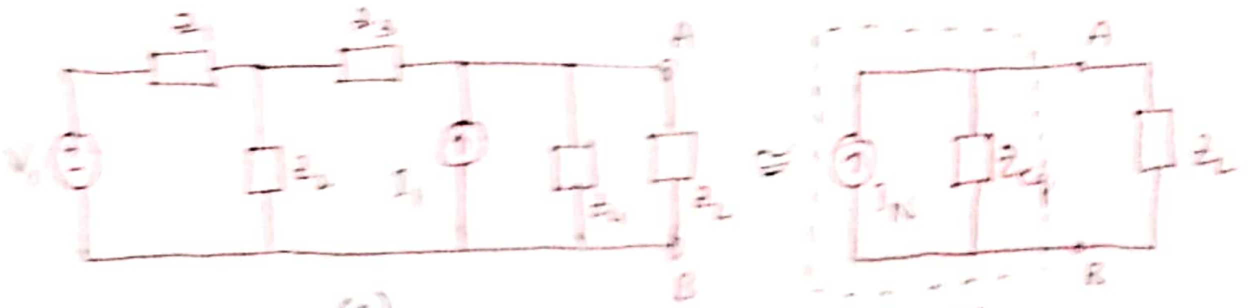
$0.75V_1 - 1.125V_2 + 0.375V_3 = 0 \rightarrow (3)$

Theorems:-

Norton's Theorem:-

Statement:- Any combination of linear bilateral circuit elements and active sources, regardless of connection or complexity, connected to a given load Z_L, can be replaced by a simple two terminal network, consisting of a single current source of I_N amperes and a single impedance Z_{eq} in parallel with it, across the two terminals of the load Z_L. The I_N is the short circuit current flowing through the short circuited path, replaced instead of Z_L. It is also called Norton's current. The Z_{eq} is the equivalent impedance of the given network as viewed through the load terminals, with Z_L removed and all the active sources are replaced by their internal impedances. If internal impedances are unknown then the independent voltage sources must be replaced by short circuit while the independent current sources must be replaced by open circuit, while calculating Z_{eq}.

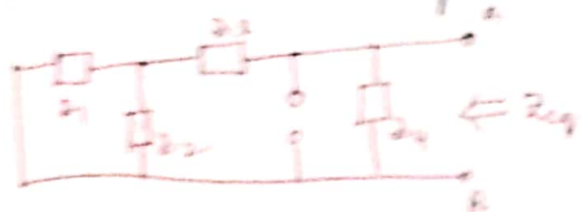
Explanation of Norton's theorem:- Consider a network shown in fig (a) below. The terminals A-B are load terminals where load impedance Z_L is connected. According to Norton's theorem, the entire network can be replaced by a current source I_N, and an equivalent impedance Z_{eq} in parallel with it, across the load terminals A-B as shown in fig (b)



(a) (b)

For obtaining current I_N, short the load terminals A-B. Calculate the current through the short circuited path by using any of the network simplification techniques, This is Norton's current I_N It is shown in Fig Below

While the equivalent impedance Z_{eq} is to be obtained by the same procedure as in case of Thevenin's theorem



When the circuit is replaced by Norton's equivalent across the load terminals, then the load current can be easily obtained by using current division in a parallel circuit as.

$$I_L = I_N \times \frac{Z_{eq}}{Z_L + Z_{eq}}$$

This theorem is also called dual of Thevenin's theorem. This is because, if the Thevenin's equivalent voltage source is converted to an equivalent current source, the Norton's equivalent is obtained. This is shown in fig.

From source transformation we can write

$$I_N = \frac{V_{th}}{Z_{eq}}$$

or $Z_{eq} = \frac{V_{th}}{I_N}$

Steps to Apply Norton's theorem:-

Step1:- Short the branch, through which the current is to be Calculated by removing the impedance between the terminals

step2:- obtain the current through this short circuited branch, using any of the network simplification techniques. this current is nothing but Norton's current I_N.

step3:- Calculate the equivalent impedance Z_{eq}, as viewed through the two terminals of interest by removing the branch impedance and making all the independent sources inactive

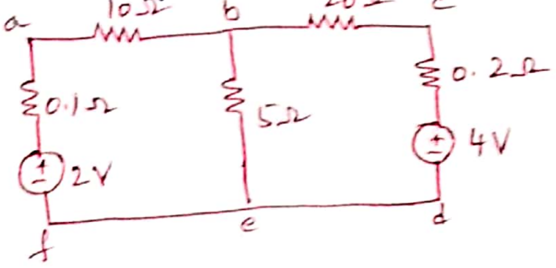
Step4:- Draw the Norton's equivalent across the terminals of interest, showing a current source I_N with the impedance Z_{eq} parallel with it. Reconnect the branch impedance now. Let it be Z_L. The current through the branch of interest is,

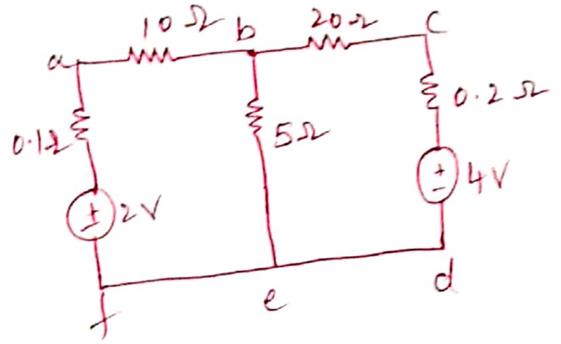
$$I = I_N \times \frac{Z_{eq}}{Z_{eq} + Z_L}$$

Note:- If dependent sources are present in the circuit then $\boxed{Z_{eq} = \frac{V_{th}}{I_N}}$

Problem1:- Find the current through branch 'b-e' using Norton's theorem.

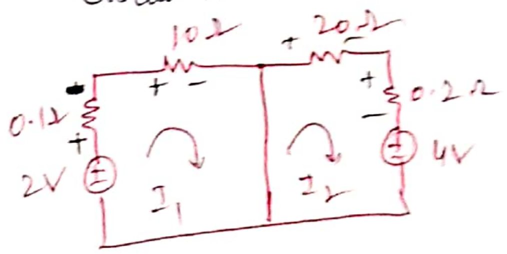
Sol:-





Step:-1

Remove 5Ω resistor and short Circuit it



Step2:- Apply KVL to loop ①

$$2 - 0.1i_1 - 10i_1 = 0 \Rightarrow -10.1i_1 = -2 \Rightarrow i_1 = \frac{2}{10.1} = 0.198 \text{ Amps}$$

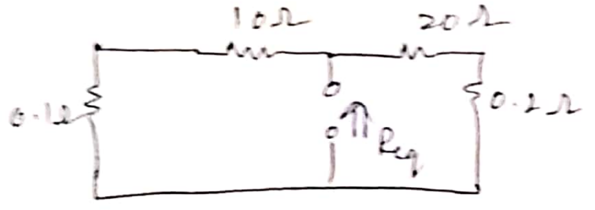
Apply KVL to loop ②

$$-20i_2 - 0.2i_2 - 4 = 0$$

$$-20.2i_2 = 4 \Rightarrow i_2 = \frac{-4}{20.2} = -0.198 \text{ amps}$$

$$I_N = I_1 - I_2 = 0.198 - (-0.198) = 0.396 \text{ amps}.$$

Step 3:- Calculate equivalent impedance.

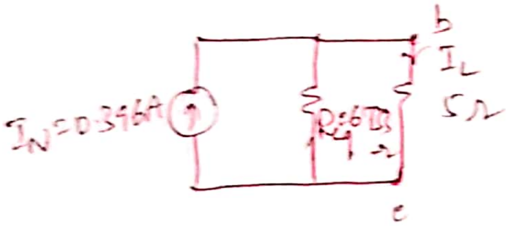


$$R_{eq} = (10+0.1) // (20+0.2)$$

$$= 10.1 // 20.2$$

$$= 6.733\,\Omega.$$

Step 4:- Draw Norton's equivalent circuit & find current through branch 'b-e'.

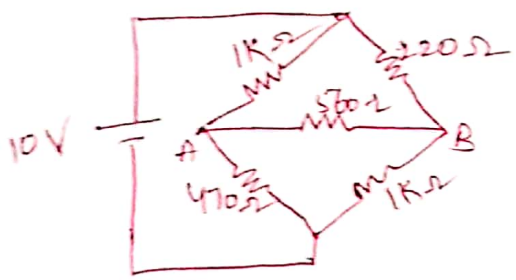


$$I_L = I_N \times \frac{R_{eq}}{R_{eq}+5}$$

$$= 0.396 \times \frac{6.733}{6.733+5} = 0.227 \text{ Amps}$$

∴ Current flowing through 5Ω Resistor = 0.227 amps

Problem 2:- Find current through 560Ω resistor using Norton's theorem



Sol: Step 1:- Remove 560Ω and short circuit A-B terminals

Step 2:- Find I_N

Apply KVL to loop ①

$10 - 1000(i_1 - i_2) - 470(i_1 - i_3) = 0$

$-1470 i_1 + 1000 i_2 + 470 i_3 = -10 \rightarrow ①$

Apply KVL to loop ②

$-220 i_2 - 1000(i_2 - i_1) = 0$

$1000 i_1 - 1220 i_2 = 0 \rightarrow ②$

Apply KVL to loop ③

$-1000 i_3 - 470(i_3 - i_1) = 0$

$470 i_1 - 1470 i_3 = 0 \rightarrow ③$

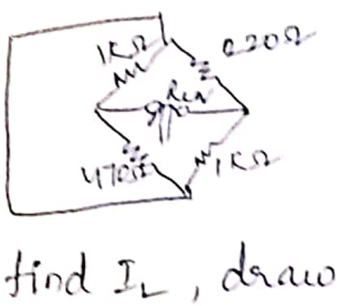
$i_1 = 19.99\,mA$, $i_2 = 16.39\,mA$, $i_3 = 6.39\,mA$.

$i_N = i_2 - i_3 = 16.39\,mA - 6.39\,mA = 10\,mA$.

$\therefore i_N = 10\,mA$

Step 3:- To find R_{eq}

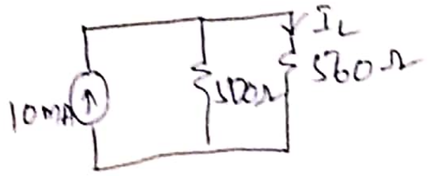
$R_{eq} = (470 // 1K\Omega) + (220 // 1K\Omega)$



$= 319.72 + 180.32$

$= 500\,\Omega$

Step 4:- To find I_L, draw Norton's equivalent circuit.

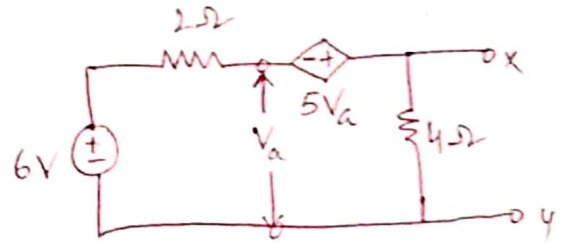


$I_L = 10 \times 10^{-3} \times \dfrac{500}{500+560}$

$= 4.716\,mA$.

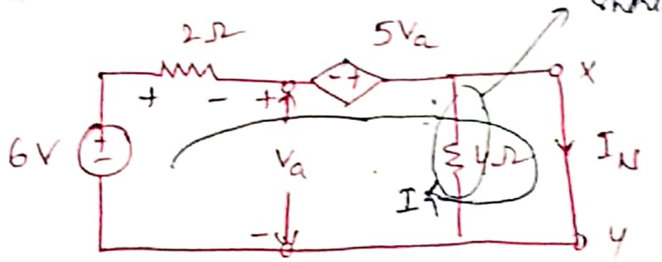
$\therefore$ Current flowing through $560\,\Omega = 4.7\,mA$

Problems:- Find Norton's equivalent of the network shown in fig. at x-y terminals. [Dependent source problem]



Sol

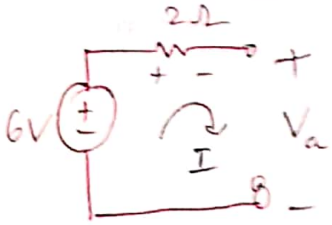
shorted, so no current flow



Step:-1 short circuit x-y terminals.

So 4Ω resistor is shorted and so it is bipassed.

$I = I_N$



$6 - 2I - V_a = 0$

$\Rightarrow V_a = 6 - 2I$

Apply KVL to the loop

$6 - 2I + 5V_a = 0$

$6 - 2I + 5(6 - 2I) = 0$

$6 - 2I + 30 - 10I = 0$

$-12I + 36 = 0$

$-12I = -36$

$\Rightarrow I = \frac{36}{12} = 3$

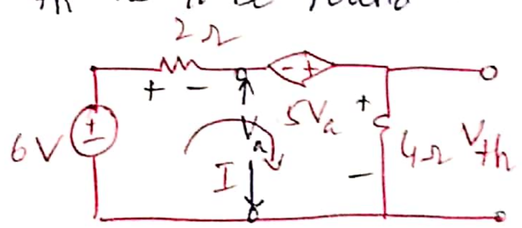
$\therefore I = 3A.$

$\Rightarrow I_N = 3$ Amps

Step 2:- To find R_{eq}.

If we have dependent sources in the network $R_{eq} = \frac{V_{th}}{I_N}$

So V_{th} is to be found.



$6 - 2I + 5V_a - 4I = 0.$

$6 - 2I + 5(6 - 2I) - 4I = 0$

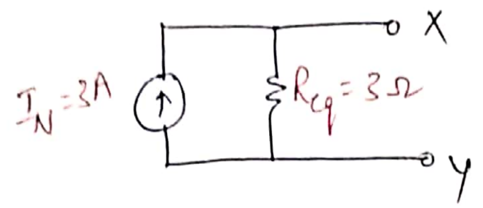
$6 - 2I + 30 - 10I - 4I = 0$

$-16I = -36 \Rightarrow I = \frac{36}{16} = 2.25A$

$V_{th} = 4 \times I = 4 \times 2.25 = 9V$

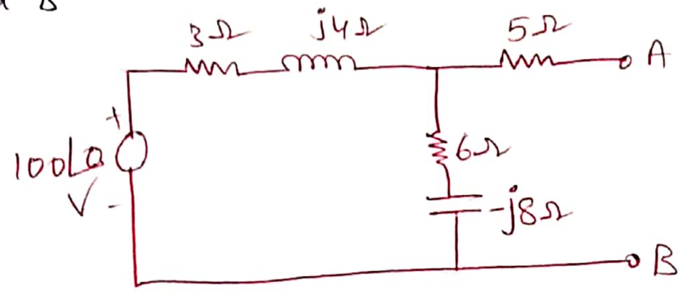
$\therefore R_{eq} = \frac{V_{th}}{I_N} = \frac{9}{3} = 3\Omega$

Step3:- Draw Norton's equivalent circuit.

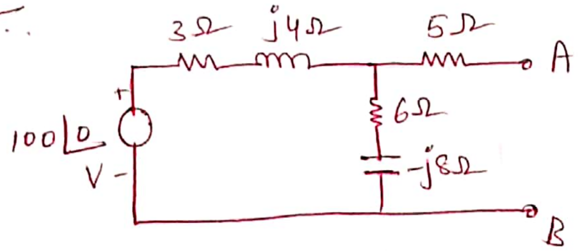


AC Excitation Norton Theorem Problems:-

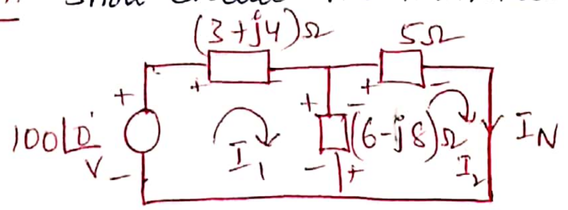
Problem1:- Obtain Norton equivalent circuit with respect to terminals A and B.



Sol:



Step1:- Short circuit AB terminals.



Step2:- To find I_N $\qquad$ $I_2 = I_N$

Apply KVL to loop ①

$$100 - (3+j4)I_1 - (6-j8)(I_1 - I_2) = 0$$

$$[-(3+j4) - (6-j8)]I_1 + (6-j8)I_2 = -100$$

$$(-9+j4)I_1 + (6-j8)I_2 = -100 \rightarrow ①$$

$$(-9+j4)I_1 = -100 - (6-j8)I_2$$

$$I_1 = \frac{-100 - (6-j8)I_2}{-9+j4} \quad ②$$

Apply KVL to loop ②

$$-5I_2 - (6-j8)(I_2 - I_1) = 0$$

$$-11I_2 + j8I_2 + 6I_1 - j8I_1 = 0$$

$$I_2(-11+j8) = (-6+j8)I_1 \rightarrow ②$$

Substitute I_1 in eq ②

$$I_2(-11+j8) = (-6+j8)\left(\frac{-100-(6-j8)I_2}{-9+j4}\right)$$

$$\cancel{\frac{I_2}{2} = \frac{(-6+j8)(-106+j8)}{(-9+j4)(-11+j8)}} = \cancel{7.927+0.35i}$$

$$I_2(-11+j8) = \frac{600-j800+(36-j48-j48+(-64))I_2}{(-9+j4)}$$

$$I_2(-11+j8)(-9+j4) = 600-j800+(-28-j96)I_2$$

$$I_2(133.9\angle -66.65^\circ) + (28+j96)I_2 = 600-j800$$

$$I_2(133.9\angle -66.65^\circ) + 100\angle 81.93\, I_2 = 600-j800$$

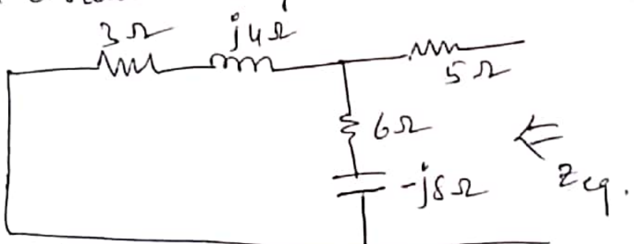
$$I_2(97.05\angle -13.17^\circ) = 600-j800$$

$$I_2 = 10.3\angle -41.27^\circ \text{ A}$$

$$\therefore I_N = 10.3\angle -41.27^\circ \text{ A}$$

Step 3:- To find Z_{eq}.

Short circuit voltage source.

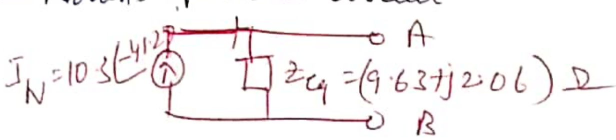


$$Z_{eq} = \frac{(3+j4)\times(6-j8)}{3+j4+6-j8} + 5$$

$$= 4.63 + 2.06i + 5$$

$$= (9.63 + j2.06)\Omega$$

Step 4:- To draw Norton's equivalent circuit.



Maximum Power Transfer theorem:-

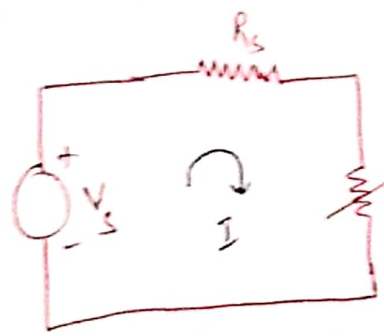
The maximum power transfer theorem can be stated as

Statement- In an active network, maximum power transfer to the load takes place when the load resistance is equal to equivalent resistance of the network as viewed from the terminals of the load (For DC Excitation)

In an active network, maximum power transfer to the load takes place when the load impedance is the complex conjugate of an equivalent impedance of the network as viewed from the terminals of the load. (For AC Excitation)

Explanation of Maximum Power transfer theorem to DC Excitation:-

Many circuits basically consist of sources supplying voltage, current or power to the load; for example a radio speaker system or a microphone supplying the input signals to voltage pre-amplifiers. Sometimes it is necessary to transfer maximum voltage, current or power from source to the load. In the simple resistive circuit shown in Fig. R_s is the source resistance. Our aim is to find the necessary conditions so that the power delivered by the source to load is maximum.



It is a fact that more voltage is delivered to the load when the load resistance is high as compared to resistance of source. On the other hand, maximum current is transferred to the load when load resistance is small compared to source resistance.

For many applications, an important consideration is the maximum power transfer to the load; for example power transfer is desirable from the output amplifier to the speaker of an audio sound system. The maximum power transfer theorem states that maximum power is delivered from source to a load when the load resistance is equal to the source resistance

For the circuit shown above

$$I = \frac{V_s}{R_s + R_L}$$

Power delivered to load R_L is $P = I^2 R_L = \frac{V_s^2}{(R_s+R_L)^2} R_L$

To determine the value of R_L for maximum power to be transferred to the load, we have to set the first derivative of the above equation with respect to R_L i.e, when $\frac{dP}{dR_L}$ is zero

$$\frac{dP}{dR_L} = \frac{d}{dR_L}\left[\frac{V_s^2 R_L}{(R_s+R_L)^2}\right] = \frac{(R_s+R_L)^2 V_s^2 - V_s^2 R_L(2(R_s+R_L))}{(R_s+R_L)^4} = 0$$

$$\Rightarrow V_s^2 (R_s+R_L)^{\cancel{2}} = V_s^2 \times 2R_L \cancel{(R_s+R_L)}$$

$$V_s^2 R_s + V_s^2 R_L = 2V_s^2 R_L \Rightarrow V_s^2 R_s = 2V_s^2 R_L - V_s^2 R_L \quad [\because V_s = V_{th}]$$

$$= V_s^2 R_L$$

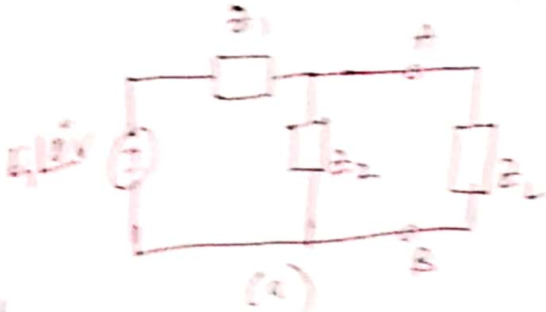
$$\therefore \boxed{R_s = R_L}, \quad P_{max} = \frac{V_s^2}{(R_s+R_L)^2} R_L = \frac{V_s^2}{4R_L^{\cancel{2}}} \times \cancel{R_L} = \frac{V_s^2}{4R_L} = \frac{V_{th}^2}{4R_L}$$

So, Maximum power is transferred to the load when load resistance is equal to source resistance.

Explanation of Maximum power Transfer Theorem for AC Excitation:-

Consider a network shown in Fig (a)

Let Z_{eq} be the equivalent impedance of the network as viewed from the terminals A-B and replacing all the independent sources by their internal impedances, as shown in Fig. (b)



Let Z_{eq} be represented as

$$Z_{eq} = R + jX$$

Then maximum power will be transferred to the load, if Z_L is complex conjugate of Z_{eq}

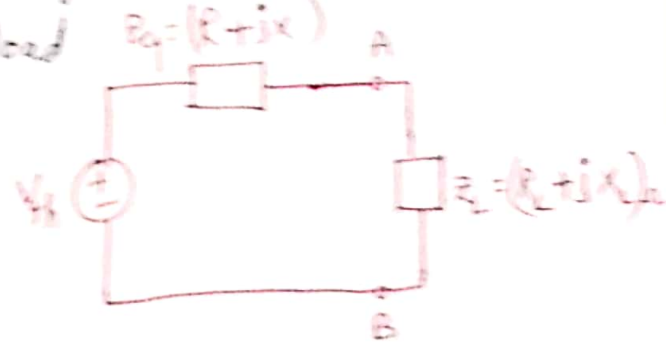
The complex conjugate is mathematically denoted as Z_{eq}^*

$$\text{So } Z_L = Z_{eq}^* = R - jX$$

Thus for maximum power transfer to the load, the resistance of load and resistance part of Z_{eq} must be same while the reactance of the load and Z_{eq} must also be same in magnitude but opposite in sign. So if Z_{eq} reactance is inductive, Z_L must be capacitive and vice versa.

Proof of Maximum Power Transfer theorem:-

Let the given network is replaced by its Thevenin's equivalent across the load terminals as shown in fig



Let $Z_{eq} = (R+jX)\,\Omega$

$Z_L = (R_L + jX_L)\,\Omega$

$$I = \frac{V_{th}}{Z_{eq}+Z_L} = \frac{V_{th}}{(R+jX)+(R_L+jX_L)}$$

The power delivered to load is $P_L = I^2 R_L$

$$I = \frac{V_{th}}{\sqrt{(R+R_L)^2+(X+X_L)^2}} \Rightarrow \cancel{P_L = \frac{V_{th}^2}{(R+R_L)^2+(X+X_L)^2}}$$

$$P_L = \frac{V_{th}^2}{(R+R_L)^2+(X+X_L)^2} \times R_L$$

Now for load impedance Z_L, both R_L and X_L are variable and are to be decided such that power will be maximum. Hence according to maximum theorem we can write that for the maximum power transfer, w.r to variable X_L and fixed R_L

$$\frac{dP}{dX_L} = 0$$

$$\frac{d}{dX_L}\left(\frac{V_{th}^2 R_L}{(R+R_L)^2+(X+X_L)^2}\right) = 0$$

$$\frac{\cancel{\left((R+R_L)^2+(X+X_L)^2\right) \times V_{th}^2} - (V_{th}^2 R_L)\left(2(X+X_L)\right)}{\left((R+R_L)^2+(X+X_L)^2\right)^2} = 0$$

$$\boxed{\begin{array}{l} \cancel{X\, V_{th}^2\left((R+R_L)^2+(X+X_L)^2\right)} = \cancel{V_{th}^2 \times \left(2R_L(X+X_L)\right)} \\ R^2+R_L^2+2RR_L+X^2+X_L^2+2XX_L = 2R_L \end{array}}$$

$$-2V_{th}^2 R_L (X+X_L) = 0$$

$$X = -X_L$$

$$\therefore \boxed{X_L = -X}$$

Thus load reactance must be same in magnitude of the reactance of Z_{eq} but opposite in sign.

Similarly power transfer will be maximum w.r to variable R_L and fixed X_L when,

$$\frac{dP_L}{dR_L} = 0$$

$$\frac{d}{dR_L}\left[\frac{V_{th}^2 R_L}{(R+R_L)^2+(X+X_L)^2}\right] = 0 \Rightarrow$$ Substitute $X_L = -X$ as already derived.

$[\because X+X_L = X-X = 0]$

$$\frac{V_{th}^2 R_L\left(2(R+R_L)\right) - (R+R_L)^2 V_{th}^2}{(R+R_L)^4} = 0$$

$$2V_{th}^2 R_L(R+R_L) = (R+R_L)^2 V_{th}^2$$

$$2R_L = R+R_L$$

$$2R_L - R_L = R$$

$$\Rightarrow R_L = R$$

$$\boxed{R = R_L}$$

Thus the resistance of the load must be same as that of equivalent impedance of the network. Thus when Z_L is the complex conjugate of Z_{eq}, the power transfer to the load is maximum and is given by

$$P_{max} = I^2 R_L = \frac{V_{th}^2}{(2R_L)^2} R_L = \frac{V_{th}^2}{4R_L} \qquad \text{as } I = \frac{V_{th}^2}{2R_L}$$

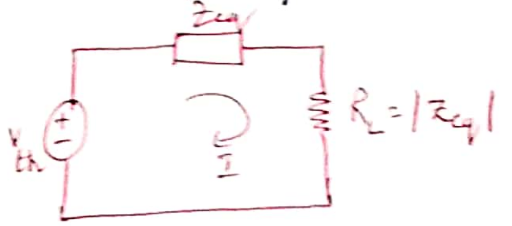
$$\boxed{\therefore P_{max} = \frac{V_{th}^2}{4R_L}}$$

Where V_{th} = Thevenin's Voltage as circuit is replaced by its Thevenin eqivalent

Corollary:- If pure resistance is to be connected as load for maximum power transfer then its value must be equal to the absolute magnitude of Z_{eq}.

$R_L = |Z_{eq}|$ for P_{max} when load is purely resistive

$$I = \frac{V_{th}}{Z_{eq}+R_L}$$

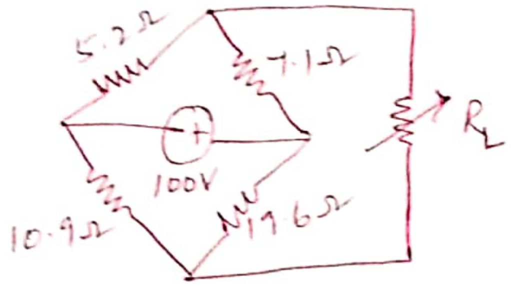


and hence maximum power delivered to the load is given by

$$P_{max} = I^2 R_L = \left[\frac{V_{th}}{Z_{eq}+R_L}\right]^2 R_L$$ and it is not given by $\frac{V_{th}^2}{4R_L}$

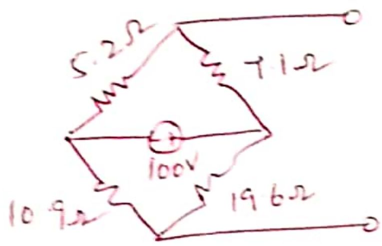
Problems-

Problem 1:- For the circuit, find the value of R_L that will receive maximum power. Determine this maximum power.

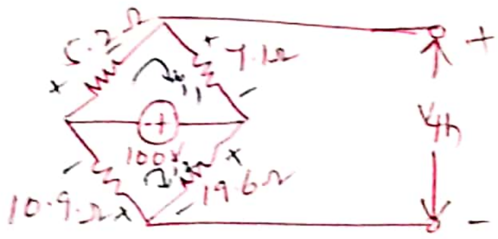


Sol:-

Step 1:- Remove R_L



Step 2:- Find V_{th} between the removed terminals



Apply KVL to loop ①

$$-5.2i_1 - 7.1i_1 - 100 = 0$$

$$-12.3i_1 = 100$$

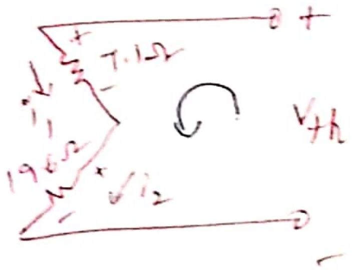
$$i_1 = \frac{-100}{12.3} = -8.13A$$

Apply KVL to loop ②

$$-19.6i_2 - 10.9i_2 + 100 = 0$$

$$-30.5i_2 = -100$$

$$\Rightarrow i_2 = \frac{100}{30.5} = 3.28A$$



Apply KVL to the path to find V_{th}

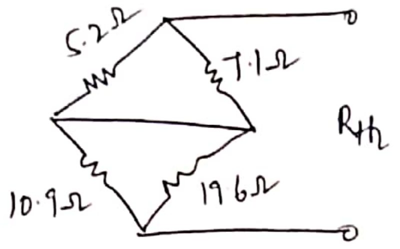
$$V_{th} - 7.1i_1 - 19.6i_2 = 0$$

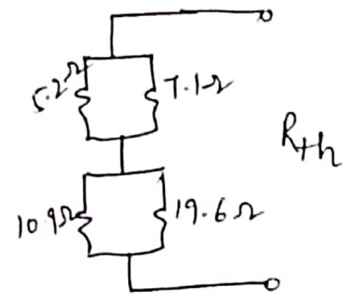
$$V_{th} - 7.1 \times -8.13 - 19.6 \times 3.28 = 0$$

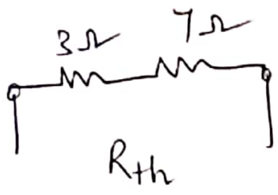
$$V_{th} = 64.262 - 57.723$$

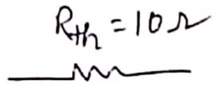
$$= \cancel{6.36V}\ 6.54V$$

Step 3: To find R_{th}

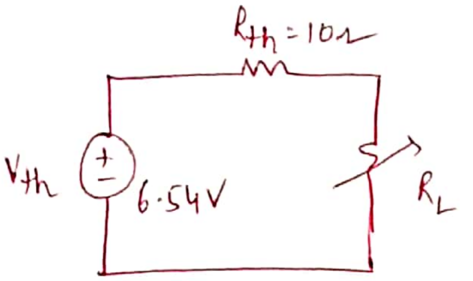






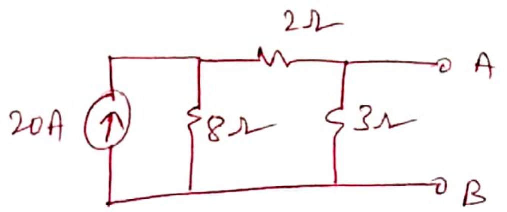


$$\therefore \boxed{R_L = R_{th} = 10\Omega}$$

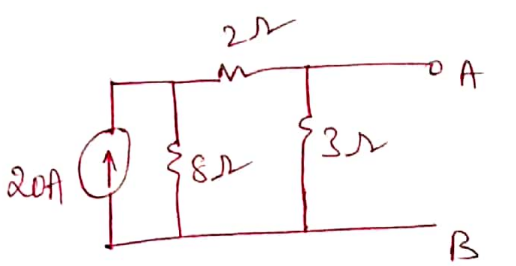


$$P_{max} = \frac{V_{th}^2}{4R_L} = \frac{6.54^2}{4 \times 10} = 1.069\,W$$

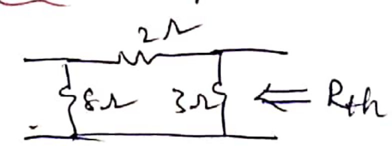
Problem 2:- Determine the value of load resistance to be connected between A-B to absorb maximum power. What is maximum power



Sol:-



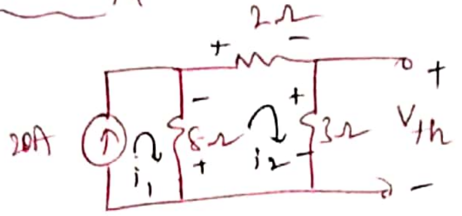
To find R_{th}:- open circuit current source.



$$R_{th} = (8+2) \parallel 3 = \frac{10 \times 3}{10+3} = 2.307\,\Omega$$

$$\boxed{R_L = R_{th} = 2.307\,\Omega}$$

To find V_{th}:-



$i_1 = 20A$

Apply KVL to loop ②

$$-2i_2 - 3i_2 - 8(i_2 - i_1) = 0$$

$$-13i_2 + 8i_1 = 0$$

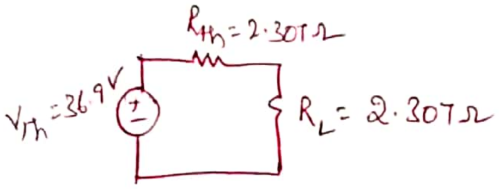
$$-13i_2 + 160 = 0$$

$$-13i_2 = -160$$

$$i_2 = \frac{160}{13} = \cancel{12.3A}\ 12.3A$$

$$V_{th} = 3i_2 = \cancel{3 \times 1.23 = 3.69V}$$

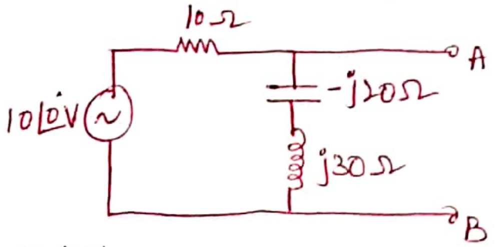
$$= 3 \times 12.3 = 36.9V$$



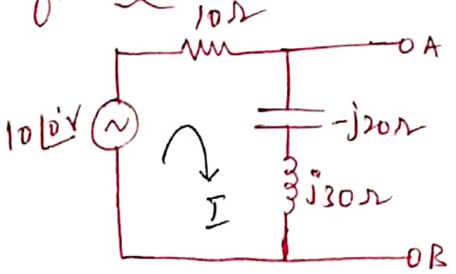
$$P_{max} = \frac{V_{th}^2}{4R_L} = \frac{36.9^2}{4 \times 2.307} = 147.55W$$

Problem 3:- (AC Excitation)

Find the load impedance required to be connected across the terminals A-B for the maximum power transfer, in the network shown. Also find maximum power delivered to the load.



Sol:- To find V_{th}:-



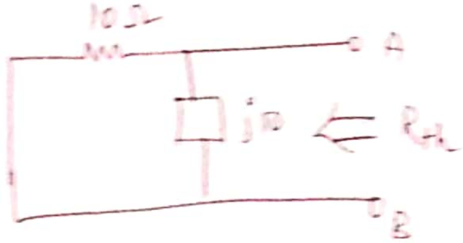
$$V_{th} = I \times (-j20 + j30)$$

$$= 0.707\angle -45° \times (j10)$$

$$= 7.07\angle 45°\ V$$

$$I = \frac{10\angle 0°}{10 - j20 + j30} = \frac{10\angle 0°}{10 + j10} = 0.707\angle -45°$$

To find R_{th} :-



$$\Rightarrow Z_{th} = 10 // j10 = \frac{10 \times j10}{10+j10} = \frac{j100}{10+j10} = 7.07 \angle 45^\circ \ \Omega$$

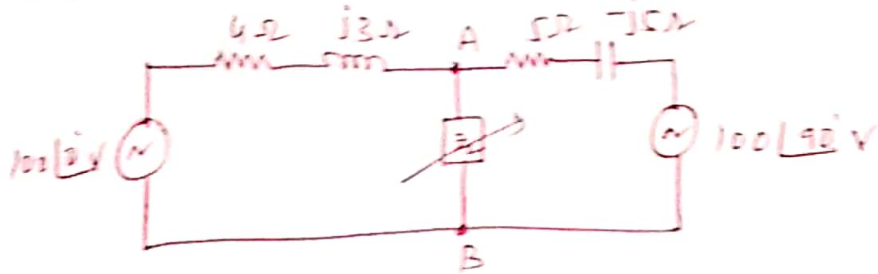
$$= 5+5i$$

$$\therefore \ \vec{Z_L} = 5+5i$$

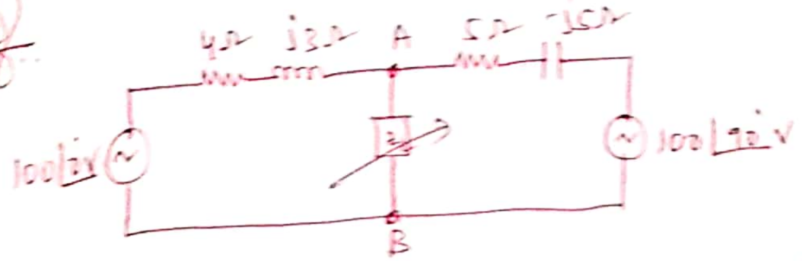
$$R_L = 5\Omega$$

$$P_{max} = \frac{V_{th}^2}{4R_L} = \frac{7.07^2}{4\times 5} = 2.499 = 2.5W$$

Problem 4:- Find Z_L for maximum power and value of P_{max}

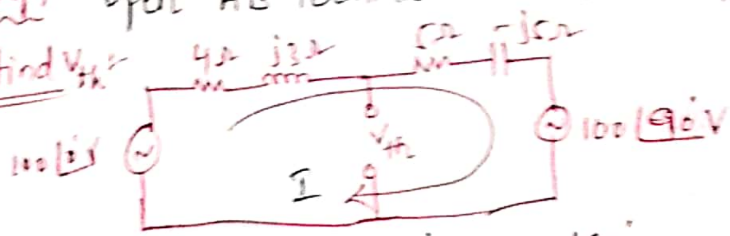


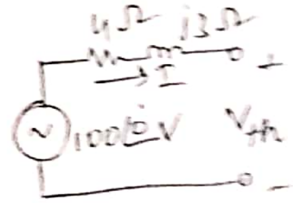
Sol:-



Step 1:- open AB terminals and find V_{th} between A & B terminals

To find V_{th}:-





$$I = \frac{100\angle 0^\circ - 100\angle 90^\circ}{9-j2}$$

$$= 15.33 \angle -32.47^\circ$$

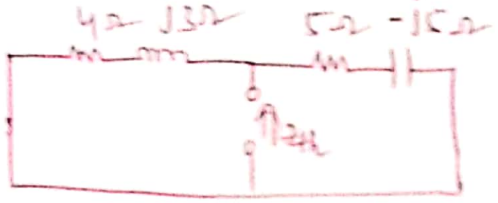
$$100\angle 0^\circ - (4+j3)I - V_{th} = 0$$

$$V_{th} = 100\angle 0^\circ - (4+j3)(15.33\angle \ldots)$$

$$= 24.29 \angle -14^\circ$$

$\therefore V_{th} = 24.29 \angle -14^\circ \; V$

To find Z_{th}:-



$Z_{th} = (4+j3) \,//\, (5-j5) = \dfrac{(4+j3)(5-j5)}{9-j2} = 3.83 \angle 4.39^\circ \,\Omega$

$= (\overset{R_{th}}{3.82} + j0.29)\,\Omega$

$R_L = 3.82\,\Omega \qquad \Rightarrow Z_L = Z_{th}^* = \overset{R_L}{3.82} - j\overset{X_L}{0.29}$

$P_{max} = \dfrac{V_{th}^2}{4R_L} = \dfrac{24.29^2}{4 \times 3.82} = 38.61\,W$

Reciprocity Theorem:-

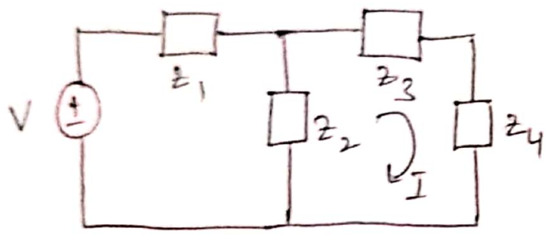
Reciprocity theorem states that In any linear network consisting of linear and bilateral elements and active sources, the ratio of voltage V introduced in one loop to the current I in other loop is same as the ratio obtained if the positions of V and I are interchanged in the network. While calculating the ratio, the sources other than one which is considered to obtain the ratio, must be replaced by their internal resistances (or impedances)

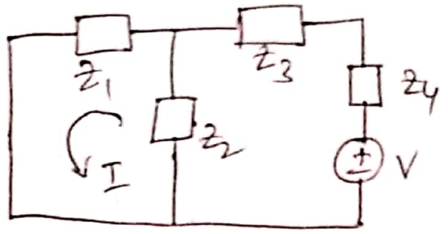
Explanation:-

Consider the network shown

V is the voltage introduced in loop 1 while I is the current in loop 2. The ratio of voltage V to I is $\frac{V}{I}$

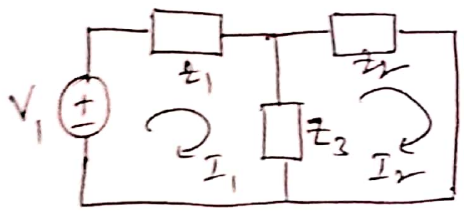
Reciprocity theorem states that the ratio $\frac{V}{I}$ remains same, if the positions of V and I are interchanged in the network, as shown in Fig.





In other words, the V and I are mutually transferable. The ratio $\frac{V}{I}$ is called transfer impedance where V is voltage introduced in loop 1 and I is the response due to V in loop 2

Proof of Reciprocity Theorem:-



Consider the network shown in fig.
Let us calculate the ratio $\frac{V_1}{I_2}$
Applying KVL to two loops

$$-I_1 Z_1 - I_1 Z_3 + I_2 Z_3 + V_1 = 0$$

$$I_1(Z_1 + Z_3) - I_2 Z_3 = V_1 \rightarrow ①$$

Apply KVL to second loop

$$-Z_2 I_2 - Z_3(I_2 - I_1) = 0$$

$$(Z_3 + Z_2) I_2 = Z_3 I_1$$

$$I_1 = \left(\frac{Z_2 + Z_3}{Z_3}\right) I_2$$

Substitute I_1 in ①.

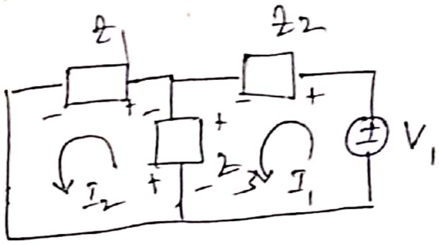
$$\left(\frac{Z_2 + Z_3}{Z_3}\right) \times (Z_1 + Z_3) I_2 - I_2 Z_3 = V_1$$

$$\left[\frac{(Z_1+Z_3)(Z_2+Z_3)}{Z_3} - Z_3\right] I_2 = V_1$$

$$\frac{Z_1Z_2 + Z_3Z_2 + Z_1Z_3 + \cancel{Z_3^2} - \cancel{Z_3^2}}{Z_3}\, I_2 = V_1$$

$$\frac{V_1}{I_2} = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_3}{Z_3} \rightarrow (A)$$

Let us interchange the positions of V_1 and I_2 as shown in fig.



Apply KVL to loop ①.

$$V_1 - Z_2 I_1 - Z_3(I_1 - I_2) = 0$$

$$V_1 - I_1(Z_2+Z_3) + Z_3 I_2 = 0 \rightarrow (1)$$

Apply KVL to loop ②

$$-Z_1 I_2 - Z_3(I_2 - I_1) = 0$$

$$Z_3 I_1 - I_2(Z_1+Z_3) = 0$$

$$Z_3 I_1 = I_2(Z_1+Z_3)$$

$$I_1 = \frac{Z_1+Z_3}{Z_3} I_2$$

Substitute I_1 in (1)

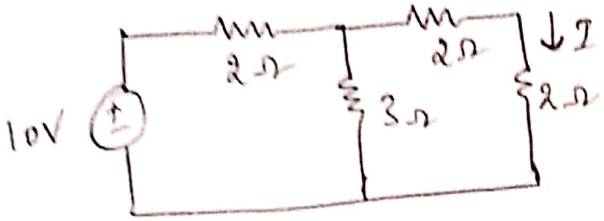
$$V_1 - \frac{(Z_1+Z_3)(Z_2+Z_3)}{Z_3} I_2 + Z_3 I_2 = 0$$

$$\frac{V_1}{I_2} = \frac{(Z_1+Z_3)(Z_2+Z_3)}{Z_3} - Z_3 = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1 + \cancel{Z_3^2} - \cancel{Z_3^2}}{Z_3}$$

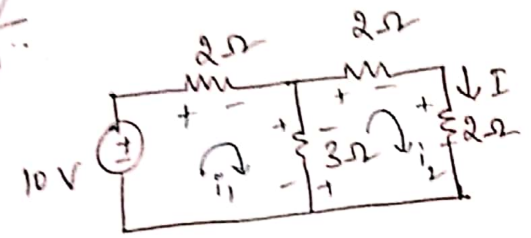
$$\therefore \frac{V_1}{I_2} = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3} \rightarrow (B)$$

Both ratio's (A) & (B) are same. So reciprocity theorem is verified.

Problem :- Verify reciprocity theorem for the voltage V and current I in the network shown.



Sol:-



Apply KVL to loop ①

$10 - 2i_1 - 3(i_1 - i_2) = 0$

$-5i_1 + 3i_2 = -10 \rightarrow (1')$

Apply KVL to loop ②

$-2i_2 - 2i_2 - 3(i_2 - i_1) = 0$

$3i_1 - 7i_2 = 0 \rightarrow (2)$

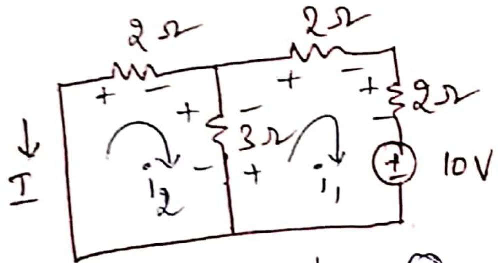
$\Rightarrow$ $i_1 = 2.69\ A$

$i_2 = 1.15\ A$

$\therefore$ $I = i_2 = 1.15\ A$

$\therefore \frac{V}{I} = \frac{10}{1.15} = 8.69 \rightarrow (A)$

Interchange the voltage source to second loop and find current in ~~second lo~~ first loop



Apply KVL to loop ①

$-2i_1 - 2i_1 - 3(i_1 - i_2) - 10 = 0$

$-7i_1 + 3i_2 = 10 \rightarrow (1)$

Apply KVL to loop ②

$-2i_2 - 3(i_2 - i_1) = 0$

$3i_1 - 5i_2 = 0 \rightarrow (2)$

$i_1 = -1.92\ A$

$i_2 = -1.15\ A$.

$I = -i_2 = 1.15\ A$.

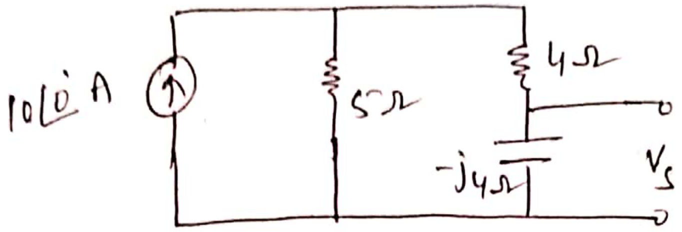
$\therefore \frac{V}{I} = \frac{10}{1.15} = 8.69 \rightarrow (B)$

Since ratio's A & B are same

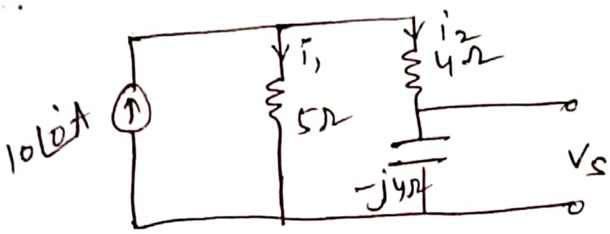
Reciprocity theorem is verified.

Problem 2:- (AC Excitation)

Verify reciprocity theorem for the network shown in fig.



Sol.

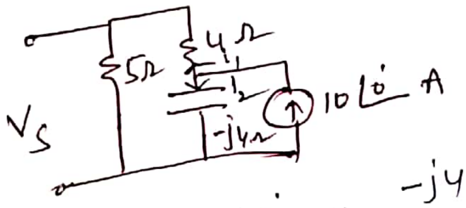


$$i_2 = 10\angle 0^\circ \times \frac{5}{5+4-j4} = 5.07\angle 23.96 \text{ A}$$

$$V_s = -j4 \times 5.07\angle 23.96 = 20.28\angle -66.04^\circ \text{ V}$$

$$\frac{V_s}{I} = \frac{20.28\angle -66.04}{10\angle 0^\circ} = 2.02\angle -66.04 \rightarrow (A)$$

Now interchange the positions of V_s and I



$$i_1 = 10\angle 0^\circ \times \frac{-j4}{-j4+4+5} = 4.06\angle -66.03^\circ \text{ A}.$$

$$V_s = 5i_1 = 5 \times 4.06\angle -66.03^\circ = 20.3\angle -66.04^\circ \text{ V}$$

$$\frac{V_s}{I} = \frac{20.3\angle -66.04}{10} = 2.03\angle -66.04^\circ \rightarrow (B)$$

The ratio's A & B are same

So reciprocity theorem is verified.

Millimann's Theorem:- Milliman's theorem states that

If n voltages sources $V_1, V_2 \ldots V_n$ having internal resistances (or impedances) $Z_1, Z_2 \ldots Z_n$ respectively are in parallel, then these sources may be replaced by a single voltage source of voltage V_m having a series impedance Z_m where V_m and Z_m are given by

$$V_m = \frac{V_1G_1 + V_2G_2 + V_3G_3 + \ldots V_nG_n}{G_1 + G_2 + G_3 \ldots G_n} = \frac{\sum_{K=1}^{n} V_K G_K}{\sum_{K=1}^{n} G_K}$$

where G_1, G_2 are conductances corresponding to resistances $R_1, R_2 \ldots R_n$

$$G_1 = \frac{1}{R_1}, \quad G_2 = \frac{1}{R_2} \quad \ldots \quad G_n = \frac{1}{R_n}$$

and $$V_m = \frac{V_1Y_1 + V_2Y_2 + V_3Y_3 + \ldots V_nY_n}{Y_1 + Y_2 + Y_3 + \ldots Y_n} = \frac{\sum_{K=1}^{n} V_K Y_K}{\sum_{K=1}^{n} Y_K}$$ (For ac excitation)

where $Y_1, Y_2 \ldots Y_n$ are admittances corresponding to impedances $Z_1, Z_2 \ldots Z_n$

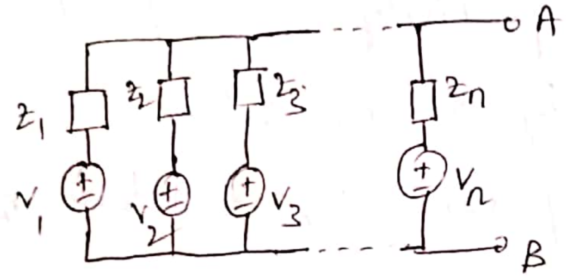
$$Y_1 = \frac{1}{Z_1}, \quad Y_2 = \frac{1}{Z_2} \quad \ldots \quad Y_n = \frac{1}{Z_n}$$

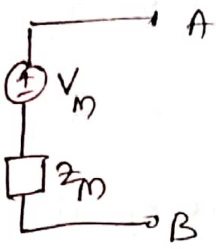
$$\cancel{G_m} = R_m = \frac{1}{G_1 + G_2 + \ldots G_n} = \frac{1}{\sum_{K=1}^{n} G_K}$$

$$Z_m = \frac{1}{Y_1 + Y_2 + \ldots Y_n} = \frac{1}{\sum_{K=1}^{n} Y_K}$$

Scanned by CamScanner

Explanation:- Consider the n voltage sources $V_1, V_2 \dots V_n$ having series impedances $Z_1, Z_2 \dots Z_n$ connected in parallel as shown.





$$Y = \frac{1}{Z}$$

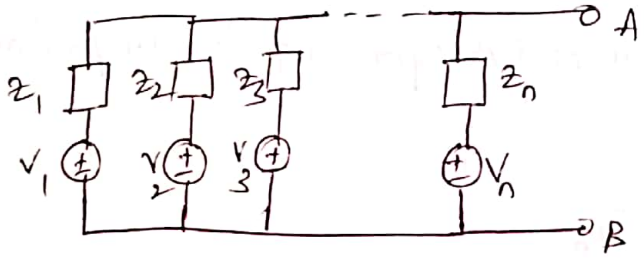
$$Y_1 = \frac{1}{Z_1}, \; Y_2 = \frac{1}{Z_2} \; \dots \; Y_n = \frac{1}{Z_n}$$

Then according to millimann's theorem, all voltage sources can be combined to get a single voltage source V_m with a series impedance Z_m as shown in fig...

$$V_m = \frac{V_1 Y_1 + V_2 Y_2 + \dots V_n Y_n}{Y_1 + Y_2 + \dots Y_n}$$

$$Z_m = \frac{1}{Y_1 + Y_2 + Y_3 + \dots Y_n}$$

Proof of Millimann's theorem:-



Consider n voltage sources in parallel as shown in fig. Let us convert each voltage source into equivalent current source. For source 1, $I_1 = \frac{V_1}{Z_1} = V_1 Y_1$

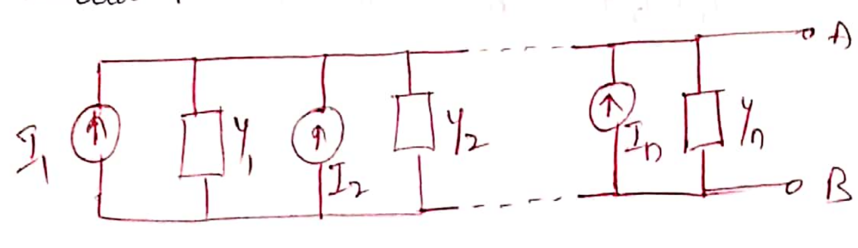
I. as $Y_1 = \frac{1}{Z_1}$

Similarly for remaining sources, we can write

$I_2 = V_2 Y_2, \; I_3 = V_3 Y_3 \; \dots \; I_n = V_n Y_n$

where $Y_1, Y_2 \dots Y_n$ are admittances to be connected in parallel

Hence circuit reduces to

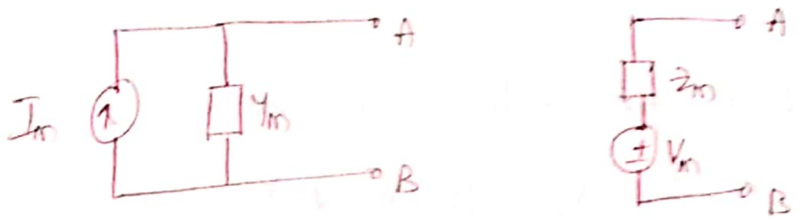


Hence the effective current source across the terminals A–B is

$$I_m = I_1 + I_2 + \cdots I_n \rightarrow (1)$$

$$Y_m = Y_1 + Y_2 + \cdots Y_n \rightarrow (2)$$

This is because admittances in parallel get added to each other. Hence circuit reduces to as shown



Converting this equivalent current source into the voltage source, we get

$$V_m = \frac{I_m}{Y_m}$$

$$Z_m = \frac{1}{Y_m}$$

$$V_m = I_m Z_m$$

Substituting I_m and Y_m from equations (1) and (2)

$$V_m = (I_1 + I_2 + \cdots I_n) \cdot \frac{1}{(Y_1 + Y_2 + \cdots Y_n)}$$

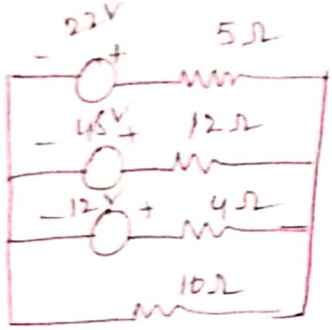
$$I_1 = \frac{V_1}{Z_1} = V_1 Y_1, \; I_2 = V_2 Y_2, \; \cdots I_n = V_n Y_n$$

$$\therefore V_m = \frac{V_1 Y_1 + V_2 Y_2 + \cdots V_n Y_n}{Y_1 + Y_2 + \cdots Y_n}$$

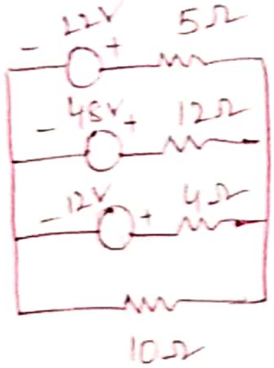
$$Z_m = \frac{1}{Y_m} = \frac{1}{Y_1 + Y_2 + \cdots Y_n}$$

Thus millman's theorem is proved.

Problem 1:- Use Millman's theorem to find the current through 10Ω resistance in the circuit.



Sol:-



From given network we can write,

$V_1 = 12V$, $V_2 = 48V$, $V_3 = 22V$

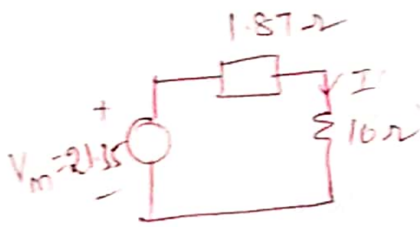
$Z_1 = 4\Omega$, $Z_2 = 12\Omega$, $Z_3 = 5\Omega$

$Y_1 = \frac{1}{4} = 0.25\mho$, $Y_2 = \frac{1}{12} = 0.083\mho$, $Y_3 = \frac{1}{5} = 0.2\mho$

$$V_m = \frac{V_1Y_1 + V_2Y_2 + V_3Y_3}{Y_1 + Y_2 + Y_3} = \frac{12 \times 0.25 + 48 \times 0.083 + 22 \times 0.2}{0.25 + 0.083 + 0.2}$$

$$= \cancel{24}\ 21.35V$$

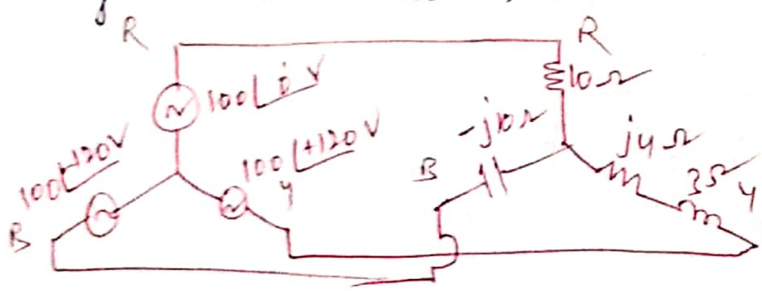
$$Z_m = \frac{1}{Y_1 + Y_2 + Y_3} = \frac{1}{0.25 + 0.083 + 0.2} = 1.87\Omega$$



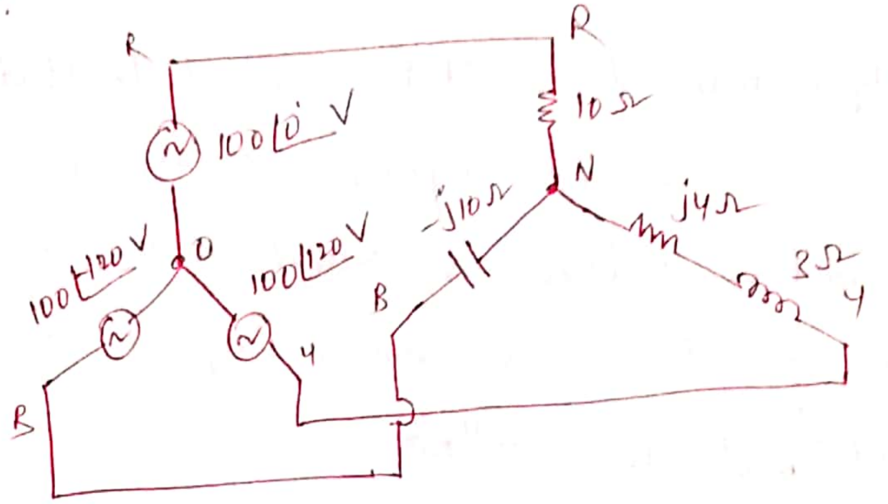
$$I = \frac{21.35}{1.87 + 10} = 1.79A$$

Problem 2:- AC excitation

Using Millman's theorem find the neutral shift voltage V_{ON}



Sol



Given that

$V_{RO} = 100\angle 0^\circ$ V

$V_{YO} = 100\angle 120$ V

$V_{BO} = 100\angle -120$ V

$Z_{RN} = 10\,\Omega$

$Z_{YN} = (3+j4)\,\Omega$

$Z_{BN} = -j10\,\Omega$

$$Y_{RN} = \frac{1}{10} = 0.1\,\mho, \quad Y_{YN} = \frac{1}{3+j4} = 0.2\angle -53.13^\circ\,\mho$$

$$Y_{BN} = \frac{1}{-j10} = 0.1\angle 90^\circ\,\mho$$

$$V_{NO} = \frac{V_{RO}Y_{RN} + V_{YO}Y_{YN} + V_{BO}Y_{BN}}{Y_{RN}+Y_{YN}+Y_{BN}} = \frac{100\angle 0^\circ \times 0.1 + 100\angle 120^\circ \times 0.2\angle -53.13^\circ + 0.1\angle 90^\circ \times 100\angle -120^\circ}{0.1 + 0.2\angle -53.13^\circ + 0.1\angle 90^\circ}$$

$$= \frac{10\angle 0^\circ + 20\angle 66.87^\circ + 10\angle -30^\circ}{0.1 + 0.2\angle -53.13^\circ + 0.1\angle 90^\circ}$$

$$= 130.27\angle 42.05\ \text{V}$$

$$Z_{NO} = \frac{1}{Y_{RN}+Y_{YN}+Y_{BN}} = \frac{1}{0.1 + 0.2\angle -53.13 + 0.1\angle 90^\circ} = 4.385\angle 15.25^\circ$$

$$= (4.23 + j1.15)\,\Omega$$

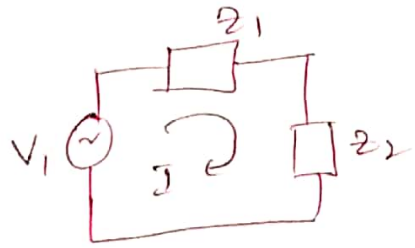
Compensation Theorem:-

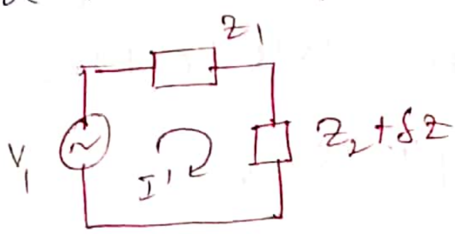
In circuit analysis many times it is required to study the effect of change in resistance (or) impedance in one of its branches on the corresponding voltages and currents of the network. The compensation theorem provides a very simple way for studying such effects. The statement is as follows.

Statement:- In any linear network consisting of linear and bilateral resistances (or) impedances and active sources, if the impedance Z of the branch carrying current I increases by δZ, then the increment or decrement of voltage or current in each branch of the network is that voltage or current that would be produced by an opposing voltage source of value V_c (= $I \cdot \delta Z$ or $I \delta R$) introduced in the altered branch after replacing original sources by their internal impedances.

Explanation:-

Consider a network shown in fig.





V_1 is voltage applied to network, I is the current flowing through Z_1 & Z_2. Consider that impedance Z_2 increases by δZ. Due to this, the current in the circuit changes to I' as shown in fig.

Then the effect of change in impedance is the change in current which is given by

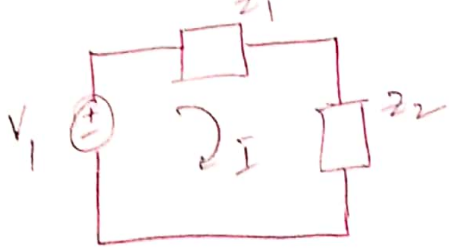
$$\delta I = I - I'$$

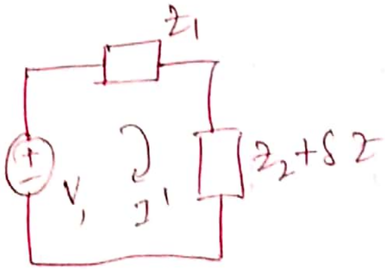
Now this current can be directly calculated by using the compensation theorem. First modify the branch of which impedance is changed, by connecting a voltage source V_c of value $I \cdot \delta Z$. The new voltage source must be connected in the branch with proper polarity. Then replace original active source V by its internal impedance as shown in Fig.

The voltage source introduced in modified branch, V_c is called compensation source with value $I \cdot \delta Z$ where I is current through impedance before impedance of branch is changed and δZ is change in impedance.

Proof of Compensation Theorem:-

Consider a network shown in Fig.





$$I = \frac{V_1}{Z_1+Z_2} \qquad I' = \frac{V_1}{Z_1+Z_2+\delta Z}$$

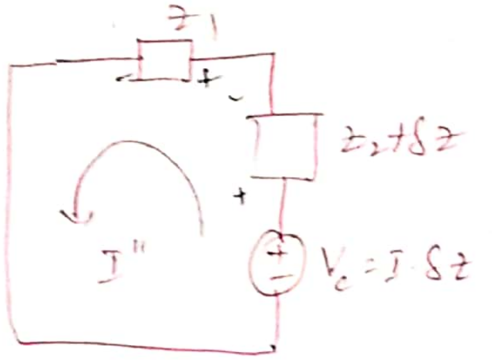
$$\delta I = I - I' = \frac{V_1}{Z_1+Z_2} - \frac{V_1}{Z_1+Z_2+\delta Z} = V_1\left[\frac{1}{Z_1+Z_2} - \frac{1}{Z_1+Z_2+\delta Z}\right]$$

$$= V_1\left[\frac{\cancel{Z_1}+\cancel{Z_2}+\delta Z - \cancel{Z_1} - \cancel{Z_2}}{(Z_1+Z_2)(Z_1+Z_2+\delta Z)}\right] = \frac{V_1}{Z_1+Z_2} \cdot \frac{\delta Z}{Z_1+Z_2+\delta Z}$$

$$\delta I = \frac{I \cdot \delta Z}{Z_1 + Z_2 + \delta Z} = \frac{V_c}{Z_1 + Z_2 + \delta Z} \rightarrow (1)$$

$\therefore$ Compensating voltage $V_c = I \cdot \delta Z$

Now Consider that the branch is modified as shown in the Fig and also original voltage source is short circuited. Let the current in circuit be I''



Apply KVL to loop

$$V_c - (Z_2 + \delta Z)I'' - Z_1 I'' = 0$$

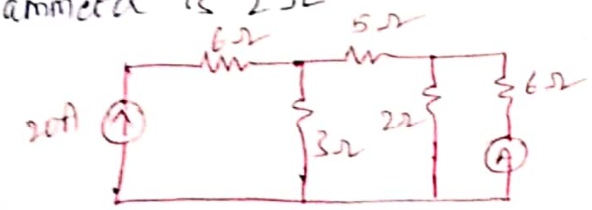
$$V_c - I''(Z_2 + \delta Z + Z_1) = 0$$

$$I'' = \frac{V_c}{Z_1 + Z_2 + \delta Z} \rightarrow (2)$$

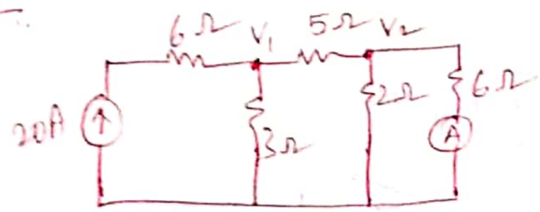
Equations (1) = (2) $\Rightarrow \delta I = I''$

Thus Compensation theorem is proved.

Problems:- Using Compensation theorem, determine the ammeter reading where it is connected to 6Ω resistor in Fig. The internal resistance of ammeter is 2Ω



Sol:



Apply KCL at node (1)

$$-20 + \frac{V_1}{3} + \frac{V_1 - V_2}{5} = 0$$

$$V_1\left(\frac{1}{3} + \frac{1}{5}\right) - V_2\left(\frac{1}{5}\right) = 20 \Rightarrow 0.53V_1 - 0.2V_2 = 20$$

Apply KCL at node (2)

$$\frac{V_2 - V_1}{5} + \frac{V_2}{2} + \frac{V_2}{6} = 0$$

$$-V_1\left(\frac{1}{5}\right) + V_2\left(\frac{1}{5} + \frac{1}{2} + \frac{1}{6}\right) = 0 \Rightarrow -0.2V_1 + 0.86V_2 = 0$$

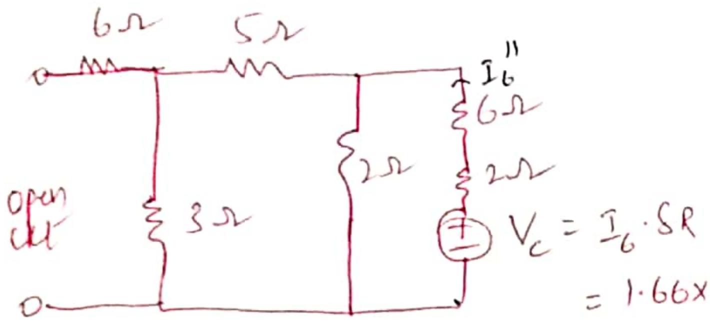
41.51V

$V_1 =$ ~~25.52V~~

$V_2 =$ ~~5.93V~~

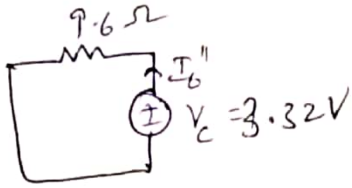
10V

$$I_6 = \frac{10}{6} = 1.66A.$$



$= 1.66 \times 2$

$= 3.32V$

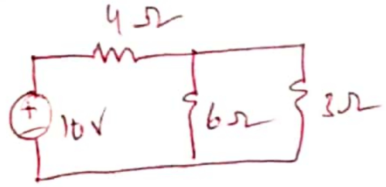


$$I_6'' = \frac{3.32}{9.6} = 0.34A$$

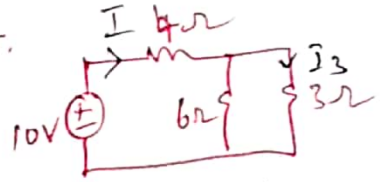
$$R_{eq} = (5 + 3 // 2) + 8$$
$$= 9.6\Omega$$

Ammeter reading $= I_6 - I_6'' = 1.66 - 0.34 = 1.31A$

Problem 2:- Determine current flowing through ammeter having 1Ω resistance in series with 3Ω

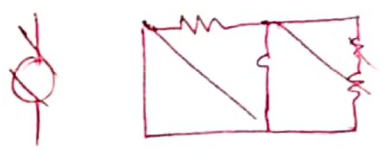


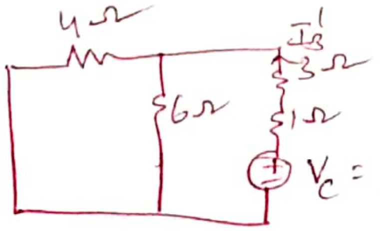
Sol:



$$I = \frac{10}{4 + 6//3} = \frac{10}{4+2} = \frac{10}{6} = 1.66A$$

$$I_3 = I \times \frac{6}{6+3} = 1.66 \times \frac{6}{9} = 1.11A$$



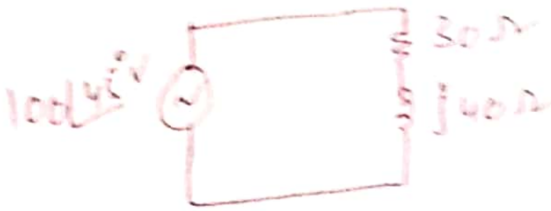


$= 1.11 \times 1 = 1.11V$

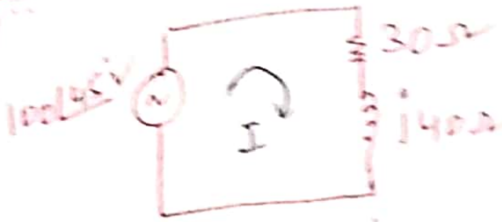
$$I_3' = \frac{V_c}{4//6 + 4} = \frac{1.11}{6.4} = 0.17A$$

Ammeter reading $= I_3 - I_3' = 1.11 - 0.17 = 0.93A$

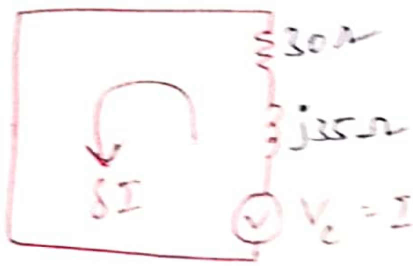
Problems :- A.C Excitation :- Calculate change in current in the network shown in Fig by using Compensation theorem when the reactance has changed to j35 Ω



Sol:



$$I = \frac{100\angle 45^\circ}{30+j40} = 2\angle{-8.13^\circ}\ A$$



$\delta Z = j40 - j35 = j5\ \Omega$

$V_c = I \cdot \delta Z$

$= 2\angle{-8.13} \times j5$

$= 10\angle 81.87^\circ$

$$\delta I = \frac{V_c}{30+j35} = \frac{10\angle 81.87^\circ}{30+j35} = 0.216\angle 32.47^\circ\ A.$$

$\therefore$ Change in current $= 0.216\angle 32.47^\circ\ A$.

magnetic circuits

Faraday's laws of Electro magnetic Induction - concept of Self & mutual inductance - Dot convention - coefficient of coupling - composite magnetic circuit - Analysis of Series & Parallel magnetic circuits, MMF calculation

---

Faraday's Laws of Electromagnetic Induction

→ Faraday's laws of Electromagnetic induction is also known as Faraday's law and it is the basic law of electromagnetism

→ The main purpose of this law is to helps us to predict how a magnetic field would interact with an electric circuit to produce an Electromotive force (EMF). This phenomenon is known electromagnetic induction.

Faraday's ~~First~~ Laws are two types. These are

1. Faraday's First Law
2. Faraday's Second Law

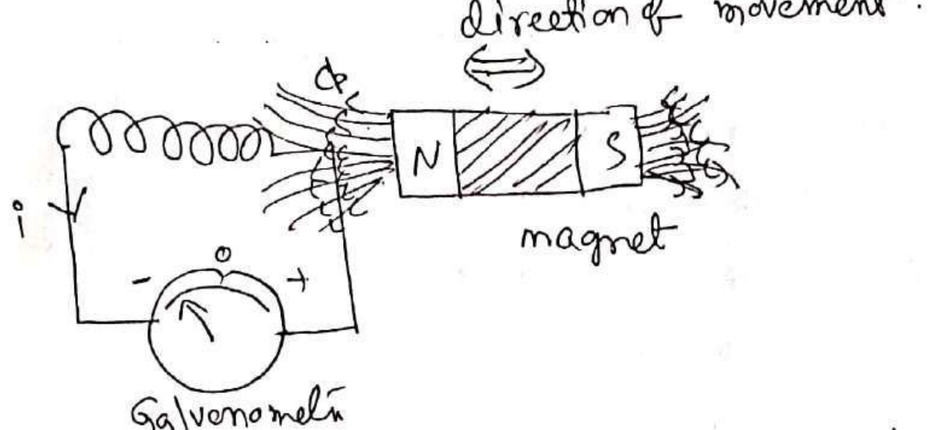
~~1. Faraday's First Law~~ :

~~"when a conductor or coil cuts the magnetic field an emf is induced in the coil".~~

Explanation: [Faraday's experiment]

Faraday takes a magnet, coil & a galvanometer. This galvanometer connects across the coil.

Step 1 : at starting, the magnet is at rest, so there is no deflection in the galvanometer needle. i.e needle at '0' position.



Step 2: when magnet is moved towards the coil, the needle of the galvanometer deflects in one direction.

Step 3: when magnet moves away from the coil, there is some deflection in the needle but opposite direction and again magnet becomes stationary, the needle of galvanometer return to zero position.

Step 4: Similarly, If the magnet is stationary and the coil moves away and towards the magnet, the galvanometer similarly shows deflection.

It also seen that the faster change in the magnetic field, the greater will be the induced emf or voltage in the coil.

Conclusion: whenever there is relative motion b/w a conductor and a magnetic field, the flux linkage with a coil changes and this change in flux induces a voltage across a coil.

Faraday's First Law

Def: "Any changes in the magnetic field of a coil of wire will cause an emf to be induced in the coil."

- This emf induced is called induced emf and if the conductor circuit is closed, the current will also circulate through the circuit and this current is called induced current.

- method to change the magnetic field
  1) By moving a magnet towards or away from the coil.
  2) By moving the coil into or out of the magnetic field.
  3) By rotating the coil relative to the magnet.

Faraday's Second Law

Def: "It states that magnitude of induced emf in the coil is equal to the rate of change of flux that linkages with the coil."

- The flux linkages of the coil is the product of the no. of turns in the coil and flux associated with the coil.

$$\psi \text{ (flux linkages)} = N\phi$$

- Rate of change of flux linkage $= \dfrac{d\psi}{dt} = \dfrac{d(N\phi)}{dt}$

→ emf = Rate of change of flux linkage $= N\dfrac{d\phi}{dt}$, ϕ = magnetic flux

According to faradays second law

$\boxed{e = N\frac{d\phi}{dt}}$ (or) $\boxed{E = -N\frac{d\phi}{dt}}$ ['−' ~~for~~ for lenz's law]

Applications of Faradays laws

Faraday's law is one of the most basic and important laws of electromagnetism. These laws have some applications in most of the electrical machines, industries & the medical field etc.

1. "Power transformer" function based on Faraday's law
2. "Electric Generator" is Faradays law of mutual induction.
3. Induction cooker.
4. Electro magnetic flow meter [Velocity measurement]
5. maxwell's equations
6. Electric guitar, Electric violin etc.

Types of Induced emf's

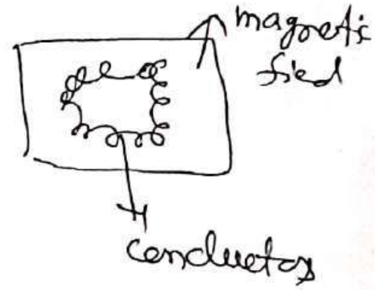
Induced emf's are two types

1. Dynamically induced emf
2. statically induced emf

1. Dynamically induced emf

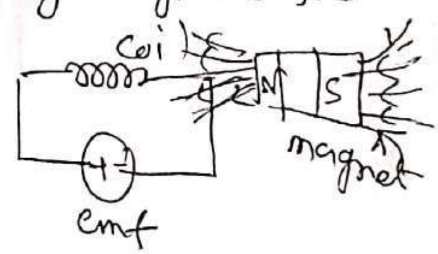
This is The emf induced in a Set of conductors which is being moved inside The stationary magnetic fied.

Eg: Generator



2. Statically induced emf

This is The emf induced in a Set of stationary conductors which are placed in a varying magnetic fied



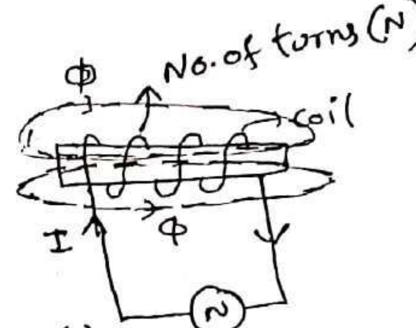
- Statically induced emf's are two types
  1. self induced emf
  2. mutually induced emf

Self induced emf

→ It is the statically induced emf

→ Def: It is the emf induced in the coil due to change of flux produced by linking it with its own turns.. This is called self induced emf.

Explanation



consider a coil having 'N' number of turns as shown in fig. when ac volteege is applied to coil, current flows through the coil, it produces flux(ϕ) linking with its own turns. If the current flowing through the coil is changed then the flux linking with it also changes. due to change in flux, emf is induced in the coil. This is called Self induced emf.

According to Faraday's second law,

$$\text{Self induced emf } (e_s) = N\frac{d\phi}{dt}$$

$$= N\frac{d\phi}{dt}\times\frac{di}{di}$$

$$= N\frac{d\phi}{di}\times\frac{di}{dt}$$

$$\boxed{e_s = L\frac{di}{dt}}$$

here L → Self inductance.

Self inductance

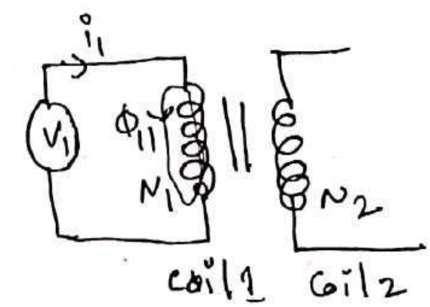
→ Self inductance or inductance of a coil.

→ It is defined as the property of coil due to which which opposes the sudden change in current flowing through it.

$$L = \frac{N d\phi}{di}$$

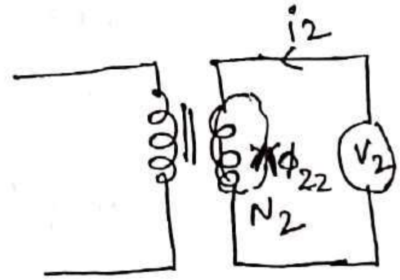
$$\boxed{L = \frac{N\phi}{i}}$$ Henry.

→ Suppose take two coils namely coil 1 & coil 2.



<u>coil 1</u> having No. of turns N_1, flux ϕ_{11}, current i_1

$$\boxed{L_1 = \frac{N_1 \phi_{11}}{i_1}}$$



$||^{ly}$ for Second coil

$$\boxed{L_2 = \frac{N_2 \phi_{22}}{i_2}}$$

Scanned with CamScanner

mutually induced emf

→ It is a statically induced emf.

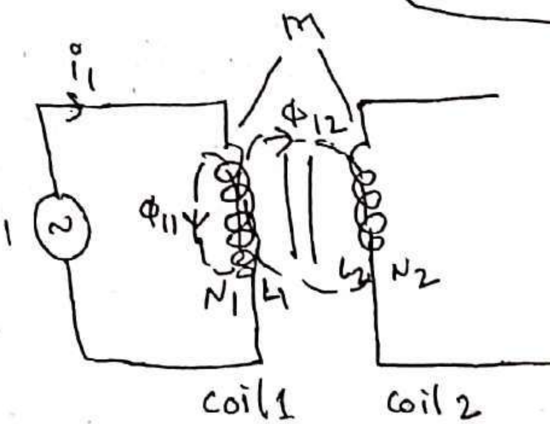
→ Def:

It is the emf induced in a coil due to change in flux produced by another neighbouring coil linking to it. It is called mutually induced emf.

$$\boxed{\begin{gathered} e_m \propto \frac{di_1}{dt} \\ e_m = M\frac{di_1}{dt} \end{gathered}}$$

Explanation

→ consider two coils with self inductances L_1 & L_2 that are closely with each other. coil 1 has N_1 turns & coil 2 has N_2 turns.



→ Suppose First coil is connected to voltage source which supplies current i_1. This current flowing through the coil 1, which produces flux ϕ_1 in coil 1.

Flux (ϕ_1) in coil 1 has two components.

$$\phi_1 = \phi_{11} + \phi_{12}$$

where ϕ_{11} → flux links with coil 1 due to current i_1

ϕ_{12} → flux links with coil 2 due to current i_1.

→ only flux ϕ_{11} links coil 1, The emf induced is

$$e_s = N_1 \frac{d\phi_{11}}{dt} \times \frac{di_1}{di_1}$$

$$= N_1 \frac{d\phi_{11}}{di_1} \times \frac{di_1}{dt}$$

$$\boxed{e_s = L_1 \frac{di_1}{dt}}$$

$$\boxed{L_1 = \frac{N_1 d\phi_{11}}{di_1} = \frac{N_1 \phi_{11}}{i_1}}$$

→ only flux ϕ_{12} links coil 2, so The emf induced in coil 2 is called mutually induced emf

$$e_m = \frac{N_2 d\phi_{12}}{dt}$$

$$= N_2 \frac{d\phi_{12}}{dt} \times \frac{di_1}{di_1}$$

$$= N_2 \frac{d\phi_{12}}{di_1} \times \frac{di_1}{dt}$$

$$\boxed{e_m = M \frac{di_1}{dt}}$$

mutual inductance (M) → mutually induced emf.

$$M = \frac{N_2 d\phi_{12}}{di_1}$$

$$\boxed{M_1 = \frac{N_2 \phi_{12}}{i_1}}$$

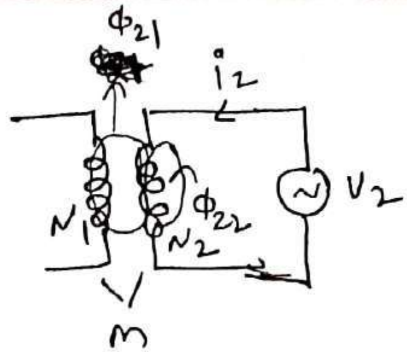
mutual inductance.

(or)

$$M = \frac{N_1 \phi_{21}}{i_2}$$ → flux in coil 1 due to current i_2

Scanned with CamScanner

III^{ly}, voltage is applied to Second coil, then.



$$\phi_2 = \phi_{22} + \phi_{21}$$

ϕ_{22} → only coil 2 flux due to i_2

ϕ_{21} → only coil I flux due i_2

<u>due to ϕ_{22}</u>

$$e_s = N_2 \frac{d\phi_{22}}{dt} \times \frac{di_2}{di_2}$$

$$= N_2 \frac{d\phi_{22}}{di_2} \times \frac{di_2}{dt}$$

$$e_s = \frac{N_2 \phi_{22}}{i_2} \times \frac{di_2}{dt}$$

$$\boxed{e_s = L_2 \frac{di_2}{dt}}$$

<u>due to ϕ_{21}</u>

$$e_{m_1} = \frac{N_1 d\phi_{21}}{dt} \times \frac{di_2}{di_2}$$

$$= N_1 \frac{d\phi_{21}}{di_2} \times \frac{di_2}{dt}$$

$$= \frac{N_1 \phi_{21}}{i_2} \times \frac{di_2}{dt}.$$

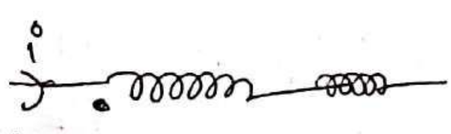
$$\boxed{e_{m1} = M \cdot \frac{di_2}{dt}}$$

Dot convention or dot rule

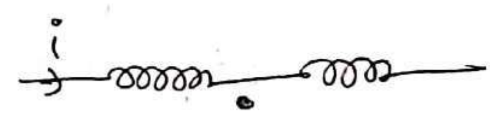
Dot convention is a technique which gives the details about voltage polarity at the dotted terminal. This information is very useful, while writing KVL equations.

* If the current enters the dotted terminal of one coil, then it induces a voltage at another coil which is having positive polarity at the dotted terminal.

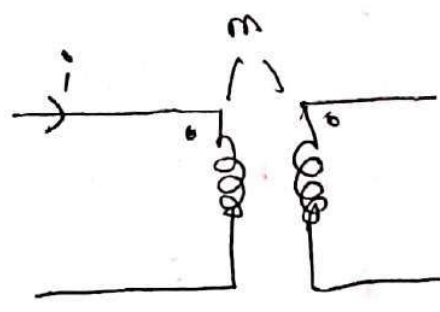
* If the current ~~enters~~ ~~the~~ leaves from the dotted terminal of one coil then it induces a voltage at another coil, which is having negative polarity at the dotted terminal.



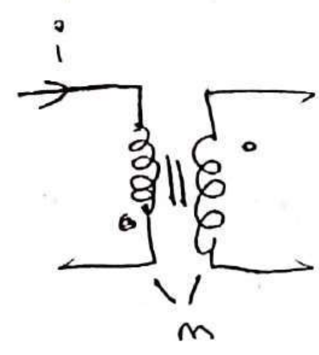
current enters the dotted terminal



current leaves the dotted terminal.



+ve



-ve

coupled circuits

An electric circuit is said to be coupled circuit, when there exists a mutual inductance b/w the coils (inductors) present in that circuit.

classification of coupling

There are two types of coupling circuits
1. Electrical coupling
2. magnetically coupling.

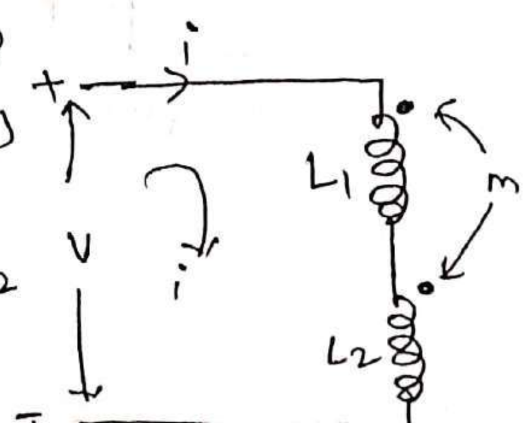
(1) Electrical coupling

Electrical coupling means a physical connection b/w two coils. This coupling can be either aiding type or opposing type. It is based on the current enters at the dotted terminal or leaves from the dotted terminal.

(A) coupling of Aiding type [Two inductors are in Series]

consider the following electrical circuit, which is having two inductors that are connected in Series.

Since inductors are connected in Series, the same current (i) flow through both inductors having self inductances L_1 & L_2 respectively.



→ In case current 'i' enters the dotted terminal of each inductor, Hence, the induced voltage in each inductor will be having +ve polarity at the dotted terminal due to the current flowing in another coil.

Apply KVL to the loop ckt

$$-V + L_1\frac{di}{dt} + M\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt} = 0$$

$$V = L_1\frac{di}{dt} + L_2\frac{di}{dt} + 2M\frac{di}{dt}$$

$$V = (L_1 + L_2 + 2M)\frac{di}{dt}$$

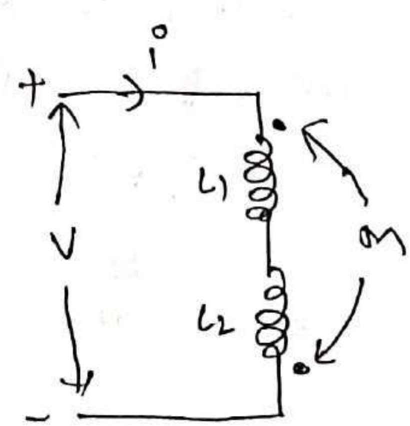
$$V = L_{eq}\frac{di}{dt}.$$

∴ Equivalent inductance of series combination of inductors is

$$L_{eq} = L_1 + L_2 + 2M$$

(B) <u>coupling of opposing type.</u>

consider the electrical circuit, which is having two inductors that are connected in series



If the current i enters the dotted terminal of L_1. Hence it induces a voltage in the other inductor (L_2). so +ve polarity of the induced voltage is present at the dotted terminal of this inductor.

and in the above ckt, the current 'i' leaves the dotted terminal of ~~L2~~ inductor L_2. Hence it induces voltage in the coil L_1. so negative polarity of induced vol is present

Apply KVL

$$-V + L_1\frac{di}{dt} - m\frac{di}{dt} + L_2\frac{di}{dt} - m\frac{di}{dt} = 0$$

$$V = L_1\frac{di}{dt} + L_2\frac{di}{dt} - 2m\frac{di}{dt}$$

$$V = \underbrace{(L_1 + L_2 - 2m)}_{L_{eq}}\frac{di}{dt}$$

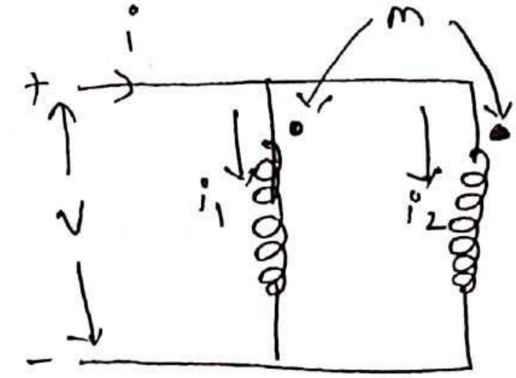
* when two inductors are series opposing

$$\boxed{L_{eq} = L_1 + L_2 - 2m.}$$

Two inductors are in parallel

(A) Parallel aiding inductors

- when two inductor are in parallel and currents are entering the dotted terminals



$$i = i_1 + i_2$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$\boxed{V = L_{eq}\frac{di}{dt}}$$

Apply KVL

$$V = L_1 \frac{di_1}{dt} + m \frac{di_2}{dt} \quad -(1)$$

$$V = L_2 \frac{di_2}{dt} + m \frac{di_1}{dt} \quad -(2)$$

From eq (2)

$$\frac{di_2}{dt} = \frac{V - m\frac{di_1}{dt}}{L_2} \quad -(3)$$

Sub eq (3) in eq (1)

$$V = L_1 \frac{di_1}{dt} + m \left[\frac{V - m\frac{di_1}{dt}}{L_2} \right].$$

$$L_1 L_2 \frac{di_1}{dt} + mV - m^2 \frac{di_1}{dt} = L_2 V$$

$$\frac{di_1}{dt} \left[L_1 L_2 - m^2 \right] = VL_2 - Vm$$

$$\frac{di_1}{dt} = \frac{V(L_2 - m)}{L_1 L_2 - m^2} \quad -(4)$$

substicute eq (4) in eq (3)

$$\frac{di_2}{dt} = \frac{V(L_1 - m)}{L_1 L_2 - m^2} \quad -(5)$$

Now add eq (4) & (5), we get

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$\frac{di}{dt} = \frac{V\left[L_1 + L_2 - 2m \right]}{L_1 L_2 - m^2}$$

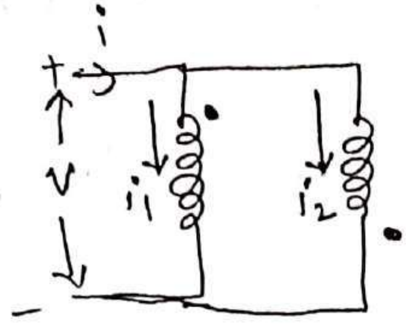
$$V = \left(\frac{L_1 L_2 - m^2}{L_1 + L_2 - 2m} \right) \frac{di}{dt}, \quad \boxed{L_{eq} = \frac{L_1 L_2 - m^2}{L_1 + L_2 - 2m}}$$

Scanned with CamScanner

(B) Parallel opposing inductors

$$V = L_{eq}\frac{di}{dt}$$

$$L_{eq} = \frac{L_1L_2 - m^2}{L_1 + L_2 + 2m}.$$



note:

Same procedure as 'A'

$$V = L_1\frac{di_1}{dt} - m\frac{di_2}{dt} \quad -(1)$$

$$V = L_2\frac{di_2}{dt} - m\frac{di_1}{dt} \quad -(2)$$

Prob 2 :

→ Two inductors whose Self inductances are of 75 mH & 55 mH respectively are connected together in parallel aiding. Their mutual inductance is given as 22.5 mH. calculate the effective inductance of the parallel combination.

sol

$$L_{effective} = \frac{L_1L_2 - m^2}{L_1 + L_2 - 2m}$$

$$= \frac{75 \times 55 - 22.5}{75 + 55 - 2 \times 22.5}$$

$$\boxed{L_{eff} = 42.6\,mH}$$

(2) magnetic coupling

- magnetic coupling occurs, when there is no ~~mutual~~ Physical connection between two coils.

→ This coupling is either aiding type or opposing type and it is based on the current enters the dotted terminals.

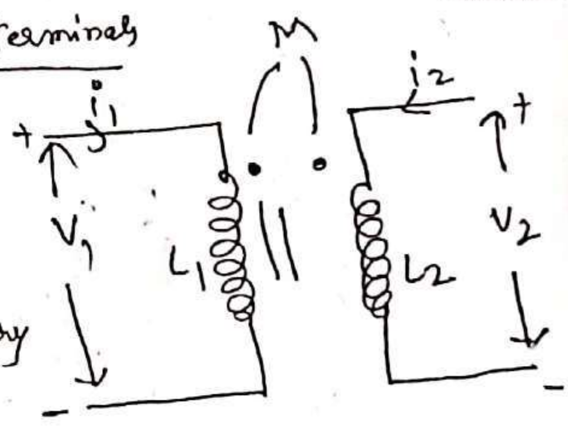
~~(A) Coupling Two~~.

(A) coupling of Aiding type [~~Two currents enter the dotted terminals~~]

(i) Two currents entering dotted terminals

→ Consider the electrical equivalent circuit of transformer.

→ It consists of two coils and these are called primary & secondary coils.



→ The currents flowing in two coils are i_1 & i_2 and both these currents enter the dotted terminal of respective coil. So ~~sign~~ sign of 'm' is +ve.

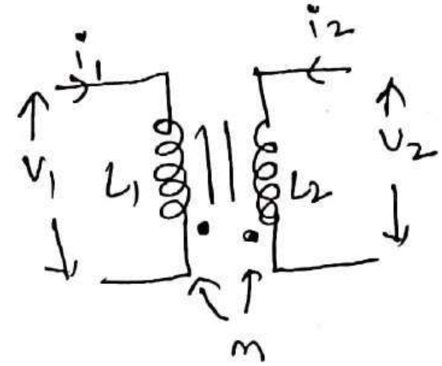
$$V_1 = L_1\frac{di_1}{dt} + m\frac{di_2}{dt}$$

$$V_2 = L_2\frac{di_2}{dt} + m\frac{di_1}{dt}$$

(ii) Two currents leaving the dotted terminals

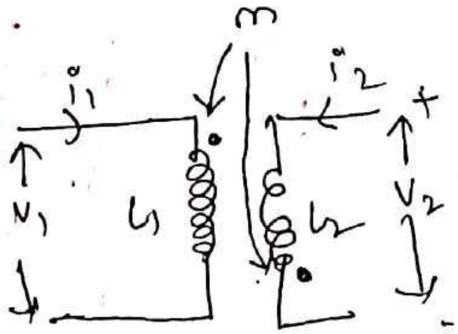
$$V_1 = L_1\frac{di_1}{dt} + m\frac{di_2}{dt}$$

$$V_2 = L_2\frac{di_2}{dt} + m\frac{di_1}{dt}$$

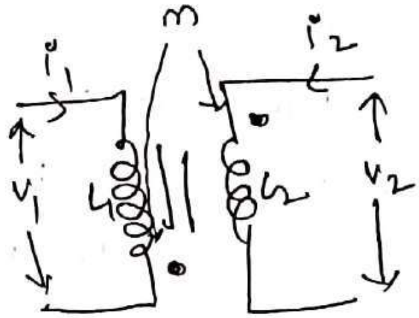


(B) <u>coupling opposing type</u>

(i) one current is entering in dotted terminal and other current is leaving the dotted terminal then sign of 'm' is −ve.



(or)



$$\boxed{\begin{aligned} V_1 &= L_1\frac{di_1}{dt} - m\frac{di_2}{dt} \\ V_2 &= L_2\frac{di_2}{dt} - m\frac{di_1}{dt} \end{aligned}}$$

coefficient of coupling (K)

→ It is defined as the ratio of mutual flux to the self flux.

→ It is denoted by symbol 'K'

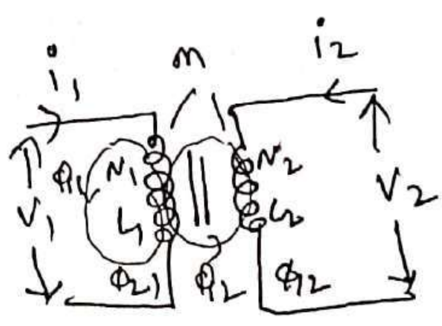
$$K = \frac{\phi_{12}}{\phi_{11}} \quad or \quad K = \frac{\phi_{21}}{\phi_{22}}$$

Properties

1. It has no units
2. If $K=0$, there is no coupling b/w two coils.
3. If $K=1$, then it is called ideal coupling.
4. The range of coefficient of coupling lies b/w 0 & 1.
5. The value of 'K' decreases when the distance b/w the coils is increases.

Derivation

Consider the two coils which are magnetically coupled as shown in fig.



For coil 1

Self inductance $L_1 = \frac{N_1 d\phi_{11}}{di_1} = \frac{N_1 \phi_{11}}{i_1}$

mutual inductance $m = \frac{N_2 d\phi_{12}}{di_1} = \frac{N_2 \phi_{12}}{i_1}$

For coil 2

self inductance $L_2 = \frac{N_2 d\phi_{22}}{di_2} = \frac{N_2 \phi_{22}}{i_2}$

mutual inductance $m = \frac{N_1 d\phi_{21}}{di_2} = \frac{N_1 \phi_{21}}{i_2}$

mutual inductance of both coils

$$m \cdot m = \frac{N_2 \Phi_{12}}{i_1} \times \frac{N_1 \Phi_{21}}{i_2} \quad -(1)$$

but we know $K = \frac{\Phi_{12}}{\Phi_{11}}$, $K = \frac{\Phi_{21}}{\Phi_{22}}$ $-(2)$

substitute eq (2) in eq (1)

$$m^2 = \frac{N_1 N_2 (K\Phi_{11})(K\Phi_{22})}{i_1 \cdot i_2}$$

$$= K^2 \left(\frac{N_1 \Phi_{11}}{i_1}\right)\left(\frac{N_2 \Phi_{22}}{i_2}\right)$$

$$m^2 = K^2 L_1 L_2$$

$$\boxed{K = \frac{M}{\sqrt{L_1 L_2}}} \quad \text{or} \quad \boxed{m = K \cdot \sqrt{L_1 L_2}}$$

Problems

→ Two inductive coupled coils have self inductance $L_1 = 50mH$ $L_2 = 200mH$. with the coefficient of coupling is 0.5

(i) Find mutual inductance.

(ii) what is the maximum possible value of m.

Sol $L_1 = 50mH$, $L_2 = 200mH$, $K = 0.5$

(i)

we know

$$K = \frac{m}{\sqrt{L_1 L_2}}$$

$$m = K \cdot \sqrt{L_1 L_2} = 0.5\sqrt{50 \times 200 \times 10^{-6}}$$

$$= 0.5 \times 0.1$$

$$= 0.05\ H$$

$$\boxed{m = 50\ mH}$$

(ii) To obtain maximum possible value of m, (put K=1)

$$m = K\sqrt{L_1 L_2}$$

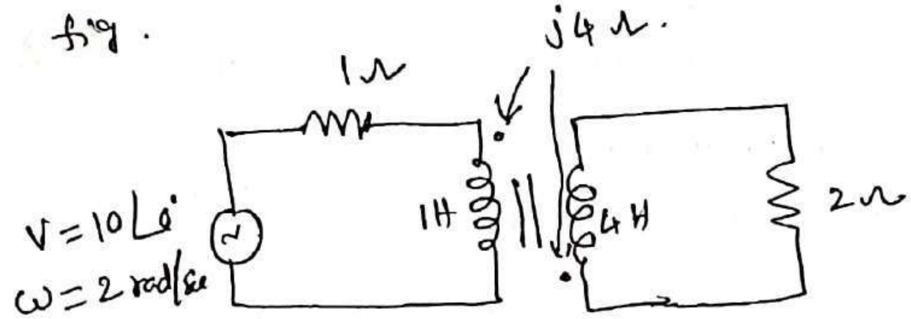
$$= 1 \times \sqrt{50 \times 10^{-3} \times 200 \times 10^{-3}}$$

$$= 0.1$$

$$\boxed{m = 100\ mH}$$

~~coefficient of coupling (K)~~

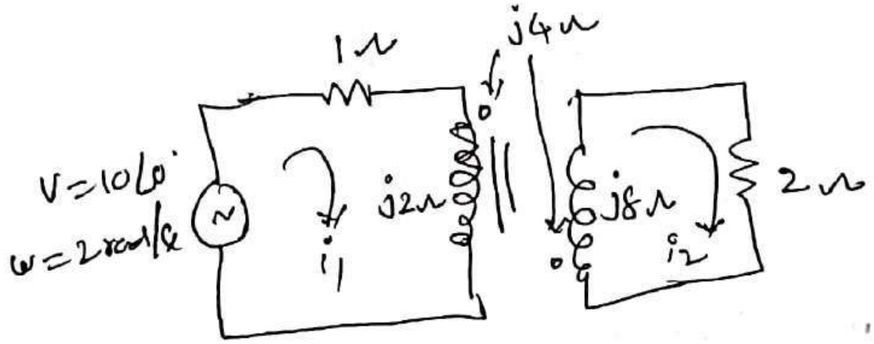
(2) Solve the mesh currents I_1 & I_2 in the circuit shown in fig.



Sol

Here $L_1 = 1H$, $L_2 = 4H$.

$X_{L_1} = \omega L_1 = 2 \times 1 = 2\,\Omega$ | $X_{L_2} = \omega L_2 = 2 \times 4 = 8\,\Omega$.



both currents are entering the dotted terminals

Apply KVL

For loop 1

$$-10 + i_1(1) + (j2)\,i_1 + (j4)\,i_2 = 0$$

$$i_1(1+j2) + i_2(j4) = 10\angle 0° \quad -(1).$$

For loop 2

$$2i_2 + (j8)\,i_2 + (j4)\,i_1 = 0$$

$$(j4)\,i_1 + i_2(2+j8) = 0 \quad -(2)$$

Use crammer rule

$$\begin{bmatrix} 10\angle 0° \\ 0 \end{bmatrix} = \begin{bmatrix} (1+2j) & 4j \\ j4 & 2+j8 \end{bmatrix} \begin{bmatrix} I_1 \\ i_2 \end{bmatrix} \quad -(3)$$

$$\Delta = (1+2j)(2+8j) - (4j)(4j)$$

$$= -14 + 12j + 16$$

$$\boxed{\Delta = 2 + j12}$$

$$\Delta_1 = \begin{bmatrix} 10\angle 0^\circ & 4j \\ 0 & 2+j8 \end{bmatrix}$$

$$= 10\angle 0^\circ \times (2+j8)$$

$$\boxed{\Delta_1 = 20 + j80}$$

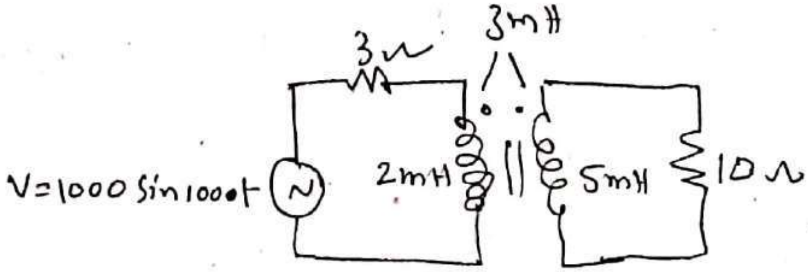
$$\Delta_2 = \begin{bmatrix} 1+2j & 10\angle 0^\circ \\ 4j & 0 \end{bmatrix}$$

$$= -40j$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{20+j8}{2+j12} = 6.75 - j0.54$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-40j}{2+j12} =$$

→ In the following coupled ckt. Determine the current supplied by the source.



Sol

$L_1 = 2mH$, $X_{L_1} = \omega L_1$

$= 100 \times 2\times10^{-3}$

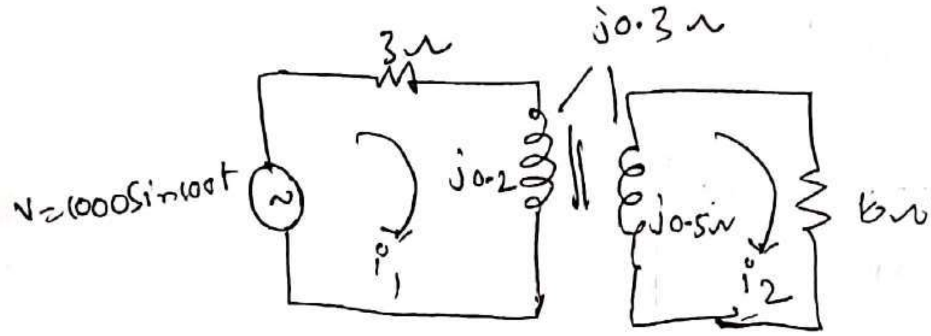
$= 0.2\,\Omega$

$L_2 = 5\,mH, \quad X_{L_2} = \omega L_2$
$$= 100 \times 5 \times 10^{-3}$$
$$= 0.5\,\Omega.$$

$$V = \frac{V_m}{\sqrt{2}} \angle \theta$$

$$= \frac{1000}{\sqrt{2}} \angle 0^\circ$$

$$\boxed{V = 707.106 \angle 0^\circ\ V}$$



Apply KVL to the ckt

for loop1

$$707.106 \angle 0^\circ = 3I_1 + I_1 (j0.2) - i_2 (j0.3)$$

$$I_1 (3 + j0.2) - j0.3\, i_2 = 707.106 \quad \text{— (1)}$$

For loop2

$$I_2 (10) + (j0.5)\, i_2 - j0.3\, i_1 = 0$$

$$-i_1 (j0.3) + (10 + j0.5)\, i_2 \quad \text{— (2)}$$

Use crammer's rule.

$$\begin{bmatrix} 707.106 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 + j0.2 & -j0.3 \\ -j0.3 & 10 + j0.5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = (3 + j0.2)(10 + j0.5) - (j0.3)(j0.3)$$

$$\boxed{\Delta = 29.9 + 3.5j}$$

$$\Delta_1 = \begin{bmatrix} 707.106 & -j0.3 \\ 0 & 10 + j0.5 \end{bmatrix}$$

$$\Delta_1 = 7071.06 + j\,353.55$$

$$\Delta_2 = \begin{bmatrix} 3 + j0.2 & 707.106 \\ -j0.3 & 0 \end{bmatrix}$$

$$= 707.106\,(j0.3)$$

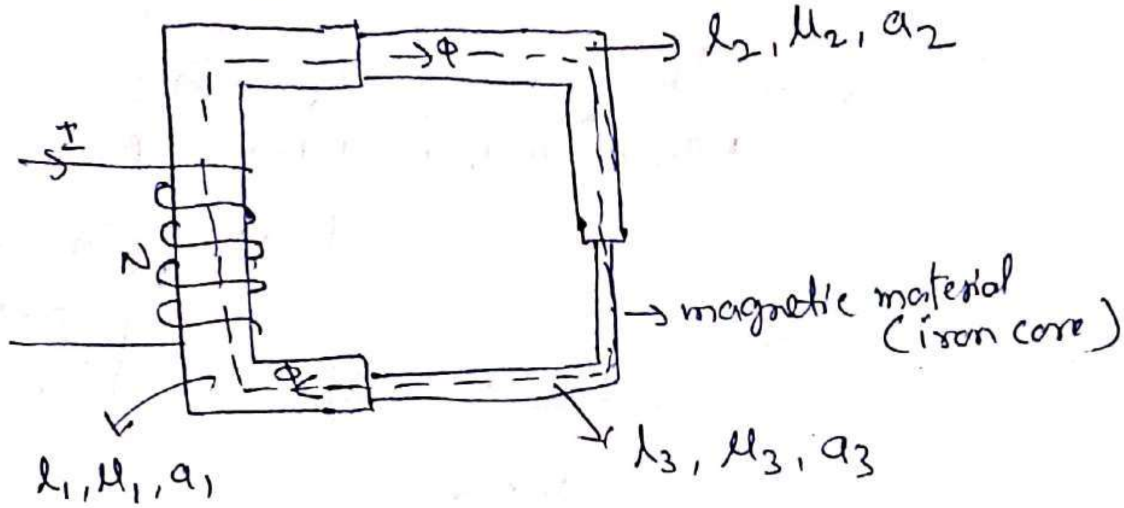
$$\Delta_2 = 212.13j$$

$$i_1 = \frac{\Delta_1}{\Delta} = 233.96 - 15.516j$$

$$i_2 = \frac{\Delta_2}{\Delta} = 0.814 + 6.978j$$

Scanned with CamScanner

Series magnetic circuit



Def :

magnetic circuit having a number of different dimensions of magnetic materials (iron core) are in series and the materials carrying the same magnetic field is called series magnetic ~~field~~ circuit.

Explanation

consider a three different dimentions of magnetic materials are connected in series which is shown in fig. current I is passed through one limb of magnetic ckt, ϕ is the flux setup in the material. In this ckt, l_1, l_2, l_3 are the length of the magnetic materials.
a_1, a_2, a_3 are the areas of three magnetic materials and μ_1, μ_2, μ_3 are the relative permeability of the three materials. l_g, a_g are lengths & area of air gap

Now, the total reluctance (S) of the magnetic circuit,

$$S = S_1 + S_2 + S_3 + S_g$$

$$S = \frac{l_1}{\mu_0 \mu_{r_1} a_1} + \frac{l_2}{\mu_0 \mu_{r_2} a_2} + \frac{l_3}{\mu_0 \mu_{r_3} a_3} + \frac{l_g}{\mu_0 a_g} \quad \text{— (1)}$$

$$\left[S = \frac{l}{\mu a} = \frac{l}{\mu_0 \mu_r a} \right.$$

$\mu_0 \rightarrow$ Permeability of free space.]

$\mu_r = 1$ for air gap.

Now,

we know

$$\text{mmF} = \text{magneto motive force} = \phi \times S$$

$$\text{mmF} = \frac{\phi l_1}{\mu_0 \mu_r a_1} + \frac{\phi l_2}{\mu_0 \mu_{r_2} a_2} + \frac{\phi l_3}{\mu_0 \mu_{r_3} a_3} + \frac{\phi l_g}{\mu_0 a_g} \quad \text{— (2)}$$

Also we know

magnetic Flux density $B = \frac{\phi}{a}$ wb/m² or Tesla. — (3)

Now, eq (2) becomes

$$\text{mmF} = \frac{B_1 l_1}{\mu_0 \mu_{r_1}} + \frac{B_2 l_2}{\mu_0 \mu_{r_2}} + \frac{B_3 l_3}{\mu_0 \mu_{r_3}} + \frac{B_g l_g}{\mu_0} \quad \text{— (4)}$$

Also

$$B = \mu H, \quad H = \frac{B}{\mu} = \frac{B}{\mu_0 \mu_r} \quad \text{— (5)}$$

sub eq(5) in eq (4), then eq (4) becomes

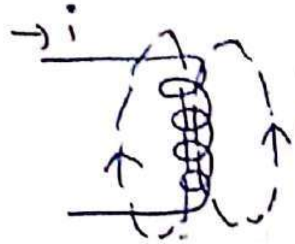
$$\boxed{\text{mmF} = H_1 \times l_1 + H_2 \times l_2 + H_3 \times l_3 + H_g \times l_g} \quad \text{— (6)}$$

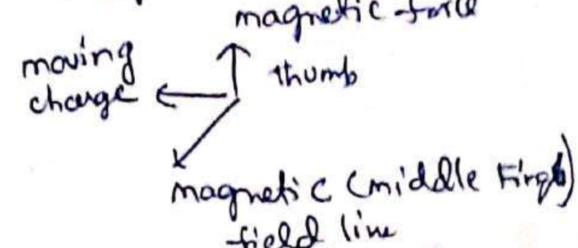
$\downarrow$

magnetic field intensity (H) $\rightarrow$ AT/m

magnetic circuit

→ Electrical current flowing along a wire creates a magnetic field arround the wire as shown in the fig.





That magnetic field can be visualized by showing lines of magnetic flux. which is represented by symbol 'Φ'

Note: Direction is determined by Right hand rule.

→ In magnetic circuit, the driving force (voltage in electrical field) is called the magneto motive force (mmF), which is designated by F.

$$mmF = Ni \; [\text{Ampere - turns}]$$

→ ohm's law of magnetic circuits is

$$F = R\Phi$$

$$R = \text{Reluctance} = \frac{l}{\mu A} \cdot (A\text{-}t/wb)$$

→

Electrical	magnetic
(1) voltage (V)	(1) mmF (F) = Ni
(2) current (i)	(2) magnetic flux (Φ)
(3) Resistance (R)	(3) Reluctance (R)
(4) conductivity $\left(\frac{1}{\rho}\right)$	(4) Permeability (μ)
(5) current density (J)	(5) magnetic flux density (B)
(6) Electric Field (E)	(6) magnetic field intensity (H)
(7) path is traced by the current is called electrical Field.	(7) path traced by the magnetic flux.
(8) EMF	(8) mmF
(9) Flow of electrons is called current	(9) The no. of magnetic lines of force
(10) Resistance opposes the flow of the current	(10) Reluctance is opposed by magnetic path to the flux

Problem on Series magnetic ckt

(1) An iron ring has a cross section area $2 cm^2$ and a mean diameter of 20cm. An airgap of 0.4 mm has been cut across the section of the ring. The ring is wound with a coil of 300 turn. The total magnetic flux is 0.20 mwb. The relative permeability of iron is 1000. Find the value of current passed in turn.

Sol

Given

$$a = 2cm^2 = 2\times10^{-4}\ m^2$$

$$D_m = 20cm = 20\times10^{-2}\ m$$

$$l_T = 2\pi r = \pi D_m = \pi\times 20\times10^{-2}$$

$$l_T = 0.628\ m, \quad l_g = 0.4\ mm = 0.4\times10^{-3}\ m$$

$$N = 300$$

$$\phi = 0.2\ mwb = 0.2\times10^{-3}\ wb.$$

$$\mu_{ri} = 1000, \quad I = ?$$

$$(mmF)_T = (mmF)_i + (mmF)_g$$

$$N\cdot I = H_i\times l_i + H_g\times l_g \quad -(1)$$

we know, $B = \mu H$, $H = \frac{B}{\mu} = \frac{B}{\mu_0\mu_r}$

for gap or air $\mu_r = 1$

$$N\cdot I = \frac{B}{\mu_0\mu_{ri}}(l_T - l_g) + \frac{B}{\mu_0} l_g \quad \left(\begin{matrix}\text{for gap}\\ \mu_r = 1\end{matrix}\right)$$

also we know, $B = \frac{\phi}{a}$

$$N\cdot I = \frac{\phi}{\mu_0\mu_{ri}a}(l_T - l_g) + \frac{\phi}{\mu_0 a} l_g$$

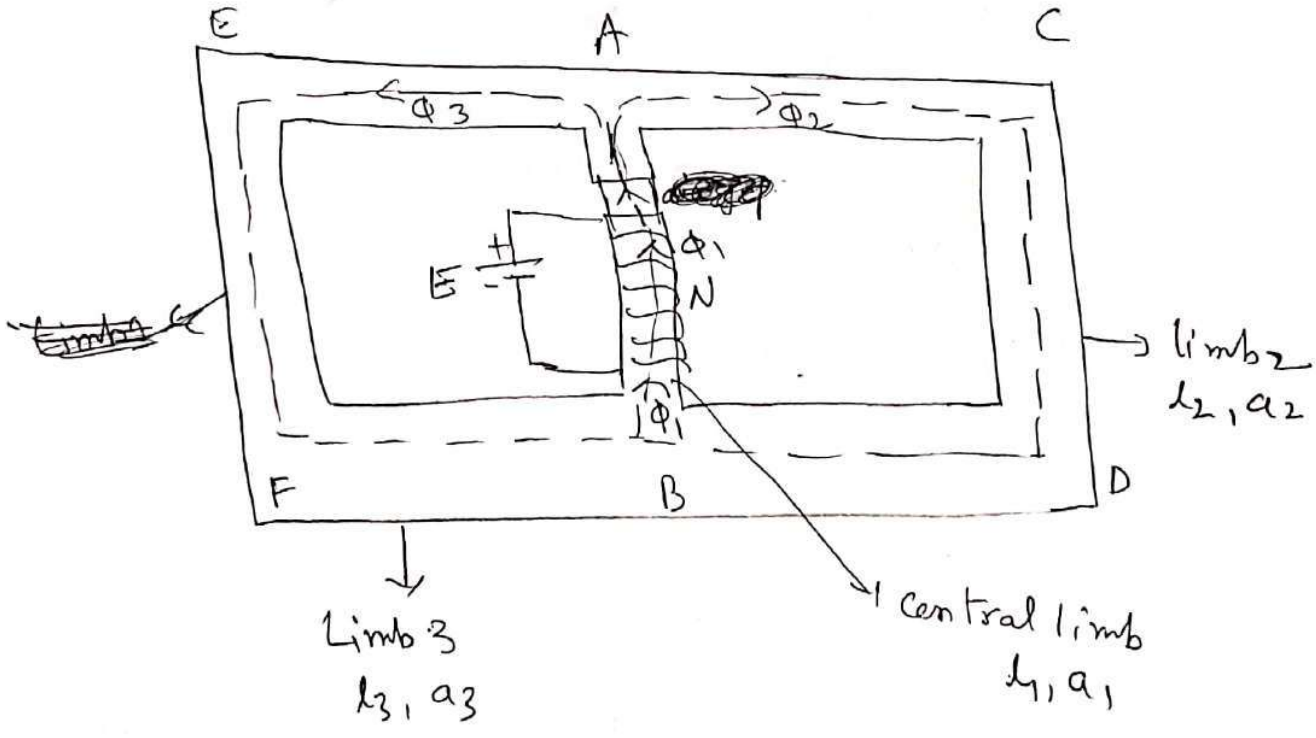
$$300 \times I = \frac{0.2\times10^{-3}\,(0.628-0.4\times10^{-3})}{4\pi\times10^{-7}\times1000\times2\times10^{-4}} + \frac{0.2\times10^{-3}}{4\pi\times10^{-7}\times2\times10^{-4}}$$

$$\boxed{I = \frac{817.73}{300} = 2.72\,A}$$

Parallel magnetic circuit

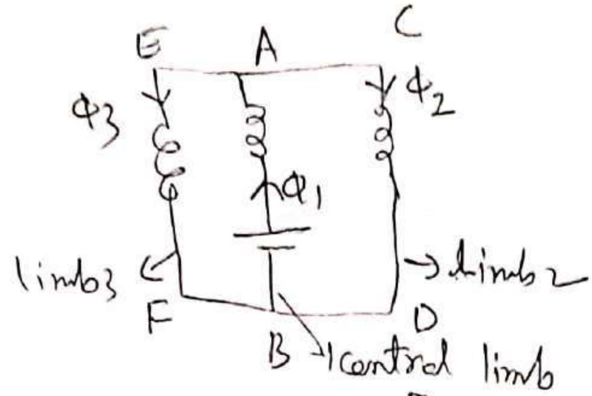
Def: A magnetic circuit having two or more than two paths for the magnetic flux is called parallel magnetic circuit.

- The parallel magnetic circuit contains different dimensional areas and materials having various number of paths.



In the above ckt, the current carrying coil is wound on the central limb. This coil sets up the magnetic flux ϕ_1 in the central limb.

This flux ϕ_1 is devided into two fluxes i.e ϕ_2 & ϕ_3. $\therefore \phi_1 = \phi_2 + \phi_3$.



~~Total MMF = MMF of~~

Reluctance of AB path $(S_{AB}) = \dfrac{l_1}{\mu_0 \mu_{r_1} a_1}$

" ABCD $(S_{ABCD}) = \dfrac{l_2}{\mu_0 \mu_{r_2} a_2}$

" AB FE $(S_{ABFE}) = \dfrac{l_3}{\mu_0 \mu_{r_3} a_3}$

Total mmF required = mmF path AB + (mmF path AB DC) or (mmF path ABFE)

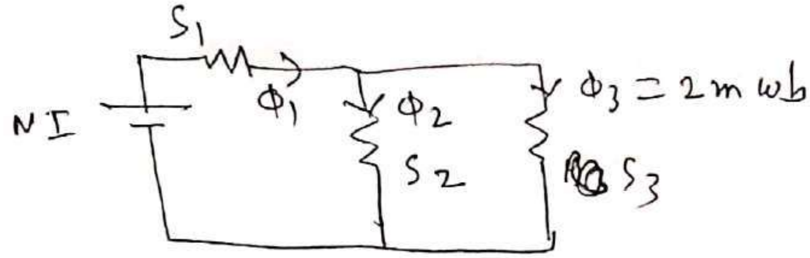
$$\boxed{MMF_{Total} = \phi_1 S_{AB} + (\phi_2 S_{ABCD} \text{ or } \phi_3 S_{ABFE})}$$

where $\phi_2 S_{ABCD} = \phi_3 S_{ABFE}$

Problems

(1) A cast steel magnetic structure made for a bar of section 2cm x 2cm. Determine the current that a 500 turn coil on the left limb should carry so that a flux of 2 mwb is produced in the right limb. Take $\mu_r = 600$. Neglect leakage.

$A = 2 \times 2cm$
$= 4cm^2$
$= 4 \times 10^{-4} m^2$



$$MMF = NI = S\phi$$

$$= \frac{l}{\mu A}\phi = \frac{l}{\mu_0 \mu_r A}\phi$$

$$\frac{S_2}{S_3} = \frac{15}{25}$$

$$\phi_3 = 2\,mwb$$

$$\phi_2 = 2 \times \frac{5}{3} = \frac{10}{3} = 3.33\ mwb$$

$$\phi_1 = \phi_2 + \phi_3$$

$$= 2 + 3.33$$

$$\boxed{\phi_1 = 5.33\ mwb}$$

$$MMF_{left} = AT_{left} = \phi_1 S_1$$

$$= \frac{5.33 \times 10^{-3} \times 25 \times 10^{-2}}{4\pi \times 10^{-7} \times 600 \times 4 \times 10^{-4}}$$

$$= 4420\ AT.$$

$$MMF_{right} = AT_{Right} = \phi_3 S_3$$

$$= \frac{2 \times 10^{-3} \times 25 \times 10^{-2}}{4\pi \times 10^{-7} \times 600 \times 4 \times 10^{-4}}$$

$$= 1658\ AT$$

$$AT_{total} = 4420 + 1658$$

$$NI_{total} = 6078\ AT.$$

If $N = 500$

$$I = \frac{6078}{500} = 12.15\,Amp$$

Unit-4

BEE Unit-2

[redacted]

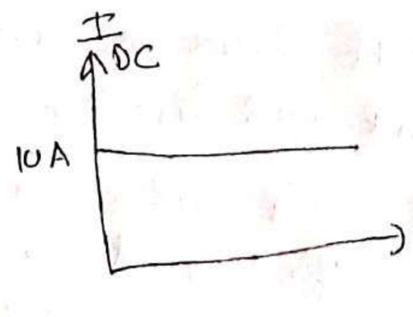
Single phase AC circuits

RMS, Average values & form factor for Different periodic wave forms – sinusoidal alternating quantities – phase & phase difference – complex & polar form of representations, j-notation, steady state analysis of R, L and C (In Series, Parallel & Series-parallel combination) with Sinusoidal Excitations – phasor diagrams – concept of power factor – concept of reactance, Impedance, Susceptance & admittance – Apparant power, active & reactive power, Examples.

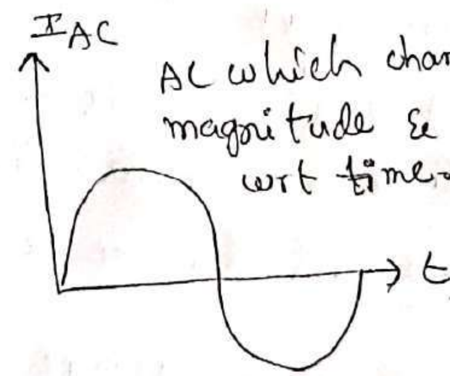
Introduction

DC : Direct current which is constant w.r.t to time.

AC : Alternating current which changes polarity or direction wrt time.



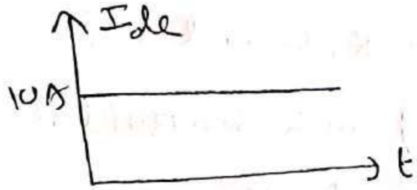
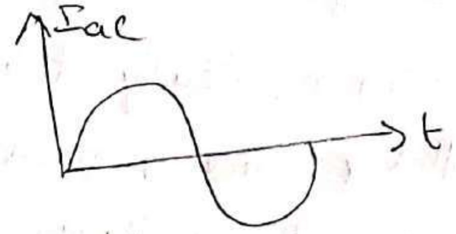
DC



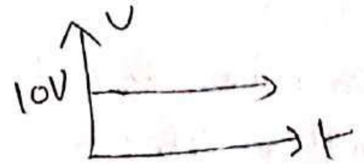
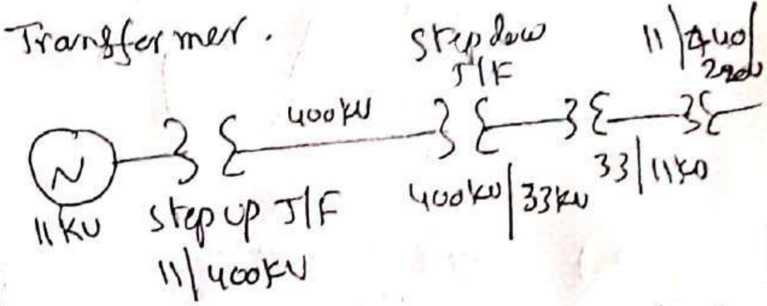
AC which changes in magnitude & direction wrt time.

There are several differences b/w AC & DC

Comparision

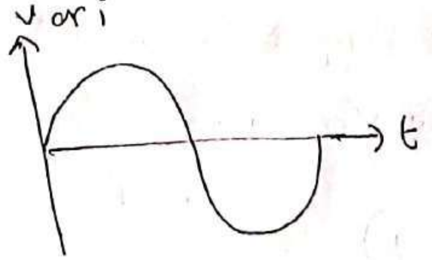
DC	AC
(1) DC Voltage can not travel very far	(1) AC voltage can travel very far distances with safety.
(2) DC current is a constant wrt time	(2) AC current is not constant at any time wrt time.
	
(3) Frequency is zero for DC	(4) Frequeny is 50Hz for Ac
(4) DC contains only Resistance (R)	(4) AC contains R, L, C i-e Impedance (Z)
(5) DC supply obtained from cell or Battery	(5) AC supply is obtained from AC Generators.
(6) DC flows only one direction in the ckt	(6) AC flows in two directions
(7) Flow of electrons in one direction (forward direction)	(7) It flows in Forward & backward direction.
(8) Power factor for DC is 1	(8) power factor for AC always lies b/w 0 & 1.
(9) DC types : (i) pure (ii) pulsating	(9) It is Sinusoidal, Trapezoidal, Traingular, Square and etc
(10) In DC, Induction is not possible	(10) In AC, Induction is possible.
(11) can not be stepped up or down	(11) step up or step down is possible with the help of T/F

Advantages of AC over DC

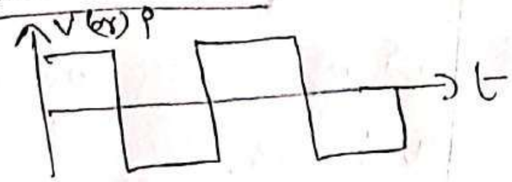
DC	AC
(1) In DC, voltage can not be stepped up or down so it is constant 	(1) AC voltage can be stepped up or stepped down with the help of Transformer. 
(2) In DC, we can obtain const losses	(2) In AC, If we go for higher voltages, currents are less & hence losses (I^2R) are less & improves the transmission efficiency.
(3) High speed DC Generators are not possible	(3) High speed AC Generators are possible & hence cost of Generators are less.
(4) DC motors are not Simple in construction & it occupies more ~~space~~ maintance.	(4) AC electric motors are Simple in construction, cheaper & requires less maintance.
(5) DC is not easy to generate	(5) AC is easy to generate
(6) DC is not cheaper because it requires rectifiers	(6) AC is cheaper.
(7) DC Generators has low efficiency	(7) AC Generators has higher efficiency.

Types of AC waveforms

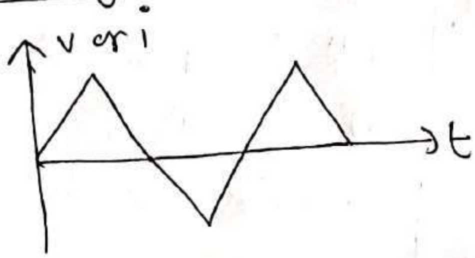
(1) Sinusoidal wave-forms



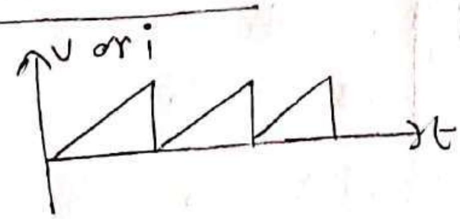
(2) Square wave



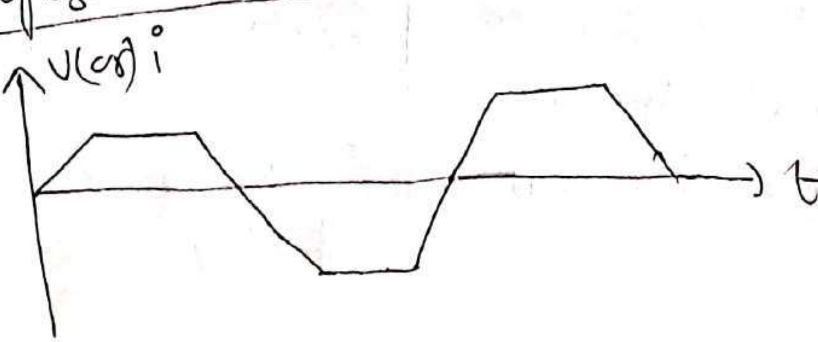
(3) Traingular wave



(4) Sawtooth wave



(5) Trapezoidal wave



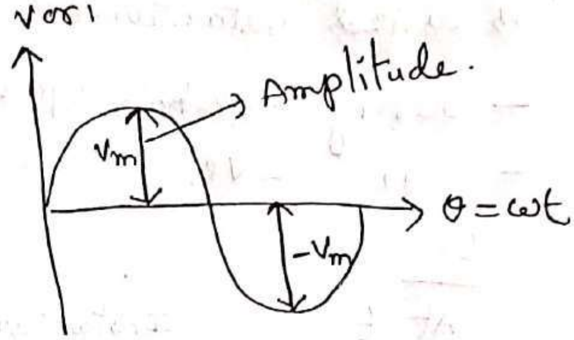
Among five Sinwave is the better. Reasons

1. Any periodic wave can be written in Sinusoidal function.
2. Sin wave integration & ~~division~~ derivation is also a sinwave.
3. It is easy to generate
4. It is easy to analyse
5. It is mostly used in power industry.

Basic Definitions

① **Amplitude [Peak value]**

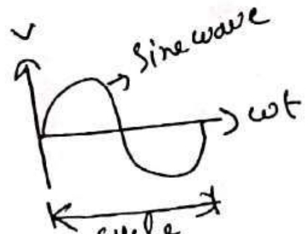
The maximum value of alternating quantity during positive or negative half cycle is called as Amplitude.

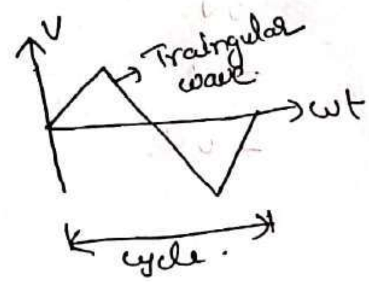


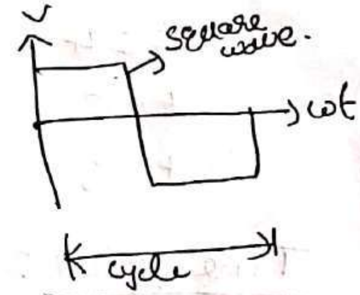
- From the fig V_m – Amplitude

② **cycle**: one complete set of positive & negative value of alternating quantity is known as cycle.

Eg:







③ **Voltage equation**

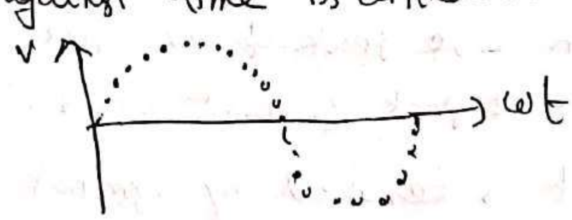
Voltage equation for sine wave is

$$v(t) = V_m \sin\omega t$$

⇒ here $v(t)$ → any point in the sine wave represents instantaneous value

④ **wave-form**

The wave-form is obtained by plotting the instantaneous values of voltage against time is called waveform

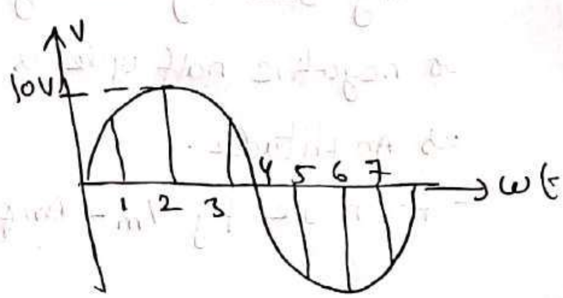


Instantaneous value:

The magnitude of waveform at any instant of time is called instantaneous value.

- During +ve half cycle, instantaneous values are positive
- " −ve " " " " " negative

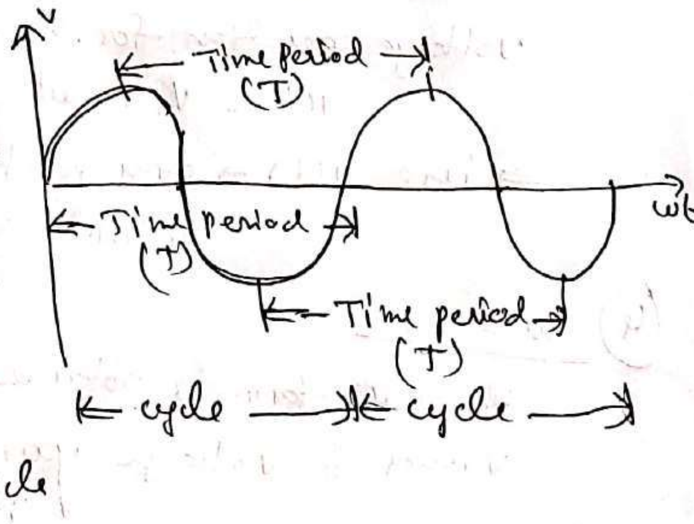
Eg:



At t	Instantaneous values (V(t))
t = 1	7V
t = 2	10V
t = 3	6V
t = 4	0V
t = 5	−7V
t = 6	−10V

Time period:

Time taken by the alternating quantity to complete one cycle is called Time period.



- From zero crossing of one cycle to zero crossing of next cycle
- From positive peak of one cycle to positive peak of next cycle
- From −ve peak of one cycle to −ve peak of next cycle.

→ It is denoted by symbol 'T'

Frequency :

The no. of cycles per second is called frequency.

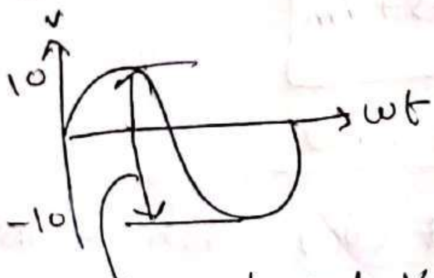
$$f = \frac{1}{T} \text{ Hz}$$

Angular frequency (ω)

$$\omega = 2\pi f \text{ rad/sec.}$$

Peak-to-peak value

The peak to peak value of a sine wave is the peak from +ve to −ve peak.



peak to peak value. $10 - (-10) = 20V$

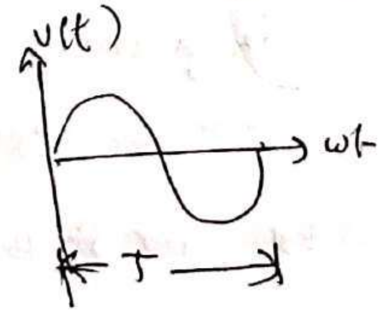
Average value:

It is defined as the "Total area of the waveform divided by the distance of waveform.

$$\text{Average} = \frac{\text{Area of waveform}}{\text{Distance of waveform}}.$$

From the fig,

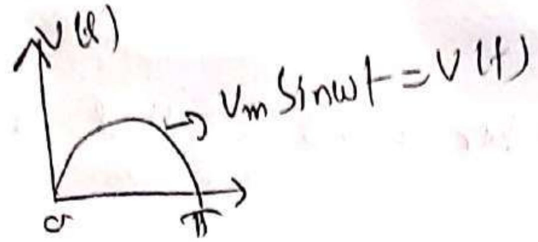
$$V_{avg} = \frac{\int_0^T v(t)\,dt}{T}$$



Note: (1) Average values of Symmetrical waveform is always zero.

(2) Average value of one full sine wave is zero.

Eg:



$$V_{avg} = \frac{\int_0^{\pi} V_m \sin\omega t \, d(\omega t)}{\pi}$$

$$= \frac{1}{\pi} V_m \cdot \left(-\cos\omega t\right)_0^{\pi}$$

$$= \frac{V_m}{\pi}(1+1) = \frac{2V_m}{\pi}$$

$$\boxed{V_{ag} = 0.637 V_m}$$

RMS value (or) effective value

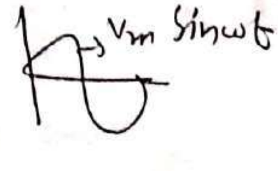
The steady current (DC) which, when flows through a resistor for a given period of time as a result same quantity of heat is produced by the AC when flows through the same resistor for the same period of time is called effective RMS value of ac.

- RMS value of any function v(t) with a period of T is given by

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^{T}(v(t))^2 dt}.$$

Ex: For sinusoidal function, find V_{rms}

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^{T}(V_m \sin\omega t)^2 d(\omega t)}$$



Scanned with CamScanner

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi}\left(\frac{1-\cos 2\omega t}{2}\right) d(\omega t)}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left[\omega t + \frac{\sin(2\omega t)}{2}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{V_m^2}{4\pi}(2\pi - 0)}$$

$$= \sqrt{\frac{V_m^2}{2}}$$

$$= \frac{V_m}{\sqrt{2}}$$

$$\boxed{V_{rms} = 0.707\, V_m}$$

Form factor

It is defined as the "ratio of Rms value to the average value of the wave".

$$\text{Form factor } K_f = \frac{\text{Rms value}}{\text{Average value}}.$$

Eg: For Sinusoidal function

$$\text{Form factor} = \frac{V_m/\sqrt{2}}{\frac{2V_m}{\pi}} = 1.11$$

Peak factor or Amplitude factor or Crest factor

It is defined as the "ratio of maximum value to rms value"

$$\text{Crest factor} = \frac{\text{maximum value}}{\text{Rms value}}$$

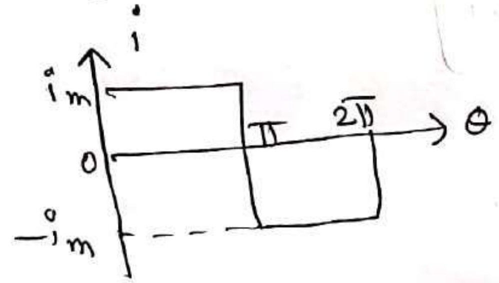
Ex: For sinusoidal functions

$$\text{crest factor} = K_p = \frac{V_m}{\left(\frac{V_m}{\sqrt{2}}\right)}$$

$$\boxed{K_p = 1.414}$$

Problems

(1) Find the form factor & peak factor of the square wave

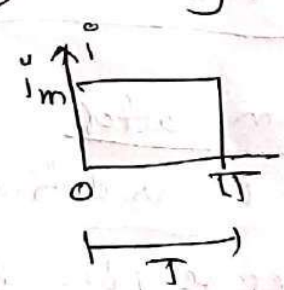


$\cancel{Avg = RMS}$

$$Avg = \frac{\int_0^T v(t)\,dt}{T}$$

Sol

$$I_{avg} = \frac{1}{\pi}\int_0^{\pi} i\,d\theta \qquad [T = \pi]$$



$0 < \theta < \pi$

$$= \frac{1}{\pi}\int_0^{\pi} i_m\,d\theta$$

$$= \frac{i_m}{\pi}\left[\theta\right]_0^{\pi}$$

$$= \frac{i_m}{\pi}\left[\pi\right] = \boxed{i_m}$$

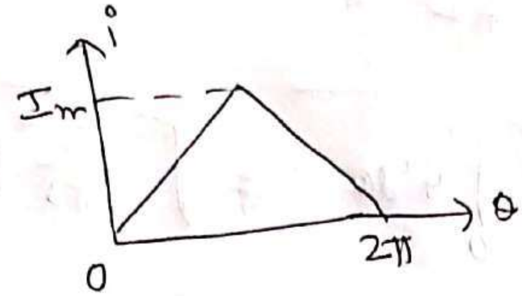
$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i_m^2\,d\theta}$$

$$= \sqrt{\frac{1}{\pi}\, i_m^2\,(\pi - 0)} = \sqrt{i_m^2} = i_m$$

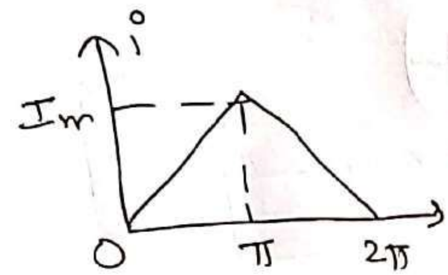
$$\text{Form factor} = \frac{I_{rms}}{I_{avg}} = \frac{i_m}{i_m} = 1$$

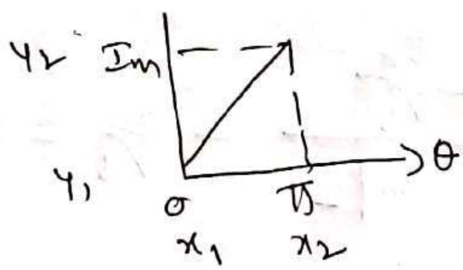
$$\text{Peak factor} = \frac{I_m}{I_{rms}} = \frac{i_m}{i_m} = 1$$

(2) Find Form-factor & Kp (Kf)



Sol





$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}\ \theta$$

$$i = \frac{I_m}{\pi}\theta.$$

↓ instantaneous current

$$I_{avg} = \frac{1}{T}\int_0^T i\,d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} \frac{I_m}{\pi}\theta\,d\theta$$

$$= \frac{I_m}{\pi^2}\left(\frac{\theta^2}{2}\right)_0^{\pi}$$

$$= \frac{I_m}{\cancel{\pi^2}}\frac{\cancel{\pi^2}}{2}$$

$$\boxed{I_{avg} = \frac{I_m}{2}}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T \frac{I_m^2 \theta^2}{\pi^2}\, d\theta}$$

$$= \sqrt{\frac{I_m^2}{\pi^3}\int_0^{\pi} \theta^2 d\theta} = \sqrt{\frac{I_m^2}{\pi^3}\left(\frac{\theta^3}{3}\right)_0^{\pi}}$$

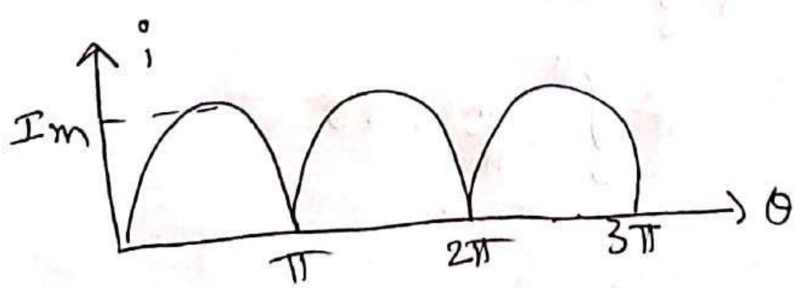
$$= \sqrt{\frac{I_m^2}{\pi^3}\left(\frac{\pi^3}{3} - 0\right)}$$

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{3}}}$$

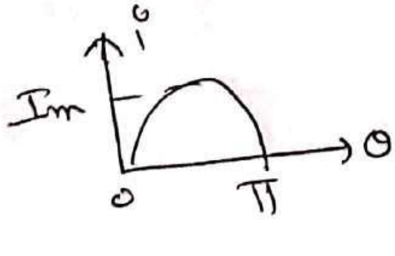
* $K_f = \frac{I_{rms}}{I_{avg}} = \frac{I_m/\sqrt{3}}{I_m/2} = \frac{2}{\sqrt{3}} = 1.155$

$K_p = \frac{I_m}{I_{rms}} = \frac{I_m}{I_m/\sqrt{3}} = \sqrt{3} = 1.732$

(3)



Find Kp & kf.



$i = I_m \sin\omega t$

$\boxed{i = I_m \sin\theta}$

$$I_{avg} = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \, d\theta$$

$$= \frac{I_m}{\pi}\left(-\cos\theta\right)_0^{\pi}$$

$$= \frac{2I_m}{\pi} = 0.637 I_m$$

$$I_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i^2 \, d\theta}$$

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta}.$$

$$= \sqrt{\frac{I_m^2}{\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta}.$$

$$= \sqrt{\frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\left[\pi - \frac{\sin 2\pi}{2}\right]}$$

$$= \frac{\sqrt{I_m^2}}{\sqrt{2}}$$

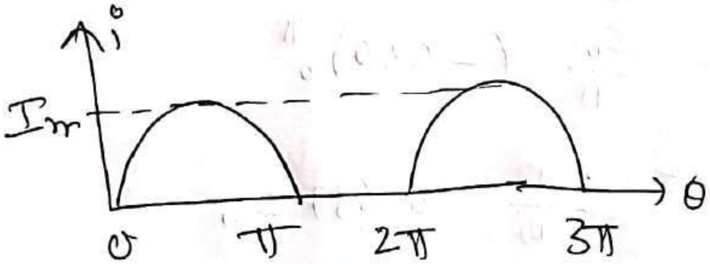
$$= \frac{I_m}{\sqrt{2}}$$

$$= 0.707 I_m$$

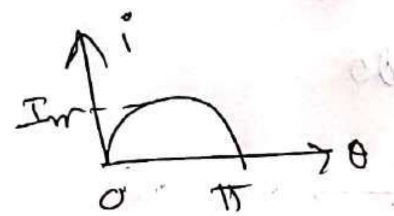
$K_f = 1.11$

$K_p = 1.414$

(4) Find K_f & K_p



Sol



This half wave is an un symmetrical waveform

$$I_{avg} = \frac{1}{T}\int_0^T i\,d\theta$$

$T = 2\pi$

wave $= \pi$

$$= \frac{1}{T}\int_0^{\pi} I_m \sin\theta\,d\theta$$

$$= \frac{I_m}{T}\left(-\cos\theta\right)_0^{\pi}$$

$$= \frac{2I_m}{T}$$

Un symmetry $I_{avg} = \frac{2I_m}{2\pi} = \frac{I_m}{\pi} = 0.318\,I_m.$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi} I_m^2 \sin^2\theta\,d\theta\right]}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\left(\frac{\theta - \sin 2\theta}{2}\right)_0^{\pi}}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left(\pi - \sin 2\pi\right)}$$

$$I_{rms} = \sqrt{\frac{I_m^2}{4\pi} \times \pi}$$

$$= \sqrt{\frac{I_m^2}{4}}$$

$$= \frac{I_m}{2}$$

$$= 0.5 I_m$$

$$K_f \text{ (Form factor)} = \frac{I_{rms}}{I_{avg}}$$

$$= \frac{0.5 I_m}{0.318 I_m}$$

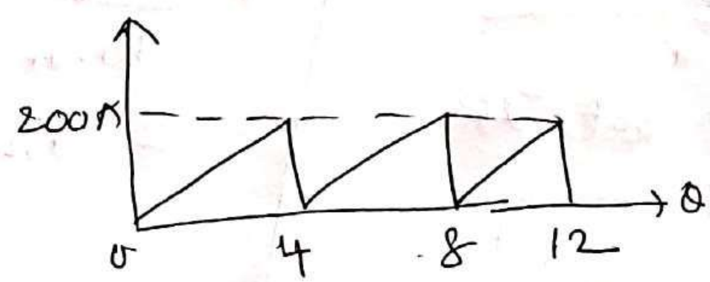
$$\boxed{K_f = 1.572}$$

$$K_p \text{ (Peak factor)} = \frac{I_m}{I_{rms}}$$

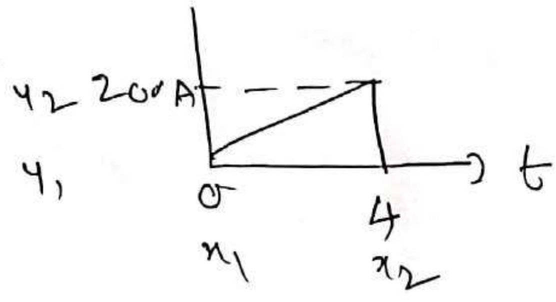
$$= \frac{I_m}{0.5 I_m}$$

$$\boxed{K_p = 2}$$

(5) Find K_f & K_p



Sol



$$i = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} t$$

$$= \frac{200}{4} t$$

$$\boxed{i = 50t}$$

(i) $I_{avg} = \frac{1}{T} \int_0^T i(t)\,dt$

$$= \frac{1}{4} \int_0^4 50t\,dt$$

$$= \frac{50}{4} \left(\frac{t^2}{2}\right)_0^4$$

$$= \frac{50}{4} \left(\frac{16}{2}\right)$$

$$I_{avg} = 100 \text{ A.}$$

(ii) $I_{rms} = \sqrt{\frac{1}{T} \int_0^T i^2\,dt}$

$$= \sqrt{\frac{1}{4} \int_0^4 (50t)^2\,dt}$$

$$= \sqrt{\frac{2500}{4} \left(\frac{t^3}{3}\right)_0^4}$$

$$= \sqrt{13333.33} = 115.47 \text{ A.}$$

$K_f = 1.155$

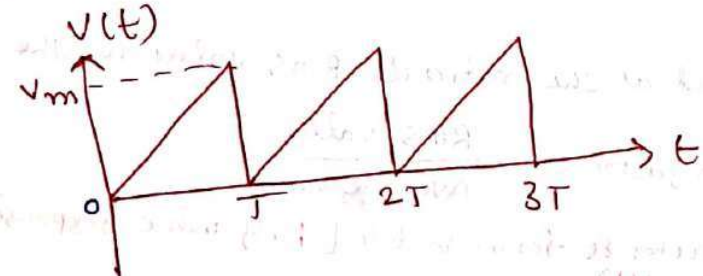
$$K_p = \frac{200}{115.47}$$

$$= 1.732$$

$$\text{Peak factor} = \frac{\text{Peak Value}}{\text{RMS Value}} = 2$$

$$\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}} = \pi/2$$

→ Find the peak factor & form factor for the waveform given below



Sol here

$$v(t) = \frac{V_m}{T} t \quad , \quad 0 \le t \le T$$

(i) $$V_{avg} = \frac{\int_0^T v(t)\,dt}{T} = \frac{\int_0^T \frac{V_m}{T} t \,dt}{T} = \frac{V_m}{T^2}\left(\frac{t^2}{2}\right)_0^T$$

$$= \frac{V_m}{2T^2}\left(T^2 - 0\right) = \frac{V_m}{2}$$

(ii) $$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{V_m}{T} t\right)^2 dt}$$

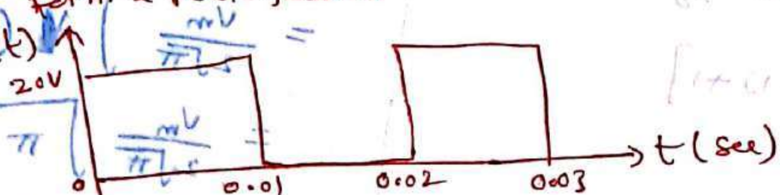
$$= \sqrt{\frac{V_m^2}{T^3}\int_0^T t^2\,dt} = \sqrt{\frac{V_m^2}{T^3} \times \left(\frac{t^3}{3}\right)_0^T}$$

$$= \sqrt{\frac{V_m^2}{T^3} \times \frac{T^3}{3}} = \frac{V_m}{\sqrt{3}}$$

$$\therefore \text{Peak factor} = \frac{\text{Peak value}}{\text{RMS value}} = \frac{V_m}{(V_m/\sqrt{3})} = \sqrt{3}$$

$$\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}} = \frac{\frac{V_m}{\sqrt{3}}}{\frac{V_m}{2}} = \frac{2}{\sqrt{3}}$$

→ Find form & peak factor



Sol $v(t) = 20V, \quad 0 \le t \le 0.01$
$= 0V, \quad 0.01 \le t \le 0.02$

(i) $V_{avg} = \dfrac{\int_0^{0.02} v(t)dt}{T} = \dfrac{\int_0^{0.01} 20\, dt}{0.02} = \dfrac{20(0.01-0)}{0.02}$
$= 10$

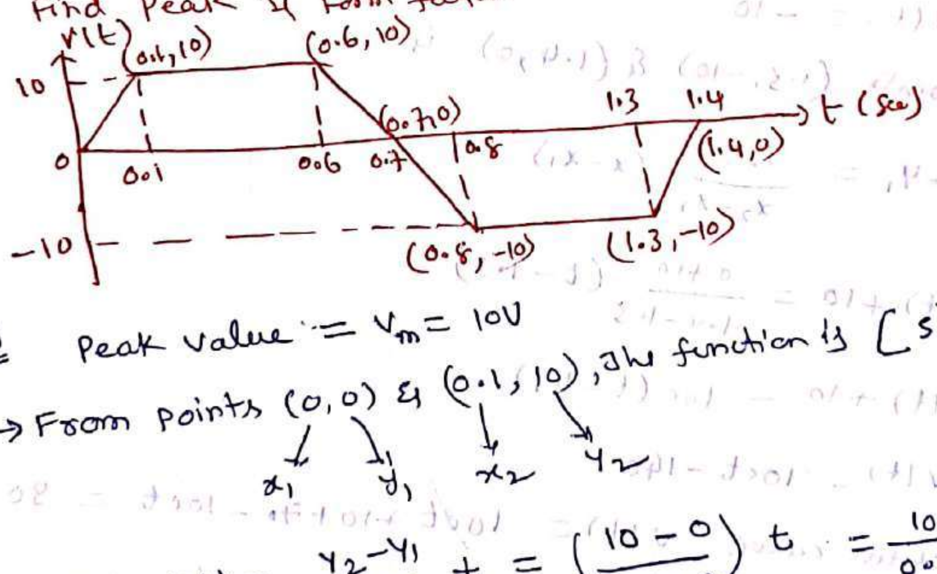
(ii) $V_{Rms} = \sqrt{\int_0^{0.02} \dfrac{(v(t))^2}{T} dt} = \sqrt{\int_0^{0.01} \dfrac{(20)^2}{T} dt} = \sqrt{\dfrac{(20)^2}{T} \times (0.01-0)}$

$= \sqrt{\dfrac{(20)^2}{\cancel{0.02}_{2}}(\cancel{0.01})} = \sqrt{\dfrac{400}{2}} = \sqrt{200} = \dfrac{20}{\sqrt{2}}$

$\therefore$ Peak factor $= \dfrac{\text{Peak value}}{\text{Rms value}} = \dfrac{20}{20/\sqrt{2}} = \sqrt{2}$

$\therefore$ Form factor $= \dfrac{\text{Rms value}}{\text{Average value}} = \dfrac{20/\sqrt{2}}{10} = \sqrt{2}$

→ Find Peak & Form factor.



Sol Peak value $= V_m = 10V$, The function is [slope $\Rightarrow y = mx$]

→ From points $(0,0)$ & $(0.1, 10)$
$x_1 \; y_1 \; x_2 \; y_2$

$v(t) = \dfrac{y_2 - y_1}{x_2 - x_1} t = \left(\dfrac{10-0}{0.1-0}\right) t = \dfrac{10}{0.1} t = 100t$

→ From points $(0.1, 10)$ & $(0.6, 10)$ [Rectangle $= 10$]

$v(t) = 10$

→ From Points $(\underset{x_1}{0.6}, \underset{y_1}{10})$ & $(\underset{x_2}{0.7}, \underset{y_2}{0})$ [towards x-axis]

$v(t) \Rightarrow \dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$

here $y = v(t)$, $x = t$

$$\frac{v(t)-10}{t-0.6} = \frac{0-10}{0.7-0.6}$$

$$v(t)-10 = -100\,(t-0.6)$$

$$= -100t+60$$

$$v(t) = 70-100t$$

→ From points $(0.7, 0)$ & $(0.8, -10)$ (downwards)
$\quad x_1 \; y_1 \quad x_2 \; y_2$

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$v(t)-0 = \frac{-10-0}{0.8-0.7}(t-0.7)$$

$$v(t) = -100\,(t-0.7)$$

$$= 70-100t$$

→ From the points $(0.8, -10)$ to $(1.3, -10)$ [Rectangle]

$$v(t) = -10$$

→ For the points $(1.3, -10)$ & $(1.4, 0)$ is

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$v(t)+10 = \frac{0+10}{1.4-1.3}(t-1.3)$$

$$v(t)+10 = 100\,(t-1.3)$$

$$v(t) = 100t-140$$

∴ For the positive curve $v(t) = 10\cancel{0t}+10+70-1\cancel{00t} = 80$

∴ For the Negative curve $v(t) = 70-1\cancel{00t}-10+1\cancel{00t}-140 = -80$

The function is

$$v(t) = \begin{cases} 80, & 0 \le t \le 0.7 \\ -80, & 0.7 \le t \le 1.4 \end{cases}$$

(i) Average value is

$$V_{avg} = \frac{\int_0^{1.4} v(t)\,dt}{1.4}$$

$$= \frac{1}{1.4}\left[\int_0^{0.7} 80\,dt + \int_{0.7}^{1.4} -80\,dt\right]$$

$$= \frac{1}{1.4}\left[80(0.7) + (-80)[1.4-0.7]\right]$$

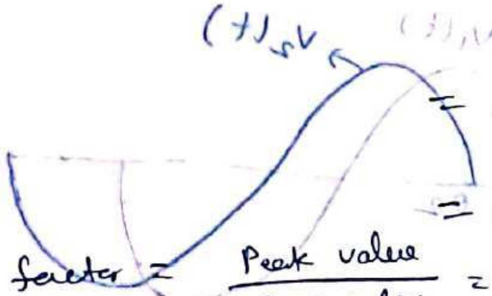
$$= 0$$

$\therefore$ <u>For half wave</u>

$$V_{ag} = \frac{1}{0.7}\int_0^{0.7} 80\,dt = \frac{80(0.7)}{0.7} = 80$$

(ii) $$V_{Rms} = \sqrt{\frac{1}{1.4}\int_0^{1.4}(v(t))^2\,dt} = \sqrt{\frac{1}{1.4}\left[\int_0^{0.7}(80)^2 dt + \int_{0.7}^{1.4}(-80)^2 dt\right]}$$

$$= \frac{80}{\sqrt{1.4}}\left[\sqrt{(0.7) + (1.4-0.7)}\right]$$



$$= \frac{80}{\sqrt{1.4}} \times \sqrt{0.7 + 0.7}$$

$$= 80$$

$\therefore$ Peak factor $= \frac{\text{Peak value}}{\text{Rms value}} = \frac{10}{80} = 0.125$

$\therefore$ Form factor $= \frac{\text{Rms value}}{\text{Average value}} = \frac{80}{80} = 1$

$$V_{eq} = \bar{V_1} + \bar{V_2} + \bar{V_3}$$
$$= 20 + j0 + 21.21 - j21.21 - 20 + j34.64$$
$$= 21.21 + j13.42$$
$$= 25.10 \angle 32.33^\circ \text{ volts.}$$

<u>Note:</u> If $V = V_m \sin(\omega t \pm \phi)$ Then In polar form $V = V_m \angle \pm\phi$ Then in Rectangular form $V = V_m(\cos\phi + j\sin\ldots$

(4) The equation of alternating quantity $i = 200 \sin 318t$. Deter
(a) maximum value (b) Frequency (c) Rms value (d) Average value
(e) Time period (f) peak factor (g) Form factor.

<u>Sol</u> Given
$$i = 200 \sin 318t. = 200 \sin \omega t.$$

(a) maximum value (I_m) = 200 A.

(b) $\omega = 2\pi f$.

From The problem $\omega = 318$

$$318 = 2 \cdot \pi f$$
$$f = \frac{318}{2\pi} = 50.61 \text{ Hz}$$

(c) Rms value (I_{rms}) $= \frac{I_m}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 141.42$ A.

(d) Average value (I_{avg}) $= \frac{2 I_m}{\pi} = \frac{2 \times 200}{\pi} = 127.32$ A.

(e) Time period (T) $= \frac{1}{f} = \frac{1}{50.61} = 0.019$ sec

(f) Peak factor (K_p) $= \frac{I_m}{I_{rms}} = \frac{200}{141.42}$

(g) Form factor (K_F) $= \frac{I_{rms}}{I_{avg}} = \frac{141.42}{127.32}$

(50) A Sine wave has a positive zero crossing at 0° and an Rms value of 10V. calculate The instantaneous values at 30° | voltage of sine wave

$V_m = \sqrt{2} \cdot V_{rms} = \sqrt{2} \times 10$

Phasor

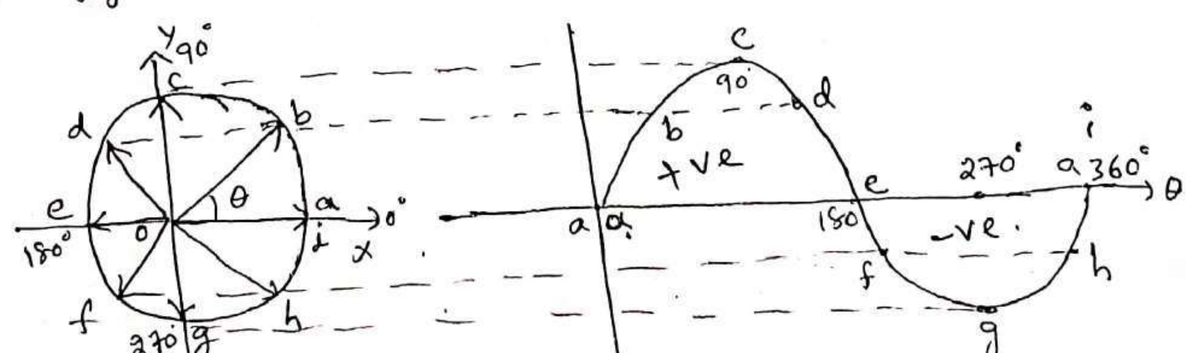
In the analysis of ac circuits, it is very difficult to solve alternating quantities in terms of waveforms and mathematical equations. Hence it is necessary to study a method which give an easier way of representing an alternating quantity. Such a representation is called Phasor.

→ The sinusoidally varying alternating quantity can be represented graphically by a straight line with an arrow. The length of line represents the magnitude of the quantity and arrow indicates its direction.

line = magnitude
direction.

Note: The phasors are assumed to be rotated in anticlockwise direction.

→ consider a phasor, rotating in anticlockwise direction, with uniform angular velocity, with its starting position 'a' as shown in fig.



From the diagram
- Oa is a phasor.

→ at position 'a', $i = 0$
→ " b, $i = I_m \sin\omega t = I_m \sin\theta$
→ " " " c, $i = I_m \sin 90° = I_m$
→ " " " d, $i = I_m \sin\theta$

} +ve cycle.

At point f , $i = -I_m \sin\theta$
" " g , $i = -I_m$
" " h , $i = -I_m \sin\theta$
" i , $i = 0$

negative cycle.

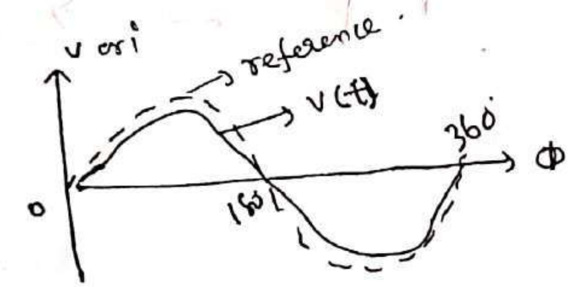
Phase

It is the fraction of angle through which an alternating quantity is delayed when compared with the reference quantity

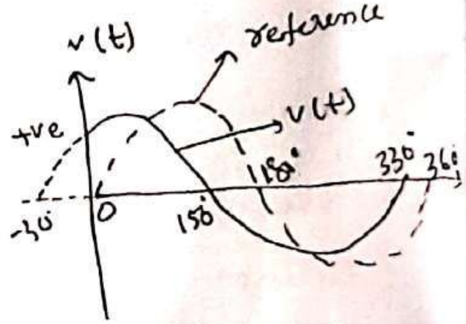
→ In General, the equation of alternating quantity in terms phase is

$$i = I_m \sin(\omega t \pm \Phi) \quad \text{or} \quad V = V_m \sin(\omega t \pm \Phi)$$

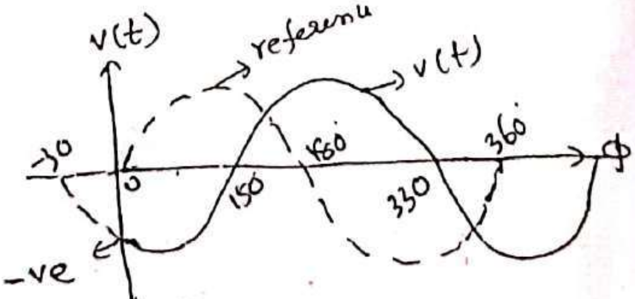
→



phase $\Phi = 0°$
$V = V_m \sin \omega t$



positive phase
$V(t) = V_m \sin(\omega t +$



Negative phase
$v(t) = V_m \sin(\omega t - \Phi)$

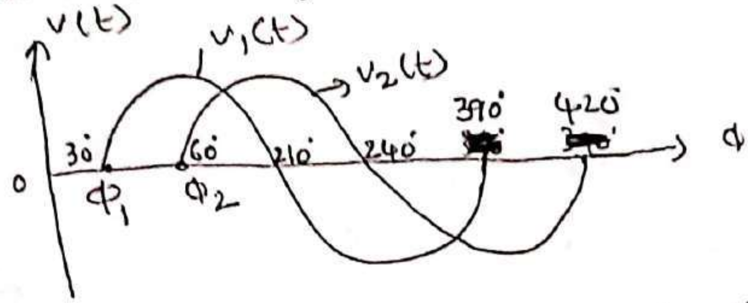
Note (1) phase is possible when the two alternating quantities have same frequency.

(2) If frequencies of two alternating quantities are diff such quantities phase is not possible

Phase difference

The difference between the phases of the two alternating quantities is called the phase difference.

→ If the two alternating quantities with same frequency have different phase angles, then they have the phase difference

→ 

For the above fig, phase difference is $\Phi_2 - \Phi_1$.

→ So, simply, phase difference is nothing but angle differen between the two phasors representing the two alternating quantities.

Phase Relationships for R, L and C

In phase (R)

When the two alternating quantities are said to be in phase, they have same frequency and same phase angles i.e phase difference is zero. Example : Resistance.

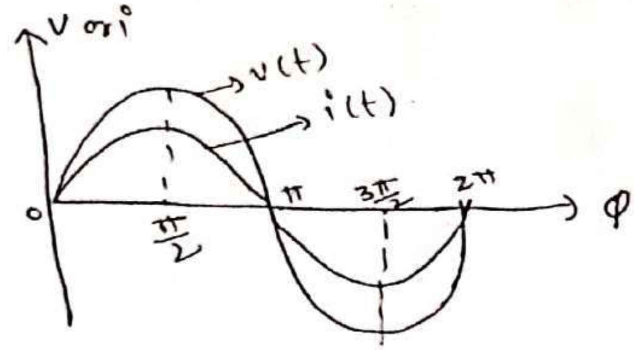
Explanation

consider the two alternating quantities having same frequency f Hz and having different maximum values.

Let $v(t) = V_m \sin\omega t$

$i(t) = I_m \sin\omega t$

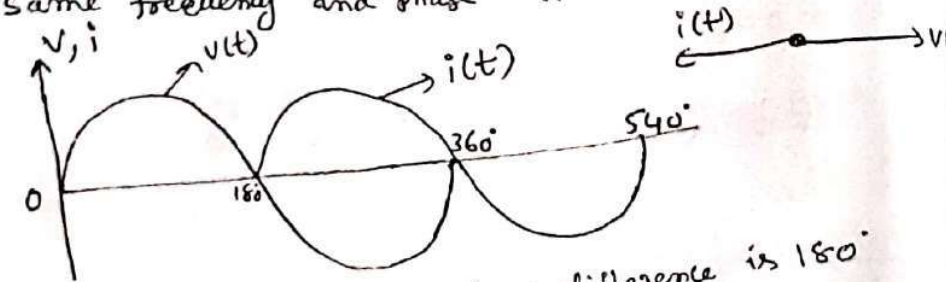
take $V_m > I_m$



Phase difference is zero.

Out of phase

When The two alternating quantities are said to be out of pha[se] if They have same frequency and phase difference is 180°.



→ From The fig, v(t) & i(t) having phase difference is 180°

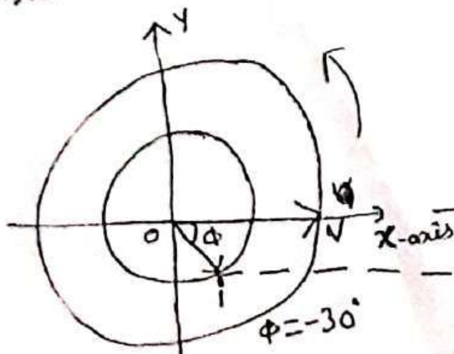
Inductance

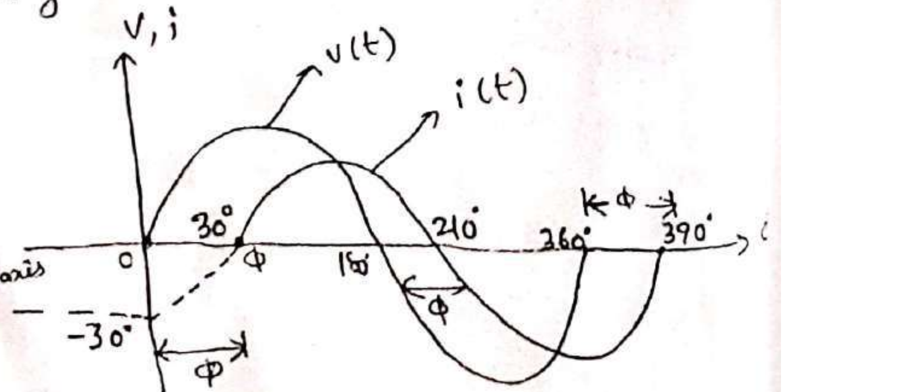
lagging phase

If an alternating quantity crosses its zero position when compare[d] with The zero position of another quantity and if it is later it is called lagging. Example : Inductance.

Explanation

Consider two alternating quantities v(t) & i(t).





here $v(t) = V_m \sin \omega t$

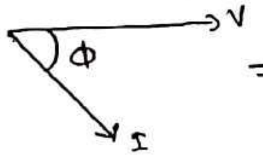
$i(t) = I_m \sin(\omega t - \phi)$

→ From the fig, ϕ is the phase difference b/w two phasors. when emf 'v' at its zero value, current 'i' has some negative value.

→ In the fig, the two Phasors are rotating in anticlockwise direction

The current is falling back with respect to voltage by an angle ϕ. This is called lagging phase difference. i.e the current i is lag the voltage by ϕ.

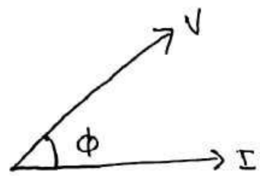
→ From the fig,



⇒ current lags the voltage by ϕ.

$\phi = -30^\circ$

(or)



⇒ Voltage leads the current by ϕ.

$\phi = 30^\circ$

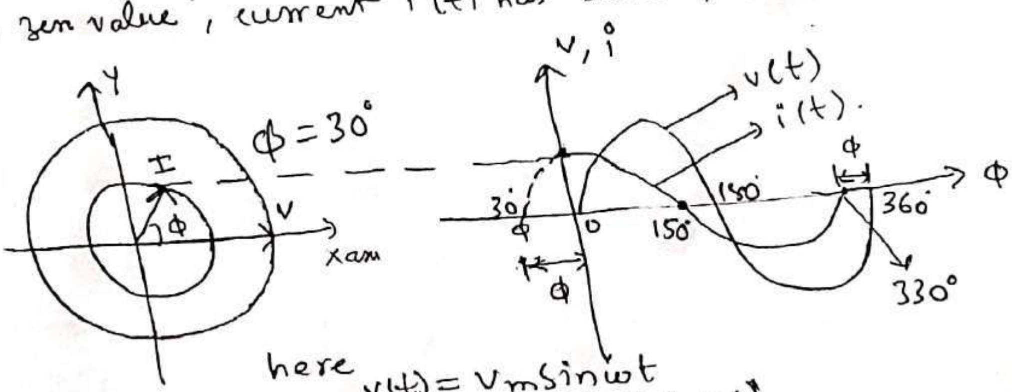
capacitance

leading

If an alternating quantity crosses its zero position when compared to zero position of another quantity then it is before/advanced, it is called leading. Example: capacitance.

Explanation

Two alternating quantities $v(t)$ & $i(t)$. when emf of voltage phasor at its zero value, current $i(t)$ has some positive value.



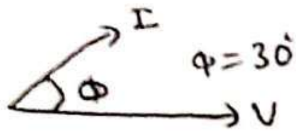
here $v(t) = V_m \sin \omega t$

$i(t) = I_m \sin(\omega t + \phi)$

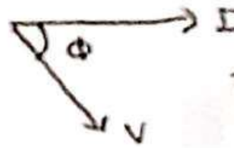
Scanned with CamScanner

→ In the fig, Two phasors are rotating anticlockwise direction, current is ahead / advance / before of voltage phasor. Thus, curr. is said to be leading with respect to the voltage and the phase difference is called leading phase difference.

→



I leads V by ϕ



$\Rightarrow$ V lags I by

$\phi = -30^\circ$

"j" operator

- In electrical circuit, The instantaneous current is represented by Symbol 'i'
- and In complex analysis, The imaginary part also represent with same 'i'
- So, Due to This, same representation of current 'i' and hence There will be a confusion.
- To avoid This, we use 'j' operator instead of 'i' operator.

→ If any vector multiplied with 'j' operator Then that vector displays 90° in anticlockwise direction.

→ Properties

$j = \sqrt{-1}$

$j^2 = -1$

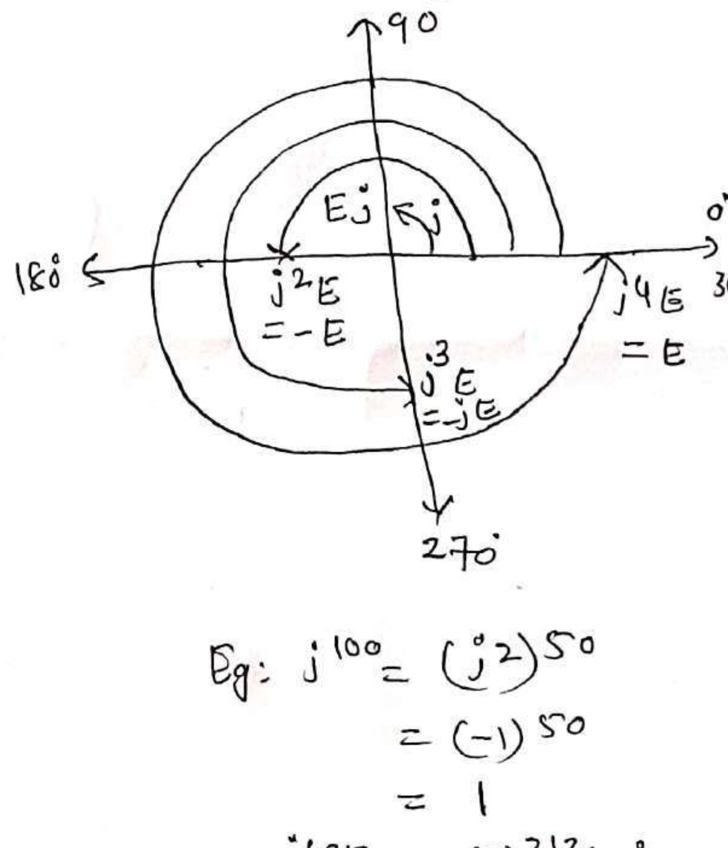
$j^3 = j^2 \times j$

$= -j$

$j^4 = (j^2)^2$

$= (-1)^2$

$= 1$



Eg: $j^{100} = (j^2)^{50}$

$= (-1)^{50}$

$= 1$

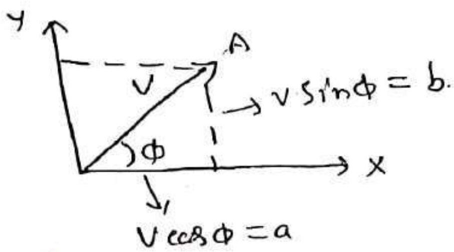
$j^{425} = (j^2)^{212} \times j$

$= (-1)^{212} \times j$

$= j$

Phasor or vector representation (Application of 'j' notation)

~~Any~~ complex vector diagram is shown below



This vector can be represented in 4 ways.

(1) Rectangular form

In rectangular form, The quantity E can be written as

$$\bar{V} = a + jb$$

$$\text{magnitude} |\bar{V}| = \sqrt{a^2 + b^2}$$

$$\text{Angle } \phi = \tan^{-1}\frac{b}{a}$$

(2) Trigonometric form

In This trigonometric form, The quantity E can be written as

$$\bar{V} = V(\cos\phi + j\sin\phi)$$

(3) polar form

In polar form, The quantity 'v' can be written as

$$\overline{V} = \underset{\text{magnitude}}{V} \angle \phi \rightarrow \text{phase angle.}$$

→ $\overline{V} = V \angle \phi$ also written as

$\overline{V} = V \cos \phi + j V \sin \phi$.

(4) Exponential form

In This form,

$$\overline{V} = V e^{j\phi}$$

→ $\overline{V} = V e^{j\phi}$ can be written as

$\overline{V} = V \cos \phi + j V \sin \phi$.

complex conjugate

If two vectors are said to be complex conjugate Then The sign of imaginary part is different.

Eg: The complex conjugate of $\overline{V} = a + jb$ is $\overline{V}^* = a - jb$

operations of complex numbers

(1) Addition of two complex numbers

If we add two complex numbers Then The result will be a complex number

Eg: $\overline{A} = 3 + j5$, $\overline{B} = 8 + 6j$

$\overline{A} + \overline{B} = (3 + j5)(8 + j6) = 11 + j11$

(2) subtraction of two complex numbers

If we subtract two complex numbers Then The resultant will be a complex number.

Eg: $\overline{A} = 6 + j7$, $\overline{B} = 3 + j8$

(3) multiplication of two complex numbers

if $z_1 = r_1 \angle \phi_1$, $z_2 = r_2 \angle \phi_2$, Then

$$z_1 \cdot z_2 = r_1 \angle \phi_1 \cdot r_2 \angle \phi_2$$

$$= r_1 r_2 \angle \phi_1 + \phi_2$$

Eg: $z_1 = 5\angle 30°$, $z_2 = 6\angle 45°$

$$z_1 z_2 = 30\angle 30+45° = 30\angle 75°$$

(4) Division of complex numbers

If $z_1 = r_1 \angle \phi_1$, $z_2 = r_2 \angle \phi_2$, Then

$$\frac{z_1}{z_2} = \frac{r_1 \angle \phi_1}{r_2 \angle \phi_2} = \frac{r_1}{r_2} \angle \phi_1 - \phi_2$$

Eg: $z_1 = 5\angle 30°$, $z_2 = 6\angle 45°$ Then

$$\frac{z_1}{z_2} = \frac{5\angle 30°}{6\angle 45°} = \frac{5}{6}\angle 30°-45° = \frac{5}{6}\angle -15°$$

Problems

(1) Find the resultant of given vector $6\angle 45° + 3\angle 65° - 5\angle 90°$

Sol $6\angle 45° + 3\angle 65° - 5\angle 90° = 6(\cos 45° + j\sin 45) + 3(\cos 65° + j\sin$
$- 5(\cos 90° + j\sin 90°)$

$$= 6\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right) + 3(0.42 + j\,0.9) - 5(0 + j1)$$

$$= 6(0.707 + j0.707) + 3(0.42 + j0.9) - 5(j1)$$

$$= 5.5 + j1.94$$

(2) Find the resultant vector of $\frac{2+j3}{4+j5}$

Sol $$\frac{2+j3}{4+j5} = \frac{2+j3}{4+j5} \times \frac{4-j5}{4-j5}$$

$$= \frac{8 - 10j + 12j + 15}{16+25}$$

$$= \frac{23+2j}{41} = \frac{23}{41} + j\frac{2}{41}$$

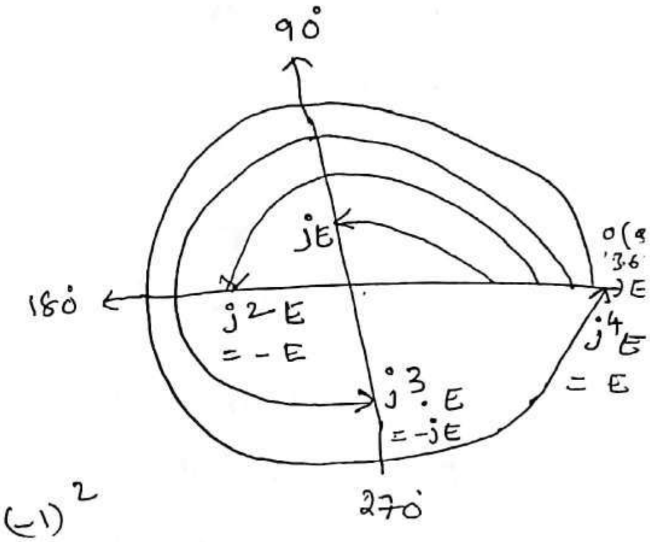
Scanned with CamScanner

Importance of j-operator

In electrical circuit, the instantaneous current represented by 'i' and in complex analysis, the imaginary also represents with same 'i'. Due to this same representation of current and i operator, there will be a confusion.

To avoid this, we use 'j' operator instead of i operator in complex analysis.

→ If any vector multiplied with j-operation then that vector displays 90° in anticlockwise direction.



Properties of j-operator

The value of $j = \sqrt{-1}$

$$j^2 = -1$$

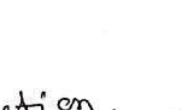
$$j^3 = j^2 . j = -j$$

$$j^4 = (j^2)^2 = (-1)^2 = 1$$

Phasor diagram

The diagram in which different alternating quantities of same frequency and same sinusoid in nature are represented by individual phasors indicating exact phase interrelationship is called phasor diagram.

→ Phasors are rotating anticlock wise direction.



Phasor diagram for Resistance(R)

If voltage as reference.

$\longrightarrow V$
i

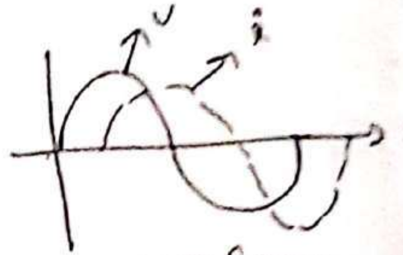
here voltage and currents are inphase.

Phasor diagram for L

→ If voltage (V) as reference.

→ For inductance, current is lagging with voltage by angle Φ.

→ The phasor diagram is



waveform.

Phasor diagram for capacitance (C)

→ If voltage (V) as reference.

→ For capacitance, current is leading with voltage by angle Φ

Phasor diagram   waveform

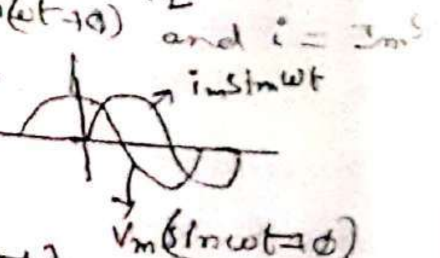
→ Two Sinusoidal currents are given by

$$i_1 = 10\sin\left(\omega t + \frac{\pi}{3}\right), \quad i_2 = 15\sin\left(\omega t - \frac{\pi}{4}\right)$$

calculate the phase difference b/w them in degree.

Sol) Phase difference $\Phi = 60 - (-45)$

$= 105°$



→ what is the meaning if $v = V_m \sin(\omega t + \Phi)$ and $i = I_m \sin \omega t$

Sol) → here, voltage is leading current by Φ.

(or)

→ current is lagging voltage by $\cancel{(-\Phi)}$ 'Φ'.

Alternating quantities

① **Impedance(Z):**

It is defined as "The measure of The opposition to flow of current in ac circuits"

- Impedance is a complex quantity

$$Z = R + jX \xrightarrow{\text{in } \Omega} \text{Reactance in } \Omega. \quad \text{[Rectangular form]}$$

R → Resistance in Ω

$X = -X_L + X_C$ or $X_L - X_C$

X_L = Inductive reactance in Ω

$= \omega L$

$\boxed{X_L = 2\pi f L}$ Ω, where L is H.

X_C = capacitive reactance in Ω

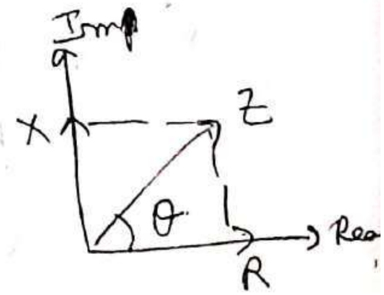
$= \dfrac{1}{\omega C}$

$= \dfrac{1}{2\pi f C}$ in Ω, where C is Farad.

→ In polar form, (we know, $Z = R + jX$)

$Z = |Z| \angle\theta$ → phase angle

$|Z|$ → magnitude



$|Z| = \sqrt{R^2 + X^2}$

$\theta = \tan^{-1}\left(\dfrac{X}{R}\right)$

→ Also from fig

$$R = |Z|\cos\theta$$

$$X = |Z|\sin\theta$$

(2) Admittance (Y)

It is defined as the "~~Inverse~~ Reciprocal of the Impedance". and its units are mho's or Seimens

→ $$Y = \frac{1}{Z}$$

$$= \frac{1}{R+jX} \times \frac{R-jX}{R-jX}$$

$$= \frac{R-jX}{R^2+X^2}$$

$$= \frac{R}{R^2+X^2} - j\frac{X}{R^2+X^2}$$

$$\boxed{Y = G + jB}$$ Rectangular form $\left[G = \frac{R}{R^2+X^2}, B = \frac{-X}{R^2+X^2}\right]$

where $G \to$ conductance $= \frac{1}{R}$

$B \to$ Susceptance $= \frac{1}{X}$

→ In Polar form

$$Y = |Y|\angle\theta$$

$$|Y| = \sqrt{G^2+B^2}$$

$$\theta = \tan^{-1}\left(\frac{B}{G}\right)$$

EXAMPLE - 31

Convert the following from polar to rectangualr form

(i) $10 \angle 45^o$ (ii) $6.71 \angle 153.43^o$

(iii) $7.52 \angle -125^o$ (iv) $4 \angle -60^o$

Solution :

(i) Let $A = 10 \angle 45^o$ .... In polar form

It is in the form of $A = |A| \angle \theta = |A| \cos \theta + j\, |A| \sin \theta$

$$A = 10 \cos 45^o + j\, 10 \sin 45^o$$

$$= 10 \times 0.707 + j\, 10 \times 0.70$$

$$= 7.07 + j\, 7.07 \text{ ..... rectangular form Ans}$$

(ii) Let $A = 6.71 \angle 153.43^o$ ...... In polar form

we knwon that $A = |A| \angle \theta = |A| \cos \theta + j\, |A| \sin \theta$

$\therefore$ $|A| = 6.71$

$$\theta = 153.43^o$$

$\therefore$ $A = 6.71 \cos 153.43^o + j\, 6.71 \sin 153.43^o$

$$= 6.71 \times (-0.894) + j\, 6.71 \times (0.447)$$

$$= -6 + j3 \text{ ...... In rectangular form} \quad \text{Ans}$$

(iii) Let $A = 7.52 \angle -125^o$

we know that, $A = |A| - \angle -\theta = |A| \cos\theta - j\, |A| \sin\theta$

$\therefore$ $|A| = 7.52,\ \theta = 125^o$

$\therefore$ $A = 7.52 \cos (125^o) - j\, 7.52 \sin (125^o)$

$$= 7.52\, (-0.574) - j7.52 \times (0.819) = -4.313 - j\, 6.160$$

(iv) Let $A = 4 \angle -60^o$

$$|A| = 4,\ \theta = 60^o$$

$\therefore$ $A = |A| \cos \theta - j\, |A| \sin\theta = 4 \cos (-60^o) - j4 \sin (60^o)$

$$= 4 \times \frac{1}{2} - j\, 4 \times \frac{\sqrt{3}}{2} = 2 - j2\, \sqrt{3}$$

Concept of Reactance (X)

→ In electrical systems, "Reactance is opposition to the flow of current (ac) due to the elements of L & C".

→ It is denoted by symbol 'x'. unit is Ω.

→ There are two types of reactances

1. Inductive reactance (X_L)
2. Capacitive reactance (X_C)

1. Inductive reactance (X_L)

→ It is opposition to the flow of ac current due to the element of 'L'

$$\boxed{X_L = \omega L}\ \Omega$$

$$\boxed{X_L = 2\pi f L}\ \Omega$$

, where L → Henry.
f → frequency in Hz

2. capacitive reactance (X_C)

$$\boxed{X_C = \frac{1}{\omega C}}\ \Omega$$

$$\boxed{X_C = \frac{1}{2\pi f C}}\ \Omega$$

where C → Farads.

Concept of Susceptance (B)

Susceptance is the imaginary part of admittance.

$$\underbrace{Y = G}_{\text{Conductance}} + j \overbrace{B}^{\text{susceptance}}$$

→ B unit is Seimens.

→ $$Y = \frac{1}{R+jX}$$

$$= \frac{R-jX}{(R+jX)(R-jX)}$$

$$= \frac{R-jX}{R^2+X^2}$$

$$= \underbrace{\frac{R}{R^2+X^2}}_{G} + j \underbrace{\left(\frac{-X}{R^2+X^2}\right)}_{B}$$

where $B = \frac{-X}{R^2+X^2}$

$$= -\frac{X}{|Z|^2}$$

Concept of powers

(i) Real power

Real power results from energy being used for work or dissipated as heat.

Def

" The power which is actually consumed or utilized in an AC circuit is called Real power ".

→ Real power also called as True power or Active power.

→ It is denoted by symbol 'P'

→ Unit of 'P' is watts (w) or kw or mw.

→ The Real power is the actual outcomes of the electrical system which runs the electrical circuits or load.

→ $P = V_{ph} I_{ph} \cos\phi$ w or kw or mw

$= VI\cos\phi$ → for single phase supply.

$= \sqrt{3} V_L I_L \cos\phi$, for $3-\phi$ supply.

where $\cos\phi$ = power factor.

(ii) Reactive power (Q)

→ "The power which flows back and front that means it moves in both the directions in the circuit (or) reacts upon itself is called Reactive power".

→ The symbol is Q.

→ units are KVAR or MVAR

→ $Q = VI \sin\phi$ KVAR or MVAR.

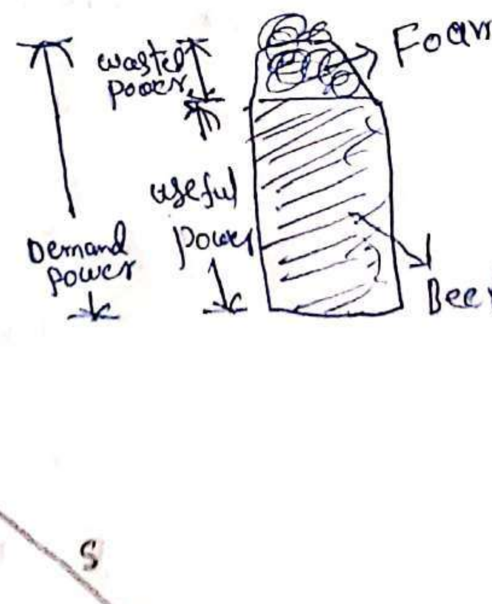
(iii) Apparent power (S)

Def:
"The product of RMS value of voltage and current is called "Apparent power".

→ The symbol is 'S'

→ units are KVA or MVA

→ $S = VI$ KVA or MVA.

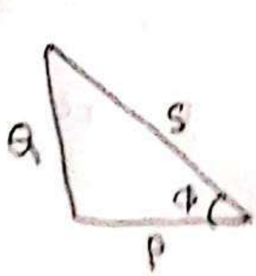


Power traingle

$S = P + jQ$

$|S|^2 = P^2 + Q^2$

$|S| = \sqrt{P^2 + Q^2}$



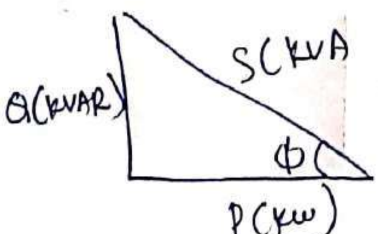
Power factor

Def I It is the ratio of Active power (P) to the Apparent power (S)

$$\text{Power factor } (\cos\phi) = \frac{P \text{ in kW}}{S \text{ in kVA}} = \frac{\text{Active power}}{\text{Apparent power}}$$

$$= \frac{\text{Active power used in a ckt}}{\text{Apparent power delived to the ckt}}.$$

From power Triangle

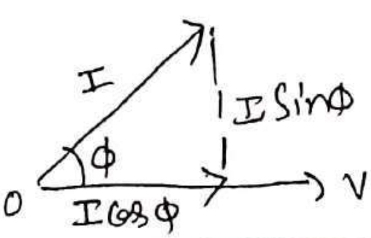


$$\boxed{\text{Power factor } (\cos\phi) = \frac{P}{S} = \frac{VI\cos\phi}{VI}}$$

→ power factor is a measure of how effectively you are using electricity.

Def 2

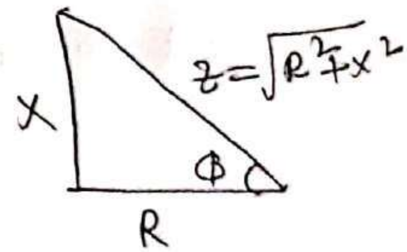
It is defined as the cosine angle ~~of phase angle~~ between phase voltage and phase current.



$$\boxed{\cos\phi = \text{Power factor}}$$

Def 3 It is defined as the ratio of Resistance to the Impedance.

$$\cos\phi = \frac{R}{Z}$$



From Impedance Triangle

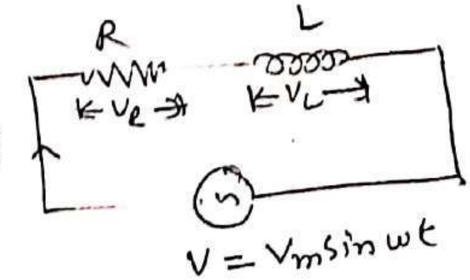
$$Z = R + jX$$

$$\cos\phi = \frac{R}{Z}, \qquad \sin\phi = \frac{X}{Z}$$

$$\boxed{pf(\cos\phi) = \frac{R}{\sqrt{R^2+X^2}}}$$

AC Through Series R-L circuit

Consider a Series RL Ckt excited by a AC source as shown in fig.



Let $v = V_m \sin \omega t$.

Apply KVL to the ckt

$$V = V_R + V_L \quad -(1)$$

$$= iR + j i X_L$$

$$= i(R + jX_L)$$

$$= i(R + j\omega L)$$

$$V = iZ.$$

$$i = \frac{V}{Z} = \frac{V}{R + j\omega L}$$

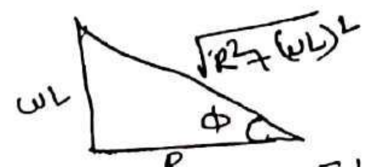
$$i = \frac{V}{\sqrt{(R^2 + \omega^2 L^2)} \angle \tan^{-1} \frac{\omega L}{R}}$$

$$i = \frac{V}{\sqrt{(R^2 + \omega^2 L^2)}} \angle \tan^{-1}\left(-\frac{\omega L}{R}\right) \quad -(2)$$

$$Z = \sqrt{R^2 + \omega^2 L^2} \angle \tan^{-1} \frac{\omega L}{R}$$

$$= \sqrt{R^2 + \omega^2 L^2} \angle \phi$$

where $\phi = \tan^{-1} \frac{\omega L}{R}$



Impedance Triangle

where $\frac{V}{\sqrt{R^2 + \omega^2 L^2}} = I_m$ ~~and~~ $\cancel{\phi = \tan^{-1}\left(-\frac{\omega L}{R}\right)}$

$\cancel{+\phi = -\tan^{-1} \frac{\omega L}{R}}$.

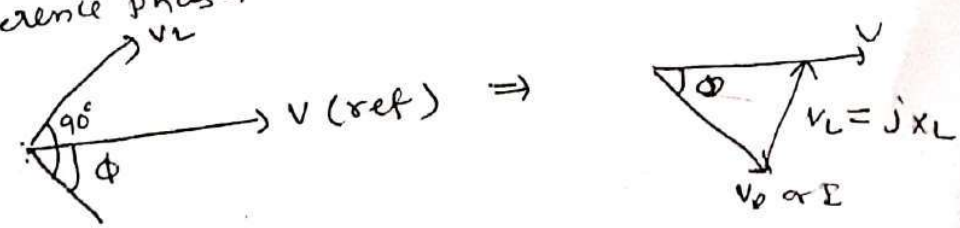
So the instantaneous value of current

$$i = I_m \sin(\omega t + \phi)$$

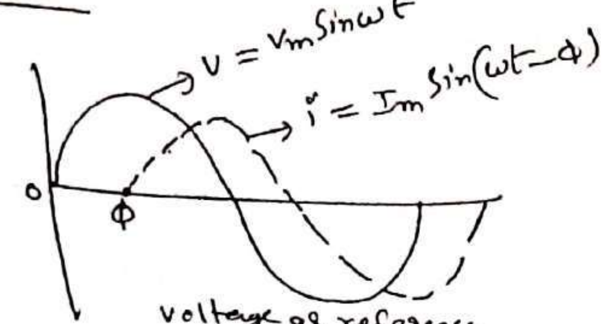
$$\boxed{i = \frac{V}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t - \tan^{-1} \frac{\omega L}{R}\right)} \quad -(3).$$

Phasor diagram

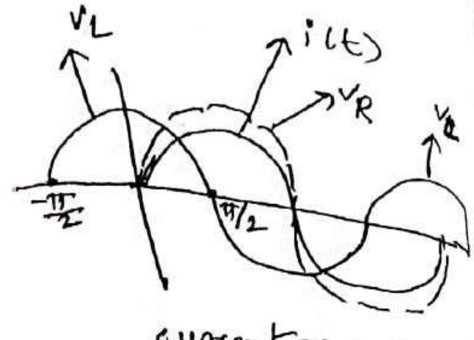
Voltage as reference phasor



Wave forms



voltage as reference



current as reference

Instantaneous power

$$P(t) = v(t)\, i(t)$$

$$= V_m \sin\omega t \times I_m \sin(\omega t - \phi)$$

$$= V_m I_m \left[\frac{\cos\phi - \cos(2\omega t - \phi)}{2} \right)$$

$2 \sin A \sin B = \cos(A-B) - c$

$$= \frac{V_m I_m}{2} \cos\phi - \frac{V_m I_m}{2} \cos(2\omega t - \phi).$$

Average power

$$P_{avg} = \frac{1}{T}\int_0^T p(t)\, dt \quad \text{or} \quad \frac{1}{T}\int_0^T P(\omega t)\, d(\omega t)$$

$$= \frac{1}{T}\int_0^T \left[\frac{V_m I_m}{2} \cos\phi - \frac{V_m I_m}{2} \cos(2\omega t - \phi)\right] d(\omega t)$$

where $T = 2$

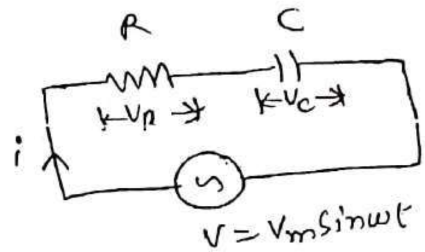
$$= \frac{V_m I_m}{2} \cos\phi$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi$$

$$\boxed{P_{avg} = V\, i \cos\phi}$$

here V & i are rms values.

AC Through series RC circuit



Consider series RC circuit excited by AC source as shown in fig.

Let $v = V_m \sin\omega t$ — (1)

Apply KVL to the closed ckt

$$v = v_R + v_C$$

$$v = iR - ijX_C$$

$$= i(R - jX_C)$$

$$\boxed{v = iZ} \quad — (2)$$

where Z = Impedance of RC ckt

$$Z = R - jX_C = R - j\frac{1}{\omega C} \quad — (3)$$

$$= \sqrt{R^2 + \frac{1}{\omega^2 C^2}} \; \angle -\tan^{-1}\left(\frac{1}{\omega CR}\right)$$

$$= \sqrt{R^2 + \frac{1}{\omega^2 C^2}} \; \angle -\phi. \qquad \text{where } \phi = \tan^{-1}\frac{1}{\omega CR}$$

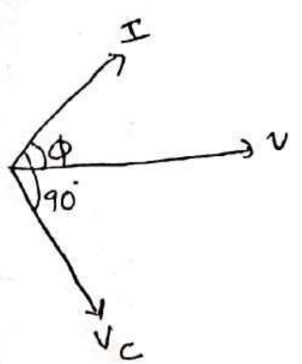
instantaneous current

$$i = \frac{v}{Z} = \frac{V}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \sin(\omega t + \phi)$$

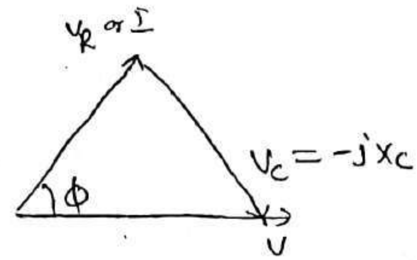
$$i = \frac{V}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \sin\left(\omega t + \tan^{-1}\frac{1}{\omega CR}\right) \quad — (4)$$

Phasor diagram

take voltage as reference phasor



$\Rightarrow$



voltage triangle.

wave forms

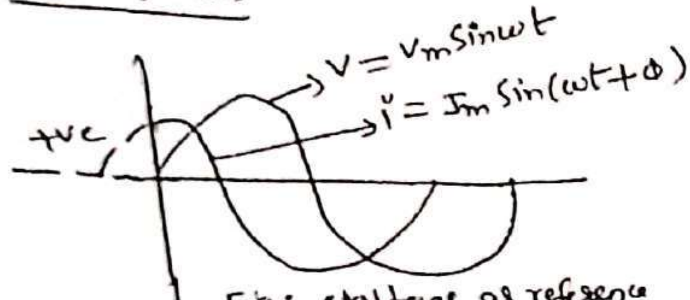


Fig: voltage as reference

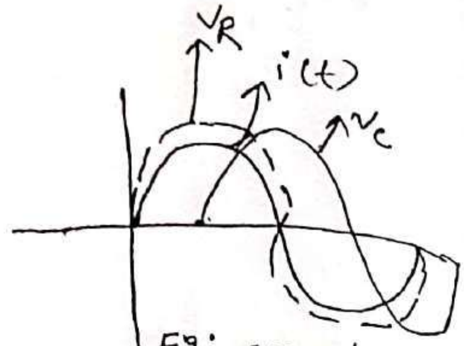


Fig: current as reference

Instantaneous power

$$P(t) = v(t)\, i(t)$$

$$= V_m \sin\omega t \times I_m \sin(\omega t + \phi)$$

$$= \frac{V_m I_m}{2}\left(\cos\phi - \cos(2\omega t + \phi)\right)$$

$$P(t) = \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t + \phi)$$

we know
$2\sin A \sin B$
$= \cos(A-B) - \cos(A$

Average power

$$P_{avg} = \frac{1}{T}\int_0^T P(t)\, dt \quad \text{or} \quad \frac{1}{T}\int_0^T P(\omega t)\, d(\omega t).$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t + \phi)\right] d(\omega t)$$

$$P_{avg} = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\phi = VI\cos\phi$$

AC Through Series RLC circuit

Consider a Series RLC circuit excited by AC source as shown in fig.

$V = V_m \sin\omega t$

case(1) $X_L > X_C$

Let $V = V_m \sin\omega t$

Apply KVL to the closed circuit

$$V = V_R + V_L + V_C \quad -(1)$$

$$= iR + i(jX_L) + i(-jX_C)$$

$$V = i\left(R + j(X_L - X_C)\right) \quad -(2)$$

where Z = Impedance of RLC circuit (28)

$$= R + j(X_L - X_C)$$

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

In polar form

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \angle \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

$$= |Z| \angle \phi \quad \text{where } \phi = \tan^{-1}\left(\frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}\right)$$

Instantaneous current

From eq (3)

$$i = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \angle -\tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

$$= \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin\left(\omega t - \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)\right)$$

case (2)

If $X_C > X_L$

$$Z = R + j(X_C - X_L) = R + j\left(\frac{1}{\omega C} - \omega L\right)$$

In polar form

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2} \angle \tan^{-1}\left(\frac{\frac{1}{\omega C} - \omega L}{R}\right)$$

$$= |Z| \angle \phi, \quad \text{where } \phi = \tan^{-1}\left[\frac{\left(\frac{1}{\omega C} - \omega L\right)}{R}\right]$$

instantaneous current

$$i = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}} \angle -\tan^{-1}\left(\frac{\frac{1}{\omega C} - \omega L}{R}\right)$$

where $\phi = \tan^{-1} \frac{\frac{1}{\omega C} - L}{R}$

$$= \frac{V}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}} \sin\left(\omega t + \tan^{-1}\left(\frac{\frac{1}{\omega C} - \omega L}{R}\right)\right)$$

case(3) If $X_L = X_C$

$$Z = R + j(X_L - X_L)$$

$$Z = R$$

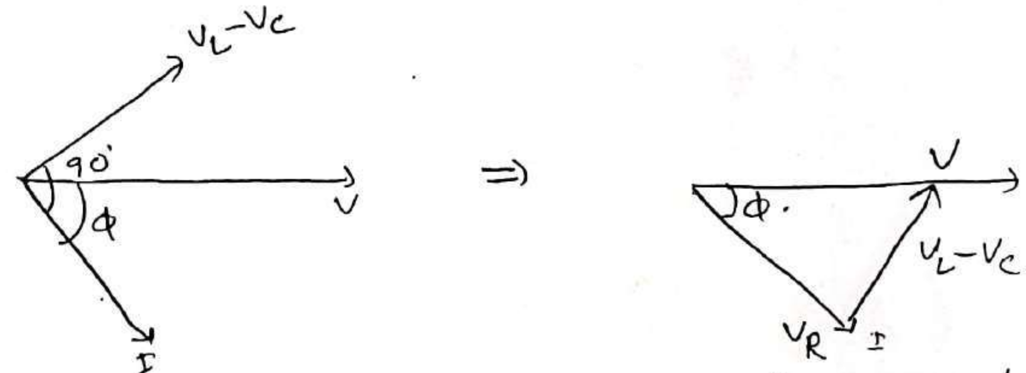
instantaneous current

$$i = \frac{V}{Z} = \frac{V}{R} \angle \tan^{-1}\left(\frac{0}{R}\right) = \frac{V}{R}\angle 0^\circ$$

Phasor diagrams

case(1) $X_L > X_C$

Take Voltage as reference phasor



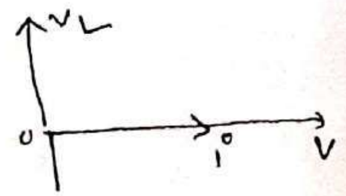
Voltage Traingle.

case(2) $X_C > X_L$

Phasor diagram

Voltage Triangle

case(3) $X_L = X_C$



Instantaneous Power

case (1) $X_L > X_C$

$$p(t) = v(t)\, i(t)$$
$$= V_m \sin\omega t \cdot I_m \sin(\omega t - \phi)$$
$$= \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)$$

[∵ similar to series RLC ckt]

case (2) $X_C > X_L$

$$p(t) = v(t) \cdot i(t)$$
$$= V_m \sin\omega t \cdot I_m \sin(\omega t + \phi)$$
$$= \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t + \phi)$$

[∵ Similar to series RC circuit]

case (3) $X_C = X_L$

$$p(t) = v(t)\, i(t)$$
$$= V_m \sin\omega t \cdot I_m \sin\omega t$$
$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t$$

$\sin\omega t \cdot \sin\omega t = \sin^2\omega t$
$= \frac{1 - \cos 2\omega t}{2}$

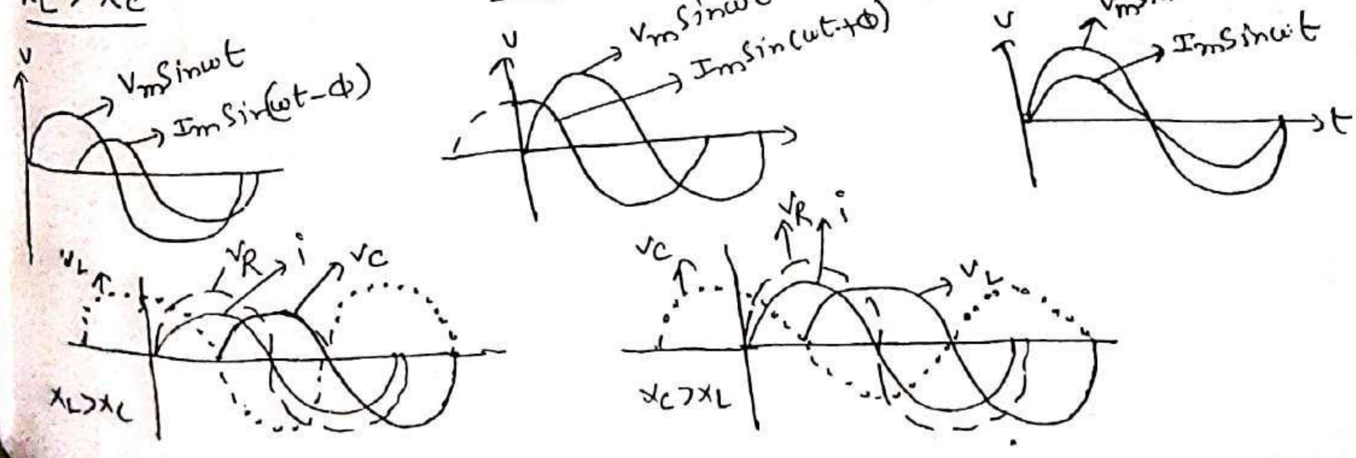
Average Power

$$P_{avg} = \underset{}{P_{avg(R)}} + \underset{\downarrow\atop 0}{P_{avg(L)}} + \underset{\downarrow\atop 0}{P_{avg(C)}}$$

$$\therefore P_{avg} = P_{avg(R)}$$
$$= V_R I = V\cos\phi \cdot I = VI\cos\phi$$

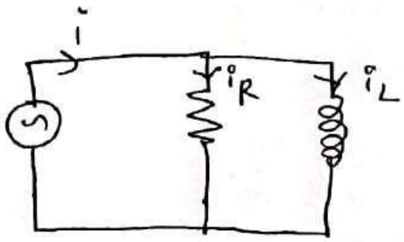
Waveforms (voltage as reference).

$X_L > X_C$ $\quad$ $X_C > X_L$ $\quad$ $X_L = X_C$



AC Through Parallel RL circuit

Consider a parallel RL circuit excited by AC source as $v = V_m \sin\omega t$ shown in fig.



Apply KCL to the circuit

$$i = i_R + i_L \quad \text{— (1)}$$

$$= \frac{v}{R} + \frac{v}{j\omega L}$$

$$= v\left(\frac{1}{R} + \frac{1}{j\omega L}\right)$$

$$i = v\left(\frac{1}{R} - j\frac{1}{\omega L}\right) \quad \text{— (2)}$$

$$i = vY \quad \text{— (3)}$$, where $Y = |Y|\angle\phi \Rightarrow |Y| = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L}\right.}$

where $Y = \frac{1}{R} - j\frac{1}{\omega L}$

$$Y = G - jB$$

From eq (2), the angle b/w voltage to current is

$$\phi = \tan^{-1}\left(\frac{-\frac{1}{\omega L}}{\frac{1}{R}}\right) = \tan^{-1}\left(\frac{-R}{\omega L}\right) \quad \text{— (4)}$$

For RL circuit

$$i = V_m \times \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L}\right)^2}\ \sin(\omega t - \phi)$$

case(1) $R >> \omega L$

if $R >> \omega L$, then $\frac{1}{R} << \frac{1}{\omega L}$ so $\frac{1}{R}$ is neglected and $\phi = 9$

instantaneous current

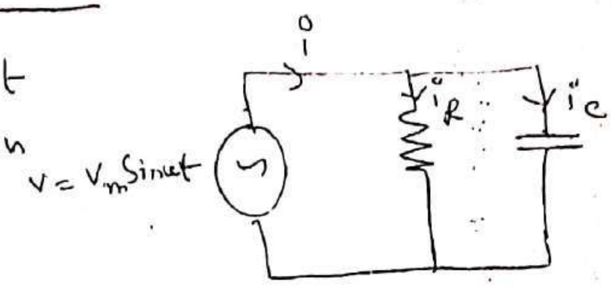
$$i = i_L = \frac{V_m}{\omega L}\sin(\omega t - 90^\circ)$$

case(2) $R << \omega L$

if $R << \omega L$, $\frac{1}{R} >> \frac{1}{\omega L}$ so $\frac{1}{\omega L}$ is neglected. only resistance is p
sented. Then $\phi = 0^\circ$ $\left[\tan^{-1}\frac{0}{\left(\frac{1}{R}\right)}\right]$

: Through Parallel RC circuit

Consider a parallel RC circuit excited by AC source as shown in fig.



Apply KCL to the circuit

$$i = i_R + i_C$$

$$= \frac{V}{R} + \frac{V}{-jX_C}$$

$$= \frac{V}{R} + j\frac{V}{\left(\frac{1}{\omega C}\right)}$$

$$i = V\left(\frac{1}{R} + j\omega C\right) \quad — (1)$$

$$i = VY \quad — (2)$$

where $Y = \frac{1}{R} + j\omega C$

$$Y = G + jB$$

$$= \sqrt{\left(\frac{1}{R}\right)^2 + (\omega C)^2} \angle \tan^{-1}\frac{\omega C}{\left(\frac{1}{R}\right)}$$

$$B = \frac{1}{X_C} = \frac{1}{\left(\frac{1}{\omega C}\right)} = \omega C$$

$$= \sqrt{\left(\frac{1}{R}\right)^2 + (\omega C)^2} \angle \tan^{-1}\omega CR$$

$$= |Y|\angle\phi, \quad \text{where } \phi = \tan^{-1}\omega CR$$

For RC circuit

instantaneous current (i)

$$i = V_m\sqrt{\left(\frac{1}{R}\right)^2 + (\omega C)^2}\,\sin(\omega t + \phi)$$

case(1) $R >> \frac{1}{\omega C}$

if $R >> \frac{1}{\omega C}$ then $\frac{1}{R} << \omega C$, $\frac{1}{R}$ is neglected, and $\phi = 90°$

$$i = V_m\,\omega C\,\sin(\omega t + 90°)$$

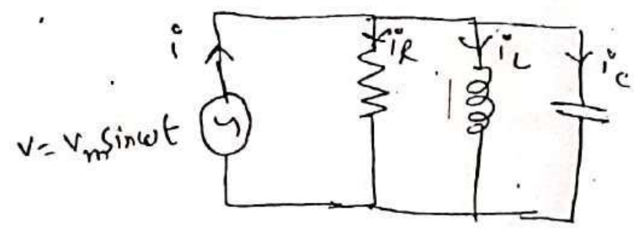
case(2) $R << \frac{1}{\omega C}$

if $R << \frac{1}{\omega C} \rightarrow \frac{1}{R} >> \omega C \rightarrow \omega C$ is neglected and $\phi = 0°$

$$i = \frac{V_m}{R}\sin(\omega t + 0°)$$

AC Through Parallel RLC circuit

Consider a Parallel RLC circuit excited by AC Source as shown in fig.



Apply KCL to the circuit

$$i = i_R + i_L + i_C$$

$$= \frac{V}{R} + \frac{1}{L}\int v\,dt + C\frac{dv}{dt}$$

$$= \frac{V_m \sin\omega t}{R} + \frac{1}{L}\int V_m \sin\omega t\, d(\omega t) + C \cdot V_m(+\cos\omega t)\,\omega$$

$$= \frac{V_m}{R}\sin\omega t + \frac{V_m}{L}\left(-\frac{\cos\omega t}{\omega}\right) + C V_m \omega \cos\omega t$$

$$\therefore i = \frac{V_m}{R}\sin\omega t + V_m\left(\omega C - \frac{1}{\omega L}\right)\cos\omega t \quad \text{— (1)}$$

Let instantaneous current $(i) = A\sin(\omega t + \phi)$

$$i = A\sin\omega t\cos\phi + A\cos\omega t\sin\phi$$

$$i = A\cos\phi \cdot \sin\omega t + A\sin\phi \cdot \cos\omega t \quad \text{— (2)}$$

compare eq (1) & (2)

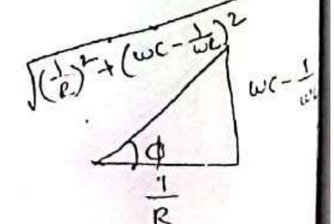
$$\frac{V_m}{R} = A\cos\phi \quad \text{and} \quad V_m\left(\omega C - \frac{1}{\omega L}\right) = A\sin\phi \quad \text{— (2.1)}$$

Then $\tan\phi = \dfrac{\omega C - \frac{1}{\omega L} \rightarrow \sin\phi}{\left(\frac{1}{R}\right) \rightarrow \cos\phi}$

$$\phi = \tan^{-1}\left(R\left(\omega C - \frac{1}{\omega L}\right)\right) \quad \text{— (3)}$$

To find A

use Impedance Triangle ⟶



From the Triangle

$$\cos\phi = \frac{\frac{1}{R}}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

From eq (2.1), $\dfrac{V_m}{R} = A\cos\phi$

$$= A\,\frac{\left(\frac{1}{R}\right)}{\quad}$$

$$A = V_m \cdot \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

∴ Thus, The instantaneous current (i)

$$= V_m \cdot \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2} \times \sin\left[\omega t + \phi\right]$$

$$= V_m \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2} \times \sin\left[\omega t + \tan^{-1}\left(\omega C - \frac{1}{\omega L}\right) R\right]$$

Problems on RL (Series) ckt

An alternating current $i = 1.414 \sin(2\pi \times 50\, t)$ A, is passed through a series ckt consisting of $100\,\Omega$ and an inductance of 0.31831 H. Find The expression for the instantaneous values of the voltage across (a) Resistance (b) inductance (c) both.

Given

$$i = \underbrace{1.414}_{I_m} \sin(2\pi \times 50\, t)$$

here $\omega = 2\pi \times 50$
$= 2\pi f$

so $f = 50$ Hz

$R = 100\,\Omega$, $L = 0.31831$ H.

$X_L = 2\pi f L = 2\pi \times 50 \times 0.31831$
$= 100\,\Omega$.

(1) $V_R = iR$

$= 1.414 \sin(2\pi \times 50 t) \times 100$

$V_R = \underbrace{141.4}_{V_{Rm}} \sin(2\pi \times 50 t)$ volts

(2) $V_L = j I X_L$

$= j\, 1.414 \sin(2\pi \times 50 t) \times 100$

$= j\, 141.4 \sin(2\pi \times 50 t)$

$V_L = 141.4 \sin(2\pi \times 50 t + 90^\circ)$

In polar form $V_L = \frac{V_m}{\sqrt{2}} \angle\phi$

$= \frac{141.4}{\sqrt{2}} \angle 90^\circ$

(3) Rms value of $V_R = \frac{V_{Rm}}{\sqrt{2}}$

$= \frac{141.4}{\sqrt{2}} = k$

$V_R = 100\angle 0^\circ = 100 + j0$

Rms value of $V_L = j\frac{V_{Lm}}{\sqrt{2}}$

$= j\,\frac{141.4}{\sqrt{2}}$

$= j\,100$ V

$V_L = 0 + j100$

Resultant voltage

$V = V_R + V_L$

$= 100 + j0 + 0 + j100$

$= 100 + j100$

$= 141.42 \angle 45^\circ$

$= V_{rms} \angle\phi$

$V_m = V_{rms} \times \sqrt{2} = \sqrt{2} \times 141.42$

$= 200$ V.

$V(t) = V_m \sin(2\pi \times 50 t + \phi)$

$= 200 \sin(2\pi \times 50 t + 45^\circ)$

(2) A voltage $e = 200 \sin 100\pi t$ is applied to a load having $R = 200\Omega$ in series with $L = 638\,mH$. Estimate (i) Expression for current (ii) power consumed by the load (iii) reactive power of the [load] (iv) V_R and V_L.

Sol

Given

$R = 200\Omega,\ L = 638\,mH.$

$e = 200 \sin 100\pi t$

here $V_m = 200$

$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 141.42\,V\angle 0^\circ$

$\omega = 100\pi$

$= 2\pi f = 2\pi \times 50$

So, $f = 50\,Hz$

$jX_L = j\omega L = j\,100\pi \times 638 \times 10^{-3} = j200.43\,\Omega$

Impedance $Z = R + jX_L$

$= 200 + j200.43$

$= 283.14\angle 45.06^\circ$

$i_{rms} = \frac{V}{Z} = \frac{141.42\angle 0^\circ}{283.14\angle 45.06}$

$= \underbrace{0.5}_{rms}\angle\underbrace{-45.06}_{\phi}\,A$

$I_m = I_{rms} \times \sqrt{2} = 0.5 \times \sqrt{2}$

$= 0.707\,A.$

$\phi = 45.06$

(i) Expression for current

$i = I_m \sin(\omega t - \phi)$

$= 0.707 \sin(100\pi t - 45.06)$

(ii) Power consumed by the load

$P = VI\cos\phi$

$P = 141.21 \times 0.5 \cos(\ldots$

$= 49.94$

$\cong 50\,W$

(iii) Reactive power of [load]

$Q = V_{rms} I_{rms} \sin\phi$

$= 141.21 \times 0.5 \times \sin\ldots$

$= 50\,Var$

(iv) $V_R = iR$

$= 0.5 \times 200\angle -45.06$

$= 100\angle -45.06^\circ$

$V_L = jiX_L$

$= j\,0.5 \times 200.43\angle -45$

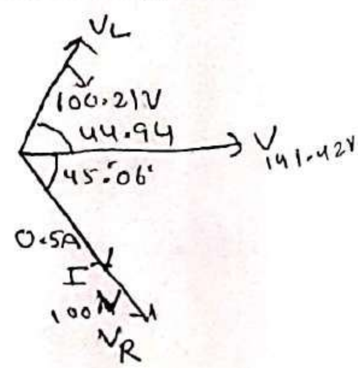
$= j\,100.21\angle -45$

$\cancel{= 100.21\angle 90^\circ\,V}$

$= 100.21\angle 90 - 45.06$

$= 100.21\angle 44.94$

Phasor diagram

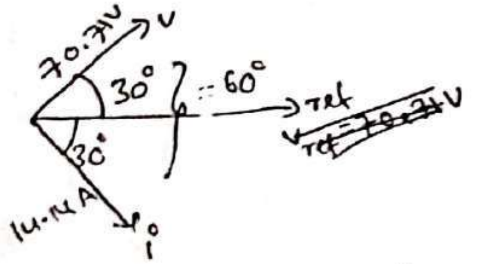


A voltage source $v = 100 \sin(300t + 30^\circ)$ is applied to a n/w containing two elements in series. The resulting current $i = 20 \sin(300t - 30)$. Determine the values of the two elements. (32)

Given

$v = 100 \sin(300t + 30^\circ)$

$i = 20 \sin(300t - 30^\circ)$



The angle b/w voltage to current is $\phi = 60^\circ$

we know that

$$Z = \frac{V_{rms}}{I_{rms}}$$

$$\sqrt{R^2 + X_L^2} = \frac{\left(\frac{100}{\sqrt{2}}\right)}{\left(\frac{20}{\sqrt{2}}\right)} = \frac{70.71}{14.14}$$

$$\sqrt{R^2 + (\omega L)^2} = 5$$

$$R^2 + (300L)^2 = 25 \quad -(1)$$

Also we know that

$$Z = R + jX_L = R + j\omega L$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$60^\circ = \tan^{-1}\frac{\omega L}{R}$$

$$\frac{\omega L}{R} = \tan 60^\circ = \frac{\sqrt{3}}{2}$$

$$\frac{R}{L} = \frac{300 \times 2}{\sqrt{3}} = 173.21$$

Substituting eq (2) in eq (1)

$$(173.21L)^2 + (300L)^2 = 25$$

Simplifying

$$L = 0.0144\,H.$$

Substituting L in eq (2)

$$R = 2.494\,\Omega.$$

The two elements are

$$\boxed{\begin{aligned} R &= 2.494\,\Omega \\ L &= 0.0144\,H \end{aligned}}$$

→ A voltage source $v = 50 \sin 100t$ is applied to a series RLC ckt with $R = 10\Omega$, $L = 0.1H$, $C = 100\mu F$. Determine the phase angle between current and voltage.

Sol

Given data

$$v = 50 \sin 100t \quad -① $$

where $V_m = 50V$, $\omega = 100V$

$R = 10\Omega$, $L = 0.1H$, $C = 100\mu F$

we know

$$X_L = \omega L = 100 \times 0.1 = 10\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 100 \times 10^{-6}} = 100\Omega \qquad \therefore X_C > X_L$$

$$Z = R + jX$$

$$= R + j(X_C - X_L)$$

$$= 10 + j(100 - 10)$$

$$= 10 + j90$$

in polar form

$$\bar{Z} = |Z| \angle \phi = \sqrt{10^2 + 10^2} \angle \tan^{-1} \frac{90}{10}$$

$$= 14.14 \angle 83.65$$

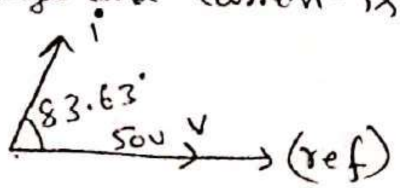
current flowing the ckt is

$$i = \frac{V_m}{Z} = \frac{50}{14.14 \angle 83.65}$$

$$i = 3.53 \angle -83.63 \quad , \phi = 83.65^\circ$$

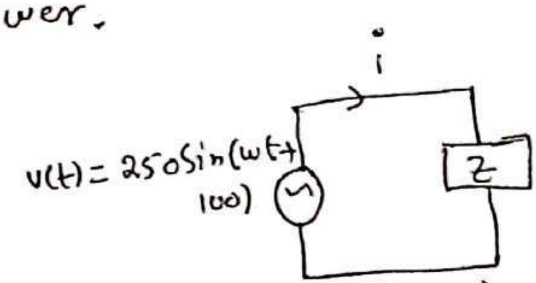
for here $X_C > X_L$, $\quad i = 3.53 \sin(100t + 83.63^\circ) \quad -②$

Angle b/w Voltage and current is $\boxed{\phi = +83.63^\circ}$



For the ckt shown in fig. a voltage $v(t)$ is applied and the resulting current in the ckt $i(t) = 15\sin(\omega t + 30^\circ)$. Determine the (1) Active power (2) Reactive power (3) power factor (4) Apparent power.

35

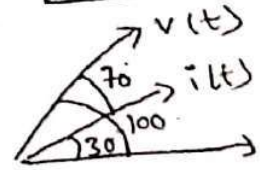


Given

$v(t) = 250\sin(\omega t + 100^\circ)$

$i(t) = 15\sin(\omega t + 30^\circ)$

Angle b/w voltage to current $\phi = 70^\circ$



(1) Active power $(P) = V_{rms} I_{rms} \cos\phi$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos\phi$$

$$= \frac{250}{\sqrt{2}} \cdot \frac{15}{\sqrt{2}} \cos 70^\circ$$

$$= 640.86 \text{ W}.$$

(2) Reactive power $(Q) = V_{rms} I_{rms} \sin\phi$

$$= \frac{250}{\sqrt{2}} \cdot \frac{15}{\sqrt{2}} \sin 70^\circ$$

$$= 1761.44 \text{ Var}$$

(3) Power factor $= \cos\phi$

$$= \cos 70^\circ$$

$$= 0.342$$

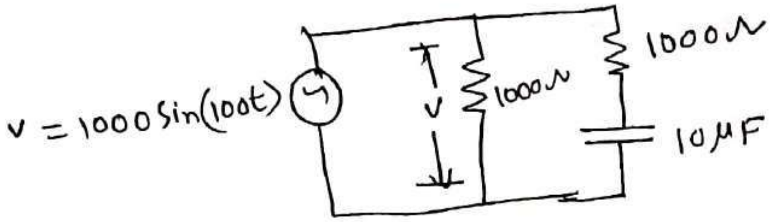
(4) Apparent power $(S) = V_{rms} I_{rms}$

$$= \frac{250}{\sqrt{2}} \cdot \frac{15}{\sqrt{2}}$$

$$= 1873.87 \text{ VA}.$$

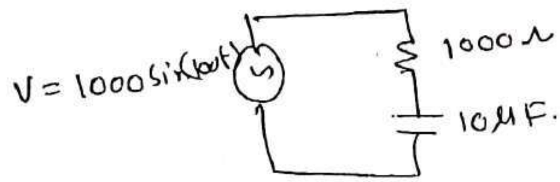
Problems on RC circuit

① In the following n/w, determine the voltage across capacitor.



Sol

From the ckt, voltage source and 1000Ω resistor are in parallel. The above ckt can be redrawn as



here $X_c = \frac{1}{\omega c} = \frac{1}{2\pi f C} = \frac{1}{100 \times 10^{-5}}$

$= \frac{1}{10^{-3}}$

$= 1000\Omega.$

$\therefore -jX_c = -j1000\Omega.$

voltage across capacitor $V_c = \frac{V_m}{\sqrt{2}} \times \frac{-j1000}{1000 - j1000}$

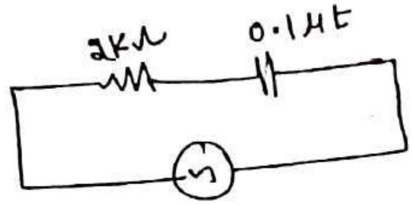
$= \frac{1000}{\sqrt{2}} \times \frac{-j\,\cancel{1000}}{\cancel{1000}(1-j)}$

$= \frac{707.10\angle -90^\circ}{1-j}$

$= \frac{707.10\angle -90^\circ}{1.414\angle -45^\circ}$

$= 499.99\angle -45^\circ$ volts.

③ A sine wave generator supplies a 10V rms, with 500Hz signal to 2KΩ resistor in series with 0.1μF capacitor as shown in fig. Det... (a) Impedance (b) current (c) phase angle (d) V_R (e) V_C (f) Draw Phasor diagram.



10 V_{rms}, 500 Hz

Sol

<u>Given</u>

$R = 2K\Omega$, $C = 0.1\mu F$

supply voltage $v = 10\ V_{rms}$.

$f = 500\ Hz$.

(a) $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$= \frac{1}{2\times 3.14\times 500\times 0.1\times 10^{-6}}$

$= 3184.71\Omega$

Impedance $(Z) = R - jX_C$

$= 2000 - j(3184.71)$

$= 3760.6\angle{-57.87}\ \Omega$

(b) current $(I) = \frac{V}{Z}$

$= \frac{10\angle 0^\circ}{3760.6\angle{-57.87}}$

$= 2.65\angle 57.87\ mA$

(c) Phase angle $\phi = \tan^{-1}\left(\frac{+X_C}{R}\right)$

$= \tan^{-1}\left(\frac{+3184.71}{2000}\right)$

$= 57.87$

(d) $V_R = iR$

$= 2.65\angle 57.87 \times 10^{-3} \times 2000$

$= 5.3\angle 57.87\ V$

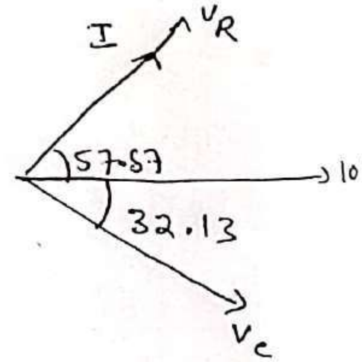
(e) $V_C = -jIX_C$

$= -j\ 2.65\times 10^{-3}\angle 57.87 \times 3184.71$

$= 2.65\times 10^{-3}\angle 57.87 \times 31..$

$= 8.43\angle{-32.13}\ V$

(f) Phasor diagram is



Problems on RLC ckt

1. A coil of resistance 10Ω and inductance 0.1H is connected in series with a 150μF capacitor across a 200V, 50Hz supply. Calculate (a) Inductive reactance (b) capacitive reactance (c) Impedance (d) current (e) voltage across coil and capacitor (f) power factor (g) power consumed (h) voltage across each element

(39)

Given data

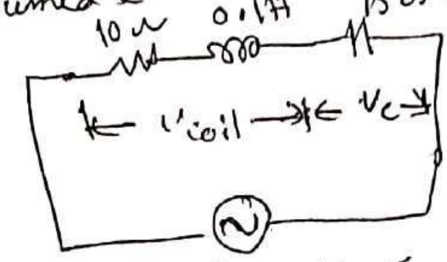
$R = 10\Omega$

$L = 0.1H$

$C = 150\mu F = 150 \times 10^{-6} F$

$V = 200V$

$f = 50Hz$



200V, 50Hz

(a) Inductive reactance

$$X_L = 2\pi f L$$
$$= 2 \times 3.14 \times 50 \times 0.1$$
$$= 31.41\Omega$$

(b) capacitive reactance

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$
$$= \frac{1}{2 \times 3.14 \times 50 \times 150 \times 10^{-6}}$$
$$= 21.22\Omega.$$

(c) Impedance

$$Z = R + j(X_L - X_C)$$
$$= 10 + j(31.41 - 21.22)$$
$$= 10 + j(10.19)$$
$$= \sqrt{10^2 + 10.19^2} \; \angle \tan^{-1}\frac{10.19}{10}$$
$$= 14.27 \angle 45.53$$

(d) current

$$i = \frac{V}{Z} = \frac{200}{14.27\angle 45.53}$$
$$i = 14.01\angle -45.53$$

(e) voltage across coil

$$V_{coil} = I\, Z_{coil}$$

where $Z_{coil} = \sqrt{R^2 + X_L^2} \angle \tan^{-1}$

$$= \sqrt{10^2 + (2\pi f \times 0.1)^2} \angle \tan^{-1}\frac{X}{R}$$
$$= \sqrt{10^2 + 31.4^2} \angle \tan^{-1}\frac{31.4}{10}$$
$$Z_{coil} = 32.96\angle 72.33$$

voltage across capacitor

$$V_C = -jX_C \cdot I$$
$$= -j\,21.22 \times 14.01\angle -45.53$$
$$= 297\angle -45.53 - 90$$
$$= 297\angle -135.53 \; V$$

(f) Power factor(ϕ) = 45.5

(g) power consumed

$$P = VI\cos\phi$$

(H) voltage across each element

$v_R = iR = 14.01 \angle{-45.53} \times 10$

$= 140.01 \angle{-45.53}$ v.

$v_L = jX_L i = j\,31.41 \times 14.01 \angle{-45.53}$

$= 31.41 \angle{90^\circ} \times 14.01 \angle{-45.53}$

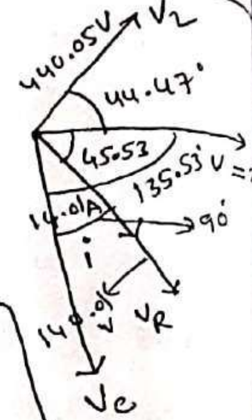
$= 440.05 \angle{44.47}$ volts

$v_C = -jX_C i = -j\,21.22 \times 14.01 \angle{-45.53}$

$= 21.22 \angle{-90} \times 14.01 \angle{-45.53}$

$= 297.29 \angle{-135.53}$

Phasor diagram



(2) A series ckt, having resistance of 10Ω and inductance of 0.25H and capacitance is connected across a 100V, 50Hz supply. If the ckt takes a current of 8A, calculate (a) Impedance (b) capacitance (c) pf and power consumed.

Sol

Given data

$R = 10\Omega$

$L = 0.25H$.

$V = 100V$ (rms).

$f = 50Hz$.

$i = 8A$ (rms)

(a) Impedance

$$Z = \frac{V}{I} = \frac{100}{8} = 12.5\Omega$$

(b) capacitance

we know

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$12.5 = \sqrt{10^2 + (2\pi \times 50 \times 0.25 - X_C)^2}$$

$$= \sqrt{10^2 + (\ldots - X_C)^2}$$

(40)

$12.5^2 = 10^2 + (78.5 - X_C)^2$

$(78.5 - X_C)^2 = 56.25$

$78.5 - X_C = \sqrt{56.25} = 7.5$

$X_C = 78.5 - 7.5 = 71 \Omega.$

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$C = \frac{1}{2\pi f X_C} = \frac{1}{2 \times \pi \times 50 \times 71}$

$= 44.83 \times 10^{-6} F$

$= 44.83 \mu F.$

(c) Power factor

$\cos\phi = \frac{R}{Z} = \frac{10}{12.5} = 0.8$ lag.

power consumed

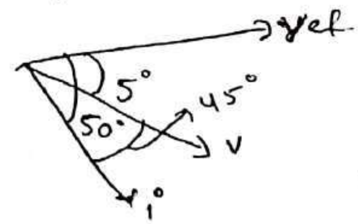
$P = VI\cos\phi$

$= 100 \times 8 \times 0.8$

$= 640 W$

3) In a series RLC ckt, L = 10 mH. The instantaneous applied voltage and current is given as $v = 100\sin(314t - 5°)$ and $i = 10\sin(314t - 50°)$. Find the resistance and capacitance.

(c) Given data



$V_m = 100V$

$L = 10mH = 10 \times 10^{-3} H.$

$I_m = 10 A$

$\omega = 314$

Angle between voltage to current is power factor

$\phi = 50 - 5 = 45°$

$X_L = 2\pi f L$

$= \omega L$

$= 314 \times 10 \times 10^{-3}$

$= 3.14\,\Omega.$

$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.71\,V$

$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.07\,A.$

$Z = \frac{V}{I} = \frac{70.71}{7.07} = 10\,\Omega.$

But we know, $\vec{Z} = R + j(X_L - X_C)$ — (1)

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$10 = \sqrt{R^2 + (3.14 - X_C)^2}$

$100 = R^2 + (3.14 - X_C)^2$ — (2)

From eq (1) $\tan\phi = \frac{X_L - X_C}{R}$

$\tan 45^\circ = \frac{3.14 - X_C}{R}$

$1 = \frac{3.14 - X_C}{R}$

$R = 3.14 - X_C$ — (3).

substicute eq (3) in eq (2).

$100 = (3.14 - X_C)^2 + (3.14 - X_C)^2$

$(3.14 - X_C)^2 = 50$

$3.14 - X_C = 7.07$

$X_C = 3.14 - 7.07$

$\boxed{X_C = -3.93}\,\Omega \Rightarrow X_C = \frac{1}{\omega C} \Rightarrow -3.93 = \frac{1}{314\,C}$

$\Rightarrow \boxed{C = -0.81\,mF}$

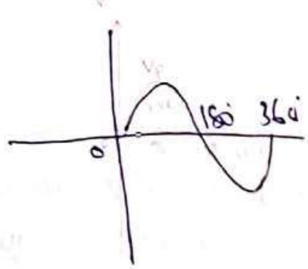
substicute X_C in eq (3)

$R = 3.14 + 3.93$

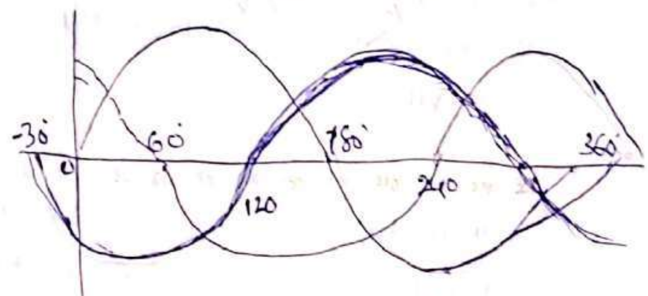
$\boxed{R = 7.07\,\Omega}$

3-Phase Circuits

Phase sequence - star & Δ connection - relationship b/w line and phase voltages & currents in balanced system - Analysis of balanced and unbalanced 3 phase circuits - measurement of active & reactive power in balanced and unbalanced 3-phase systems - loop method - application of Millimon's theorem - star-Δ transformation techique for balanced and unbalanced circuits. measurement of active and reactive power.



Single phase waveform



3-φ wave form

Difference between 3-φ & 1-φ systems:

3-φ System	1-φ System
1. Power delivered is constant 	1. Power delivered is pulsating. 
2. 3-φ induction moter is self-starting.	2. 1-φ induction motor is not self starting.
3. High starting torque	3. No starting torque
4. 3-φ can develop rotating magnetic field.	4. It is not possible to develop rotating magnetic field.
5. parallel operation is easy	5 It is difficult
6 High power factor (0.95)	6 Low powerfactor (0.75).
7. High efficiency	7. low efficiency.
8. For transmitting same amount of power & voltage, 3-φ machine gives more o/p. (×5)	8. It gives less output.
9. Little maintainance	9. Maintainance is more.
10. Less no. of turns, less insulation, Installation cost is less.	10. more no. of turns, more insulation, more cost.

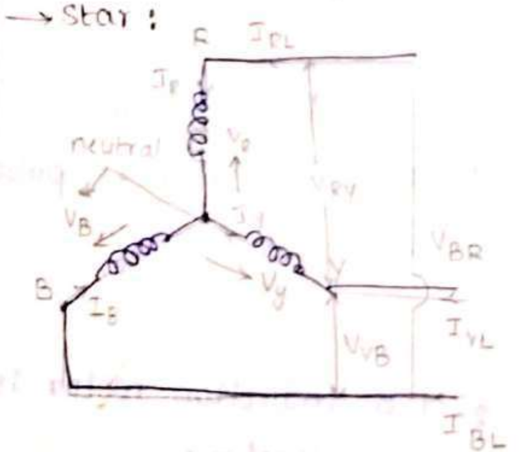
11. 3-ø motors are robust & cheap	11. 1-ø motors are not robust & cheap.
12. 3-ø motor has light in weight as compared to 1-ø motor.	12. More weight.
13. 3-ø can easily convert into 1-ø	13. It is difficult.
14. 3-ø cannot depend on 1-ø	14. It depends on 3-ø
15. 3-ø is same is used for domestic, agriculture, industries & commercial.	15. It is only suitable for domestic.
16. for 3-ø motor, the frequency of vibrations are less.	16. Vibrations are more.

Phase Sequence :

→ It is the sequence in which voltage of 3-ø reaches their maximum positive values is known as phase sequence.

* In case of 3-ø system

→ Star :

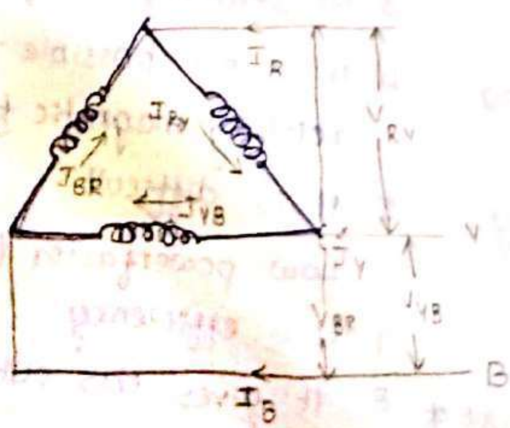


V_R, V_Y, V_B → phase vol

$I_L = I_{Ph}$

V_{RY} & V_R are different

→ delta :



I_{RY}, I_{YB}, I_{BR} → phase current

I_R & I_{RY} are different

$V_{RY} = V_{Ph}$

* Relation between phase & line voltages, phase & line currents in star connected system.

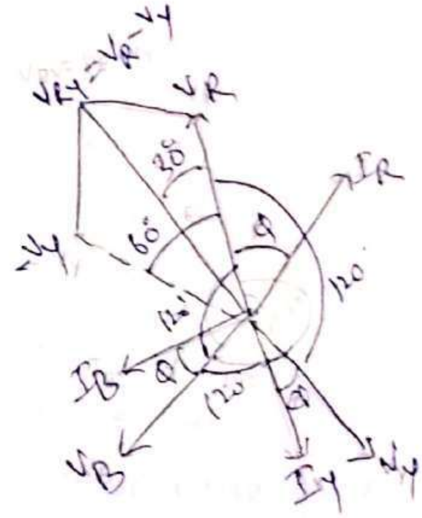
→ I_{RL}, I_{YL}, I_{BL} → line currents

I_R, I_B, I_Y → phase current

In this case,

line current = phase current.

$$\boxed{I_L = I_{Ph}} \longrightarrow (1)$$



V_{RY} → line voltage

$V_{RY} = V_R - V_Y$

$V_{YB} = V_Y - V_B$

$V_{BR} = V_B - V_R$

vector relation :

$$V_{RY} = \sqrt{V_R^2 + V_Y^2 + 2V_RV_Y\cos(V_R \hat{} V_Y)}$$

$$= \sqrt{V_R^2 + V_Y^2 + 2V_RV_Y\cos 60°}$$

$$= \sqrt{V_{Ph}^2 + V_{Ph}^2 + 2V_{Ph}V_{Ph}\,1/2}$$

$$= \sqrt{3V_{Ph}^2}$$

$$= \sqrt{3}\,V_{Ph}$$

$$\boxed{V_{Ph} = \frac{V_{RY}}{\sqrt{3}} = \frac{V_L}{\sqrt{3}}}$$

Note :

→ From the phasor diagram, the phase angle between phase voltage and line voltage is 30°.

→ The phase angle between line current (I_R) and line voltage (V_{RY}) is $30° + \phi$

→ The phase angle between I_Y & V_{RY} is $150° + \phi$.

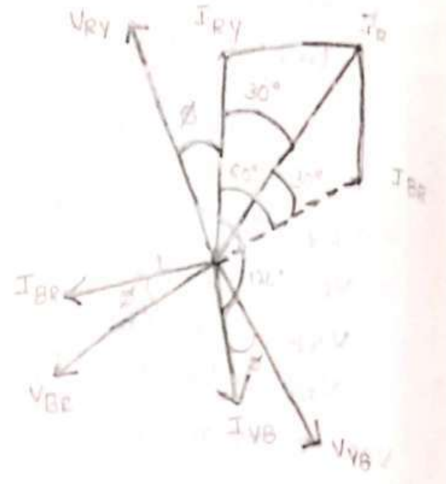
* Relation between Phase & line voltages, phase & line currents in delta connected systems.

$I_{RY}, I_{YB}, I_{BR} \longrightarrow$ phase currents

$I_R, I_Y, I_B \longrightarrow$ line currents

V_{RY}, V_{YB}, V_{BR} = line voltages = phase voltages

$$\boxed{V_L = V_{Ph}} \longrightarrow (1)$$



Apply KVL at R junction

$$I_R + I_{BR} = I_{RY}$$

$$\boxed{I_R = I_{RY} - I_{BR}}$$

Using vector relation,

$$I_R = \sqrt{I_{RY}^2 + I_{BR}^2 + 2 I_{RY} I_{BR} \cos 60^\circ}$$

$$= \sqrt{I_{Ph}^2 + I_{Ph}^2 + 2 I_{Ph} I_{Ph} \tfrac{1}{2}}$$

$$= \sqrt{3}\, I_{Ph}$$

$$I_{Ph} = \frac{I_R}{\sqrt{3}} = \frac{I_L}{\sqrt{3}} \longrightarrow (2)$$

Note :

→ In case of delta connected system the phase voltage is equal to line voltage and

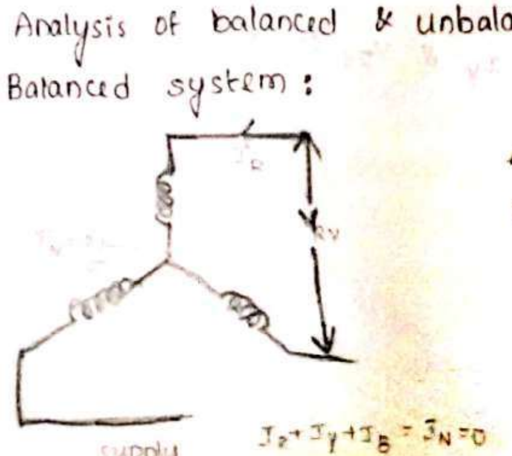
$$\text{phase current} = \frac{\text{line current}}{\sqrt{3}}$$

→ The phase angle between line current and line voltage is that means angle between I_R & V_{RY} is $30^\circ + \phi$

→ The angle between phase current (I_{RY}) and line current (I_R) is 30°.

* Analysis of balanced & unbalanced system :

1. Balanced system :

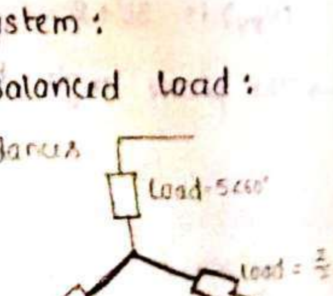


$I_R + I_Y + I_B = I_N = 0$

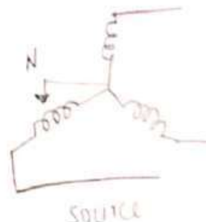
supply

2. Balanced load :

All impedances are

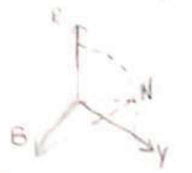


3. Unbalanced system:



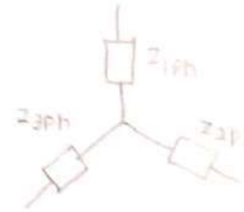
$V_{RY}, V_{YB}, V_{BR} \rightarrow$ different

$I_R, I_Y, I_B \rightarrow$ different



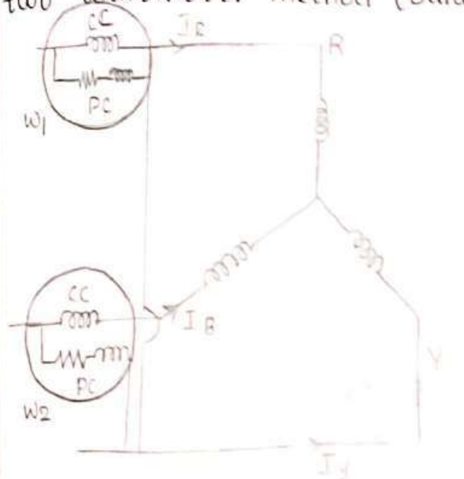
$$\boxed{I_R + I_Y + I_B = I_N}$$

4. Unbalanced Load:



$Z_{1ph}, Z_{2ph}, Z_{3ph} \rightarrow$ all are different loads.

* Measurement of active & Reactive power & power factor by two wattmeter method (balanced load).



current in current coil of $w_1 = I_R$

current in current coil of $w_2 = I_B$

voltage across pressure coil of $w_1 = V_{RY}$

voltage across pressure coil of $w_2 = V_{BY}$

Reading (power) of $w_1 = V_{RY} I_R \cos(30+\phi) \longrightarrow (1)$

Reading of $w_2 = V_{BY} I_B \cos(30^\circ-\phi) \longrightarrow (2)$

Total active power measurement $= w_1 + w_2$.

$$P = w_1 + w_2$$
$$= V_{RY} I_R \cos(30+\phi) + V_{BY} I_B \cos(30^\circ-\phi)$$
$$= V_L I_L [\cos(30+\phi) + \cos(30^\circ-\phi)]$$
$$= 2 V_L I_L \cos 30^\circ \cdot \cos\phi$$
$$= 2 V_L I_L \frac{\sqrt{3}}{2} \cos\phi$$
$$= \sqrt{3} V_L I_L \cos\phi.$$

Similarly,

$$W_2 - W_1 = V_L I [\cos(30^\circ-\phi) - \cos(30^\circ+\phi)]$$

$$\omega_2 - \omega_1 = V_L I \left[\cancel{\cos 30^\circ \cos\phi} + \sin 30 \sin\phi - \cancel{\cos 30^\circ \cos\phi} + \sin 30^\circ \sin\phi\right.$$

$$= 2 V_L I \sin 30^\circ \sin\phi \longrightarrow (4)$$

The reactive power for $3\text{-}\phi$:

$$\sqrt{3}\,(\omega_2 - \omega_1) = \frac{\sqrt{3}\, V_L I_L \sin\phi}{Q}$$

$$Q = = \sqrt{3}\,(\omega_2 - \omega_1)$$

Power factor :

$$\tan\phi = \frac{\sin\phi}{\cos\phi}$$

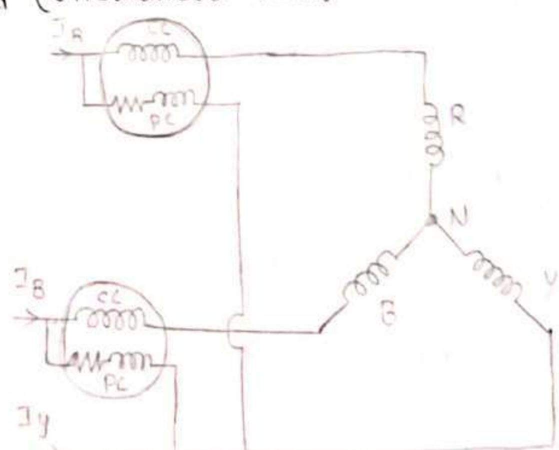
$$\tan\phi = \frac{Q}{P}$$

$$= \frac{\sqrt{3}\,(\omega_2 - \omega_1)}{\omega_1 + \omega_2}$$

$$\phi = \tan^{-1}\left(\frac{\sqrt{3}\,(\omega_2 - \omega_1)}{\omega_1 + \omega_2}\right)$$

$$\cos\phi = \cos\left[\tan^{-1} \frac{\sqrt{3}\,(\omega_2 - \omega_1)}{\omega_1 + \omega_2}\right]$$

*** Measurement of active power by using two wattmeter method (Unbalanced Load) :



Current through Load of $\omega_1 = I_R$

Current through current coil of $\omega_2 = I_B$

Voltage of PC of $\omega_1 = V_{RY}$

voltage of PC of $\omega_2 = V_{BY}$

watt meter reading $\omega_1 = V_{RY} I_R$

watt meter reading $\omega_2 = V_{BY} I_B$

Here, the circuit is unbalanced, so

$$P = \omega_1 + \omega_2$$

$$P = V_{RY} I_R + V_{BY} I_B$$

$$P = (V_{RN} - V_{YN}) I_R + (V_{BN} - V_{YN}) I_B$$

$$P = V_{RN} I_R + V_{BN} I_B - V_{YN}(I_R + I_B) \longrightarrow (1)$$

apply KCL at N,

$$I_R + I_Y + I_B = 0$$

$$I_R + I_B = -I_y \longrightarrow (2)$$

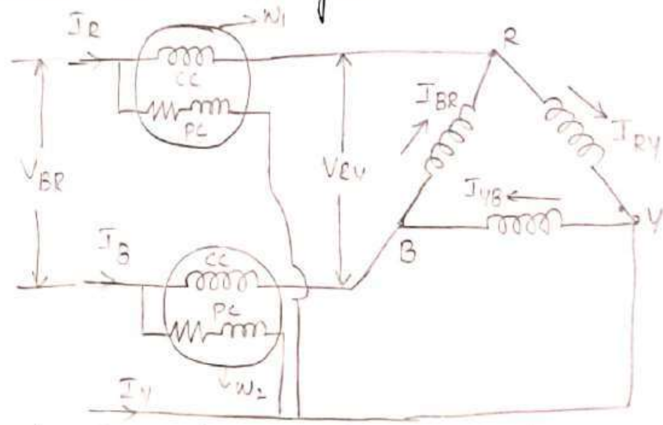
substitute eqn (2) in eqn (1)

$$P = V_{RN} I_R + V_{BN} I_B + V_{YN} I_Y$$

Therefore, these are the three instantaneous powers of all the 3-ø system.

Nill

Measurement of active power by two wattmeter method for delta connected system (unbalanced node):



Current through CC of $W_1 = I_R$

Current through CC of $W_2 = I_B$

Voltage of PC of $W_1 = V_{RY}$

Voltage of PC of $W_2 = V_{BY}$

Watt meter reading $W_1 = V_{RY} I_R$

Watt meter reading $W_2 = V_{BY} I_B$

Here, the circuit is unbalanced, so

$$P = W_1 + W_2$$

$$P = V_{RY} I_R + V_{BY} I_B \longrightarrow (1)$$

At point R:

apply KCL

$I_R + I_{BR} = I_{RY}$

$I_R = I_{RY} - I_{BR}$

At point B:

$I_B + I_{YB} = I_{BR}$

$I_B = I_{BR} + I_{YB}$

$$P = V_{RY}[I_{RY} - I_{BR}] + V_{BY}[I_{BR} - I_{YB}]$$

$$= V_{RY} I_{RY} - V_{RY} I_{BR} + V_{BY} I_{BR} - V_{BY} I_{YB}$$

$$= V_{RY} I_{RY} - V_{RY} I_{BR} - V_{YB}[I_{BR} - I_{YB}] \qquad \because V_{BY} = -V_{YB}$$

$$P = V_{RY} I_{RY} + V_{YB} I_{YB} - I_{BR}(V_{RY} + V_{YB}) \longrightarrow (2)$$

We know, summation of all the voltages (phase) are equal to zero.

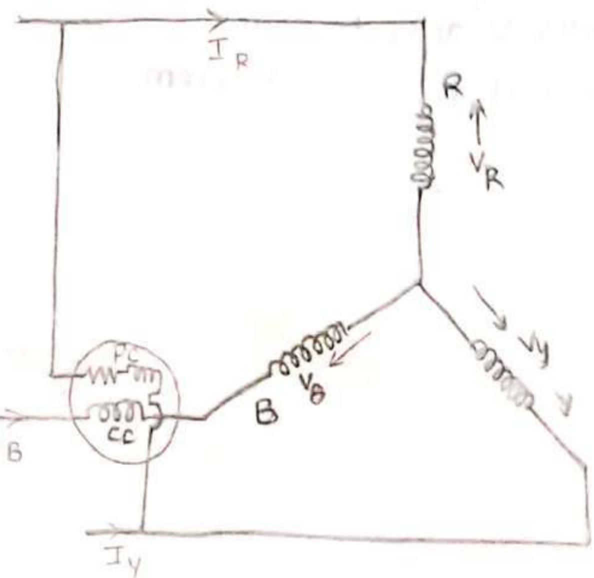
$$V_{BR} + V_{RY} + V_{YB} = 0$$

$$V_{RY} + V_{YB} = -V_{BR}$$

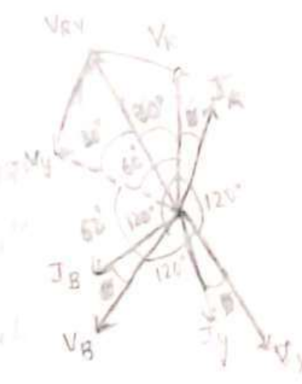
$$P = V_{RY} I_{RY} + V_{YB} I_{YB} + V_{BR} I_{BR}$$

power = instantaneous power of all the phases.

Measurement of reactive power by single wattmeter method (Balanced load):



$V_{RY} = V_R - V_Y$



$$\text{wattmeter reading} = V_{RY} I_B \cos(V_{RY} \hat{\ } I_B)$$

$$= V_L I_L \cos(V_{RY} \hat{\ } I_B)$$

$$= V_L I_L \cos(90^\circ - \theta)$$

$$= V_L I_L \sin\theta.$$

$$\text{Reactive power } (Q) = 3V_{ph} I_{ph} \sin\theta \text{ (ph)}$$

$$= \sqrt{3} V_L I_L \sin\theta \text{ (line)}$$

$$\boxed{Q = \sqrt{3}\,(\text{wattmeter reading})}$$

Analysis of unbalanced 3-ϕ loads:

→ unbalanced 3-ϕ, 3-wire star connected

→ unbalanced 3-ϕ, 4-wire star connected

→ Unbalanced delta connected load.

fig 1:

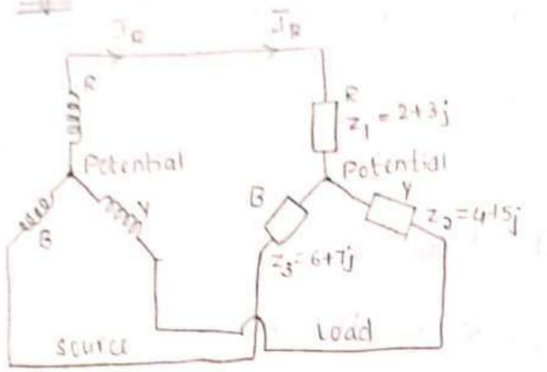
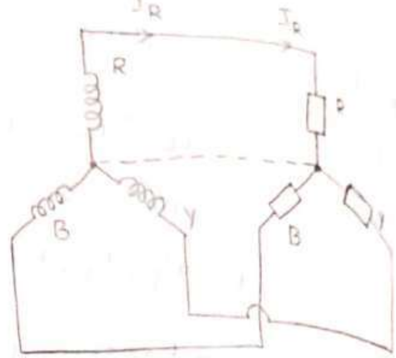


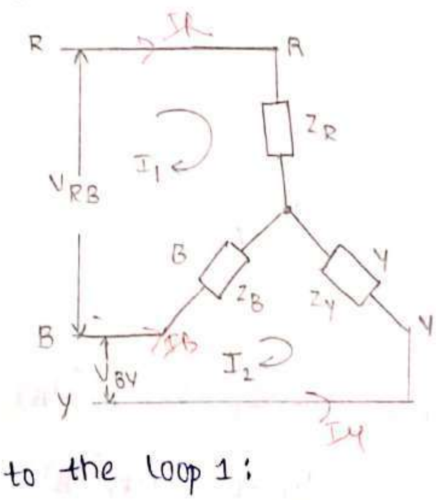
fig 2:



Source solution methods:

1. Loop method
2. Star to delta & delta to star
3. Applications of milliman's theorem

1. Loop method or Mesh method:



Apply KVL to the loop 1:

$$V_{RB} - I_1 Z_R - I_1 Z_B + I_2 Z_B = 0$$

$$V_{RB} = I_1 Z_R + I_1 Z_B - I_2 Z_B$$

$$I_1(Z_R + Z_B) + I_2(-Z_B) = V_{RB} \longrightarrow (1)$$

Apply KVL to the loop 2:

$$V_{BY} - I_2 Z_B + I_1 Z_B - I_2 Z_Y = 0$$

$$-I_1 Z_B + I_2 (Z_B + Z_Y) = V_{BY} \longrightarrow (2)$$

In matrix form,

$$\begin{bmatrix} Z_R+Z_B & -Z_B \\ -Z_B & Z_B+Z_Y \end{bmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{bmatrix} V_{RB} \\ V_{BY} \end{bmatrix} \longrightarrow (3)$$

$$\Delta = (Z_R+Z_B)(Z_B+Z_Y) - (-Z_B)(-Z_B)$$

$$\Delta_1 = \begin{bmatrix} V_{RB} & -Z_B \\ V_{BY} & Z_B+Z_Y \end{bmatrix} \qquad \Delta_2 = \begin{bmatrix} Z_R+Z_B & V_{RB} \\ -Z_B & V_{BY} \end{bmatrix}$$

$=$ $\qquad$ $=$

$$I_1 = \frac{\Delta_1}{\Delta} \qquad I_2 = \frac{\Delta_2}{\Delta}$$

from this,

$I_R = I_1$

$I_Y = -I_2$

$I_B = I_2 - I_1$

phase voltage,

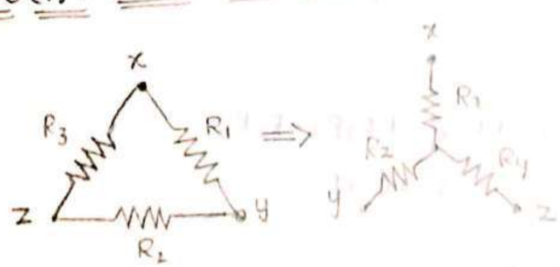
$V_R = I_R Z_R$

$V_Y = I_Y Z_Y$

$V_B = I_B Z_B$

2. Star to delta & delta to star:

Case (i): Delta to Star



From star network,

Resistance b/w x & y is $R_{xy} = R_x + R_y \longrightarrow (1)$

Resistance b/w y & z is $R_{yz} = R_y + R_z \longrightarrow (2)$

Resistance b/w z & x is $R_{zx} = R_z + R_x \longrightarrow (3)$

From delta network,

$$R_{xy} = R_1 // (R_2 + R_3)$$

$$= \frac{R_1(R_2+R_3)}{R_1+R_2+R_3} \longrightarrow (4)$$

similarly,

$$R_{yz} = \frac{R_2(R_3+R_1)}{R_1+R_2+R_3} \longrightarrow (5)$$

$$R_{zx} = \frac{R_3(R_1+R_2)}{R_1+R_2+R_3} \longrightarrow (6)$$

Equating (1,4) ; (2,5); & (3,6)

$$R_x + R_y = \frac{R_1(R_2+R_3)}{R_1+R_2+R_3} \longrightarrow (7)$$

$$R_y + R_z = \frac{R_2(R_3+R_1)}{R_1+R_2+R_3} \longrightarrow (8)$$

$$R_z + R_x = \frac{R_3(R_1+R_2)}{R_1+R_2+R_3} \longrightarrow (9)$$

Substract eqn (5) from eqn (7)

$$R_x + R_y - R_y - R_z = \frac{R_1(R_2+R_3)}{R_1+R_2+R_3} - \frac{R_2(R_3+R_1)}{R_1+R_2+R_3}$$

$$R_x - R_z = \frac{R_1R_2 + R_1R_3 - R_2R_3 - R_1R_2}{R_1+R_2+R_3}$$

$$R_x - R_z = \frac{R_1R_3 - R_2R_3}{R_1+R_2+R_3} \longrightarrow (10)$$

Add eqn (9) & eqn (10)

$$2R_x = \frac{R_1R_3 + R_2R_3 + R_1R_3 - R_2R_3}{R_1+R_2+R_3}$$

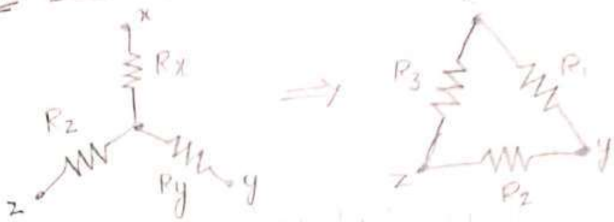
$$2R_x = \frac{2R_1R_3}{R_1+R_2+R_3}$$

$$R_x = \frac{R_1R_3}{R_1+R_2+R_3} \longrightarrow (11)$$

lly,
$$R_y = \frac{R_1R_2}{R_1+R_2+R_3}$$

$$R_z = \frac{R_2R_3}{R_1+R_2+R_3}$$

Case (ii): Star to delta



We know R_x, R_y & R_z values.

$$R_xR_y + R_yR_z + R_xR_z = \frac{R_1^2R_2R_3 + R_1R_2^2R_3 + R_1R_2R_3^2}{(R_1+R_2+R_3)^2}$$

$$= \frac{(R_1+R_2+R_3)(R_1R_2R_3)}{(R_1+R_2+R_3)^2}$$

$$R_xR_y + R_yR_z + R_xR_z = \frac{R_1R_2R_3}{R_1+R_2+R_3} \longrightarrow (12)$$

divide the eqn with R_x

$$R_y + \frac{R_yR_z}{R_x} + R_z = R_2$$

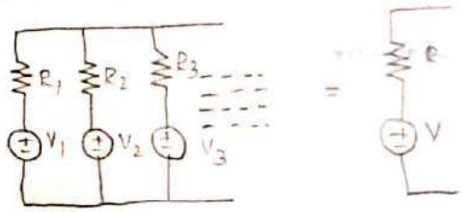
$$R_2 = R_y + R_z + \frac{R_yR_z}{R_x}$$

lly,
$$R_1 = R_x + R_y + \frac{R_xR_y}{R_z}$$

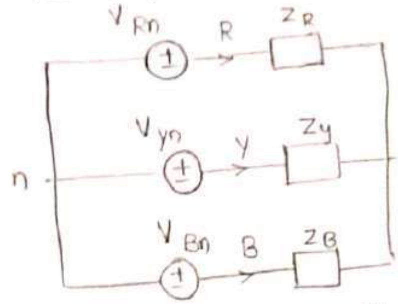
$$R_3 = R_x + R_z + \frac{R_xR_z}{R_y}$$

* Application of milliman's theorem:

→ In any network, there are no. of sources in series with resistances each are connected in parallel are replaced by single voltage source in series with single resistance.



→ The below figure shows the application of millimann's theorem to 3-ϕ unbalanced system



Potential voltage across nn' by millimann's theorem:

$$V_{nn'} = \frac{V_{Rn}Y_R + V_{yn}Y_y + V_{Bn}Y_B}{Y_R + Y_y + Y_B}$$

Here $Y_R = \frac{1}{Z_R}$, $Y_y = \frac{1}{Z_y}$, $Y_B = \frac{1}{Z_B}$

voltage across phase of load is

$$V_{Rn'} = V_{Rn} - V_{nn'}$$

$$= V_{Rn} - \left[\frac{V_{Rn}Y_R + V_{yn}Y_y + V_{Bn}Y_B}{Y_R + Y_B + Y_y}\right]$$

$$V_{yn'} = V_{yn} - V_{nn'}$$

$$V_{Bn'} = V_{Bn} - V_{nn'}$$

Phase currents :

$$I_R = \frac{V_{Rn}}{Z_R}$$

$$I_Y = \frac{V_{yn}}{Z_Y}$$

$$I_B = \frac{V_{Bn}}{Z_B} \qquad \therefore \frac{I}{Y} = V$$

Variation of wattmeter reading with PF (2 wattmeter method) :

We know,

$$W_1 = V_L I_L \cos(30+\phi)$$

$$W_2 = V_L I_L \cos(30-\phi)$$

ϕ is the power factor

Case 1 : If power factor (PF) = 0°

$$W_1 = V_L I_L \cos(30°)$$

$$W_1 = 0.866\, V_L I_L$$

lly,

$$W_2 = V_L I_L \cos(30°)$$

$$W_2 = 0.866\, V_L I_L$$

Case 2 : PF (ϕ) = 30°

$$W_1 = V_L I_L \cos 60°$$

$$= 0.5\, V_L I_L$$

$$W_2 = V_L I_L \cos 0°$$

$$= V_L I_L$$

Case 3 : PF (ϕ) = 60° , cos 60° ≐ ½ = 0.5 lagging

$$W_1 = V_L I_L \cos(30+60)$$

$$W_1 = 0$$

$$W_2 = V_L I_L \cos(-30°)$$

$$= 0.866\, V_L I_L$$

Case 4 : PF (ϕ) = 90°

$$W_1 = V_L I_L \cos(30°+90°)$$

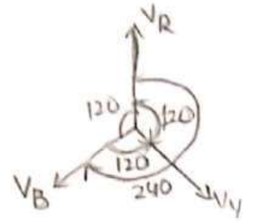
$$= -0.5\, I_L V_L$$

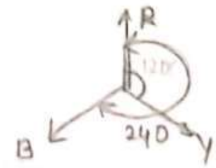
$$W_2 = V_L I_L \cos(30°-60°)$$

$$= 0.5\, V_L I_L$$

S.No	PF angle	Power Factor	W_1	W_2
1.	0°	cos 0° = 1° lag	0.866	0.866
2.	30°	cos 30° = 0.866 lag	0.5	1
3.	60°	cos 60° = 0.5 lag	0	0.866
4.	90°	cos 90° = 0° lag	−0.5	0.5

Generation of 3-ϕ voltage :





$$V_R = V_R \angle 0°$$

$$V_Y = V_R \angle -120°$$

$$V_B \Rightarrow V_R \angle -240 = V_B \angle 120$$

Problems on relationship between phase and line voltage.

1. Three inductive coils each having a resistance of 16Ω and reactance of 12Ω are connected in Y across a 400 V in 3-ϕ, 50 Hz supply, calculate :

(a) Line voltage
(b) Phase voltage
(c) Line current
(d) Phase current
(e) Power factor
(f) Power absorbed
(g) Draw the phasor diagram

sol:

(a) Line voltage :

Line voltage = 400 V

$V_{RY} = 400\angle 0$, $V_{YB} = 400\angle -120°$, $V_{BR} = 400\angle 120°$.

(b) Phase voltage :

In star connection

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.9\,V$$

(c) Phase current (I_{ph}) :

$$I_{ph} = \frac{V_{ph}}{Z_{ph}}$$

$$= \frac{230.9}{16+j12}$$

$$I_{ph} = \frac{230.9}{\sqrt{16^2+12^2}\ \tan^{-1}\left(\frac{12}{16}\right)}$$

$$= \frac{230.9}{20 \angle 36.86}$$

$$= 11.54 \angle -36.86$$

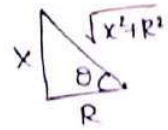
(d) Line current:

In star connection, $I_L = I_{ph}$

$$I_L = 11.54 \angle -36.86^\circ$$

(e) Power factor ($\cos\phi$):

$$\cos\phi = \frac{R}{\sqrt{R^2+X^2}}$$



$$= \frac{16}{20}$$

$$= 0.8$$

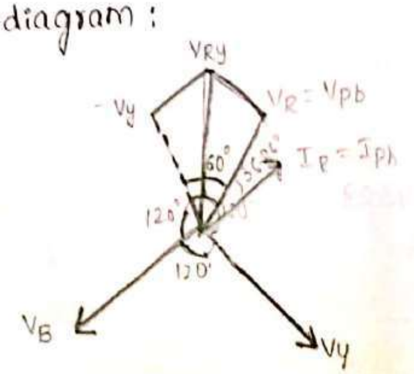
(f) Power absorbed:

$$P = \sqrt{3} V_L I_L \cos\phi$$

$$= \sqrt{3} \times 400 \times 11.54 \times 0.8$$

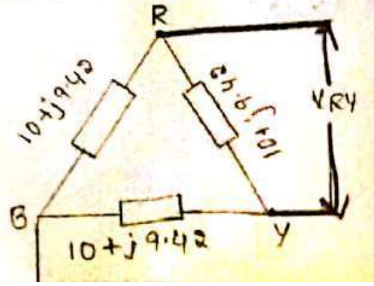
$$= 6396.11 \text{ W}$$

(g) Phasor diagram:



2 Three identical coils each having resistance of 10Ω and inductance of 0.03 H are connected in Δ across a 3-φ 400V, 50 Hz ac supply. Calculate

(a) Line voltage
(b) Phase voltage
(c) Line current
(d) Phase current
(e) Power factor
(f) Power absorbed
(g) Draw the phasor diagram



Sol. Given data,

$R = 10\Omega$

$L = 0.03\,H$

$X_L = 2\pi f L$

$= 2\pi \times 50 \times 0.03$

$= 9.42$

$Z = R + jX_L$

$= 10 + j9.42$

Star connection:

$I_{line} = I_{ph}$

$V_{ph} = \frac{V_L}{\sqrt{3}}$

Δ connection:

$V_{ph} = V_L$

$I_{ph} = \frac{I_L}{\sqrt{3}}$

(a) Line voltage : 3

$V_L = 400\,V$

(b) Phase voltage :

$V_{ph} = V_L$

$V_{ph} = 400\,V$

(c) Line current :

$I_L = \sqrt{3} I_{ph}$

$= \sqrt{3} \times 29.11 \angle -43.28$

$= 50.41 \angle -43.28$

(d) Phase current ;

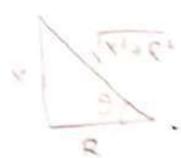
$I_{ph} = \frac{V_{ph}}{Z_{ph}}$

$= \frac{400}{10 + 9.42j}$

$= 50.41 \angle -43.28$

(e) Power Factor ($\cos\phi$) :

$\cos\phi = \frac{R}{\sqrt{R^2 + X^2}}$



$= \frac{10}{\sqrt{10^2 + (9.42)^2}}$

$= 0.727$

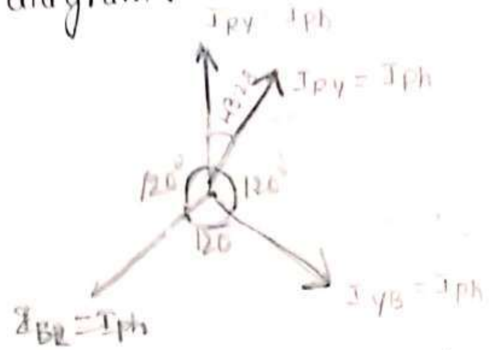
(f) Power absorbed :

$P = \sqrt{3} V_L I_L \cos\phi$

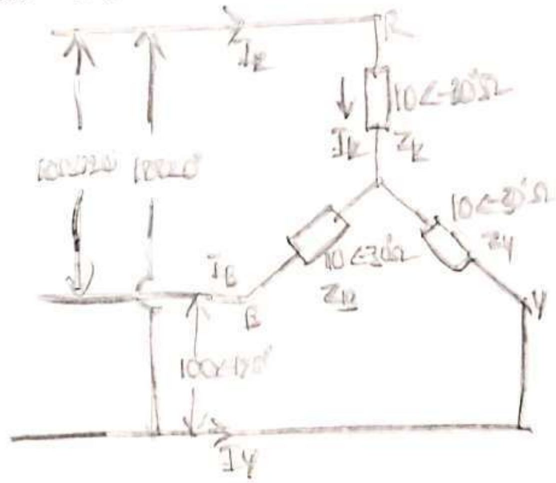
$P = \sqrt{3} \times 400 \times 50.41 \times 0.727$

$P = 25390.52\,W$

(9) Phasor diagram :



3. A 3-Ø 4 wire 100 W (L-L) the system supplied a balanced star connected load having impedances of an 10 an angle ∠-30° at each phase. Find the line currents and also draw the phasor diagram and how much current is flowing in the neutral.



RYB phase sequence,

Let $V_{RY} = 100\angle 0°$

$V_{YB} = 100\angle -120°$

$V_{BR} = 100\angle 120°$

In case of star,

Phase current $I_R = \frac{V_{ph}}{Z}$

$= \frac{\frac{V_{RY}\angle 0}{\sqrt{3}}}{10\angle -30°}$

$I_R = 5.77\angle 30°$

Phase current $I_Y = \frac{\left(\frac{V_{YB}}{\sqrt{3}}\right)}{10\angle -30°}$

$= 5.77\angle -90°$.

Phase current $I_B = \frac{V_{BR}}{\sqrt{3}10\angle-30^\circ}$

$= 5.77\angle150^\circ$

In case of star :

Line current = Phase currents

Current in the neutral wire :

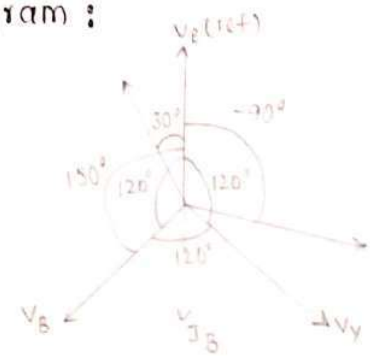
$I_R + I_y + I_B + I_N = 0$

$I_N = -(I_R + I_y + I_B)$

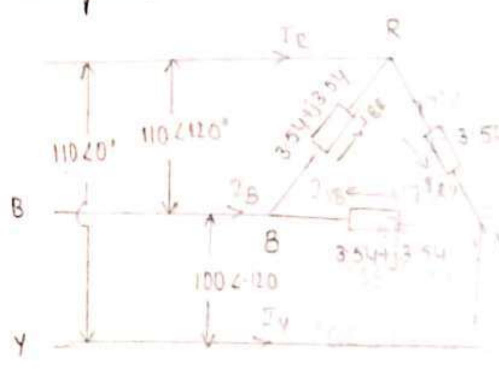
$I_N = -(5.77\angle30^\circ + 5.77\angle-90^\circ + 5.77\angle150^\circ)$

$I_N = 0$

Phasor diagram :



4. A 3-Ø balanced system supplied a 110V to delta connected load, phase impedances are equal to $3.54 + j3.54\Omega$ Determine the phase current, line currents and draw the phasor diagram.



Take RYB phase sequence.

In case Δ connected load :

$V_L = V_{ph}$ $\qquad V_{BR} = 110\angle120^\circ$

$V_{RY} = 110\angle0^\circ$

$V_{YB} = 110\angle-120^\circ$

$$Z = 3.54 + j\,3.54$$

$$= \sqrt{3.54^2 + 3.54^2} \angle \tan^{-1}\frac{3.54}{3.54}$$

$$= 5 \angle 45^\circ$$

Phase current :

$$I_{RY} = \frac{V_{RY} \angle 0^\circ}{Z}$$

$$= \frac{110 \angle 0^\circ}{5 \angle 45^\circ}$$

$$= 22 \angle -45^\circ$$

lly, phase current $I_{YB} = \frac{V_{YB}}{Z}$

$$= \frac{110 \angle -120^\circ}{5 \angle 45^\circ}$$

$$= 22 \angle -165^\circ$$

Phase current $I_{BR} = \frac{V_{BR}}{Z}$

$$= \frac{110 \angle 120^\circ}{5 \angle 45^\circ}$$

$$= 22 \angle 75^\circ$$

Line current :

At point R, apply KCL

$$I_R + I_{BR} = I_{RY}$$

$$I_R = I_{RY} - I_{BR}$$

$$I_R = 22 \angle -45^\circ - 22 \angle 75^\circ$$

$$I_R = 22\sqrt{3} \angle -75^\circ$$

$$I_R = 38.10 \angle -75^\circ$$

At point Y, apply KCL

$$I_Y + I_{RY} = I_{YB}$$

$$I_Y = I_{YB} - I_{RY}$$

$$= 22 \angle -165^\circ - 22 \angle -45^\circ$$

$$= 38.1 \angle 165^\circ$$

At point B apply KCL

$$I_B + I_{YB} = I_{BR}$$

$$I_B = I_{BR} - I_{YB}$$

$$= 22\angle 75^\circ - 22\angle -165^\circ$$

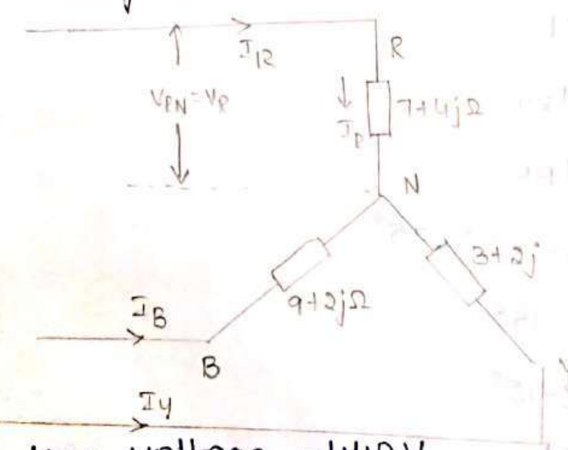
$$= 38.1\angle 45^\circ$$

Phasor diagram :

5. The impedance of $7+4j\,\Omega$, $3+2j\,\Omega$, and $9+2j\,\Omega$ are connected between neutral and RYB phases, the line voltage is 440V. Calculate the

(a) Line currents (b) current in the neutral line

(c) Find the power consumed in each phase and total power drawn by the circuit.



Given line voltage $= 440\,V$

phase voltage $= \frac{440}{\sqrt{3}}$

$= 254\,V$

$V_{RN} = 254\angle 0^\circ$ ref

$V_{YN} = 254\angle -120^\circ$

$V_{BN} = 254\angle 120^\circ$

In case of star,

phase currents = line currents

$$I_{ph} = I_L$$

$$I_R = \frac{V_{RN}}{Z_R} = \frac{254\angle 0^\circ}{7+4j}$$

$$I_R = 31.5\ \angle -29.7$$

$$I_y = \frac{V_{yN}}{Z_y} = \frac{254\angle -120^\circ}{3+2j}$$

$$I_y = 70.4\ \angle -153.6$$

$$I_B = \frac{V_{BN}}{Z_B} = \frac{254\angle 120}{9+2j}$$

$$= 27.5\ \angle 107.47$$

2. Current in the neutral wire

$$I_N = -(I_R + I_y + I_B)$$

$$I_N = 48.53\angle 25.2$$

3. Power consumed at each phase

$$P_R = I_R^2 \times R_R$$

$$= 31.5^2 \times 7$$

$$= 6945.75\ W$$

$$P_y = I_y^2 \times y_R$$

$$= 70.4^2 \times 3$$

$$= 14868.48\ W$$

$$P_B = I_B^2 \times B_R$$

$$= 27.5^2 \times 9$$

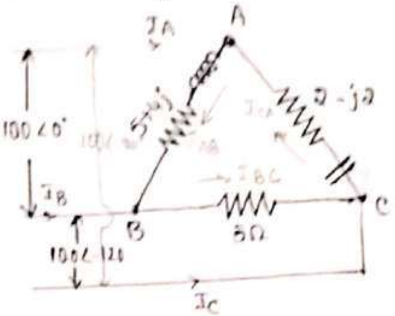
$$= 6806.25\ W$$

4. Total power:

$$P_T = P_R + P_y + R_B$$

$$= 28620.48$$

6 The network shown in figure, calculate the line currents and phase currents and also find power consumed in each phase. If the phase sequence is ABC



June. 2013

Sol: Let us take,

I_{AB}, I_{BC}, I_{CA} are the phase currents.

I_A, I_B, I_C are line Currents

$V_{AB} = 100\angle 0^\circ$, $V_{BC} = 100\angle -120^\circ$, $V_{CA} = 100\angle 120^\circ$.

Phase currents:

$$I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{100\angle 0^\circ}{5+4j} = 15.61\angle -38.6$$

$$I_{BC} = \frac{V_{BC}}{Z_{BC}} = \frac{100\angle -120}{5} = 20\angle -120^\circ$$

$$I_{CA} = \frac{V_{CA}}{Z_{CA}} = \frac{100\angle 120^\circ}{2-2j} = 35.35\angle 165$$

Line currents:

At A, use KCL

$$I_A + I_{CA} = I_{AB}$$

$$I_A = I_{AB} - I_{CA}$$

$$I_A = 50\angle -22.17$$

lly, $I_B = I_{BC} - I_{AB}$

$= 23.45\angle -161.14$

$I_C = I_{CA} - I_{BC}$

$= 35.828\angle 132.37$

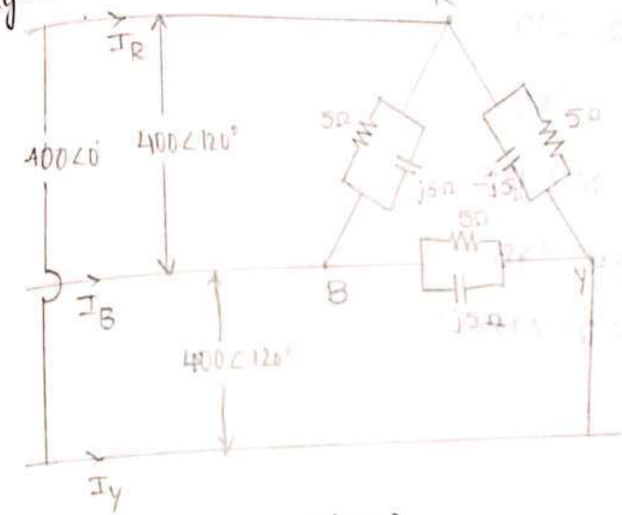
Power Consumed in each phase:

$P_{AB} = I_{AB}^2 R_{AB}$

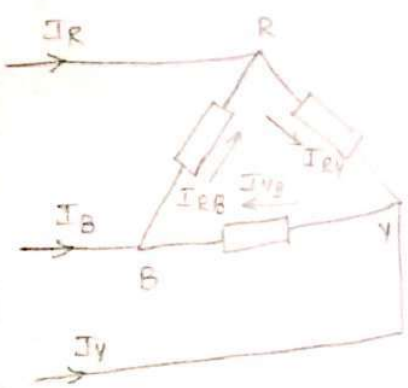
$= 1218.36\ W$

$$P_{BC} = I^2_{BC} R_{BC}$$
$$= 2000\ W$$
$$P_{CA} = I_{CA}^2 R_{CA}$$
$$= 2499.24\ W$$

A Δ connected has a parallel combination of resistance 5Ω & capacitance of $-j5\Omega$ in each phase. If a balanced $3\text{-}\phi$ 400 V supply is applied between lines. Find the phase currents & line currents and draw the phasor diagram. Phase sequence is RYB.



$$Z_{RY} = Z_{Ph} = 5 \| (-j5)$$
$$= \frac{5 \times (-j5)}{5-5j}$$
$$= \frac{-25j}{7.07\angle{-45}}$$
$$= 3.5361\ \angle{-45}$$



Phase currents:

$$I_{RB} = \frac{V_{RB}}{Z_{RB}} = \frac{400\angle 120^\circ}{3.53\angle -45^\circ} = 113.314\angle 165$$

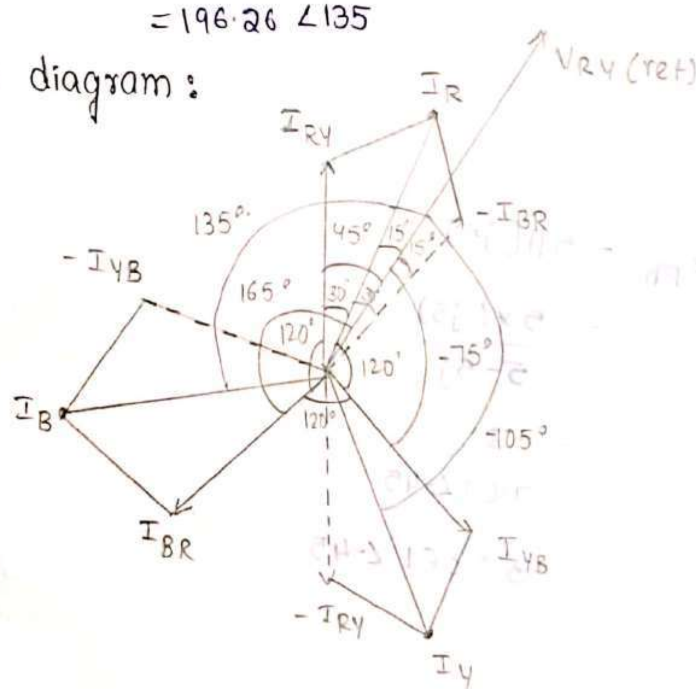
$$I_{YB} = \frac{V_{YB}}{Z_{YB}} = \frac{400\angle -120^\circ}{3.53\angle -45^\circ} = 113.314\angle -75$$

$$V_{RY} = \frac{V_{BRY}}{Z_{BRY}} = \frac{400\angle 0^\circ}{3.53\angle -45} = 113.314\angle 45$$

Line currents:

$$I_R = I_{RY} - I_{BR}$$
$$= 196.26\angle 15$$
$$I_Y = I_{YB} - I_{RY}$$
$$= 196.26\angle -105$$
$$I_B = I_{BR} - I_{YB}$$
$$= 196.26\angle 135$$

Phasor diagram:



Problems on Two-Wattmeter method:

1. Two-wattmeters are connected to the measure ~~input~~ output of 15Hp, 50 Hz 3-ϕ induction motor at full load. Full load efficiency and power factors are 0.9 & 0.8 lagging respectively. Find the readings of wattmeter.

Given data,

$$\text{Efficiency } (\eta) = 0.9$$

$$P.F = 0.8$$

$$\cos\phi = 0.8$$

$$\phi = \cos^{-1}(0.8)$$

$$= 36.86^\circ$$

$$P_{out} = 15 \text{ Hp}$$

$$= 15 \times 735.5 \text{ W}$$

$$\% \eta = \frac{P_{out}}{P_{in}}$$

$$0.9 = \frac{15 \times 735.5}{P_{in}}$$

$$P_{in} = 12258.33 \text{ W}$$

We know,

$$P_{in} = \sqrt{3} V_L I_L \cos\phi$$

$$12258.33 = \sqrt{3} V_L I_L (0.8)$$

$$V_L I_L = \frac{12258.33}{\sqrt{3}(0.8)}$$

$$\boxed{V_L I_L = 8846.68 \text{ W}}$$

Readings of wattmeters,

$$W_1 = V_L I_L \cos(30 + \phi)$$

$$= 3476.5 \text{ W}$$

$$W_2 = V_L I_L \cos(30^\circ - \phi)$$

$$= 8783.17 \text{ W}$$

Scanned with CamScanner

2. Two wattmeters are connected to the measure input balanced three phase circuit indicates 2000 W and 500 W respectively. Find the power factor of the circuit

(a) When both readings are +ve

(b) When the later is obtained after reversing the connection to the current coil of one instrument.

Sol. Highest $W_2 = 2000\ W$

Lowest $W_1 = 500\ W$

Case (a): When both readings are +ve

$$W_1 = 500\ W,\ W_2 = 2000\ W$$

$$\text{Power Factor}(\cos\phi) = \cos\left[\tan^{-1}\left(\frac{\sqrt{3}(W_2-W_1)}{W_2+W_1}\right)\right]$$

$$= \cos(46.1021)$$

$$= 0.6934$$

Case (b): When later is reversing

$$W_2 = 2000\ W,\ W_1 = -500\ W$$

$$\text{Power Factor}(\cos\phi) = \cos\left[\tan^{-1}\left(\frac{\sqrt{3}(2500)}{1500}\right)\right]$$

$$= \cancel{\cos(0.0378)}$$

$$= \cos(70.8934)$$

$$= 0.3273$$

3. The two wattmeter method is used to measure power in a 3-ϕ Load. Supply voltage is 440 V. The wattmeter readings are 400 W and -35 W respectively. Calculate:

(a) Total active power

(b) Power factor

(c) Reactive power

(d) Line current

Sol: Given,

$W_2 = 400\ W$

$W_1 = -35\ W$, Supply voltage = 440 V

(a) Total active power $(P) = W_2 + W_1$

$$= 365\ W$$

(b) power factor ($\cos\phi$).

$$\phi = \tan^{-1}\left(\frac{\sqrt{3}(W_2 - W_1)}{W_1 + W_2}\right)$$

$$= 64.1524$$

$$\cos\phi = 0.436$$

(c) Reactive power $(Q) = \sqrt{3}(W_2 - W_1)$

$$= 753.4421 \text{ Var}$$

(d) $P = \sqrt{3} V_L I \cos\phi$

$$I = \frac{P}{\sqrt{3} V_L \cos\phi}$$

$$= \frac{365}{\sqrt{3} \times 440 \times 0.436}$$

$$= 1.0984 \text{ A}$$

4 A 3-ϕ 400 V load has a power factor of 0.4. Two wattmeters are connected to measure the power. If the input power be 10 KW. Find the reading of each instrument.

Sol: Given,

$$\text{input power} = W_2 + W_1 = 10 \text{ KW} \longrightarrow (1)$$

$$\text{Power factor } (\cos\phi) = 0.4$$

$$\phi = 66.422$$

$$\tan\phi = \frac{\sqrt{3}(W_2 - W_1)}{10}$$

$$\frac{2.291 \times 10}{\sqrt{3}} = W_2 - W_1$$

$$13.229 = W_2 - W_1 \longrightarrow (2)$$

adding (1) & (2)

$$W_2 = \frac{23.229}{2} \text{ KW} = 11.615$$

$$W_1 = -1.615 \text{ KW}.$$

Dr. A. Hema Sekhar

Electrical circuit Analysis([redacted])

Unit-1: Locus Diagrams and Resonance

Series R-L, R-C, R-L-C and Parallel Combination with variation of various Parameters - Resonance - Series, Parallel circuits, Frequency Response, concept of bandwidth and Q Factor

Introduction:

For a particular circuit like R-L, R-C, R-L-C. If any one of the element is variable then depending upon the value of the variable element circuit characteristic changes then circuit parameters like voltage, current and power consumed by the element is also changes.

Def of Locus diagram:

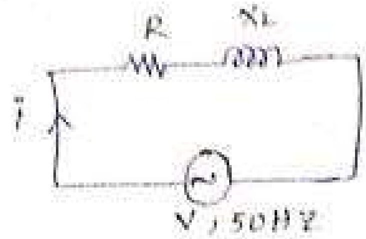
"It is defined as the Locus of the current obtained for various values of the variable element."

classification

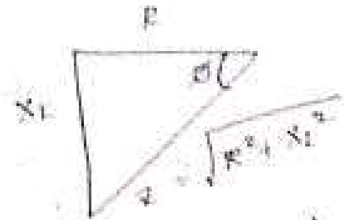
Locus diagrams are classified into two types

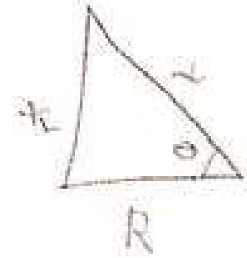
1. Series R-L, R-C and R-L-C circuits.
2. Parallel combination of circuits.

Locus diagram of R-L Circuit :-



Impedence diagram





According to KVL

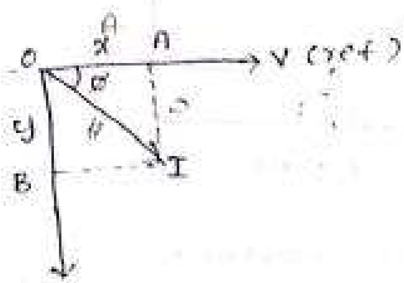
$V = I(R + jX_L)$

$V = IZ \quad ; \quad Z = R + jX_L$

$I = \frac{V}{Z} \longrightarrow (1)$

Now, phasor diagram for R-L circuit is

$(x = R, \; y =)$



From the phasor diagram

$\cos\phi = \frac{x}{I}$

$x = I\cos\phi \longrightarrow (2)$

$\sin\phi = \frac{OB}{I} = \frac{y}{-I}$

$y = -I\sin\phi \longrightarrow (3)$

$x^2 + y^2 = I^2\cos^2\phi + I^2\sin^2\phi$

$x^2 + y^2 = I^2(\cos^2\phi + \sin^2\phi)$

$$x^2+y^2 = I^2$$

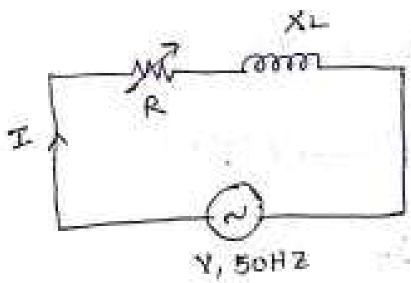
$$x^2+y^2 = \left(\frac{V}{Z}\right)^2$$

$$x^2+y^2 = \frac{V^2}{Z^2}$$

$$x^2+y^2 = \frac{V^2}{\left(\sqrt{R^2+X_L^2}\right)^2}$$

$$\boxed{\therefore x^2+y^2 = \frac{V^2}{R^2+X_L^2}} \longrightarrow (4)$$

Case(i):- Variable R, constant X_L



Here X_L is constant

$$y = -I\sin\phi$$

$$= -\frac{V}{Z} \times \frac{X_L}{Z}$$

$$= \frac{-VX_L}{R^2+X_L^2}$$

$$y = -X_L \times \frac{V}{R^2+X_L^2} \longrightarrow (5)$$

$$\frac{V}{R^2+X_L^2} = \frac{-y}{X_L} \longrightarrow (6)$$

Substitute Eqn (6) in Eqn (4)

$$x^2+y^2 = V \times \frac{V}{R^2+X_L^2}$$

$$x^2+y^2 = V \times \frac{-y}{X_L}$$

$$x^2+y^2+2 \cdot y \cdot \frac{V}{2X_L} + \left(\frac{V}{2X_L}\right)^2 - \left(\frac{V}{2X_L}\right)^2 = 0$$

$$x^2+\left(y^2+\frac{V}{2X_L}\right)^2 = \left(\frac{V}{2X_L}\right)^2$$

$$\therefore x^2+\left(y+\frac{V}{2X_L}\right)^2 = \left(\frac{V}{2X_L}\right)^2 \longrightarrow (7)$$

Now we know the circle equation

$$(x-x_1)^2+(y-y_1)^2 = r^2 \longrightarrow (8)$$

comparing eqn's (7) & (8), we get.

$x_1 = 0,$

$$y_1 = \frac{-V}{2X_L}$$

$$r = \frac{V}{2X_L}$$

$$\therefore \text{centre} = (x_1, y_1) = \left(0, \frac{-V}{2X_L}\right)$$

$$\text{radius } (r) = \frac{V}{2X_L}$$

Construction of locus diagram :-

$$\cos\phi = \frac{R}{Z} \quad \& \quad I = \frac{V}{Z} \Rightarrow I = \frac{V}{\sqrt{R^2+X_L^2}}$$

Suppose if $\phi = 30^\circ$ then $\cos 30^\circ = 0.866$

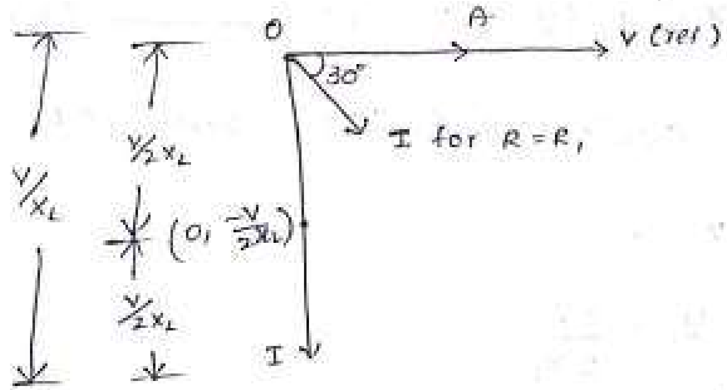
$\phi = 45^\circ$ then $\cos 45^\circ = 0.707$

$\phi = 60^\circ$ then $\cos\phi = 0.5$

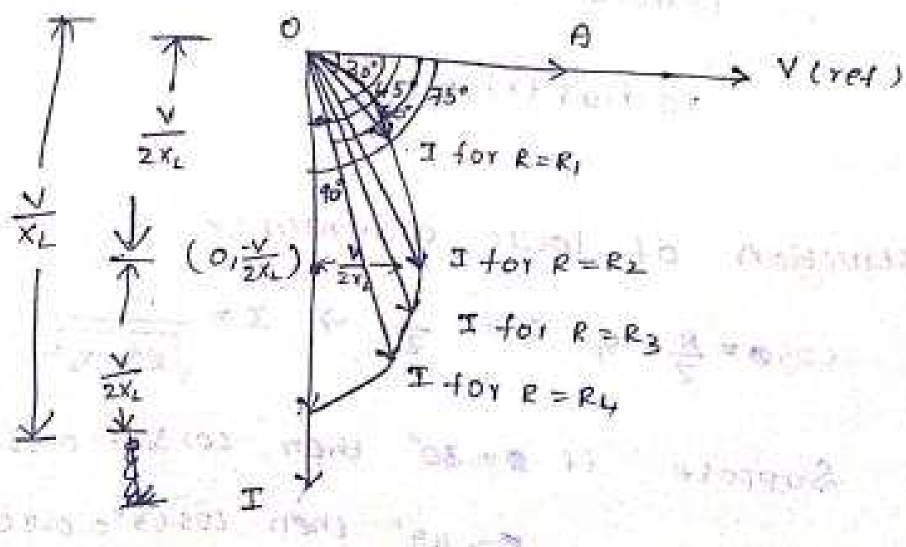
$\phi = 75°$ then $\cos 75° = 0.25$

$\phi = 90°$ then $\cos 90° = 0$

If $\cos 30° = 0.866$ and compared due to other angle it is high value and we know that R is directly proportional to $\cos\phi$ and 'R' value is high then if we put high value of 'R' in current eqn then the magnitude of current is less. that means at 30° magnitude of current is less.



If angle increases 'R' value decreases then current values increases.



Case (ii) :- constant R, variable X_L

We know

$$x^2 + y^2 = \frac{V^2}{R^2+X_L^2} \longrightarrow (4).$$

Here constant 'R'.

$$x = I\cos\phi$$

$$= \frac{V}{\sqrt{R^2+X_L^2}} \times \frac{R}{\sqrt{R^2+X_L^2}}$$

$$x = \frac{V \cdot R}{R^2+X_L^2}$$

$$\frac{V}{R^2+X_L^2} = \frac{x}{R} \longrightarrow (5)$$

sub eqn (5) in eqn (4)

$$x^2 + y^2 = \frac{Vx}{R}$$

$$x^2+y^2 - Vx/R = 0$$

$$x^2+y^2 - 2x \cdot \frac{V}{2R} = 0$$

$$x^2+y^2 - 2x \cdot \frac{V}{2R} + \left(\frac{V}{2R}\right)^2 - \left(\frac{V}{2R}\right)^2 = 0$$

$$\left(x \mp \frac{V}{2R}\right)^2 + y^2 - \left(\frac{V}{2R}\right)^2 = 0$$

$$\left(x \mp \frac{V}{2R}\right)^2 + y^2 = \left(\frac{V}{2R}\right)^2 \longrightarrow (6)$$

Now we know the circle equation

$$(x - x_1)^2 + (y - y_1)^2 = r^2 \longrightarrow (7)$$

comparing eqn's (6) & (7), we get

$$\text{centre} = (x_1, y_1) = \left(\frac{V}{2R}, 0\right)$$

$$\text{Radius } (r) = \frac{V}{2R}.$$

Here also one of the element in the centre is zero. So locus diagram is a semicircle.

Construction of locus diagram :-

$$\sin\phi = \frac{X_L}{\sqrt{R^2 + X_L^2}}, \quad I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

Case (i):- Constant R, variable X_L

$$\text{Centre} = \left(\frac{V}{2R}, 0\right)$$

$$\text{radius} = \frac{V}{2R}$$

here variable is X_L, so

$$\sin\phi = \frac{X_L}{Z} \qquad I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

$\phi = 30°$, $\sin 30 = 0.5$

$\phi = 45°$, $\sin 45° = 1/\sqrt{2} = 0.707$

$\phi = 60°$, $\sin 60° = 0.866$

$\phi = 75°$, $\sin 75° = 0.965$

$\phi = 90°$, $\sin 90° = 1$

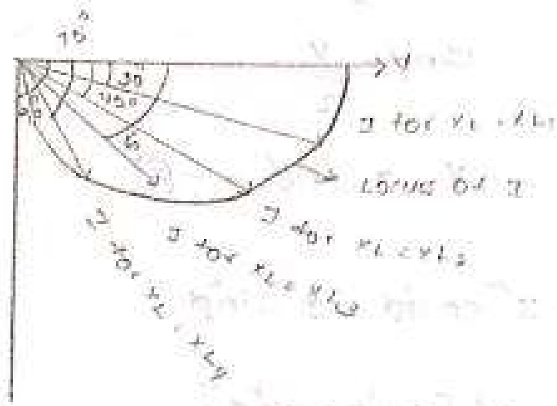
$\phi = 30°$, X_L is small, I is large

$\phi = 45°$, X_L is high, I is small

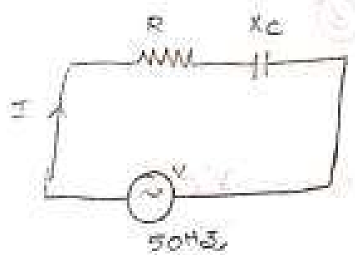
$\phi = 60°$, X_L is very high, I is very small

$\phi = 75°$, X_L is vv high, I is vv small

$\phi = 90°$,



* Locus diagram of R-C circuit :-

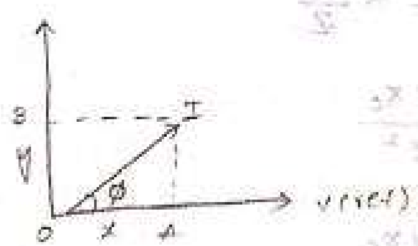


$$V = I(R + jX_C)$$

$$V = IZ$$

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + X_C^2}}$$

Phasor diagram :-



$$\cos\phi = \frac{OA}{I} = \frac{x}{I}$$

$$x = I\cos\phi \quad \text{—} \ ①$$

$$\sin\phi = \frac{OB}{I}$$

$$\sin\phi = \frac{y}{I}$$

$$y = I\sin\phi \quad — ②$$

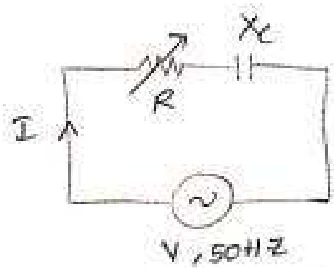
$$x^2 + y^2 = I^2\cos^2\phi + I^2\sin^2\phi$$

$$= I^2[\cos^2\phi + \sin^2\phi]$$

$$= I^2(1)$$

$$\boxed{x^2 + y^2 = \frac{V^2}{R^2 + X_C^2}} \quad — ③$$

Case(i) :- <u>Variable R or constant X_C :-</u>



Here X_C is constant

$$y = I\sin\phi$$

$$y = \frac{V}{Z} \times \frac{X_C}{Z}$$

$$y = \frac{V X_C}{Z^2}$$

$$y = \frac{V X_C}{R^2 + X_C^2}$$

$$y = X_C \times \frac{V}{R^2 + X_C^2} \longrightarrow (4)$$

$$\frac{V}{R^2+X_c^2} = \frac{Y}{X_c} \longrightarrow (5)$$

Sub Eqn (5) in Eqn (3)

$$X^2+Y^2 = V \times \frac{Y}{X_c}$$

$$X^2+Y^2 - V \times \frac{Y}{X_c} = 0$$

$$X^2+Y^2 - 2V \times \frac{Y}{2X_c} + \left(\frac{V}{2X_c}\right)^2 - \left(\frac{V}{2X_c}\right)^2 = 0$$

$$X^2+Y^2 - 2Y \times \frac{V}{2X_c} + \left(\frac{V}{2X_c}\right)^2 - \left(\frac{V}{2X_c}\right)^2 = 0$$

$$X^2 + \left(Y - \left(\frac{V}{2X_c}\right)\right)^2 = \left(\frac{V}{2X_c}\right)^2 \longrightarrow (6)$$

Now we know the circle Eqn

$$(x-x_1)^2 + (y-y_1)^2 = r^2 \longrightarrow (7)$$

comparing eqn's (6) & (7)

$$x_1 = 0$$

$$y_1 = \frac{V}{2X_c}$$

$$r = \frac{V}{2X_c}$$

$\therefore$ centre $= (x_1, y_1) = \left(0, \frac{V}{2X_c}\right)$

radius $(r) = \frac{V}{2X_c}$

<u>Construction of locus diagram</u>:-

$$\cos\phi = \frac{R}{Z}, \quad I = \frac{V}{Z} \Rightarrow I = \frac{V}{\sqrt{R^2+X_c^2}}$$

Suppose

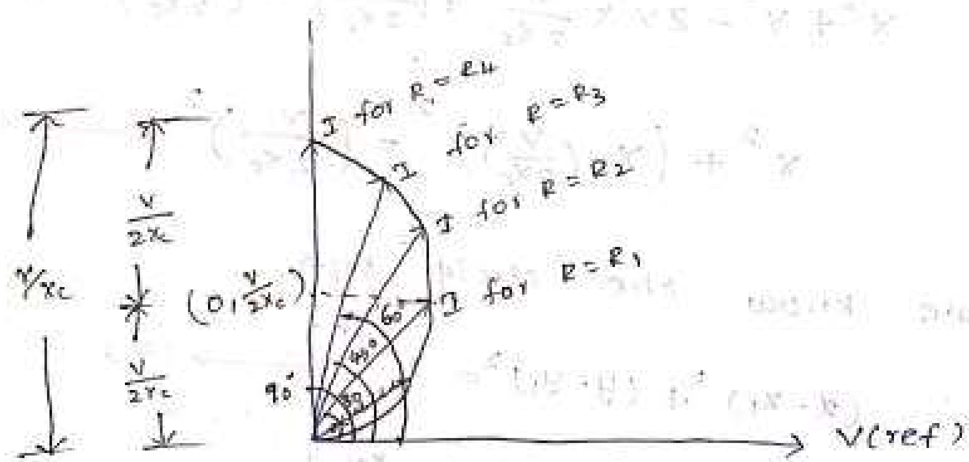
$\phi = 30^\circ$, Then $\cos\phi = 0.868$

$\phi = 45^\circ$, Then $\cos\phi = 0.707$

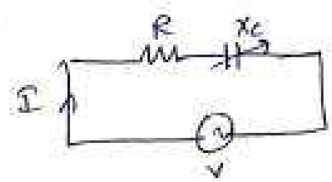
$\phi = 60^\circ$, Then $\cos\phi = 0.5$

$\phi = 75^\circ$, Then $\cos\phi = 0.25$

$\phi = 90^\circ$, Then $\cos\phi = 0^\circ$



Case(ii) :- constant R, variable X_c



$x = I\cos\phi$

$$x = \frac{V}{Z} \times \frac{R}{Z}$$

$$x = \frac{VR}{R^2 + X_c^2}$$

$$\frac{V}{R^2 + X_c^2} = \frac{x}{R} \longrightarrow (8)$$

Sub eqn (8) in eqn (3)

$$x^2+y^2 = V \times \frac{x}{R}$$

$$x^2+y^2 - V\times\frac{x}{R} = 0$$

$$x^2+y^2 - 2x\times\frac{V}{2R} + \left(\frac{V}{2R}\right)^2 - \left(\frac{V}{2R}\right)^2 = 0$$

$$y^2 + \left(x - \frac{V}{2R}\right)^2 = \left(\frac{V}{2R}\right)^2$$

comparing above eqn w.r.t.eq $(x-x_1)^2 + (y-y_1)^2 = r^2$

$x_1 = \frac{V}{2R}$, $y_1 = 0$, $r = \frac{V}{2R}$, $\left(\frac{V}{2R}, 0\right)$, $\frac{V}{2R}$

construction of locus diagram

$\sin\phi = \frac{X_C}{\sqrt{R^2+X_C^2}}$, $I = \frac{V}{\sqrt{R^2+X_C^2}}$

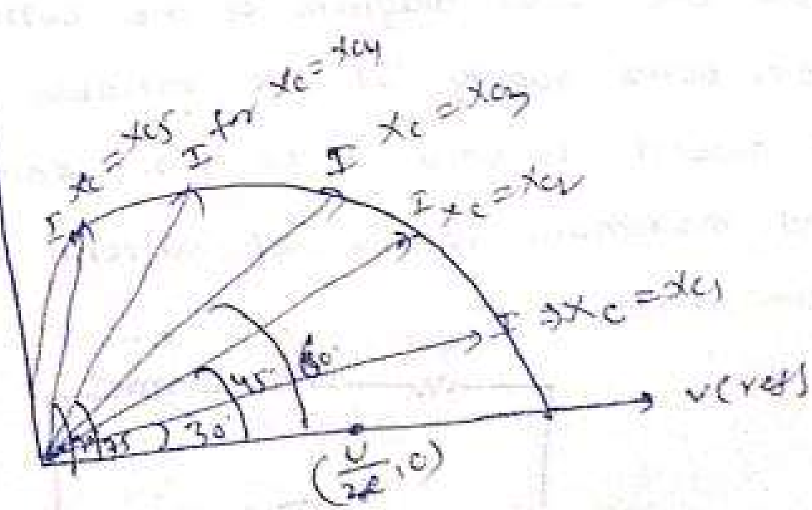
$\phi = 30^\circ$ the $\sin\phi = 0.5$

$\phi = 45^\circ$ then $\sin\phi = 0.707$

$\phi = 60^\circ$ then $\sin\phi = 0.866$

$\phi = 75^\circ$ then $\sin\phi = 0.965$

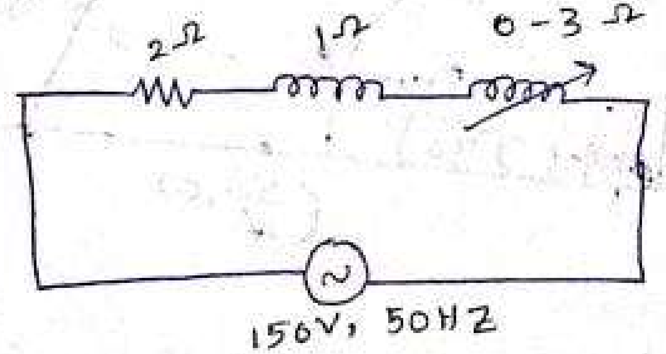
$\phi = 90^\circ$ then $\sin\phi = 1$



PROBLEMS :-

1. A circuit consisting of a choke coil with resistance of $2\,\Omega$ and reactance of $1\,\Omega$ at 50 Hz. It is connected in series with other choke coil with negligible resistance and variable inductive reactance. Draw the locus diagram of the current drawn from a 150 V, 50 Hz supply. If the variable inductive reactance is allowed to vary 0 to $3\,\Omega$. Calculate the minimum and maximum values of current and corresponding power factor.

Sol:-



In this problem,

variable reactance (X_L) & constant R

$$\text{radius} = \frac{V}{2R} = \frac{150}{2 \times 2}$$

$$\boxed{\text{radius} = 37.5\ m}$$

$$\text{centre}\left(\frac{V}{2R}, 0\right) = (37.5, 0)$$

X_L is varies from 1 to 1+3

1 to 4 Ω

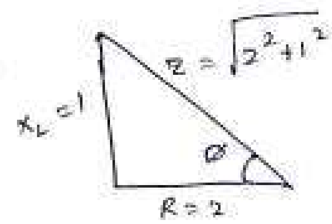
case(i):- Put $X_L = 0\ \Omega$

Total reactance (X_L) = $1+0 = 1\ \Omega$

$R = 2\ \Omega$

$$I = \frac{V}{\sqrt{R^2+X_L^2}} = \frac{150}{\sqrt{2^2+1^2}}$$

$$\boxed{I = 67.08\ A}$$



$$\tan\phi = \frac{X_L}{R}$$

$$\phi_{min} = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$= \tan^{-1}\left(\frac{1}{2}\right)$$

$$\phi_{min} = 26.56^\circ$$

$$\cos\phi_{min} = \cos 26.56$$

$$\boxed{\cos\phi_{min} = 0.894}$$

case(ii):- $X_L = 3\Omega$

$R = 2\Omega$

Total $(X_L) = 1+3 = 4\Omega$

$$I = \frac{V}{\sqrt{R^2+X_L^2}} = \frac{150}{\sqrt{2^2+4^2}}$$

$I = 33.5\,A$

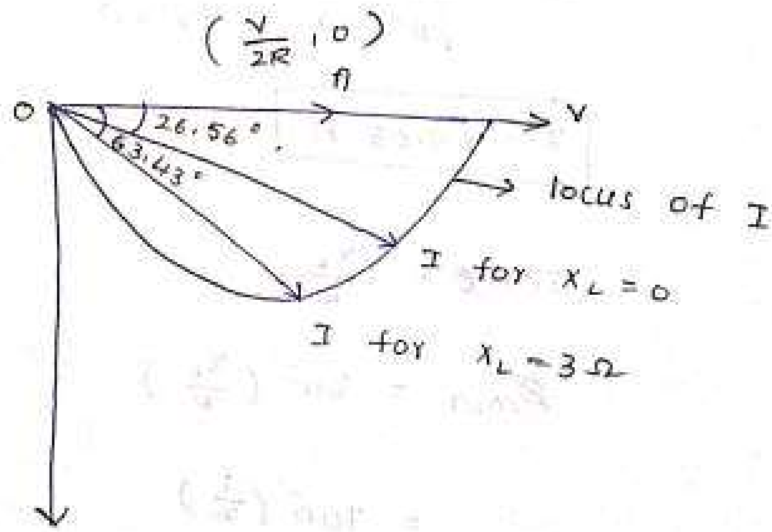
$$\phi_{max} = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$= \tan^{-1}\left(\frac{4}{2}\right)$$

$\phi_{max} = 63.43$

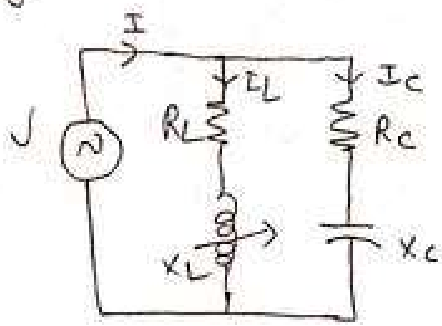
$\cos\phi_{max} = \cos(63.43)$

$$\boxed{\cos\phi_{max} = 0.447}$$



Parallel RLC circuit

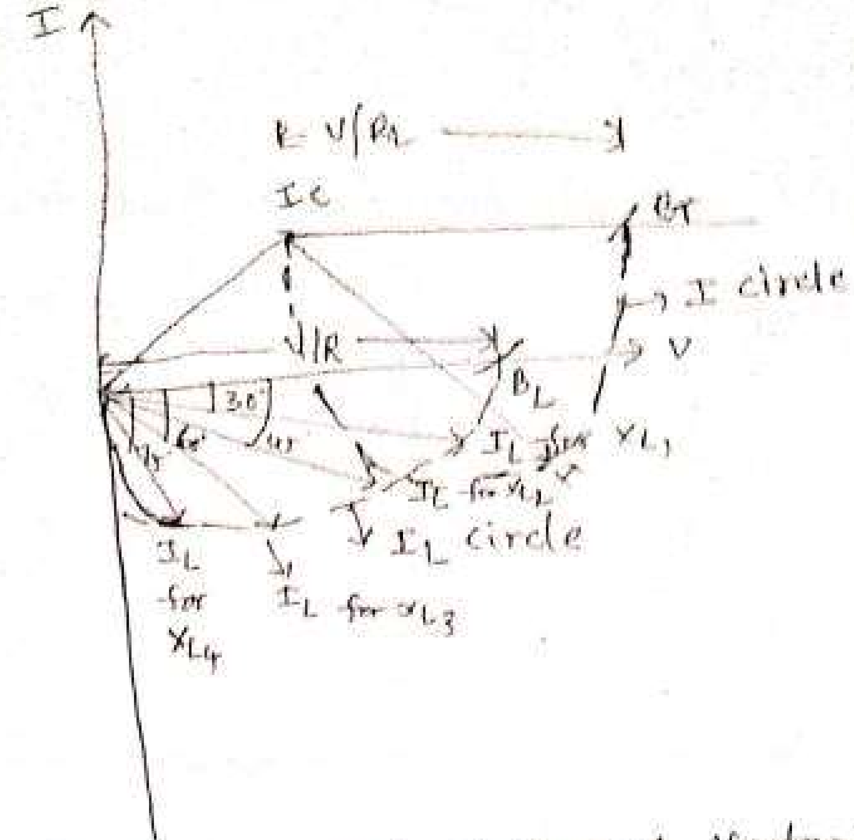
Parallel LC circuit along with internal resistances as shown in fig



In the above circuit, there are two branch currents i_L & i_C along with total current i

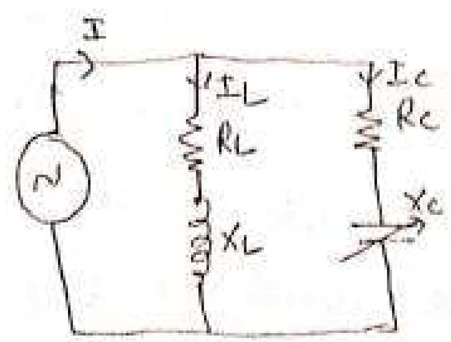
Case(1): Varying X_L :

- In this case, X_L is variable, X_C, R_L, R_C are fixed. and I_C is through capacitor is constant since R_C, R_L are fixed and it leads the voltage vector OV by an angle θ_C $\left[\theta_C = \tan^{-1}\frac{X_C}{R_C}\right]$
- The current I_L through the inductance is vector OI_L and its amplitude is maximum and is equal to $\frac{V}{R_L}$ when X_L is zero and it is in phase with applied voltage V.
- $I = \frac{V}{\sqrt{R^2+X_L^2}}$, $\sin\phi = \frac{X_L}{Z}$
- when X_L is increased from 0 to infinity and current is decreased from higher value to lower value. and its phase angle will be $\theta_L = \tan^{-1}\left(\frac{X_L}{R_L}\right)$ and it is same as series R-L circuit.



- To get Total current circle add Vectorially the current I_C and I_L.

<u>Case (2): Varying X_C</u>



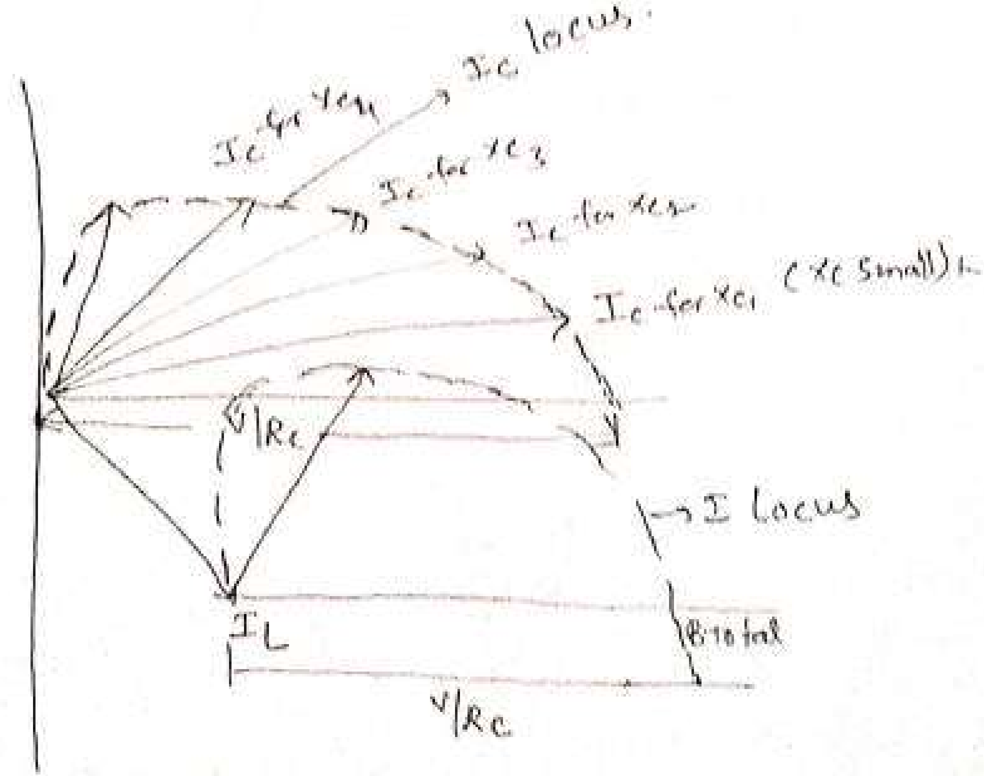
- here current I_L through inductor is constant since R_L and X_L are fixed and it lags the voltage vector OV by an angle $\theta_L = \tan^{-1}\left(\frac{X_L}{R_L}\right)$
- The current I_C through the capacitance is the vector OI_C. its amplitude is maximum and equal to V/R_C when $X_C = 0$

and it is in phase with applied voltage v.

$\rightarrow$ $I = \frac{V}{\sqrt{R^2+X_C^2}}$ $\quad \sin\theta = \frac{X_C}{Z}$

$\sin 30 = 0.5$
$\sin 45 = 0.707$
$\sin 60 = 0.866$

- when X_C is increased from 0 to infinity, it's amplitude is decreases to lower value and phase will be lead by 90°
- Phase angle $\theta_C = \tan^{-1}\left(\frac{X_C}{R_C}\right)$
- The ~~cur~~ locus of current is a semicircle with diameter of length equal to $\frac{V}{RC}$



Case(3): Varying R_L :

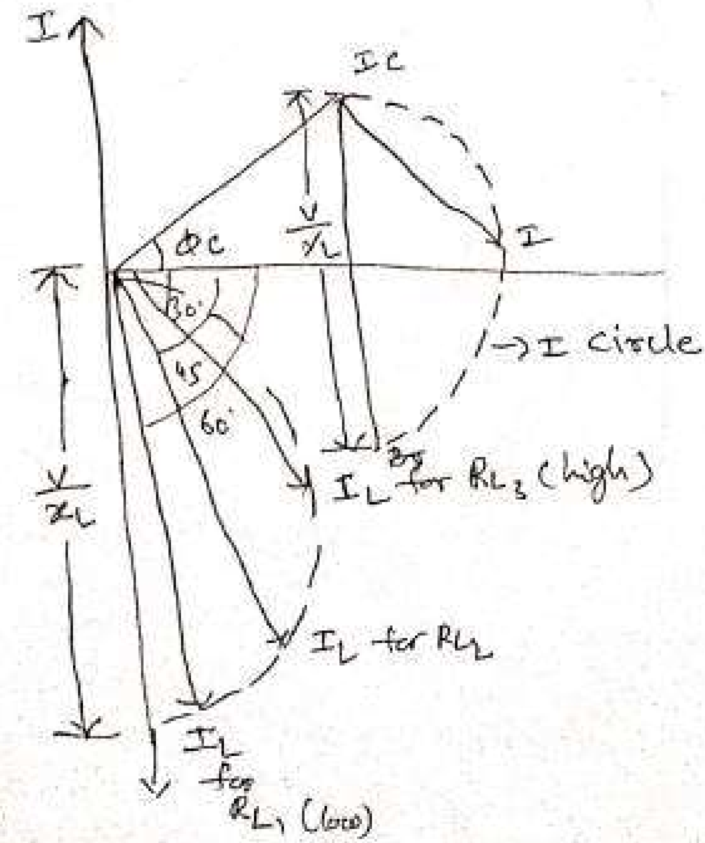
- The current I_C through capacitance is constant since R_C & C are fixed and it leads the voltage vector OV by an angle $\theta_C = \tan^{-1}\left(\frac{X_C}{R_C}\right)$.

- The current I_L through the inductance is OI_L. its amplitude is maximum and its equal to $\frac{V}{X_L}$ where R_L is zero. It's phase will be lagging the voltage by 90°

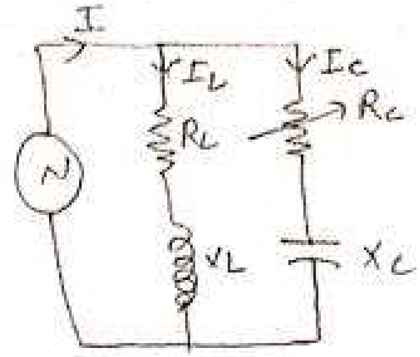
- when R_L is increased from '0' to infinity, its amplitude decreases to lower value $I = \frac{V}{\sqrt{R_L^2 + X_L^2}}$ $\cos\theta = \frac{R_L}{Z}$

- phase angle is lagging the voltage 'V' by an angle $\theta_L = \tan^{-1}\left(\frac{X_L}{R_L}\right)$

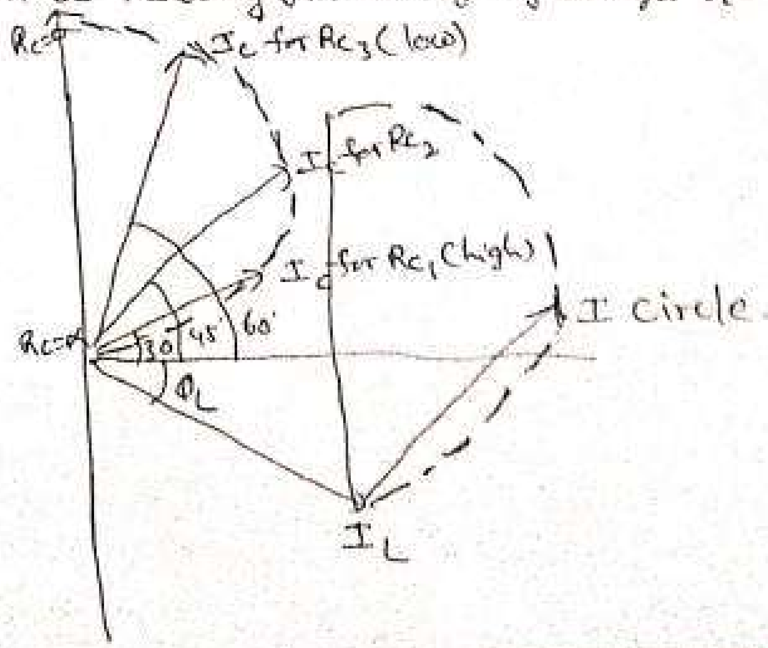
- Locus of current is a semi circle with diameter is equal to V/R_L



Case (4): Varying R_c:



- The current I_L through the inductor is constant since R_L & L are fixed and it lags the applied voltage vector OV by an angle $\theta_L = \tan^{-1}\left(\frac{X_L}{R_L}\right)$
- The current I_c through the capacitance is the vector OI_c. Its amplitude is maximum and is equal to $\frac{V}{X_c}$ when R_c is 0 and its phase will be leading the voltage by 90°
- when R_c is increased from 0 to infinity its amplitude is decreases to lower value (or '0') and it will be in phase with applied voltage 'V'.
- phase angle will be leading the voltage by an angle $\theta_c = \tan^{-1}\left(\frac{X_c}{R_c}\right)$



- $\cos\phi = \frac{R_c}{Z}$
- $I = \frac{V}{\sqrt{R_c^2 + X_c^2}}$

Introduction :-

Resonance :- Resonance is the phenomenon in which applied voltage and resultant current are always inphase with each other.

[or]

An AC circuit is said to be in resonance if it exhibits unity power factor.

[or]

At resonance impedance becomes resistance only. i.e, $Z = R$.

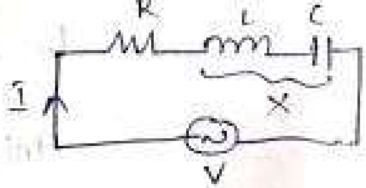
[or]

At resonance net ~~reacten~~ reactances become zero.

Series Resonance :-

In Series RLC circuit, if any one of the parameter is varied such that applied voltage & resultant current are in phase with each other.

$$Z = R + jX$$
$$= R + j(X_L - X_C)$$



At resonance , $X = 0$

$$X_L - X_C = 0$$

$$\boxed{X_L = X_C}$$

$$\boxed{Z = R}$$

At resonance ,

$$X_L = X_C$$

$$\omega L = \frac{1}{\omega C}$$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r^2 = \frac{1}{4\pi^2 LC}$$

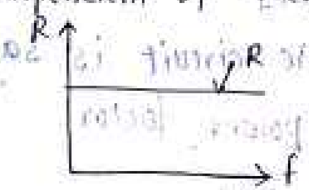
$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}} \ Hz} \rightarrow \text{Resonant frequency.}$$

PF = 1 → power facto

** Imp * Graphical representation of Various elements in Series resonanc

(1) Resistance :-

R is independent of frequency (f).

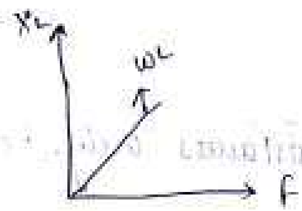


(2) Inductive reactance :-

$$jX_L = j\omega L$$

$$jX_L = j2\pi fL$$

$$\boxed{X_L \propto f}$$



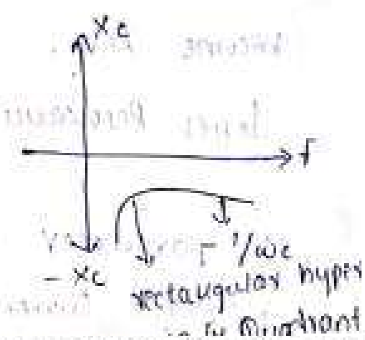
(3) capacitance :-

$$-jX_C = -j \times \frac{1}{\omega C}$$

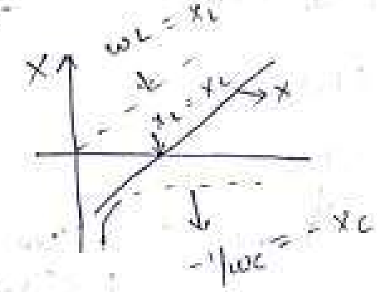
$$= j(-1/\omega C)$$

$$= j\left(\frac{-1}{2\pi fC}\right)$$

$$\boxed{X_C \propto 1/f}$$

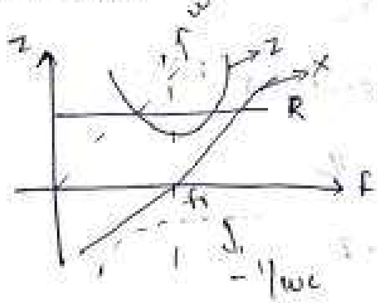


(4) Reactance (X):-



(5) Impedence:-

$Z = R + i(X_L - X_C)$



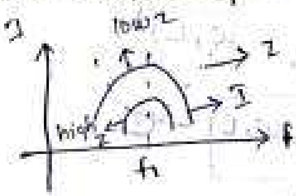
At resonant frequency, Z is minimum.

(6) Current:-

$I = V/Z$

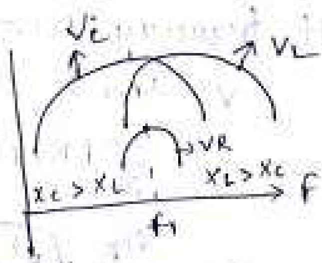
$I \propto 1/Z$

At resonant frequency, Z is minimum, I is maximum



(7) Voltage:-

$V_R = iR$



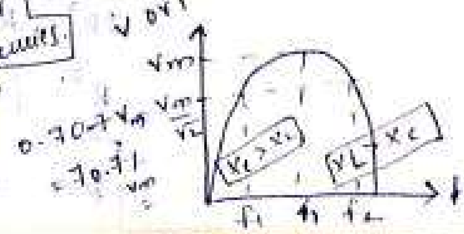
Bandwidth:-

Bandwidth is the difference in frequencies at which Voltage or Current is equal to 70.7% of its maximum Value. Half power frequencies.

$B.W = f_2 - f_1$

$f_1 \to$ lower cut off frequency

$f_2 \to$ Upper " " "



Imp ***

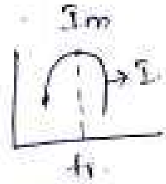
Relationship b/w Bandwidth & RLC :-

At resonant frequency (f_r),

we know, $V = I Z$

$\downarrow \quad \downarrow$

$V_{rms} \quad I_{rms}$



At f_r :-

$V = I_m R$

$$\boxed{I_m = \frac{V}{R}} \rightarrow (1)$$

At lower cut off frequency (f_1):-

we know, $V_{rms} = I_{rms} R$

$$V = \frac{I_m}{\sqrt{2}} \cdot Z$$

$$V = \frac{V}{\sqrt{2} R} \left(R + j \left(X_{C_1} - X_{L_1} \right) \right)$$

$$\sqrt{2} R = \sqrt{R^2 + (X_{C_1} - X_{L_1})^2}$$

both sides on squarring

$$2R^2 = R^2 + \left(\frac{1}{\omega_1 C} - \omega_1 L \right)^2$$

$$R^2 = \left(1/\omega_1 C - \omega_1 L \right)^2$$

$$\boxed{R = \frac{1}{\omega_1 C} - \omega_1 L} \rightarrow (2)$$

At high cut off frequency (f_2):-

$$V = I_{rms} Z$$

$$= \frac{I_m}{\sqrt{2}} \left(R + j \left(X_{L_2} - X_{C_2} \right) \right)$$

$$V = \frac{V/R}{\sqrt{2}} \sqrt{R^2 + (X_{L_2} - X_{C_2})^2}$$

$$\sqrt{2} R = \sqrt{R^2 + (X_{L_2} - X_{C_2})^2}$$

Squarring on both sides:

$$2R^2 = R^2 + (X_{L_2} - X_{C_2})^2$$

$$R^2 = \left(\omega_2 L - \frac{1}{\omega_2 C} \right)^2$$

$$\boxed{R = \omega_2 L - \frac{1}{\omega_2 C}} \rightarrow (3)$$

Equating eqn ② & ③

$$R = \omega_2 L - \frac{1}{\omega_2 C} = \frac{1}{\omega_1 C} - \omega_1 L$$

$$\omega_2 L + \omega_1 L = \frac{1}{\omega_1 C} + \frac{1}{\omega_2 C}$$

$$L(\omega_1 + \omega_2) = \frac{1}{C}\left(\frac{\omega_2 + \omega_1}{\omega_1 \omega_2}\right)$$

$$L = \frac{1}{C}\left(\frac{1}{\omega_1 \omega_2}\right)$$

$$\boxed{\omega_1 \omega_2 = \frac{1}{LC}} \rightarrow ④$$

we know, $f_r = \frac{1}{2\pi\sqrt{LC}}$

$$2\pi f_r = \frac{1}{\sqrt{LC}}$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$\boxed{\omega_r^2 = \frac{1}{LC}} \rightarrow ⑤$$

Substitute eqn ⑤ in eqn ④

$$\omega_1 \omega_2 = \omega_r^2$$

$$\boxed{\omega_r = \sqrt{\omega_1 \omega_2}}$$

Hence $\boxed{f_r = \sqrt{f_1 f_2}}$

∴ Resonant frequency is the geometrical mean of f_1 & f_2.

Adding eqn ② & ③

$$R + R = \frac{1}{\omega_1 C} - \omega_1 L + \omega_2 L - \frac{1}{\omega_2 C}$$

$$2R = \frac{1}{C}\left(\frac{1}{\omega_1} - \frac{1}{\omega_2}\right) + L(\omega_2 - \omega_1)$$

$$2R = \frac{1}{C}\left(\frac{\omega_2 - \omega_1}{\omega_1 \omega_2}\right) + L(\omega_2 - \omega_1)$$

we know $\omega_1 \omega_2 = 1/LC$ (from eqn 4).

$$2R = \frac{1}{C}\cdot\left(\frac{\omega_2 - \omega_1}{1/LC}\right) + L(\omega_2 - \omega_1)$$

$$2R = \frac{1}{\cancel{C}}(\omega_2 - \omega_1)L\cancel{C} + L(\omega_2 - \omega_1)$$

$$2R = (\omega_2 - \omega_1)(L + L)$$

$$2R = 2L(\omega_2 - \omega_1)$$

$$\omega_2 - \omega_1 = \frac{R}{L}$$

$$2\pi f_2 - 2\pi f_1 = R/L$$

$$2\pi(f_2 - f_1) = \frac{R}{L}$$

$$(f_2 - f_1) = \frac{R}{2\pi L}$$

$$\boxed{\text{Bandwidth} = \frac{R}{2\pi L}}$$

* Quality factor:- Quality factor (Q) is defined as the ratio of voltage across inductor (or) capacitor to be the supply voltage.

It is the indication of quality of coil (L (or) ~~R~~ ...

$$\therefore Q = \frac{V_L}{V}$$

At resonance,

$$V = V_R$$

$$Q = \frac{V_L}{V_R}$$

$$= \frac{I X_L}{I R}$$

$$\boxed{Q = \frac{X_L}{R}}$$

$$\boxed{Q = \frac{\omega L}{R}}$$

$$Q = \frac{V_C}{V}$$

At resonance,

$$V = V_R$$

$$Q = \frac{V_C}{V_R}$$

$$= \frac{I X_C}{I R}$$

$$\boxed{Q = \frac{X_C}{R}}$$

$$\boxed{Q = \frac{1}{\omega C R}}$$

Imp. *** Quality factor = $\boxed{Q = 2\pi \times \frac{\text{Energy stored/cycle}}{\text{Energy dissipated/cycle}}}$

Relationship b/w Bandwidth, Quality factor & Resonant frequency:-

we know, Bandwidth $= \frac{R \times f_r}{2\pi f_r L}$

$$= \frac{R \times f_r}{\omega_r L}$$

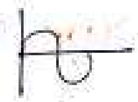
$$= \frac{R \times f_r}{X_L}$$

$$B.\omega = \frac{f_r}{(X_L/R)} = \frac{f_r}{Q} \Rightarrow \boxed{B.\omega = \frac{f_r}{Q}}$$

Half power frequencies in terms of resonance frequency:-

For Symmetrical wave,

$$f_r = \frac{f_1 + f_2}{2} \rightarrow (1)$$



From eqn (1),

$$f_1 = 2f_r - f_2 \rightarrow (2)$$

we know, Bandwidth $= f_2 - f_1 = \frac{R}{2\pi L} \rightarrow (3)$

Substitute (f_1) in eqn (3)

$$f_2 - 2f_r + f_2 = \frac{R}{2\pi L}$$

$$2f_2 - 2f_r = \frac{R}{2\pi L}$$

$$2(f_2 - f_r) = \frac{R}{2\pi L}$$

$$\boxed{f_2 = \frac{R}{4\pi L} + f_r} \rightarrow (4)$$

|||ly from eqn (1),

$$f_2 = 2f_r - f_1 \rightarrow (5)$$

Substitute eqn (5) in eqn (3)

$$2f_r - f_1 - f_1 = \frac{R}{2\pi L}$$

$$2f_r - 2f_1 = \frac{R}{2\pi L}$$

$$2(f_r - f_1) = \frac{R}{2\pi L}$$

$$f_r - f_1 = \frac{R}{4\pi L}$$

$$\boxed{f_1 = f_r - \frac{R}{4\pi L}} \rightarrow (6)$$

Selectivity (S):- It is reciprocal of Quality factor (Q).

$$S = \frac{1}{Q}$$

$$= \frac{1}{\frac{R}{2\pi L}}$$

$$\boxed{S = \frac{2\pi L}{R}} \quad (\text{or}) \quad S = \frac{1}{Q}$$

$$= \frac{1}{f_r / B.\omega} = \frac{B.\omega}{f_r}$$

Voltage across L and C at resonance:-

$$V_L = I(jX_L)$$

at resonance $\Rightarrow I = V/R$.

$$V_L = \frac{V}{R} \cdot j(\omega_r L)$$

$$= j \cdot \frac{\omega_r L}{R} \cdot V$$

$$= j \cdot \frac{X_L}{R} \cdot V$$

$$\therefore \boxed{V_L = jQV}$$

$$V_C = I(-jX_C)$$

At resonance $\rightarrow I = V/R$

$$V_C = V/R(-jX_C) = -j \cdot \left(\frac{X_C}{R}\right)V$$

$$\therefore \boxed{V_C = -jQV}$$

Frequency for voltage across Inductor is maximum:-

we know,

$$V_L = IX_L \quad (\text{in magnitude}).$$

$$= \frac{V}{Z} X_L$$

$$= \frac{V\omega L}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$= \frac{V\omega L}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\frac{V^2\omega^2L^2 \times \omega^2C^2}{R^2\omega^2C^2 + (\omega^2LC - 1)^2} \rightarrow (1)$$

$\omega^2C^2 \times \omega^2L^2C^2 + 1 - 2\omega^2LC$

$$V_L^2 = \frac{V^2\omega^2L^2}{R^2 + (\omega L - 1/\omega C)^2}$$

$$V_c = \frac{V}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

$$V_c^2 = \frac{V^2}{\omega^2 C^2\left[R^2 + (\omega L - 1/\omega C)^2\right]}$$

$$V_c^2 = \frac{V^2 \omega^2 C^2}{\omega^2 C^2\left(R^2\omega^2 C^2 + (\omega^2 LC - 1)^2\right)}$$

$$V_c^2 = \frac{V^2}{R^2\omega^2 C^2 + (\omega^2 LC - 1)^2}$$

$$\omega^4 L^2 C^2 + 1 - 2\omega^2 LC$$

To get frequency, V_c is maximum.

$$\frac{dV_c^2}{d\omega} = 0$$

$$\left(R^2\omega^2 C^2 + (\omega^2 LC - 1)^2\right)(0) - V^2\left(2\omega R^2 C^2 + 4\omega^3 L^2 C^2 - 4\omega LC\right) = 0$$

$$2\omega R^2 C^2 + 4\omega^3 L^2 C^2 - 4\omega LC = 0$$

$$2\omega C\left(R^2 C + 2\omega^2 L^2 C - 2L\right) = 0$$

$$R^2 C + 2\omega^2 L^2 C - 2L = 0$$

$$2\omega^2 L^2 C = 2L - R^2 C$$

$$\omega^2 = \frac{2L}{2L^2 C} - \frac{R^2 C}{2L^2 C}$$

$$\omega^2 = \frac{1}{LC} - \frac{R^2}{2L^2}$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$$

$$\boxed{f_c = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}}$$

$f_r = 71.18$ Hz
$f_L = 72.11$ Hz
$f_c = 70.01$ Hz

1) A Series ckt with $R = 10\,\Omega$, $L = 0.1$ H, & $C = 50\ \mu F$ as an applied voltage $V = 50$ at an angle $0°$ with a variable frequency. find (a) Resonant frequency, (b) frequency at which voltage across inductor is maximum, (c) frequency at which voltage across capacitor is max. d) voltage across inductor at resonant

8) An inductance of 0.5 H, Resistances of 5 Ω & Capacitances 8 μF are in Series across a 220 V Ac Supply, (a) calculate the frequency at which ckt resonance, (b) find the current at resonance, bandwidth of power frequency & Voltage across inductance & Capacitances.

A) Given data,

$R = 5\Omega$,

$L = 0.5 H$,

$C = 8 \mu F = 8 \times 10^{-6} F$

(a) frequency at resonance :- $f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.5 \times 0.8 \times 10^{-6}}}$

$= 79.58 \text{ Hz}$.

(b) $V = IRZ$

at resonance $Z = R$,

$V = IR$

$I = V/R$

$= \frac{220}{5} = 44 A$.

Bandwidth $= B.W = \frac{R}{2\pi L}$

$= \frac{5}{2 \times 3.14 \times 0.5}$

$= 1.59 \text{ Hz}$.

half power frequencies $= f_1 = f_r - \frac{R}{4\pi L}$

$= 79.58 - \frac{5}{4 \times 3.14 \times 0.5}$

$= 78.79 \text{ Hz}$.

$f_2 = f_r + \frac{R}{4\pi L}$

$= 79.58 + \frac{5}{4 \times 3.14 \times 0.5} = 80.40 \text{ Hz}$.

Quality factor :- $Q = \frac{f_r}{B.W}$

$= \frac{79.58}{1.59}$

$= 50$

Voltage across Inductor (V_L) :-

we know, $Q = \frac{V_L}{V_R} = \frac{V_L}{V}$

$V_L = Q \times V$

$V_L = 50 \times 220$

$= 11 \text{ kV}$

Voltage across capacitor (V_C) :-

$Q = \frac{V_C}{V_R} = \frac{V_C}{V}$

$V_C = Q V$

$= 11 \text{ kV}.$

Note:-

In Series resonance, Current $I = I_m$ & Voltage $V = V_R$

3) In Series RLC ckt has quality factor of 5 at 50 Radians/sec. The current flowing through ckt at resonance is 10 amperes & applied voltage 100 V. The total impedence of the ckt is 20Ω, find the ckt elements.

A) Given data,

Quality ~~packing~~ factor = 5

$\omega = 50$ rad/sec.

At resonance $I = I_m = 10A$

Scanned with CamScanner

$V = 100V$

$Z = 20\Omega$

we know,

$V = IZ$

At resonance, $V = I_m R$

$$R = \frac{V}{I_m} = \frac{100}{10} = 10\Omega$$

we know, $Q = \frac{X_L}{R} = \frac{\omega L}{R}$

$$L = \frac{QR}{\omega}$$

$$= \frac{5 \times 10}{50}$$

$$= 1\,H$$

& Also

we know, $Q = \frac{X_C}{R} = \frac{1}{\omega C R}$

$$Q\omega R = 1/C$$

$$C = \frac{1}{Q\omega R}$$

$$= \frac{1}{5 \times 50 \times 10}$$

$$= 400\,\mu F$$

4) In the Series RLC ckt with $L = 0.5$ H, as an instantaneous Voltage $v = 70.7 \sin(500t + 30°)$ v & instantaneous current $i = 1.5 \sin(500t)$ A. find the value's of R & C. At what frequency will the ckt be resonance.

A) Given data, $L = 0.5H$

$v = 70.7 \sin(500t + 30°)$ v, $\omega = 500$

$i = 1.5 \sin(500t)$ A

$V = 70.7 \angle 30° \Rightarrow V_m = 70.7$

$i = 1.5 \angle 0° \Rightarrow I_m = 1.5$

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{70.7}{\sqrt{2}} = 49.9\,V \cong 50\,V$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{1.5}{\sqrt{2}} = 1.06\,A$$

we know,

$$Z = \frac{V_{rms}}{I_{rms}} = \frac{49.9\angle 30^\circ}{1.06\angle 0^\circ}$$

$$= 47.07\angle 30$$

$$= 47.07(\cos 30 + j\sin 30)$$

$$= 47.07\left(\sqrt{3}/2 + j\cdot 1/2\right)$$

$$= 40.76 + j\,23.53$$

$$= R + jX$$

where $R = 40.76\,\Omega$, $X = 23.53\,\Omega$

we know,

$$X = X_L - X_C$$

$$23.53 = \omega L - \frac{1}{\omega C}$$

$$23.53 = 500(0.5) - \frac{1}{500 \times C}$$

$$23.53 = 250 - \frac{1}{500 \times C}$$

$$23.53 - 250 = -\frac{1}{500C}$$

$$-226.47 = -\frac{1}{500C}$$

$$C = \frac{1}{500 \times 226.47}$$

$$= \frac{1}{113235}$$

$$\boxed{C = 8.83\,\mu F}$$

(b) $f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3.14\sqrt{0.5 \times 8.83 \times 10^{-6}}} = 75.78\,Hz$

5) A Series RLC ckt with $R = 25\,\Omega$, $L = 0.6\,H$, results in a leading phase angle of 60° at frequency of 40 Hz. find the value of the C & at what frequency the ckt will be resonant.

A) Given data,

$R = 25\,\Omega$.

$L = 0.6\,H$

leading phase angle $(\phi) = 60^\circ$.

$f = 40\,Hz$.

here leading phase angle, so $Z = R + j(X_C - X_L)$.

then $\tan\phi = \dfrac{X_C - X_L}{R}$

$$\tan 60^\circ = \frac{\frac{1}{\omega C} - \omega L}{25}$$

$2 \times 3.14 \times 40$

$$\sqrt{3}\,(25) = \frac{1}{2\pi f C} - 2\pi f L$$

$$= \frac{1}{251.2C} - 251.2\,(0.6)$$

$$43.30 = \frac{1}{251.2C} - 150.72$$

$$43.3 + 150.72 = \frac{1}{251.2\,C}$$

$$\frac{194.02}{\cancel{107.41}} = \frac{1}{251.2C}$$

$$C = \frac{1}{251.2\,(194.02)}$$

$$\boxed{C = 2.05 \times 10^{-5} F}$$

Resonant frequency $= f_r = \dfrac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2 \times 3.14\sqrt{0.6 \times 2.05 \times 10^{-5}}}$$

$$\boxed{f_r = 45.40\,Hz}$$

Admittance :- [Y]

It is the reciprocal of impedence (z)

$$Y = \frac{1}{Z}$$

Impedence parameter	Admittance parameter
Z	$Y = 1/Z$
R	$1/R = G$ (conductance)
X_L	$\frac{1}{X_L} = B_L$ (susceptive Inductance)
X_C	$\frac{1}{X_C} = B_C$ (susceptive capacitance)
$Z = R + j(X_L - X_C)$	$Y = G + j(B_C - B_L)$.

Impedence :- $V = IZ$

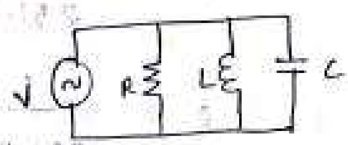
$I = V/Z$

$= V \cdot 1/Z$

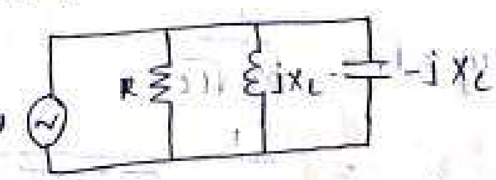
$\boxed{I = V \cdot Y} \rightarrow$ Admittance.

Parallel Resonance:-

Consider parallel RLC ckt excited by alternating voltage as shown in figure.



Now express all the parameters in terms of ohms & ckt is redrawn as



Take $\underline{Y_1 = \frac{1}{R}}$, $\underline{Y_2 = \frac{1}{jX_L} = \frac{-j}{X_L}}$, $\underline{Y_3 = \frac{1}{-jX_C} = \frac{j}{X_C}}$

The admittance $Y = Y_1 + Y_2 + Y_3$.

$$= \frac{1}{R} - \frac{j}{X_L} + \frac{j}{X_C}$$

$$= \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_L}\right)$$

$$= \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

$$= \frac{1}{R} + j(B_C - B_L).$$

$$Y = G + j(B_C - B_L) \rightarrow (1)$$

$$Y = G + jB \rightarrow (1)$$

At resonance, $B = 0$

$$B_C - B_L = 0$$

$$B_C = B_L$$

$$\frac{1}{X_C} = \frac{1}{X_L}$$

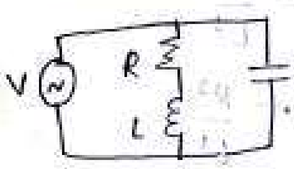
$$\omega_r C = \frac{1}{\omega_r L}$$

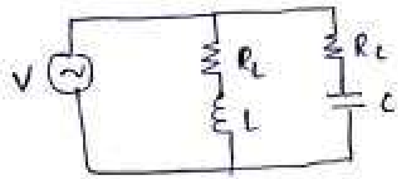
$$\omega_r^2 = \frac{1}{LC}$$

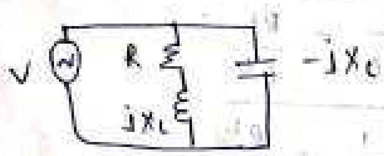
$$\omega_r = \frac{1}{\sqrt{LC}}$$

Resonant frequency $\Rightarrow f_r = \frac{1}{2\pi\sqrt{LC}}$ Hz.

1) find the resonant frequency for the following ckt's.

(A) 

(B) 

A) 

$$Y_1 = \frac{1}{R + jX_L}$$

$$Y_2 = \frac{1}{-jX_C} = \frac{j}{X_C}$$

Total admittance : $Y = Y_1 + Y_2$.

$$Y = \frac{1}{R+jX_L} + \frac{j}{X_C}$$ [on Rationalization]

$$= \frac{R-jX_L}{R^2+X_L^2} + \frac{j}{X_C}$$

$$Y = \frac{R}{R^2+X_L^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2}\right) \rightarrow (1)$$

At resonance,

Imaginary part $= 0$.

$$\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2} = 0$$

$$\omega_r C - \frac{\omega_r L}{R^2+(\omega_r L)^2} = 0$$

$$\omega_r C = \frac{\omega_r L}{R^2+(\omega_r L)^2}$$

$$R^2+(\omega_r L)^2 = \frac{L}{C}$$

$$(\omega_r L)^2 = \frac{L}{C} - R^2$$

$$\omega_r^2 L^2 = \frac{L}{C} - R^2$$

$$\omega_r^2 = \frac{L}{CL^2} - \frac{R^2}{L^2}$$

$$\omega_r^2 = \frac{1}{CL} - \frac{R^2}{L^2}$$

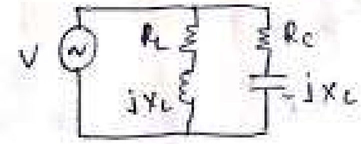
$$\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$2\pi f_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Resonant frequency $= \boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$

Scanned with CamScanner

B)



$$Y_1 = \frac{1}{R_L + jX_L}, \quad Y_2 = \frac{1}{R_C - jX_C}$$

Total admittance :- $Y = Y_1 + Y_2$

$$Y = \frac{1}{R_L + jX_L} + \frac{1}{R_C - jX_C}$$

$$= \frac{R_L - jX_L}{R_L^2 + X_L^2} + \frac{R_C + jX_C}{R_C^2 + X_C^2}$$

$$= \frac{R_L}{R_L^2 + X_L^2} - j\frac{X_L}{R_L^2 + X_L^2} + \frac{R_C}{R_C^2 + X_C^2} + j\frac{X_C}{R_C^2 + X_C^2}$$

$$= \frac{R_L \cancel{R_L}}{(R_L^2 + X_L^2)} + \frac{R_C}{(R_C^2 + X_C^2)} + j\left(\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2}\right)$$

At resonance,

imaginary part = 0

$$\frac{X_C}{R_C^2 + X_C^2} - \frac{X_L}{R_L^2 + X_L^2} = 0$$

$$\frac{X_C}{R_C^2 + X_C^2} = \frac{X_L}{R_L^2 + X_L^2}$$

$$\frac{1}{\omega_r C\left(R_C^2 + \left(\frac{1}{\omega_r C}\right)^2\right)} = \frac{\omega_r L}{R_L^2 + (\omega_r L)^2}$$

$$\frac{1}{\cancel{\omega_r C}}\left(R_L^2 + (\omega_r L)^2\right) = \cancel{\omega_r L}\left(R_C^2 + \left(1/\omega_r C\right)^2\right)$$

$$\frac{\cancel{\omega_r} C}{\left(R_C^2 (\omega_r C)^2 + 1\right)} = \frac{\cancel{\omega_r} L}{R_L^2 + (\omega_r L)^2}$$

$$C\left(R_L^2 + (\omega_r L)^2\right) = L\left(R_C^2 \omega_r^2 C^2 + 1\right).$$

Scanned with CamScanner

$$CR_L^2 + C\omega_r^2 L^2 = L R_L^2 \omega_r^2 C^2 + L$$

or $$\omega_r^2 (CL^2 - LR_L^2 C^2) = L - CR_L^2$$

$$\omega_r^2 = \frac{L - CR_L^2}{CL^2 - LR_L^2 C^2}$$

$$\omega_r^2 = \frac{C\left[L/C - R_L^2\right]}{C^2 L\left[\frac{L}{C} - R_C^2\right]}$$

$$\omega_r^2 = \frac{R_L^2 - L/C}{LC(R_C^2 - L/C)}$$

$$\omega_r = \sqrt{\frac{R_L^2 - L/C}{LC(R_C^2 - L/C)}}$$

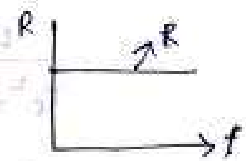
$$2\pi f_r = \sqrt{\frac{R_L^2 - L/C}{LC(R_C^2 - L/C)}}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}} \ Hz}$$ → Resonant frequency.

Graphical representation of Various elements in parallel Resonance

(1) Resistance (R):-

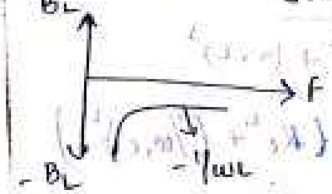
R is independent of frequency



(2) Inductive Reactance :- (B_L)

$$B_L = \frac{1}{jX_L} = \frac{-j}{X_L} = j\left(\frac{-1}{X_L}\right) = j\left(\frac{-1}{\omega L}\right)$$

$$= j\left(\frac{-1}{2\pi f L}\right)$$



$$\boxed{B_L \propto \frac{1}{f}}$$

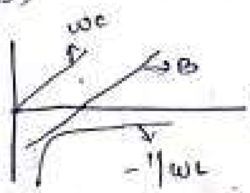
(3) Capacitive reactance :-

$$B_C = \frac{1}{-jX_C} \cdot \frac{j}{j} = \frac{j}{X_C}$$

$$B_C = j\omega_r C = j2\pi f C$$

$\boxed{B_C \text{ v/s } F}$

(4) Susceptance (B)



$Z = R + j(X_L - X_C)$

$Y = G + j(B_C - B_L)$

$= G + j\left(\omega C - \frac{1}{\omega L}\right).$

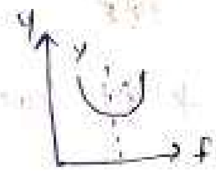
(5) Impedence & admitance:

we know,

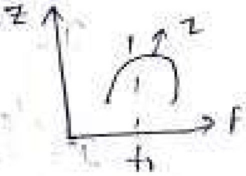
$Y = G + jB$

At resonance, $B = 0$

$\therefore Y = G$, i.e, Y is minimum



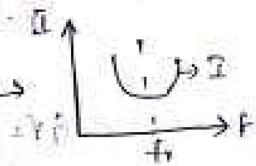
$Z = 1/Y \Rightarrow$



(6) Voltage & current :-

we know, $I = VY$

$I \propto Y$ →



$I = VY$

$V = I/Y$ →

$V \propto 1/Y$

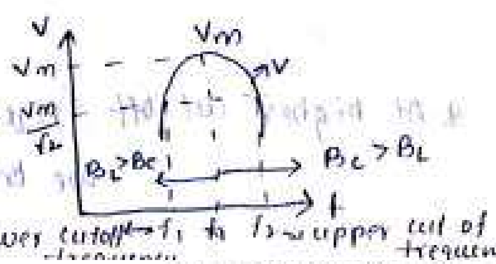
Note :- In parallel resonance, $V = V_m$

$I = I_R$

Bandwidth (BW) :-

Like series resonance, bandwidth of parallel resonance also defined as the difference in frequency at which voltage is equal to 70.7% is of maximum value.

Bandwidth $= f_2 - f_1$ Hz.



Relation b/w Bandwidth & RLC:

* At f_r:-

We know, $i = VY$

At resonance,

$$I = V_m \cdot G \qquad [\because Y = G + jB^{\nearrow 0}]$$

$$Y = G.$$

$$\boxed{V_m = \frac{I}{G}} \rightarrow ①$$

* At lower cut off frequency (f_1):

we know,

$$I = VY$$

$$I = \frac{V_m}{\sqrt{2}}(G + j(B_L - B_C))$$

$$I = \frac{I/G}{\sqrt{2}}(G + j(B_L - B_C))$$

$$1 = \frac{1}{\sqrt{2}G}(G + j(B_L - B_C))$$

$$G\sqrt{2} = G + j(B_L - B_C) \Rightarrow G\sqrt{2} = \sqrt{G^2 + (B_L - B_C)^2}$$

Squaring on both sides.

$$2G^2 = G^2 + (B_L - B_C)^2$$

$$G^2 = (B_L - B_C)^2$$

$$G = B_L - B_C$$

$$1/R = \frac{1}{X_L} - \frac{1}{X_C}$$

$$\frac{1}{R} = \frac{1}{\omega_1 L} - \frac{1}{\omega_1 C}$$

$$1/R = \frac{1}{\omega_1 L} - \omega_1 C \rightarrow ②$$

* At higher cut off frequency:-(f_2)

we know;

$$I = VY$$

$$I = \frac{V_m}{\sqrt{2}} \cdot (G + j(B_C - B_L))$$

$$I = \frac{I/G}{\sqrt{2}} (G + j(B_C - B_L))$$

$$G\sqrt{2} = G + j(B_C - B_L)$$

$$G\sqrt{2} = \sqrt{G^2 + (B_C - B_L)^2}$$

squarring on both sides

$$2G^2 = G^2 + (B_C - B_L)^2$$

$$G^2 = (B_C - B_L)^2$$

$$G = (B_C - B_L)$$

$$\frac{1}{R} = \frac{1}{X_C} - \frac{1}{X_L}$$

$$\frac{1}{R} = \omega_2 C - \frac{1}{\omega_2 L} \rightarrow (3)$$

Equating (2) & (3)

$$\frac{1}{\omega_1 L} - \omega_1 C = \omega_2 C - \frac{1}{\omega_2 L}$$

$$\frac{1}{\omega_1 L} + \frac{1}{\omega_2 L} = \omega_1 C + \omega_2 C$$

$$\frac{1}{L}\left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right) = (\omega_1 + \omega_2) C$$

$$\frac{1}{L}\left(\frac{\omega_2 + \omega_1}{\omega_1 \omega_2}\right) = (\omega_1 + \omega_2) C$$

$$\frac{1}{L(\omega_1 \omega_2)} = C$$

$$\omega_1 \omega_2 = \frac{1}{LC}$$

we know from RLC parallel ckt $\omega_r^2 = \frac{1}{LC}$

$$\omega_1 \omega_2 = \omega_r^2$$

$$\omega_r = \sqrt{\omega_1 \omega_2} \Rightarrow \boxed{f_r = \sqrt{f_1 f_2}} \rightarrow (5)$$

$\therefore$ Resonant frequency (f_r) is the geometrical mean of f_1 & f_2.

Adding eq ② & ③

$$\frac{1}{R}+\frac{1}{R} = \frac{1}{\omega_1 L} - \omega_1 C + \omega_2 C - \frac{1}{\omega_2 L}$$

$$\frac{2}{R} = \frac{1}{L}\left[\frac{1}{\omega_1} - \frac{1}{\omega_2}\right] + C(\omega_2 - \omega_1)$$

$$\frac{2}{R} = \frac{1}{L}\left(\frac{\omega_2-\omega_1}{\omega_1\omega_2}\right) + C(\omega_2-\omega_1)$$

$$= (\omega_2-\omega_1)\left[\frac{1}{L(\omega_1\omega_2)} + C\right]$$

$$= (\omega_2-\omega_1)\left(\frac{1}{\cancel{L}\times\frac{1}{\cancel{L}C}} + C\right)$$

$$= (\omega_2-\omega_1)(C+C)$$

$$\frac{\cancel{2}}{R} = (\omega_2-\omega_1)(\cancel{2}C)$$

$$\omega_2-\omega_1 = 1/RC$$

$$f_2 - f_1 = \frac{1}{2\pi RC}$$

$$\boxed{\text{Band width} = \frac{1}{2\pi RC}}$$

Half power frequencies interms of resonant frequency (f_r):-

for Symmetrical wave,

$$f_r = \frac{f_1+f_2}{2}$$

$$\text{let } f_1 = 2f_r - f_2 \rightarrow ①$$

we know,

$$B.W = f_2 - f_1 = \frac{1}{2\pi RC} \rightarrow ②$$

② in ①

$$f_2 - 2f_r + f_2 = \frac{1}{2\pi RC}$$

$$2f_2 = \frac{1}{2\pi RC} + 2f_r$$

Scanned with CamScanner

$$\boxed{f_2 = \frac{1}{4\pi RC} + f_r}$$

Similarly, $f_r = \frac{f_1 + f_2}{2}$

$$f_2 = 2f_r - f_1 \rightarrow (3)$$

(3) in eqn (2)

$$f_2 \rightarrow 2f_r - f_1 - f_1 = \frac{1}{2\pi RC}$$

$$2f_r - 2f_1 = \frac{1}{2\pi RC}$$

$$f_r - f_1 = \frac{1}{4\pi RC}$$

$$-f_1 = \frac{1}{4\pi RC} - f_r$$

$$\boxed{f_1 = f_r - \frac{1}{4\pi RC}\ Hz}$$

Quality factor:-

It is defined as "The ratio of current through the inductor (or) capacitor to the Supply current.

$$Q = \frac{I_L}{I}$$

At resonance, $I = I_R$

$$Q = \frac{I_L}{I_R}$$

$$= \frac{V/X_L}{V/R}$$

$$\boxed{Q = \frac{R}{X_L}}$$

$$\boxed{Q = \frac{R}{\omega L}}$$

$$Q = \frac{I_C}{I}$$

At resonance, $I = I_R$

$$Q = \frac{I_C}{I_R}$$

$$= \frac{V/X_C}{V/R}$$

$$\boxed{R = \frac{R}{X_C}}$$

$$R = \frac{R}{1/\omega C}$$

$$\boxed{Q = R\omega C}$$

Relation b/w Bandwidth, Quality factor & Resonant frequency:-

we know, Bandwidth $= \frac{1}{2\pi RC}$

Scanned with CamScanner

Multiply numerator & dinaminator by f_r

$$B.\omega = \frac{f_r}{2\pi f_r RC}$$

$$\boxed{B.\omega = \frac{f_r}{\omega_r RC}}$$

$$\boxed{B\omega = \frac{f_r}{Q}}$$ were where = Q = R...

Also,
$$B\text{-}\omega = \frac{f_r}{\omega_r RC}$$

$$= f_r \cdot \frac{X_C}{R}$$

Selectivity :- (S) =

Selectivity is the reciprocal of Quality factor.

$$S = 1/Q \qquad S = 1/Q$$

$$= \frac{1}{R/\omega L} \qquad S = \frac{1}{\omega RC}$$

$$S = \frac{\omega L}{R}$$

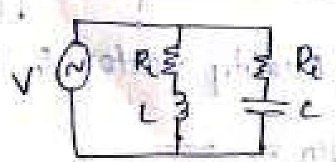
1) What is dynamic impedence?

A) Dynamic imdepence :- Dynamic impedence ~~are~~ or simply dynamic resistance is a pure resistance which is defined only at resonant frequency.

$$\boxed{\text{Dynamic impedence :- } Z_{dy} \text{ (or) } R_{dy} = \frac{L}{CR}}$$

2) Write the Equation for R_L & R_C which causes the ckt to resonant to at all frequencies.

A)



$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}$$

if R_L & R_C are very small,

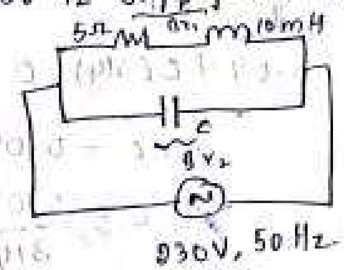
$$R_L^2 - L/C = 0 \qquad R_C^2 - L/C = 0$$

$$R_L^2 = \frac{L}{C} \qquad R_C^2 = L/C$$

$$R_L = \sqrt{\frac{L}{C}} \qquad R_C = \sqrt{L/C}$$

problems on Parallel Resonance:-

1) A parallel ckt has 2nd branches, 1st branche has a resistances of 5 Ω connected in series with an inductance of 10mH. A capacitor is connected in 2nd branch. The parallel ckt is connected across a 230V, 50 Hz supply. If the ckt to be in resonance, find the value of Capacitance & also Current drawn from the supply & also find Currents in branchs 1 & 2.



$$\text{let } Y_1 = \frac{1}{R+jX_L} = \frac{1}{R+j\omega L}$$

$$= \frac{1}{R+j2\pi fL}$$

$$= \frac{1}{5+j(2\times3.14\times50\times10\times10^{-3})}$$

$$= \frac{1}{5+j(3.14)}$$

$$Y_2 = \frac{1}{-jX_C} = \frac{j}{X_C} = j\omega C$$

$$= j(2\pi f)C$$

$$= j(2\times3.14\times50)(C)$$

$$= j(314)C$$

$\cancel{Y} = Y_1 + Y_2$

$$Y_1 = \frac{1}{5+j(3.14)}$$

$$= \frac{5-j(3.14)}{5^2+(3.14)^2}$$

$$= \frac{5-j(3.14)}{25+9.85}$$

$$= 0.14 - j(0.09).$$

$$Y = Y_1 + Y_2$$

$$= 0.14 - j(0.09) + j(314)C$$

$$= 0.14 + j(-0.09 + C\,314)$$

At resonance, imaginary part $= 0$

$$-0.09 + C(314) = 0$$

$$314C = 0.09$$

$$C = \frac{0.09}{314}$$

$$= 2.86 \times 10^{-4}$$

$$= 286 \times 10^{-6}$$

$$= 286\ \mu F$$

Current in branch 1,

$$I_1 = \frac{V}{Z_1} = \frac{230\angle 0}{R + j\omega L}$$

$$= \frac{230\angle 0}{5 + j(314)}$$

$$= \frac{230\angle 0}{5.9\angle 32.12}$$

$$= 38.98\angle -32.12.$$

Current in branch 2. $I_2 = \frac{V}{Z_2} = \frac{V}{-jX_C}$

$$= \frac{230\angle 0}{-j11.10}$$

$= \frac{230 \angle 0}{11.10 \angle -90^\circ}$

$= 22.52 \angle 90^\circ$

Supply Current $(I) = I_1 + I_2$

$= 38.98 \angle -32.12 + 22.52 \angle 90^\circ$

$= 3898 \left(\cos(-32.12) + j \sin(-32.12) \right) + 22.52 \left(\cos 90 + j \sin 90 \right)$

$= 38.98 \left[0.84 + j(-0.53) \right] + 22.52 (0 + j)$

$= 32.74 - j(20.65) + j(22.52)$

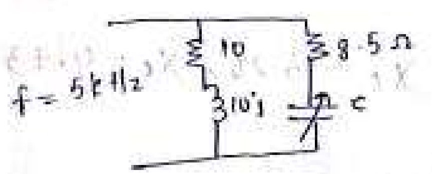
$= 32.74 + j(22.52 - 20.65)$

$= 32.74 + j(1.87)$

$= 32.79 \angle 3.26$

2) An impedence of $Z_1 = 10 + 10j \, \Omega$ is connected with ||el to another impedence of resistances 8.5Ω & Variable capacitance connected in series. find c such that the ckt is in resonance at 5 kHz.

A)



$Y_1 = \frac{1}{10 + 10j}$, $Y_2 = \frac{1}{8.5 - jX_c}$

$Y_1 = \frac{10 - j10}{10^2 + 10^2}$, $Y_2 = \frac{1}{8.5 - jX_c}$

$Y = Y_1 + Y_2$

$= \frac{10 - j10}{200} + \frac{8.5 + jX_c}{(8.5)^2 + (X_c)^2}$

$= \frac{10}{200} - j\frac{10}{200} + \frac{8.5 + jX_c}{72.25 + X_c^2}$

Scanned with CamScanner

$$= \frac{10}{200} + \frac{8.5}{72.25+X_c^2} + j\left(\frac{-10}{200} + \frac{X_c}{72.25 * X_c^2}\right)$$

$$= \frac{1}{20} + \frac{8.6}{72.25} + j\left(\frac{-1}{20} + \frac{X_c}{72.25+X_c^2}\right)$$

$$= 0.16 + j\left(-0.05 + \frac{X_c}{72.25+X_c^2}\right)$$

At resonance, imaginary part = 0

$$-0.05 + \frac{X_c}{72.25+X_c^2} = 0$$

$$\frac{X_c}{72.25+X_c^2} = 0.05$$

$$X_c = (0.05)(72.25+X_c^2)$$

$$X_c = 3.61 + 0.05X_c^2$$

$$3.61 + 0.05X_c^2 - X_c = 0$$

$$3.61 + X_c(0.05X_c - 1) = 0$$

$$3.61 = -X_c(0.05X_c - 1)$$

$$3.61 = X_c(1 - 0.05X_c)$$

$$X_{c_1} = 15.26, \; X_{c_2} = 4.73.$$

$$C_1 = X_c = \frac{1}{\omega C_1}$$

$$15.26 = \frac{1}{\omega C_1}$$

$$15.26 = \frac{1}{2\pi f C_1}$$

$$15.26 = \frac{1}{2 \times 3.14 \times 5 \times 10^3 C_1} \Rightarrow 15.26 = \frac{3.18 \times 10^{-5}}{C_1}$$

$$\cancel{15.26 = \frac{3.18 \times 10^{-5}}{C_1}} \Rightarrow C_1 = \frac{3.18 \times 10^{-5}}{15.26}$$

$$C_1 = \frac{1}{3.18 \times 10^{-5} \times 15.26} = 2.08 \times 10^{-6}$$

$$= 2.08\,\mu F.$$

$$C_2 = X_c = \frac{1}{\omega c}$$

$$4.73 = \frac{1}{2\pi f c}$$

$$4.73 = \frac{1}{2 \times 3.14 \times 5 \times 10^3 \times c}$$

$$C_2 = \frac{1}{2 \times 3.14 \times 5000 \times 4.73}$$

$$= 6.73 \times 10^{-6}$$

$$\boxed{C_2 = 6.73\ \mu F}$$