Basic Maxwell's Equations:-

Maxwells Equation can be written in differential & integral forms.

Electric field intensity → E
Electric flux density → D
Magnetic field intensity → H
Magnetic flux density → B.
Current density → J.
Charge density → ρ.

ε → permittivity
μ → permeability
σ → Conductivity
ρ → Resistivity ($\rho = \frac{1}{\sigma}$)

$$\nabla \times H = J + \frac{\partial D}{\partial t} \quad — (1)$$

$$\nabla \times H = \frac{\partial D}{\partial t} \ (\text{if } J = 0) \quad \& \quad \nabla \times H = J \ (\text{for dc field}). — (2)$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad \& \ \nabla \times E = 0 \ (\text{for static field}). \quad — (3)$$

$$\nabla \cdot D = \rho \quad \& \quad \nabla \cdot D = 0 \ (\text{for free-charge region i.e., } \rho = 0). — (4)$$

$$\nabla \cdot B = 0. \quad — (5)$$

The field quantities are Connected by the following relations:

$$D = \varepsilon E \quad \rightarrow (6)$$

$$B = \mu H \quad \rightarrow (7)$$

$$J = \sigma E = E/\rho. \rightarrow (8)$$

The other relavent equations,

$$V = \int \frac{\rho\, dl}{4\pi\varepsilon R} = \iint \frac{\rho_s\, ds}{4\pi\varepsilon R} = \iiint \frac{\rho_v\, dv}{4\pi\varepsilon R} \quad \rightarrow (9)$$

$$E = -\nabla V \quad \rightarrow (10)$$

$$\cancel{\nabla^2 (\text{or})} \ \nabla^2 V = -\rho/\varepsilon \quad \& \quad \nabla^2 V = 0 \text{ if } \rho = 0. \rightarrow (11)$$

V → scalar electric potential
... source & the point at which V is to be evaluated

$$A = \int \frac{\mu I\,dl}{4\pi R} = \iint \frac{\mu K\,ds}{4\pi R} = \iiint \frac{\mu J\,dv}{4\pi R} \quad - (12)$$

$$B = \nabla \times A \quad - (13)$$

$$\nabla^2 A = -\mu J \ \& \ \nabla^2 A = 0 \text{ for } J = 0. \quad - (14)$$

$A \rightarrow$ vector magnetic potential

$I \rightarrow$ Current

$K \rightarrow$ Surface Current density

Retarded Potential (Time Varying):-

Radiation is a time varying phenomena.

$$\nabla \times E = \nabla \times (-\nabla V) \equiv 0. \quad - (a)$$

By vector identity the curl of a gradient is identically zero.

But from eq. (3), $\nabla \times E = -\frac{\partial B}{\partial t}$ for a time-varying field.

let $E = -\nabla V + N \quad \rightarrow (b)$

$$\nabla \times E = \nabla \times (-\nabla V) + \nabla \times N$$

$$= 0 + \nabla \times N$$

$$\therefore \nabla \times N = -\frac{\partial B}{\partial t}$$

$$= -\frac{\partial (\nabla \times A)}{\partial t}$$

$$\nabla \times N = \nabla \times \left(-\frac{\partial A}{\partial t}\right)$$

$$N = -\frac{\partial A}{\partial t} \quad - (c)$$

Sub. eq. (c) in (b),

$$E = -\nabla V - \frac{\partial A}{\partial t} \quad - (d)$$

The equation (4) is to be tested by using the relation,

$$D = \varepsilon E$$

$$\nabla\cdot D = \nabla\cdot(\varepsilon E)$$

$$= \varepsilon(\nabla\cdot E)$$

$$\cancel{\varepsilon\,\nabla\cdot\left(-\frac{\partial A}{\partial t}\right)} = \varepsilon\,\nabla\cdot\left(-\nabla V - \frac{\partial A}{\partial t}\right)$$

$$= \varepsilon\left(-\nabla\cdot\nabla V - \frac{\partial}{\partial t}(\nabla\cdot A)\right) = \rho$$

$$\boxed{\nabla\cdot D = \rho}$$

$$\therefore\ \nabla^2 V + \frac{\partial(\nabla\cdot A)}{\partial t} = -\frac{\rho}{\varepsilon} \quad -(e)$$

The eq-(e) can be written as,

$$\nabla^2 V = -\rho/\varepsilon \quad \text{for static Conditions.} \quad \rightarrow (f)$$

$$\nabla^2 V = -\rho/\varepsilon - \frac{\partial}{\partial t}(\nabla\cdot A) \quad \text{for time-varying Conditions.} \rightarrow (g)$$

To validate the eq.

$$\nabla\times H = J + \frac{\partial D}{\partial t}$$

$$B = \mu H \quad (\text{or}) \quad H = B/\mu$$

LHS can be written as,

From identity,
$\nabla\times\nabla\times A = \nabla(\nabla\cdot A) - \nabla^2 A$

$$\Rightarrow (\nabla\times B/\mu) = (\nabla\times\nabla\times A)/\mu$$

$$= [\nabla(\nabla\cdot A) - \nabla^2 A]/\mu \quad \rightarrow (h)$$

RHS;

$$\Rightarrow\ J + \frac{\partial D}{\partial t} = J + \varepsilon\frac{\partial E}{\partial t}$$

$$= J + \varepsilon\,\partial\left(-\nabla V - \partial A/\partial t\right)/\partial t$$

$$= J + \varepsilon\left[-\nabla\left(\frac{\partial V}{\partial t}\right) - \frac{\partial^2 A}{\partial t^2}\right]$$

$$= J - \varepsilon\left[\nabla\left(\frac{\partial V}{\partial t}\right) + \frac{\partial^2 A}{\partial t^2}\right] \quad -(I)$$

Equating LHS & RHS,

$$\nabla(\nabla\cdot A) - \nabla^2 A = \mu J - \mu\varepsilon\left[\nabla\left(\frac{\partial V}{\partial t}\right) + \frac{\partial^2 A}{\partial t^2}\right] \rightarrow (J)$$

$\nabla^2 A$ is defined from eq. (14), where as term $\nabla\cdot A$ is yet to be defined.

Helmholtz Theorem:-

A vector field is Completely defined only when both its curl & divergence are known.

There are some Conditions which specify divergence of A. Two of these Conditions known as Lorentz gauge condition & Coulomb's guage condition, are given by,

$$\nabla\cdot A = -\mu\varepsilon\frac{\partial V}{\partial t} \rightarrow (K)$$

$$\nabla\cdot A = 0 \rightarrow (L)$$

Using the Lorentz gauge Condition, eq. (g) & (J) Can be written as,

$$\nabla^2 V = -\rho/\varepsilon - \partial\left(\mu\varepsilon\frac{\partial V}{\partial t}\right)/\partial t = -\rho/\varepsilon - \mu\varepsilon\left(\frac{\partial^2 V}{\partial t^2}\right)$$

$$\nabla^2 A = -\mu J + \mu\varepsilon\left(\frac{\partial^2 A}{\partial t^2}\right)$$

$$\nabla^2 V = -\rho/\varepsilon + \omega^2\mu\varepsilon V$$

$$\nabla^2 A = -\mu J + \omega^2\mu\varepsilon J.$$

$$\left.\begin{aligned} V &= \int_V \frac{[\rho]\,dv}{4\pi\varepsilon R} \\ A &= \int_V \frac{\mu[J]\,dv}{4\pi R} \end{aligned}\right\} \text{Retarded potentials.}$$

Radian:-

→ Radian is a measure of plane angle.

→ one radian is defined as the plane angle with its vertex at the
centre of radius 'r' that is subtended by an arc whose length is
equal to 'r'.

As the Circumference of a Circle is $C = 2\pi r$, there are 2π Radians $\left(\frac{2\pi r}{r}\right)$ in a full Circle.

Steradian:-

→ Steradian is a measure of Solid angle.

→ One steradian is defined as "the Solid angle with its
vertex at centre for a sphere of radius 'r' which is subtended
by a spherical surface area equal to the area of a square with side
length 'r'.

As the area of the sphere $A = 4\pi r^2$, there are 4π steradians $\left(\frac{4\pi r^2}{r^2}\right)$ in a Closed sphere.

The differential areads on the Surface of the sphere of radius 'r' is $ds = r^2 \sin\theta \, d\theta \, d\phi$ (m^2).

Applications of Reciprocity Theorem:-

The reciprocity Theorem may be used to derive the following very important properties of Transmitting & Receiving antennas.

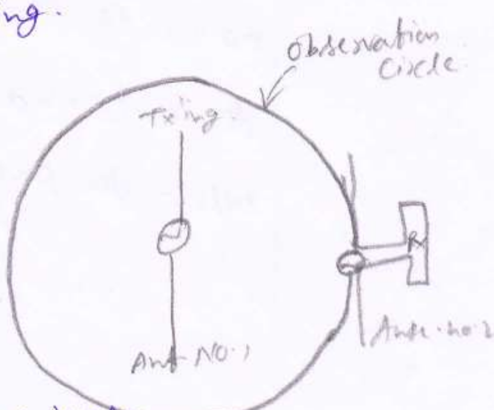
1. Equality of Directional patterns.
2. Equality of Directivities.
3. Equality of Effective lengths.
4. Equality of Antenna Impedances.

Equality of Directional Patterns:-

The directional patterns of Tx'ing & Rx'ing antennas are identical if all the media are linear, passive, isotropic then the Reciprocity Theorem holds goods.

Proof:-

Under the mentioned Conditions it is to be proved that the Tx'ing & Rx'ing antennas patterns are identical. For this Consider fig-(a) in which two antennas no.1 (Test antenna) & antenna no.2. (Exploring antenna) are shown. Let antenna no.1 is Tx'ing and antenna.no.2 is Rx'ing.



Measurement of pattern on observation circle.

The pattern may be either field pattern or power pattern which itself is proportional to square of field pattern.

Considering field pattern, keeping the Tx'ing antenna no.1 at the centre of the observation circle. The Rx'ing antenna no.2 is moved along the surface of the great observation circle.

Now if a voltage E is applied at Transmitting antenna No.1 & the resultant Current I at the terminals of Rx'ing antenna No.2 is measured which will be the indication of electric field at the location of antenna No.2.

If the process is reversed i.e., the same voltage E is applied to antenna no.2 & resulting Current I is measured at the test antenna no.1 measures the electric field at the location of antenna no.1.

A/c'ing to Reciprocity theorem, for every position of test antenna no.1 the ratio E/I is the same as was in the previous case.

Thus, it is proved that Radiation pattern of test antenna no.1 (Tx'ing) observed by moving Rx'ing antenna no.2 is identical with the radiation pattern obtained when the antenna no.2 is Tx'ing & antenna no.1 is Rx'ing i.e., when the process is Reversed.

2. Equality of Directivities:-

The Directivity 'D' is defined as,

$$\text{Directivity} = \frac{\text{Max. Radiation Intensity.}}{\text{Avg. Radiation Intensity.}}$$

$$D = \frac{\phi(\theta,\phi)_{max}}{\phi_{avg}} = \frac{\phi_m}{\phi_{avg}} \quad \text{— (1)}$$

But, $\text{Avg. Radiation Intensity} = \dfrac{\text{Total power radiated in watts}}{4\pi \text{ in steredian}}$

$$\phi_{avg} = \frac{W}{4\pi} \left(W/Sr\right) \quad \text{— (2)}$$

Sub. (2) in (1),

$$D = \frac{\phi(\theta,\phi)_{max}}{W/4\pi} = \frac{4\pi \cdot \phi(\theta,\phi)_{max}}{W} \quad \text{— (3)}$$

But the Total power radiated W is given by radiation intensity $\phi(\theta,\phi)$, integrated over solid angle 4π steradian i.e.,;

$$W = \iint_{4\pi} \phi(\theta,\phi)\, d\Omega \quad \text{— (4)}$$

$$\therefore D = \frac{4\pi \cdot \phi(\theta,\phi)_{max}}{\iint_{4\pi} \phi(\theta,\phi)\, d\Omega} = \frac{4\pi\, \phi_m}{\iint_{4\pi} \phi\, d\Omega}$$

$$= \frac{4\pi}{\iint_{4\pi} \left(\frac{\phi}{\phi_m}\right) d\Omega}$$

$$\therefore D = \frac{4\pi}{\quad} \quad \text{— (5)}$$

where $d\Omega$ = solid angle = $\sin\theta \cdot d\theta \cdot d\phi$ sr.

$$\phi_n = \phi_n(\theta,\phi) = \frac{\phi(\theta,\phi)}{\phi(\theta,\phi)_{max}} = \frac{\phi}{\phi_m} = \text{Normalized Power pattern.}$$

∴ Since Radiation intensity is a function of ϕ & θ,

it can be expressed as,

$$\phi = \phi_m f(\theta,\phi)$$

$$\frac{\phi}{\phi_m} = f(\theta,\phi).$$

$$\frac{\phi}{\phi_m} = f_n(\theta,\phi) = \text{Normalized three dimensional Power pattern}$$

$$\therefore D = \frac{4\pi}{\iint f(\theta,\phi)\, d\Omega} \quad \text{— (6)}$$

Directivity 'D' depends on the shape of the power pattern only. So, The Radiation pattern of an antenna is same whether Tx'ing or Rx'ing.

∴ Directivities will be same whether it is Calculated from antennas Tx'ing pattern or Rx'ing pattern. Hence the Directivity Can be applied to both Tx'ing & Rx'ing antennas if the value of the Directivity is same in both the Cases.

Problems

1) The Radiation intensity of a certain antenna is 95%. The maximum radiation intensity is 0.5 W/sr. Calculate the directivity of the antenna if i) Pinput = 0.4 W and ii) Prad = 0.3 W.

given $\eta = 0.95$ & $U_{max} = 0.5$ W/sr.

$$\eta = \frac{P_{rad}}{P_{input}}$$

$$P_{rad} = \eta \times P_{input}$$

$$= (0.95) \times 0.4 = 0.38W$$

$$D = \frac{U_{max}}{\left(\frac{P_{rad}}{4\pi}\right)} = \frac{0.5}{\left(\frac{0.38}{4\pi}\right)} = 16.5346$$

$$D = \frac{U_{max}}{\left(\frac{P_{rad}}{4\pi}\right)} = \frac{0.5}{\left(\frac{0.3}{4\pi}\right)} = 20.9439.$$

2) An antenna has radiation resistance of 72 Ω, loss resistance of 8 Ω & a power gain of 12 dB. Determine antenna efficiency and directivity.

$$R_r = 72\Omega, \quad R_l = 8\Omega, \quad G_p(dB) = 12$$

$$\eta = \frac{R_r}{R_r + R_l} = \frac{72}{72+8} = 0.9 \qquad \%\eta = 90\%$$

$$G_p(dB) = 12$$

$$10 \log(G_p) = 12 \Rightarrow G_p = (10)^{1.2} = 15.8489.$$

$$\eta = \frac{G_p}{G_d} \Rightarrow G_d = \frac{G_p}{\eta} = \frac{15.8489}{0.9} = 17.6099$$

3) The radiation resistance of an antenna is 72Ω and the loss resistance is 8Ω. What is the directivity if the Power gain is 16.

$$R_r = 72\Omega, \quad R_l = 8\Omega, \quad G_p = 16.$$

$$\eta = \frac{R_r}{R_r + R_l} = \frac{72}{72+8} = 0.9. \qquad \therefore \eta = 90\%$$

$$\eta = \frac{G_p}{G_d} \Rightarrow G_d = \frac{G_p}{\eta} = \frac{16}{0.9} = 17.7778.$$

$$G_d(dB) = 10\log_{10}(17.7778) = 12.4987\text{ dB}.$$

4) A loss resistance of antenna is 25Ω, Calculate its radiation resistance if power gain is 30 & directivity is 42.

$$R_l = 25\Omega$$

$$G_d = 42;\ G_p = 30$$

$$\eta = \frac{R_r}{R_r + R_l}$$

$$\eta = \frac{G_p}{G_d} = \frac{30}{42} = 0.71428.$$

$$\%\eta = 71.428\%. \qquad \eta = \frac{R_r}{R_r + R_l}$$

$$0.71428 = \frac{R_r}{R_r + 25}$$

$$R_r = 62.4982\Omega.$$

5) An half wave dipole is $\lambda/15$ m long. If its loss resistance is 1.5Ω. Calculate i) Radiation resistance ii) Antenna efficiency.

$$dL = \lambda/15\text{ m}, \quad R_l = 1.5\Omega.$$

$$R_{rad} = 80\pi^2\left(\frac{dL}{\lambda}\right)^2$$

$$= 80\pi^2\left(\frac{\lambda/15}{\lambda}\right)^2 = 3.5091\Omega.$$

$$\eta = \frac{R_r}{R_r+R_l} = \frac{3.5091}{3.5091+5} = 0.70$$

$$\% \eta = 70.05\%$$

6) Calculate the power gain of an half wave dipole whose ohmic losses and directive gain are 7Ω and 1.64 respectively.

$$G_d = 1.64, \quad R_l = 7\Omega.$$

for an half wave dipole, $R_r = 73\Omega$

$$\eta = \frac{R_r}{R_r+R_l} = \frac{73}{73+7\Omega} = 0.9125.$$

$$\% \eta = 91.25\%$$

$$\eta = \frac{G_P}{G_d} \Rightarrow G_P = \eta \times G_d = (0.9125)(1.64) = 1.4965.$$

$$G_P(dB) = 10\log_{10}(1.4965) = 1.7507\,dB.$$

7) Calculate the radiation resistance of an antenna which is drawing 15A current and radiating 5W.

$$W = 5KW = 5000\,W.$$

$$I_{rms} = 15A.$$

$$W = I_{rms}^2 \cdot R_r.$$

$$(5000) = (15)^2 \cdot R_r$$

$$R_r = \frac{5000}{15\times15} = 22.22\Omega.$$

8) An antenna operating at a wavelength of 2.5m has directivity of 90. Determine maximum effective aperture.

$$\lambda = 2.5\,m, \quad G_d = D = 90.$$

$$(A_e)_{max} = \frac{\lambda^2}{4\pi} \times D$$

$$= \frac{(2.5)^2}{4\pi} \times 90 = 44.7623\,m^2.$$

(9) Determine maximum effective aperture of an antenna having small side lobes. The HPBW's in the perpendicular planes intersecting in beam axis are $35°$ & $40°$.

$$\theta_E = 35°, \theta_H = 40°$$

$$D = \frac{41,257}{\theta_E \times \theta_H} = \frac{41,257}{35 \times 40} = 29.4692.$$

$$(A_e)_{max} = \frac{\lambda^2}{4\pi} \times D = \frac{\lambda^2}{4\pi} \times 29.4692.$$

$$= 2.34\,\lambda^2\ m^2$$

(10) Calculate the gain of an antenna with circular aperture of diameter 3m at 5GHz frequency assuming zero losses.

$$f = 5GHz \Rightarrow 5 \times 10^9\,Hz.$$

$$d = 3m\ ;\ P_{loss} = 0.$$

As zero losses are assumed, $\eta = 1$

$$\therefore G_p = G_d = D$$

$$D = \frac{4\pi}{\lambda^2} \cdot (A_e)_{max}$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{5 \times 10^9} = 0.06\,m.$$

$$A_{em} = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \pi \left(\frac{3}{2}\right)^2 = 7.0685\,m^2.$$

11) An isotropic antenna radiates equally in all directions. The total power delivered to the radiator is 100 kW. Calculate the power density at distances (i) 100 m, (ii) 1000 m (iii) 100,000 m

given, $W_t = 100\,kW = 100 \times 10^3\, W$.

$$i)\ P_r = \frac{W_t}{4\pi r^2} = \frac{100 \times 10^3}{4\pi \times (100)^2} = 0.796178\ W/m^2.$$

$$ii)\ P_r = \frac{W_t}{4\pi r^2} = \frac{100 \times 10^3}{4\pi \times (1000)^2} = 7.9617\ mW/m^2.$$

$$iii)\ P_r = \frac{W_t}{4\pi r^2} = \frac{100 \times 10^3}{4\pi \times (100000)^2} = 0.7961\ \mu W/m^2.$$

12) Determine the electric field intensity at a distance of 10 km from an antenna having dielectric gain of 5 dB and radiating a total power of 20 kW.

$$r = 10\,km = 10 \times 10^3\, m.$$

$$P_{rad} = 20\,kW = 20 \times 10^3\, W.$$

$$G_d(\theta, \phi) = 5\,dB.$$

$$10 \log G_d(\theta, \phi) = 5\,dB.$$

$$G_d(\theta, \phi) = (0.5)^{10} = 3.1622.$$

$$\text{But, } G_d(\theta, \phi) = \frac{P_d(\theta, \phi)}{P_{avg}} = \frac{P_d(\theta, \phi)}{\left(\frac{P_{rad}}{4\pi r^2}\right)}$$

$$\text{Power density, } P_d(\theta, \phi) = \frac{1}{2} \frac{E^2}{\eta_0}$$

$$= \frac{1}{\ } |E|^2.$$

$$G_d(\theta,\phi) = \frac{\frac{1}{240\pi}|E|^2}{\left(\frac{P_{rad}}{4\pi r^2}\right)}$$

$$|E|^2 = G_d(\theta,\phi) \cdot \left(\frac{P_{rad}}{4\pi r^2}\right)(240\pi).$$

$$|E| = \frac{\sqrt{60 \cdot G_d(\theta,\phi) \cdot P_{rad}}}{r}$$

$$= \frac{60\,(3.1622)\,(20\times10^3)}{10\times10^3}$$

$$= 0.1948 \text{ V/m.}$$

13) Calculate the electric field E_{rms} due to an isotropic radiator radiating 3KW power at a distance of 2Km from it

$P_{rad} = 3\,KW = 3\times10^3\,W$

$d = 2\,km = 2\times10^3\,m.$

for an isotropic radiator,

$$E_{rms} = \frac{\sqrt{30\times P_{rad}}}{r} = \frac{\sqrt{30\times(3\times10^3)}}{2\times10^3} = 0.15 \text{ V/m.}$$

Note:- For an grounded antenna,

$$E_{rms} = \frac{\sqrt{90\times P_{rad}}}{r}$$

14) Evaluate the directivity of i) An isotropic source and ii) Source with bidirectional $\cos\theta$ power pattern.

$$P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} U(\theta,\phi)\sin\theta\, d\theta\, d\phi.$$

i) For an isotropic radiator, $U(\theta,\phi) = U_{max}$.

$$P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} U_{max}\cdot\sin\theta\, d\theta\, d\phi.$$

$$= U_{max}\int_{\phi=0}^{2\pi} d\phi \cdot \int_{\theta=0}^{\pi}\sin\theta\, d\theta.$$

$$= U_{max}(2\pi)(-\cos\theta)_0^{\pi}$$

$$= U_{max}(2\pi)[-(\cos\pi - \cos 0)]$$

$$= U_{max}(2\pi)[-((-1)-(1))]$$

$$= U_{max}\, 4\pi.$$

$$\therefore D = \frac{4\pi U_{max}}{P_{rad}}$$

$$= \frac{4\pi U_{max}}{4\pi U_{max}} = 1.$$

ii) $$P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} U_{max}\cdot\cos\theta\cdot\sin\theta\, d\theta\, d\phi.$$

$$= U_{max}\int_{\phi=0}^{2\pi} d\phi\cdot\frac{1}{2}\int_{\theta=0}^{\pi} 2\sin\theta\cos\theta\, d\theta.$$

$$= U_{max}\cdot(2\pi)\cdot\frac{1}{2}\int_{\theta=0}^{\pi}\sin 2\theta\, d\theta.$$

$$\pi/2$$

$$= U_{max}\,(2\pi)\left[\frac{-\cos 2\theta}{2}\right]_0^{\pi/2}$$

$$= U_{max}\,(2\pi)(1)$$

$$\therefore D = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi U_{max}}{2\pi U_{max}}$$

$$D = 2.$$

15) For a source with radiation intensity $U = 6\cos\theta$, find the directivity and HPBW when its pattern is unidirectional.

$$P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} U(\theta,\phi)\cdot\sin\theta\, d\theta\, d\phi.$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} 6\cos\theta\sin\theta\, d\theta\, d\phi.$$

$$= 6\int_{\phi=0}^{2\pi} d\phi \cdot \frac{1}{2}\int_{\theta=0}^{\pi/2} 2\sin\theta\cos\theta\, d\theta$$

$$= 6(2\pi)\cdot\frac{1}{2}\int_0^{\pi/2}\sin 2\theta\, d\theta.$$

$$= 6(2\pi)\cdot\frac{1}{2}\left[\frac{-\cos 2\theta}{2}\right]_0^{\pi/2}$$

$$= 6(2\pi)\cdot\frac{1}{2}(1)$$

$$= 6\pi.$$

$$D = \frac{4\pi\cdot U_{max}}{P_{rad}} = \frac{4\pi\cdot U_{max}}{6\pi} = \frac{4\pi(6)}{6\pi} = 4.$$

$$\Rightarrow D = \frac{4\pi}{\Omega}.$$

$$\Omega = \frac{4\pi}{D} = \frac{4\pi}{4} = \pi \text{ rad.}$$

$$HPBW = \frac{\Omega}{2} = \frac{\pi}{2} \text{ rad.}$$

16) Calculate the minimum distance required to measure the field pattern of an antenna of diameter 2m at a freq. of 3 GHZ.

$$f = 3\,GHZ = 3\times10^{9}\,HZ.$$

$$d = 2m.$$

$$\lambda = \frac{c}{f} = \frac{3\times10^{8}}{3\times10^{9}} = 0.1m$$

The minimum distance required to measure the field pattern is given by,

$$r = \frac{2d^2}{\lambda} = \frac{2(2)^2}{0.1} = 80m.$$

17) An antenna with effective temperature of 15K is fed into a microwave amplifier with effective noise temperature of 20K. Calculate available noise power per unit bandwidth at the input for this particular antenna temperature. Calculate noise power for a noise bandwidth of 4MHZ.

$$T_A = 15K, \quad T_R = 20K, \quad B = 4MHZ.$$

$$P = K(T_A + T_R)B$$

$$\frac{P}{B} = K(T_A + T_R)$$

$$= 1.38\times10^{-23}(15+20)$$

$$= 48.3\times10^{-3}\ W/HZ.$$

$$P = K(T_A + T_R)B$$

$$= 4\times10^{6}(15+20)(1.38\times10^{-23})$$

$$= 193.2\times10^{-17}\ W.$$

18) Calculate $D(\theta,\phi)$, the directivity for the three unidirectional sources with following power patterns.

i) $\phi = \phi_m \sin\theta \sin^2\phi$

ii) $\phi = \phi_m \sin\theta \sin^3\phi$

iii) $\phi = \phi_m \sin^2\theta \sin^3\phi$.

Where ϕ lies between $0 \le \theta \le \pi$ and $0 \le \phi \le \pi$.

i) $\phi = \phi_m \sin\theta \sin^2\phi$.

$$P_{rad} = \int_{\phi=0}^{\pi}\int_{\theta=0}^{\pi} \phi_m \sin\theta \cdot \sin^2\phi \cdot \sin\theta \, d\theta \, d\phi.$$

$$= \phi_m \int_{\phi=0}^{\pi} \sin^2\phi \, d\phi \int_{\theta=0}^{\pi} \sin^2\theta \, d\theta.$$

$$= \phi_m \cdot 2 \cdot \int_{\phi=0}^{\pi/2} \sin^2\phi \, d\phi \cdot 2 \int_{\theta=0}^{\pi/2} \sin^2\theta \, d\theta.$$

$$= \phi_m \cdot 2 \cdot \left(\frac{1}{2} \cdot \pi/2\right) \cdot 2 \cdot \left(\tfrac{1}{2} \cdot \pi/2\right)$$

$$= \phi_m \cdot \frac{\pi^2}{4}.$$

$$\therefore D = \frac{4\pi \, \phi_m}{P_{rad}} = \frac{4\pi \, \phi_m}{\phi_m \frac{\pi^2}{4}} = \frac{16}{\pi}.$$

ii) $\phi = \phi_m \sin\theta \sin^3\phi$.

$$P_{rad} = \int_{\phi=0}^{\pi}\int_{\theta=0}^{\pi} \phi_m \sin\theta \sin^3\phi \sin\theta \, d\theta \, d\phi.$$

$$= \phi_m \int_{\phi=0}^{\pi} \sin^3\phi \, d\phi \cdot \int_{\theta=0}^{\pi} \sin^2\theta \, d\theta.$$

$$= \phi_m \cdot 2 \cdot \int^{\pi/2} \sin^3\phi \, d\phi \cdot 2 \cdot \int^{\pi/2} \sin^2\theta \, d\theta.$$

$$= \phi_m \cdot \cancel{2}\left(\frac{2}{3}\right) \cdot \cancel{\pi} \cdot \left(\frac{\pi}{\cancel{2}} \cdot \frac{1}{\cancel{\pi}}\right)$$

$$= \frac{2\pi \phi_m}{3}$$

$$D = \frac{4\pi \cdot \phi_m}{P_{rad}}$$

$$= \frac{4\pi \cdot \phi_m}{\frac{2\pi \cdot \phi_m}{3}} = \frac{12 \cdot \pi}{2\pi} = 6.$$

$$\therefore D = 6$$

Note: $\int_0^{\pi} \sin^2\theta \, d\theta = 2\int_0^{\pi/2} \sin^2\theta \, d\theta$

$= 2\left(\frac{1}{2} \cdot \frac{\pi}{2}\right)$

$\int_0^{\pi} \sin^3\theta \, d\theta = 2\int_0^{\pi/2} \sin^3\theta \, d\theta.$

$= 2\left(\frac{2}{3}\right)$

iii) $\phi = \phi_m \cdot \sin^2\theta \, \sin^3\phi.$

$$P_{rad} = \int_{\phi=0}^{\pi} \int_{\theta=0}^{\pi} \phi \cdot \sin\theta \, d\theta \, d\phi.$$

$$= \int_{\phi=0}^{\pi} \int_{\theta=0}^{\pi} \phi_m \sin^2\theta \, \sin^3\phi \, \sin\theta \, d\theta \, d\phi.$$

$$= \phi_m \int_{\phi=0}^{\pi} \sin^3\phi \, d\phi \cdot \int_{\theta=0}^{\pi} \sin^3\theta \, d\theta$$

$$= \phi_m \cdot 2 \int_{\phi=0}^{\pi/2} \sin^3\phi \, d\phi \cdot 2 \cdot \int_{\theta=0}^{\pi/2} \sin^3\theta \, d\theta.$$

$$= \phi_m \cdot 2 \cdot \left(\frac{2}{3}\right) \cdot 2 \cdot \left(\frac{2}{3}\right)$$

$$= \frac{16}{9} \phi_m.$$

$$\therefore D = \frac{4\pi \cdot \phi_m}{P_{rad}} = \frac{4\pi \cdot \phi_m}{16/9 \cdot \phi_m} = \frac{\cancel{4}\pi \times 9}{\cancel{16}\,4} = \frac{9\pi}{4}.$$

$$\therefore D = \frac{9\pi}{4}.$$

Power gain:-

Power gain Compares the radiated power density of the actual antenna and that of an isotropic antenna on the basis of the same input power to both.

$$G_p = \frac{\text{Power density radiated in a particular direction by the Test antenna.}}{\text{Power density radiated in that direction by an isotropic antenna.}}$$

$$G_p = \frac{\text{Radiation intensity in the given direction}}{\text{Average Total power input.}}$$

$$G_p = \frac{\phi(\theta,\phi)}{(W_T/4\pi)}$$

$$= \frac{4\pi \cdot \phi(\theta,\phi)}{W_T}$$

$$W_T = W_r + W_l$$

Total i/p power = Power radiated + Ohmic losses in antenna.

$$G_p = \frac{4\pi \cdot \phi(\theta,\phi)}{W_T} = \frac{4\pi \cdot \phi(\theta,\phi)}{W_r + W_l}$$

$P = \frac{V^2}{R}$

$$G_p(dB) = 10\log_{10} G_p = 10\log_{10}\left(\frac{P_1}{P_2}\right)$$

$$= 10\log_{10}\left(\frac{V_1}{V_2}\right)^2$$

Power gain (G_p) depends on,

i) Sharpness of the lobe

ii) Volume of the Solid Radiation pattern

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Directivity (D):-

The maximum directive gain is called as directivity (D) of an antenna.

$$D = \frac{\text{Maximum Radiation Intensity of Test antenna}}{\text{Average Radiation Intensity of Test antenna}}$$

$$D = \frac{\phi(\theta,\phi)_{max}}{\phi_{avg}}$$

In term of Radiated power,

$$D = \frac{\text{Power radiated from Test antenna}}{\text{Power radiated from an isotropic antenna.}}$$

$$D = \frac{4\pi}{B.A.} \quad \text{where } B.A = \frac{\iint f(\theta,\phi)\, d\Omega}{f(\theta,\phi)_{max}}$$

$$D = \frac{4\pi}{\Omega_A} \; ; \; \Omega_A = \text{Beam Solid angle}$$

$$= \frac{\iint f(\theta,\phi)\, d\Omega}{f(\theta,\phi)_{max}}$$

Relation between Directive gain (G_d) and Directivity (D):-

$$\text{Directive gain } (G_d) = \frac{\phi(\theta,\phi)}{\frac{W_r}{4\pi}} = \frac{4\pi \cdot \phi(\theta,\phi)}{W_r}$$

$$\text{Directivity } (D) = \frac{\phi(\theta,\phi)_{max}}{\phi_{avg}} = \frac{\phi(\theta,\phi)}{\phi_0}$$

For an lossless isotropic antenna, G_d & D are same.

$G_0 = KD$. $K \Rightarrow$ Efficiency factor. ; $K = 1$ for 100% efficiency
$K < 1$ if any losses are present

w.r.t isotropic antenna, ϕ_{max}

Antenna Efficiency:-

The Efficiency of an Antenna is defined as the ratio of Power radiated to the total input power supplied to the antenna. [η] (or) [K].

$W_r \rightarrow$ Radiated power
$W_l \rightarrow$ Ohmic losses.

$$\eta = \frac{\text{Power radiated}}{\text{Total input power}}$$

$$\eta = \frac{W_r}{W_T} \qquad [W_T = W_r + W_l].$$

$$= \frac{W_r}{W_r + W_l}$$

$$\eta = \frac{W_r}{W_T} \times \frac{4\pi \cdot \phi(\theta,\phi)}{4\pi \cdot \phi(\theta,\phi)}$$

$$= \frac{4\pi\,\phi(\theta,\phi)}{W_T} \cdot \frac{W_r}{4\pi \cdot \phi(\theta,\phi)}$$

$$= \frac{4\pi \cdot \phi(\theta,\phi)}{W_T} \cdot \frac{1}{\left(\frac{4\pi \cdot \phi(\theta,\phi)}{W_r}\right)}$$

$$= G_p \cdot \frac{1}{G_d}$$

$$\eta = \frac{G_p}{G_d}$$

$G_p \rightarrow$ Power gain
$G_d \rightarrow$ directive gain.

$$\eta = \frac{W_r}{W_T} = \frac{I^2 R_r}{I^2 R_r + I^2 R_l} = \frac{R_r}{R_r + R_l}$$

$R_r \rightarrow$ Radiation resistance.
$R_l \rightarrow$ loss resistance.

$$\%\eta = \frac{R_r}{R_r + R_l} \times 100.$$

$R_r + R_l \rightarrow$ Total effective resistance.

Loss Resistance may consists:
i) Ohmic loss in the antenna Conductor.
ii) Directive loss
iii) [illegible] loss in antenna & ground system.

Effective area (or) Capture Area (or) Effective Aperture:-

A Transmitting antenna Transmits Electromagnetic waves and a receiving antenna receives a fraction of the same. The concept of effective area or aperture is best understood by considering an antenna to have an effective area or aperture over which it extracts electromagnetic energy from the travelling electromagnetic waves.

$$\text{Effective Area (or) Effective Aperture} = \frac{\text{Power Received}}{\text{Poynting vector of incident wave}}$$

$$A_e = \frac{W}{P} = A$$

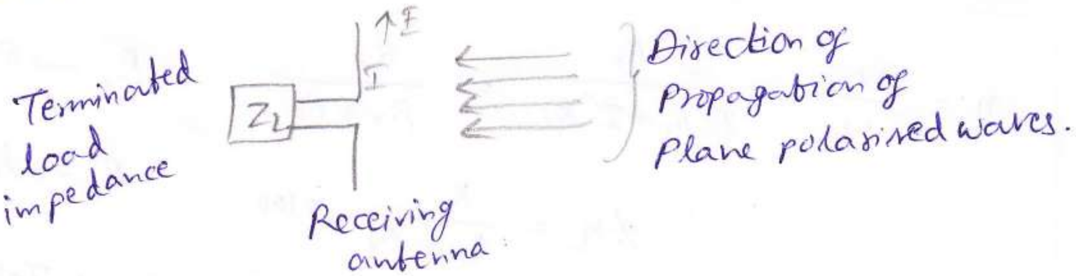
$W \Rightarrow$ Power received in watts

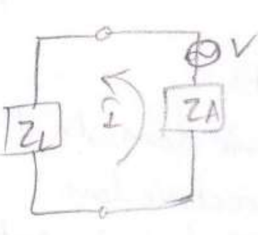
$P \Rightarrow$ Poynting vector of incident wave watt/m^2

$$W = PA$$

$A \Rightarrow$ Aperture in m^2

→ Let a receiving antenna be placed in the field of plane polarised travelling waves as shown in figure having an effective area A and the receiving antenna is terminated at the load impedance $Z_L = R_L + jX_L$.





⇒ Thevinin's Equivalent circuit.

If 'I' be the terminal current, then received power is,

$$W = I_{rms}^2 \cdot R_L .$$

where R_L = Load resistance in Ω.

I_{rms} = Terminal rms current.

$$\therefore A = \frac{W}{P} = \frac{I_{rms}^2 \cdot R_L}{P} .$$

Since the antenna extracts energy from the incident EM waves, It delivers the same to the terminated load impedance 'Z_L' and the poynting vector 'P'.

A/c to Thevinin's theorem the entire system can be replaced by an equivalent circuit.

V = Equivalent Thevinin's voltage.

Z_A = Equivalent Thevinin's Impedance.

The voltage 'V' is induced by passing EM waves which produces current I_{rms} through terminal load impedance 'Z_L'.

$$I_{rms} = \frac{\text{Equivalent voltage}}{\text{Equivalent Impedance}}$$

$$I_{rms} = \frac{V}{Z_A + Z_L} \text{ Amp.}$$

$Z_A = R_A + jX_A$. $\longrightarrow$ Complex antenna impedance

$Z_L = R_L + jX_L$ $\longrightarrow$ Complex Load impedance.

$$R_A = R_r + R_l .$$

$$I_{rms} = \frac{V}{(R_L + jX_L) + (R_A + jX_A)}$$

$R_L \rightarrow$ load resistance

$$= \frac{V}{(R_L + R_A) + j(X_L + X_A)}$$

$R_A =$ loss (ohmic)

$$|I_{rms}| = \frac{|V|}{\sqrt{(R_L + R_A)^2 + (X_L + X_A)^2}}$$

$\downarrow$

$R_r + R_l$.

$X_L \rightarrow$ Load reactance

$X_A \rightarrow$ Antenna reactance.

$$W = I_{rms}^2 \cdot R_L$$

$$W = \frac{V^2 \cdot R_L}{\sqrt{(R_L + R_A)^2 + (X_L + X_A)^2}}$$

This is the power delivered by the antenna at the terminal load impedance 'Z_L'.

Now, by definition,

$$A_e = \frac{W}{P} = \frac{V^2 \cdot R_L}{[(R_L + R_A)^2 + (X_L + X_A)^2] \cdot P} \quad m^2 \text{ (or) } \lambda^2.$$

$\because$ 'W' is in watts, 'P' is in watt/m^2 & hence the unit of A_e is m^2. But if the unit of 'P' is taken interms of wave-length λ as watt/λ^2. then A_e is 'λ'.

$$A_e = \frac{V^2 \cdot R_L}{[(R_L + R_A)^2 + (X_L + X_A)^2] \cdot P} \lambda^2.$$

$P \rightarrow$ Poynting vector $\quad V \rightarrow$ induced voltage.

$R_L \rightarrow$ Load resistance. $\quad Z_A \rightarrow$ Antenna impedance.

load impedance.

The induced ~~impedance~~ voltage is maximum when antenna is oriented for maximum response and the antenna as well as the incident wave both have the same polarization.

A/c to maximum power transfer theorem, maximum power will be transferred from antenna to the antenna terminating load if,

$$X_L = -X_A.$$

$$R_L = R_A = R_r + R_l$$

$$\text{If } R_l = 0 \text{ then } R_L = R_r.$$

$$\text{then, } W_{max} = \frac{V^2 R_L}{4R_L^2} = \frac{V^2}{4R_L}$$

$$W_{max} = \frac{V^2}{4R_L} = \frac{V^2}{4R_r} \quad (R_L = R_r).$$

This is the maximum power received in antenna terminating load impedance Z_L' under the condition of maximum power transfer and without antenna loss the corresponding effective Aperture is known as maximum effective Aperture.

$$\text{Maximum effective aperture} = \frac{\text{Maximum Power received}}{\text{Power density of incident wave}}$$

$$(A_e)_{max} = \frac{W_{max}}{P} = \frac{V^2}{4R_r \cdot P}$$

$$\therefore (A_e)_{max} = \frac{V^2}{4R_L \cdot P} = \frac{V^2}{4PR_r} \quad \lambda^2 \text{ (or) } m^2.$$

Effectiveness Ratio:-

The ratio of effective area to the maximum effective area is known as "Effectiveness Ratio".

$$\alpha = \frac{A_e}{(A_e)_{max}}$$

α lies between 0 & 1.

$\alpha = 1$ indicates the perfectly matched antenna having 100% efficiency.

Scattering loss Aperture:-

Besides Effective Aperture, there are other apertures also like scattering Aperture (A_s), loss Aperture (A_l) corresponding to considerable losses in radiation or re-radiation resistance (R_r) and antenna loss resistance (R_l) respectively and accordingly they are called as scattering and loss Apertures.

$$\text{Scattering Aperture } (A_s) = \frac{I_{rms}^2 \cdot R_r}{P}$$

$$A_s = \frac{V^2 R_r}{[(R_L+R_A)^2+(X_L+X_A)^2] \cdot P}$$

$$\text{Loss Aperture } A_l = \frac{I_{rms}^2 \cdot R_l}{P}$$

$$A_l = \frac{V^2 R_L}{[(R_L+R_A)^2+(X_L+X_A)^2] \cdot P}$$

If the Conditions of maximum transfer of energy is introduced then,

$$(A_s)_{max} = \frac{V^2}{4 R_r \cdot P} = (A_e)_{max}$$

$X_L = -X_A$
$R_L = R_r + R_l$
if $R_l = 0$, $R_L = R_r$.

$$(A_l)_{max} = \frac{V^2}{4 R_l \cdot P}$$

$$(A_s)_{max} = (A_e)_{max}.$$

Scattering Ratio:-

The ratio of scattering Aperture to effective Aperture is known as "scattering Ratio" and it is denoted by 'β'.

$$\text{Scattering ratio } (\beta) = A_s/A_e$$

β lies between 0 to ∞.

Collecting Aperture:-

Out of the power collected by antenna, there are losses as heat in load resistance (R_L), Radiation resistance (R_r) and antenna loss resistance (R_l) and Correspondingly there three apertures are effective, scattering and loss apertures.

By the law of Conservation of energy these three apertures are collectively known as "Collecting aperture" and is given by,

$$\text{Collecting aperture } (A_c) = A_e + A_s + A_l.$$

$$A_c = \frac{I_{rms}^2 R_L}{P} + \frac{I_{rms}^2 R_r}{P} + \frac{I_{rms}^2 \cdot R_l}{P}.$$

$$= \frac{V^2 R_L}{P[(R_L+R_A)^2+(X_L+X_A)^2]} + \frac{V^2 R_r}{P[(R_L+R_A)^2+(X_L+X_A)^2]}$$

$$+ \frac{V^2 R_l}{P[(R_L+R_A)^2+(X_L+X_A)^2]}$$

$$A_c = \frac{V^2(R_L+R_r+R_l)}{P[(R_L+R_r+R_l)^2+(X_L+X_A)^2]}$$

Physical Aperture:-

The physical aperture is related to the actual physical size of the antenna and is denoted by the symbol 'A_p'. It is more meaningful for the antennas of larger physical size interms of wavelength like horn, parabolic Reflector etc, i.e., for some particular antennas only while effective aperture is a unique quantity for any antenna.

→ Therefore physical aperture is defined as the physical cross section ⊥er to the direction of propagation of incident electromagnetic wave with antenna set for maximum response.

→ In larger cross section antennas like horn, parabolic reflectors, physical aperture is greater than effective aperture i.e., $A_p > A_e$ where as incase of short diople, physical aperture is less than effective aperture i.e., $A_p < A_e$.

→ In an ideal ~~case~~ size when there is no thermal losses and field is inphase, the physical aperture 'A_p' and effective aperture 'A_e' are equal.

$A_p = A_e$ (when no losses)

But directivity and effective aperture are related as,

$$D = \frac{4\pi}{\lambda^2} A_e$$

$$\therefore D_{max} = \frac{4\pi}{\lambda^2} A_p$$

Absorption ratio:-

The ratio of maximum effective aperture to the physical aperture is known as "Absorption ratio" and is denoted as 'γ'.

$$\text{Absorption ratio} (\gamma) = \frac{(A_e)_{max}}{A_p}$$

it has no units.

γ lies between 0 and ∞ and it has no units.

Relation between Maximum effective aperture and gain(or)directivity

Let us Consider two antennas 'A' and 'B' then,

Let us Consider two antennas 'A' and 'B'.

(i) D_a and D_b are the directivities of antenna A and B.

ii) A_{ea} and A_{eb} are the effective apertures of antenna A & B.

iii) $(A_{ea})_{max}$ and $(A_{eb})_{max}$ are the maximum effective apertures of Antenna A & B.

iv) α_a and α_b are the effectiveness ratio of Antenna A & B.

v) G_{oa} & G_{ob} are the gains of antenna A and B w.r.t isotropic antenna.

Directivity of the receiving antenna is directly proportional to their maximum effective aperture.

i.e., $D \propto (Ae)_{max}$

So, $D_a \propto (Aea)_{max}$

$D_b \propto (Aeb)_{max}$

$$\frac{D_a}{D_b} = \frac{(Aea)_{max}}{(Aeb)_{max}}$$

Gain and directivity with respect to isotropic antenna is related as,

$G_0 = KD$.

Where K is efficiency factor.

If losses of efficiency factor 'K' and any mismatch are included then 'K' can be replaced by effectiveness ratio 'α'.

$G_0 = \alpha D$.

$G_{oa} = \alpha_a D_a$; $G_{ob} = \alpha_b D_b$.

$$\frac{G_{oa}}{G_{ob}} = \frac{\alpha_a D_a}{\alpha_b D_b} = \frac{\alpha_a \cdot (Aea)_{max}}{\alpha_b \cdot (Aeb)_{max}}$$

But by definition, $\alpha = \frac{Ae}{(Ae)_{max}}$

$$\alpha_a = \frac{Aea}{(Aea)_{max}}, \quad \alpha_b = \frac{Aeb}{(Aeb)_{max}}$$

$$\therefore \frac{G_{oa}}{G_{ob}} = \frac{\alpha_a \cdot (Aea)_{max}}{\alpha_b \cdot (Aeb)_{max}}$$

$$= \frac{\frac{A_{ea}}{(A_{ea})_{max}} \cdot (A_{ea})_{max}}{\frac{A_{eb}}{(A_{eb})_{max}} \, (A_{eb})_{max}}$$

$$= \frac{A_{ea}}{A_{eb}}.$$

$$\therefore \frac{G_{oa}}{G_{ob}} = \frac{A_{ea}}{A_{eb}}.$$

Assume that Antenna 'A' is an isotropic antenna then $D_a = 1$.

$$\frac{D_a}{D_b} = \frac{1}{D_b} = \frac{(A_{ea})_{max}}{(A_{eb})_{max}}.$$

$$\therefore (A_{ea})_{max} = \frac{(A_{eb})_{max}}{D_b}.$$

The above equation states that the ratio of the maximum effective aperture of a Test antenna to the directivity of that antenna will gives the maximum effective aperture of an isotropic antenna.

$$\text{Also, } D_b = \frac{(A_{eb})_{max}}{(A_{ea})_{max}}.$$

The above equation states that the ratio of the maximum effective aperture of a test antenna to the maximum effective aperture of an isotropic antenna will gives directivity of the Test antenna.

Suppose that in case of short dipole antenna whose maximum effective aperture is $\left(\frac{3}{8\pi}\right)\lambda^2$ and directivity is $3/2$ then,

$$(Aea)_{max} = \frac{(Aeb)_{max}}{D_b} = \frac{\frac{3\lambda^2}{8\pi}}{3/2}$$

$$= \frac{\lambda^2}{4\pi}.$$

$$D_b = \frac{(Aeb)_{max}}{(Aea)_{max}} = \frac{(Aeb)_{max}}{(\lambda^2/4\pi)}$$

$$= \frac{4\pi}{\lambda^2}(Aeb)_{max}$$

In general, $D = \frac{4\pi}{\lambda^2}.(Ae)_{max}.$

The above equation is the relation between directivity 'D' and maximum effective aperture of the antenna.

Effective length:-

The term effective length of an antenna represents the effectiveness of an antenna as radiator or collector of electromagnetic wave energy. In other words, effective length indicates how far an antenna is effective in transmitting or receiving the electromagnetic wave energy.

For an receiving antenna, the effective length may be defined interms of Induced voltage 'V' and incident field. Effective length is nothing but the ratio of induced voltage at the terminal of the receiving antenna under open circuited condition to the incident electric field intensity 'E'.

$$\therefore \text{Effective length} = \frac{\text{open Circuited Voltage} \quad (V)}{\text{Incident electric field Intensity } (V/m) \text{ (or) } (V/\lambda)}$$

$$l_e = \frac{V}{E} \; m \text{ (or) } \lambda.$$

As induced voltage 'V' also depends on the effective aperture and hence effective length and effective aperture of an antenna are related to each other as

$$A_e = \frac{V^2 \cdot R_L}{[(R_A+R_L)^2+(X_A+X_L)^2] \cdot P}$$

$$V^2 = \frac{A_e \cdot [(R_A+R_L)^2+(X_A+X_L)^2] \cdot P}{R_L}$$

$$V = \frac{\sqrt{A_e \, [(R_A+R_L)^2+(X_A+X_L)^2] \cdot E^2}}{} \qquad \left\{ P = \frac{E^2}{Z} \right\}$$

But, $l_e = \frac{V}{E}$

$$= \frac{\sqrt{A_e\left[(R_A+R_L)^2+(X_A+X_L)^2\right]}\cdot \cancel{E}}{\sqrt{2R_L}\cdot \cancel{E}}$$

$$= \frac{\sqrt{A_e\left[(R_A+R_L)^2+(X_A+X_L)^2\right]}}{\sqrt{2R_L}}$$

$$l_e = \sqrt{\frac{A_e\left[(R_A+R_L)^2+(X_A+X_L)^2\right]}{2R_L}}$$

→ under the Conditions of maximum transfer of energy,

$$X_A = -X_L, \quad R_A = R_L = R_r + R_l$$

If $R_l = 0$ then $R_L = R_A = R_r$

$$l_e = \sqrt{\frac{A_{e\,max}(2R_L)^2}{2R_L}} = \sqrt{\frac{A_{e\,max}\,4R_L^{\cancel{2}}}{2\cancel{R_L}}}$$

$$= 2\sqrt{\frac{A_{e\,max}\,R_L}{2}}$$

$$= 2\sqrt{\frac{A_{e\,max}\cdot R_r}{2}}$$

$$l_e = 2\sqrt{\frac{A_{e\,max}\,R_r}{2}}$$

$$l_e^2 = 4\cdot\frac{A_{em}\,R_r}{2}$$

$$A_{e\,max} = \frac{2\,l_e^2}{4R_r}$$

$$A_{e\,max} = A_{em} = \frac{l_e^2\cdot 2}{4R_r}$$

The above eq. is the relation b/w maximum effective aperture and effective length

For an transmitting antenna, the effective length is that length of an equivalent linear antenna that has the same Current $I(c)$ at all the points along its length and that radiates the same field intensity 'E' as the actual antenna.

$I(c)$ = Current at the terminals of actual antenna.

$I(z)$ = Current at the point z of the antenna.

l_e = effective length.

l = Actual length.

$$I(c) \cdot l_{ef} = \int_{-l/2}^{l/2} I(z) \cdot dz$$

$$l_{ef} = \frac{1}{I(c)} \int_{-l/2}^{l/2} I(z) \cdot dz.$$

$$\therefore l_{ef} = \frac{2}{I(c)} \int_{0}^{l/2} I(z) \cdot dz.$$

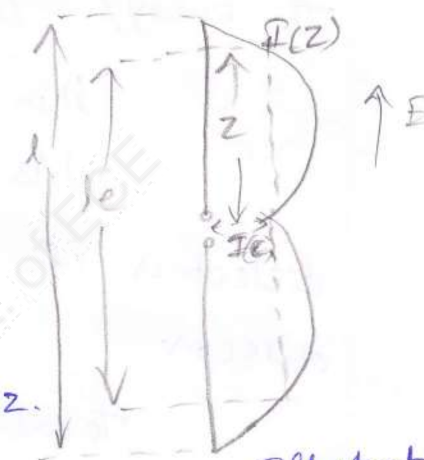


Illustration of effective length for Tx' antenna.

Radiation resistance:-

Antenna is a radiating device in which the power (i.e., energy per unit time) is radiated into space in the form of electromagnetic waves. Hence there must be power dissipation which may be expressed as,

$$W' = I^2 R.$$

If it is assumed that all power appears as electromagnetic waves then the power (W') can be divided by square of the Current.

$$R_r = \frac{W'}{I^2}.$$

... it is fed to the antenna and obtains a fictitious resistance called as Radiation resistance. It is normally denoted by R_r (or) R_a (or) R_0.

The radiation resistance represents a relation b/n total energy radiated from a transmitting antenna and the current flowing in the antenna.

The energy supplied to the antenna is dissipated

i) In the form of electromagnetic waves.

ii) As ohmic losses in the antenna wire and nearby dielectrics i.e., insulators, ground and other surrounding objects.

Total power loss = Ohmic loss + Radiation loss

$$W = W' + W''$$

$$= I^2 R_r + I^2 R_l$$

$$= I^2 (R_r + R_l)$$

$$W = I^2 R \quad \text{if } R = R_r + R_l.$$

The value of radiation resistance depends on

→ Configuration of antenna.

→ The point where radiation resistance is considered.

→ Location of antenna w.r.t grounds and other objects.

→ Ratio of length of diameter of the conductor used.

→ Corona discharge - A luminous discharge round the surface of antenna due to ionization of air etc.

Front to Back Ratio (FBR):-

It is defined as the ratio of power radiated in desired direction to the power radiated in opposite direction.

$$FBR = \frac{\text{Power radiated in the desired direction}}{\text{Power radiated in opposite direction.}}$$

FBR Changes if frequency of operation of antenna system shifts or changes.

1) The FBR decreases if spacing between elements of antenna increases.

2) The FBR depends on the tuning conditions or electrical length of the parasitic elements. The higher FBR is achieved by diverting the gain of the opposite direction (i.e, backward response) to the forward or desired direction by adjusting or tuning the length of parasitic elements. Hence, higher value of FBR is achieved at the cost of sacrifying gain from the opposite direction.

3) In practice, for receiving purposes adjustments are made to get maximum front to back ratio rather than maximum gain.

Antenna Bandwidth:-

There is no unique definition of bandwidth of an antenna or antenna system. It is because for the operation of antenna many factors like gain, side lobe level, SWR (or Front to back ratio, pattern, impedance and polarisation characteristics etc, ... may change when the

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1) Therefore, the functional bandwidth of an antenna is limited by one or more of these factors and accordingly antenna bandwidth may be specified in many different ways,

i) Bandwidth over which the gain is higher than some acceptable value.

ii) Bandwidth over which atleast a given FBR is achieved,

iii) Band width over which the SWR on the Transmission line Can be maintained below a Chosen value.

2) In other words, it can be said that "Antenna bandwidth" is a width (i.e., range) of frequency over which the antenna maintains Certain required Characteristics like gain, FBR, Polarization and impedance.

3) In practice, however these requirements Change with the operation of the antenna, therefore specifications are set to meet the needs of a particular application.

4) For antennas, where increase in SLL decrease in gain and Change in impedance value, pattern and polarization chara-cteristics are important, then one of these factors determines the low frequency limit and the other factor (eg: Change of pattern - shape or direction) high frequency limit.

5) Hence the bandwidth of a particular antenna, in general Can then be defined as "the bandwidth with in which the antenna maintains a given set of specifications".

6) A Considerable mathematical analysis will show that two frequency limits (i.e., ω_2 & ω_1) or bandwidth is given by,

$$\Delta\omega = \omega_2 - \omega_1 = \frac{\omega_r}{Q}$$

$$\Delta\omega = \frac{\omega_r}{Q} \qquad \therefore \omega_r = 2\pi f_r$$
$$\Delta\omega = 2\pi\Delta f.$$

$$2\pi\Delta f = \frac{2\pi f_r}{Q}$$

$$\Delta f = \frac{f_r}{Q}$$

$$\Delta f \propto \frac{1}{Q}.$$

Where f_r = Centre (or) resonant (or) design frequency.

$$Q = 2\pi \frac{\text{(Total energy stored by antenna)}}{\text{(Energy dissipated or radiated per cycle)}}.$$

Thus the lower 'Q' of the antenna the higher bandwidth and vice-versa.

7) For narrow band antennas, bandwidth is generally expressed interms of percentage of centre frequency as,

$$B.W\% = \frac{\text{operating range}}{\text{Centre frequency}} \times 100.$$

8) For broad band antennas, bandwidth is represented by a ratio of highest to lowest operating frequency.

$$B.W = \frac{\text{Highest operating frequency}}{\text{Lowest operating frequency}}.$$

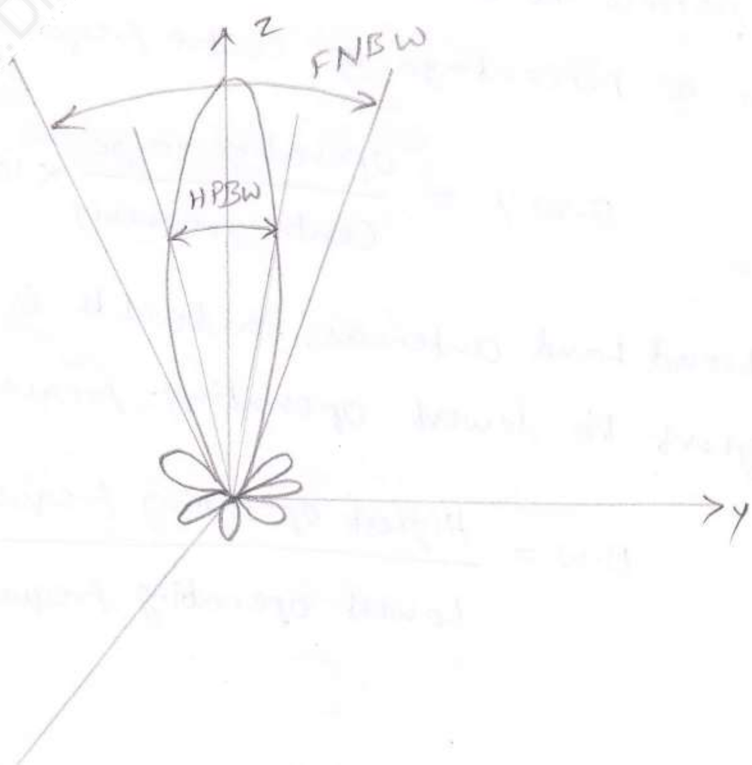
Antenna Beamwidth :-

1) Antenna Beamwidth is a measure of directivity of an antenna.

2) Antenna Beamwidth is an angular width in degrees, measured on the radiation pattern (major lobe) between points where the radiated power has fallen to half its maximum value. This is called as beamwidth between half power point or half power beamwidth because the power at half power points is just half.

3) Half power beamwidth is also known as "3-dB beamwidth" because at half power points, the power is 3-dB down of the maximum power value of the major lobe.

4) Further at these points, the field intensity (i.e., voltage) equals $1/\sqrt{2}$ (or) 0.707 times its maximum value (or) 3-dB down from the maximum value.



5) Therefore, Antenna beamwidth can also be defined as the angular width of the major lobe between the two directions at which the radiated or received power is one half the maximum power."

6) Some times radiation pattern is also described in terms of angular width between first Nulls or first side lobes, known as beamwidth between first Nulls and is abbreviated as BWFN (or) beamwidth - down from the antenna pattern maximum.

$$FNBW = 2\,(HPBW)$$

(or)

$$HPBW = \tfrac{1}{2}(FNBW).$$

7) The directivity (D) is related with beam solid angle (Ω_A) or beam area (B) as,

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{B}.$$

Since the radiation pattern or lobe is a 3-dimensional and hence the major lobe area approximately may be given by the product of beamwidths in horizontal and vertical planes (or) E-plane & H-planes i.e.,

$$B = (HPBW) \text{ in Horizontal plane} \times HPBW \text{ in vertical Plane}$$

$$\simeq HPBW \text{ in E-plane} \times HPBW \text{ in H-plane square radians.}$$

$$\therefore B \simeq \theta_E \times \theta_H \text{ . Square radians if } \theta_E \times \theta_H \text{ in radians.}$$

$$\therefore D = \frac{4\pi}{\theta_E\,\theta_H}, \text{ where } \theta_E \,\&\, \theta_H \text{ in radians.}$$

$$D = \frac{4\pi \times (57.3)^2}{\theta_E^\circ \times \theta_H^\circ} \text{ Square degrees.} \quad \because 1 \text{ radian} = 57.3 \text{ degrees.}$$

$$\therefore D = \frac{41257}{\theta_E^\circ \times \theta_H^\circ}.$$

The factors affecting the beamwidth of an antenna are:

i) The shape of radiation pattern.

ii) The wavelength.

iii) Dimensions.

Antenna Beam Efficiency :-

Antenna Beam efficiency is the parameter that is frequently used to judge the quality of transmitting and receiving antennas.

The beam efficiency is,

$$B.E = \frac{\text{Power transmitted or received with in Cone angle } \theta_1}{\text{Power transmitted or received by the antenna.}}$$

Where θ_1 = half angle of the Cone with in which the percentage of the total power is to be found.

$$BE = \frac{\int_0^{2\pi}\int_0^{\theta_1} \phi(\theta,\phi) \sin\theta \, d\theta \, d\phi}{\int_0^{2\pi}\int_0^{\pi} \phi(\theta,\phi) \sin\theta \, d\theta \, d\phi.}$$

Antennas having very high beam efficiency are used in radiometry, astronomy, radar and other applications where the received signal through the minor lobes must be minimised.

In terms of beam area (Ω_A), the beam efficiency is defined as the ratio of the main beam area (Ω_M) to the total beam area (Ω_A).

$$BE \text{ (or) } \varepsilon_M = \frac{\Omega_M}{\Omega_A} = \frac{\text{Main beam area}}{\text{Total beam area}}$$

Where the total beam area Ω_A consists of main beam area Ω_M and the minor lobe area Ω_m.

$$\text{i.e., } \Omega_A = \Omega_M + \Omega_m$$

Total beam area = Main beam area + Minor lobe area.

divide the above eq. by Ω_A,

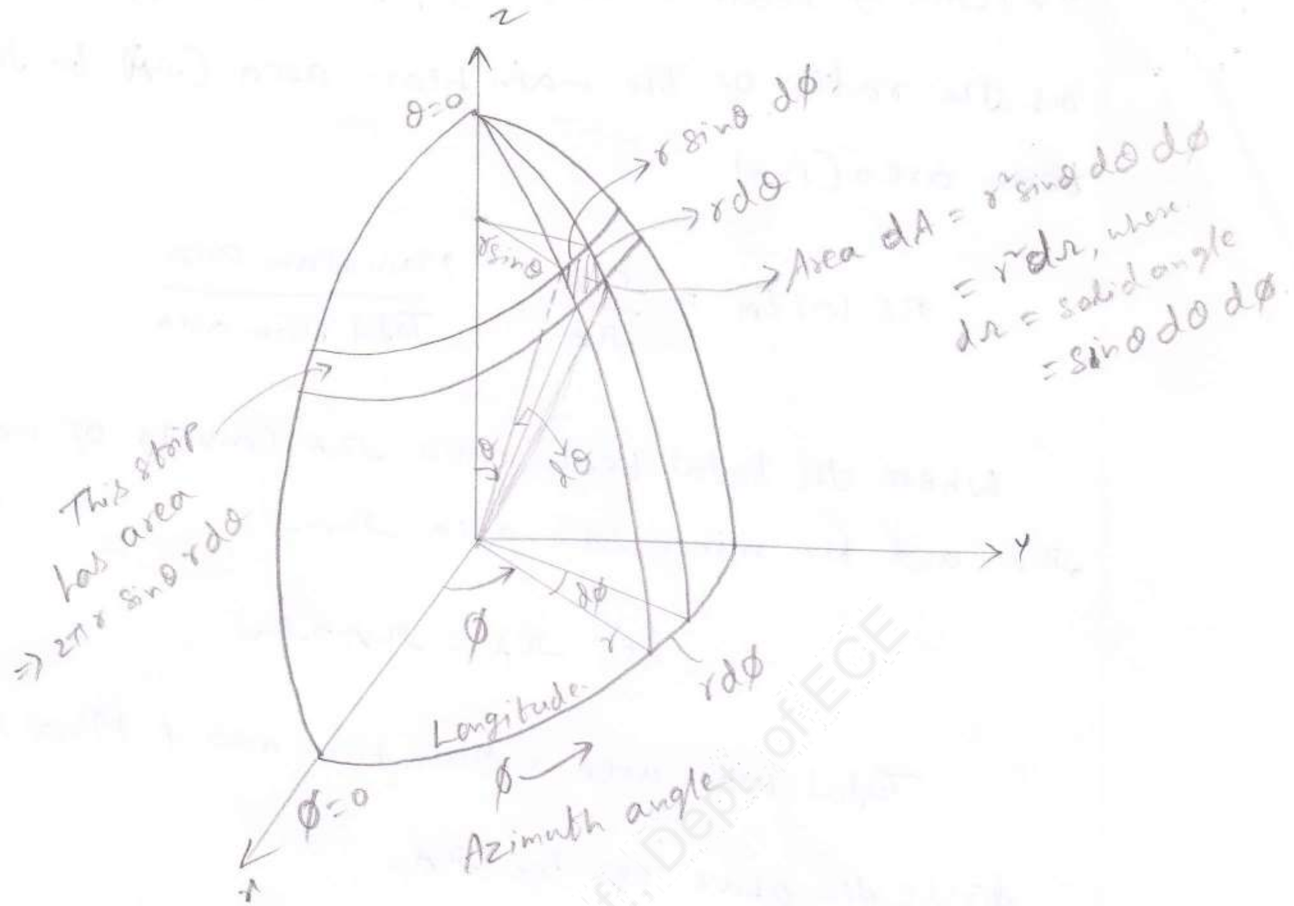
$$1 = \frac{\Omega_M}{\Omega_A} + \frac{\Omega_m}{\Omega_A} \qquad \varepsilon_M = \text{Beam efficiency.}$$

$$1 = \varepsilon_M + \varepsilon_m$$

$$\text{Where } \varepsilon_m = \frac{\Omega_m}{\Omega_A} = \text{Stray factor} = \frac{\text{Minor lobe area}}{\text{Total beam area}}.$$

Antenna beam area (or) Beam Solid angle (Ω):-

An area ds on surface of a sphere as seen from the Centre of the sphere subtends a solid angle Ω. The total solid angle subtended by the sphere is 4π steradians. (or square radians) abbreviated as Sr.



The increamental area ds of the surface of sphere is given by,

$$ds = (r \sin\theta \, d\phi)(r \, d\theta)$$

$$= r^2 \sin\theta \, d\theta \, d\phi.$$

$$\therefore ds = r^2 \cdot d\Omega \text{ m}^2.$$

$d\Omega$ = solid angle subtended by area ds.

$$d\Omega = \frac{ds}{r^2} \cdot Sr.$$

$$d\Omega = \frac{4\pi r^2}{r^2} Sr.$$

$$d\Omega = 4\pi \, Sr.$$

$$1 \text{ Steradian} = \frac{d\Omega}{4\pi}$$

$$= \frac{\text{Solid angle of sphere}}{4\pi}$$

$$= 1 \text{ (radian)}^2$$

$$= \left(\frac{180}{\pi}\right)^2 \text{(degrees)}^2$$

$$= \frac{180 \times 180}{3.146 \times 3.146} \text{ (deg)}^2$$

$$= \frac{32400}{3.146 \times 3.146} \text{ (deg)}^2$$

$$1 \text{ Sr} = 3282.7909 \text{ (deg)}^2$$

$$4\pi \text{ Sr} = 4\pi \times 3282.7909 \text{ (deg)}^2$$

$$= 41252.861 \text{ (deg)}^2$$

$$\therefore 4\pi \text{ Sr} \simeq 41,253 \text{ (deg)}^2 = \text{solid angle in a sphere.}$$

⇒ Beam Solid angle is defined as the "Solid angle through which all the power of the antenna will flow if its radiation intensity is Constant and is equal to maximum value of 'U' for all angle with in Ω_A."

The beam solid angle Ω_A for an antenna is therefore is given by the integral of the Normalised power pattern over a sphere (4π Sr).

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} P_n(\theta, \phi)\, d\Omega \text{ Sr.}$$

$$\Omega_A = \int_0^{2\pi}\int_0^{\pi} P_n(\theta, \phi) \sin\theta \, d\theta \, d\phi \text{ Sr.}$$

Where $P_n(\theta, \phi)$ = Normalised power pattern

$$= \frac{P(\theta,\phi)}{P(\theta,\phi)_{max}}$$

$$P(\theta,\phi) = \text{poynting vector}$$

$$= \frac{E_\theta^2(\theta,\phi) + E_\phi^2(\theta,\phi)}{Z_0}$$

$P(\theta,\phi)_{max}$ = Maximum value of $P(\theta,\phi)$

$$Z_0 = 120\pi \approx 377\Omega.$$

Solid angle is also described approximately interms of the angle subtended by the half-power points of the main lobe in principal planes.

$$\cancel{\Omega_A = \theta_{HP}}$$

$$\Omega_A = \theta_{HP}\,\phi_{HP}\ Sr.$$

Where θ_{HP} = HPBW in E-plane (or) θ-plane.

ϕ_{HP} = HPBW in H-plane (or) ϕ-plane.

The relation between Directivity and beam area is,

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{B.A}. \quad \text{where } \Omega_A = \theta_{HP}\,\phi_{HP}\ Sr$$

Antenna resolution:-

The ability of an antenna to distinguish between two closely placed transmitters is known as "Antenna resolution".

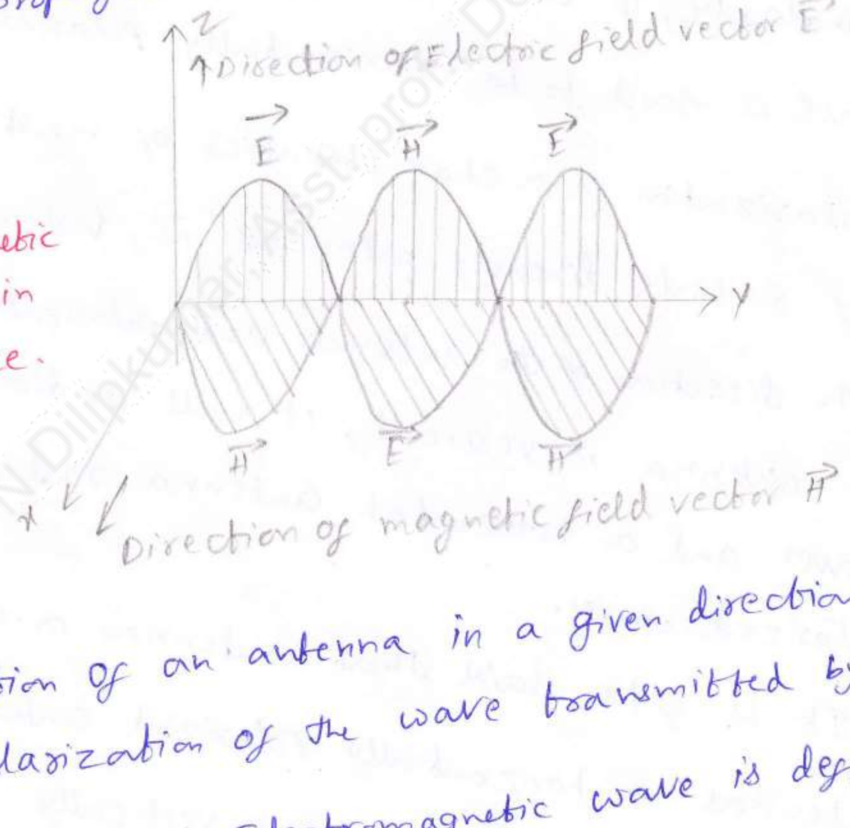
$$\text{Antenna resolution} = \frac{1}{2}(\text{FNBW})$$

$$= \frac{1}{2}(2 \times \text{HPBW})$$

Polarization:-

Polarization is often described interms of electric vector $\bar{E}$. Polarization (or plane of polarization) of a radio wave can be defined by "the direction in which the electric vector $\bar{E}$ is aligned during the passage of atleast one full cycle".

1) Since Electric vector $\bar{E}$ and magnetic vector $\bar{H}$ are mutually perpendicular and this electromagnetic wave propagate in the perpendicular direction as shown in which mutually perpendicular directions of electric vector, magnetic vector and its propagation are shown.



Electromagnetic waves in free space.

2) Polarization of an antenna in a given direction is defined as the polarization of the wave transmitted by the antenna.

3) Polarization of an Electromagnetic wave is defined as "the curve traced by the end point of the arrow representing the instantaneous electric field".

4) Polarization refers to the physical orientation of the radiated electromagnetic waves in space.

5) An electromagnetic wave is said to be linearly polarized if they all have the same alignment in space.

6) The EM wave is said to be linearly polarized (or) vertically polarized, if all the electric field vectors are vertical. That is if the electric field vector $\bar{E}$ is vertical or lies in the vertical plane, the wave is said to be vertically polarized.

7) Similarly, if electric field vector $\bar{E}$ is in horizontal plane, the wave is said to be "Horizontally polarised".

8) Polarization is a characteristics of most of the antenna that they radiate linearly polarized (i.e., vertically or horizontally) waves

9) The direction of the antenna and polarization is alike i.e., if an antenna is vertical, it will radiate vertically polarized waves and a horizontal antenna radiates horizontally polarized waves.

10) It is often said that antennas are vertically polarized antennas or horizontally polarized antenna according to whether they are producing vertically polarized or horizontally polarized wave respectively.

11) The initial polarization of EM waves is determined by the orientation of antenna itself in the space. Hence in the design of an antenna, the type of polarisation is one of the factor. Different types of polarizations are useful in diff. types of application.

12) Besides linear polarization, antenna may also radiate Circularly or elliptically polarized waves.

13) If two linearly polarized waves produced simultaneously in the same direction from the same antenna provided that the two linear polarization are mutually perpendicular to each other with a phase difference of 90°, then Circularly polarized waves are produced.

14) Circular polarization may be right handed or left handed depending up on the sense of rotation i.e., phase difference is +ve or -ve.

15) Circular polarization results only when the amplitudes of two linearly polarized waves are equal. If the amplitudes are not equal, then combination of two linearly polarized waves will produce "Elliptically Polarized waves".

16) The undesired radiation from an antenna is called as cross polarization. The cross polarization for linearly polarized antennas, is perpendicular to the intended radiation.

17) polarization from an antenna may be linearly, elliptically or circularly but polarization in different portion of the total antenna pattern may be different i.e., polarization of major and minor lobes even be different.

Antenna Temperature (T_A):-

1) Every object with a physical temperature above absolute zero ($0^\circ K = -273^\circ c$) radiates energy.

2) The parameter antenna temperature is not the inherent property of the antenna because it is not all related to the physical temperature of the antenna.

3) The antenna temperature (T_A) is defined as the temperature of the surroundings coupled to the antenna through radiation resistance.

4) Antenna temperature depends on the temperature of the regions to which the antenna is radiating.

5) So, the receiving antenna may be regarded as a remote sensing (or) temperature measuring device.

6) Both the antenna temperature (T_A) and radiation resistance (R_r) are single valued scalar quantities.

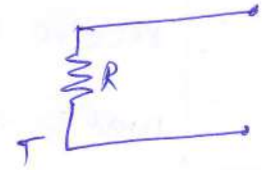
7) The noise per unit bandwidth available at the terminals of a resistor of resistance R and temperature T is given by,

$$P = KT \text{ watt/Hertz} \quad \text{— (1)}$$

Where, P = Power per unit bandwidth in watt/Hertz

K = Boltzman's Constant $= 1.38 \times 10^{-23}$ J/K.

T = Absolute Temperature of Resistor in °K.



Resistor R at temperature T.

8) If the power per unit bandwidth 'p' is independent of frequency, the total power (P) is obtained by multiplying by the bandwidth (B) i.e.,

$$P = KTB \text{ watts.} \quad -(2)$$

Where P = Total Power in watt.

B = Bandwidth in Hertz.

9) Let an antenna has effective Aperture A_e and that its beam is directed at a source of radiation which produce a power density per unit bandwidth or flux density (S) at the antenna. Then the power received from the source is given by,

$$P = S A_e B \text{ watts} \quad -(3)$$

$S \rightarrow$ Power density per unit bandwidth in W/m^2 Hz

$A_e \rightarrow$ Effective Aperture in m^2.

$B \rightarrow$ Bandwidth in Hz.

10) By equating equations (2) & (3),

$$S A_e B = KTB$$

$$S = \frac{K T_A}{A_e} \quad W/m^2 Hz$$

$$T_A = \frac{SA_e}{k} \ {}^\circ K$$

11) Any antenna irrespective of polarization characteristics can receive only half the incident power of an unpolarized wave, so the actual flux density should be twice.

$$S = \frac{2kT_A}{A_e}$$

$T \rightarrow$ Antenna Noise Temperature - °K.

$k = 1.38 \times 10^{-23}$ J/K.

A_e = Effective Aperture of antenna - m².

12) If the angular size of the source Ω_S is small when compared with antenna Beam Ω_A and if their magnitudes are known, then it is possible to determine the source temperature T_S,

$$T_A = \frac{\Omega_S}{\Omega_A} T_S \quad \text{(or)} \quad T_S = \frac{\Omega_A}{\Omega_S} T_A.$$

$\Omega_A \rightarrow$ Antenna Beam solid angle in Sr.

$\Omega_S \rightarrow$ Source solid angle in Sr.

$T_A \rightarrow$ Antenna Noise temperature in °K.

$T_S \rightarrow$ Source Temperature in °K.

13) Incase if the receiver itself has a certain noise temperature T_r due to the thermal noise in the receiver components the system noise power at the receiver terminals is given by,

$$P_S = k(T_A + T_r)B_N.$$

$$T_{sys} = T_A + T_r.$$

P_S = System Noise power at receiver Terminals.

T_A = Antenna Noise Temperature at receiver Terminals.

T_r = Receiver Noise Temperature at receiver Terminals.

[illegible] System Noise Temperature at receiver terminals

Effective Noise Temperature of Antenna:-

The noise introduced by the network may be expressed as effective Noise Temperature, T_e and is defined as the "fictional Temperature at the input of the network which would account for the Noise ΔN at the output. ΔN is the additional noise introduced by the network itself.

Sometimes another parameter noise figure is used. The noise figure, F is related with effective noise Temperature T_e as,

$$F = 1 + \frac{T_e}{T_0} \quad (\text{or}) \quad (F-1) = \frac{T_e}{T_0}$$

$$\therefore T_e = (F-1) T_0$$

T_e = Effective Noise Temperature in °K.

T_0 = 290°K (273 + 17°)°K.

F = Noise figure (dimensionless).

$$F(dB) = 10 \log_{10}(F)$$

Note:-

The fundamental form of Radar equation is,

$$R_{max} = \left(\frac{W_t \cdot G \cdot \sigma \cdot A_{er}}{(4\pi)^2 P_{min}} \right)^{1/4}$$

$$R_{max} = \left(\frac{W_t \cdot G_t^2 \cdot \sigma \cdot \lambda^2}{(4\pi)^3 P_{min}} \right)^{1/4}$$

$$\frac{W_r}{W_t} = \frac{\sigma \cdot A_{er}^2}{4\pi \lambda^2 R^4}$$

Transmission Between Two Antennas:-

The utility of aperture Concept will be shown in deriving a simple formula popularly known as FRIIS Transmission formula or FRISS formula, after the name H.T. Friis. This formula helps in determining the transmission loss between two antennas in free space.

R = Distance b/n Transmitting & receiving antennas,

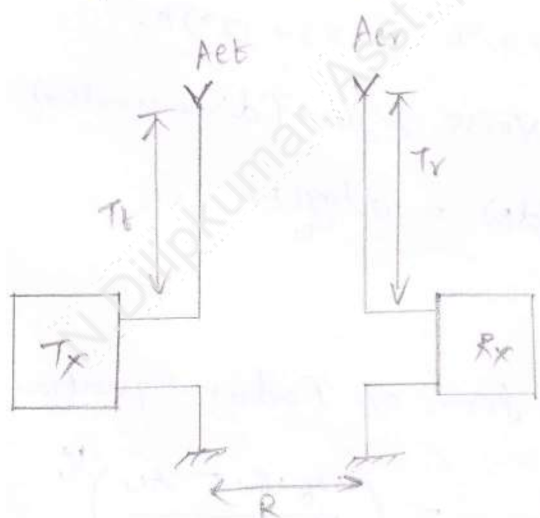
A_{et}, A_{er} = Effective apertures of Tx & Rx antennas,

W_t = Total power radiated by Isotropic antenna,

W_r = Power received at antenna terminal,

W_r' = " " at receiver.

λ = wavelength in metre.



Transmission reception circuit of Isotropic transmitting antenna and a receiving antenna.

Since power radiated by Transmitting antenna is W_t, therefore the power received per unit area at receiving antenna is,

$$P = \frac{W_t}{4\pi R^2} \quad -(1)$$

Power received at the receiver,

$$W_r' = A_{er} \cdot P$$

$$= A_{er} \cdot \frac{W_t}{\ldots} \quad -(2)$$

If it is assumed that the transmitting antenna is a practical antenna, instead of isotropic antenna then the directivity D_t of the practical antenna is,

$$D_t = \frac{4\pi}{\lambda^2} A_{emt} \quad -(3)$$

A_{emt} = Maximum effective aperture of the Transmitting antenna, then the power received at receiver is,

$$W_r = W_r' \cdot D_t$$
$$= A_{er} \cdot \frac{W_t}{4\pi R^2} \cdot \frac{4\pi}{\lambda^2} A_{emt}$$

$$\frac{W_r}{W_t} = \frac{A_{er} A_{emt}}{R^2 \lambda^2} \quad -(4)$$

This is known as Friis transmission formula which represents the ratio of power received to power transmitted for a direct path.

The ratio of power received to power transmitted is also given the name "power Transfer ratio" which represents the power input to a transmitting antenna which is ultimately made to receive at the receiving antenna terminals at a distance 'R'. This eq. is applied as long as the Fraunhofer Condition is satisfied.

i.e., $R \geq \frac{2d^2}{\lambda}$.

d → Largest dimension of each antenna.

λ → wave length.

In Friis transmission formula, if maximum effective Apesture of transmitting antenna A_{emt} is replaced by effective apesture A_{et} then,

$$\frac{W_r}{W_t} = \frac{A_{er} \cdot A_{et}}{R^2 \lambda^2} \quad \text{--- (5)}$$

Friis Transmission formula is expressed interms of effective Apestures for Transmitting antenna, the seperation b/n the two antennas and the wave length,

but, $D_t = G_t = \frac{4\pi}{\lambda^2} \cdot A_{et}$

$$D_r = G_r = \frac{4\pi}{\lambda^2} \cdot A_{er}$$

$$\therefore A_{et} = \frac{G_t \cdot \lambda^2}{4\pi} \quad \& \quad A_{er} = \frac{G_r \cdot \lambda^2}{4\pi}$$

from eq. (5):

$$\frac{W_r}{W_t} = \frac{A_{er} \cdot A_{et}}{R^2 \lambda^2}$$

$$= \left(\frac{G_t \cdot \lambda^2}{4\pi}\right) \left(\frac{G_r \cdot \lambda^2}{4\pi}\right) \left(\frac{1}{R^2 \lambda^2}\right)$$

$$\frac{W_r}{W_t} = \left(\frac{\lambda}{4\pi R}\right)^2 G_t \cdot G_r$$

$$\frac{W_r}{W_t} = \frac{G_t \cdot G_r}{\left(\frac{4\pi R}{\lambda}\right)^2}.$$

This is also known as Friis formula which relates the power W_r (delivered to the receiver load) to the input power of the Tx antenna W_t. The term $\left(\frac{\lambda}{4\pi R}\right)^2$ is called the "free space loss factor" and it takes into account the losses due to the spherical spreading of the energy by the antenna. R is assumed ... Compared to the dimensions of the antenna &

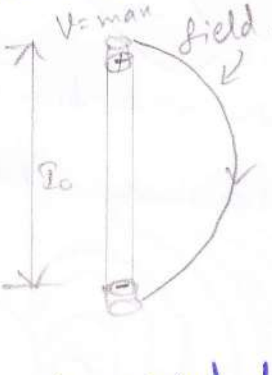
Fields from oscillating dipole :-

A charge moving with uniform velocity along a straight conductor does not radiate, a charge moving back and forth in simple harmonic motion along the conductor is subjeq to acceleration (and deceleration) and radiates.

When an electric field is applied to these charges, their seperation varies with the change in period 'T'. Let us consider a dipole antenna having two equal charges of opposite sign oscillating up and down in harmonic motion with instantaneous seperation l (maximum seperation l_0).

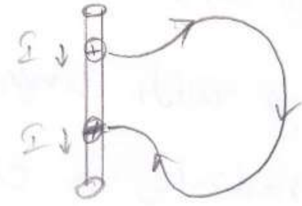
For simplicity, let us consider a single electric field line and its variation with change in seperation of charges at different time 't'.

At time $t=0$, the charges are at maximum seperation and underge maximum acceleration V. At this instant of time current I is zero. The corresponding propagation of an electric field line is shown in fig(i).



$t=0$:
Electric field line or wavefront with charges at ends of dipole.

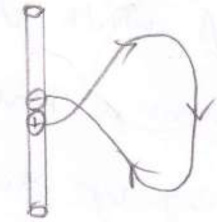
At an $\frac{1}{8}$ period later i.e., at $t=\frac{1}{8}T$, the charges move towards each other and the field line variations are shown in fig (ii)



t = 1/8 T.

Wavefront moves out as Charges go in.

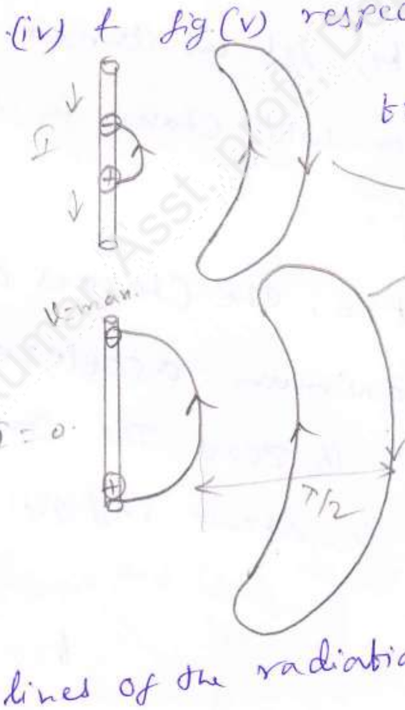
At a 1/4 Period i.e., $t = \frac{1}{4}T$, the Charges pass at the mid point. As this happens, the field lines seperate to form new field lines of opposite sign as shown in fig (iii). At this time the equivalent Current I is maximum & the Charge acceleration $\dot{v}$ is zero.



$t = \frac{1}{4}T$.

As Charges Pass the midPoint the field lines cut Close.

As the time progress to $t = \frac{3}{8}T$ & $t = \frac{1}{2}T$, the field lines continue to move out as shown in fig (iv) & fig (v) respectively.

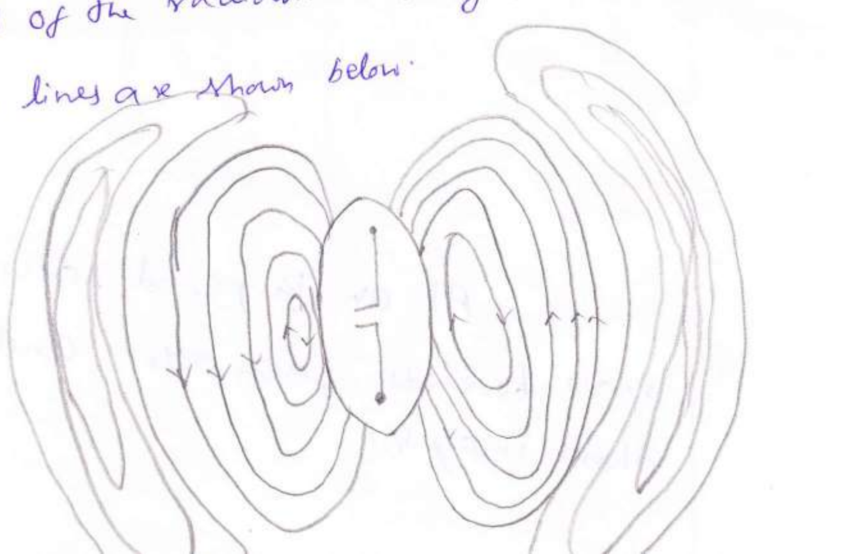


$t = \frac{3}{8}T$

wave fronts moving out

$t = \frac{1}{2}T$.

The electric field lines of the radiation moving out from λ/2 dipole antenna for more no. of feed lines are shown below.



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only coming for labs.
Present 8/7, & 14/7 & 25/7.

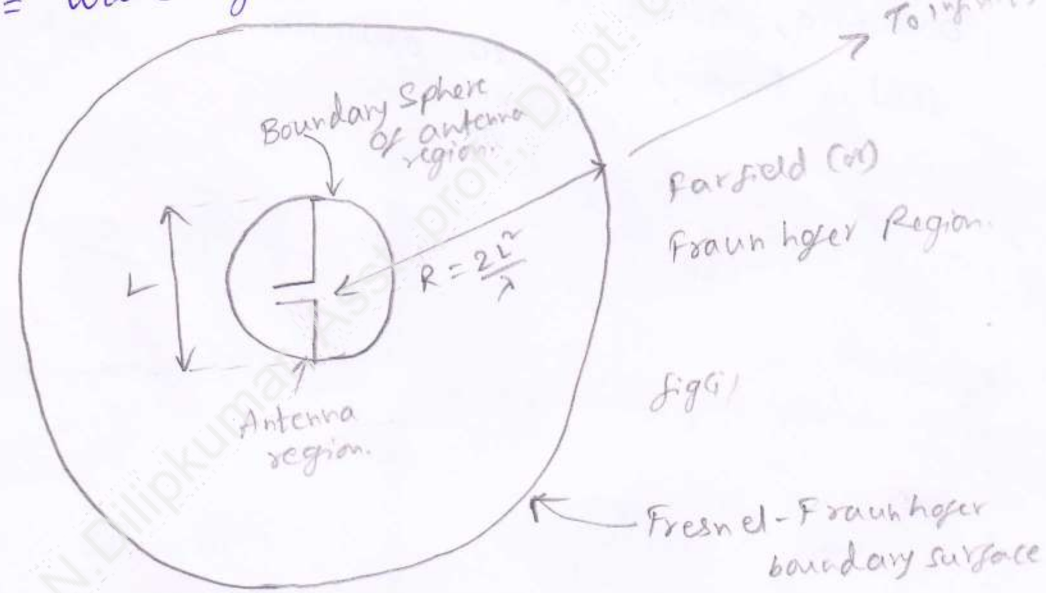
Antenna Field Zones:-

The fields around an antenna may be divided in to two principle regions, one near the antenna called the Near field (or Fresnel zone" & one at a large distance called far field (or Fraunhofer zone". The boundary b/n the two may be arbitararily taken to be at a radius,

$$R = \frac{2L^2}{\lambda} \text{ (m)}.$$

where, L = max. dimension of the antenna (m)

λ = wavelength (m).



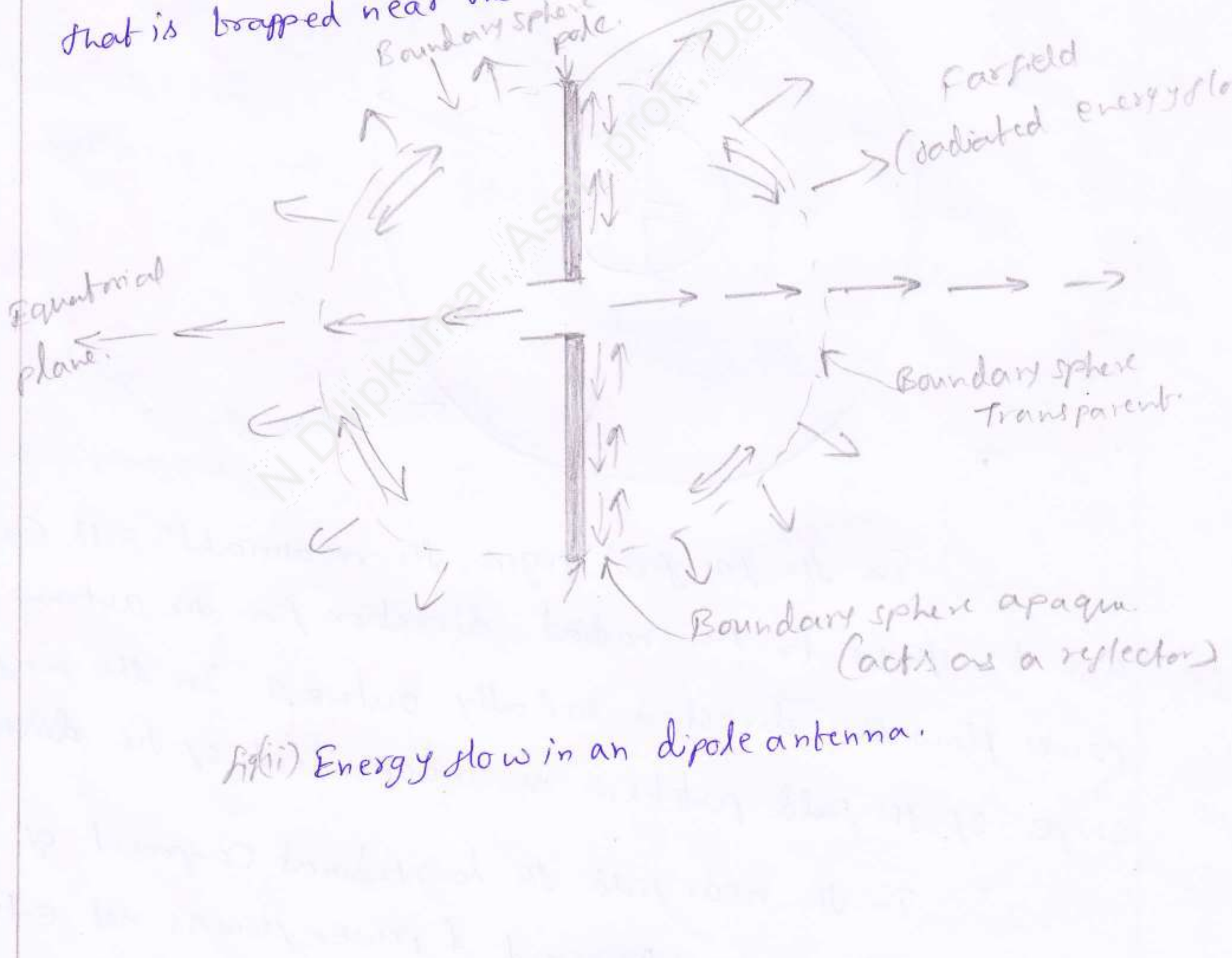
fig(i)

In the far field region, the measurable field Components are transverse to the radial direction from the antenna & all power flow is directed radially outward. In the far field the shape of the field pattern is independent of the distance.

In the near field the longitudinal Component of the electric field may be significant & power flow is not entirely radial. In the near field, the shape of the field pattern depends, in general, on the distance.

Let us consider an antenna in an imaginary boundary sphere as shown in fig (ii): The region near the poles of the sphere acts as a reflector. On the other hand, the waves expanding ter to the dipole in the equatorial region of the sphere result in power leakage through the sphere as if partially transparent in this region.

This results in reciprocating energy flow near the antenna accomplished by outward flow in the equatorial region. The outflow accounts for the power radiated from the antenna, while the reciprocating energy represents reactive power that is trapped near the antenna.

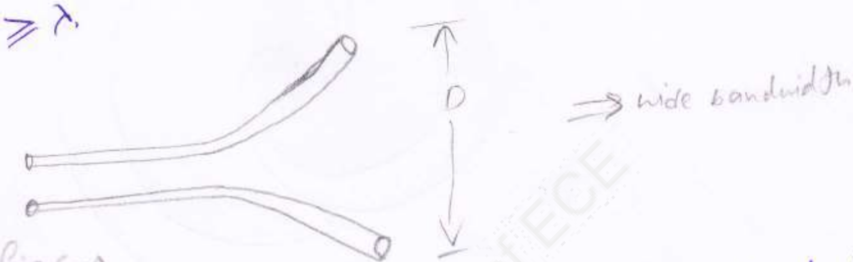


fig(ii) Energy flow in an dipole antenna.

Shape- Impedance Considerations:-

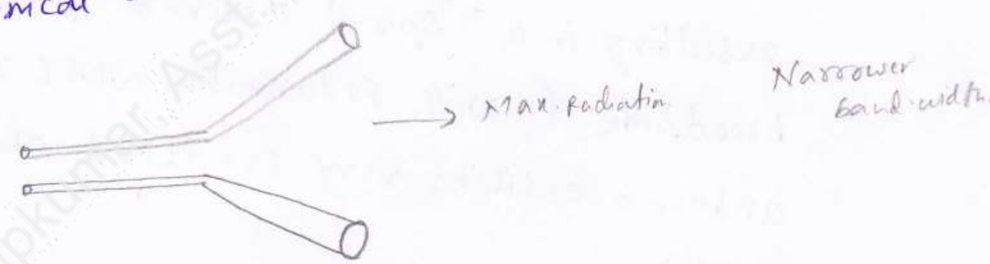
Generally the shape of antenna is responsible to easily understood the qualitative behaviour.

Starting with the opened-out two Conductor transmission line shown in fig.(a). If it is extended far enough, a nearly Constant impedance will be provided at the i/p end for for $d \ll \lambda$ & $D \geq \lambda$.

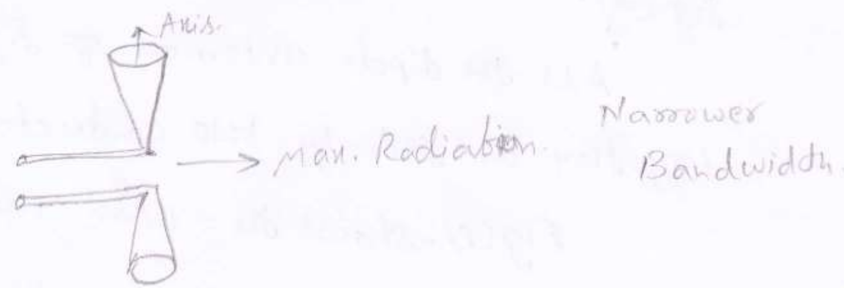


fig(a)

In fig.(b) the curved Conductors are straightened into rectangular Cones & in fig(c), the Cones are aligned Collinearly forming a "biconical antenna".



fig(b)



fig(c)

Whereas in fig(d), the Cones degenerate into the straight wires.

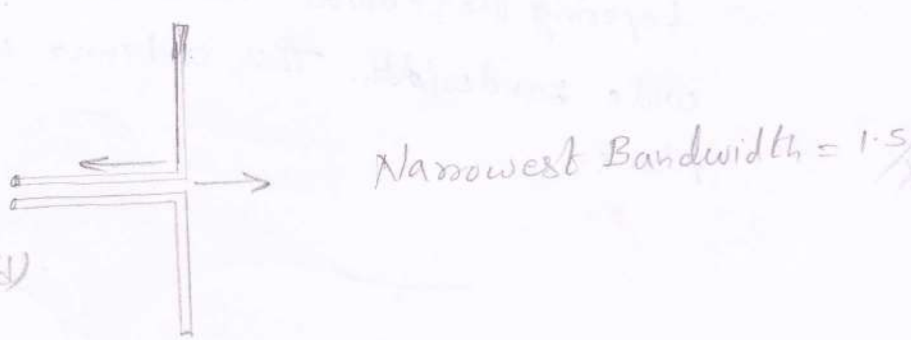


fig (d)

From fig (a) to fig (d), the bandwidth of relatively constant impedance tends to decrease. Another difference is that the antennas of fig. (a) & fig. (b) are unidirectional with beams to the right, while the antennas of fig (c) & fig (d) are omnidirectional in the horizontal plane. (⊥er to the wire or cone axis).

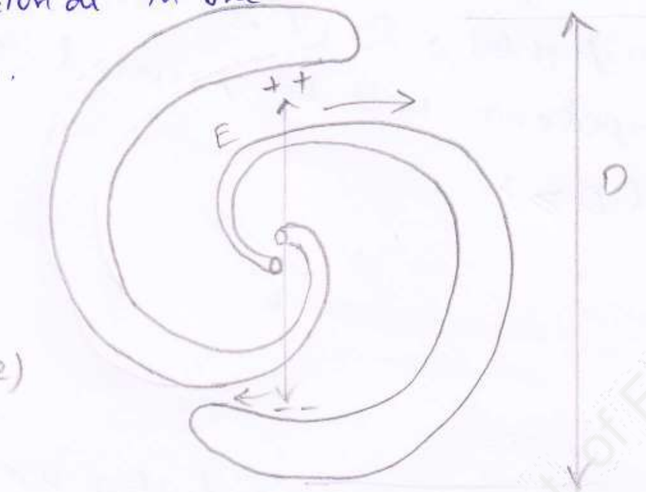


fig (e)

A different modification is shown in fig (e). Here the two conductors are curved more sharply & in opposite directions resulting in a "spiral antenna" with maximum radiation broadside & with polarization which rotates clockwise. This antenna exhibits very broadband characteristics like the one in fig (a).

All the dipole antennas of fig (a), (b), (c), (d), & (e) are balanced i.e., they are fed by two conductor (balanced) transmission lines.

Fig (f). shows the basic monopole antenna called "volcano-smoke antenna". Such an antenna is obtained by tapering the coaxial transmission line gradually. It has a very wide bandwidth. This antenna is omnidirection in the horizontal plane.

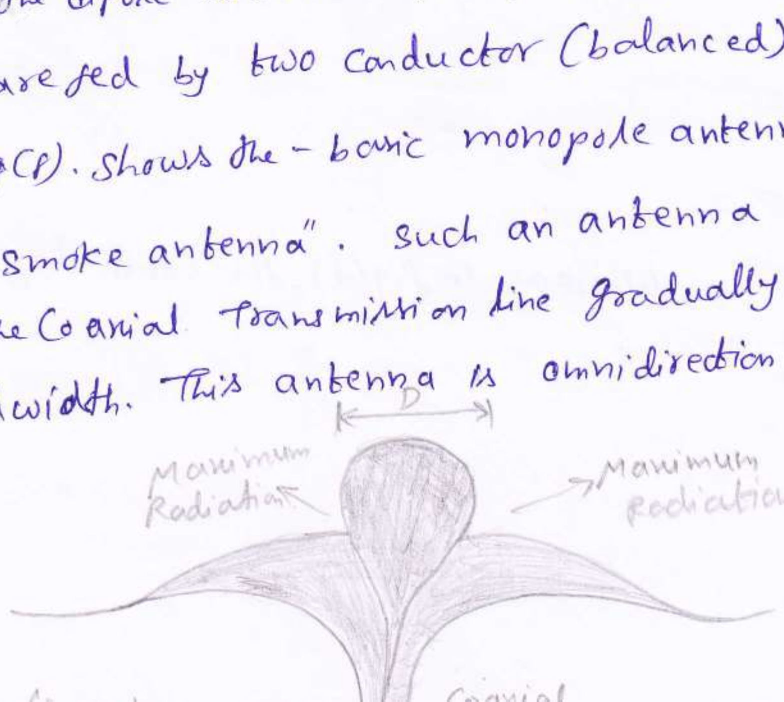
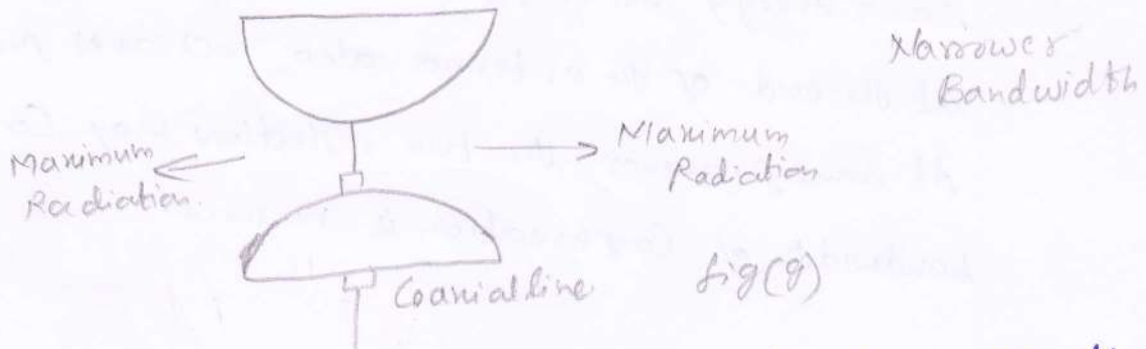
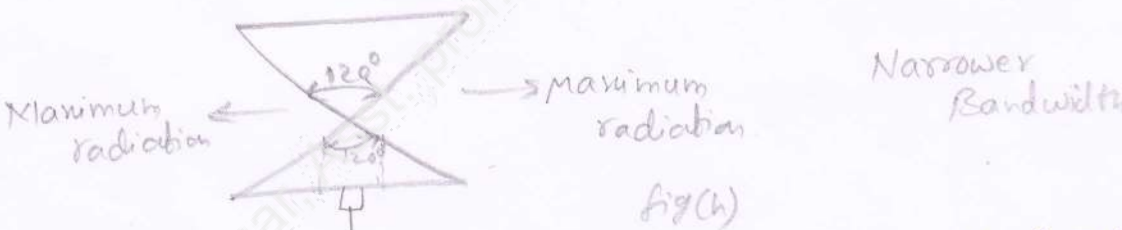


fig.(g) is a double dish antenna which is obtained by applying changes to the volcano-smoke antenna. This antenna is also omnidirectional in the horizontal plane.



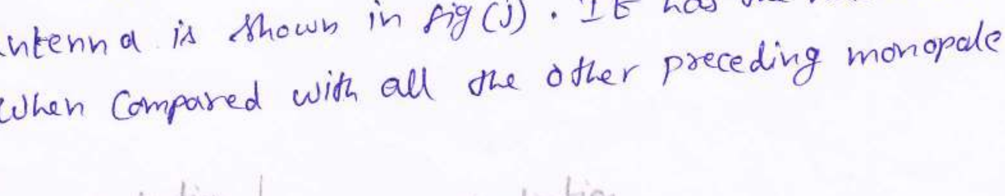
fig(g)

The volcano form of the monopole antenna is also modified into two wide angle cones as shown in fig.(h). It is a biconical antenna that has a full cone angle of 120°.



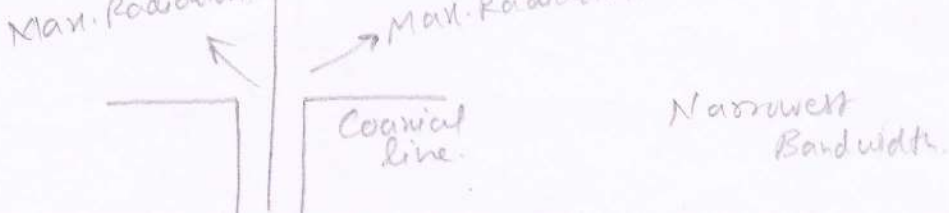
fig(h)

If the lower cone angle is made 180° i.e., flat & reducing the upper cone angle then the resulting antenna is shown in fig (i).



fig(i)

When the upper cone is degenerated into a thin stub then the resulted antenna is shown in fig (j). It has the narrowest bandwidth when compared with all the other preceding monopole antennas.



As we depart further, the discontinuity in the Transmission line becomes more abrupt & eventually becomes the junction of the ground plane & the Coaxial line. This discontinuity results in some energy being reflected back into the line. The reflection at the end of the antenna also increases for thinner antennas. At some frequency, the two reflections may Compensate but the bandwidth of Compensation is narrow.

UNIT-II

THIN LINEAR WIRE ANTENNAS

The basic antenna Elements are:

1. Alternating Current element (or) Hertzian dipole.
2. Short dipole.
3. Short Monopole.
4. Half - wave dipole.
5. Quarter - Wave Monopole.

Alternating Current Element (or) Hertzian Dipole:-

It is a short Linear antenna in which the current along its length is assumed to be constant.

Short dipole:-

It is a Linear antenna whose length is less than $\lambda/4$ and the Current distribution is assumed to be Triangular.

~~Half wave dipole:-~~

~~It is a linear antenna~~

Short Monopole:-

It is a linear antenna whose length is less than $\lambda/8$ and the Current distribution is assumed to be Triangular.

Half wave dipole:-

It is a linear antenna whose length is $\lambda/2$ and the Current distribution is assumed to be sinusoidal. It is usually Centre feed.

Quarter wave Monopole:-

It is a Linear antenna whose length is $\lambda/4$ and the ...tion is assumed to be sinusoidal. It is fed

1. Current Element

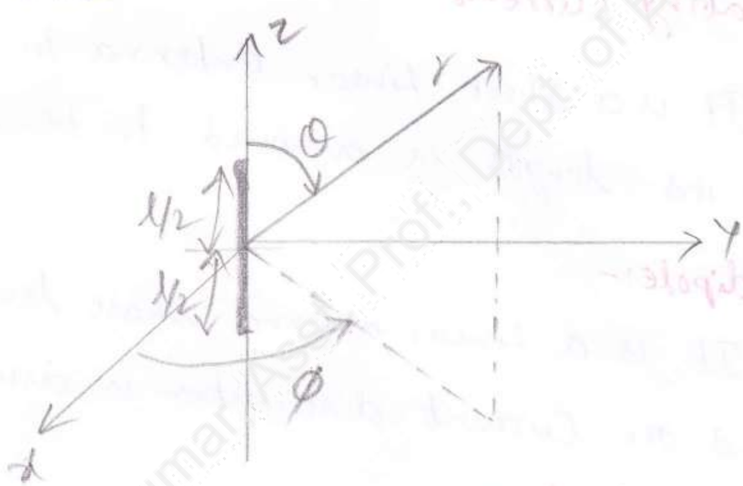
→ Current element is an infinitesimal linear wire whose length is very small and is very thin.

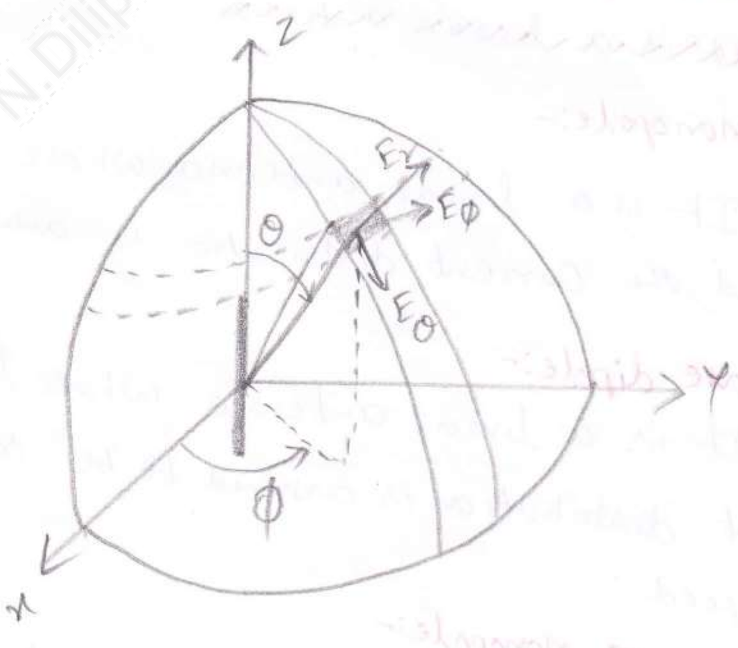
i.e., $l << \lambda$.

→ Let the Current element is positioned symmetrically at the origin of the coordinate system and is oriented along z-axis.

→ Current is assumed to be constant along its length.

$\therefore I = I_0 \, a_z$.





Vector potential (A):-

The vector potential of the current element is given by,

$$A(x,y,z) = a_z \frac{\mu I_0}{4\pi r} e^{-jkr} \int_{-l/2}^{+l/2} dz.$$

$$= a_z \frac{\mu I_0 l}{4\pi r} e^{-jkr} \quad (\because A_x = A_y = 0)$$

The transformation between rectangular and spherical components in matrix form is,

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \cos\theta \, A_z \\ -\sin\theta \, A_z \\ 0 \end{bmatrix}$$

$$A_r = A_z \cos\theta = \frac{\mu I_0 l}{4\pi r} e^{-jkr} \cos\theta$$

$$A_\theta = -A_z \sin\theta = \frac{-\mu I_0 l}{4\pi r} e^{-jkr} \sin\theta$$

$$A_\phi = 0.$$

Magnetic Field Strength (H):-

$$H = \frac{1}{\mu}\left[\nabla \times A\right] \quad \text{— (1)}$$

$$\nabla \times A = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta\,A_\phi \end{vmatrix}$$

$$= \frac{1}{r^2\sin\theta}\left\{ \hat{a}_r\left[\frac{\partial}{\partial\theta} r\sin\theta A_\phi - \frac{\partial}{\partial\phi} rA_\theta\right] + r\hat{a}_\theta\left[\frac{\partial}{\partial r} r\sin\theta A_\phi - \frac{\partial}{\partial\phi}A_r\right] + r\sin\theta\,\hat{a}_\phi\left[\frac{\partial}{\partial r} rA_\theta - \frac{\partial}{\partial\theta}A_r\right]\right\}$$

Using the symmetry, i.e., no variations in ϕ component as from above vector potential A_ϕ,

Hence, $\hat{a}_r = \hat{a}_\theta = 0$

$$= \frac{1}{r^2\sin\theta}\left\{ \hat{a}_r\left[-\frac{\partial}{\partial\phi} rA_\theta\right] + r\hat{a}_\theta\left[-\frac{\partial}{\partial\phi}A_r\right] + r\sin\theta\,\hat{a}_\phi\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial\theta}A_r\right]\right\}$$

$$= \frac{1}{r^2\sin\theta}\left[0 + 0 + r\sin\theta\,\hat{a}_\phi\left(\frac{\partial}{\partial r} rA_\theta - \frac{\partial}{\partial\theta}A_r\right)\right]$$

$$= \frac{1}{r^{\cancel{2}}\cancel{\sin\theta}}\left[\cancel{r}\cancel{\sin\theta}\,\hat{a}_\phi\left(\frac{\partial}{\partial r} rA_\theta - \frac{\partial}{\partial\theta}A_r\right)\right]$$

$$\nabla \times A = \frac{\hat{a}_\phi}{r}\left[\frac{\partial}{\partial r} rA_\theta - \frac{\partial}{\partial\theta}A_r\right]$$

Substitute in (1),

$$H_\phi = \frac{1}{\mu r}\left[\frac{\partial}{\partial r} rA_\theta - \frac{\partial}{\partial\theta}A_r\right]$$

$$= \frac{1}{\mu r}\left[\frac{\partial}{\partial r}\cancel{(r)}\left(-\frac{\mu I_0 d\, e^{-jkr}\sin\theta}{4\pi \cancel{r}}\right) - \frac{\partial}{\partial\theta}\left(\frac{\mu I_0 d\, e^{-jkr}}{4\pi r}\cos\theta\right)\right]$$

$$= \frac{1}{\mu r}\left[\frac{jk\mu I_0 d\sin\theta}{4\pi} + \frac{\mu I_0 d\, e^{-jkr}\sin\theta}{4\pi r}\right]$$

$$\therefore H_r = 0$$
$$H_\theta = 0$$
$$\boxed{H_\phi = \frac{jk I_o dl \sin\theta}{4\pi r}\left[1 + \frac{1}{jkr}\right] e^{-jkr}}$$

Electric Field Strength:-

$$E = \frac{1}{j\omega\varepsilon}\left[\nabla \times H\right]$$

$$\nabla \times H = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ H_r & r H_\theta & r\sin\theta\, H_\phi \end{vmatrix}$$

$$= \frac{1}{r^2 \sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & 0 & r\sin\theta\, H_\phi \end{vmatrix}$$

$$= \frac{1}{r^2 \sin\theta}\left\{\left[\hat{a}_r\left(\frac{\partial}{\partial \theta} r\sin\theta H_\phi\right)\right] - \left[r\hat{a}_\theta\left(\frac{\partial}{\partial r}(r\sin\theta H_\phi)\right)\right] + 0\right\}$$

$$\therefore E_r = \frac{1}{j\omega\varepsilon}\left[\frac{1}{r^{\cancel{2}}\sin\theta}\cdot\frac{\partial}{\partial\theta}(\cancel{r}\sin\theta\; H_\phi)\right]$$

$$= \frac{1}{j\omega\varepsilon}\;\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\frac{jk I_o dl \sin^2\theta}{4\pi r}\left(1 + \frac{1}{jkr}\right)e^{-jkr}\right)\right]$$

$$= \frac{1}{j\omega\varepsilon\, r\sin\theta}\cdot\frac{jk I_o dl\, 2\sin\theta\cos\theta}{4\pi r}\left(1 + \frac{1}{jkr}\right)e^{-jkr}$$

$$\therefore E_r = \frac{\eta I_o dl \cos\theta}{2\pi r^2}\left(1 + \frac{1}{jkr}\right)e^{-jkr} \quad \left\{\because \frac{k}{\omega\varepsilon} = \eta\right\}$$

$$E_\theta = \frac{1}{j\omega\epsilon}\left[\nabla \times H\right]_\theta$$

$$= \frac{1}{j\omega\epsilon}\left[\frac{1}{r^2 \cancel{\sin\theta}}\left(-\cancel{r}\,\cancel{\sin\theta}\,\frac{\partial}{\partial r}\, r H_\phi\right)\right]$$

$$= \frac{1}{j\omega\epsilon}\cdot\frac{1}{r}\left[-\frac{\partial}{\partial r}\left(\frac{\cancel{r}\, jk I_0 l \sin\theta}{4\pi\, \cancel{r}}\left(1+\frac{1}{jkr}\right)e^{-jkr}\right)\right]$$

$$= -\frac{jk I_0 l \sin\theta}{j\omega\epsilon\, r\, 4\pi}\cdot\frac{\partial}{\partial r}\left[e^{-jkr} + \frac{e^{-jkr}}{jkr}\right]$$

$$= -\frac{\eta I_0 l \sin\theta}{4\pi r}\left[-jk\cdot e^{-jkr} + \left(\frac{(jkr)(-jk)\,e^{-jkr} - e^{-jkr}\cdot jk}{(jkr)^2}\right)\right]$$

$$\cancel{= -\frac{\eta I_0 l \sin\theta}{4\pi r}\left[-jk e^{-jkr} + (jkr)\right.}$$

$$= \frac{\eta I_0 l \sin\theta\, jk}{4\pi r}\left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right]e^{-jkr}$$

$$\therefore E_r = \frac{\eta I_0 l \cos\theta}{2\pi r^2}\left[1 + \frac{1}{jkr}\right]e^{-jkr}$$

$$E_\theta = \frac{jk\eta I_0 l \sin\theta}{4\pi r}\left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right]e^{-jkr}$$

$$E_\phi = 0$$

Power density and Radiation Resistance:-

The input impedance of an antenna, which consists of real and imaginary impedances.

$$Z_A = R_A + jX_A.$$

$$\text{where } R_A = R_r + R_l.$$

→ For a lossless antenna, the real part of the input impedance was designated as radiation resistance. It is through the mechanism of the radiation resistance that power is ~~transformed~~ transferred from the guided wave to the free space wave.

→ To find the i/p ~~impedance~~ resistance for a loss less antenna, the poynting vector is formed interms of E & H fields radiated by the antenna.

→ By integrating the poynting vector over a closed surface (usually a sphere of constant radius), the total power radiated by the source is found. The real part of it is related to the input resistance.

→ For an Infinitesimal dipole, the Complex poynting vector can be written using H_r, H_θ, H_ϕ & E_r, E_θ, E_ϕ as,

$$P = \frac{1}{2}\left[\bar{E}\times\bar{H}^*\right].$$

$$\begin{bmatrix} H_r = H_\theta = 0 \\ E_\phi = 0 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} \hat{a}_r & \hat{a}_\theta & \hat{a}_\phi \\ E_r & E_\theta & E_\phi \\ H_r & H_\theta & H_\phi \end{bmatrix}$$

In equation (1) first term ~~terms~~ represents radial component P_r and second terms represents transverse components P_θ.

Power density:-

$$P = \frac{1}{2}\left[E \times H^*\right]$$

$$P = \frac{1}{2}\left[\hat{a}_r E_\theta H_\phi^* - \hat{a}_\theta E_r H_\phi^*\right]$$

From this, $P_r = \frac{1}{2}\hat{a}_r E_\theta H_\phi^*$

$$P_\theta = -\frac{1}{2}\hat{a}_\theta E_r H_\phi^*$$

$$P_r = \frac{1}{2}\hat{a}_r \left\{ \overbrace{\frac{jk\eta I_0 l \sin\theta}{4\pi r}\left(1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right)\cancel{e^{-jkr}}}^{E_\theta} \right\} \underbrace{\left\{\frac{(-j)kI_0 l\sin\theta}{4\pi r}\left(1 + \frac{1}{(-j)kr}\,\cancel{e^{jkr}}\right)\right\}}_{H_\phi^*}$$

$$= \frac{\eta k^2 I_0^2 l^2 \sin^2\theta}{32\pi^2 r^2}\left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right]\left[1 - \frac{1}{jkr}\right]$$

$$= \frac{\eta k^2 I_0^2 l^2 \sin^2\theta}{32\pi^2 r^2}\left[1 - \cancel{\frac{1}{jkr}} + \cancel{\frac{1}{jkr}} + \cancel{\frac{1}{k^2r^2}} - \cancel{\frac{1}{k^2r^2}} + \frac{1}{jk^3r^3}\right]$$

$$= \frac{\eta k^2 I_0^2 l^2 \sin^2\theta}{32\pi^2 r^2}\left[1 - \frac{j}{k^3r^3}\right] \quad \text{[multiply \& divide by } j\text{]}.$$

$$= \frac{\eta}{8}\frac{k^2(I_0 l)^2}{4\pi^2}\frac{\sin^2\theta}{r^2}\left[1 - \frac{j}{(kr)^3}\right]$$

let $\lambda = \frac{2\pi}{k}$.

$$= \frac{\eta}{8}\left(\frac{I_0 l}{\lambda}\right)^2\frac{\sin^2\theta}{r^2}\left[1 - \frac{j}{(kr)^3}\right]$$

$$P_r = \frac{\eta}{8}\left(\frac{I_0 l}{\lambda}\right)^2\frac{\sin^2\theta}{r^2}\left[1 - \frac{j}{(kr)^3}\right]$$

$$P_\theta = -\frac{1}{2} E_r H_\phi^*$$

$$= -\frac{1}{2}\left\{\frac{\eta I_0 d \cos\theta}{2\pi r^2}\left(1+\frac{1}{jkr}\right)e^{-jkr}\right\}\left\{\frac{(-j)kI_0 d\sin\theta}{4\pi r}\left[1+\frac{1}{jkr}\right]e^{+jkr}\right\}$$

$$= \frac{j\eta (I_0 d)^2 k \sin\theta\cos\theta}{16\pi^2 r^3}\left[(1)^2 - \frac{1}{(jkr)^2}\right]$$

$$P_\theta = \frac{j\eta k (I_0 d)^2 \cos\theta \sin\theta}{16\pi^2 r^3}\left[1+\frac{1}{(kr)^2}\right]$$

$$\therefore P_{rad} = \int_0^{2\pi}\int_0^{\pi} \underbrace{P_r}_{\text{w.r.to } P_r} r^2 \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{\eta}{8}\left(\frac{I_0 d}{\lambda}\right)^2 \frac{\sin^2\theta}{\cancel{r^2}}\left[1-\frac{j}{(kr)^3}\right]\sin\theta \, \cancel{r^2}\, d\theta \, d\phi.$$

Consider,

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

$$\sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$$

integrate,

$$= \frac{1}{4}\int_0^{2\pi}\int_0^{\pi}\left[3\sin\theta - \sin 3\theta\right] d\theta \, d\phi$$

$$= \frac{1}{4}(2\pi)\left[-3\cos\theta + \frac{\cos 3\theta}{3}\right]_0^{\pi}$$

$$= \frac{\pi}{2}\left[-3\cos\pi + \frac{\cos 3\pi}{3} - \left(-3+\frac{1}{3}\right)\right]$$

$$= \frac{\pi}{2}\left[3 - \frac{1}{3} + 3 - \frac{1}{3}\right]$$

$$= \frac{\pi}{2}\left[6 - \frac{2}{3}\right] \Rightarrow \frac{8\pi}{3}.$$

$$\therefore P_{rad} = \frac{\eta}{\cancel{8}}\left(\frac{I_0 d}{\lambda}\right)^2\left[1-\frac{j}{(kr)^3}\right] \cdot \frac{\cancel{8}\pi}{3}.$$

$$P_{rad} = P_{(real\ part)} = \frac{\eta\pi}{3}\left(\frac{I_0 dl}{\lambda}\right)^2 = \frac{1}{2}(I_0^2) R_r.$$

where $R_r = \frac{2\eta\pi}{3}\left(\frac{dl}{\lambda}\right)^2$

$$= \cancel{120}^{40}\pi \cdot \frac{2\pi}{\cancel{3}}\left(\frac{dl}{\lambda}\right)^2$$

$\left[\because \eta = \text{intrinsic impedance}, \quad \eta = 120\pi = 377\Omega\right]$

$$R_r = 80\pi^2\left(\frac{dl}{\lambda}\right)^2$$

Average poynting vector (or power density (P_{avg})):-

$$P_{avg} = \frac{1}{2} Re\left[\bar{E} \times \bar{H}^*\right]$$

$\left[\eta = \frac{E}{H} \Rightarrow H = \frac{E}{\eta}\right]$

$$= \frac{1}{2}\left[E_\theta H_\phi\right] a_r.$$

$$= \frac{|E_\theta|^2}{2\eta} a_r.$$

$$= \frac{1}{2\cancel{\eta}}\left|\frac{\eta^{\cancel{2}} K I_0 dl \sin\theta}{4\pi r}\right|^2 a_r$$

$$= \frac{\eta}{2}\left[\frac{K I_0 dl}{4\pi r}\right]^2 \sin^2\theta\, a_r.$$

Radiation Intensity:-

$$U = P_{avg}\, r^2$$

$$= r^2 \cdot \frac{\eta}{2}\left[\frac{K I_0 dl}{4\pi r}\right]^2 \sin^2\theta\, a_r$$

$$U = \frac{\eta}{2}\left[\frac{K I_0 dl}{4\pi}\right]^2 \sin^2\theta\, a_r.$$

$\left[\because \theta = 90^\circ, \ \sin 90^\circ = 1\right]$

$$U_{max} = \frac{\eta}{2}\left[\frac{K I_0 dl}{4\pi}\right]^2 a_r.$$

Average radiated power:-

$$P_{avg\,rad} = \int_0^{2\pi}\int_0^{\pi} P_{avg}\, r^2 \sin\theta \, d\theta \, d\phi \; \hat{a}_r$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{\eta}{2}\left[\frac{k I_0 d}{4\pi \cancel{r}}\right]^2 \sin^2\theta \; \cancel{r^2} \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{\eta}{2}\left[\frac{k I_0 d}{4\pi}\right]^2 \sin^3\theta \, d\theta \, d\phi.$$

Consider $\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$

$$\sin^3\theta = \frac{1}{4}\left[3\sin\theta - \sin 3\theta\right] \longrightarrow \text{integrate.}$$

$$= \frac{1}{4}\left[-3\cos\theta + \frac{\cos 3\theta}{3}\right]_0^{\pi}$$

$$= \frac{1}{4}\left[3 - \frac{1}{3} + 3 - \frac{1}{3}\right] \Rightarrow \frac{1}{4}\left[6 - \frac{2}{3}\right] \Rightarrow \frac{4}{3}.$$

$$\int_0^{2\pi}\int_0^{\pi} = \frac{\eta}{2}\left[\frac{k I_0 d}{4\pi}\right]^2 (2\pi)\,\frac{4}{3}$$

$$= \frac{\eta}{2}\left[\frac{k I_0 d}{4\pi}\right]^2 \frac{8\pi}{3}.$$

Directivity:-

$$D = \frac{U_{max}}{U_0} = \frac{U_{max}}{P_{avg}} = \frac{U_{max}}{\frac{P_{rad}}{4\pi}}$$

$$= \frac{4\pi\, U_{max}}{P_{rad}}$$

$$\Rightarrow \frac{\cancel{4\pi}\,\cancel{\frac{\eta}{2}}\cancel{\left[\frac{k I_0 d}{4\pi}\right]^2}}{\cancel{\frac{\eta}{2}}\cancel{\left[\frac{k I_0 d}{4\pi}\right]^2}\frac{\cancel{8}^{2}\cancel{\pi}}{3}} = 3/2 = 1.5.$$

$$D_{(dB)} = 10\log(1.5) = 1.76\text{ dB}.$$

Maximum Effective Aperture (A_{emax}) :-

$$A_{emax} \text{ (or) } A_{em} = \frac{\lambda^2}{4\pi} D.$$

$$= \frac{\lambda^2}{4\pi} \times \frac{3}{2} = \frac{3\lambda^2}{8\pi} = 0.119\lambda^2.$$

$$A_{em} = \frac{3\lambda^2}{8\pi} = 0.119\lambda^2.$$

Effective length:-

As Current is Constant through out the length, effective length is equal to physical length.

$$\underline{l_e = l.}$$

Prob:- Find the radiation resistance of an Infinetesimal dipole ~~whereas~~ as overall length is $l = \lambda/50$.

$$R_r = 80\pi^2 \left(\frac{l}{\lambda}\right)^2 = 80\pi^2 \left(\frac{1}{50}\right)^2 = 0.316\,\Omega.$$

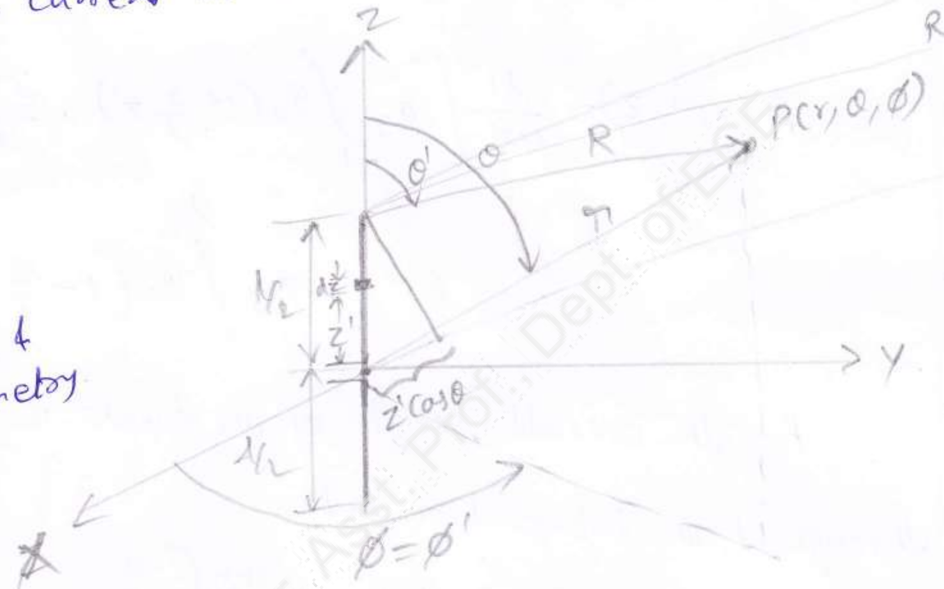
Since the R_r of Infinetesimal dipole is about $0.3\,\Omega$ it will present a very large mismatch when Connected to practical Tx'sion lines, many of which have Co-axial Transmission impedance of 50 or $75\,\Omega$, The Reflection efficiency (e_r) & hence the overall efficiency (e_t) will be very small.

2. Small Dipole

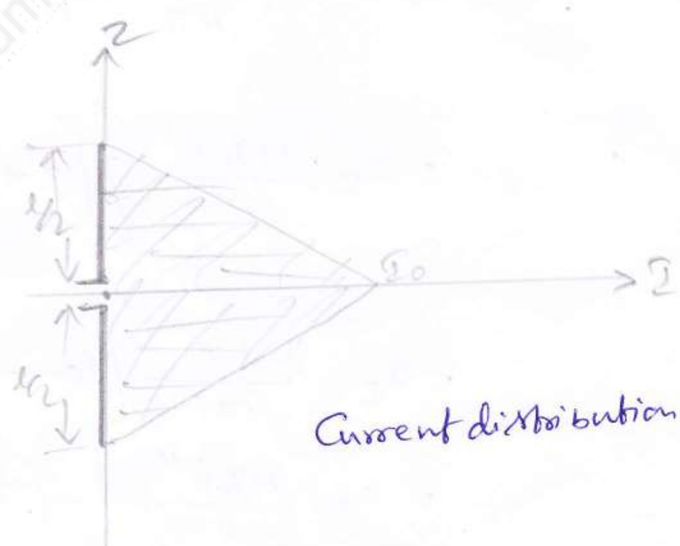
Small dipole is a linear wire antenna whose length is usually $\lambda/50 < l \leq \lambda/10$

Small dipole is oriented along z-axis & positioned Symmetrically at the origin of spherical Coordinate system.

The current is assumed to be Triangular variation.



Dipole & geometry



Current distribution.

Let dz' be the length of the element which is at a distance of dz' from o. 'p' is remote distant point which is at a distance 'R' from dz' & r from o.

→ Triangular current distribution in small dipole can be expressed as,

$$I = \begin{cases} a_z \cdot I_0\left(1 - \frac{2}{l} z'\right) & 0 \le z' \le l/2 \\ a_z \cdot I_0\left(1 + \frac{2}{l} z'\right) & -l/2 \le z' \le 0. \end{cases}$$

Vector potential (A) :-

$$A(x,y,z) = \frac{\mu}{4\pi}\left[a_z \int_{-l/2}^{0} I_0\left(1+\frac{2}{l}z'\right) \cdot \frac{e^{-jkR}}{R}\, dz' + a_z \int_{0}^{l/2} I_0\left(1-\frac{2}{l}z'\right)\frac{e^{-jkR}}{R}\, dz'\right]$$

As the overall length of the dipole is small, $R \simeq r$ throughout the integration path.

$$A(x,y,z) = a_z \cdot \frac{\mu I_0}{4\pi} \cdot \frac{e^{-jkr}}{r}\left[\int_{-l/2}^{0}\left(1+\frac{2}{l}z'\right) dz' + \int_{0}^{l/2}\left(1-\frac{2}{l}z'\right) dz'\right]$$

$$= a_z \cdot \frac{\mu I_0 e^{-jkr}}{4\pi r}\left\{\left[z' + \frac{2}{l}\frac{z'^2}{2}\right]_{-l/2}^{0} + \left[z' - \frac{2}{l} \cdot \frac{z'^2}{2}\right]_{0}^{l/2}\right\}$$

$$= a_z \cdot \frac{\mu I_0 e^{-jkr}}{4\pi r}\left[(0+0) - \left(-l/2 + l/4\right) + \left(l/2 - l/4\right) - (0-0)\right]$$

$$= a_z \cdot \frac{\mu I_0 e^{-jkr}}{4\pi r}\left[-\left(-l/2 + l/4\right) + \left(l/2 - l/4\right)\right]$$

$$= a_z \cdot \frac{\mu I_0 e^{-jkr}}{4\pi r}\left[l/2\right]$$

$$A(x,y,z) = a_z \cdot \frac{\mu I_0 l e^{-jkr}}{8\pi r}.$$

$$\therefore\ A_x = 0$$

$$A_y = 0$$

$$A_z = \frac{\mu I_0 dl\, e^{-jkr}}{8\pi r}.$$

It is observed that the vector potential of small dipole is one half of that of current element.

$$\therefore\ A_r = A_z \cos\theta = \frac{\mu I_0 dl\, e^{-jkr}}{8\pi r}\cos\theta.$$

$$A_\theta = -A_z \sin\theta = \frac{-\mu I_0 dl\, e^{-jkr}}{8\pi r}\sin\theta.$$

$$A_\phi = 0.$$

Magnetic Field strength (H):-

$$H = \frac{1}{\mu}\left[\nabla \times A\right]$$

Because of the symmetry (No variation in ϕ), H can be written in simplified form as,

$$H_\phi = \frac{1}{\mu}\left[\nabla \times A\right]_\phi$$

$$= \frac{1}{\mu r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial \theta}(A_r)\right]$$

$$= \frac{1}{\mu r}\left[\frac{\partial}{\partial r}\left(-\frac{r\mu I_0 dl\, e^{-jkr}\sin\theta}{8\pi r}\right) - \frac{\partial}{\partial \theta}\left(\frac{\mu I_0 dl\, e^{-jkr}\cos\theta}{8\pi r}\right)\right]$$

$$= \frac{1}{\mu r}\left[\frac{jk\mu I_0 dl\, e^{-jkr}\sin\theta}{8\pi} + \frac{\mu I_0 dl\, e^{-jkr}\sin\theta}{8\pi r}\right]$$

$$= \frac{jk I_0 dl \sin\theta}{8\pi r}\left[1 + \frac{1}{jkr}\right] e^{-jkr}.$$

$$\therefore \ H_r = 0$$

$$H_\theta = 0$$

$$H_\phi = \frac{jkI_0 dl \sin\theta}{8\pi r}\left[1 + \frac{1}{jkr}\right] e^{-jkr}$$

Radiation field. Component of 'H' for which $(kr >> 1)$ is,

$$H_\phi = \frac{jkI_0 dl \sin\theta}{8\pi r} e^{-jkr}$$

Electric field strength (E):-

$$E = \frac{1}{j\omega\varepsilon}[\nabla \times H]$$

As $H_r = 0$, & $H_\theta = 0$, $E_\phi = 0$

$$E_r = \frac{1}{j\omega\varepsilon}[\nabla \times H]_r$$

$$= \frac{1}{j\omega\varepsilon}\left[\frac{1}{r^2 \sin\theta} \cdot \frac{\partial}{\partial\theta}(r \sin\theta H_\phi)\right]$$

$$= \frac{1}{j\omega\varepsilon} \frac{1}{r\sin\theta} \cdot \frac{\partial}{\partial\theta}(H_\phi \sin\theta)$$

$$= \frac{1}{j\omega\varepsilon} \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\frac{jkI_0 dl \sin^2\theta}{8\pi r}\left(1 + \frac{1}{jkr}\right) e^{-jkr}\right)\right]$$

$$= \frac{\eta I_0 dl \cos\theta}{4\pi r^2}\left[1 + \frac{1}{jkr}\right] e^{-jkr} \qquad \left\{\frac{k}{\omega\varepsilon} = \eta\right\}$$

$$E_\theta = \frac{1}{j\omega\varepsilon}[\nabla \times H]_\theta$$

$$= \frac{1}{j\omega\varepsilon}\left[\frac{1}{r^2 \sin\theta}\left(-r\sin\theta \cdot \frac{\partial}{\partial r} r H_\phi\right)\right]$$

$$= \frac{1}{j\omega\varepsilon} \cdot \frac{1}{r}\left[\frac{-\partial}{\partial r}(r H_\phi)\right]$$

$$= \frac{1}{j\omega\varepsilon}\left(\frac{1}{r}\right)\left[-\frac{jkI_0 dl \sin\theta}{8\pi} \ldots \left(1 + \frac{1}{\ldots}\right) e^{-jkr}\right]$$

$$= \frac{-jkI_0 l\sin\theta}{j\omega\varepsilon \cdot r \cdot 8\pi} \cdot \frac{\partial}{\partial r}\left[e^{-jkr} + \frac{e^{-jkr}}{jkr}\right]$$

$$= \frac{\eta I_0 l\sin\theta \cdot jk}{8\pi r}\left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right]e^{-jkr}$$

$$\therefore E_r = \frac{\eta I_0 l\cos\theta}{4\pi r^2}\left[1 + \frac{1}{jkr}\right]e^{-jkr}$$

$$E_\theta = \frac{jk\eta I_0 l\sin\theta}{8\pi r}\left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right]e^{-jkr}$$

$$E_\phi = 0.$$

Radiation field Component of E (for which $kr \gg 1$) is,

$$E_r \approx 0.$$

$$E_\theta \approx \frac{\eta I_0 l\sin\theta\, jk}{8\pi r}\, e^{-jkr}$$

In radiation field,

$$Z_W = \frac{E_\theta}{H_\phi} = \frac{\left[\frac{\eta I_0 l\sin\theta\, jk}{8\pi r}\right]e^{-jkr}}{\left[\frac{jkI_0 l\sin\theta}{8\pi r}\right]e^{-jkr}} = \eta.$$

$= 120\pi$ (or) $377\,\Omega$.

Radiation resistance (R_r):-

As field Components becomes half when Compared to Current element then Radiation resistance becomes onefourth of Current element.

$$\text{i.e., } R_r = \frac{1}{4}\left[80\pi^2\left(\frac{l}{\lambda}\right)^2\right]$$

$$R_r = 20\pi^2\left(\frac{l}{\lambda}\right)^2 \text{ ohms.}$$

Average poynting vector (or) Power density (P) :-

$$P_{avg} = \frac{1}{2} Re\left[E * H^*\right]$$

$$= \frac{1}{2} E_\theta H_\phi^* a_r$$

-ve

$$= \frac{1}{2}\left[\frac{\eta I_0 l \sin\theta \cdot jk\, e^{-jkr}}{8\pi r}\right]\left[\frac{+jk I_0 l \sin\theta}{8\pi r} e^{-jkr}\right] \cdot a_r$$

$$= \frac{1}{2} \frac{\eta k^2 I_0^2 l^2 \sin^2\theta}{(8\pi r)^2} \cdot a_r.$$

Radiation Intensity (U):-

$$U = r^2 \cdot P_{avg}$$

$$= r^2\left[\frac{1}{2} \cdot \frac{\eta k^2 I_0^2 l^2 \sin^2\theta}{(8\pi r)^2}\right] a_r.$$

$$= \frac{1}{2} \cdot \frac{\eta k^2 I_0^2 l^2 \sin^2\theta}{(8\pi)^2} a_r$$

$$U_{max} = \frac{\eta k^2 I_0^2 l^2}{2(8\pi)^2}$$

Power radiated (W_{rad}) :-

$$W_{rad} = \int_0^{2\pi}\int_0^{\pi} U \cdot \sin\theta \cdot d\theta\, d\phi \cdot a_r.$$

$$= \int_0^{2\pi}\int_0^{\pi}\left[\frac{1}{2} \frac{\eta k^2 I_0^2 l^2}{(8\pi)^2} \sin^2\theta\, a_r\right] \sin\theta\, d\theta\, d\phi \cdot a_r.$$

$$= \frac{1}{2} \frac{\eta k^2 I_0^2 l^2}{(8\pi)^2} \int_0^{2\pi}\int_0^{\pi} \sin^3\theta \cdot d\theta\, d\phi.$$

$$= \frac{1}{2} \frac{\eta k^2 I_0^2 l^2}{(8\pi)^2} (2\pi) \int_0^{\pi} \sin^3\theta\, d\theta.$$

$$= \frac{1}{2} \cdot \frac{\eta k^2 I_0^2 l^2}{(8\pi)^2} \cdot (2\pi)\left(\frac{4}{3}\right)$$

$$= \frac{1}{2} \cdot \frac{\eta k^2 I_0^2 l^2}{(8\pi)^2}\left(\frac{8\pi}{3}\right) = \frac{\eta k^2 I_0^2 l^2}{48\pi}.$$

Directivity (D):-

$$D = \frac{4\pi \cdot U_{max}}{W_{rad}}$$

$$= \frac{4\pi\left[\frac{\eta}{2}\,\frac{k^2 I_0^2 l^2}{(8\pi)^2}\right]}{\frac{1}{2}\,\frac{\eta k^2 I_0^2 l^2}{(8\pi)^2}\left(\frac{8\pi}{3}\right)} = \frac{2}{\frac{1}{2}*\frac{8}{3}} = 1.5.$$

$$D = 10\log_{10} D = 10\log_{10}(1.5) = 1.76\,dB.$$

Maximum Effective aperture (A_{em}):-

$$A_{em} = \frac{\lambda^2}{4\pi} \cdot D$$

$$= \frac{\lambda^2}{4\pi} \cdot \left(\frac{3}{2}\right)$$

$$A_{em} = 0.119\,\lambda^2.$$

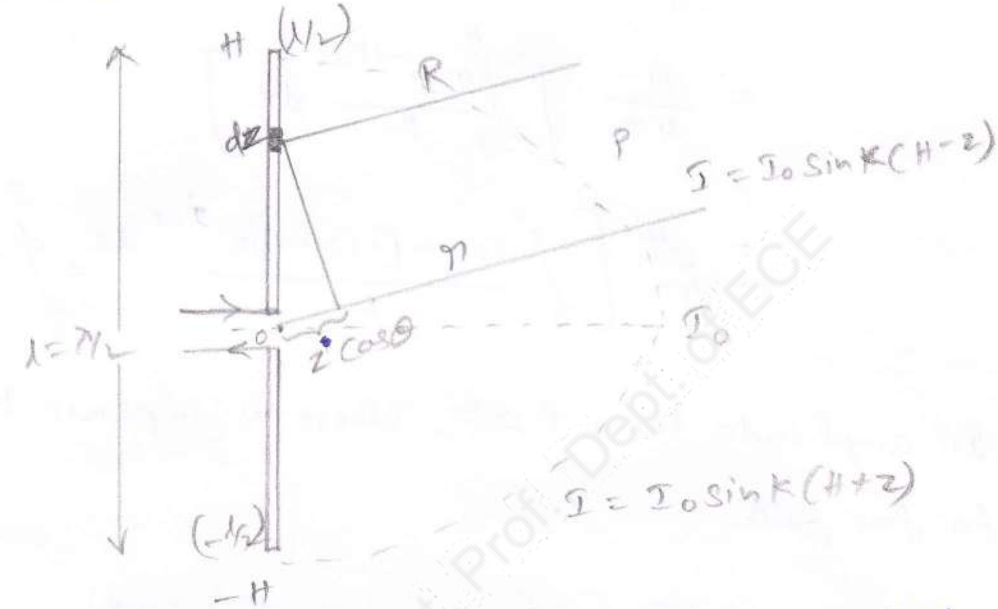
Effective length (l_e):-

Effective length in small dipole is equal to the half of its physical length.

i.e, $l_e = \frac{l}{2}.$

3. Half wave dipole:-

A linear wire antenna whose length is $l = \lambda/2$ and ungrounded is called as "Half wave dipole".



Ungrounded half wave dipole produces radiation fields in all directions. That is it radiates through entire surface of the sphere. Here antenna is oriented along z-axis and current distribution is assumed to be sinusoidal with I_0 as peak value.

$$I(x,y,z) = \begin{cases} a_z \cdot I_0 \sin K(H-z) & 0 \le z \le H \\ a_z \cdot I_0 \sin K(H+z) & -H \le z \le 0. \end{cases}$$

Here $H = l/2$.

Vector potential (A):-

Vector potential at 'P' due to current element dz is,

$$dA_z = \frac{\mu I}{4\pi} \frac{e^{-jkR}}{R} dz.$$

Total vector potential at 'P' due to all current elements in the dipole is,

$$A_z = \int_{-H}^{H} dA_z$$

$$= \int_{-H}^{H} \frac{\mu I}{4\pi} \cdot \frac{e^{-jkR}}{R} dz$$

$$= \frac{\mu}{4\pi} \left[\int_{-H}^{H} \frac{I e^{-jkR}}{R} dz \right]$$

$$= \frac{\mu}{4\pi} \left[\int_{-H}^{0} \frac{I_0 \sin[k(H+z)] e^{-jkR}}{R} dz + \int_{0}^{H} \frac{I_0 \sin[k(H-z)] e^{-jkR}}{R} dz \right]$$

In amplitude term $R \simeq r$, whereas in phase term $R \simeq r - z\cos\theta$ for far field.

$$\therefore A_z = \frac{\mu I_0}{4\pi r} \left[\int_{-H}^{0} \sin[k(H+z)] \cdot e^{-jk(r - z\cos\theta)} dz + \int_{0}^{H} \sin[k(H-z)] e^{-jk(r - z\cos\theta)} dz \right]$$

~~A_z~~ On simplifying the eq,

$$A_z = \frac{\mu I_0 e^{-jkr}}{2\pi r k} \frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin^2\theta}$$

& $A_x = 0$, $A_y = 0$ (from figure)

Now converting the vector potential from Cartesian co-ordinate system into spherical co-ordinate system gives,

$$A_r = A_z \cos\theta = \frac{\mu I_0 e^{-jkr}}{2\pi r \cdot k} \frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin^2\theta} \cos\theta$$

$$A_\theta = -A_z \sin\theta = \frac{\mu I_0 e^{-jkr}}{2\pi r \cdot k} \frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}$$

Magnetic field strength:-

$$H = \frac{1}{\mu}\left[\nabla \times A\right]$$

Because of symmetry, (No variations in ϕ)

$$H_r = H_\theta = 0.$$

$$H_\phi = \frac{1}{\mu}\left[\nabla \times A\right]_\phi$$

$$= \frac{1}{\mu r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial \theta}(A_r)\right]$$

$$H_\phi = \frac{I_0}{2\pi k}\,\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\,\frac{e^{-jkr}}{r}\left[\frac{1}{r} + jk\right]$$

$$\therefore H_r = 0$$

$$H_\theta = 0$$

$$H_\phi = \frac{I_0}{2\pi k}\,\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\,\frac{e^{-jkr}}{r}\left[\frac{1}{r} + jk\right]$$

The magnetic field Component (H_ϕ) for which $kr \gg 1$ is,

$$H_\phi = \frac{jI_0 e^{-jkr}}{2\pi r}\,\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}.$$

Electric field strength:-

$$E = \frac{1}{j\omega\epsilon}\left[\nabla \times H\right]$$

$$E_r = \frac{1}{j\omega\epsilon}\left[\nabla \times H\right]_r$$

$$= 0.$$

$$E_\theta = \eta H_\phi.$$

$$= \frac{\eta I_0 e^{-jkr}}{2\pi r k}\,\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\left[\frac{1}{r} + jk\right]$$

$$\therefore E_r = 0$$

The electric field component (E_θ) for which $kr \gg 1$ is,

$$E_\theta \approx \frac{j\eta I_0 e^{-jkr}}{2\pi r} \cdot \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$$

Average poynting vector (P):-

$$P = \frac{1}{2} \mathrm{Re}\left[E \times H^*\right]$$

$$= \frac{1}{2} E_\theta H_\phi^*$$

$$= \frac{1}{2}\left[\frac{j\eta I_0 e^{-jkr}}{2\pi r} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right] \times \left[\frac{jI_0 e^{-jkr}}{2\pi r} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^*$$

$$= \frac{1}{2} \frac{\eta I_0^2}{(2\pi r)^2}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2$$

$$= \frac{\eta}{2} \frac{I_0^2}{4\pi^2 r^2} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

Radiation Intensity (U):-

$$U = r^2 P_{avg}$$

$$U = r^2 \frac{\eta}{2} \frac{I_0^2}{4\pi^2 r^2} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

$$= \frac{\eta}{2} \frac{I_0^2}{4\pi^2} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

$$\therefore U_{max} = \frac{\eta}{2} \frac{I_0^2}{4\pi^2}$$

Power radiated (P_{rad}):-

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U \sin\theta \, d\theta \, d\phi.$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{\eta}{2} \frac{I_0^2}{4\pi^2} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} \sin\theta \, d\theta \, d\phi$$

$$= \frac{\eta}{2} \frac{I_0^2}{4\pi^2} (2\pi) \int_0^{\pi} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta$$

$$= \frac{\eta}{2} \frac{I_0^2}{4\pi^2} (2\pi) [1.2175]$$

$$= \frac{\eta I_0^2}{4\pi} (1.2175)$$

Directivity (D):-

$$D = \frac{4\pi \cdot U_{max}}{W_{rad}}$$

$$= \frac{4\pi \left[\frac{\eta}{2} \frac{I_0^2}{4\pi^2}\right]}{\frac{\eta I_0^2}{4\pi}(1.2175)} = \frac{2}{1.2175} = 1.643.$$

$$D(dB) = 10\log_{10} D = 10\log_{10}(1.643) = 2.156 dB.$$

Maximum Effective aperture (A_{em}):-

$$A_{em} = \frac{\lambda^2}{4\pi} D.$$

$$= \frac{\lambda^2}{4\pi}(1.643)$$

$$A_{em} = 0.13 \lambda^2$$

Radiation Resistance (R_r):-

$$P_{rad} = I^2 R_r = \frac{1}{2} I_0^2 R_r$$

$$\frac{\eta I_0^2}{4\pi}(1.2175) = \frac{1}{2} I_0^2 R_r$$

Effective length (l_e):-

$$l_e = \frac{2\sqrt{A_{em} R_r}}{\sqrt{Z}}$$

$$Z = \eta = 120\pi.$$

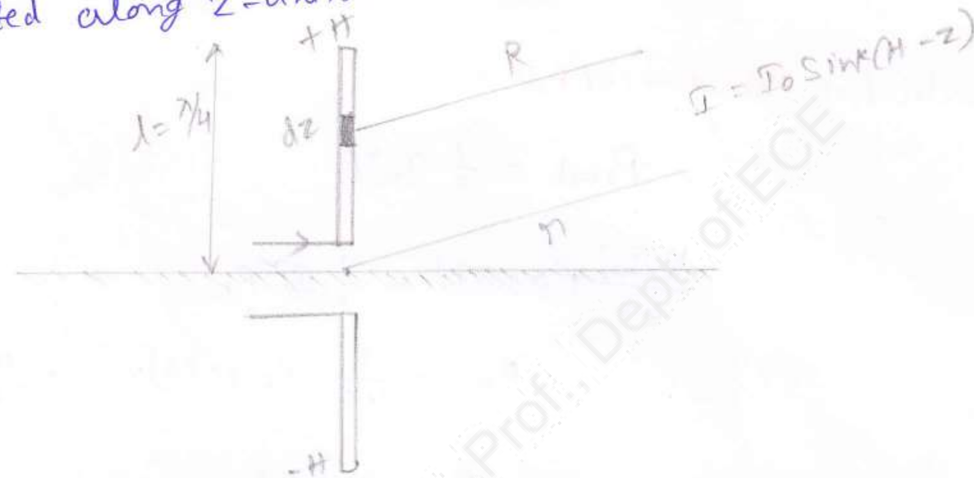
$$l_e = \frac{2\sqrt{(0.13\lambda^2)(73.05)}}{\sqrt{120\pi}}$$

$$= \frac{2(3.08)\lambda}{19.41}$$

$$l_e = 0.317\lambda.$$

Quarter wave Monopole:-

A linear wire antenna whose length is $\lambda/4$ and grounded is called as "Quarter wave Monopole". Grounded Quarter wave monopole produces the radiation fields in all directions. That is it produces radiation through hemisphere only. Here current is assumed to be sinusoidal & wire is oriented along Z-axis.



→ A, H, E, P and U are same as $\lambda/2$ dipole.

Radiated power:-

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi/2} U \cdot \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi}\int_0^{\pi/2} \frac{1}{2} \cdot \frac{\eta I_0^2}{4\pi^2} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} \sin\theta \, d\theta \, d\phi$$

$$= \frac{1}{2} \frac{\eta I_0^2}{4\pi^2}(2\pi) \int_0^{\pi/2} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta$$

$$= \frac{1}{2} \frac{\eta I_0^2}{4\pi^2}(2\pi)[0.6087] = \frac{\eta I_0^2}{4\pi}(0.6087).$$

Directivity (D):-

$$D = \frac{4\pi U_{max}}{P_{rad}}$$

$$U_{max} = \frac{\eta}{2}\frac{I_0^2}{4\pi^2}$$

$$\therefore D = \frac{4\pi\left[\frac{\eta}{2}\frac{I_0^2}{4\pi^2}\right]}{\dots} = \frac{2}{0.6087} = 3.28$$

$$D(dB) = 10\log_{10} D$$

$$= 10\log_{10}(3.28)$$

$$= 5.16\ dB.$$

Maximum Effective Aperture (A_{em}):-

$$A_{em} = \frac{\lambda^2}{4\pi} \cdot D$$

$$= \frac{\lambda^2}{4\pi}(3.28) = 0.26\lambda^2.$$

Radiation Resistance (R_r):-

$$P_{rad} = \frac{1}{2} I_0^2 R_r.$$

$$\frac{\eta I_0^2}{4\pi}(0.6087) = \frac{1}{2} I_0^2 R_r$$

$$R_r = \frac{\eta}{4\pi}(0.6087)\times 2 = \frac{120\pi}{2\pi}(0.6087)$$

$$\therefore R_r = 36.5\Omega.$$

Effective length (l_e):-

$$l_e = \frac{2\sqrt{A_{em} \cdot R_r}}{\sqrt{Z}}$$

$$= \frac{2\sqrt{(0.26\lambda^2)(36.5)}}{\sqrt{120\pi}}$$

$$= 0.317\lambda.$$

VHF, UHF & MICROWAVE Antennas - I

→ The antennas which are used (or) operated in very high frequency range (30 to 300MHZ) & ultra high frequency range (300 to 3000MHZ) respectively are known as VHF antennas & UHF antennas.

→ The corresponding wavelengths of the above antennas a 10 to 1 metre & 1 to 0.1 metre.

→ Antennas operating in microwave frequency range (generall above 2000MHZ) are called as "Microwave Antennas".

→ VHF & UHF antennas are used in land mobile Commun in the Coastal areas, public safety, public Communication Industry.

→ They also used for vehicle, aircraft & ship uses. Usu for in these mobile services is "whip antenna" mounted vertically over the vehicle. The body of the vehicle acts as ground.

→ Typical antennas in VHF & UHF bands are:

i) Yagi-Uda antenna,
ii) Folded Dipole antenna.
iii) Ground Plane Corner Reflector antenna.

→ Space wave propagation is used for effective propagation of VHF & UHF waves. For effective propagation, antennas have mount on the top of vertical Mast at greater heights.

Arrays with Parasitic Elements:-

→ The element supplied power directly from source (or Tx'ter) usually through Transmission line is called as "Driven element".

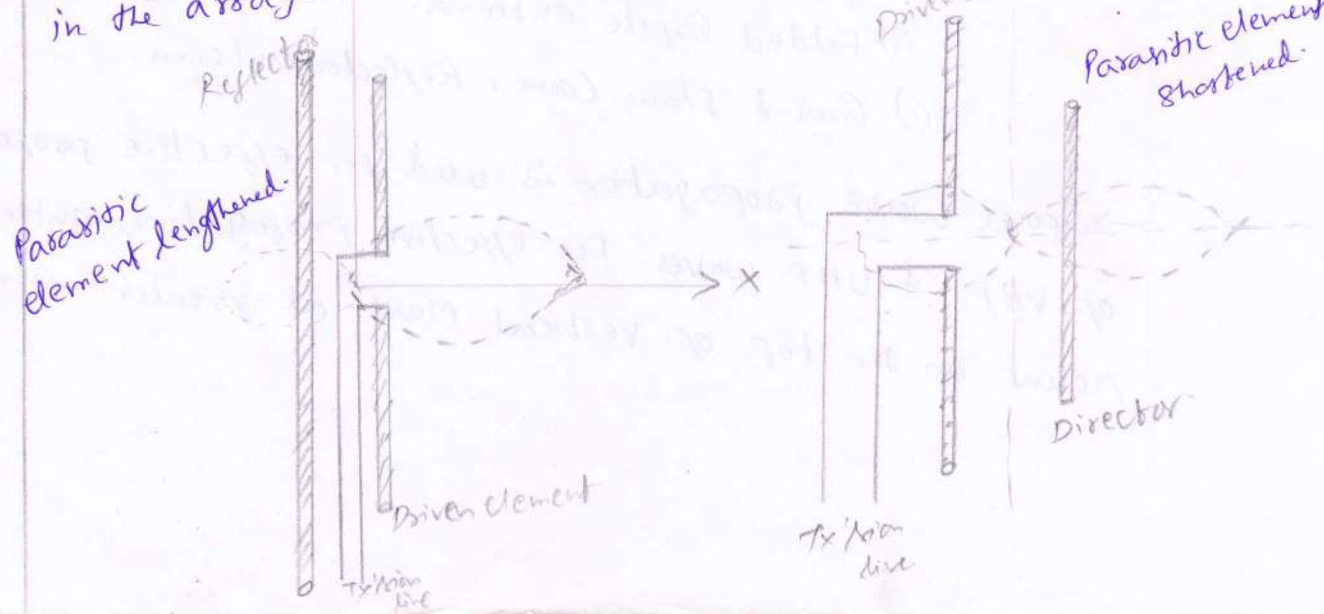
→ But a parasitic element is not fed directly instead parasitic element derives power by radiation from near by driven element.

→ In other words, parasitic element obtains power without any other source included solely through ~~...~~ electromagnetic coupling with a driven element because of its proximity to that driven element.
↳ The property of being close together

→ The simplest case of a parasitic array is one driven element & one parasitic element & this may be considered as an "Two element Array".

→ "Multi-element Arrays" having number of parasitic elements are called "Parasitic Arrays" whether the driven element is one or more.

→ Hence, in parasitic Arrays there is one or more parasitic elements & atleast one driven element to introduce power in the array.



→ A parasitic Array with linear half-wave dipole as elemen
is normally called as "Yagi-Uda" (or) simply "Yagi" Antenn
after the name of Inventors S. Uda & H. Yagi.

→ The Amplitude & phase of the Current induced in a
Parasitic element depends on its tuning & the spacing b/n
Parasitic element & driven element to which it is Coupled.

→ Variations in the distance between driven element & paras
element Changes the relative phases & it helps in making
radiation pattern unidirectional.

→ A Parasitic element lengthened by 5% w.r.t driven elemen
acts as "Reflector" & Shortened by 5% acts as director

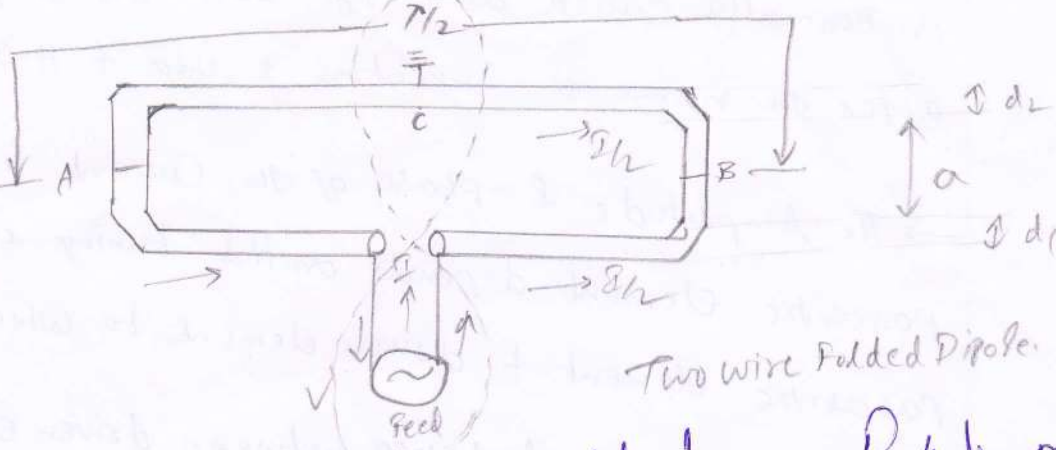
→ A Reflector makes the Radiation pattern maximum in the direc
from parasitic element towards driven element.

→ A Director makes the Radiation pattern maximum in the dire
from driven elements towards parasitic element.

→ These parasitic elements are used specially at higher
frequencies b/n 100 - 1000 MHZ.

→ A properly designed parasitic array can provide a larger
Front-to-Back ratio (FBR).

Folded Dipole Antenna:-



Two wire Folded Dipole.

Two wire F.D with (i) Current distribution (ii) Radiation pattern

A, B – Minimum Current (or) Maximum Voltage points.

C – Maximum current (or) Minimum voltage point.

8 – Radiation pattern.

To provide the good directional characteristics & at same time to provide good Matching Characteristics, variation of a single Conventional half wave dipole must be used.

→ one simple geometry that can acheive this is a folded dipole as shown in fig. in which two half wave dipoles - one Continuous & other split at the centre - have been folded & joined together in parallel at the ends.

→ The split ~~mod~~ dipole is fed at the centre by a balanced Transmission line. The two dipoles, therefore, have the same voltage at their ends.

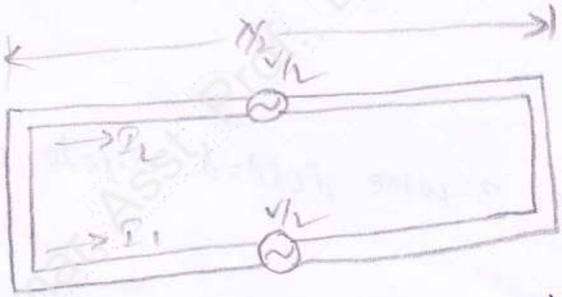
→ The Radiation pattern of a folded dipole & a Conventional half wave dipole is same but the i/p impedance of the Folded dipole is higher.

It differs from Conventional half wave dipole in two aspec i.e., directivity & broadness in bandwidth.

→ If the radii of the two Conductors are equal, then equal Currents flow in both the conductors in the same direction i.e., Currents are equal in magnitude & phase in the two dipoles.

Input impedance:-

The equation for i/p impedance (or) Terminal impeda (or) Radiation resistance of a folded dipole can be derived from equivalent diagram shown in fig. where 'V' be the applied Emf that is equally divided in each dipole



Equivalent diagram of two wire folded λ/2 wave dipol

→ From Nodal Analysis:-

$$\frac{V}{2} = Z_{11} I_1 + Z_{12} I_2 .$$

I_1, I_2 are the Currents flowing at the terminals of dip & dipole 2.

Z_{11} is self Impedance of dipole 1.

Z_{12} is mutual impedance b/n dipole1 & dipole2.

As, $I_1 = I_2$,

$$\frac{V}{2} = I_1 (Z_{11} + Z_{12})$$

As two poles are very close to each other, then,

$Z_{11} \simeq Z_{12}$

$\frac{V}{2} = I_1 (Z_{11} + Z_{11})$

Halfwavedipole

$Z_{in} = 73 + j42$

$\frac{V}{2} = I_1 (2 Z_{11})$

$\frac{V}{I_1} = 2^2 \cdot Z_{11}$

$\therefore Z = \frac{V}{I_1} = 2^2 Z_{11} = 2^2 (73\,\Omega) = 292\,\Omega.$

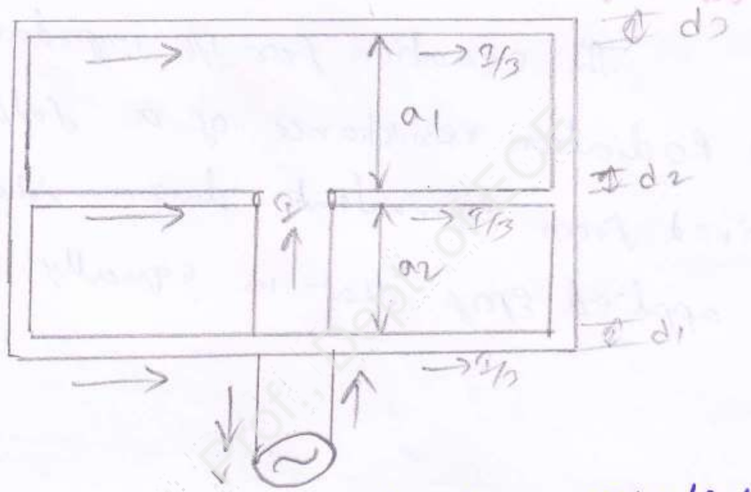
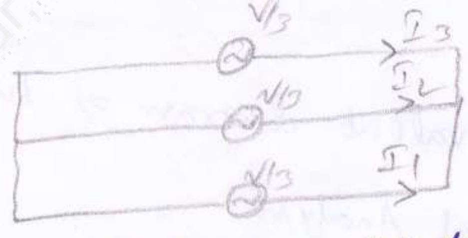


fig: 3-wire folded dipole (or) Tripole when $d_1 = d_2 = d$, $a_1 = a_2$ & $a \ll \lambda$



Equivalent of 3-wire folded dipole.

Similarly, for a folded dipole of 3 wires (Called as Tripole) it can be proved that,

$\frac{V}{3} = I_1 (Z_{11} + Z_{12} + Z_{13})$

As $Z_{11} = Z_{12} = Z_{13}$

$\frac{V}{3} = I_1 (3 Z_{11})$

$Z = \frac{V}{I_1} = 3^2 Z_{11} = 3^2 \cdot (73\,\Omega) = 657\,\Omega$

$\therefore$ For a folded dipole of 'n' $\lambda/2$ dipoles,

$Z = n^2 (73\,\Omega).$

i.e., The terminal impedance (or) input impedance (or) Radiation resistance of a folded dipole antenna is equal to the square of the number of $\lambda/2$ dipoles compositing the antenna times the impedance at the terminals of a conventional half wave dipole (i.e., 73Ω).

Instead of changing the number of dipoles, impedance transformation is possible by making radii (or diameter) of two dipoles unequal. So that i/p impedance of a two wire folded dipole is,

$$Z = Z_{11}\left[1+\frac{d_2}{d_1}\right]^2$$

$$= 73\left[1+\frac{r_2}{r_1}\right]^2.$$

If $r_2 = 2r_1$, then,

$$Z = 73\left[1+\frac{2r_1}{r_1}\right]^2 = 73\times3^2 = 657\Omega.$$

The impedance transformation not only depends upon the relative radii of the conductors but also on the relative spacing.

A/c'ing to Uda & Mushiake, i/p impedance is,

$$Z = Z_{11}\left[1+\frac{\log\left(\frac{a}{r_1}\right)}{\log\left(\frac{a}{r_2}\right)}\right]^2 = Z_{11}\cdot Z_{ratio}.$$

Z_{ratio} is known as Impedance Transformation ratio.

Uses of Folded dipole:-

→ In Conjunction with parasitic elements, folded dipole is used in wide band operation such as Television.

→ In yagi antenna, the driven element is Folded dipole & remaining are reflector & director.

Advantages:-

i) High I/P impedance.

ii) wide band in frequency.

iii) Acts as an built in reactance Compensation network.

Yagi-UDA Antenna:-

Yagi-uda antennas are having high gain & are kn after the professors S.Uda & H.Yagi. It consists of a driven element, a reflector & one or more directors.

→ That is the Yagi-uda antenna is an array of

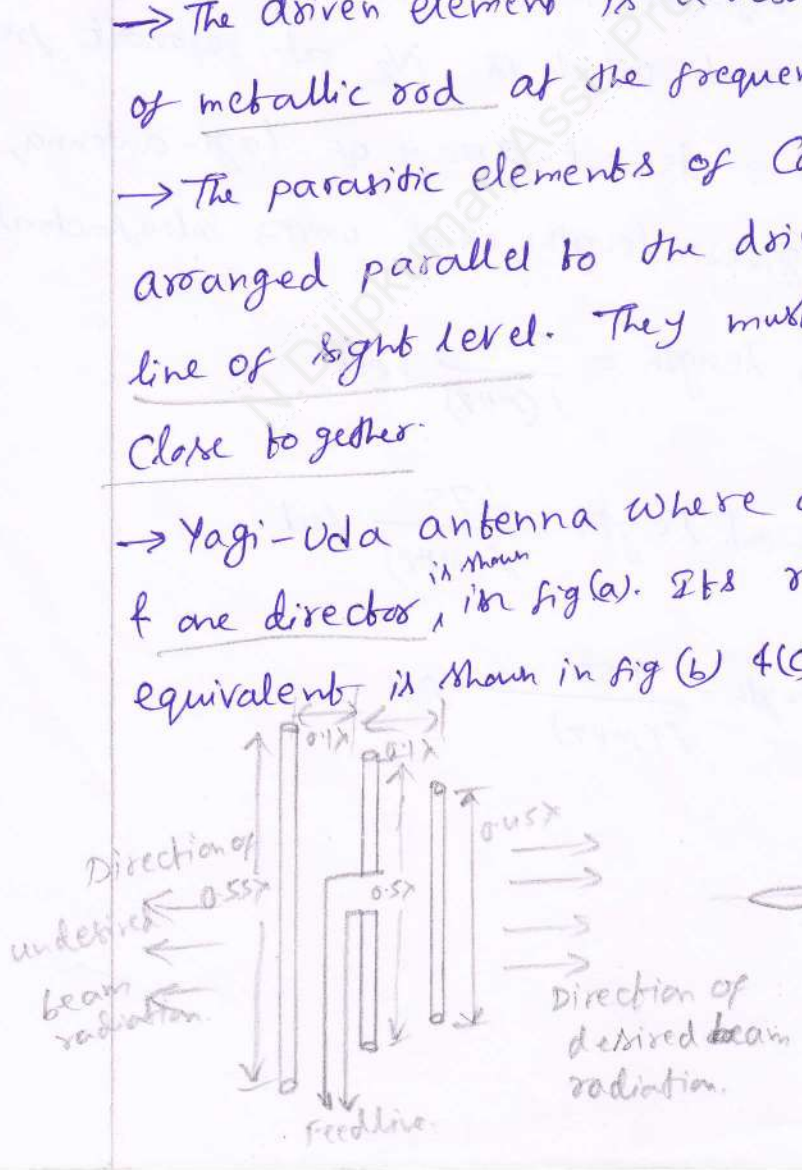
i) Driven element (or) active element where the power from the Transmitter is fed or which feeds the received power to the receiver.

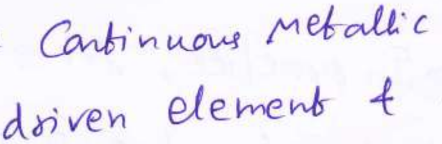
ii) One or more parasitic elements (or) Passive elements which are not Connected directly to the Transmission li but electrically Coupled.

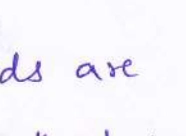
→ The driven element is a resonant half-wave dipole us of metallic rod at the frequency of operation.

→ The parasitic elements of Continuous Metallic rods are arranged parallel to the driven element & at the sam line of sight level. They must be arranged Collinearly & Close together.

→ Yagi-Uda antenna where driven element with one re & one director, is shown in fig (a). Its radiation pattern & optical equivalent is shown in fig (b) & (c).







The parasitic elements receive their excitation from the voltages induced in them by the current flow in the driven element.

The phase & currents flowing due to the induced voltage depend on the spacing between the elements & upon the reactance of the elements (i.e, length).

The reactance may be varied by dimensioning the length of the parasitic element.

The spacing between driven & parasitic elements that are usually used in practice, are of the order of $\lambda/10$ i.e., 0.10λ to

The parasitic element infront of driven element is known director & its number may be more than one, where as the element in back of it is known as "reflector".

Generally, the reflector is 5% more & director is 5% l than the driven element which is $\lambda/2$ at resonant frequen

→ In practice, for 3-element array of Yagi-antenna, the following formulae gives lengths which works satisfactorily.

$$\text{Reflector length} = \frac{500}{f(MHz)} \text{ feet}$$

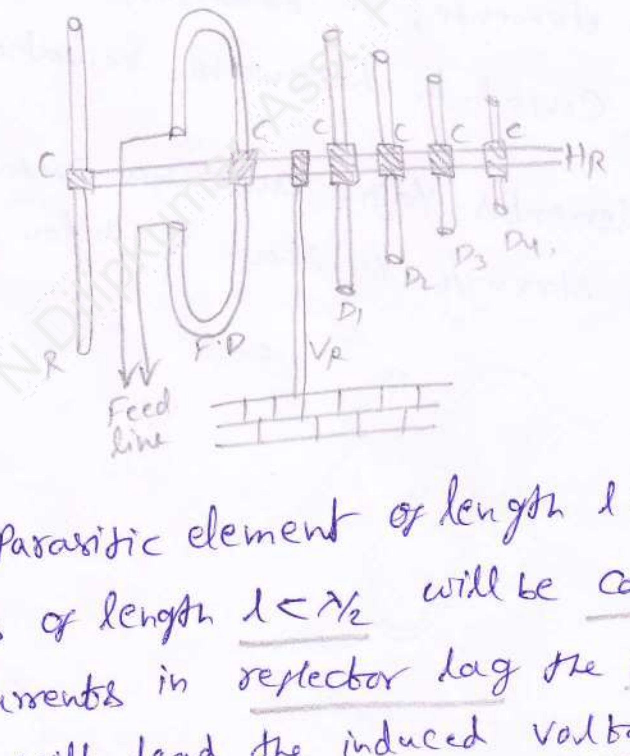
$$\text{Driven element length} = \frac{475}{f(MHz)} \text{ feet}$$

$$\text{Director length} = \frac{455}{f(MHz)} \text{ feet}$$

The above equations provide the average length of Yagi-ant determined experimentally for elements of length/diameter ratio of 200 to 400 & spacing/seperation from 0.10λ to 0.20λ.

When parasitic elements are used in Conjunction with driven element Causes the dipole impedance to fall well bel 73Ω. It may be as low as 25Ω & hence it become necessary to use either shunt feed or folded dipole. So the input impedance could be raised to a suitable value to ma the feed Cable as shown in figure below.

6-elements Yagi antenna with folded dipole.



R = Reflector.
FD = Folded dipole.
D1, D2, D3, D4 = Directors.
VR = vertical rod to support horizontal ro
HR = Horizontal rod to support
C = clamps.

A Parasitic element of length $l \geq \lambda/2$ will be inductive wh elements of length $l < \lambda/2$ will be capacitive. Hence the phase of the Currents in reflector lag the induced Voltage where as i directer will lead the induced voltage.

Properly spaced dipoles shorter than λ/2 acts as direct & add the fields of driven element in the direction away from driven element. If more than one directors are employed, the each director will excite the next.

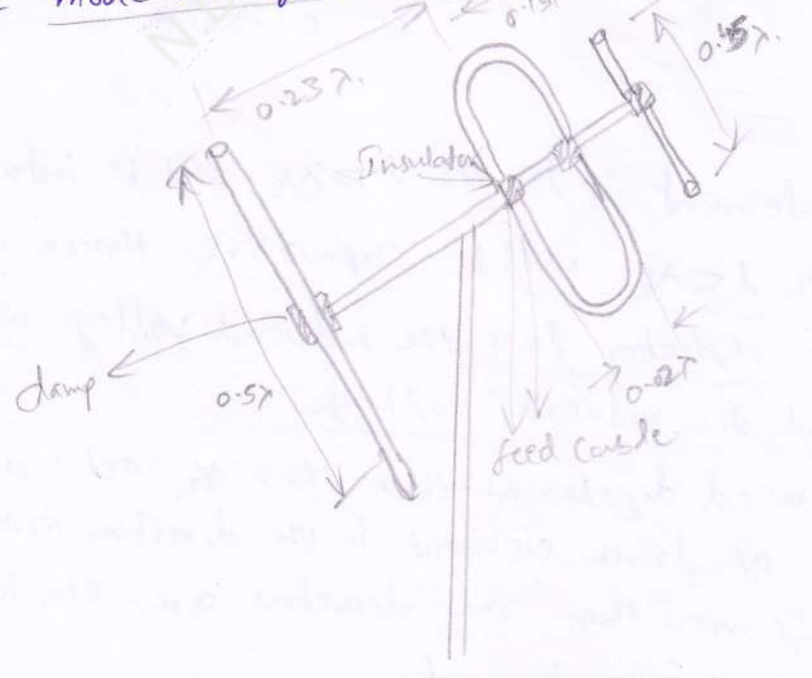
On the other hand an element of length equal or greater than λ/2 acts as reflector & add up the fields of driven element in the direction from reflector towards driven element, if properly spaced.

To achieve additional gain, more directors are used in the beam direction. The distance b/n the two elements may range from 0.1λ to 0.3λ, close spacing of elements are used in parasitic arrays to get a good excitation.

Now the driven element radiates from front to rear (i.e., from reflector to directors) part of this radiation ~~instead~~ induces current in the parasitic elements which in turn re-radiate virtually all the radiation.

By suitably dimensioning the lengths of parasitic elements & spacing b/n two elements, the radiated energy is added up in front & tend to cancel the backward radiation.

→ A typical 3 elements Yagi- antenna suitable for TV reception of moderate field strength is shown in below fig.



Further addition of directors can be done at intervals of 0.15λ i.e., to increase the gain even upto 12dB as is required in for fringe area reception.

In a 11 elements Yagi antenna the lengths of D_2, D_3, D_4, D_5, D_6, D_7, D_8, D_9 are respectively 0.427λ, 0.40λ, 0.38λ, 0.36λ, 0.32λ, 0.304λ, & 0.29λ.

General Characteristics:-

→ If three elements array (i.e., R, Dr & D) is used, then such of Yagi-Uda antenna is generally referred to as "Beam anten

→ It has unidirectional beam of moderate directivity with light weight, low cost & simplicity in feed system design.

→ If the spacing b/n the elements is of the order of 0.1λ 0.15λ, a frequency bandwidth of the order of 2% is obta which is sufficient ~~of the order of 2%~~ for TV Reception.

→ It provides gain of the order of 8dB (or) FBR of about 20d

→ It is also known as "Super directive" (or) Super gain ant due to its high gain & bandwidth per unit area of the arra

→ If greater directivity is desired, further elements may be used, Arrays up to 40 can be constructed.

→ It is a fixed frequency device i.e., frequency sensitive & a bandwidth of about 3% is obtainable which is sufficient Television Reception.

Voltage & Current Relations in Parasitic Elements:-

One or more passive elements coupled magnetically to driven element is known as "parasitic Antenna".

The presence of parasitic element effects the directional pattern.

The effect on the directional pattern produced depends upon the magnitude & phase of the induced current in the parasitic element i.e., on the spacing of the antenna & tuning of the parasitic antenna.

The quantitive relations b/n voltages & currents of an antenna system involving parasitic antennas can be given by considering the general equation.

$$V_1 = I_1 Z_{11} + I_2 Z_{12} + \cdots + I_n Z_{1n}$$
$$V_2 = I_1 Z_{21} + I_2 Z_{22} + \cdots + I_n Z_{2n}$$
$$V_3 = I_1 Z_{31} + I_3 Z_{32} + \cdots + I_n Z_{3n}$$
$$\vdots$$
$$V_n = I_1 Z_{n1} + I_2 Z_{n2} + I_3 Z_{n3} + \cdots + I_n Z_{nn}$$

where,

$V_1, V_2 \cdots V_n$ = Voltage applied to antenna no. $1, 2, \cdots n$.

$I_1, I_2 \cdots I_n$ = Current flowing in antenna no. $1, 2, \cdots n$.

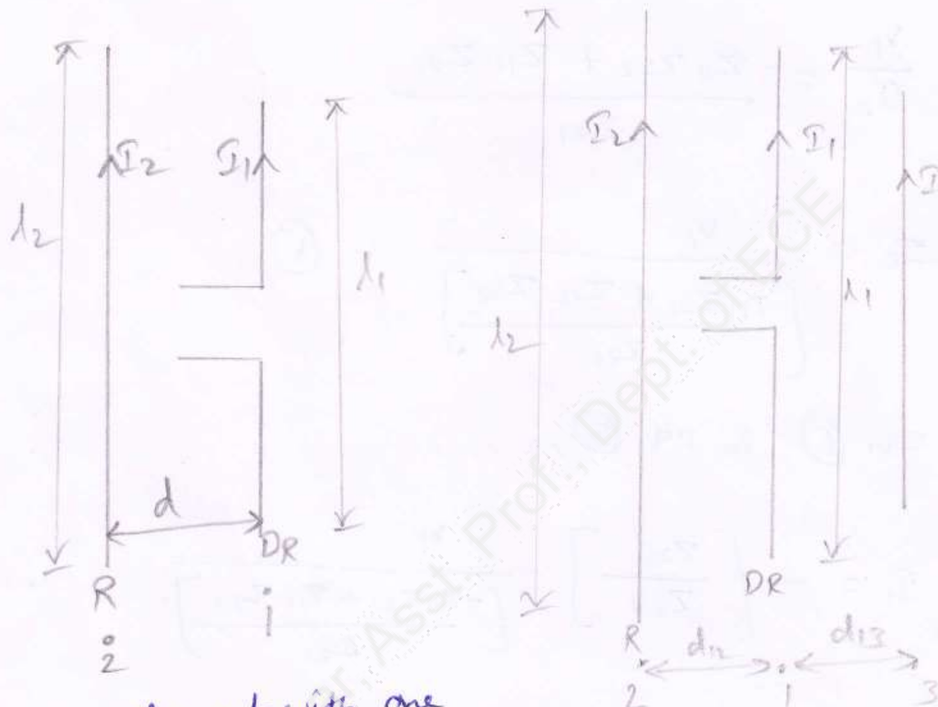
$Z_{11}, Z_{22} \cdots Z_{nn}$ = Self impedances of antenna no. $1, 2, \cdots n$.

$Z_{12}, Z_{21}, Z_{13} \cdots Z_{31}$ = Mutual impedances b/n antennas i.e., Z_{1n} means mutual impedance b/n antenna no 1 & n.

→ If individual antennas are not excited then the corresponding applied voltages V_1, V_2, V_3 etc., are zero.

Thus in an antenna system, involving parasitic antenna's, the voltages are zero in case of Transmitting while in receiving case these applied voltage are the voltages induced in each parasitic antenna by the electromagnetic waves.

→ Now considering the simplest case with one driven element & one parasitic antenna as shown in the figure below.



Driven element with one parasitic

Driven element with two parasitic

$$V_1 = I_1 Z_{11} + I_2 Z_{12} \longrightarrow (1)$$

$$V_2 = I_1 Z_{21} + I_2 Z_{22} \longrightarrow (2)$$

→ I_1 & I_2 are the currents flowing in D_R & R respectively.

→ V_1 be the voltage fed at D_R & $V_2 = 0$ as it is parasitic.

→ Z_{11} & Z_{22} are self impedances of D_R & R, Z_{12} & Z_{21} are mutual impedance b/n them.

$$Z_{12} = Z_{21}$$

$$V_1 = I_1 Z_{11} + I_2 Z_{12} \longrightarrow (3)$$

$$0 = I_1 Z_{21} + I_2 Z_{22} \longrightarrow (4)$$

from eq. (4),

$$I_2 Z_{22} = -I_1 Z_{21}$$

$$\therefore I_1 = -\left(\frac{Z_{22}}{Z_{21}}\right) I_2 \rightarrow (5)$$

sub. eq. (5) in eq. (3),

$$V_1 = Z_{11}\left(\frac{-Z_{22}}{Z_{21}}\right) I_2 + Z_{12} I_2$$

$$\frac{V_1}{I_2} = \frac{-Z_{11}Z_{22} + Z_{12}Z_{21}}{Z_{21}}$$

$$I_2 = \frac{V_1}{\left[\frac{-Z_{11}Z_{22} + Z_{12}Z_{21}}{Z_{21}}\right]} \quad -(6)$$

Sub. eq. (6) in eq. (5),

$$I_1 = -\left[\frac{Z_{22}}{Z_{21}}\right] \frac{V_1}{\left[\frac{-Z_{11}Z_{22} + Z_{12}Z_{21}}{Z_{21}}\right]}$$

$$\therefore I_1 = \frac{V_1}{\left[Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22}}\right]} \longrightarrow (7)$$

Hence i/p impedances of Driven element (DR) & Parasitic elem (R) are given by,

$$Z_{DR} = \frac{V_1}{I_1} = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22}} \quad (\text{or}) \quad Z_{11} - \frac{Z_{12}^2}{Z_{22}} \quad -(8$$

$$Z_R = \frac{V_1}{I_2}$$

→ Eq. (8) indicates that i/p impedance of the Driven element decreases as the number of parasitic element increases. Hences the use of folded dipole avoids this problem.

→ Eq. (9) indicates that i/p impedance of the parasitic element depends on the self impedance of the driven element.

→ Electric field produced by the antenna can be obtained by ~~a~~ assuming a Constant Current in D_R & field pattern.

$$E_\theta = K\left(I_1 + I_2 / \beta d \cos\psi'\right)$$

where ψ is the phase difference between field at observation point.

Note:-

→ Length of the elements are as follows:

$l_R = 0.5\lambda$.

$l_{FD} = 0.475\lambda$.

$l_{D_1} = 0.45\lambda$.

$l_{D_2} = 0.427\lambda$.

$l_{D_3} = 0.40\lambda$.

$l_{D_4} = 0.38\lambda$.

$l_{D_5} = 0.36\lambda$.

$l_{D_6} = 0.32\lambda$.

$l_{D_7} = 0.304\lambda$.

$l_{D_8} = 0.29\lambda$.

→ Distance (or) Spacing between the elements are follows:

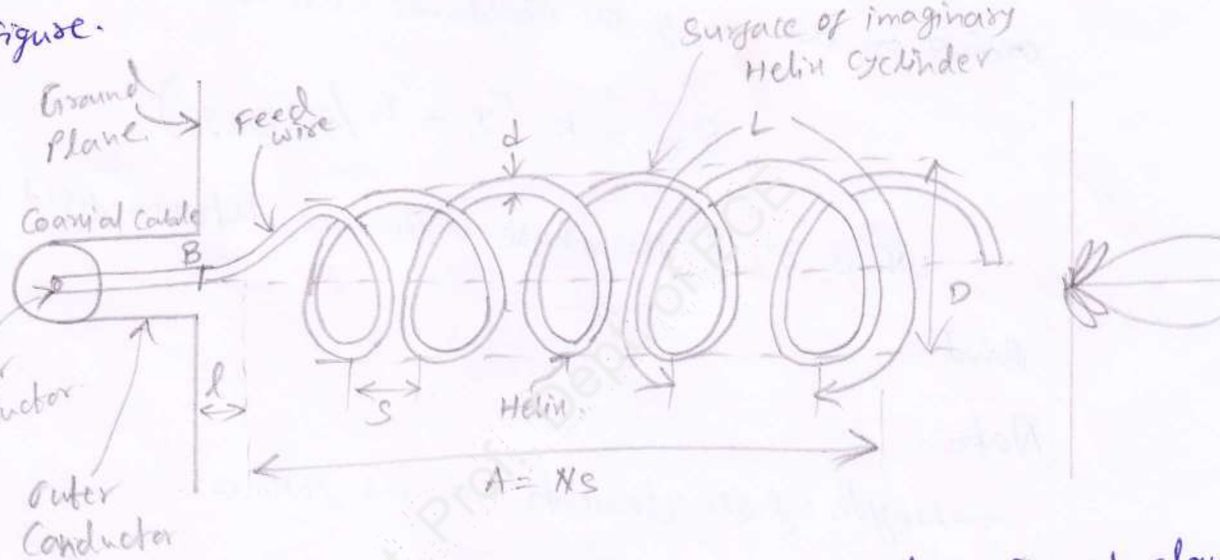
$$d_{R\&FD} = 0.23\lambda.$$

$$d_{FD\&D_1} = d_{D_1\&D_2} = d_{D_2\&D_3} \text{ - - - - } = 0.15\lambda.$$

Helical Antenna:-

Helical antenna is one of the most popular broadban[d] VHF & UHF antenna Providing circular polarization Characterist[ics]

It consists of a helix of thick Copper wire or tubing woun[d] in the shape of a Screw thread & used as an antenna in Conju[nction] with a flat metal plate called as "Ground plane" as shown in the figure.



Helical antenna is fed between one end and a ground plan[e]. The ground plane is simply made of sheet or of screen or of radial & Concentric Conductors.

→ The helix is fed by a Coaxial Cable, generally the one end [of] the helix is Connected to the Centre Conductor of the Cable & Outer Conductor is Connected to the ground plane.

→ The parameters on which the mode of radiation depend are diameter of Helix (D) & turn spacing (S).

The dimensions of the helix are as shown below:

C = Circumference of helix (πD)

α = Pitch angle = $\tan^{-1}\left(\frac{S}{\pi D}\right)$

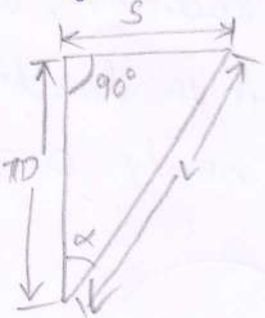
d = diameter of helix Conductor.

A = Axial length = NS.

N = Number of turns.

L = Length of one turn.

l = spacing of helix from ground plane.



For N turn of helix, the total length of the antenna is equ to NS & circumference πD. If one turn of helix is unrolled on plane surface, the circumference (πD), spacing S, turn length L & pitch angle α are related by the Triangle as shown above.

$$\text{i.e., } L = \sqrt{S^2 + C^2} = \sqrt{S^2 + (\pi D)^2}.$$

The pitch angle is the angle b/n a line tangent to the helix wire & the plane normal to the helix axis. Pitch angle is an important parameter of the helix & can be calculated from the Triangle as,

$$\tan\alpha = \frac{S}{C} = \frac{S}{\pi D}.$$

$$\therefore \alpha = \tan^{-1}\left(\frac{S}{\pi D}\right)$$

The properties of the helical antenna can be described inter of all the above geometric parameters. The different radiation characteristics are obtained by changing these parameters in relation to wavelength (λ).

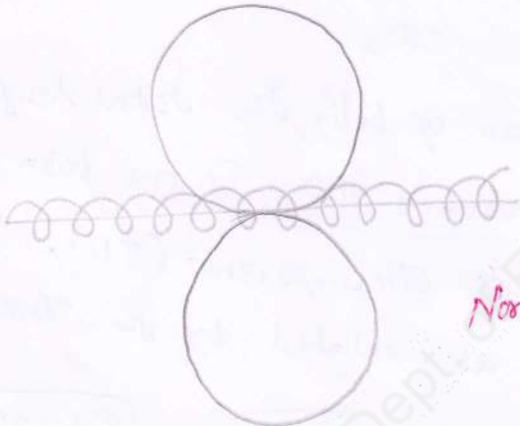
The variation of the feed wire geometry affects the input impedance of the antenna. A helical antenna may radiate in ma modes but prominent modes of radiations are two i.e.,

⟹ Normal mode (or) Broad side mode (or) Perpendicular mode of rad

⟹ Axial mode (or) End-fire mode (or) Beam mode of Radiation

Normal mode of radiation:-

→ In normal mode of radiation, the radiation field is maximu in the broadside i.e., in the direction normal to the helix axi and is circularly or nearly circularly polarised waves.

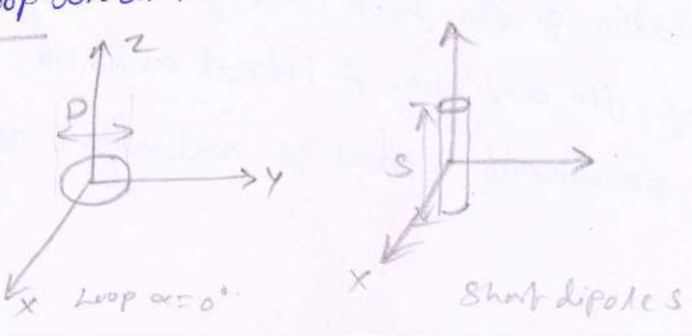


Normal mode of radiation.
Circular polariz

This mode of radiation is obtained if the dimensions of t helix is small when compared with wavelength i.e., $NL << \lambda$. However the bandwidth of such a small helix is very narrow & radiation efficiency is low.

The bandwidth & radiation efficiency can be increased increasing the size of the helix to have the current in phase along the helix axis. Some type of phase shifters at interval are required which put practical limitations.

The Radiation Pattern is a combination of the equivalent radiation from a short dipole positioned on the same helix axis & small loop which is also coaxial with the helix axis.



→ The geometry of the helix reduces to a loop of diameter D when the pitch angle $\alpha = 0$ & to a linear wire of length when pitch angle $\alpha = 90°$. i.e., limiting geometries of the helix are a loop & a dipole.

→ Thus in a helix of fixed diameter,

- if $S \to 0$, helix collapses to a loop.
- if $D \to 0$, for S = constant, helix straightens to a short d

→ In the normal mode the helix consists of N small loops & N short dipoles connected together in the series & hence far field radiation can be described in terms of E_θ & E_ϕ components of the dipole & loop respectively.

→ The radiation pattern of the above two conditions are same but the polarizations are at right angles & the phase angle at any point in space are at 90° apart.

→ Hence the resultant field is either circularly polarized or elliptically polarized depending upon the field strength ratio or amplitudes of the two components which inturn depends on the pitch angle α.

→ If α is small, loop type of radiation pre-dominates & when α becomes very large the dipole polarization pre-dominates. In limiting conditions ~~are~~ polarizations are linear i.e., loop polarization & dipole polarization. For in b/n value of α the polarization is elliptical & ~~the~~ the polarization is circular at a particular value of α which has been calculated by H. A. wheeler

→ A helix may be Considered of having a no. of small loops & short dipoles connected in series in which loop diameter is same as helix diameter and helix spacing S is same as dipole length.

→ Therefore, far field of small loop is given by,

$$E_\phi = \frac{120\pi^2 [I] \sin\theta}{r} \cdot \frac{A}{\lambda^2}.$$

where I = Retarded current
r = distance.
A = Area of loop = $\frac{\pi D^2}{4}$.

Also, far field of a short dipole is given by,

$$E_\theta = \frac{j 60\pi [I] \sin\theta}{r} \cdot \frac{S}{\lambda}.$$

$S = L$ = length of dipole.

j indicates $90°$ phase difference b/n E_θ & E_ϕ.

→ The ratio of magnitudes of these equations provides Axial ratio (AR) of elliptical polarization.

$$A_R = \frac{|E_\theta|}{|E_\phi|} = \frac{\left|\frac{j 60\pi [I] \sin\theta \cdot S}{\lambda r}\right|}{\left|\frac{120\pi^2 [I] \sin\theta \cdot A}{r \cdot \lambda^2}\right|} = \left|\frac{S\lambda}{2\pi A}\right|$$

$$= \frac{2S\lambda}{\pi^2 D^2}$$

$$\left[\because A = \frac{\pi D^2}{4}\right]$$

$\therefore$ Axial ratio(AR) $= \frac{2S\lambda}{\pi^2 D^2}$.

When $AR \to 0$, Elliptical polarization becomes Linear horizontal polar

$AR \to \infty$, Elliptical polarization becomes Linear vertical polariz

$AR \to 1$, Elliptical polarization becomes Circular polarization.

→ Therefore, for Circular polarization,

$$AR = 1,$$

$$\frac{|E_\theta|}{|E_\phi|} = 1.$$

$$|E_\theta| = |E_\phi|$$

$$|2S\lambda| = |\pi^2 D^2|$$

$$S = \frac{\pi^2 D^2}{2\lambda}.$$

$$\boxed{S = \frac{C^2}{2\lambda}.}$$

Also, we have $\tan\alpha = \frac{S}{C} = \frac{S}{\pi D}$.

$$\alpha = \tan^{-1}\left(\frac{S}{\pi D}\right)$$

$$= \tan^{-1}\left(\frac{\pi^2 D^2}{2\lambda \cdot \pi D}\right)$$

$$\alpha = \tan^{-1}\left(\frac{\pi D}{2\lambda}\right) \quad (\text{or})$$

$$\alpha = \tan^{-1}\left(\frac{C}{2\lambda}\right)$$

This is the Condition for pitch angle to get circular pola

→ The Radiation Pattern is thus equal to the super-posing o the fields from the elemental radiators (loops & dipoles) whe planes of loops are parallel to each other & ⊥er to the axes of the dipoles. The axes of loop & dipole Coincides with the hel axis.

Axial Mode of Radiation:-

→ In axial mode of operation, the radiation field is maximum in the end-fire direction i.e., along the helix axis & the polarization is Circular (or) nearly Circular polarization.

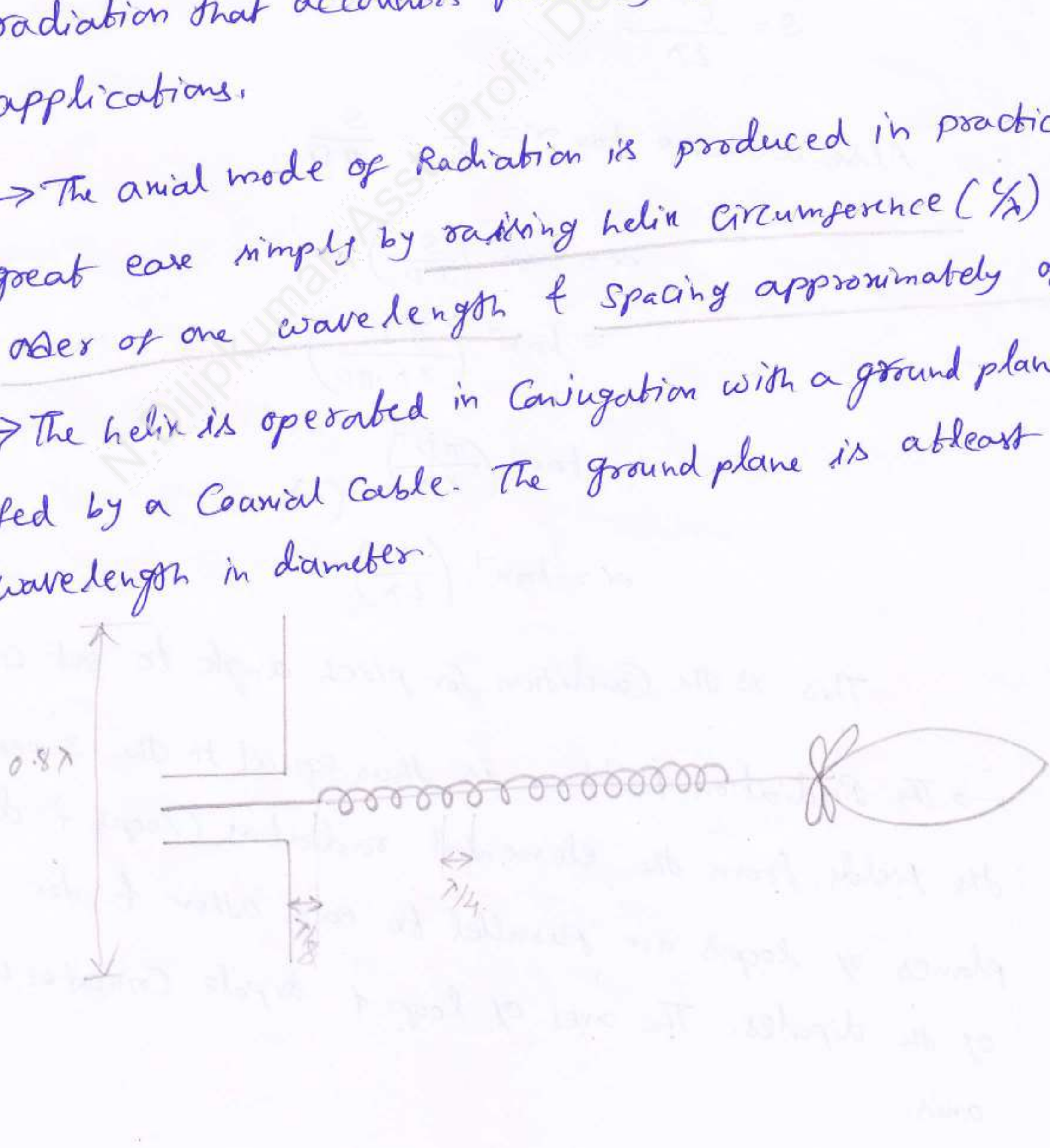
→ This mode occurs when the helix circumference (D) & Spacing (S) & appreciable of the order of one wave length (λ).

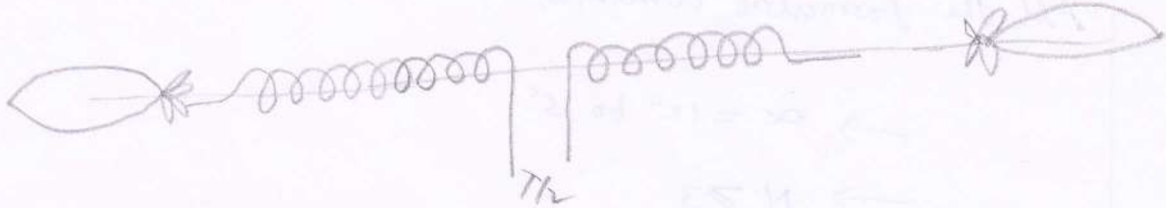
→ This mode is more intresting as it produces a broad and fairly directional beam in the axial direction with minor lobes at oblique angles.

→ It is this feature of the helical antenna in axial mode of radiation that accounts probably for most of the practical applications.

→ The axial mode of Radiation is produced in practice with great ease simply by raising helix circumference (C/λ) of the order of one wavelength & Spacing approximately of $\lambda/4$.

→ The helix is operated in Conjugation with a ground plane and is fed by a Coaxial Cable. The ground plane is atleast half wavelength in diameter





→ The pitch angle α varies from $12°$ to $18°$ & about $14°$ is optimum pitch angle.

→ The antenna gain & beam width depends upon the helix length.

→ In axial mode, the terminal impedance of helical antenna lies b/n 100Ω & 200Ω. The terminal impedance is given by,

$$R = \frac{140C}{\lambda}\ \Omega.$$

→ The Beamwidth b/n half power points is given by

$$HPBW = \frac{52}{C}\sqrt{\frac{\lambda^3}{NS}} \text{ degrees.}$$

→ The beam width b/n first Nulls is given by,

$$BWFN = \frac{115}{C}\sqrt{\frac{\lambda^3}{NS}} \text{ degrees.}$$

→ The max. directive gain (or) directivity is given by,

$$D = \frac{15NSC^2}{\lambda^3}.$$

→ Axial ratio is given by,

$$AR = 1 + \frac{1}{2N}.$$

→ Normalised field pattern is given by,

$$E = \sin\left(\frac{\pi}{2N}\right)\cos\theta \cdot \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}$$

where $\psi = 2\pi\left[\frac{S}{\lambda}(1-\cos\theta) + \frac{1}{2N}\right]$

All the formulae assumes,

$\rightarrow \alpha = 12^\circ$ to 15°.

$\rightarrow N \geq 3$

$\rightarrow NS \leq 10$

$\rightarrow C = \frac{3}{4}\lambda$ to $\frac{4}{3}\lambda$.

→ The helix may be wound with a right handed or left hand pitch & accordingly the wave is right handed or left handed circularly polarized.

→ A Receiving antenna meant of receiving right handed circular polarized waves cannot receive left handed circularly polarized waves.

Applications:-

1. Single or an array of helical antenna is used to receive or Transmit the VHF signals through Iono sphere.
2. For space Communication, it is used to transmit teleme data from moon to earth.
3. Because of Circular polarization, helical antenna is capabl of receiving signals of arbitary polarization.

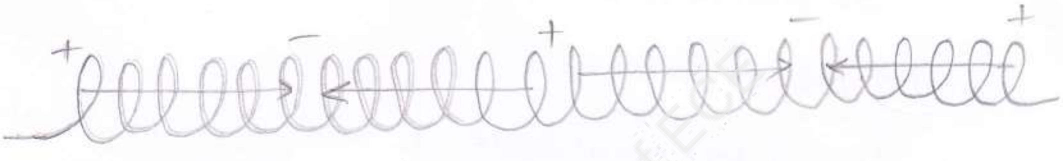
Helix modes:-

a) Transmission (T) mode.

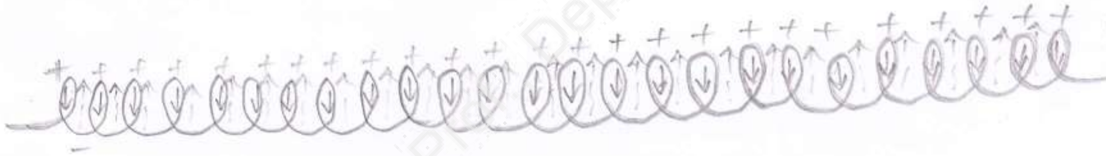
b) Radiation (R) mode.

Transmission (T) mode:-

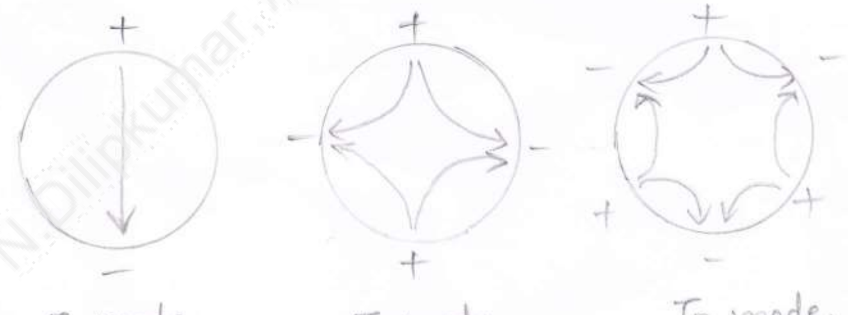
It describes the manner in which the electromagnetic wa... propagates along an infinite helix as though the helix constitutes a... infinite transmission line or waveguide.



T_0 mode.



T_1 mode.



T_1 mode	T_2 mode	T_3 mode.
180°	90°	60°

Seperation between + & - polarities along the circumference on each turn of helix mode T_m is higher then T_{m-1} where $m = 1, 2, 3 \dots$

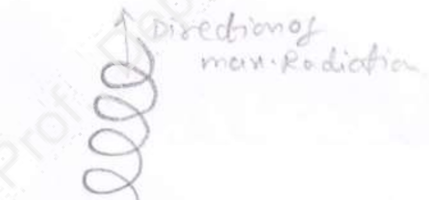
In view of the mode order, T_0 is the lowest mode wher
in the Charges are seperated by several turns. T_1 is a higher
mode than T_0 & the Charges are seperated by only one turn. Th
modes T_2, T_3 etc.., are still higher modes. In T_2, Charges cha
their polarity twice in one turn or are seperated by 90°, in T_3 by
& in T_m by π/m degrees where m is the order of the mode.

Radiation mode:-

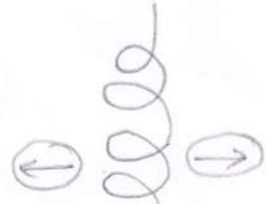
In describes the general form of the far field pattern of a
finite helical antenna.

1. Normal (or) omni mode of Radiation is denoted by R_0.
2. Axial (or) beam mode " " " R_1.

R_0 mode



R_1 mode.



$0 < \alpha < 90°$.
T_0 R_1 mode.

$\alpha < \alpha < 90°$
T_1 R_1 mode

Practical design Considerations for the Monofilar Axial-mode Helical Antenna:-

The monofilar axial-mode helical antenna is very non-critical & one of the easiest of all antennas to build.

The important parameters are:

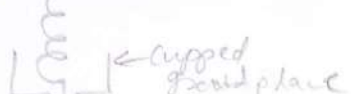
1. Beam width
2. Gain
3. Impedance
4. Axial Ratio

Gain & Beam width, are interdependent $[G \propto (1/HPBW^2)]$, & other parameters are all functions of number of turns, the turn spacing or pitch angle & the freq. The nominal center freq. of this bandwidth corresponds to a helix circumference of 1λ $(C_\lambda = 1)$.

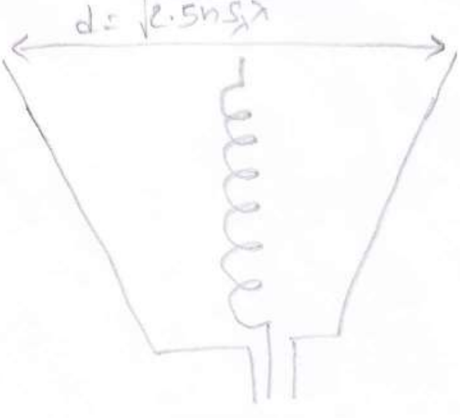
The parameters are also functions of the ground plane size & shape, the helical conductor diameter, the helix support structure & the feed arrangement.

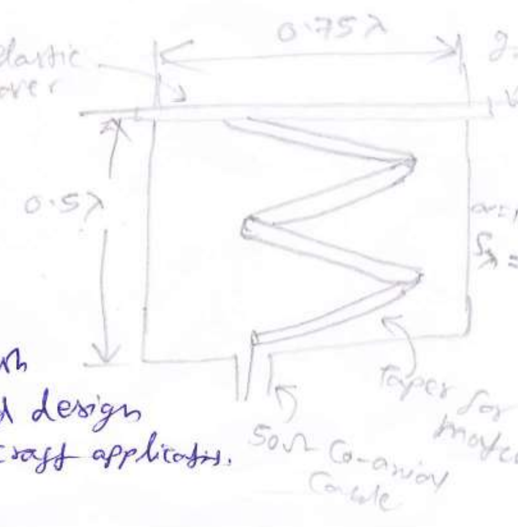
→ Flat Ground plane with side dimension $3\lambda/4$.

→ Cup-shaped ,,



→ Deep conical ground plane enclosed for reducing side & back lobes





→ Conductor size may range from 0.005λ ~~to~~ less to 0.05λ.

Two flush mounted design for air craft applications.

1) Design a 5 element Yagi antenna operating at 475 MHz with all its structural Requirements.

Operating wavelength, $\lambda = c/f = \frac{3*10^8}{475\times10^6} = 0.6315$ m.

A 5-element Yagi-Uda antenna Consists of:

→ one reflector (R)
→ one folded dipole (FD) &
→ three directors (D1, D2, D3)

lengths of the elements are as follows:-

$l_R = 0.5\lambda = 0\cdot$

$l_{FD} = 0.475\lambda =$

$l_{D_1} = 0.45\lambda =$

$l_{D_2} = 0.427\lambda =$

$l_{D_3} = 0.40\lambda =$

Distance (or) Spacing b/n the elements are follows:

$d_{R \& FD} = 0.23\lambda =$

$d_{FD \& D_1} = d_{D_1 \& D_2} = d_{D_2 \& D_3} = 0.15\lambda =$

The following figure shows the design of 5-element Yagi-Uda anten

2) Design Yagi-uda antenna of 6 elements to provide a gain of 12 if its operating freq. is 200 MHz.

Operating wavelength, $\lambda = c/f = \frac{3\times10^8}{200\times10^6} = 1.5$ m.

→ R
→ FD
→ D1, D2, D3, D4.

$l_R =$

$l_{FD} =$

$l_{D_1} =$

$l_{D_2} =$

$l_{D_3} =$

$l_{D_4} =$

$d_{R \& FD} =$

$d_{FD \& D_1} = d_{D_1 \& D_2} = d_{D_2 \& D_3} = d_{D_3 \& D_4} = 0.15\lambda =$

The following fig. shows the 6-element Yagi-Uda ant

3) Design 5 turn helical antenna which is operated at 300 MHz in the axial mode & possess circular polarization in the major lobe. Determine circumference spacing b/n each turn for near optimum pitch angle. Also find HPBW & Max. directivity?

$N = 5.$ $\qquad C = \pi D.$

$f = 300MHz.$

Optimum pitch angle. $\qquad$ In axial mode

let $\alpha = 14°$

$\lambda = c/f = \frac{3*10^8}{300 \times 10^6} = 1$ metre.

$S = \lambda/4 = 1/4 = 0.25m.$

$\tan\alpha = \frac{S}{C}$

$$C = \frac{S}{\tan\alpha} = \frac{\lambda/4}{\tan 14°} = \frac{0.25}{\tan 14°} = \frac{0.25}{0.249} = 1.0026.$$

$$HPBW = \frac{52}{C}\sqrt{\frac{\lambda^3}{NS}} = \frac{52}{1} * \sqrt{\frac{1^3}{5 * 1/4}}$$

$$D = \frac{15NSC^2}{\lambda^3} = \frac{15 * 5 * 1 * 1/4}{(1)^3} = \frac{75}{4} = 18.75.$$

4) A 5 turn helical antenna is operated at 400MHz in the normal mode. The spacing b/n the turns is $\lambda/50$. It is desired to have circular polarization. Determine the circumference, length of the single turn L_0, pitch angle & overall length of the helix.

$N = 5, f = 400MHz, S = \lambda/50.$

$C = \lambda, L_0 = ?, \alpha = ?, L_n = ?$ $\qquad S = \frac{\pi^2 D^2}{2\lambda}.$

$\lambda = c/f = \frac{3 \times 10^8}{400 \times 10^6} = 0.75m.$ $\qquad D = \frac{\sqrt{2\lambda S}}{\pi}$

$S = \lambda/50 = \frac{0.75}{50} = 0.015m.$

$C = \pi D = \cancel{\pi} \times \frac{\sqrt{2\lambda S}}{\cancel{\pi}} = 0.15m.$

Length of single turn = $L_0 = \sqrt{S^2 + C^2}$

$$= \sqrt{(0.015)^2 + (0.15)^2}$$

$$= 0.151\,m$$

$$\alpha = \tan^{-1}\left(\frac{S}{C}\right) = \tan^{-1}\left(\frac{0.015}{0.15}\right) = 5.67^\circ$$

overall length = $N \cdot L_0$

$$= 5 * (0.151)$$

$$= 0.755\,m$$

5) Design a helical antenna operating in the axial mode that give[s] a directivity of 14dB at 2.4GHZ. For this helical antenna

Calculate:

a) i/p impedance.

b) HPBW.

c) Beam width b/n Nulls.

d) Axial ratio.

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{2.4\times10^9} = 0.125\,m.$$

(optimum) $\alpha = 14^\circ$ $\qquad S = \lambda/4 = \frac{0.125}{4} = 0.03\,m,$

$$C = \frac{S}{\tan\alpha} = \frac{0.03}{\tan 14^\circ} = 0.12\,m.$$

Also, $C/\lambda = 0.96.$

$\therefore \frac{3}{4} < C/\lambda < \frac{4}{3}$. Therefore 'C' satisfies the Conditi[on] for antenna to be in axial mode.

Now, $D = \dfrac{15NSC^2}{\lambda^3} \Rightarrow \quad = \dfrac{15N(0.03)(0.12)^2}{(0.125)^3}$

$N = 8.$

a) i/p impedance, $R = \dfrac{140C}{\lambda} = \dfrac{140 * 0.12}{0.125} = 134.4\Omega$

b) $HPBW = \dfrac{52}{C}\sqrt{\dfrac{\lambda^3}{NS}} = \dfrac{52}{0.12}\sqrt{\dfrac{(0.125)^3}{8*0.03}} = 39.09^\circ$

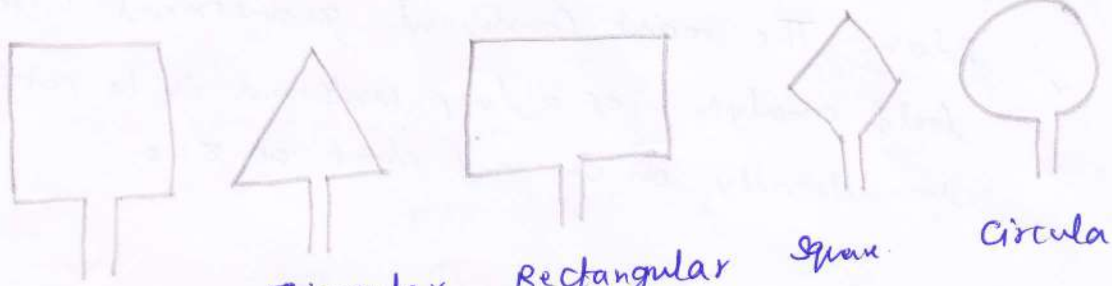
c) $FNBW = \dfrac{115}{C}\sqrt{\dfrac{\lambda^3}{NS}} = \dfrac{115}{0.12}\sqrt{\dfrac{(0.125)^3}{8*0.03}}$

$= 86.4$

d) $AR = 1 + \dfrac{1}{2N} = 1 + \dfrac{1}{16} = 1.0$

Loop antennas:-

Loop antenna is very simple and inexpensive. They take many different forms like Rectangle, square, Triangle, Ellipse, Circle and many other configurations.



Square Triangular Rectangular Square Circular.

Because of the simplicity, in analysis and construction, the circular loop is the most popular.

→ A small loop (circular or square) is equivalent to an infinitesimal magnetic dipole whose axis is ⊥er to the plane of the loop.

Loop antennas are usually classified in to two categories,

i) Electrically small

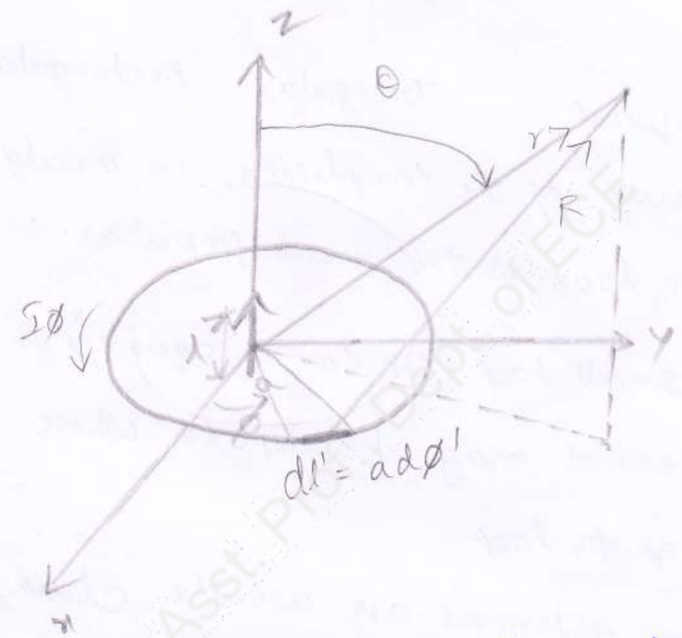
ii) Electrically large.

Electrically small antennas are those whose overall length (No. of turns times circumference) is usually less than about one tenth of a wave length i.e., $(N \times C) < \lambda/10$.

→ Where as Electrically large loops are those whose circumference is about a free space length $(C \sim \lambda)$.

→ The field pattern of electrically small antennas of any shape (circular, Elliptical, Rectangular, square etc) is similar to that of an infinitesimal dipole with a null perpendicular to the loop and with its maximum along the plane of the loop.

As the ~~loop~~ overall length of the loop increases and its circumference approaches the free space wavelength, then the maximum of the pattern shifts from the plane of the loop to the axis of the loop which is perpendicular to its plane. The most convinient geometrical arrangement for the field analysis of a loop antenna is to position the antenna symmetrically on the x-y plane at $z=0$.



The wire is assumed to be very thin and the current distribution is,

$I_\phi = I_0$ where I_0 is a constant.

Vector potential:-

The vector potential is given by,

$$A(x,y,z) = \frac{\mu}{4\pi}\int_C I_e(x',y',z')\,\frac{e^{-jkR}}{R}\,dl'.$$

$$A_\phi = \frac{a^2 \mu I_0}{4}\, e^{-jkr}\left[\frac{jk}{r} + \frac{1}{r^2}\right]\sin\theta.$$

$$A_r = 0$$

$$A_\theta = 0.$$

Magnetic field strength:-

$$H_r = \frac{jka^2 I_0 \cos\theta}{2r^2}\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

$$H_\theta = -\frac{(ka)^2 I_0 \sin\theta}{4r}\left[1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right]e^{-jkr}$$

$$H_\phi = 0.$$

Electric field strength:-

$$E_r = 0$$

$$E_\theta = 0.$$

$$E_\phi = \frac{\eta (ka)^2 I_0 \sin\theta}{4r}\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

Radiation resistance (R_r):-

$$P_{rad} = \eta\left(\frac{\pi}{12}\right)(ka)^4 |I_0|^2$$

$$R_r = \eta\left(\frac{\pi}{6}\right)(k^2a^2)^2 = \eta\,\frac{2\pi}{3}\left(\frac{kS}{\lambda}\right)^2 = 20\pi^2\left(\frac{C}{\lambda}\right)^4 \simeq 31,171 ($$

$S = \pi a^2 \rightarrow$ area.

$C = 2\pi a \rightarrow$ Circumference of the loop.

$$R_r = \eta\left(\frac{\pi}{6}\right)(ka)^4$$

$$= \eta\left(\frac{\pi}{6}\right)\left(\frac{2\pi}{\lambda}\right)^4\left(\frac{S}{\pi}\right)^2$$

$$= \eta\left(\frac{\pi}{6}\right)\left(\frac{16\cdot\pi^4}{\lambda^4}\right)\left(\frac{S^2}{\pi^2}\right)$$

$$= \eta\left(\frac{8\pi^3}{3}\right)\left(\frac{S^2}{\lambda^4}\right)$$

$$= \eta\cdot\frac{S^2}{\lambda^2}\cdot\frac{2\pi}{3}\cdot\left(\frac{2\pi}{\lambda}\right)^2$$

$$= \eta\,\frac{2\pi}{3}\left(\frac{Sk}{\lambda}\right)^2$$

$S = \pi a^2$

$a = \sqrt{\frac{S}{\pi}}$

$C = 2\sqrt{\pi S}.$

$k = \frac{2\pi}{\lambda}$

$\eta = 120\pi.$

$$S = \frac{C^2}{4\pi}$$

$$C = 2\pi a$$

$$a = \frac{C}{2\pi} \Rightarrow S = \pi \frac{C^2}{4\pi^2}$$

$$S = \frac{C^2}{4\pi}$$

$$R_r = \eta \left(\frac{2\pi}{3}\right)\left(\frac{kS}{\lambda}\right)^2$$

$$= 120\pi \cdot \left(\frac{2\pi}{3}\right)\left(\frac{1}{\lambda}\right)^2 \left(\frac{2\pi}{\lambda}\right)^2 \left(\frac{C^2}{4\pi}\right)^2$$

$$= \cancel{120\pi} \cdot \frac{2\pi}{3} \left(\frac{C}{\lambda}\right)^4 \frac{1}{16\pi^2} \cdot 4\pi^2$$

$$R_r = 20\pi^2 \left(\frac{C}{\lambda}\right)^4.$$

$$R_r = 20\pi^2\, C^2 \cdot C^2 \cdot \frac{1}{\lambda^4}$$

$$= 20\pi^2 \cdot (S \cdot 4\pi)^2 \cdot \frac{1}{\lambda^4}$$

$$= 20\pi^2 \cdot 16 \cdot \pi^2 \left(\frac{S^2}{\lambda^4}\right)$$

$$= 320\pi^4 \cdot \left(\frac{S^2}{\lambda^4}\right)$$

$$= 31,170.90913 \left(\frac{S^2}{\lambda^4}\right)$$

$$R_r \simeq 31,171 \left(\frac{S^2}{\lambda^4}\right)$$

The radiation resistance is given in above eq. is only for a single-turn loop. If the loop antenna has N turns wound, so that the magnetic field passes through all the loops, the radiation resistance is equal to that of single turn multiplied by N^2.

$$R_r = \eta \left(\frac{2\pi}{3}\right)\left(\frac{kS}{\lambda}\right)^2 N^2 = 20\pi^2 \left(\frac{C}{\lambda}\right)^4 N^2 \simeq 31,171 N^2 \left(\frac{S^2}{\lambda^4}\right)$$

Prob:- Find the R_r of 8 turn small circular loop antenna if R of loop is $\lambda/25$ & medium is free space. Derive the expression used.

$$S = \pi a^2 = = \pi \left(\frac{\lambda}{25}\right)^2 = \frac{\pi \lambda^2}{625}.$$

$$R_r(\text{single turn}) = 120\pi \left(\frac{2\pi}{3}\right)\left(\frac{KS}{\lambda}\right)^2$$

$$= 120\pi \left(\frac{2\pi}{3}\right)\left(\frac{2\pi}{\lambda} \cdot \frac{S}{\lambda}\right)^2$$

$$= 120\pi \left(\frac{2\pi}{3}\right)\left(\frac{2\pi}{\cancel{\lambda}} \cdot \frac{\pi \cancel{\lambda^2}}{625}\right)^2$$

$$= 120\pi \left(\frac{2\pi}{3}\right)\left(\frac{2\pi^2}{625}\right)^2 = 0.788 \text{ ohms}.$$

$$R_r\,(8\text{ turns}) = 0.788(8)^2 = 50.43 \text{ ohms}.$$

Advantages:-

→ A small loop is generally used as a Magnetic dipole.

→ A loop antenna has Directional properties where as a simple vertical antennas not have the same.

→ The radiation pattern of the loop antenna does not depend upon the shape of the loop.

→ The Currents are at same magnitude and phase throughout the loop.

Disadvantages:-

→ Transmission Efficiency of the loop is very poor.

→ It is suitable for low, Medium frequencies but not for high frequencies.

Applications :-

→ They are used as Radio receivers.

→ They are used for aircraft Receivers.

→ For finding directions.

→ As UHF Transmitters.

Small dipole :-

Small dipole is a linear wire antenna whose length is usually $\lambda/50 < l \leq \lambda/10$.

Small dipole is oriented along Z-axis and positioned symmetrically at the origin of spherical co-ordinate system. The current is assumed to be Triangular variation.

Comparision between small loop & short dipole :-

Short dipole	Small loop
1) It is a linear wire antenna whose length is usually, $\lambda/50 < l \leq \lambda/10$	1) The wire is assumed to be very thin & is in circular form current distribution is constant. (or) It is a electrically small antenna whose length (No. of turns times circumference) is usually less than about one ~~length of~~ tenth of wave length. i.e., $(N \times C) < \lambda/10$.
2) It is placed along Z-axis.	2) It is placed symmetrically on the x-y plane at z=0.

3) $R_r = 20\pi^2 \left(\frac{l}{\lambda}\right)^2$	3) $R_r = 20\pi^2 \left(\frac{C}{\lambda}\right)^2 \approx 31,171 \left(\frac{S^2}{\lambda^4}\right)$
4) R_r is high.	4) R_r is low.
5) Current distribution is Triangular.	5) It has doughnut shape radiati pattern.
6) R_r is high, therefore it increases antenna efficiency $\eta = \frac{R_r}{R_r + R_l}$	6) R_r is low, therefore it decrea antenna efficiency.
7) Q-factor of a dipole is high when Compared to loop.	7) Q-factor for the loop is half of that of dipole.
8) Dipole utilizes the physical dimensional less than loop.	8) Loop utilizes the physical dimens more efficiently than dipole.

Horn Antennas:-

One of the Simplest & most widely used Microwave antenna is Horn Antenna. A radiating element (or) an antenna which has the Shape of a horn is called as "Horn antenna".

→ It is a waveguide one end of which is flared out.

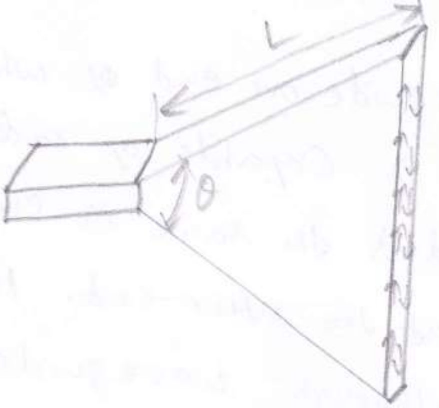
→ A waveguide is Capable of radiating radiation in to open space provided the same is exited at one end opened at the other end. ~~at the other end~~. How ever, the radiation is much greater through waveguide than the two wire Tx'sion line.

→ In a waveguide a small portion of the incident wave is radiated and the large portion is ~~ea~~ Reflected back by the openciscuit. Hence radiation is poor because of the mismatch b/n waveguide & freespace which results in a non-directive radiation pattern.

→ In order to over Come these difficulties, the mouth of the horn is flared out so that the abrupt disContinuity is replaced by a gradual transformation provided that the impedance matching is proper & all the incident energy will be radiated in the forward direction in the wave guide. This improves directivity & reduces diffraction (No standing waves)

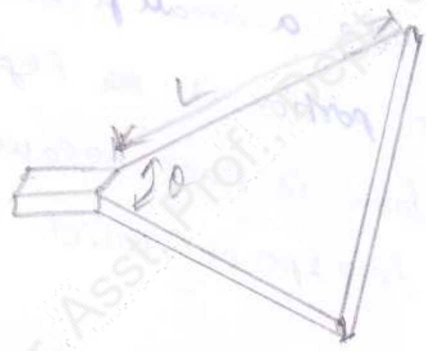
→ If flaring is done only in one direction then sectorial horn is produced. Flaring in the direction of Electric vector & magnetic vector, the sectorial E-plane horn & sectorial H-plane horn are obtained respectively.

→ If flaring is in the direction of Electric field vector it is Called as "sectorial E-plane horn". as shown in fig.

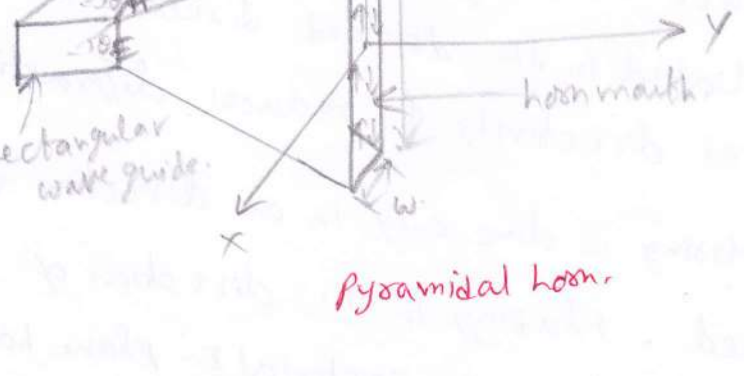


Sectorial E-plane horn

→ If the flaring is in the direction of Magnetic field vector, it is Called as "Sectorial H-plane Horn" as shown in fig.

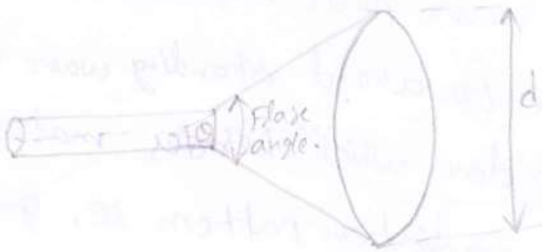


Sectorial H-plane horn



Pyramidal horn.

If the walls of a circular waveguide is flared it is called as "Conical Horn" as shown in figure below.



To minimise reflections sometimes exponentially tapered horns can be used as shown in the below figures.

Exponentially Tapered pyramidal Horn.

Exponentially Tapered Conical Horn.

→ The fields inside the wave guide propagate in the same manner as in free space, but with the difference that in waveguide the waves are constrained by the walls of waveguide from being spherically spreading.

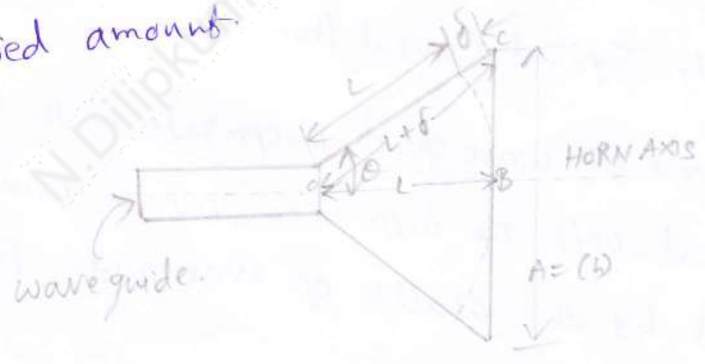
→ on reaching at the waveguide mouth, these propagating fields continue to propagate in the same general direction but now also starts spreading laterally according to Huygen's principle & the wavefront eventually results spherical.

→ This may be treated as Transition Region where the change over from the guided propagation to free space propagation occurs.

→ Since the wave guide impedance & free space impedance not equal, hence to avoid standing wave ratio flaring at walls of waveguide is done which besides matching of impedance, also provides Concentrated radiation pattern i.e, greater directivity & narrower beamwidth. It is the flared structure that is given the name "Electromagnetic horn radiator."

→ The function of the Electromagnetic Horn is to produce a uniform phase front with a larger aperture in comparision to waveguide & thus the directivity is greater.

→ Although the principle of equality of path length (fermat's principle) is applicable to horn design but in different sense i.e., instead of specifying that the wave over the plane of the horn mouth is inphase exactly, we allow that phase may deviate but by an amount less than the specified amount.



A = Apertuse
θ = Flare Angle
δ = path difference.

From the geometry of the above fig,

$$\cos\theta = \frac{L}{L+\delta} \quad \& \quad \tan\theta = \frac{h/2}{L}$$

$$\theta = \tan^{-1}\left(\frac{h}{2L}\right) = \cos^{-1}\left(\frac{L}{L+\delta}\right) \quad \text{— (1)}$$

Where, δ is permissible phase angle variation expressed as a fraction of 360°.

From right angled Triangle OBC,

$$(L+\delta)^2 = L^2+(h/2)^2$$

$$\cancel{L^2}+\delta^2+2L\delta = \cancel{L^2}+\frac{h^2}{4}.$$

As δ is small, δ^2 can be neglected,

$$2L\delta = (h/2)^2$$

$$\therefore L = \frac{h^2}{8\delta}.$$

The above equation is the design equation of the horn antenna.

→ If flare angle (2θ) is very large, the wavefront on the mouth of the horn will be curved rather than plane. This will result in non-uniform phase distribution over the aperture, resulting increased bandwidth & decreased directivity.

→ If the flare angle is small as the small angle results in small aperture area for a specified length 'L'.

→ The directivity is proportional to the aperture size for a given aperture distribution. Thus there is optimum aperture angle given by,

$$\theta = \tan^{-1}\left(\frac{h}{2L}\right) = \cos^{-1}\left(\frac{L}{L+\delta}\right)$$

→ The maximum directivity is achieved at the largest flare angle for which δ doesnot exceed a value. Typical values of δ are 0.25, 0.32, 0.4 for plane horn, Conical horn & H-plane horn respectively.

→ Directivity with pyramidal horn (or) Conical horn antenna increases as they have more than one flare angle when Compared with E-plane (or) H-plane Horn antenna.

→ As there is no resonant elements involved in the horn antenna & hence they can be operated over a broad band of frequency.

→ Approximate formulae for the half power beamwidth of the optimum flare horns are as follows:

$$\theta_E = \frac{56\lambda}{h} \text{ degrees.}$$

$$\theta_H = \frac{67\lambda}{w} \text{ degrees.}$$

where θ_E & θ_H are HPBW in E & H directions.

→ Directivity is given by,

$$D = \frac{7.5\,hw}{\lambda^2} = \frac{7.5A}{\lambda^2}.$$

where $A = h * w$ = area of horn mouth opening (aperture).

→ Power gain is given by,

$$G_p = \frac{4.5\,hw}{\lambda^2} = \frac{4.5A}{\lambda^2}.$$

Merits:-

1) Horn antennas are extensively used at microwave frequencies under the Condition that power gain needed is moderate.

2) Horn antennas are used as primary radiations for Reflector antennas.

3) Horn antennas are also used as a universal standard for Calibration & gain measurement of other high gain antennas.

Demerits:-

For high power gain, since the horn dimensions becomes large, so the other antennas like lens (or) Parabolic Reflector etc., are preferred rather than horn antenna.

Applications:-

1) The transmission & Reception of the RF Microwave signals is done by horn antenna.

2) Horn antennas are used in short range ~~Raadar~~ Radar system when enforce speed limits on automobiles.

3) In case of earth stations, horn antennas are employed as feeds for reflectors when higher & narrower bandwidths are required.

Prob:-

The length of an E-plane Sectorial horn is 15cms, Design the horn dimensions such that it is optimum at 10 GHZ.

$L = 15\text{cms}, f = 10\text{GHZ}.$

$h = ?, \theta = ?, \delta = ?$

for an E-plane sectorial horn antenna, $\delta = 0.2\lambda$

where, operating wavelength $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10 \times 10^9} = 0.03m$ (or) $3cms$.

Permissible path difference, $\delta = 0.2\lambda = 0.2(3) = 0.6cms$.

Height of E-plane sectorial horn is,

$$h = \sqrt{8L\delta} = \sqrt{8(15)(0.6)} =$$

Flare angle of the E-plane sectorial horn is given by,

$$\theta = \cos^{-1}\left(\frac{L}{L+\delta}\right)$$

$$\theta = \cos^{-1}\left(\frac{15}{15+0.6}\right) = 15.94^\circ$$

Prob:- The pyramidal horn is required to have a half power width of 10° in both the vertical & horizontal planes. Determine the dimensions of the horn mouth.

given $\theta_E = \theta_H = 10^\circ$.

$$\theta_E = \frac{56\lambda}{h} \text{ degrees.}$$

$$h = \frac{56\lambda}{\theta_E} = \frac{56\lambda}{10} = 5.6\lambda \text{ degrees.}$$

Similarly, $\theta_H = \frac{67\lambda}{w}$.

$$w = \frac{67\lambda}{\theta_H} = \frac{67\lambda}{10} = 6.7\lambda \text{ degrees.}$$

$$\text{Directivity, } D = \frac{7.5A}{\lambda^2} = \frac{7.5 \times h \times w}{\lambda^2} = \frac{7.5 \times 5.6\lambda \times 6.7\lambda}{\lambda^2}$$

$$= 7.5 \times 37.52$$

$$=$$

$$\therefore D(dB) = 10\log_{10}(\;) = \_$$

UNIT - 3
Antenna Arrays

→ Array of antennas is an arrangement of several individ antennas so spaced & phased that their Contributions Coming in one preferred direction and Cancel in all other directions to get greater directive gain & directivity.

→ There fore, an antenna is a system of similar antennas oriented similarly to get greater directivity in a desi direction.

→ Also, it is defined as "a radiating system Consisting of several spaced and properly phased radiators".

→ An antenna array is said to be linear if the individ antennas of the array are equally spaced along a st. l

→ The individual antennas of an antenna array system also termed as elements.

→ Thus linear antenna array is a system of equally space elements.

→ Uniform Linear array is "one in which the elements are f with a Current of equal magnitude with uniform progre phase shift along the line".

→ The term phase in an antenna arrays and ordinary ckt same meaning i.e, "two Currents in two elements are said to be in phase if they reach their maximum values flowing in the same direction at the same instant".

→ Antennas may be put in various Configurations i.e., Strai
line, Circles, Triangles, rectangles, etc., & hence there a
large number of possible Configurations.

Various forms of antenna arrays:-

Various antenna arrays used in practice are:

1) Broad side array
2) End-fire array
3) Collinear array
4) Parasitic array.

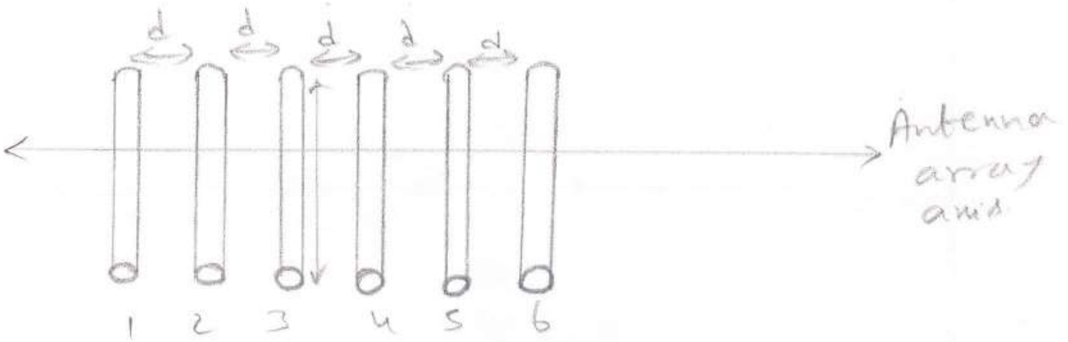
Broad side array:-

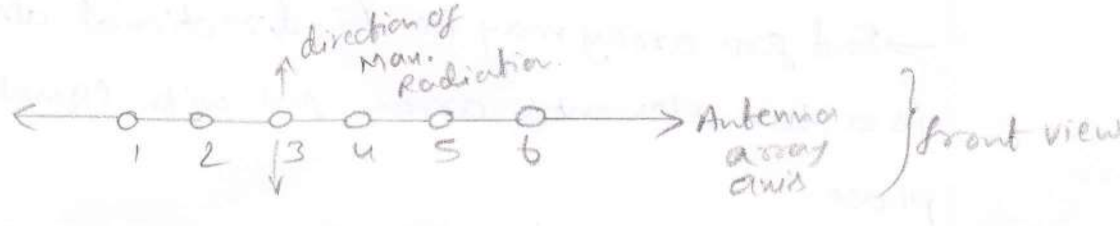
The broad side array is the array of antennas in whic
all the elements are placed ||le to each other & the direc
of maximum radiation is always ⊥er to the plane consist
elements.

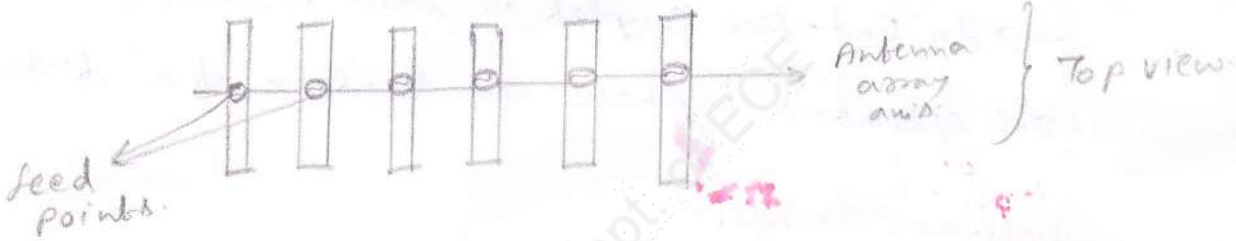
→ In the broad-side array, individual antennas (elements)
equally spaced along a line and each element is fed w
current of equal amplitude, all in the same phase.

→ Then this arrangement fires in broad-side directions (i.e.,
⊥er to the line of array axis) where these are maximum
radiations and relatively a little radiations in other
direction and hence the radiation pattern of broad side
array is bi-directional.

→ Therefore, broad side array may be defined as "an arrangem
in which the principal direction radiation is ⊥er to the array
axis and also to the plane containing the array elements".







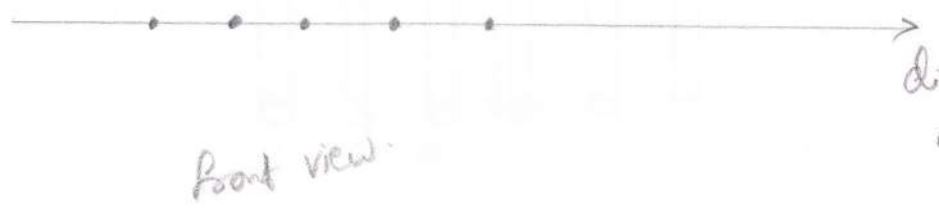
→ Broad side Couplet is said to form if two isotropic radiators operate in phase there by they reinforce each othe most strongly in the plane right angles to the line joining the

End-fire Arrays :-

→ The end-fire arrays is nothing but broadside array except that individual elements are fed in, out of phase (usually

→ Thus in the end-fire array, a number of identical antenn are spaced equally along a line and individual element are fed with Currents of equal magnitude but their pha varies progressively along the line in such a way as to ma the entire arrangement substantially unidirectional.

→ The endfire array may be defined as the arrangement in which the principal direction of radiation Coincides with th direction of the array axis.

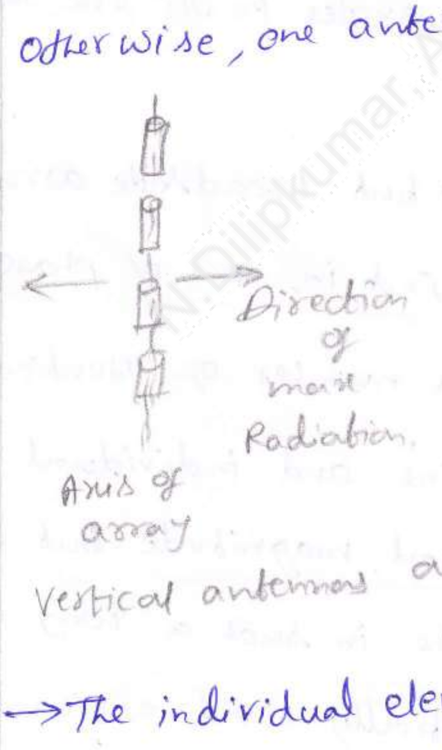


→End fire array may be bi-directional also - one such exam is a two elements array fed with equal current 180° ou phase.

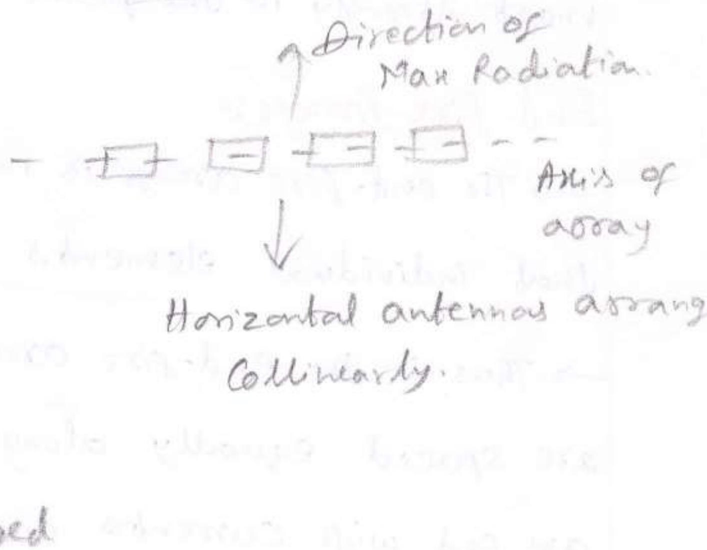
→An End-fire couplet is said to form, if two equal radiat are operated inphase quadrature at a distance of $\lambda/4$ or

Collinear arrays:-

→In Collinear array, the antennas are arranged co-axially i.e., antennas are mounted end to end in a single line otherwise, one antenna is stacked over another antenna



Vertical antennas arranged Collinearly.



Horizontal antennas arranged Collinearly.

→The individual elements are fed with equal inphase curren as in the broadside arrays.

→A Collinear array is a broadside array in which the directi of maximum radiation is ⊥er to the line of antenna.

→Collinear is also called as broad cast or omni directional arra

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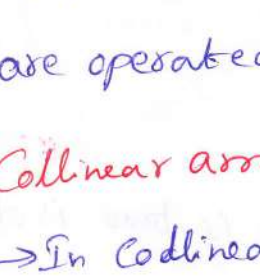
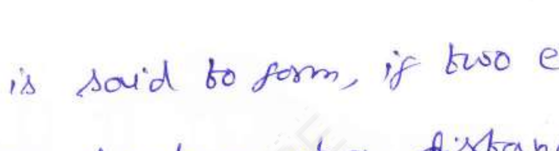


Vertical antennas arranged
Collinearly.



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→ Collinear is also called as broad cast or omni directional arra

Parasitic arrays:-

→ The element supplied power directly from source (Tx) usua through Tx line is called "Driven element".

→ But parasitic element is not fed directly, instead, parasi element derives power by radiation from nearby driven ele

→ Otherwise, parasitic element obtains power solely throu electromagnetic coupling with a driven element because o its proximity to that driven element.

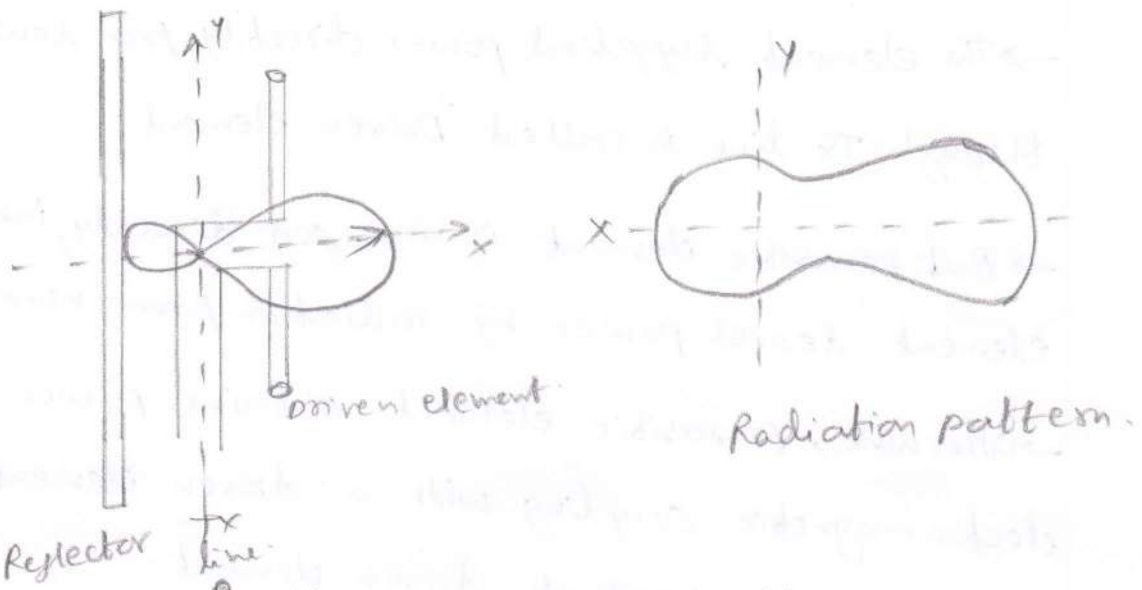
→ Parasitic array is nothing but one driven element and o parasitic element and this may be considered as two-elem array.

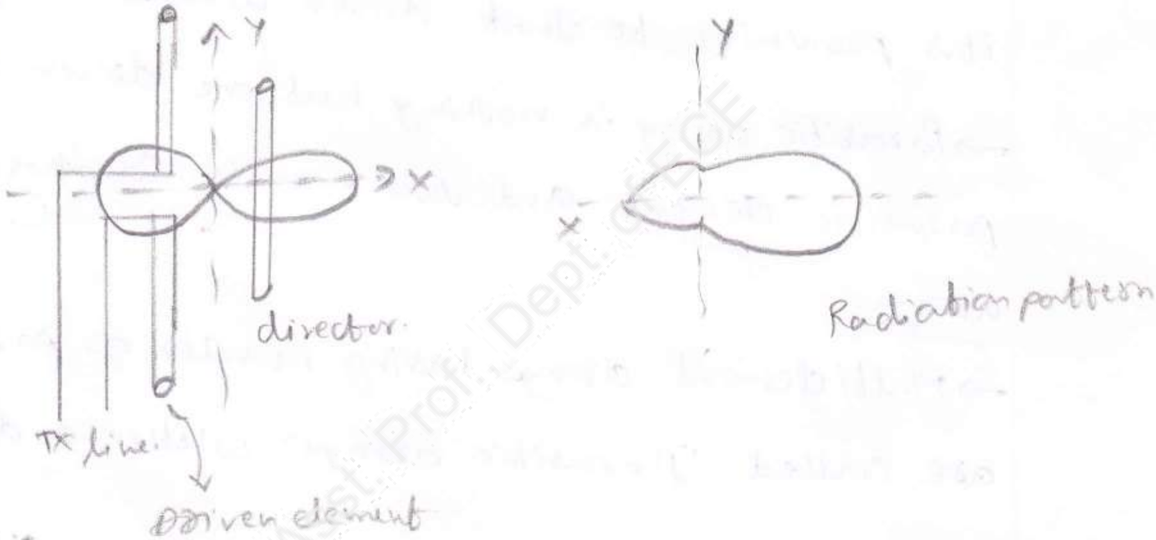
→ Multielement arrays having number of parasitic element are called "parasitic arrays" whether a driven element i one or more.

→ Hence in parasitic arrays there is one or more parasitic el ments & atleast one driven element to introduce powe in the array.

→ A parasitic array with linear half-wave dipole element is normally called as "Yagi-Uda" or simply Yagi antenn after the name of inventors S. Uda (Japanese) & H. Yagi (Engl

→ A parasitic element lengthened by 5% with respect to driven element acts as reflector and shortened by 5% acts director.





Parasitic element shortened.

→ A reflector makes the radiation maximum in ter directio
from parasitic element towards driven element

→ A director helps in making maximum radiation in ter dire
from driven element to the parasitic element.

Arrays of Point Sources:-

→ In this an antenna is regarded as a point source (or) volume less radiator and is a hypothetical antenna (or) isotropic (or) omni-directional (or) non-directional antenna which occupies zero volume.

→ First we consider only two isotropic point sources seperated by a distance with different phasing conditions and then this idea is extended for more and finally for n isotropic point sources.

→ Further, the case of non-isotropic but similar to point sources will also be taken which will lead to the "principle of pattern multiplication".

Array of Two point Sources:-

Two isotropic point sources are seperated by a distance 'd' and have the same polarization. The three cases to be studied

i) equal amplitude & phase.
ii) equal amplitude & opposite phase.
iii) unequal amplitude & opposite phase.

Case(i):- Equal amplitude & phase (i.e., Arrays of two point sources with equal amplitude & phase).

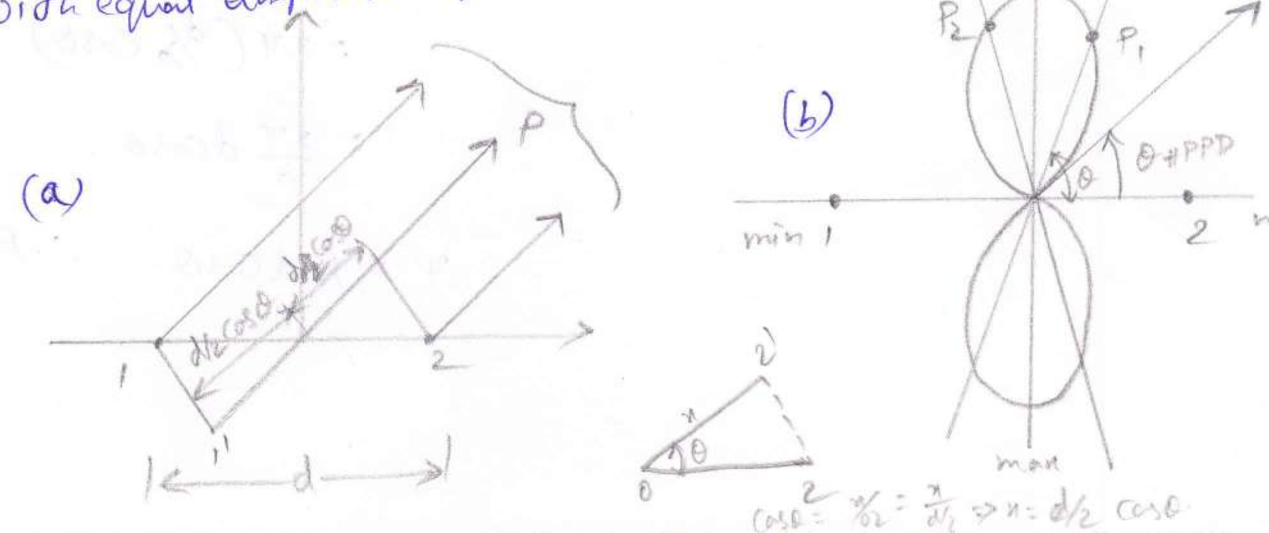


fig. (a): Two isotropic point sources situated symmetrically w.r.t origin 'o' with same amplitude & same phase.

fig. (b): Field pattern of fig (a). with $d = \lambda/2$.

→ Two isotropic point sources situated symmetrically w.r.t origin in the Cartesian Co-ordinate system as shown in fig. (a)

→ 'p' is a point at great distant 'R' is the distance from origin 'o' to point 'p'. The origin 'o' is taken as reference point for phase calculation.

→ Obviously waves from source 1 reaches the point 'p' at the later time than the waves from source 2. This is because the path difference involved b/n the two waves.

Thus fields due to source 1 lags while fields due to source 2 leads.

$$\text{path difference } (1'2') = \left(\frac{d}{2}\cos\theta + \frac{d}{2}\cos\theta\right)$$

$$= d\cos\theta \text{ meters.}$$

$$= \frac{d}{\lambda}\cos\theta \text{ wave lengths.}$$

$$\text{Phase angle } (\psi) = 2\pi\,(\text{path difference})$$

$$= 2\pi\left(\frac{d}{\lambda}\cos\theta\right)$$

$$= \frac{2\pi}{\lambda} d\cos\theta$$

$$\psi = \beta d\cos\theta \quad \therefore \beta = \frac{2\pi}{\lambda}.$$

→ Let E_1, E_2 and E be the far electric fields at distant point 'p' due to source 1, source 2 & their sum in the direction of θ.

$$E = E_1 e^{-j\psi/2} + E_2 e^{j\psi/2}$$

Since Amplitudes are same : $E_1 = E_2 = E_0$

$$E = E_0 \cdot e^{-j\psi/2} + E_0 e^{j\psi/2}$$

$$= E_0 \left[e^{-j\psi/2} + e^{j\psi/2} \right]$$

$$= 2E_0 \cos(\psi/2)$$

$$E = \underbrace{2E_0}_{\text{amp.}} \underbrace{\cos\left[\frac{\beta d \cos\theta}{2}\right]}_{\text{Phase.}}$$

where array factor, $AF = 2\cos\left(\frac{\beta d \cos\theta}{2}\right)$

$$\therefore E = E_0 (AF)$$

on Normalizing with $2E_0$ gives,

$$E_{norm} = \cos\left[\frac{\beta d \cos\theta}{2}\right]$$

$$= \cos\left[\frac{2\pi/\lambda \cdot \lambda/2 \cos\theta}{2}\right]$$

$$E_{norm} = \cos\left[\frac{\pi}{2}\cos\theta\right]$$

Maximum direction (θ_{max}) :-

$$\cos\left(\frac{\pi}{2}\cos\theta\right) = \pm 1$$

$$\pi/2 \cos\theta = \pm n\pi \; ; \; n = 0, 1, 2, 3 \ldots$$

$$\pi/2 \cos\theta_{max} = 0 \quad \text{for } n = 0.$$

$$\cos\theta_{max} = 0 \quad \Rightarrow \theta_{max} = 90^\circ \text{ (or) } 270^\circ \left[\pi/2 \text{ (or) } 3\pi/2 \right.$$

Minimum direction (θ_{min}):-

$$\cos\left(\frac{\pi}{2}\cos\theta\right) = 0$$

$$\pi/2 \cos\theta = \pm(2n+1)\pi/2 \quad \text{for } n=0,1,2\ldots$$

$$\pi/2 \cos\theta_{min} = \pm \pi/2 \quad \text{for } n=0.$$

$$\cos\theta_{min} = \pm 1 \Rightarrow \theta_{min} = 0 \text{ \& } 180^\circ$$
$$= [0 \text{ \& } \pi]$$

Half power point direction (θ_{HPPD}):-

$$\cos\left(\frac{\pi}{2}\cos\theta\right) = \pm 1/\sqrt{2}$$

$$\pi/2 \cos\theta = \pm(2n+1)\pi/4 \quad \text{for } n=0,1,2\ldots$$

$$\pi/2 \cos\theta_{HPPD} = \pm \pi/4 \quad \text{for } n=0.$$

$$\cos\theta_{HPPD} = \pm 1/2 \Rightarrow \theta_{HPPD} = 60^\circ \text{ and } 120^\circ$$
$$= \pi/3 \text{ \& } 2\pi/3$$

→ The field pattern (E vs θ) for $d = \lambda/2$ is shown in fig (b). Which is bi-directional, figure of eight 360° rotation of th figure around the Z-axis will generate a 3-dimensional sp pattern known as "Doughnut shape".

→ As the array fires along the broad side of the array axis this is known as broad side array of size 2.

→ If the reference point is shifted to source1, then

$$E = E_1 + E_2 e^{j\psi}.$$

$$E_1 = E_2 = E_0.$$

$$E = E_0\left[1 + e^{j\psi}\right]$$

$$= E_0\left[e^{-j\psi/2}\, e^{j\psi/2} + e^{j\psi/2}\, e^{j\psi/2}\right]$$

$$= 2E_0\, e^{j\psi/2}\left[\frac{e^{-j\psi/2} + e^{j\psi/2}}{2}\right]$$

$$= 2E_0\, e^{j\psi/2}\, \cos(\psi/2)$$

$$= E_0(AF) \quad \text{where, Array factor} = 2\,e^{j\psi/2}\cos(\psi/2)$$

On Normalising with $2E_0$ gives,

$$E_{norm} = e^{j\psi/2}\cos(\psi/2) \qquad \beta = \frac{2\pi}{\lambda}$$

$$= e^{\frac{j\beta d\cos\theta}{2}}\cos\left(\frac{\beta d\cos\theta}{2}\right) \qquad d = \lambda/2$$

$$= e^{j\pi/2\cos\theta}\cos\left(\pi/2\cos\theta\right)$$

$$= \cos\left(\pi/2\cos\theta\right) . \angle \pi/2\cos\theta.$$

Therefore it is observed that there is no change in amplitude & phase.

Case(ii):- Arrays of Two point sources with equal amplitude & opposite phase. ($180°$)

→ Here the source 1 & source 2 are out of phase (or opposite phase) with each other.

$$\text{i.e.,}\quad E = \left[-E_1\, e^{-j\psi/2}\right] + \left[E_2\, e^{j\psi/2}\right]$$

$$\text{let } E_1 = E_2 = E_0$$

$$E = -E_0 e^{-j\psi/2} + E_0 e^{+j\psi/2}$$

$$= E_0 \left[e^{j\psi/2} - e^{-j\psi/2} \right]$$

$$= E_0 \cdot 2j \sin\psi/2$$

$$= E_0 \cdot (AF)$$

where AF = Array factor = $j2 \sin \psi/2$

On Normalising with $j2E_0$ gives,

$$E_{Norm} = \sin\psi/2$$

$$= \sin\left(\frac{\beta d \cos\theta}{2}\right) = \sin\left(\pi/2 \cos\theta\right)$$

Maxima direction (θ_{max}):-

$$\sin\left(\pi/2 \cos\theta\right) = \pm 1$$

$$\pi/2 \cos\theta = \pm(2n+1)\pi/2 \quad \text{for } n=0,1,2 \dots$$

$$\pi/2 \cos\theta_{max} = \pm \pi/2 \quad \text{for } n=0$$

$$\cos\theta_{max} = \pm 1$$

$$\theta_{max} = 0 \;\&\; 180^\circ \text{ (or) } 0^\circ \;\&\; \pi.$$

Minima direction (θ_{min}):-

$$\sin\left(\pi/2 \cos\theta\right) = 0$$

$$\pi/2 \cos\theta = \pm n\pi \quad \text{for } n=0,1,2 \dots$$

$$\pi/2 \cos\theta_{min} = 0 \quad \text{for } n=0$$

$$\cos\theta_{min} = 0 \Rightarrow \theta_{min} = 90^\circ \;\&\; 270^\circ \text{ (or) } \pi/2 \;\&$$

Half power point directions (θ_{HPPD}) :-

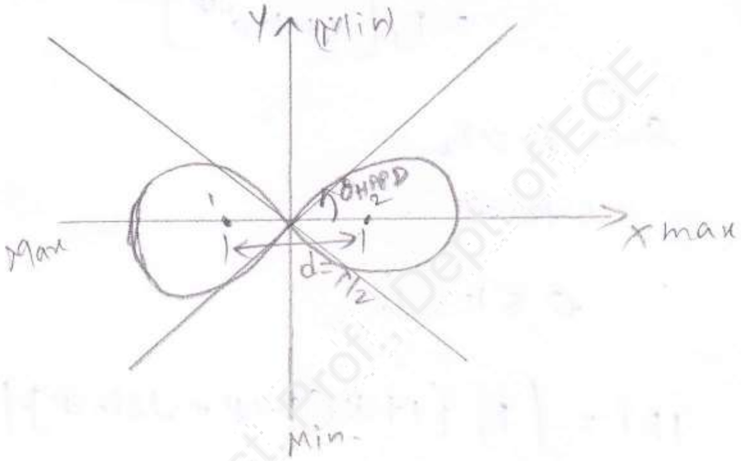
$$\sin\left(\tfrac{\pi}{2}\cos\theta\right) = \pm \tfrac{1}{\sqrt{2}}$$

$$\tfrac{\pi}{2}\cos\theta = \pm(2n+1)\tfrac{\pi}{4} \quad \text{for } n=0,1,2\ldots$$

$$\tfrac{\pi}{2}\cos\theta_{HPPD} = \pm \tfrac{\pi}{4} \quad \text{for } n=0$$

$$\cos\theta_{HPPD} = \pm \tfrac{1}{2}$$

$$\theta_{HPPD} = 60^\circ \;\&\; 120^\circ \;(\text{or})\; \pi/3 \;\&\; 2\pi/3$$



→ The field pattern (E vs θ) for $d=\lambda/2$ is shown in figure which is bi-directional & figure of eight 360° rotation of this figure around the Z-axis will generate a 3-dimensional space pattern known as "doughnut shape".

→ As the array fires along the axis it is known as End-fire array of size 2.

Case (iii): Arrays of two point sources with unequal amplitude & any phase.

→ Here amplitude of the two point sources are not same.

$$\text{i.e., } E_1 \neq E_2 \;\&\; \text{let } K = E_2/E_1$$

& α be the phase difference b/n E_1 & E_2.

Now the total phase difference b/n the far electric field Component at point 'P' is,

$$\psi = \beta d \cos\theta + \alpha = \frac{2\pi}{\lambda} d\cos\theta + \alpha$$

If $\alpha = 0$ then Case(i), $\alpha = 180°$ then Case(ii)

$$\text{Now, } E = E_1 \cdot e^{j0} + E_2\, e^{j\psi}$$

$$= E_1 \left[1 + \frac{E_2}{E_1} e^{j\psi} \right]$$

$$= E_1 \left[1 + k\, e^{j\psi} \right]$$

Since $E_1 > E_2$

$\therefore k < 1$

$0 \leq k \leq 1.$

$$|E| = \left| E_1 \{ 1 + k(\cos\psi + j\sin\psi) \} \right|$$

$$E = E_1 \sqrt{(1 + k\cos\psi)^2 + (k\sin\psi)^2}$$

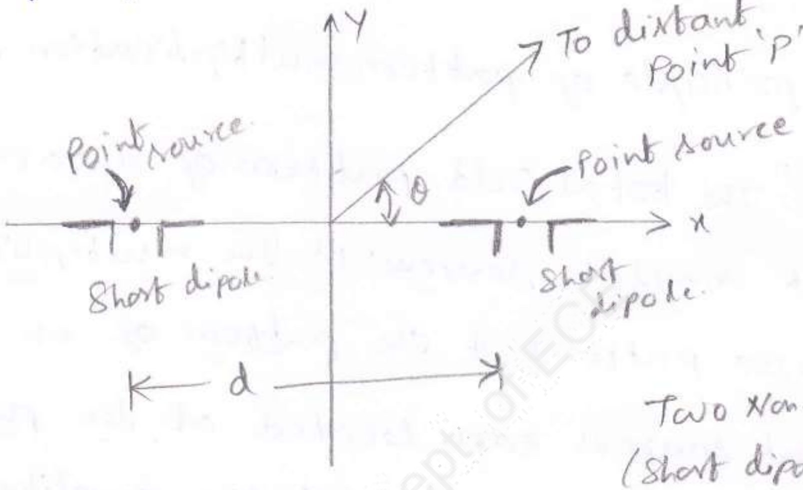
$$\therefore \angle E \text{ (or) } \phi = \tan^{-1}\left[\frac{k\sin\psi}{1 + k\cos\psi} \right]$$

Non-Isotropic but similar point sources:

The analysis of array of two isotropic point sources may be extended to the non-isotropic (directional) sources provided that their field patterns are similar to that of isotropic point source.

→ If the amplitudes of individual non-isotropic source are un-equal then they are said to be similar otherwise if the amplitudes are equal they are said to be identical.

→ Let us consider two short dipoles which are superimposed on the two isotropic point sources & are seperated by a distance



Two Non-isotropic (Short dipoles & point sou

→ Here field pattern of each non-isotropic point source is,

$$E_0 = E_1 \sin\theta$$

→ Now field pattern of two identical isotropic source is,

$$E = 2E_0 \cos(\psi/2)$$

where $\psi = \beta d \cos\theta + \alpha$

→ Field pattern of two non-isotropic but similar sources is,

$$E = 2(E_1 \sin\theta) \cos \psi/2$$

on Normalising with $2E_1$ gives,

$$E_{norm} = (\sin\theta) \cdot \cos(\psi/2)$$

$$= \begin{Bmatrix} \text{Pattern of individual} \\ \text{Non-isotropic source} \end{Bmatrix} \times \begin{Bmatrix} \text{Pattern of array} \\ \text{two isotropic point} \\ \text{sources} \end{Bmatrix}$$

(primary pattern) (secondary patt

→ This lead to the principle of multiplication of pattern o multiplication of individual point source & pattern of ar of isotropic point sources gives the field pattern of non-isotrop but similar point sources.

Principle of Pattern Multiplication:-

The principle of pattern multiplication states that:

→ "The total field pattern of an array of non-isotropic but similar sources is the multiplication of the individu source patterns & the pattern of an array of isotropic point sources each located at the phase centre of individ source & having the relative amplitude & phase, where as the total phase pattern is the addition of the phase patte of the individual sources & that of the array of isotropi point sources."

→ Here $E = E(\theta) \cdot \cos(\psi/2)$ where $\psi = \beta d \cos\theta$.

→ This principle can be applicable for 2 & 3-dimensional patterns.

→ Let $E_i(\theta,\phi)$ - field pattern of individual non-isotropic sou

$E_a(\theta,\phi)$ - field pattern of array of isotropic point sour

$\angle E_i(\theta,\phi)$ - phase pattern of individual non-isotropic source.

$\angle E_a(\theta,\phi)$ - phase pattern of array of isotropic point sources

E - Total field pattern of an array of non-isotropic but similar sources.

Now, $E = \left[E_i(\theta,\phi) \angle E_i(\theta,\phi)\right]\left[E_a(\theta,\phi) \angle E_a(\theta,\phi)\right]$

$$= \underbrace{\left[E_i(\theta,\phi) \times E_a(\theta,\phi)\right]}_{\text{Multiplication of field pattern}} \underbrace{\left[\angle E_i(\theta,\phi) + \angle E_a(\theta,\phi)\right]}_{\text{Addition of Phase pattern}}$$

Here θ & ϕ indicates polar & azimuth angles respectively.

→ This principle provides a speedy method for sketching the pattern of Complicated array just by inspection. So that it is an useful tool in the design of antenna arrays.

→ The width of the principle lobe (i.e., width between Nulls) & the Corresponding width of the array pattern are same.

→ The secondary lobes are determined from the number of nulls in the resultant pattern.

→ In the resultant pattern the Number of Nulls are the sum of nulls of individual pattern & array pattern.

→ This method provides exact Resultant pattern.

Array of point sources:-

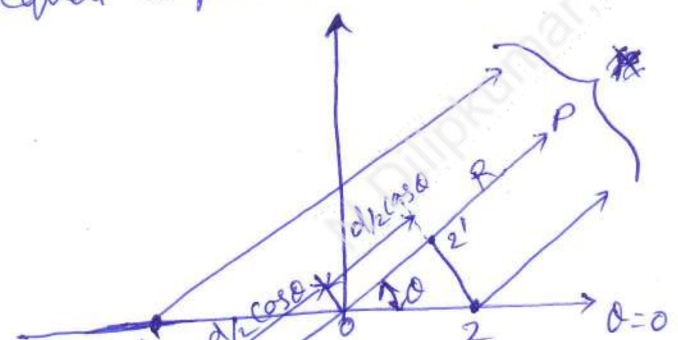
In this an antenna is regarded as a point source or volume less radiator and is a isotropic (or) omni-directional (or) non-directional antenna which occupies zero volume.

Array of Two point sources:-

Two isotropic point sources are seperated by a distance 'd' & have the same polarization. The three cases to be study are,

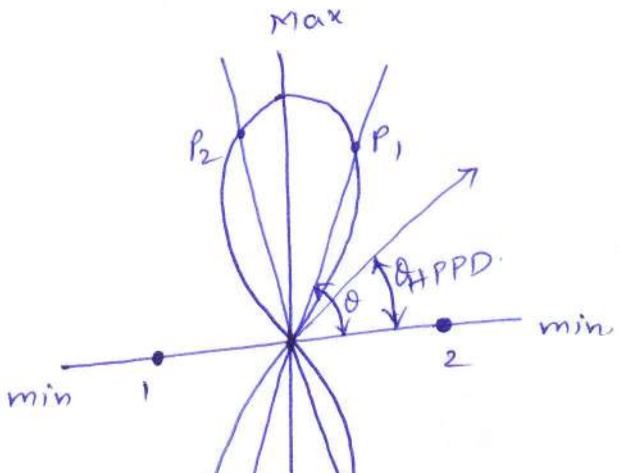
i) Equal amplitude & phase.
ii) Equal amplitude & opposite phase.
iii) Unequal amplitude & opposite phase.

Case(i):-
Equal amplitude and phase (i.e., Arrays of two point sources with equal amplitude and phase).



fig(a).

Two isotropic point sources situated symmetrically w.r.t origin 'o' with same amp & same phase.

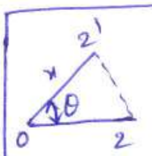


field pattern of fig(a). with $d = \lambda/2$.

→ Two isotropic point sources situated symmetrically w.r.t origin in the Cartesian co-ordinate system as shown in fig. (a).

→ P is a point at great distant 'R' is the distance from 'o' to point. 'o' is taken as reference point for phase calculation.

→ Obviously waves from source 1 reaches the point 'P' at later time than waves from source 2. This is because of the path difference involved b/n the two waves. Thus fields due to source 1 lags while fields due to source 2 leads.



$\cos\theta = \frac{x}{o2}$

$\cos\theta = \frac{x}{d/2}$

$x = \frac{d}{2}\cos\theta$

Path difference (1' 2')

$= \left(\frac{d}{2}\cos\theta + \frac{d}{2}\cos\theta\right)$

$= d\cos\theta$ meters

$= \frac{d}{\lambda}\cos\theta$ wavelengths.

phase angle $(\psi) = 2\pi$ (Path difference).

$= 2\pi\left(\frac{d}{\lambda}\cos\theta\right)$

$= \frac{2\pi}{\lambda} d\cos\theta.$

$= \beta d \cos\theta. \quad \because \beta = \frac{2\pi}{\lambda}$

Let E_1, E_2 & E be the far electric fields at distant point 'P' due to source 1, source 2 & their sum in the direction of θ,

$$E = E_1 e^{-j\psi/2} + E_2 e^{j\psi/2}$$

$\because$ Amplitudes are same, $E_1 = E_2 = E_0$.

$$E = E_0\left[e^{-j\psi/2} + e^{j\psi/2}\right]$$

$$= 2E_0\cos(\psi/2)$$

$$= \underbrace{2E_0}_{\text{amp.}} \underbrace{\cos\left(\frac{\beta d\cos\theta}{2}\right)}_{\text{phase}}$$

Array factor (AF) $= 2\cos\left(\frac{\beta d\cos\theta}{2}\right)$

$\therefore E = E_0(AF).$

on Normalizing with $2E_0$ gives,

$$E_{norm} = \cos\left[\frac{\beta d\cos\theta}{2}\right]$$

$$= \cos\left[\frac{\frac{2\pi}{\lambda}\frac{\lambda}{2}\cos\theta}{2}\right] \Rightarrow E_{norm} = \cos\left(\frac{\pi}{2}\cos\theta\right)$$

Maximum direction (θ_{max}):-

$$\cos\left(\frac{\pi}{2}\cos\theta\right) = \pm 1$$

$\frac{\pi}{2}\cos\theta = \pm n\pi$ for $n = 0, 1, 2, 3 \ldots$

$\pi/2 \cos\theta_{max} = 0$ for $n = 0$.

$\cos\theta_{max} = 0$

$\theta_{max} = 90°$ & $270°$
(or)
$\pi/2$ & $3\pi/2$.

Min. direction (θ_{min}):-

$$\cos\left(\frac{\pi}{2}\cos\theta\right) = 0.$$

$\frac{\pi}{2}\cos\theta = \pm(2n+1)\pi/2$ for $n = 0, 1,$

$\frac{\pi}{2}\cos\theta_{min} = \pm\pi/2$ for $n = 0$

$\cos\theta_{min} = \pm 1.$

$\theta_{min} = 0$ & $180°$
(or)
0 & π

Half power point direction (θ_{HPPD}):-

$$\cos\left(\frac{\pi}{2}\cos\theta\right) = \pm\frac{1}{\sqrt{2}}$$

$\pi/2 \cos\theta = \pm(2n+1)\pi/4$ for $n = 0, 1, 2 \ldots$

$\pi/2 \cos\theta_{HPPD} = \pm\pi/4$ for $n = 0$.

$\cos\theta_{HPPD} = \pm 1/2$

$\theta_{HPPD} = 60°$ & $120°$
(or)
$\pi/3$ & $2\pi/3$.

→ The field pattern (E vs θ) for $d = \lambda/2$ is shown in fig (b) which is bi-direction[al] figure of eight. 360° rotation of this figure around the Z-axis will generate a 3-dimensional space pattern known as "Doughnut shape".

→ As the array fires along the broad side of the array axis, this is known broad side array of size 2.

→ If the reference point is shifted to Source 1 then,

$$E = E_1 + E_2\, e^{j\psi}$$
$$= E_0 + E_0\, e^{j\psi}$$
$$= E_0\left[1 + e^{j\psi}\right]$$
$$= E_0\left[e^{-j\psi/2}\cdot e^{j\psi/2} + e^{j\psi/2}\, e^{j\psi/2}\right]$$
$$= 2E_0\, e^{j\psi/2}\left[\frac{e^{-j\psi/2} + e^{j\psi/2}}{2}\right]$$
$$= 2E_0\, e^{j\psi/2}\cos(\psi/2)$$
$$= E_0\,(AF)$$

Array factor,
$$AF = 2e^{j\psi/2}\cos(\psi/2)$$

on Normalizing with $2E_0$ gives,

$$E_{norm} = e^{j\psi/2}\cos(\psi/2)$$
$$= e^{j\frac{\beta d\cos\theta}{2}}\cos\left(\frac{\beta d\cos\theta}{2}\right)$$
$$= e^{j\frac{\pi}{2}\cos\theta}\cos\left(\frac{\pi}{2}\cos\theta\right)$$
$$= \cos\left(\frac{\pi}{2}\cos\theta\right)\ \angle\frac{\pi}{2}\cos\theta.$$

Therefore it is observed that there is no change in Amp. & phase.

ii) Arrays of two points sources with equal amp. & opp. phase.

→ Here the source 1 & Source 2 are out of phase (opposite phase or 180°). with each other

$$E = \left[-E_1\, e^{-j\psi/2}\right] + \left[E_2\, e^{+j\psi/2}\right].$$

but $E_1 = E_2 = E_0$

$$E = -E_0\, e^{-j\psi/2} + E_0\, e^{j\psi/2}$$
$$= E_0\left[e^{j\psi/2} - e^{-j\psi/2}\right]$$
$$= E_0\, 2j\sin\psi/2$$
$$= E_0\,(AF)$$

$$AF = j2\sin\psi/2.$$

on Normalizing with $j2E_0$ gives,

$$E_{norm} = \sin \Psi/2$$
$$= \sin\left(\frac{\beta d \cos\theta}{2}\right)$$
$$= \sin\left(\tfrac{\pi}{2}\cos\theta\right).$$

max. direction (θ_{max}):-

$\sin\left(\frac{\pi}{2}\cos\theta\right) = \pm 1.$

$\frac{\pi}{2}\cos\theta = \pm(2n+1)\frac{\pi}{2}$ for 0, 1, 2 ---

$\frac{\pi}{2}\cos\theta_{max} = \pi/2$ for $n=0$.

$\cos\theta_{max} = \pm 1.$

$\theta_{max} = 0$ & $180°$ (or) 0 & π.

min direction (θ_{min}):-

$\sin\left(\frac{\pi}{2}\cos\theta\right) = 0.$

$\frac{\pi}{2}\cos\theta = \pm n\pi$ for n=0, 1, 2

$\frac{\pi}{2}\cos\theta_{min} = 0$ for n=0.

$\cos\theta_{min} = 0$

$\theta_{min} = 90°$ & $270°$.
$\pi/2$ & $3\pi/2$.

HPPD (θ_{HPPD}):-

$\sin\left(\frac{\pi}{2}\cos\theta\right) = \pm 1/\sqrt{2}$

$\frac{\pi}{2}\cos\theta = \pm(2n+1)\frac{\pi}{4}$ for n=0, 1, 2 ---

$\frac{\pi}{2}\cos\theta_{HPPD} = \pm\frac{\pi}{4}$ for n=0,

$\cos\theta_{HPPD} = \pm 1/2$

$\theta_{HPPD} = 60°$ & $120°$ (or) $\pi/3$ & $2\pi/3$.

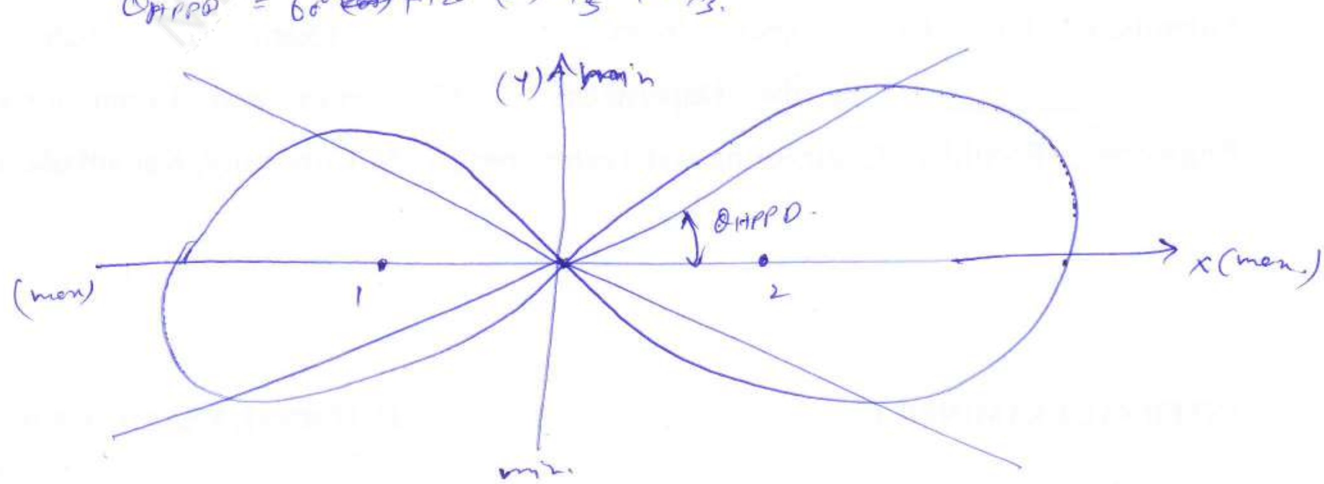


fig (b).

→ The field pattern (E vs θ) for $d = \lambda/2$ is shown in fig. which is bidirectional & figure of eight 360° rotation of this figure around the z-axis will generate a 3-dimensional space pattern known as doughnut shape

→ As the array fires along the axis, it is known as End-fire array of sizes.

Case iii):-

Array of point sources with unequal amplitude & any phase.

→ Here amplitudes of two point sources are not same.

i.e., $E_1 \neq E_2$ & Let $k = E_2/E_1$.

& α be the phase difference b/w E_1 & E_2.

Now the Total phase difference b/w the far electric field component at point

$$\psi = \beta d\cos\theta + \alpha = \frac{2\pi}{\lambda} d\cos\theta + \alpha$$

If $\alpha = 0$ then Case(i), $\alpha = 180°$ then Case(ii)

Now,
$$E = E_1 e^{j0} + E_2 e^{j\psi}$$
$$= E_1\left[1 + \frac{E_2}{E_1} e^{j\psi}\right].$$
$$= E_1\left(1 + k e^{j\psi}\right).$$

Since $E_1 > E_2$,

$\therefore k < 1$

$0 \le k \le 1.$

$$|E| = |E_1 \{1 + k(\cos\psi + j\sin\psi)\}|$$

(or)

$$E = E_1\sqrt{(1 + k\cos\psi)^2 + (k\sin\psi)^2}$$

$$\therefore \angle E = \tan^{-1}\left[\frac{k\sin\psi}{1 + k\cos\psi}\right].$$

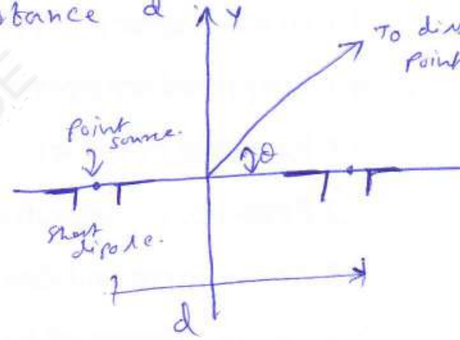
(or) ϕ.

Non-Isotropic but similar point sources :-

→ The analysis of array of two isotropic point sources may be extend to the non-isotropic (directional) sources provided that their field patterns are similar to that of isotropic point source.

→ If the amplitudes of individual non-isotropic source are unequal then they are said to be similar otherwise if the amp.'s are equal they are said to be identical

→ Let us consider two short dipoles which are superimposed over the two isotropic point sources & are seperated by a distance 'd'



Non-isotropic (Short dipole & point source)

→ Here the field pattern of each non-isotropic point source is,

$$E_0 = E_1 \sin\theta .$$

→ Field pattern of two identical isotropic source is,

$$E = 2E_0 \cos(\psi/2).$$

$$\psi = \beta d \cos\theta + \alpha.$$

→ Field pattern of Two non-isotropic but similar sources is,

$$E = 2(E_1 \sin\theta) \cos \psi/2 .$$

on Normalizing. with $2E_1$,

$$E_{norm} = \sin\theta \cos(\psi/2).$$

$$= \begin{Bmatrix} \text{pattern of individual} \\ \text{non-isotropic source} \end{Bmatrix} \times \begin{Bmatrix} \text{pattern of array of} \\ \text{two isotropic point sources} \end{Bmatrix}$$

(primary pattern) (secondary pattern).

→ This lead to the principle of multiplication of pattern as multiplication pattern of individual point source & pattern of array of isotropic point sources gives the field pattern of non-isotropic but similar point sources.

Principle of Pattern multiplication:-

The total field pattern of an array of non-isotropic but similar sources is the multiplication of the individual source pattern, and the pattern of an array of isotropic point sources each loca at the phase centre of individual source & having the relative amplitude & phase, where as the total phase pattern is the addit of phase pattern of the individual sources & that of the array of iso point sources.

$$E = E(\theta)\cos \psi/2 . \quad ; \psi = \beta d \cos\theta .$$

$E_i(\theta,\phi)$ - field pattern of individual non-isotropic source.

$E_a(\theta,\phi)$ - field pattern of array of isotropic point sources.

$\angle E_i(\theta,\phi)$ - phase pattern of individual non-isotropic source.

$\angle E_a(\theta,\phi)$ - phase pattern of array of isotropic point sources.

E = Total field pattern of an array of non-isotropic but similar sources

$$E = [E_i(\theta,\phi)\angle E_i(\theta,\phi)]\,[E_a(\theta,\phi)\angle E_a(\theta,\phi)].$$

$$= \underbrace{[E_i(\theta,\phi) \times E_a(\theta,\phi)]}_{\text{multiplication of field pattern}}\;\underbrace{[\angle E_i(\theta,\phi) + \angle E_a(\theta,\phi)]}_{\text{Addition of phase pattern}}$$

This principle provides a speedy method for sketching the pattern of Compli array just by inspection. So that it is an useful tool in the design of antenna arrays. → The width of the principle lobe (i.e., width b/w Nulls) & the Corre width of the array pattern are same. → The secondary lobes are determined from the number of nulls in the resultant pattern → In the resultant pattern the no. of nulls are the sum of nulls of individual pattern & array pattern.

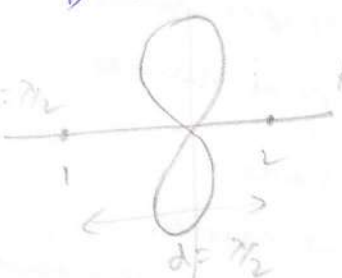
$\cos\left(\frac{\beta d \cos\theta}{2}\right)$ (2)

$\cos\left(\frac{\frac{2\pi}{\lambda} \cdot \lambda \cdot \cos\theta}{2}\right) = \cos(\pi \cos\theta)$

Note:- Sketch the radiation pattern of two isotropic point sources seperat[ed]

by, i) $d = \lambda/2$ (ii) $d = \lambda$ (iii) $d = 2\lambda$.

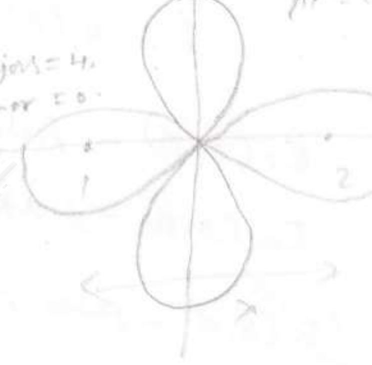
i) $d = \lambda/2$



$AF = \cos\left(\frac{\pi}{2}\cos\theta\right)$

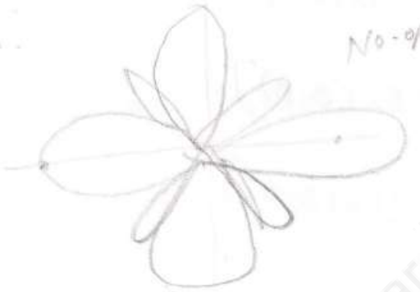
No. of majors = 2
Minors = 0

ii) $d = \lambda$
No of majors = 4,
minor = 0.



$AF = \cos(\pi \ldots$

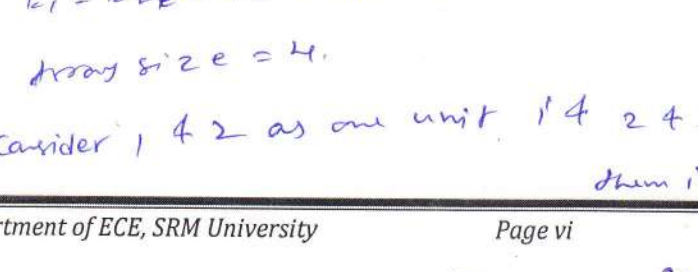
iii) $d = 2\lambda$.



No. of majors = 4
minors = 4.

Radiation pattern of 4-isotropic elements fed in phase spaced $\lambda/2$ apart:-

Consider 4 isotropic sources 1, 2, 3, 4 which are feed in phase & spacing b/n them is $d = \lambda/2$ along z-axis as shown,



4-isotropic elements spaced $\lambda/2$.

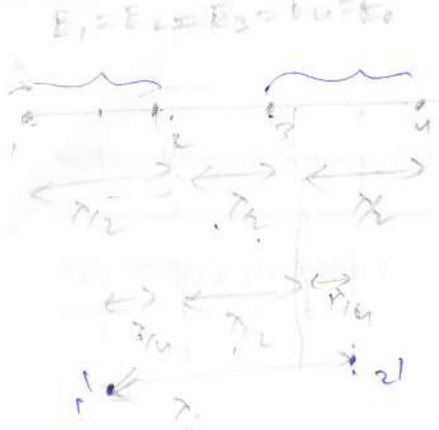
$E_1 = E_2 = E_3 = E_4 = E_0$.

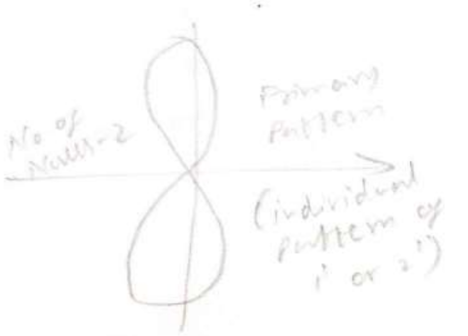
Array size = 4.

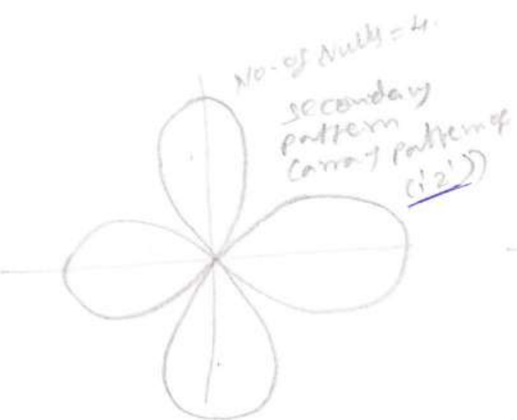
Now consider 1 & 2 as one unit 1' & 2 & 3 as another unit 2' & spacing b/n them is now, λ, as shown below.

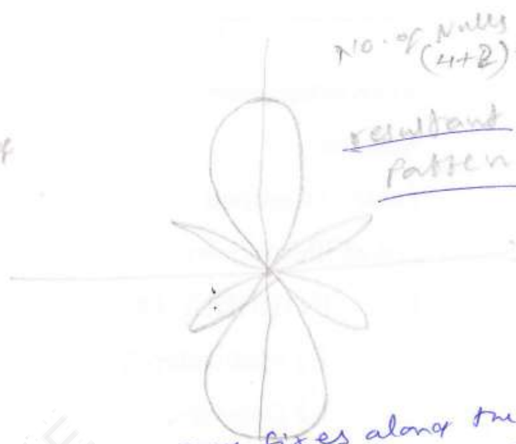
4E → 2, 5,
83, 5K).
8)
72, 6

Now consider array (1'2') where primary pattern, secondary pattern & the resultant pattern from principle of pattern multiplication is shown in figure.





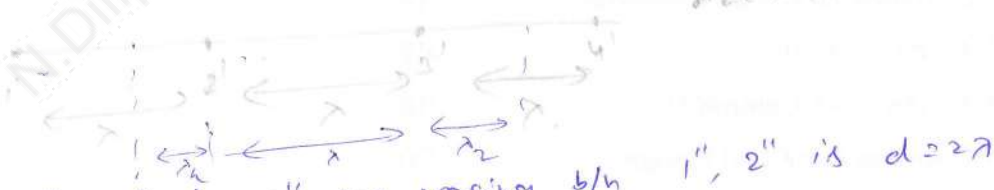




As the array fires along the broad side it is called as B... of size

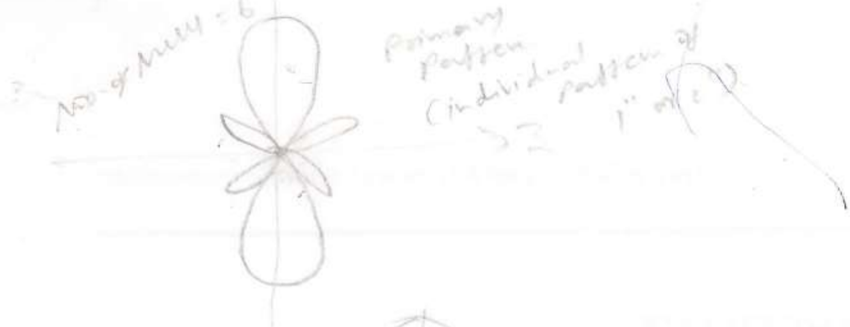
Radiation pattern of 8-isotropic elements fed in phase, spaced $\lambda/2$ apart:-

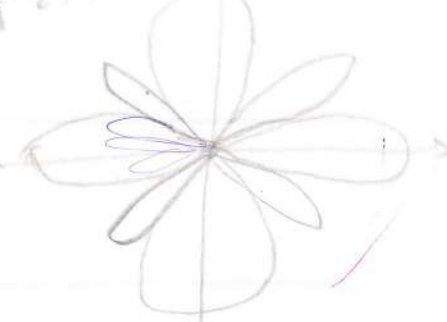
Consider 1, 2, as 1'; 3, 4 as 2'; 5, 6 as 3' & 7, 8 as 4' where spacing b/n 1', 2', 3', ... is $d = \lambda$.

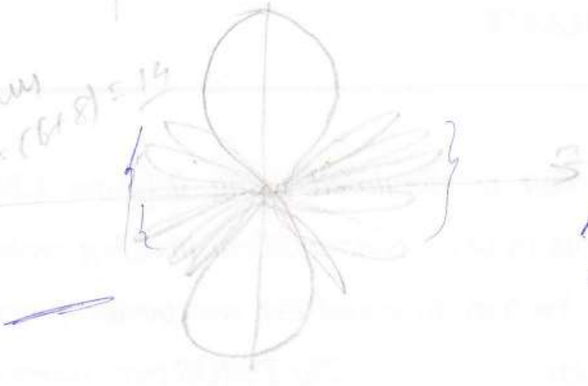


Consider 1', 2' as 1'', 3', 4' as 2'' where spacing b/n 1'', 2'' is $d = 2\lambda$.

The individual pattern of 1'' or 2'' is primary pattern & pattern of array of 2-isotropic sources is secondary pattern ⇒ From the principle of pattern multiplication, the resultant field pattern of (1'' 2'') (or) (1' 2' 3' 4') (or) (1 2 3 4 5 6 7 8) is multiplication of primary pattern & secondary pattern.



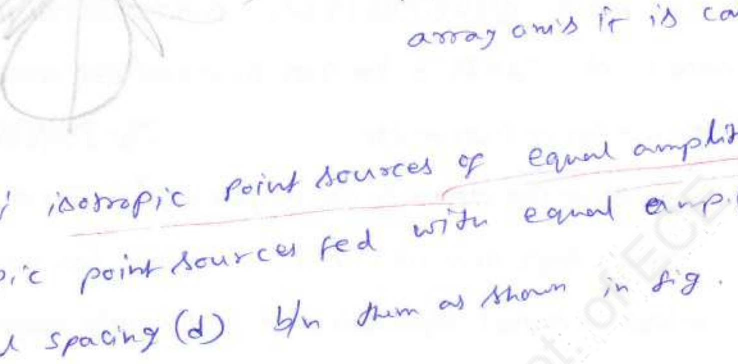




As the array fires along the broad side of array axis it is called as B.S.A of size 8.

Linear array with 'N' isotropic point sources of equal amplitude & spacing :-

→ Consider N isotropic point sources fed with equal amp. (E_0) & placed along Z-axis with equal spacing (d) b/n them as shown in fig. Take source 1 as reference point for phase.



Linear array with 'N' isotropic point sources with equal amp. & Spacing

$E_1 = E_2 = E_3 = \cdots E_N = E_0$.

& $\psi = \beta d\cos\theta + \alpha$.

ψ = Total phase difference of the fields at point 'p'.

α = phase difference two adjacent point sources

$$E_1 = E_1 e^{j0} = E_0.$$
$$E_2 = E_2 e^{j\psi} = E_0 e^{j\psi}$$
$$E_3 = E_3 e^{2j\psi} = E_0 e^{2j\psi}$$
$$\vdots$$
$$E_N = E_N e^{j(N-1)\psi} = E_0 e^{j(N-1)\psi}.$$

Total far electric field pattern at a distant point 'p' is obtained by adding ~~vertically~~ vectorially the fields of individual sources as,

$$E = E_0 + E_0 e^{j\psi} + E_0 e^{j2\psi} + \cdots + E_0 e^{j(N-1)\psi}.$$

$$= E_0\left[1 + e^{j\psi} + e^{j2\psi} + \cdots + e^{j(N-1)\psi}\right]$$

$$= E_0\left[\frac{1-\left(e^{j\psi}\right)^N}{1-\left(e^{j\psi}\right)}\right] \qquad \because S_{GP'N'} = a\left[\frac{1-r^N}{1-r}\right].$$

$$= E_0\left[\frac{1-\left(e^{j\psi}\right)^N}{1-e^{j\psi}}\right] \Rightarrow E_0\left[\frac{1-e^{jN\psi}}{1-e^{j\psi}}\right]$$

$$= E_0\left[\frac{e^{jN\psi/2}e^{-j\psi N/2} - e^{jN\psi/2}e^{jN\psi/2}}{e^{j\psi/2}e^{-j\psi/2} - e^{j\psi/2}\cdot e^{j\psi/2}}\right]$$

$$= E_0\,\frac{-e^{j\psi N/2}}{-e^{j\psi/2}}\left[\frac{e^{jN\psi/2} - e^{-jN\psi/2}}{e^{j\psi/2} - e^{-j\psi/2}}\right]$$

multiply & divide by $2j$,

$$= E_0\, e^{\frac{j(N-1)\psi}{2}}\left[\frac{\left(e^{jN\psi/2} - e^{-jN\psi/2}\right)}{\left(e^{j\psi/2} - e^{-j\psi/2}\right)}\right]\times\frac{j2}{j2}$$

$$= E_0\, e^{\frac{j(N-1)\psi}{2}}\left[\left(\frac{e^{jN\psi/2} - e^{-jN\psi/2}}{2j}\right)\Big/\left(\frac{e^{j\psi/2} - e^{-j\psi/2}}{j2}\right)\right]$$

$$= E_0\, e^{\frac{j(N-1)\psi}{2}}\left[\frac{\sin N\psi/2}{\sin \psi/2}\right]$$

→ If the reference point is shifted to the centre of the array then $\frac{(N-1)\psi}{2} =$

$$E = E_0\left[\frac{\sin\left(\frac{N\psi}{2}\right)}{\sin \psi/2}\right]$$

$$= E_0\,(AF).$$

$$AF = \left[\frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\psi/2\right)}\right].$$

$$(AF)_{Norm} = \frac{AF}{(AF)_{max}}$$

$$(AF)_{max} = N.$$

$$(A.F)_{Norm} = \frac{\sin\left(\frac{N\psi}{2}\right)}{N\sin\left(\psi/2\right)}$$

$$E_{max} = E_0\cdot(A.F)_{max} = E_0\cdot N.$$

Array of 'N' isotropic sources of equal amp. & spacing (B.S Case)

→ A Linear array is said to be B.S. array if it fires (max. radiation occurs) along the Broad side (Normal to the array axis) i.e., $\theta = 90°$

→ In Broad side array all the sources are in phase. i.e., $\alpha = 0$,

$$\psi = \beta d \cos\theta + \alpha.$$

$$= \beta d \cos\theta + 0.$$

for maxima to occur,

$$\psi = 0 \Rightarrow \beta d \cos\theta = 0$$

$$\cos\theta = 0$$

$$\theta = 90° \text{ (or) } 270°.$$

Directions of principal maxima:-

principal Maxima (or) major lobe maxima occurs at,

$$\theta = (\theta_{max})_{major} = \pm 90°.$$

Directions of pattern maxima:- also called as Minor lobe maxima (or) $(\theta_{max})_{minor}$

Pattern maxima occurs b/n first Nulls & high order Nulls..

The minor lobe maxima

$$\text{Now, } E = E_0 \cdot \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\psi/2\right)}$$

from S.A. Schelkunoff procedure, E is maximum only when $\sin\left(\frac{N\psi}{2}\right)$ is maximum provided that $\sin\left(\frac{\psi}{2}\right) \neq 0$.

$$\therefore \sin\left(\frac{N\psi}{2}\right) = 1.$$

$$\frac{N\psi}{2} = \pm (2n+1)\pi/2 \quad \text{for } n = 0, 1, 2, \ldots$$

$$\psi = \pm \frac{(2n+1)\pi}{N}.$$

$$\beta d \cos\theta + \alpha = \pm \frac{(2n+1)\pi}{N}.$$

$$\beta d \cos(\theta_{max})_{minor} = \pm \frac{(2n+1)\pi}{N} - \alpha.$$

$$\cos\left(\theta_{max}\right)_{minor} = \frac{1}{\beta d}\left[\pm\frac{(2n+1)\pi}{N} - \alpha\right]$$

$$\left(\theta_{max}\right)_{minor} = \cos^{-1}\left\{\frac{1}{\beta d}\left[\pm\frac{(2n+1)\pi}{N} - \alpha\right]\right\}$$

For broad side array, $\alpha = 0$,

$$\left(\theta_{max}\right)_{minor} = \cos^{-1}\left\{\frac{1}{\beta d}\left[\pm\frac{(2n+1)\pi}{N}\right]\right\}$$

$$= \cos^{-1}\left\{\frac{\lambda}{2\pi d}\left[\pm\frac{(2n+1)\pi}{N}\right]\right\}$$

$$\left(\theta_{max}\right)_{minor} = \cos^{-1}\left\{\pm\frac{(2n+1)\lambda}{2Nd}\right\}$$

for $n = 1, 2, \ldots$

Note :- for $n=0 \Rightarrow \theta = \left(\theta_{max}\right)_{major}$

for $n = 1, 2, 3 \ldots \Rightarrow \theta = \left(\theta_{max}\right)_{minor}$.

Direction of pattern minima:-

A/c to S.A. Schelkunoff procedure, the directions of minima of mi lobes in the array of N isotropic sources of equal amp. & phase is giv

$$E = \frac{E_0 \sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}$$

$\sin\left(\frac{N\psi}{2}\right) = 0$ provided that $\sin\left(\frac{\psi}{2}\right) \neq 0$.

$$\sin\left(\frac{N\psi}{2}\right) = 0.$$

$$\frac{N\psi}{2} = 0.$$

$$\frac{N\psi}{2} = \pm n\pi \quad \text{for } n = 1, 2, 3 \ldots$$

$$\psi = \pm\frac{2n\pi}{N}.$$

$$\beta d\cos\theta + \alpha = \pm \frac{2n\pi}{N}$$

$$\beta d\left(\cos\theta_{min}\right)_{minor} + \alpha = \pm \frac{2n\pi}{N} \quad \left\{\because \alpha = 0^\circ\right\}$$

$$\cos\left(\theta_{min}\right)_{minor} = \frac{1}{\beta d}\left\{\pm \frac{2n\pi}{N} - \alpha\right\}$$

$$\left(\theta_{min}\right)_{minor} = \cos^{-1}\left\{\frac{1}{\beta d}\left[\pm \frac{2n\pi}{N} - \alpha\right]\right\}$$

for Broad side, $\alpha = 0^\circ$

$$\left(\theta_{min}\right)_{minor} = \cos^{-1}\left[\frac{1}{\beta d}\left(\pm \frac{2n\pi}{N}\right)\right]$$

$$= \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(\pm \frac{2n\pi}{N}\right)\right]$$

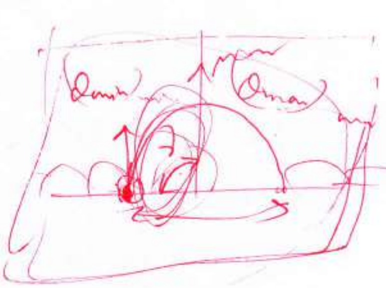
$$= \cos^{-1}\left[\pm \frac{2n\lambda}{2Nd}\right]$$

$$\left(\theta_{min}\right)_{minor} = \cos^{-1}\left[\pm \frac{n\lambda}{Nd}\right].$$

$\therefore$ At $\left(\theta_{min}\right)_{minor}$ minor lobe minima occurs.

Beam width of major lobe (FNBW & HPBW):-

It is defined as the angle b/n the first nulls (or) Twice the angle b/n the first null & major lobe maxima direction (γ)



i.e., $FNBW = 2\gamma$ & $HPBW = \frac{FNBW}{2}$.

$\therefore \gamma = 90 - \left(\theta_{min}\right)_{minor}$. apply sine on b.s.

$$\sin\gamma = \sin\left[90 - \left(\theta_{min}\right)_{minor}\right]$$

$$= \cos\left(\theta_{min}\right)_{minor}.$$

$$= \pm \frac{1}{\beta d} \frac{2n\pi}{N}$$

[$\because n = 1$ for first null].

$$k = \frac{2\pi}{\lambda}$$

$$= \pm \frac{2\pi}{\beta d N}$$

$$= \pm \frac{2\pi \cdot \lambda}{2\pi \cdot d \cdot N} = \pm \frac{\lambda}{Nd}$$

If array size is large such that $L = (N-1)d \approx Nd$. then $\pm \frac{\lambda}{Nd}$ is small,

$$\sin\gamma \simeq \gamma = \pm \frac{\lambda}{L} \text{ radians.}$$

$$= \pm 57.3^\circ \frac{\lambda}{L}$$

$$\frac{114.6^\circ}{16 \cdot (\lambda/4)/\lambda}$$

$$FNBW = 2\gamma$$

$$= 2\left(\pm 57.3^\circ \frac{\lambda}{L}\right)$$

$$FNBW = \pm \frac{114.6^\circ}{(L/\lambda)}$$

$$HPBW = \frac{FNBW}{2} = \pm \frac{57.3^\circ}{(L/\lambda)}$$

$$\frac{57.3}{(Nd/\lambda)}$$

$$HPBW = \pm \frac{57.3^\circ}{(L/\lambda)}$$

$$\pm \frac{57.3}{64 \cdot (\lambda/4)/\lambda}$$

First null occurs when $n=1$,

$$\pm \frac{57.3}{16}$$

$$\boxed{\gamma_1 = \pm \frac{\lambda}{Nd}}$$

$$\pm 3.58$$

$$\pm 7.16$$

$$FNBW = 2\gamma_1 = \frac{2\lambda}{Nd}$$

Directivity:-

$$E = E_0 \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}$$

$$E_{max} = E_0 \cdot N. \qquad \left[\because E_{max} = E_0 (AF)_{max}\right].$$

$$E_{norm} = \frac{E}{E_{max}} = \frac{\sin\left(\frac{N\psi}{2}\right)}{N \sin\left(\frac{\psi}{2}\right)}$$

$$\psi = \beta d \cos\theta$$

$$L = (N-1) d \simeq Nd.$$

If the spacing b/n elements is small when compared to the overall length of the array of large size (N), then 'd' & hence ψ also small

$$\sin\frac{\psi}{2} \simeq \frac{\psi}{2}.$$

$$E_{norm} = \frac{\sin\left(\frac{N\psi}{2}\right)}{N \cdot \frac{\psi}{2}}$$

$U = Pr^2$

$U \propto P$

$P = \frac{E^2}{2\eta}$ / $P \propto$

$$|E_{norm}|^2 = \left[\frac{\sin\left(\frac{N\psi}{2}\right)}{\frac{N\psi}{2}}\right]^2$$

$$W_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} U \sin\theta \, d\theta \, d\phi.$$

$$= \int_0^{2\pi}\int_0^{\pi} \left[\frac{\sin\left(\frac{N\beta d\cos\theta}{2}\right)}{\left(\frac{N\beta d\cos\theta}{2}\right)}\right]^2 \sin\theta \, d\theta \, d\phi.$$

$$= 2\pi \int_0^{\pi} \left[\frac{\sin\left(\frac{N\beta d\cos\theta}{2}\right)}{\left(\frac{N\beta d\cos\theta}{2}\right)}\right]^2 \sin\theta \, d\theta$$

$$Z = \frac{N\beta d\cos\theta}{2} \text{ then}$$

$$dZ = -\frac{N\beta d\sin\theta}{2}\, d\theta$$

$$\sin\theta\, d\theta = \frac{-2}{N\beta d}\, dZ$$

upper limit $\Rightarrow \theta = 0 \Rightarrow Z = \frac{N\beta d}{2}$

lower limit $\Rightarrow \theta = \pi \Rightarrow Z = -\frac{N\beta d}{2}$

$$W_{rad} = 2\pi \int_{\frac{N\beta d}{2}}^{-\frac{N\beta d}{2}} \left(\frac{\sin Z}{Z}\right)^2 \left(\frac{-2}{N\beta d}\right) dZ$$

$\int \sin\theta\, d\theta = \int \frac{-2}{N\beta d} dZ$

$-\cos\theta = \frac{-2Z}{N\beta d}$

$\theta = 0, \quad Z = \frac{N\beta d}{2}$

$\theta = \pi, \quad Z = -\frac{N\beta d}{2}$

If L is very large then N d & hence,

$$\pm \frac{N\beta d}{2} \longrightarrow \infty$$

Now,

$$W_{rad} = 2\pi \left(\frac{-2}{N\beta d}\right) \int_{\infty}^{-\infty} \left(\frac{\sin Z}{Z}\right)^2 dZ$$

$$= 2\pi \left(\frac{2}{N\beta d}\right) \int_{-\infty}^{\infty} \left(\frac{\sin Z}{Z}\right)^2 dZ$$

$$= 2\pi \left(\frac{2}{N\beta d}\right) (\pi)$$

$$= 2\pi \left(\frac{2}{N\left(\frac{2\pi}{\lambda}\right) d}\right) \pi$$

$$= \frac{2\pi\lambda}{Nd}$$

$$\simeq \frac{2\pi\lambda}{L}$$

Now, $D = \frac{4\pi(1)}{\frac{2\pi\lambda}{Nd}} = \frac{2Nd}{\lambda} \simeq \frac{2L}{\lambda}$

$$\therefore D_{BSA} = \frac{2Nd}{\lambda} \approx \frac{2L}{\lambda}$$

$\frac{2(\cancel{64}^{16})\cancel{\lambda}/\cancel{4}}{\cancel{\lambda}}$

32

$10\log 32 =$

Array of N Sources of equal Amplitude & spacing (END-fire case):-

→ A Linear array is said to be an end fire array if it fires (max. radiation occurs) along the end fires (along array axis) of the array i.e., $\theta = 0°$ & $180°$.

Principal maxima ~~occurs~~ direction $(\theta_{max})_{major}$:-

principal maxima occurs at $\psi = 0$.

i.e., $\beta d \cos\theta + \alpha = 0$

if $\theta = 0°$, $\beta d \cos(0) + \alpha = 0 \Rightarrow \alpha = -\beta d$

if $\theta = 180°$, $\beta d \cos(180) + \alpha = 0 \Rightarrow \alpha = \beta d$.

$(\theta_{max})_{major} \Rightarrow$ Principal maxima (or) Major lobe maxima occurs,

$$(\theta_{max})_{major} = 0° \ \& \ 180°$$

→ Directional of pattern maxima (or) Minor lobe maxima (or) $(\theta_{max})_{minor}$:-

$$\psi = \beta d \cos\theta + \alpha$$

If $\theta = 0°$ then $\alpha = -\beta d$.

$$\psi = \beta d \cos\theta - \beta d = \beta d [\cos\theta - 1].$$

$$E = E_0 \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\psi/2\right)}$$

from S.A Schelkanoff procedure, E is maximum when, $\sin(\psi/2) \neq 0$.

$$\sin\left(\frac{N\psi}{2}\right) = 1.$$

$$\frac{N\psi}{2} = \pm(2n+1)\pi/2$$

$$\psi = \pm\frac{(2n+1)\pi}{N}$$

$$\beta d[\cos\theta - 1] = \pm\frac{(2n+1)\pi}{N}$$

$$\cos\theta - 1 = \frac{1}{\beta d}\left(\pm\frac{(2n+1)\pi}{N}\right)$$

$$\cos\theta = \pm\frac{(2n+1)\pi}{N\beta d} + 1.$$

$$\cos\left(\theta_{max}\right)_{minor} = \frac{\pm(2n+1)\pi}{N\beta d} + 1$$

$$\left(\theta_{max}\right)_{minor} = \cos^{-1}\left[\pm\frac{(2n+1)\pi}{N\beta d} + 1\right] \quad \text{for } n = 1, 2, \ldots$$

$\theta = 180°$, then $\alpha = +\beta d$ then,

$$\left(\theta_{max}\right)_{minor} = \cos^{-1}\left[\pm\frac{(2n+1)\pi}{N\beta d} - 1\right] \quad \text{for } n = 1, 2, \ldots$$

Directions of pattern minima (or) $\left(\theta_{min}\right)_{minor}$:-

$$E = E_0 \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}$$

From S.A. Schelkunoff procedure, E is max. when,

$$\sin\frac{N\psi}{2} = 0$$

$$\frac{N\psi}{2} = \pm n\pi \quad \text{for } n = 0, 1, 2, \ldots$$

$$\psi = \pm\frac{2n\pi}{N}$$

if $\theta = 0°$, $\alpha = -\beta d$.

$$\beta d\cos\theta - \beta d = \pm\frac{2n\pi}{N}$$

$$[\cos\theta - 1] = \left(\pm\frac{2n\pi}{N}\right)\left(\frac{1}{\beta d}\right)$$

$$\cos\theta = \pm\frac{2n\pi}{N\beta d} + 1$$

$$\theta = \cos^{-1}\left[\pm\frac{2n\pi}{N\beta d} + 1\right]$$

$\theta = \left(\theta_{min}\right)_{minor}$. minor lobe minima occurs,

$$\left(\theta_{min}\right)_{minor} = \cos^{-1}\left[\pm\frac{2n\pi}{N\beta d} + 1\right] \quad \text{for } n = 1, 2, \ldots$$

If $\theta = 180°$, $\alpha = +\beta d$ then,

$$\left(\theta_{min}\right)_{minor} = \cos^{-1}\left[\pm\frac{2n\pi}{N\beta d} - 1\right] \quad \text{for } n = 1, 2, \ldots$$

Beamwidth of major lobes:-

$$FNBW = \pm 114.6^\circ \sqrt{\frac{2}{(L/\lambda)}}$$

$$HPBW = \pm 57.3^\circ \sqrt{\frac{2}{(L/\lambda)}}$$

$57.3 \sqrt{\frac{2}{\frac{16\times\lambda}{\lambda}}}$

$57.3 \sqrt{2/16}$

$\sqrt{\frac{1}{8}}$

$\Rightarrow \pm 20.25$

Directivity:-

$$D = \frac{4Nd}{\lambda} \simeq \frac{4L}{\lambda}$$

$$D_{EFA} = 2\left(\frac{2L}{\lambda}\right) \simeq 2\, D_{BSA}.$$

Type of array — Directions of minor lobes maxima.

General case, $\left(\theta_{max}\right)_{min} = \cos^{-1}\left[\pm \frac{(2N+1)\pi}{N} - \alpha\right]\frac{1}{\beta d}$

Broad side $(\alpha = 0)$ $\left(\theta_{max}\right)_{min} = \cos^{-1}\left(\left[\pm \frac{(2n+1)\pi}{N}\right]\frac{1}{\beta d}\right)$

$$\left(\theta_{min}\right)_{min} = \cos^{-1}\left(\left[\pm \frac{2n\pi}{N}\right]\frac{1}{\beta d}\right)$$

BSA

$$\left(\theta_{max}\right)_{minor} = \cos^{-1}\left[\left(\pm\frac{(2n+1)\pi}{N}\right)\frac{1}{\beta d}\right]$$

$$\left(\theta_{min}\right)_{minor} = \cos^{-1}\left[\left(\pm\frac{2n\pi}{N}\right)\frac{1}{\beta d}\right]$$

$$FNBW = \frac{114.6^\circ}{(L/\lambda)} \quad ; \quad HPBW = \frac{53.6}{(L/\lambda)}$$

End fire case :-

$$\left(\theta_{max}\right)_{minor} = \cos^{-1}\left[\pm\frac{(2n+1)\pi}{N\beta d}+1\right] \qquad (\because \alpha = -\beta d)$$

$$= \cos^{-1}\left[\pm\frac{(2n+1)\pi}{N\beta d}-1\right] \qquad (\because \alpha = \beta d$$

$$\left(\theta_{min}\right)_{minor} = \cos^{-1}\left[\pm\frac{2n\pi}{N\beta d}+1\right] \qquad (\because \alpha = -\beta d)$$

$$= \cos^{-1}\left[\pm\frac{2n\pi}{N\beta d}-1\right] \qquad (\because \alpha = \beta d).$$

Prob's :- (B.S.A).

$N = 4 \quad d = \lambda/2$

$\left(\theta_{max}\right)_{minor}$:-

for $n=1 \Rightarrow \left(\theta_{max}\right)_{minor} = \cos^{-1}\left[\pm\frac{(2(1)+1)\pi}{4}\left(\frac{1}{\frac{2\pi}{\lambda}\left(\frac{\lambda}{2}\right)}\right)\right] = \cos^{-1}(\pm$

$= \pm 41.$
(or)
$\pm 138.$

$n=2 \Rightarrow \left(\theta_{max}\right)_{minor} = \cos^{-1}\left[\frac{1}{\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{2}\right)}\left(\pm\frac{(2\times2+1)\pi}{4}\right)\right] = \cos^{-1}[\pm 1.5]$

$\left(\theta_{min}\right)_{minor}$:-

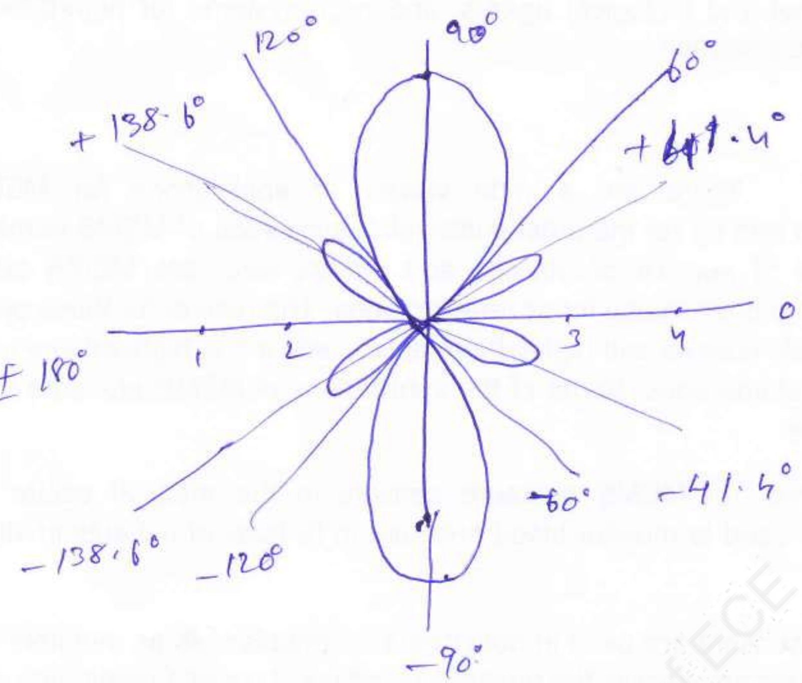
for $n=1$, $\Rightarrow \left(\theta_{min}\right)_{minor} = \cos^{-1}\left[\pm\frac{2(1)\pi}{4}\frac{1}{\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{2}\right)}\right] = \cos^{-1}\left(\frac{1}{2}\right) = \pm 60^\circ$ &
± 12

for $n=2$, $\left(\theta_{min}\right)_{minor} = \cos^{-1}\left[\frac{\pm 2(2)\pi}{4}\frac{1}{\pi}\right] = \cos^{-1}(\pm 1) = \pm 0^\circ$ & ± 180

for $n=3$, $\left(\theta_{min}\right)_{minor} = \cos^{-1}\left[\pm\frac{2(3)\pi}{4}\frac{1}{\pi}\right] = \cos^{-1}[\pm 1.5]$ = doesn't exi

Here there are six minor lobe minima occurs at $\theta = 0, 180°, 60°, 120°$

$-60°, -120°$ for the array of 4 isotropic point sources spaced $\lambda/2$ a



$N = 8;\ d = \lambda/2.$

$(\theta_{max})_{minor}$ for $n=1 \Rightarrow (\theta_{max})_{minor} = \cos^{-1}\left[\frac{1}{\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{2}\right)}\left(\pm\frac{(2(1)+1)\pi}{8}\right)\right]$

$= \cos^{-1}\left(\pm\frac{3}{8}\right)$

$n=2 \Rightarrow (\theta_{max})_{minor} = \cos^{-1}\left(\pm\frac{5}{8}\right) = \pm 51.3, \pm 128.6.$

$n=3 \Rightarrow (\theta_{max})_{minor} = \cos^{-1}\left(\pm\frac{7}{8}\right) = \pm 28.9°, \pm 151.0°$

$n=4 \Rightarrow (\theta_{max})_{minor} = \cos^{-1}\left(\pm 9/8\right) =$ Does not exist.

$(\theta_{min})_{minor}$:-

for $n=1 \Rightarrow (\theta_{min})_{minor} = \cos^{-1}\left(\frac{1}{\pi}\left(\pm\frac{2\times1\times\pi}{8}\right)\right)$

$= \cos^{-1}(\pm 1/4) \Rightarrow \pm 75.5, \pm 104.4°$

$n=2 \quad \Rightarrow \cos^{-1}\left(\pm\frac{1}{2}\right) = \pm 60°, \pm 120°$

$n=3 \quad \Rightarrow \cos^{-1}\left(\pm 3/4\right) = \pm 41.4°, \pm 138.5°$

$n=4 \quad \Rightarrow \cos^{-1}(\pm 1) = \pm 0, \pm 180°$

$n=5 \quad \Rightarrow \cos^{-1}(\pm 1.25) =$ does not exist.

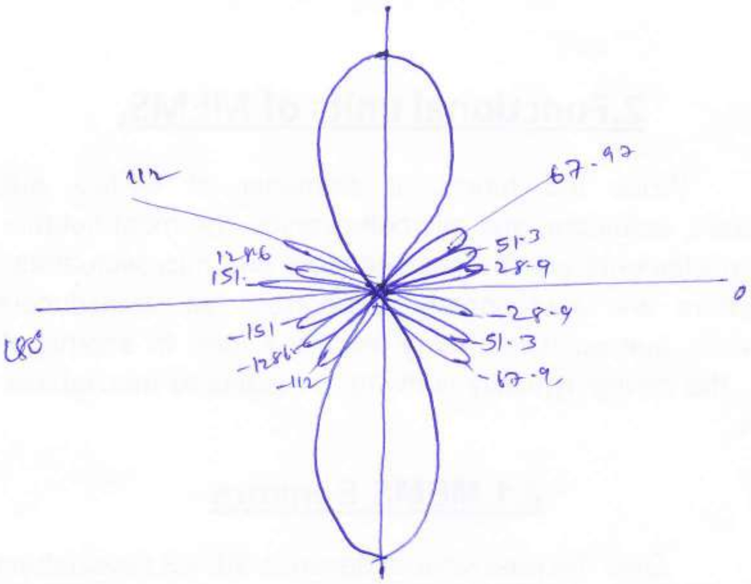
17

± 67.97

$\pm 112.0^\circ$

$\pm 51.3, \pm 128$

$\pm 28.9, \pm 151$



Array factor :-

$$(AF)_{Norm} = \frac{\sin\frac{N\psi}{2}}{N\sin\left(\frac{\psi}{2}\right)}$$

for BSA :- [90° or 270°]

$$\psi = kd\cos\theta + \beta \Rightarrow 0$$

$$(AF)_\theta = max. = N$$

$$\beta =$$

$$kd\cos\theta = 0$$

$$\cos\theta = 0$$

$$\theta = 90^\circ$$

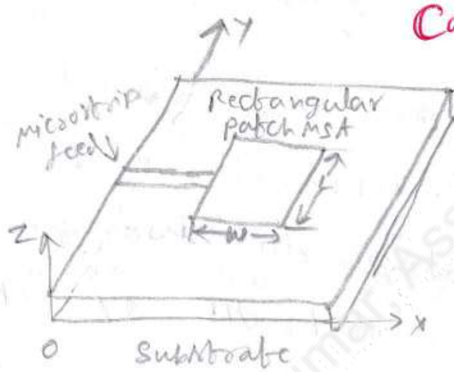
Micro Strip Antennas:-

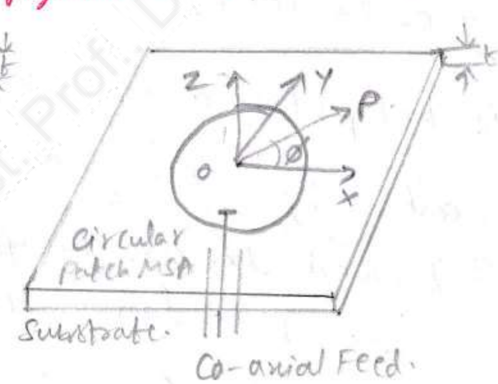
Patch antennas are assigned different names such as printed antennas, microstrip patch antennas or Simply microstrip antennas (MSA). MSA are often used where thickness & Conformability to the host surfaces are the key requirements.

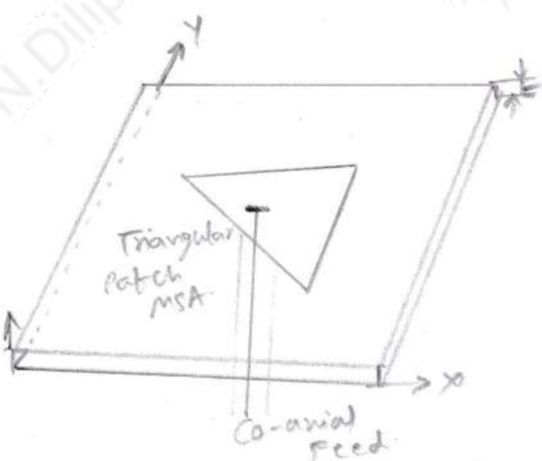
Since Patch antennas can be directly printed onto a circuit board, these are becoming increasing popular with in the mobile phone market. They are low cost, have a low profile & easily fabricated.

Sailent Features of Microstrip antennas:-

Configurations of Rectangular Circular & Triangular





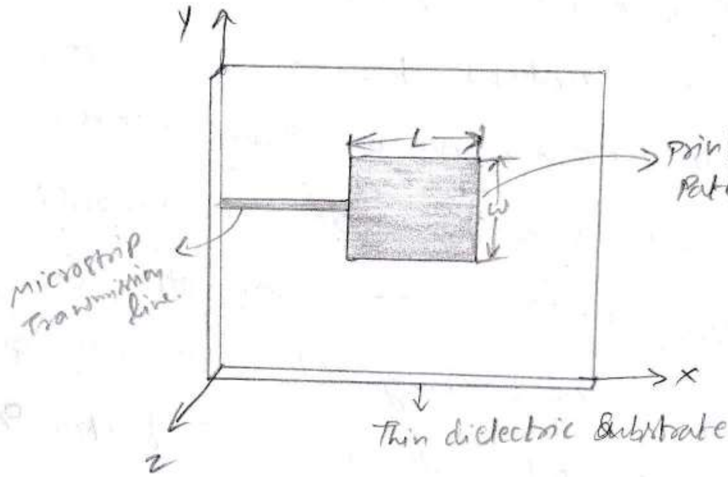


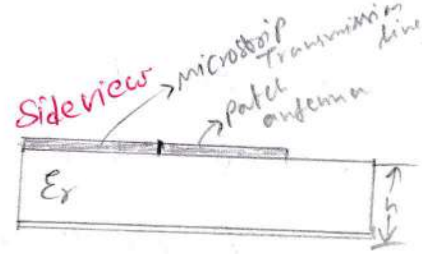
1) A patch antenna basically a metal patch suspended over a ground plane. The assembly is usually contained in a plastic radome, which protects the structure from damage. Patch antennas are simple to fabricate, easy to modify, & Customize & Closely related to MSA. These are ... a dielectric substrate, usually employing the same sort of

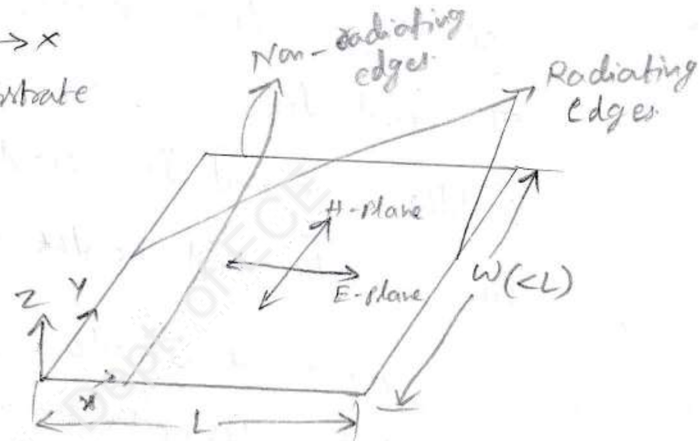
2) In its most basic form, a microstrip patch antenna consists of a radiating patch on one side of a dielectric substrate which has a ground plane on the other side. The simplest patch antenna uses a half-wavelength-long patch with a larger ground plane to give better performance but at the cost of larger antenna size. The ground plane is normally modestly larger than the active patch. The current flow is along the direction of the feed wire, so the vector potential & thus the electric field E follows the current. Such a simple patch antenna radiates a linearly polarized wave. The radiation at top & bottom or equivalently as a result of the current flowing on the patch & the ground plane.

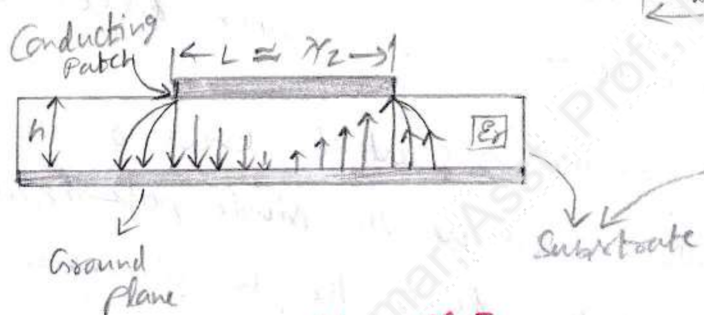
3) A patch antenna is a narrowband, wide-beam antenna fabricated by etching the antenna element pattern in metal trace bonded to an insulating dielectric substrate with a continuous metal layer bonded to the opposite side of the substrate which forms a ground plane.

Rectangular Microstrip Antennas:-

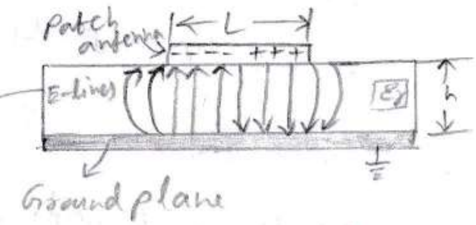








Sinusoidal variation of E



Uniform variation of E.

Microstrip (or) patch antennas are popular for low profile applications at frequencies above 100MHZ (or) $\lambda_0 < 3m$. In space (or) aircraft applications where size, cost, weight, performance, ease of installation & aerodynamic profile are constraints low profile antennas are required. In order to meet these specifications, Microstrip (or) patch antennas are used.

These antennas can be flush-mounted to metal or other existing surfaces & they only require space for the feed line which is normally placed behind the ground plane.

They usually consists of a Rectangular Metal patch on a dielectric - Coated ground plane. Microstrip antenna consists of a very thin metallic strip (patch) $t << \lambda$ placed on a small fraction of wavelength $h << \lambda$. above a ground plane.

The Radiating patch may be square, Rectangular, thin strip (or) dipole, circular, Elliptical, Triangular or any other Configuration. The feed line is also a conducting strip normally of smaller width. Coaxial line feeds where the inner Conductor of the co-axial line is attached to the radiating path are widely used.

Linear & Circular polarization can be achieved with microstrip (or) patch antennas & arrays of microstrip elements with single or multiple feeds may be used for greater directivity.

The radiating edges are at the ends of the L-dimension of the rectangle, which sets up the single polarization. Radiation (if any) that occurs at the ends of the W-dimension is far less & is referred to as the cross polarization.

Due to half-wave nature of the patch, the fields under the L-edges are of opposite polarity & when the field lines curve out they are now in the same direction & finally propagate out into the direction normal to the substrate.

In the far field ter to the substrate, the radiation from the two sides adds up because the fields are in phase. It can be seen that in direction of off-bore-sight the intensity drops as the fields of the two edges go farther & farther and out of phase.

At two angles, the fields exactly cancel. Thus, the microstrip patch radiation intensity depends on the direction it is viewed from as it has gain & directivity. For effective radiation from a microstrip antenna, the structure needs to be a half-wavelength resonator with a thicker dielectric material of low dielectric constant under the patch but the height still needs to be a fraction of a wavelength.

The wave generated with in the dielectric substrate (b/n the patch & ground plane) undergo reflections to some extent when they arrive at the edge of the strip, resulting radiation of only small fraction of the incident energy. ∴, the antenna is considered to be very inefficient & it behaves more like a cavity rather than a radiator.

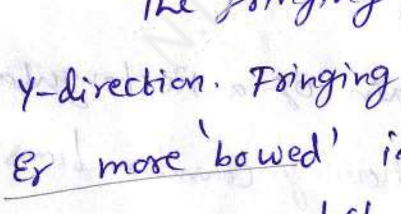
Characteristics of Microstrip Antenna:-

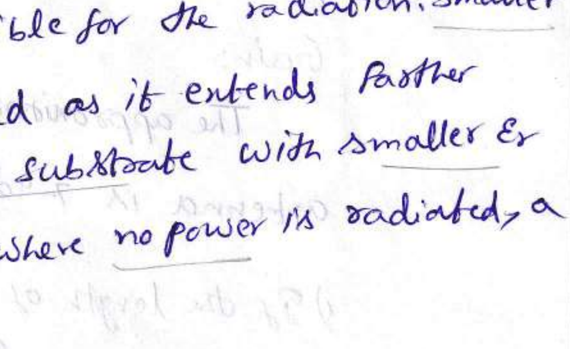
Radiation Pattern:-

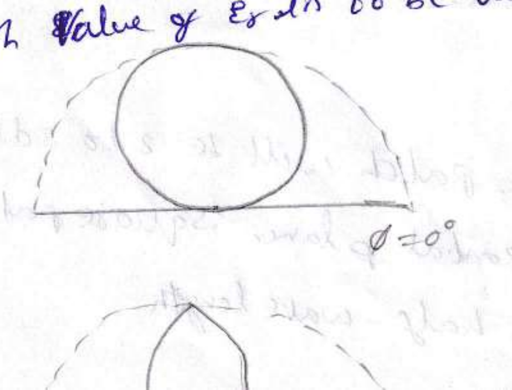
The figure shows two Radiation Patterns in $\phi = 0°$ (in azimuth) & $\phi = 90°$ (in elevation). The power fig (c) shows the Radiation pattern of linearly polarized patch Antenna. The power radiated at 180° is about 15dB less than the power in the center of the beam i.e., at 90°. The beamwidth is about 65° & the gain is about 9dBi. An infinitely large ground plane would prevent any back radiation, but the real antenna has a fairly small ground plane, & the power in the back ~~board~~ is only about 20dB. down from that in the main direction beam.

The current will be zero (theoretically) zero at the (open circuit) ends of the patch & maximum at the center of the Half-wave patch. Since the patch is a conductor, the voltage & current are out of phase. Voltage will be maximum at the end of the patch & minimum at ~~the~~ mid-point.

The fringing field near the surface of the patch is in the Y-direction. Fringing field is responsible for the radiation. Smaller the ε_r more 'bowed' is the fringing field as it extends farther away from the patch. ∴ use of a substrate with smaller ε_r yeilds better radiation. The locations where no power is radiated, a high ~~value~~ value of ε_r is to be used.







Beam width:-

The microstrip antennas in azimuthal & elevation planes have greater beam width.

Directivity:-

By the method of cavity model of the microstrip antenna, directivity for dominant mode $TM_{1,0}$ mode can be mathematically represented as,

$$D = \frac{2h^2 E_0^2 W'^2 K_0^2}{P_r \pi \eta_0}$$

$h \rightarrow$ Thickness of substrate
$P_r \rightarrow$ Radiated Power
$W' \Rightarrow W+h$
$K_0 \rightarrow$ Wave number.

E_0 is the magnitude of the Z-directed electric field intensity inside the cavity given by,

$$E_Z = E_0 \cos\frac{m\pi x}{L} \cos\frac{n\pi y}{W}$$

Where W - width of the patch along Y-axis.
L - Length of the patch along X-axis.

Gain:-

The approximate value of gain of a Rectangular microstrip antenna is 7-9dB under the following considerations.

i) If the length of patch is half of wave length, then gain of the patch from the directivity relative to the vertical axis is normally 2dB.

ii) Gain of the square shape patch will be 2 to 3dB & pattern will be directional in horizontal plane. Square patch is equivalent to the pair of dipoles with half-wave length.

iii) The average power of over all directions is reduced by the factor '2' because all the radiations behind the antenna neglected by the additional ground plane. Thus reduction of average power results increase in gain by 3dB.

Bandwidth:-

The bandwidth of the microstrip antenna is given as,

$$\text{Bandwidth} = \frac{S-1}{Q_0\sqrt{S}}$$

where, S = Voltage Standing Ratio

Q_0 = Unloaded Radiation Quality factor.

Quality factor 'Q' represents losses associated with antenna. If Quality factor increases, the bandwidth will be decreased.

Efficiency :-

Efficiency of the microstrip antenna is given by,

$$\eta = \frac{P_r}{P_c + P_d + P_r}$$

$P_r \rightarrow$ Radiated Power

$P_c \rightarrow$ Power dissipated due to loss.

$P_d \rightarrow$ Power dissipated due to dielectric.

Polarization:-

Patch antennas can be designed to exhibit various types of polarization such as horizontal polarization, vertical polarization, left hand circular polarization, Right hand circular polarization. Due to this unique inherent property microstrip ... in many types of Communication Links.

Return loss:-

It is reffered as the ratio of the fourier transforms of the incident signal to the reflected signal. The absolute value of magnitude of reflection Co-efficient is nothing but return loss expressed in dB.

i.e., $RL = 10 \log |\Gamma|$

where Γ is reflection Co-efficient

Impact of different parameters on Characteristics:-

Characteristics of the Microstrip antenna influenced by various parameters. They are L, W, h, A & ε_r. These parameters should be Considered properly for an efficient design.

Width 'W' of the patch Controls the input impedance & Radiation pattern & width can be given as,

$$W = \frac{C}{2f_0 \sqrt{(\varepsilon_R+1)/2}}$$

Length 'L' of the patch can be given as,

$$L \approx \frac{1}{2} f_c \sqrt{\varepsilon_0 \varepsilon_r \mu_0}$$

The fringing field of the substrate is Controlled by the permittivity ε_r. If ε_r is less, then more will be fringing, better will be the radiation pattern. It is also responsible for increase in Badwidth of the antenna & Can be given as,

$$B \propto \frac{(\varepsilon_r - 1)}{\varepsilon_r^2} \cdot \frac{W}{L} h.$$

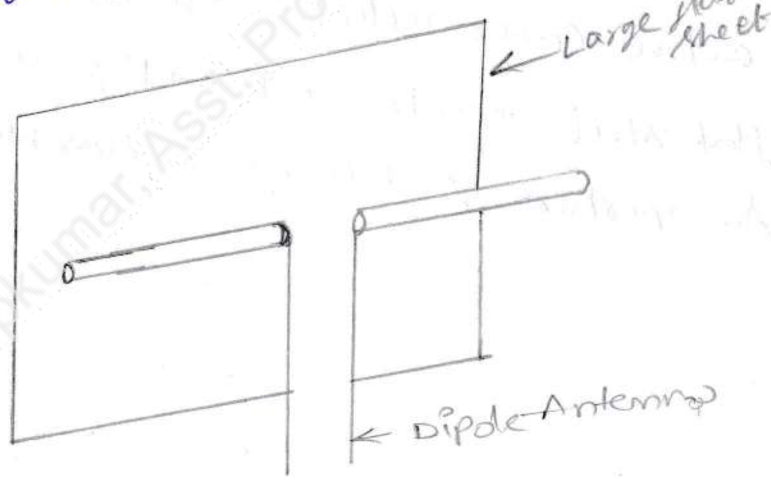
Reflectors:-

Reflector is a device that alters the radiation pattern of a radiating element in order to produce the desired beam with suitable Characteristics.

Types of Reflectors:

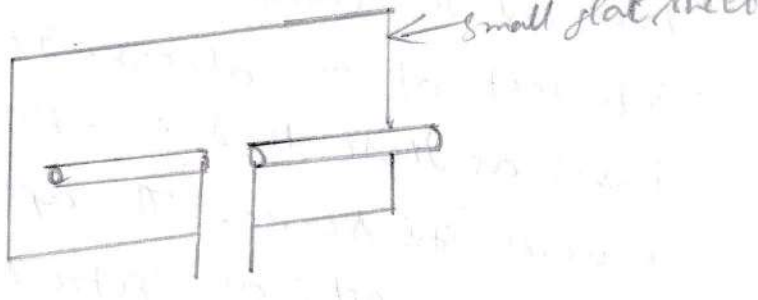
1) Large flat sheet Reflector:-

It Comprises of a large flat sheet Reflector placed with a dipole antenna. Increased gain can be achieved in the forward radiations, if the antenna is placed close to the sheet. The dipole antenna also helps in reducing Backward Radiations. It is sensitive to small frequency changes.



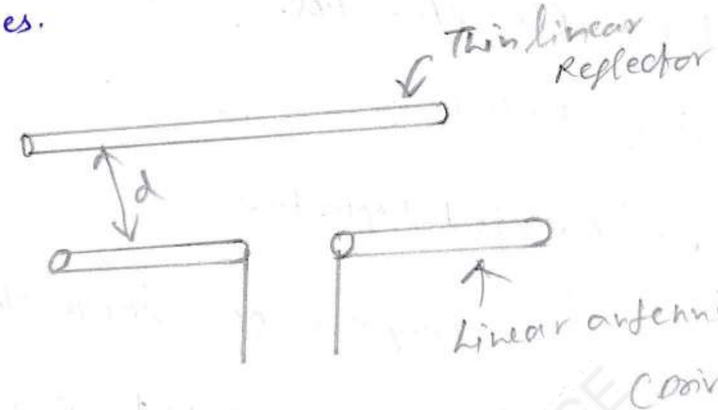
2) Small flat sheet Reflector:-

In this type, the size of the Reflector sheet is reduced in order to achieve the desired Radiation pattern.



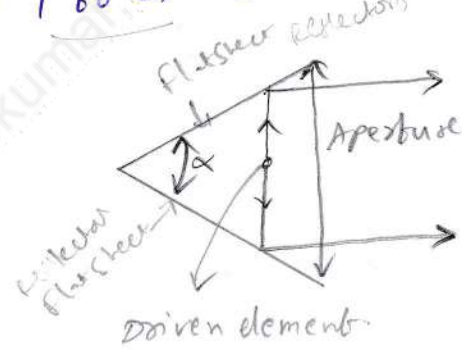
iii) Thin Linear Reflector:-

The reflector sheet is decomposed or reduced to a thin reflector element. This reflector is very sensitive to frequency changes.



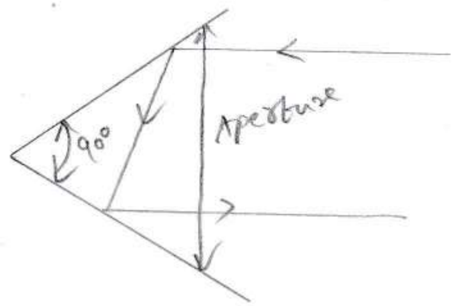
iv) Active Corner Reflector:-

In order to obtain a sharp radiation pattern, active Corner reflectors are used. These comprise of two flat sheet reflectors intersecting at an angle less than 180°. An aperture of 1 to 2λ is feasible in this type of reflector.



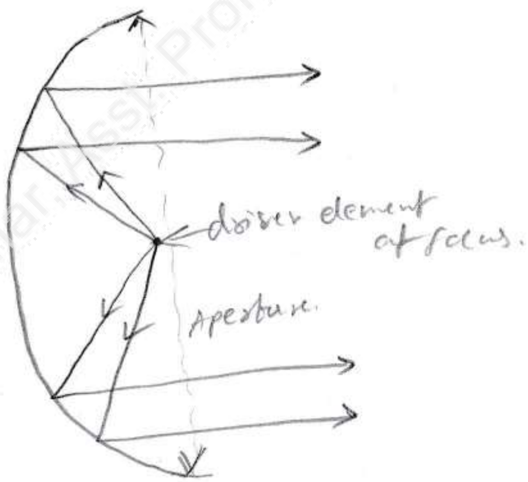
v) Passive Corner Reflector (or) Retro Reflector:-

It is similar to active Corner Reflector, the difference being that the exciting antenna is absent & the two reflectors intersect at 90° always. It is used as a target for radar waves as these tend to reflect back the incident wave towards its source. The aperture can be of many wavelengths. Thus it is also called as ... acts as "Retro Reflector".



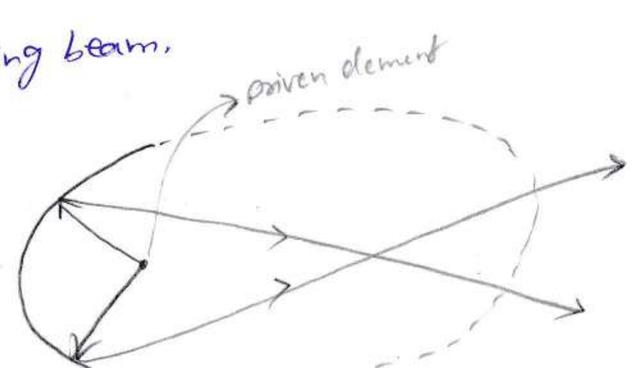
vi) Parabolic Reflector:-

These provide highly directional antennas as they can have aperture with many wave lengths. The parabolic reflectors basically transform a curved wavefront coming from the feed antenna which is placed at the focus to the plane wavefront as it reflects the waves that originate from the source at the focus.



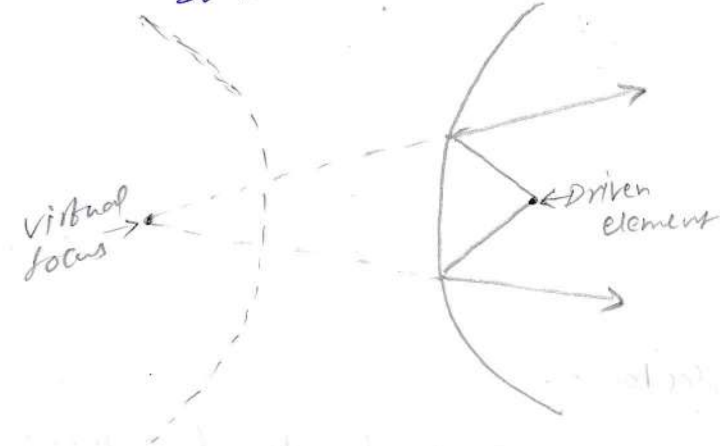
vii) Elliptical Reflector:-

In an elliptical reflector, all the reflected waves pass through the second focus of the ellipse leading to the production of a diverging beam.



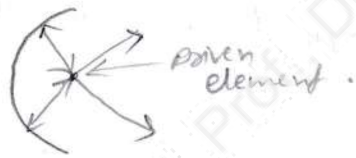
viii) Hyperbolic Reflector:-

It is also called as stavis-l Reflector.



ix) Circular reflector:-

Similar to Parabolic ~~fo~~ hyperbolic reflectors, circular reflectors are also used to obtain desired Radiations.

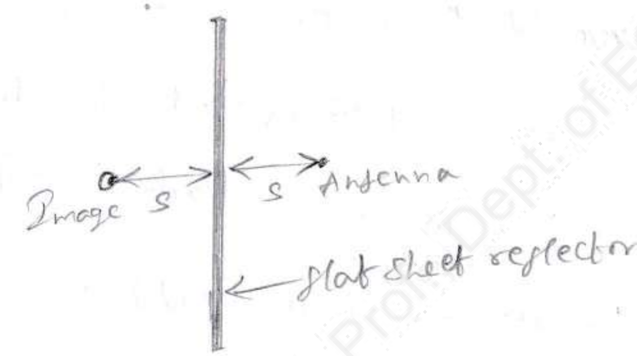


Flat Sheet Reflectors:-

(8)

These are some drawbacks associated with flat sheet reflectors when antenna is placed at a distance 's'. These drawbacks can be overcome by using the method of images.

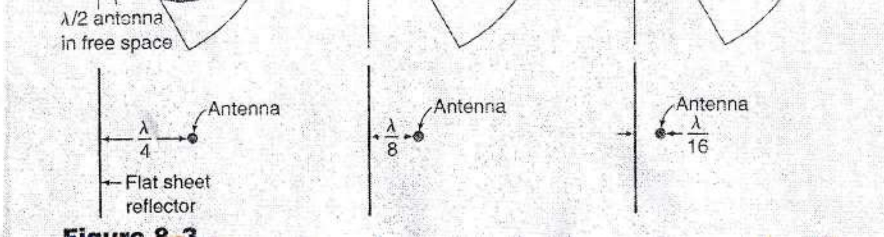
In this method the reflector is placed in between the antenna & image of the antenna in such a way that the image and antenna are '2s' apart & the reflector is at a distance 's' from both as shown.



Now, for an $\lambda/2$ dipole antenna, assuming zero reflector losses, the gain in field intensity of a $\lambda/2$ dipole antenna at a distance 's' from an infinite plane reflector is expressed as,

$$G_f(\phi) = 2\sqrt{\frac{R_{11}+R_L}{R_{11}+R_L-R_{12}}}\,\left|\sin(S_r \cos\phi)\right|$$

→ Now, the field patterns of $\lambda/2$ antennas at distance $S = \lambda/4, \lambda/8$ & $\lambda/16$ from the flat sheet reflector is shown in figure.



A large flat sheet reflector can convert a bidirectional antenna array into a unidirectional system.

Suppose when the reflecting sheet is reduced in size, the analysis is less simple. There are three principle angular regions.

Region 1 (above or front of the sheet):-

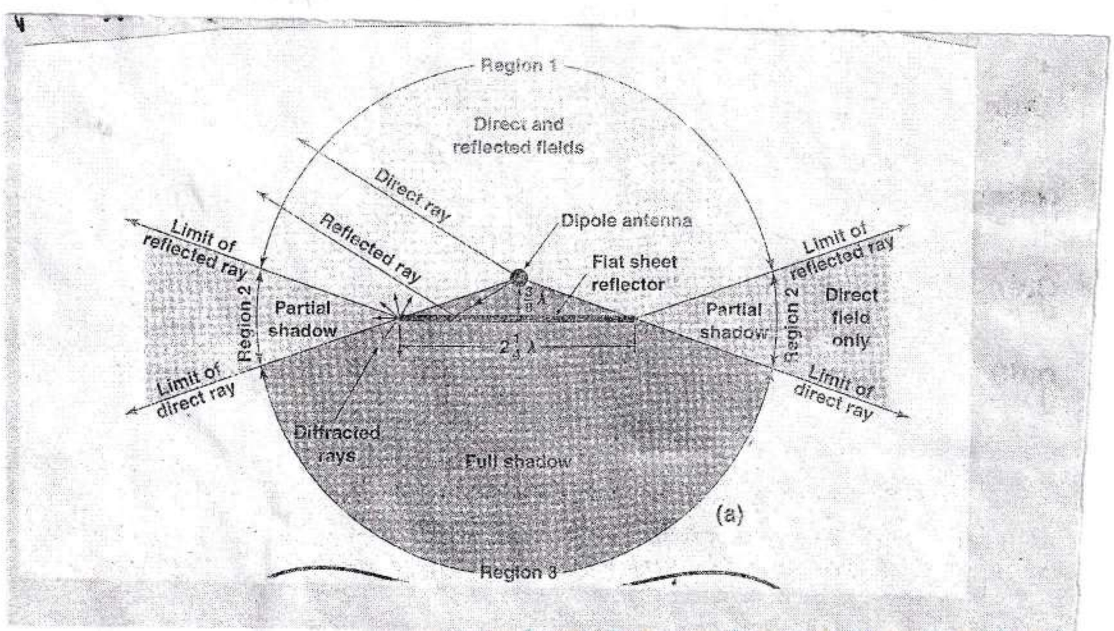
In this region the radiated field is given by the resultant of the direct field of the dipole & the reflected field from the sheet.

Region 2 (above & below at the sides of the sheet):-

In this region there is only the direct field from the dipole. This region is in the shadow of the reflected field.

Region 3 (below or behind the sheet):-

In this region the sheet acts as a shield, producing full shadow (No direct or Reflected fields, only diffracted fields).



(a)

(9)

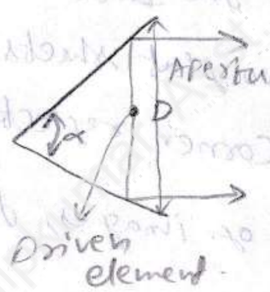
Corner Reflector Antenna:-

Two flat reflecting sheets intersecting at an angle 'α' (or) Corner are used as reflectors. Then such an arrangement is called as "Corner Reflector".

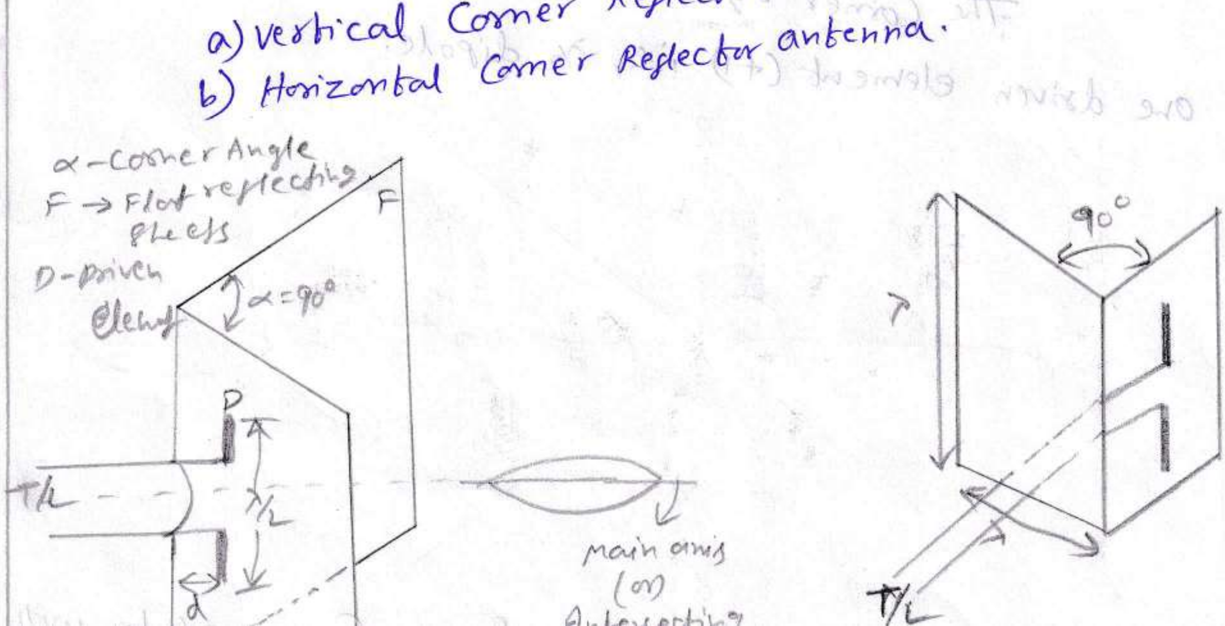
Types:-
1) Active Corner Reflector.
2) Passive Corner Reflector.

1) Active Corner Reflector:-

Active Corner Reflector antenna consists of a driven element & two flat conducting sheets. When the driven antenna is used in conjugation with a corner reflector, the arrangement is an effective directional antenna (Active antenna) for a wide range of corner angle $0 < \alpha < \pi$ as shown in fig.



Active Corner reflectors are classified into
a) vertical Corner Reflector antenna.
b) Horizontal Corner Reflector antenna.



When the Corner angle is π radians (or) $180°$, Corner reflector is equivalent to flat sheet as limiting case. The Corner Reflector antenna may be analysed by using the method of images for Corner angle

$$\alpha = \frac{180}{n}.$$

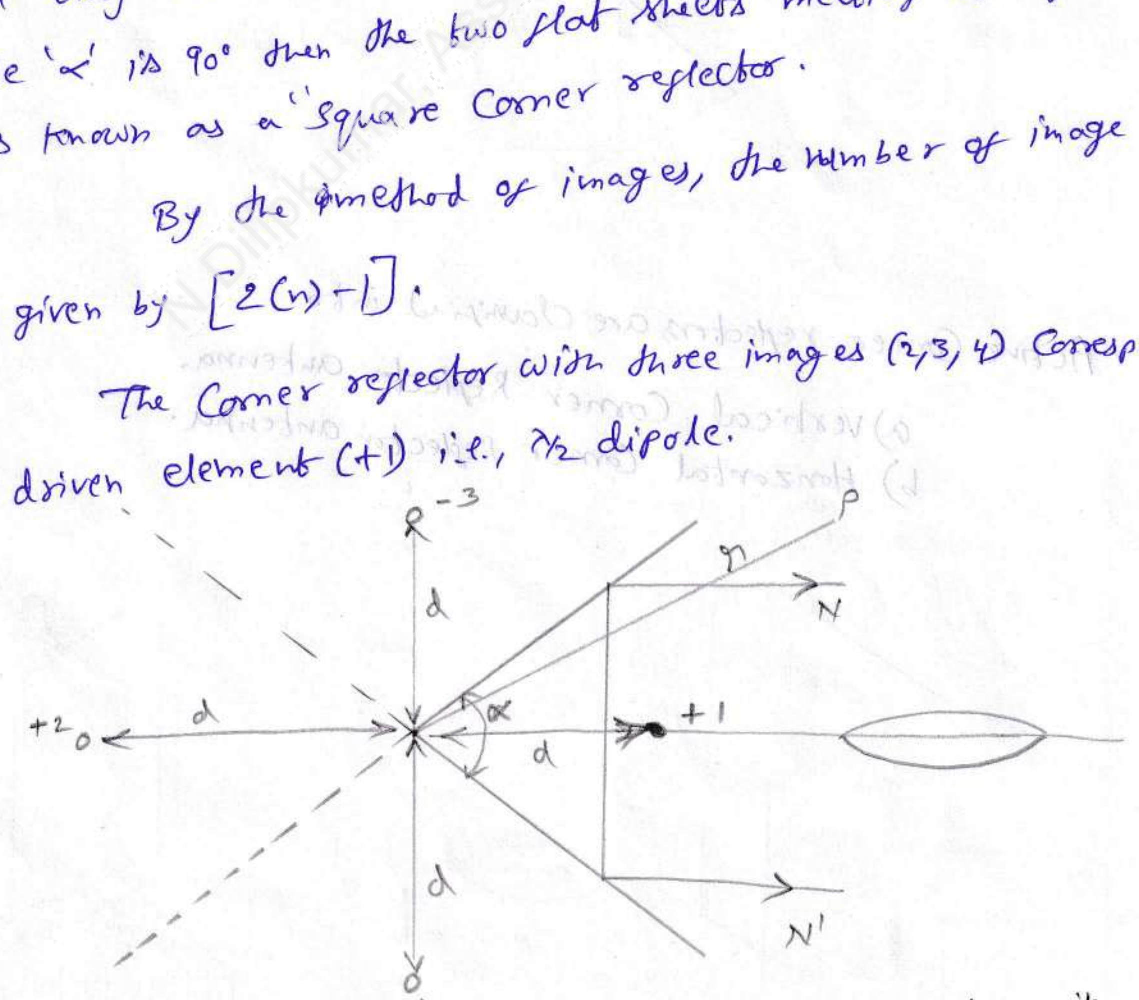
where 'n' is an integer i.e., 1, 2, 3, 4 - - - - -

n	α	Reflector
1	$180°$	flat sheet
2	$90°$ (or) $\pi/2$	Square Corner Reflector
3	$60°$ (or) $\pi/3$	Corner angle $60°$.
4	$45°$ (or) $\pi/4$	Corner angle $45°$.
5	$30°$ (or) $\pi/6$	Corner angle $30°$.

∴ By the method of images, Corner angle of π, $\pi/2$, $\pi/3$ ---etc Can only be used & not the Intermediate angles. If corner angle 'α' is $90°$ then the two flat sheets meeting at right angles & is known as a "Square Corner reflector.

By the method of images, the number of image elements are given by $[2(n)-1]$.

The Corner reflector with three images (2, 3, 4) Corresponding to one driven element (+1) i.e., $\lambda/2$ dipole.



with driven

The driven element & all the three images carry equal currents. However the driven element (+1) & image element (+2) are in same phase & (−3), (−4) are in same phase. But phase difference between (+1) & (+2) & (−3) & (−4) is 180°.

(−3) & (−4) → Negative images corresponding to single reflection of rays N & N'. (+2) is a positive image corresponds to driven element D.

The field pattern $E_\phi(\theta)$ in horizontal plane is,

$$E_\phi(\theta) = K' I_1 \left[\cos(\beta d \cos\theta) - \cos(\beta d \sin\theta) \right] \quad -①$$

K' → Constant
I_1 is current flowing in each element
d → Distance b/n each element
β → propagation constant $\left(\frac{2\pi}{\lambda}\right)$.

The terminal voltage at the centre of the dipole (+1) is,

$$V_1 = I_1 Z_{11} + I_1 Z_{12} - I_1 Z_{13} - I_1 Z_{14}.$$

$$= I_1 (Z_{11} + Z_{12} - Z_{13} - Z_{14})$$

$$= I_1 (Z_{11} + Z_{12} - 2 Z_{14}) \quad -② \qquad \because Z_{13} = Z_{14}.$$

Z_{11} → self impedance of $\lambda/2$ dipole (+1) i.e., 73Ω.
Z_{12} → Mutual " between (+1) & (+2)
Z_{13} → " " " (+1) & (−3)
Z_{14} → " " " (+1) & (−4)

~~Similarly terminal voltage of image elements~~

→ If 'P' is the power supplied to the driven antenna (also for each image antenna) then,

$$P = I_1^2 R$$

$$I_1 = \sqrt{\frac{P}{R}} = \sqrt{\frac{P}{R_{11} + R_{12} - 2R_{14}}} \quad -③$$

Substitute I_1 in eq. ①,

If reflector is removed then $R_{12} = R_{13} = R_{14} = 0$.

$$\therefore E_\phi(\theta)_{\lambda/2} = K' \sqrt{\frac{P}{R_{11}}} \quad — (5)$$

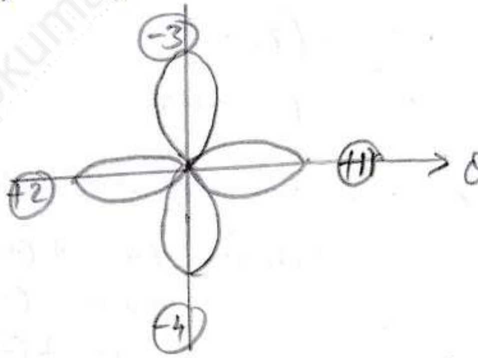
Eq. (5) gives the field pattern of an isolated driven antenna in free space which provides convinient reference for Corner reflector antenna.

$$\text{Gain, } G = \frac{E_\phi(\theta)}{E_\phi(\theta)_{\lambda/2}}$$

$$= \underbrace{\sqrt{\frac{R_{11}}{R_{11}+R_{12}-2R_{14}}}}_{\text{Pattern factor}} \underbrace{\left[\cos(\beta d\cos\theta) - \cos(\beta d\sin\theta)\right]}_{\text{Coupling factor}}$$

The field pattern has four lobes among which, one is real & remaining three are virtual i.e.,

(+1) is real,
(+2), (-3), (-4) are virtual.

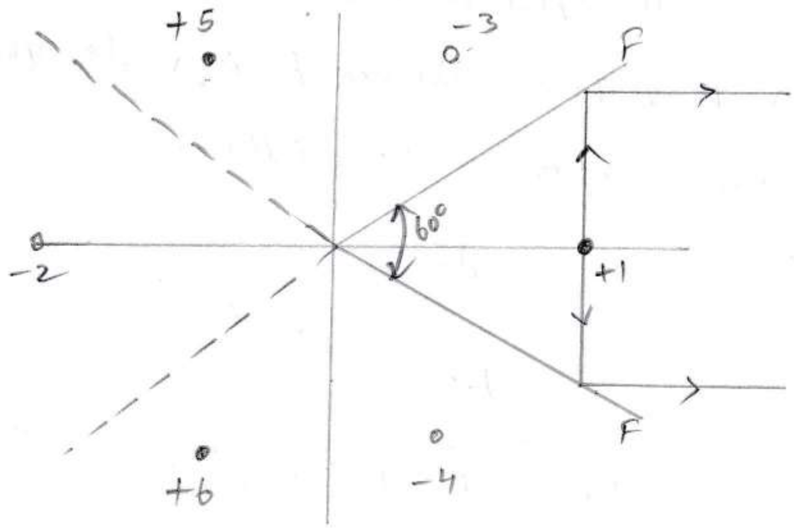


Field pattern due to driven element & its 3 images.

Maximum Radiation from the Corner Reflector antenna is in the direction of $\theta = 0°$.

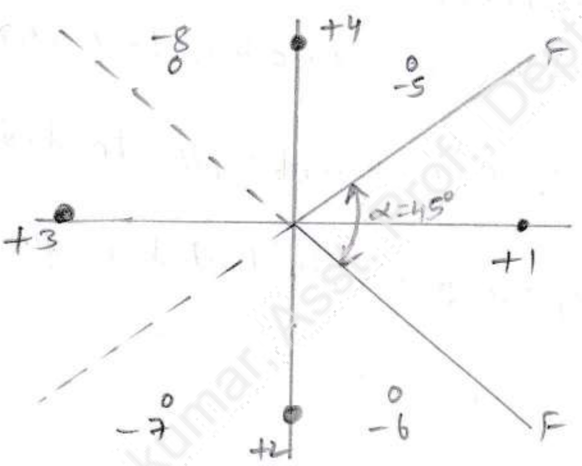
$$G_0 = \sqrt{\frac{R_{11}}{R_{11}+R_{12}-2R_{14}}} \left[\cos(\beta d) - 1\right].$$

→ 60° Corner Reflector Antenna:-



Here n = 3
Image elements is
2(3) − 1 = 5

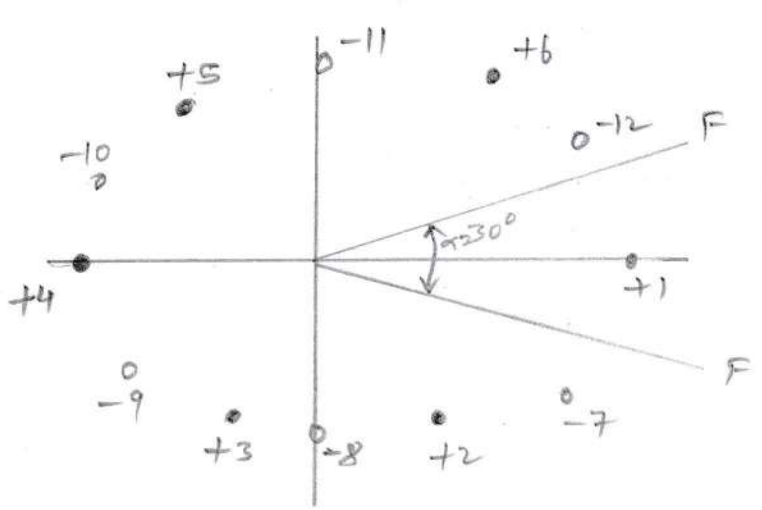
→ 45° Corner Reflector Antenna:-



Here n = 4
Image elements is
2(4) − 1 = 7.

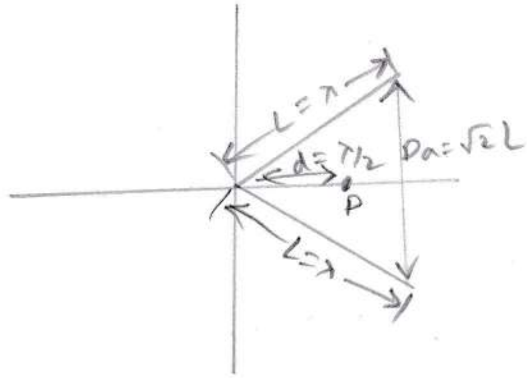
→ 30° Corner Reflector Antenna:-

Here n = 6,
Image element is
2(6) − 1 = 11.



Design Considerations:-

The aperture width 'Da' lies between 1 to 2 wavelength. The feed to vertex distance 'd' is made equal to side length. Hence design equations are as follows.



$$d = L/2$$

$$L = 2d.$$

$$D_a = \sqrt{L^2 + L^2} = L\sqrt{2}.$$

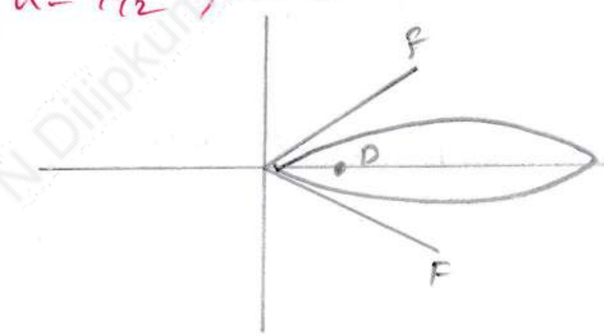
$$D_a = 1.414L = 2.828d.$$

The dimensions of square corner reflector that provides greater bandwidth are $L = 0.7\lambda$, $d = 0.35\lambda$, $D_a = 0.99\lambda$.

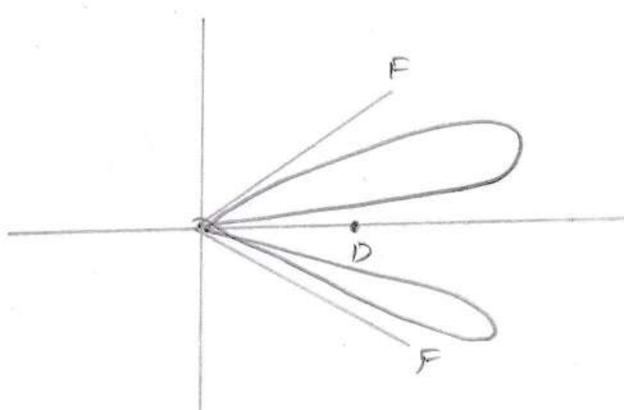
Further spacing effects the bandwidth and gain.

The effect of spacing is illustrated in the following figures:.

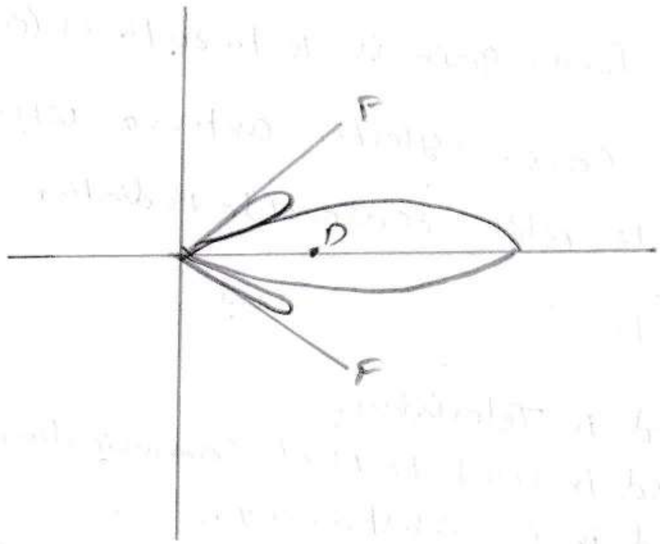
i) $L = \lambda$, $d = \lambda/2$, $\alpha = 90°$.



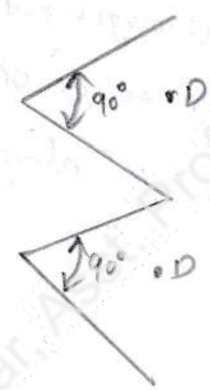
ii) $L = 2\lambda$, $d = \lambda$, $\alpha = 90°$



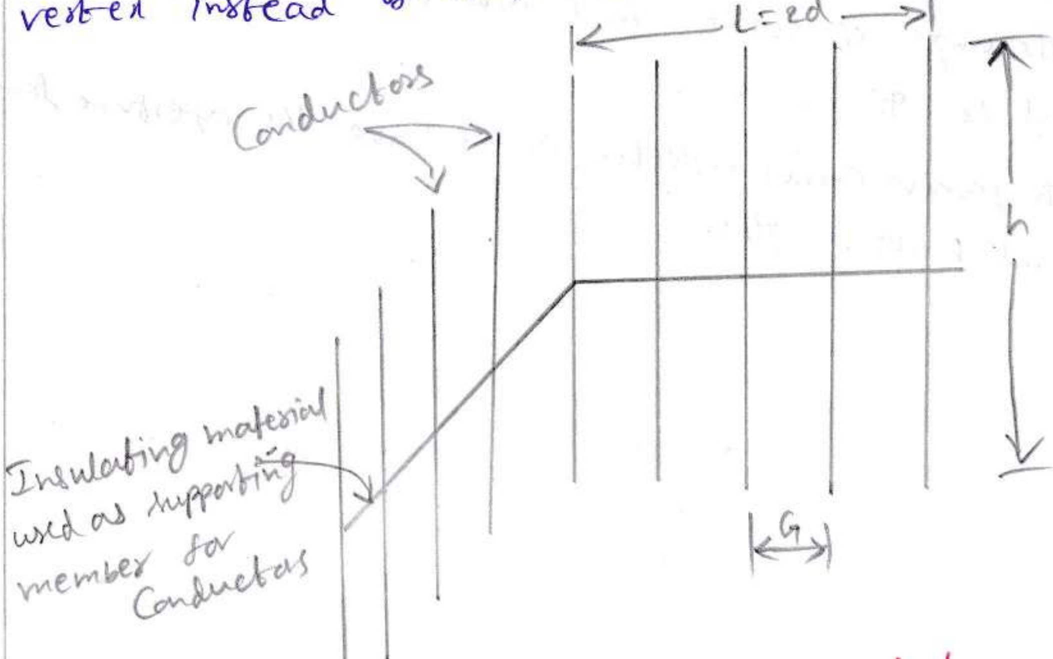
iii) $L = 3\lambda$, $d = 0.5\lambda$, $\alpha = 90°$



To achieve greater directivity array of square corner reflectors (or) stacking of two corner reflectors can be used.



→ Grid Type reflectors are corner reflectors in which the surfaces are usually made of spaced wires (or) tubes parallel to the vertex instead of solid metal sheet as shown in the figure.



Merits of Active Corner reflector:-

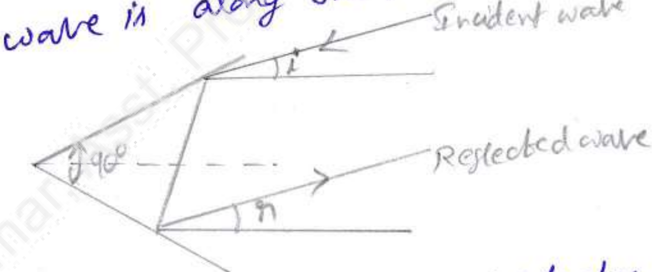
Power gain is 10 to 20 times (or) 10 to 13 dB is obtained from a Corner reflector antenna when Compared with an isolated λ/2 dipole with reasonable Radiation resistance.

Applications:-

→ Used in Televisions.
→ Used in point to point Communications.
→ Used in Radio astronomy.

Passive Corner Reflector:-

When the driven element is not used in Conjuction with a Corner reflector, the arrangement is an effective reflector (Passive antenna) over a wide range of incident angle $0 < i < \pm \pi/4$ where the reflected wave is along the path direction of incident ray.



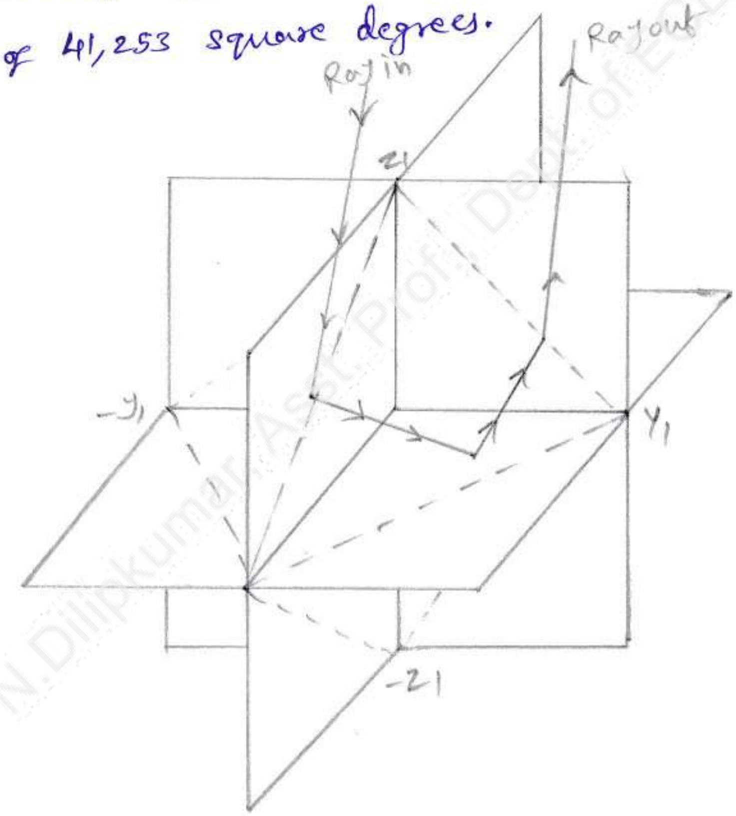
The difference between active & passive Corner reflectors are:

→ Absence of driven element in Passive reflector.
→ The angle of corner in passive Corner reflector should always be equal to 90° ($\alpha = 90°$).
→ The passive Corner reflector should have the aperture length equal to several wave lengths.

Retro Reflectors:-

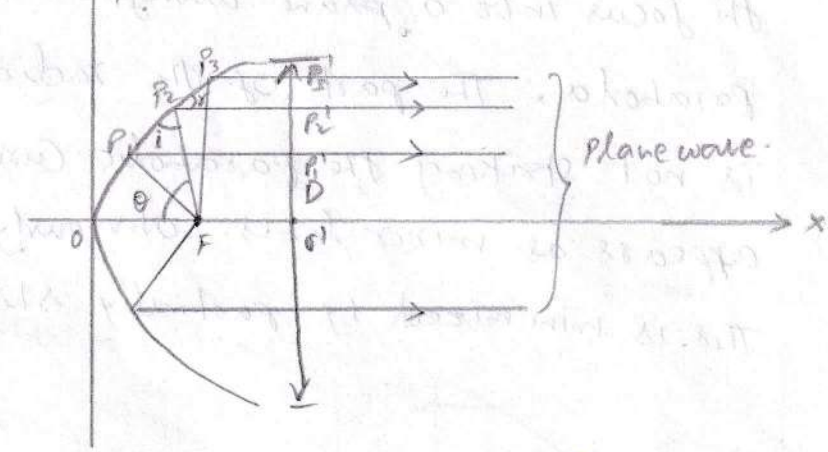
Reflectors with Corner angle 90° in which the incident wave is reflected back towards its source is Called as "Retro Reflector". It consists of three mutually perpendicular reflecting sheets intersecting each other at the centre producing eight three dimensional square corner reflectors.

Any ray incident is reflected back with in a full solid sphere (4π steradians). Each square Corner reflector occupies one octant (5157 square degrees) & 8 Square Corners occupies a full sphere solid angle of 41,253 square degrees.



Retroreflector of 8 Square Corners for reflecting back waves from any direction. Path of ray returning via triple bounce is shown.

Antennas with Parabolic Reflectors:-



A parabola may be defined as the locus of a point which moves in such way that its distance from the fixed point (called focus) plus its distance from a straight line (called directrix) is Constant.

OF = Focal length = f.
F = Focus, O = vertex.
OO' = Axis of parabola.

By def.,

$$FP_1 + P_1P_1' = FP_2 + P_2P_2' = \text{Constant}. \ (\text{Say } k).$$

The mouth (D) of the parabola is known as the Aperture.
The ratio of focal length to Aperture size (f/D) known as f over D ratio. is an important Characteristic of parabolic reflector & its value usually varies b/n 0.25 to 0.50.

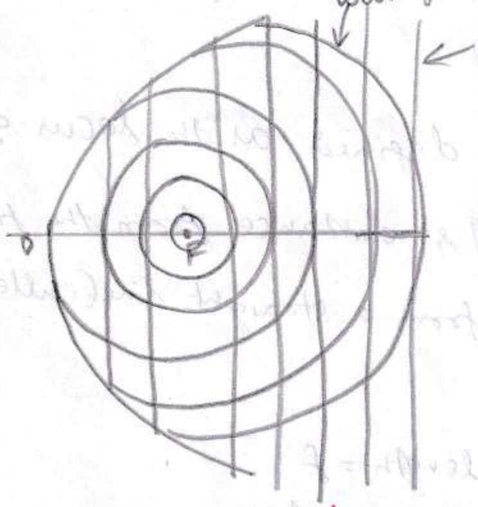
→ It follows ~~angle of~~ law of reflection.

→ All the wave originating from focus will be reflected parallel to the parabolic axis.

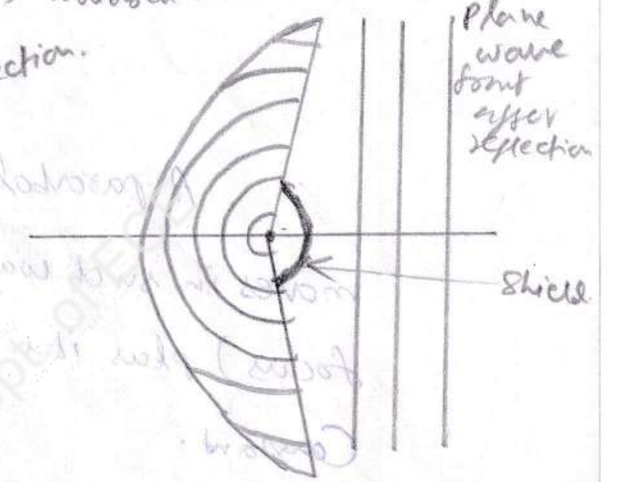
→ All the wave reaching at the aperture are inphase.

Hence, the geometrical properties of parabola provide excellent microwave reflectors that lead to the production of concentrated

Parabola Converts a spherical wavefront Coming from the focus into a plane wavefront at the mouth of the parabola. The part of the radiation from the focus which is not striking the parabolic Curve as spherical wave appears as minor lobes. obviously this is a waste of power. This is minimized by partially shielding the source.



Production of plane wavefront by Parabolic reflector with omnidirectional source at focus.



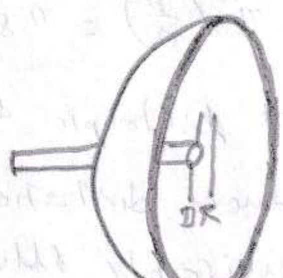
with partially Shield source

Further if a beam of parallel rays is incident on the Parabolic surface, they will be focussed at a point i.e., Focus. This is in effect due to principle of reciprocity theorem.

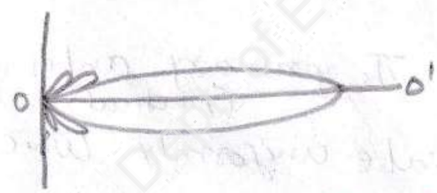
The rays Coming ter to the directrix will be focussed at the focus & not others due to path length difference. Parallel rays are known as Callimated

Paraboloidal reflector (or) Microwave dish:-

A parabola is a two dimensional plane curve. A practical reflector is a three dimensional curved surface. ∴ A practical reflector is formed by rotating a parabola about its axis (OO'). The surface so generated is known as paraboloid which is often called "microwave dish" (or) "Parabolic reflector".



Full paraboloidal reflector with dipole source at the focus.



Radiation pattern of a paraboloid of aperture $D = 10\lambda$.

The intersection of any plane per to x-axis with the paraboloid surface is a circle. In conventional automobile (Eg. motor-car headlight, in search light), this beam property is utilized.

The actual shape would be like a fat cigar.

If the feed (or) primary antenna is isotropic, then the paraboloid will produce a beam of radiation.

$$BWFN = \frac{140\lambda}{D} \text{ degree.} \quad [\text{For circular aperture}]$$

λ = Free space wave length in m.

D = Diameter of aperture in m i.e, mouth diameter.

$$BWFN = \frac{115\lambda}{L} \text{ degree.}$$

[rectangular aperture]

L = length of aperture.

$$HPBW = \frac{58\lambda}{D} \text{ degree.}$$

Directivity, $D = \frac{4\pi A}{\lambda^2}$

For circular aperture,

$$D = \frac{4\pi}{\lambda^2}\left(\frac{\pi D^2}{4}\right) = \pi^2\left(\frac{D}{\lambda}\right)^2 = 9.87\left(\frac{D}{\lambda}\right)^2.$$

The primary antenna [practical] is not isotropic & thus does not radiate uniformly which introduces distortion. Besides the surface of paraboloid is not uniformly illuminated, as there is gradual tapering towards edge.

This result in less capture area which is small than the actual area i.e.,

$$A_0 = KA.$$

A_0 = Capture area

A = Actual area ~~of~~ mouth

K = Const. depends on type of antenna used for feed.

$= 0.65$ (approx) for dipole antenna.

Thus power gain of circular aperture paraboloid, w.r.t half wave dipole is,

$$G_p = \frac{4\pi A_0}{\lambda^2} = \frac{4\pi KA}{\lambda^2} \qquad A = \frac{\pi D^2}{4} \text{ for circular aperture.}$$

$$= \frac{4\pi K}{\lambda^2}\left(\frac{\pi D^2}{4}\right)$$

$$= 0.65\left(\frac{\pi D}{\lambda}\right)^2$$

The gain is a function aperture ratio (D/λ) of the paraboloid. The Effective Radiated power [ERP] of an antenna is multiplication of Input power fed to antenna & its power gain.

With the help of paraboloid reflector, extremely large gain & narrow beam widths can be achieved. For the effective useful use, a paraboloid reflector must have a open circular mouth aperture of min. 10λ.

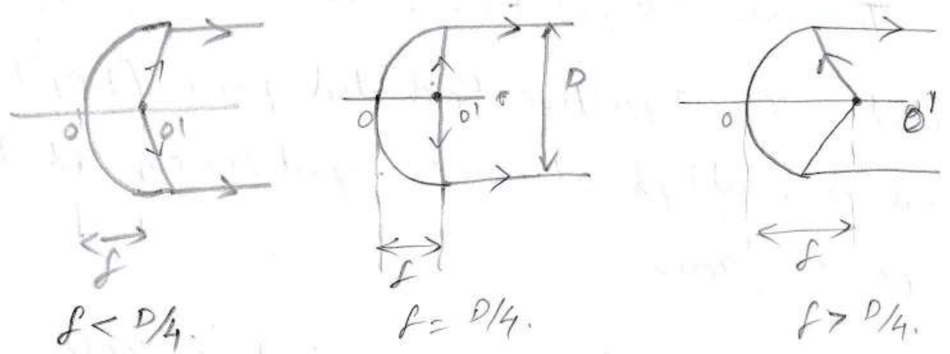
Paraboloid reflector can be designed by keeping the mouth diameter fixed & varying the focal length (f) also.

→ Three cases possible, the focal length is small such that the focus lies inside the mouth aperture. In this case it is difficult to get a source giving adequately uniform illumination over such a wide angle.

→ When the focal length is large such that the focus lies beyond the open mouth, it becomes difficult to focus all the radiation from the source on the reflector.

→ The focus lies in the plane of the open mouth. When the focus lies in the plane of open mouth, the focal length (f) is equal to the one fourth of open mouth diameter (i.e., $D/4$).

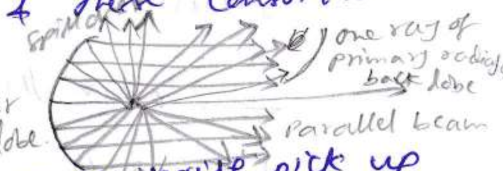
The radiation beam from an antenna employing paraboloid reflector should be theoritically a pencil-shape. The pencil shape beam is almost equal in horizontal & vertical [...]. However a little amount of control of beam shape [...] of paraboloid.



The most widely used antenna for microwave is the Paraboloidal reflector antenna. This consists of primary antenna such as dipole (or) horn situated at the focal point of a paraboloidal reflector.

* The important practical implication of this property is that reflector can focus parallel rays on to the focal point or conversely it can produce a parallel beam from radiations originating from the focal point.

For an isotropic source is assumed to be situated at the focal point. It is seen that some of the desired rays are not captured by the reflector & these constitute "spill-over".

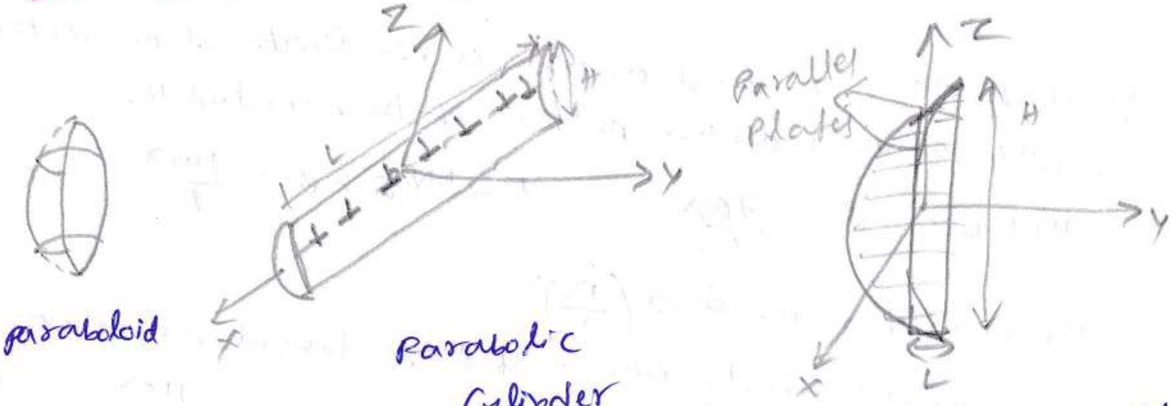


while receiving spill over increases noise pick up which is particularly trouble some in satellite ground stations.

Further, some radiations from the primary radiator occur in the forward direction in addition to the desired parallel beam. This is known as "backlobe" radiation as it is from the backlobe of the primary radiator.

Backlobe radiations are not desirable as it can ... with the reflected beam & hence practical

Other Types of Parabolic Reflectors:-



Cut paraboloid (or) Truncated paraboloid.

Parabolic Cylinder Reflector with line source of eight dipoles.

Pill box (or) Cheese antenna.

→ It is not circular when viewed from a point on the paraboloid axis.

→ Plane sheet for parab

→ focal line instead of focal point.

→ uniform illumination.

→ beam in vertical plane.

→ beam wide in E-plane (i.e., x y) plane.
narrow in H-plane (i.e., Y Z).

→ Short Parabolic Cylinder enclosed with parallel plates.

→ Excited with coaxial line.

All three are smaller in size. (Advantage).

Primary & secondary pattern:-

Feed radiator (or) primary radiator (or) simply feed. } & its radiation pattern is known as primary pattern.

Parabolic reflector (or) secondary radiator } " secondary pattern. (or) Antenna pattern.

Feed Systems:-

From G.S.N Raju

~~Other Types of Parabolic Reflectors~~

Formulas for Parabolic Reflector

If the primary or feed antenna is non-directional or isotropic, the beam width of the radiation pattern of the paraboloid is,

$HPBW = \phi = \frac{70\lambda}{D_a}$. $\quad BWFN = 2\phi = \frac{140\lambda}{D_a}$.

Directivity $= D = 9.87\left(\frac{D_a}{\lambda}\right)^2$

For a large, uniformly illuminated rectangular aperture,

$\phi = \frac{57.5\lambda}{L}$ (degrees) $\quad BWFN = \phi_0 = \frac{115\lambda}{L}$

$D = \frac{4\pi A}{\lambda^2}$. $\quad L$ = length of the aperture in λ

The capture area,

$A_c = bA$.

$A \rightarrow$ actual area.

$b \rightarrow$ constant (Depends on Type of antenna). ≈ 0.65 for dipole antenna.

Power gain, $g_p = \frac{4\pi}{\lambda^2} A_c$.

$= \frac{4\pi}{\lambda^2} bA$.

For circular aperture paraboloid,

$A = \frac{\pi D_a^2}{4}$.

$g_p = \frac{4\pi K}{\lambda^2} \cdot \frac{\pi D_a^2}{4}$

$= \frac{\pi^2 K D_a^2}{\lambda^2}$ (for dipole feed). $K = 0.65$

$= 0.65\,\pi^2 \left(\frac{D_a}{\lambda}\right)^2$

$g_p = 6.4 \left(\frac{D_a}{\lambda}\right)^2$

Ionospheric Abnormalities

Feed systems:-

The entire parabolic reflector antenna Consists of two basic Components e.g., the reflector + a source of primary radiation at the focus. The source is called the primary radiation or feed radiator while the reflector, the secondary radiator.

An ideal feed would be that radiator which radiates towards reflector in such a way that illuminates entire surface of reflector & no or zero energy is radiated in other directions.

An isotropic antenna as feed would not be a better Choice. As far as the secondary radiator is Concerned, the best Choice is the paraboloid which is inherited with compactness & simplicity.

Dipole antenna is also not very much suitable for the feed but occasionally used. The simplest & generally used is a dipole with parasitic reflector (i.e., Yagi-Uda) (or) a small plane reflector, with is fed with a co axial line. Typically the spacing b/n driven element & parasitic element is 0.125λ & for a plane reflector it may be around 0.4λ.

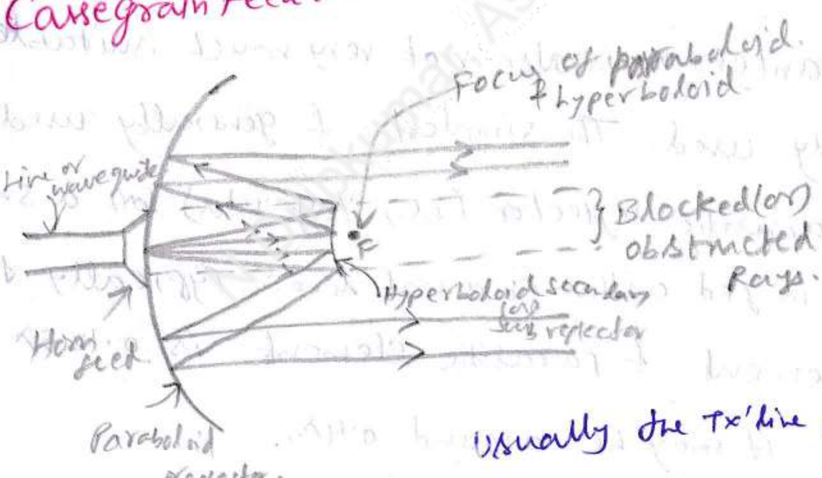
The double dipoles are so spaced & phased that endfire pattern is produced which illuminates the paraboloid reflector. The feeding with a dipole involves changing from unbalanced system to a balanced system.

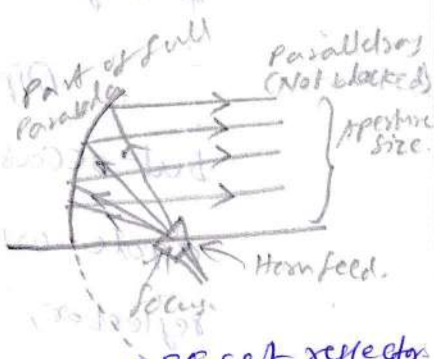
A most Common feed radiation for paraboloid reflector antenna is a 'waveguide horn'. The horn feed is a waveguide feed. The horn antenna pointing the paraboloid & radiation pattern of horn antenna is mild, in the same direction. Thus, the direct radiation from the horn antenna is minimum.

If circular polarization is required then, Conical horn antenna or helix antenna Can be used as feed at the focus of Paraboloid.

For getting maximized beam pattern along the parabolic axis, feed is placed at the focus. But if the feed is moved laterally from the focus i.e., perpendicular to axis, then beam deteriorates i.e., limited beam motion Can be obtained. If it is moved along the axis, then pattern is broadend.

Cassegrain Feed:-





Usually the Tx'line is required to be short.

Advantages of Cassegrain Feed:-

1) Reduction in spill over & minor lobe radiation.
2) Ability to get an equivalent focal length, much greater than the physical length.
3) Ability to place the feed in a Convenient location.
4) Capability for scanning (or) broadening of the beam by [illegible] surfaces.

Salient features of Corner reflector:

1) It is simple to construct.

2) It is used as a passive target for radar & Communication applications to return the signal exactly in the same direction by choosing $\alpha = 90°$. Due to this unique feature, most of defense ships & vehicles are designed with minimum sharp corners to reduce the chances of their detection by enemy radars.

3) It is also used in home television antennas.

4) The most preferred value of α is $90°$.

5) The spacing b/n the vertex & feed element position is increased if α is decreased & viceversa, in order to improve efficiency.

6) When α is small, gain is increased by increasing the length of the sides of the reflector.

7) The feed element can be a dipole or an array of collinear dipoles.

8) When the feed elements are cylindrical or biconical dipoles instead of thin wires, bandwidth & radiation resistance are high.

9) When the reflector is large with high λ, the surfaces of Corner reflectors are made of grid wires to reduce aerodynamic drag due to wind speeds & overall sys. wt.

10) For small included angle, the side lengths should be longer.

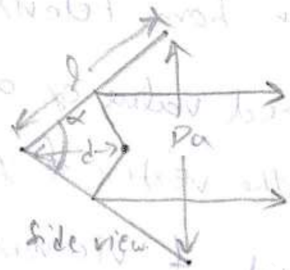
11) If the spacing 'd' is small, radiation resistance becomes small & hence efficiency is reduced.

12) If the spacing is very large, the sys. produces undesirable multiple lobes & it loses its directional properties.

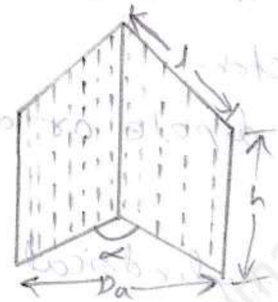
Corner reflector:-

A corner reflector is a reflecting object which consists of two or three mutually intersecting, conducting flat surfaces.

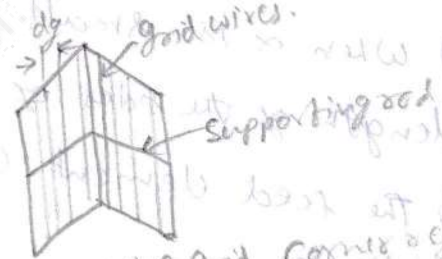
Dihedral forms of corner reflectors are frequently used in antennas. However trihedral forms with mutually ter surfaces are used as radar targets.



side view



Perspective view.



wire-grid Corner reflector

A corner reflector is designed to improve the collimation of electromagnetic energy in the forward direction & to eliminate radiation in the back & side directions.

D_a = aperture size.
l = length.
h = height.
d = spacing b/n the vertex & feed point location.
d_g = spacing b/n grid wires.

The ranges of the above parameters for a good corner reflector are:

1) $\lambda < D_a < 2\lambda$

4) h is 1.2 to 1.5 times greater than the total length of feed element.

1. Cal. gain, BWFN of a 2m paraboloid at 6GHZ. (20)

For paraboloid, $D = 2m$.

$f = 6\,GHZ = 6 * 10^9$

$\lambda = \frac{c}{f} = 0.05m$

$BWFN = \frac{140\lambda}{D} = \frac{140(0.05)}{2} = 3.5$

$G = 6.389\left(\frac{D}{\lambda}\right)^2 = 6.389\left(\frac{2}{0.05}\right)^2 = 10222$

$G_{dB} = 40.09.$

$G_{dB} \approx 40.$

2. A Paraboloid reflector has radiated characteristics whose HPBW is 5°. Find out its null to null beam width & power gain.

$HPBW = 5°.$

$HPBW = \frac{70\lambda}{D}.$

$D = 14\lambda.$

$BWFN = \frac{140\lambda}{D} = 10°.$

$G_p = \frac{4\pi K A}{\lambda^2}$

$A = \frac{\pi D^2}{4}.$

$G_p = \frac{\pi^2 D^2 K}{\lambda^2}.$

$K = 0.65.$

$G_p = \frac{\pi^2(14\lambda)^2(0.65)}{\lambda^2} = 1257.39.$

$(G_p)_{dB} = 31dB.$

Lens Antennas:-

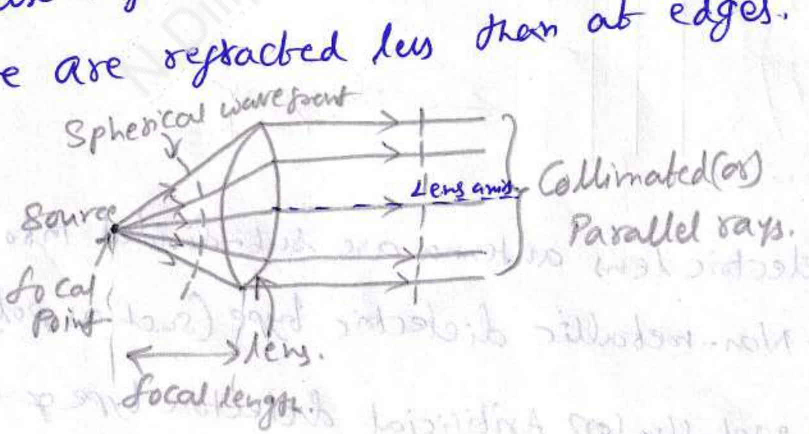
The parabolic reflector is an example of an optical device which can be applied to centimeter wavelengths (Eg: Microwaves) antennas & lens antenna is yet another. Although lens, paraboloid reflector, & parabolic cylinder have the same applications.

In fact the frequency range of lens antenna starts at 1000MHz but its greatest use is at & above 3000MHz. At lower freq.'s lens antenna become bulky & heavy. They act just as a glass lens as used in optics.

Principle:-

The collimating action of a simple optical lens can be illustrated with the help of fig. Assuming a source at focal length, along lens axis, it is seen that collimated (or) parallel rays are obtained (plane wave front) on the right hand side of lens.

From optical point of view a divergent beam is collimated because refraction takes place as a result of which ray at centre are refracted less than at edges.



Lens is, made of dielectric material operates for radio frequencies.

An EM source at the focus on the LHS will produce collimated i.e., parallel rays on the RHS. Converse is also true i.e., incoming parallel rays on RHS will converge to a point on the focus at LHS.

Further, it is possible to construct a medium having refractive index less than unity at radio frequencies. So that focusing properties could be achieved.

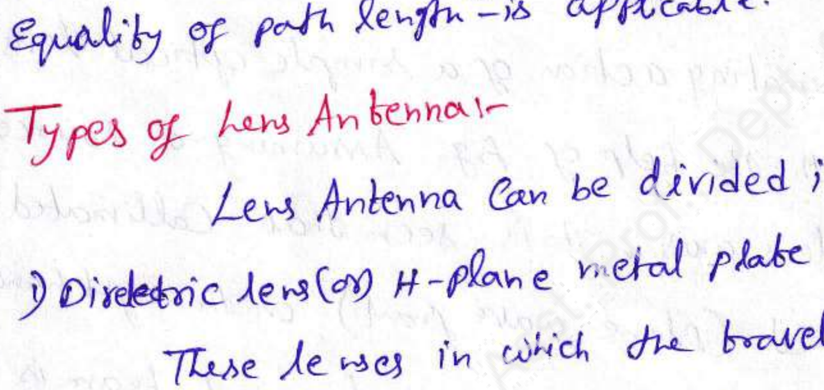
Thus, whatever may be the refractive index of the lens material the purpose is to straighten out the wavefront. At microwave frequencies, the microwave lens is used in conjunction with a horn antenna for correction into plane waveform (or) to focus parallel rays at one point. For lens antenna also the Geometric optics principle - "the principle of Equality of path length" - is applicable.

Types of Lens Antenna:-

Lens Antenna can be divided into two distinct types.

1) Dielectric lens (or) H-plane metal plate lens or Delay lens:-

These lenses in which the travelling wave-fronts are retarted or delayed by lens media.



Dielectric lens antenna are sub-divided into

→ Non-metallic dielectric type (such as polystyrene (or) Lucite) of lens.

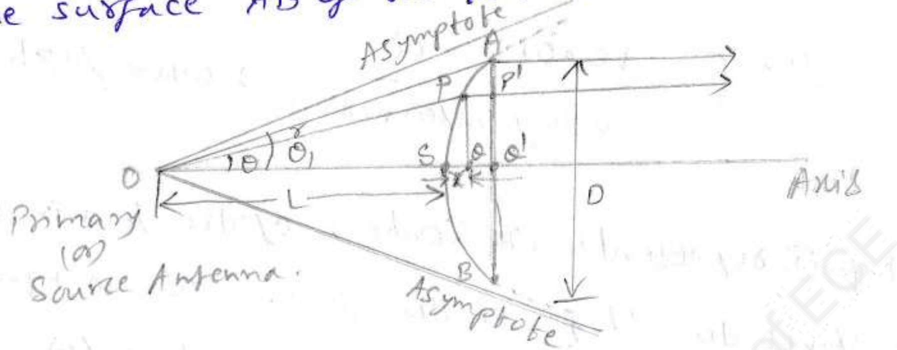
→ Metallic (or) Artificial dielectric type of lens.

2) E-Plane Metal plate lens:-

These lenses in which the travelling wave-fronts are speed up by the lens media.

Non-metallic dielectric lens:-

This may be designed by the ray analysis methods of geometrical optics as dielectric lens is identical to optical lens. If this lens is to Convert a spherical wave from a source placed at focus F to a plane wavefront, then all the rays paths from O to the plane surface AB of the lens should have equal electrical length.



According to principle of equality of electrical path lengths all the paths from the source to the plane surface of lens should be equal to so that field over entire plane surface is in phase. Hence the wave emerging from source will have Constant phase across aperture (D). The time taken by a ray to travel from O to aperture plane (AB) is same for all possible rays paths.

Let the velocity of wave in air & in lens medium be c & v respectively, then for equal time Condition,

$$OP + PP' = OS + SQ'$$

$$OP + PP' = OS + SQ + QQ'$$

from fig, $PP' = QQ'$,

$$OP = OS + SQ.$$

$$\frac{r}{c} = \frac{L}{c} + \frac{x}{v}$$

$$r = L + \left(\frac{c}{v}\right)x$$

$$\because \mu = c/v \ \& \ OS + SQ = OQ.$$

$\cos\theta = \frac{OQ}{OP}$

$\cos\theta = \frac{L+x}{r}$

$(L+x) = r\cos\theta$

$x = r\cos\theta - L$

$$r = L + \mu x$$

$$r = L + \mu(r\cos\theta - L)$$

$$r(1 - \mu\cos\theta) = L(1-\mu)$$

$$r = \frac{L(\mu - 1)}{(\mu\cos\theta - 1)} \quad \text{— (1)}$$

$$\mu = \frac{c}{v} = \frac{\text{velocity in air}}{\text{velocity in lens medium}} \text{ ; where } \mu > 1.$$

Eq. (1) represents the contour of the lens in polar co-ordinates. This is equation of hyperbola & hence gives the shape of the lens.

Whose focal length is L & radius of curvature (R).

$R = L(\mu - 1)$ provided θ is small.

The asymptote of the hyperbola is at angle θ w.r.t axis & can be obtained by tending to ∞ (i.e., $r \to \infty$)

$$(\mu\cos\theta - 1) = \frac{L(\mu-1)}{r} = \frac{L(\mu-1)}{\infty} = 0.$$

$$\cos\theta = \frac{1}{\mu}.$$

One of the focus of hyperbola is at 0. $R = L(\mu - 1)$ is the optical formula.

Polystyrene dielectric constant $K = 2.5$, $\mu = 1.6$.

Polyethylene " $K = 2.2$, $\mu = 1.5$

The wavelength is comparable to the lens aperture at radio frequencies. Radiation pattern is a characteristic of lens aperture & of the uniformity of illumination just like a paraboloid reflector. However, it is difficult to get uniform illumination for a lens antenna. A long focal length lens provides more uniform illumination rather than a short focal

(9) (23)

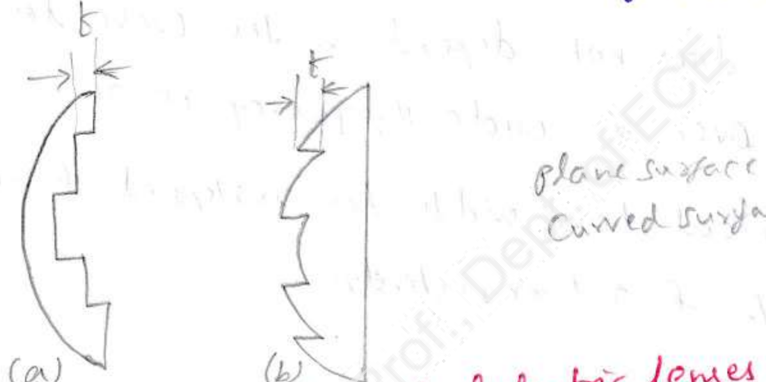
It may be noted that at a frequency less than 10,000 MHz, lens antenna have excessive thickness which is undesirable. This problem, however is cured by designing the lens antenna of zoned or stepped dielectric. The thickness t of such a stepped lens is given by,

$$t = \frac{\lambda}{\mu - 1}.$$

$\mu = c/v$ = Refractive index.

λ = freespace wave length.

$t \rightarrow$ Thickness.



Zoned (or) stepped dielectric lenses.

As a special case, if $\mu = 1.5$, the thickness would be twice the free space wavelength. Figure (b) is generally preferred as it is mechanically strong.

→ Zoned dielectric lens antenna becomes frequency sensitive i.e., it is dependent on wave length(λ).

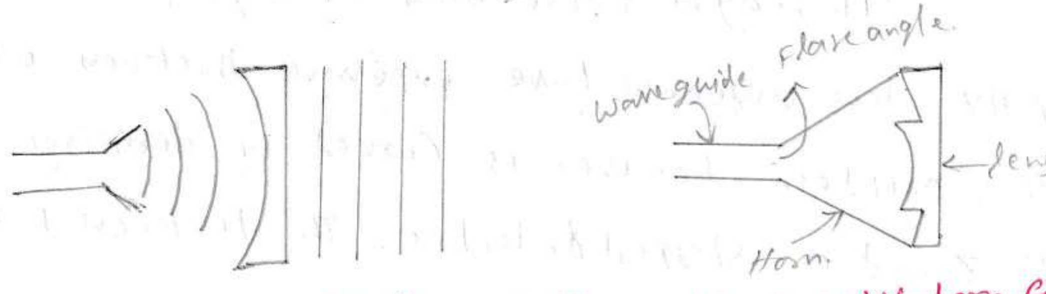
→ Stepped lens antenna has the benifit of reduced weight & less power dissipation.

Primary feed of Lens Antenna:-

→ Lens Antenna is usually fed with horn antenna.

→ The focal length is approx. equal to aperture dimension then it is more directive than a reflector.

→ To avoid stray radiation from horn, the sides of horn



Feeding of lens antenna with horn feed.

Uses of lens Antennas:-

→ Unstepped dielectric lens is a wide band antenna as its shape does not depend on the wave length & hence can be used over a wide frequency range.

→ Typical band width for unstepped & stepped lens antenna are 12% & 5% respectively.

→ Both reflectors & lens antennas are commonly used above 1000MHz. Lens antenna being a microwave device, is preferred to be used usually above 3000 MHz & not below it.

E-Plane metal plate lens Antenna:-

A metal plate lens, makes use of wave guide theory which states that the guide wavelength λ_g is related to the free-space wavelength λ,

$$\frac{1}{\lambda_g^2} = \frac{1}{\lambda^2} - \frac{1}{(2a)^2}$$

$\lambda_g \rightarrow$ Guide wavelength

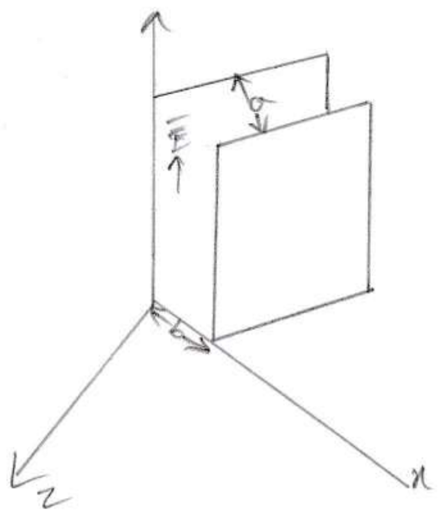
$\lambda \rightarrow$ free space wavelength

$a \rightarrow$ wider internal dimension of the rectangular waveguide or spacing of plates.

The phase velocity of the wave in the guide is given by,

$$v = \frac{C\lambda_g}{\lambda}$$

& this velocity is always greater than c.



Let us consider a wave propagated between two infinite parallel planes spaced a distance 'a' apart as shown in figure. & Electric vector parallel to the plates, & the dimensions are 'b'.

A structure consisting of many such parallel plates with spacing 'a' can be regarded as a uniform medium with an equivalent or refraction index μ is ratio of velocities.

$$\mu = \frac{c}{v} = \frac{c\cdot\lambda}{c\lambda_g} = \frac{\lambda}{\lambda_g}$$

$$\left(\frac{\lambda}{\lambda_g}\right)^2 = \left(\frac{\lambda}{\lambda}\right)^2 - \left(\frac{\lambda}{2a}\right)^2$$

$$\frac{\lambda}{\lambda_g} = \sqrt{1-\left(\frac{\lambda}{2a}\right)^2}$$

$$\mu = \frac{c}{v} = \frac{\lambda}{\lambda_g} \sqrt{1-\left(\frac{\lambda}{2a}\right)^2}$$

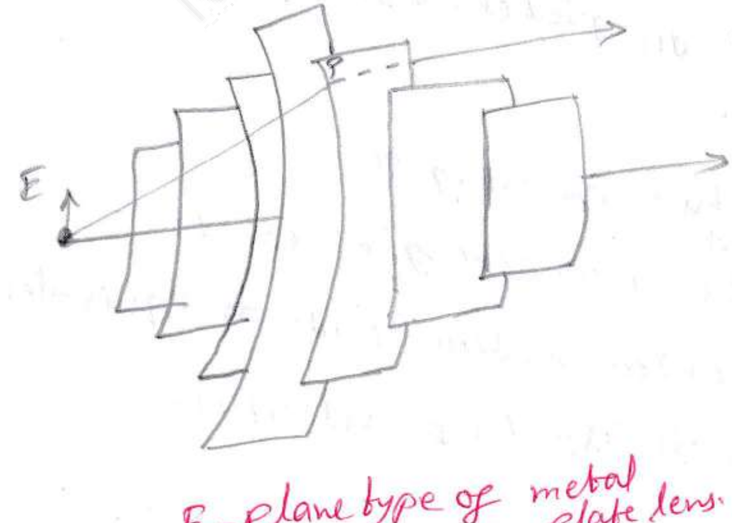
$$\mu = \sqrt{1-\left(\frac{\lambda}{2a}\right)^2}$$

It is apparent that refractive index is always less than unity. Clearly the value of 'a' must not be less than its critical value i.e.,

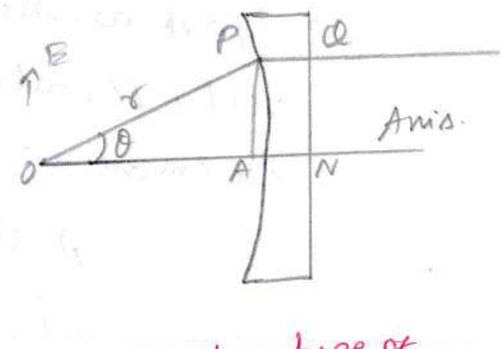
$$\sqrt{1-\left(\frac{\lambda}{2a}\right)^2} = 0.$$

$$\frac{\lambda}{2a} = 1.$$

$$a = \frac{\lambda}{2}.$$



E-plane type of metal plate lens.



E-plane type of metal antenna.

In this type, lens may be constructed from parallel metal plates. It differs fundamentally from the dielectric lens that where as the dielectric lens corrects the phases by slowing down a wavefront, the metal plate lens operates by speeding up the wavefront. A convergent metal plate lens must therefore have plates which are concave in shape & the arrangement of such a lens is shown in figure. Such lens are used at the higher ranges of radio frequencies.

The shape of the plate can be determined by the principle of equality of electrical path length according to Fermat's principle.

$$OPQ = OAN$$

$\lambda_g \rightarrow$ wave length in lens.

$\lambda_0 \rightarrow$ " in free space

$$\frac{L}{\lambda_0} = \frac{r}{\lambda_0} = \frac{L - r\cos\theta}{\lambda_g}$$

multiplying by λ_0,

$$L = r + \frac{\lambda_0}{\lambda_g}(L - r\cos\theta)$$

$$L = r + \mu(L - r\cos\theta)$$

$$L(1-\mu) = r(1-\mu\cos\theta)$$

$$r = \frac{L(1-\mu)}{1-\mu\cos\theta}$$

Demerits:-

→ Frequency sensitive

→ Small Band width.

$$B = \frac{50\mu}{1+K\mu}$$

$K \rightarrow$ No. of zones, the zone on the axis of the lens is counted as the first zone.

Tolerances of lens antennas:-

Type of Antenna.	Type of Tolerance	Amount of tolerance (rms).
Parabolic reflector	Surface Contour	0.016λ.
Dielectric lens plus unzoned.	Thickness	$\frac{0.03\lambda}{n-1}$.
	Index of refraction.	$\frac{3}{nt}$ %
Dielectric lens plus zoned.	Thickness	3%
	Index of refraction.	$\frac{3(n-1)}{n}$ %
E-plane metal plane lens plus unzoned.	Thickness	$\frac{0.03\lambda}{1-n}$.
	plate spacing.	$\frac{3n}{1-n^2}$ t %
E-plane metal plate lens plus zoned.	Thickness	3%
	plate spacing.	$\frac{3n}{1+n}$ %

Wave Propagation:-

Free Space is the Space which doesnot interfere with the norma radiation & propagation of radio waves. In other words, in free space, no magnetic or gravitational fields or solid bodies or ionized particles are assumed to exist. The Concept of free Space propagation simplifies the approach to wave propaga

Propagations → Ground wave (or) surface wave propa
→ Sky wave (or) ionospheric propagation.
→ space wave propagation.

These modes of propagation is largely depend on frequen

Propagation of radio waves are not only used in radio Communication for the transmission of intelligence over short long distance, but also in radar, radio ~~distance aid~~ direction finding, Control of machine from a distance. etc.,

wave propagation is restrict only to unguided propag & all others will be omitted.

In addition to these, special propagation i.e., supe refraction (or) duct propagation, & MUF, Critical frequency, s distance fading.

EM (or) Radio waves:-

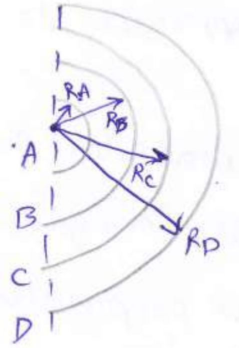
EM waves are nothing but oscillations ~~of~~ which propagate with the velocity of light in free Space. EM wa Consist of moving fields of electric & magnetic forces.

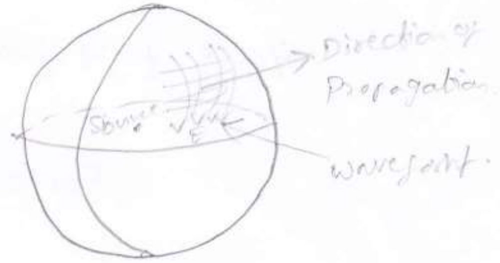
Electric field
Magnetic field
Propagation

} mutually ler (⊥) in EM waves.

→ Initial polarization is determined by orientation of antenna.

→ Since no obstacle or interference in free space is assumed to be present so e.m. waves spread uniformly in all directions from point source.





A, B, C, D → wave fronts.

wave fronts & rays from a point source in free space.

Modes of propagation:-

The radio waves from the transmitting antenna may reach to the receiving antenna following any of the following modes of propagations depending upon several factors like frequency of operation, distance between transmitting & receiving antennas etc.

Ground wave propagation:-

→ Up to 2MHZ.

→ It is important at broadcast & lower frequencies i.e. medium waves, long waves & very long waves.

Sky wave (or) Ionospheric wave propagation:-

→ Between (2 to 30MHZ).

→ It is at medium & high frequencies-[medium waves & Short waves]

→ 50 km to 400 km [propagation is by no. of reflections].

Sky wave (or) Ionospheric wave propagation:-

The sky waves are of practical importance at me & high frequencies (i.e., at medium waves & short waves) very long distance radio communications. In this mode of propagation EM waves reach the receiving point after refle from the ionized region in the upper atmosphere called ionosphere - situated between 50 km to 400 km above ea surface - under favourable conditions.

The ionosphere acts like a reflecting surface & able to reflect back the EM waves of frequencies between 2 to 30 MHZ. EM waves of frequency more than 30 MHz a not reflected back from the ionosphere rather they penetra Mostly sky wave propagation is suitable for frequencies bet 2 to 30 MHZ, so this mode of propagation is also called as 'short wave propagation'.

Shows multiple reflection radio waves from ionosp

Further, since sky wave propagation takes place after reflection from the ionosphere, so it is also called as "ionospheric propagation." Since long distance point to poi communication is possible with sky wave propagation so it also called as point to point propagation.

Extremely long distance i.e., round the globe Communica is also possible with the multiple reflections of sky waves In a single reflection from the ionosphere the radio waves co a distance not more than 4000 km.

The signals received due to sky wave propagation a however, subjected to fading in which signal strength vari with time. It is because at the receiving point a large waves follow a different number of paths. Hence provis has to be made to overcome the fading.

Space wave propagation (above 30 MHZ):-

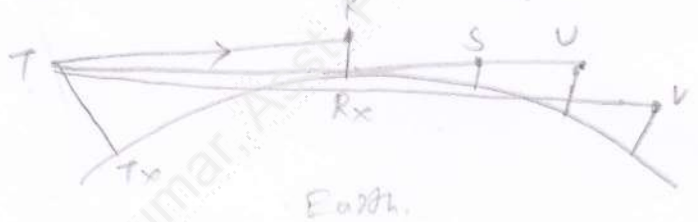
The space wave propagation of practical importance at VHF bands (between ~~to~~ 30 MHz to 300 MHz), UHF & microwaves Communications like televisions, radar, frequency modulations et utilize this mode of propagation. In this mode of propagati EM waves from the transmitting antenna reach the ~~rx~~ing ant either directly or after reflections from ground in the earth's tropo region.

Troposphere is that portion of the atmosphere which ext upto 16 Km from the earth surface. Space wave consists, of atl two Components e.g. direct Component & indirect Component i.e., ground reflected Components. It means in the former, wave reach directly from the transmitting antenna to receiving antenna & in latter, the wave reaches the receiving antenna after reflection the ground, where the phase change of 180° is also introduced du to reflection at the ground, in the ground reflected wave.

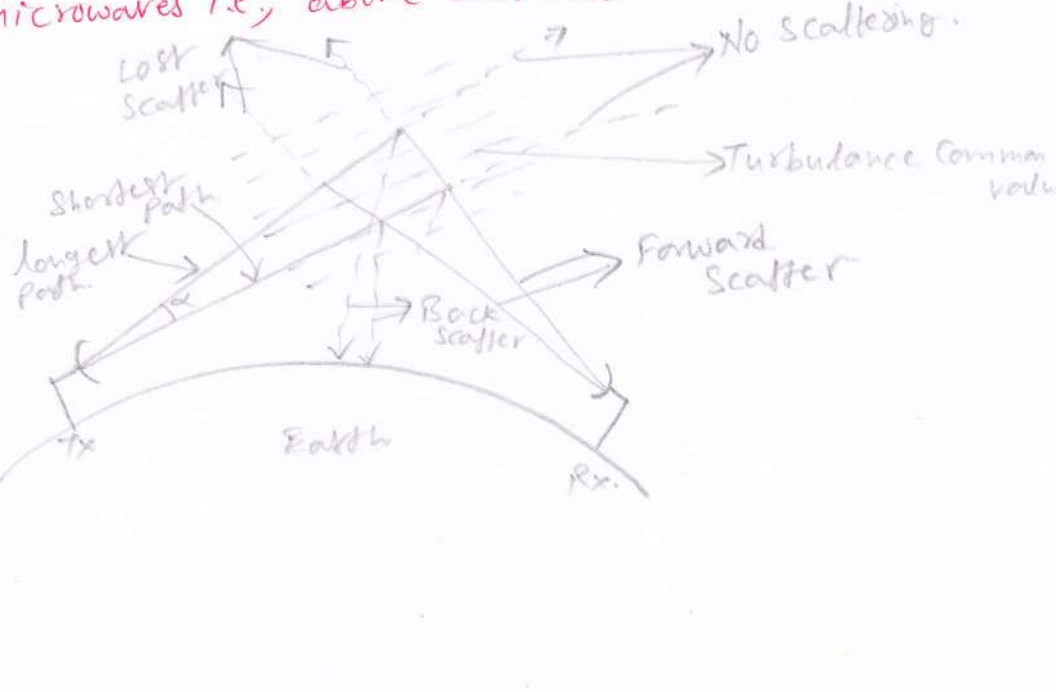
Space wave propagation is also called as Tropospheric propagation because space wave propagates through troposphere.

Space wave propagation is mainly in VHF, & higher frequencies because at such frequencies sky wave & ground wave propagation both fail. Beyond 30 MHZ sky wave fails as the wavelength becomes too shorts to be reflected from the ionosphere & ground waves are propagating close to the antenna only, as the attenuation is very high.

Space wave propagation is also called as line of sight propagation because VHF, UHF & microwave frequencies, this mode propagation is limited to the line of sight distance and is also limited by the curvature of the earth.



Tropospheric Scatter propagation (or) Forward Scatter propagation (UHF & Microwaves i.e, above 300 MHZ) :-



UHF & microwaves signals were found to be propagated much beyond the line of sight propagation through the forward scattering in the tropospheric irregularities.

Communication range → 160 Km to 1600 Km.

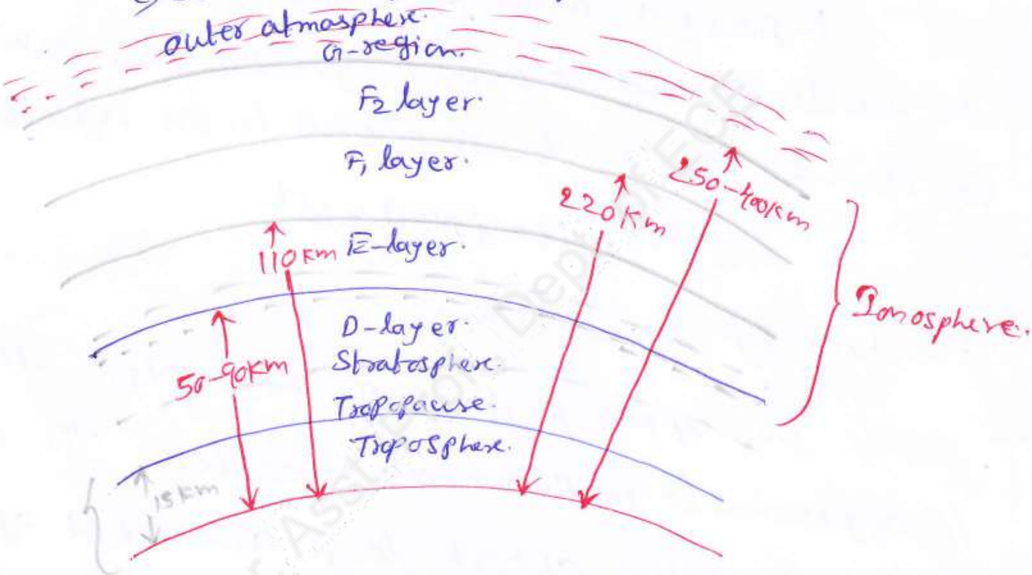
Structure of Atmosphere:-

Since the medium between transmitting & receiving antennas plays an important role, therefore it is necessary to study the medium above earth, through which the radio waves propagate before further study on modes of propagation is taken.

a) Structure of troposphere.

b) Structure of ionosphere.

c) Structure of atmosphere.



Structure of Troposphere:-

It is a portion of earth's atmosphere which from earths surface extends up to a height of 8 to 10km in polar latitude, 10 to 12 km in moderate latitude, 16 to 18 km at equator.

On average it is extended up to 15km from earth surface.

The entire belt is called as troposphere or region of change. Although in the troposphere, the % of gas components remains almost constant with increase of height, yet the water vapour components sharply decrease with height. The other important property of the troposphere is that temperature decreases with increase of height & falls to a minimum temp. of $-68°F$.

After the top of troposphere, tropopause starts & ends the begining of 'stratopause'. Above a critical height - called Tropopause - the temperature remains uniform through the nar belt & begins to increase afterwards.

In the Stratosphere, no intermixing due to air curr take place & the composition of the atmosphere varies with height & stratification begins.

Refractive index μ of the troposphere at the surface the earth is just 0.0003% greater than unity, it is there convenient to refer to variations in the refractive index in N

i.e., $N = (\mu - 1) * 10^{6}$.

Structure of Ionosphere:-

The upper part of the atmosphere where the ionizat is appreciable is known as ionosphere. The upper part of earth's atmosphere absorb large quantities of radiant energ from sun. This not only heats the atmosphere but also prod ionizations i.e., formation of +ve & -ve ions occurs.

The most important ionisation agents are UV, α, β Cosmic rays & meteors.

Since in a gas under very low pressure it is possible knock off one or two electrons out of a gas molecule. When the molecule donate one electron it becomes +vely charged ion & molecule which accept it becomes -vely charged lion.

Electrons can be knocked off not only by fast moving electrons but also by certain type of radiations such as U cosmic rays.

Further, the ions, electrons & atoms in a gas are const
in motion so frequent Collisions occur b/n them & Consequent
the process of recombination Continued all the time.

The time of recombination depends on many factors &
such is the average distance b/n the particles of the gas. In
lower part of the earth atmosphere — Collisions are very frequ
& hence air molecules donot remain ionized for a longer
Besides the UV rays from the sun are greatly absorbed by t
upper parts of the atmosphere & so there is relatively little
ionization in the lower part of the earth's atmosphere & ve
little ionization below about 50 km. On the other hand
above the height of 400 km the air particles present are so
that the density of ionization is again very low.

However, Considerable ionization exists in the inter
height (i.e., between 50km to 400km, & this region is, there
has the most influence on the sky wave propagation.

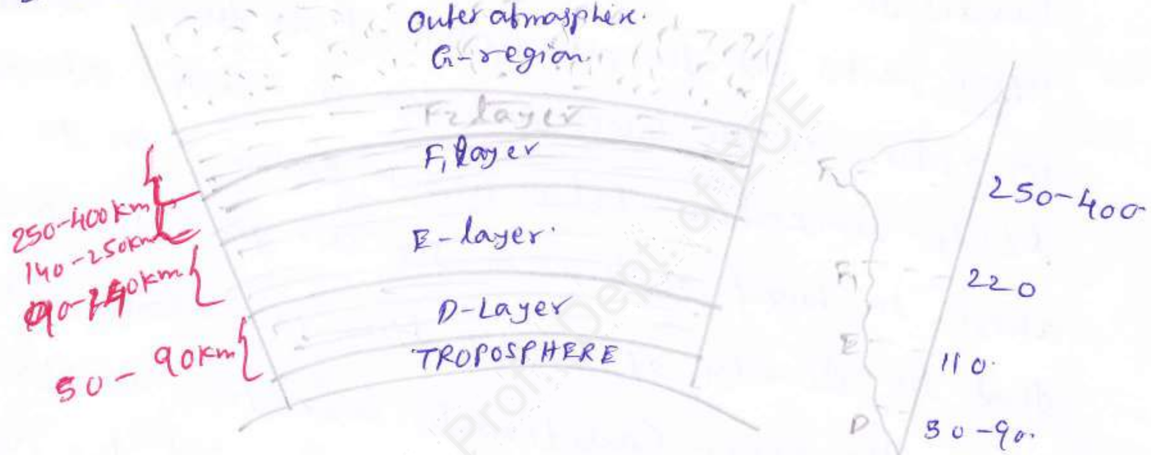
Since sky waves of different frequencies are found to
return to earth from different heights it means ionosphere
is not having one but several layers. The different layers
the ionosphere are due to fact that different gases in the
earths atmosphere ionized at different pressures. & there a
different ionizing agents to do the ionization.

Thus due to different ionizing agents and different
physical properties of the atmosphere at different heights, the
ionization in the ionosphere ~~[illegible]~~ stratified & the levels a
which the electron density reaches a maximum are called as la

The number of layers, their heights & the amount of sky wave that can be bent by them, will vary from day to month to month & year to yer.

For each such layers there is a critical frequency, ab which if a radio wave is sent vertically upward, will not return back to the earth but will penetrate it.

There are 3 principle layers during day time called E, F
F_2 layers as shown.



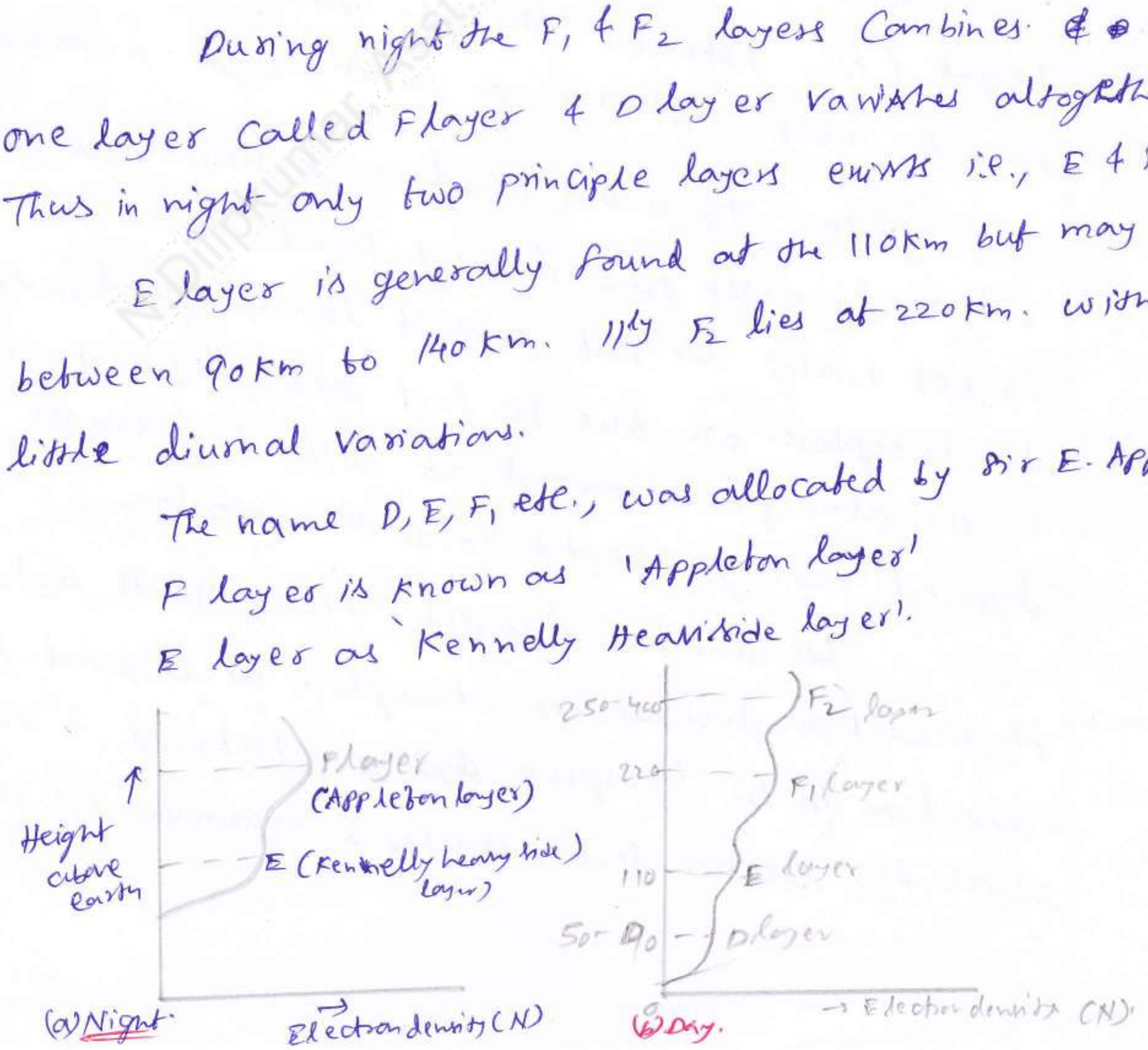
During night the F_1 & F_2 layers combines & form one layer called F layer & D layer vanishes altogether. Thus in night only two principle layers exists i.e., E & F layer

E layer is generally found at the 110 km but may vary between 90 km to 140 km. 11ly F_2 lies at 220 km. with little diurnal variations.

The name D, E, F_1 etc., was allocated by Sir E. Appleton

F layer is known as 'Appleton layer'

E layer as 'Kennelly Heaviside layer'.



Characteristics of Different Ionized Regions:-

D-region:-

D-region is the lower most region of the ionosphere & located in the height range of 50 to 90km. This layer is present only during the day light hours & disappears at night because recombination rate is high. This is due to the fact the ionization depends on the altitude of the Sun & on Sunset recombination increases resulting vanishing of D-region all toge

→ The ionization density, is max. at noon.

→ Electron density is ranging from 10^4 to 10^6 per cubic centimete

→ D-region is believed to be ionized by photo-ionization of
This ionization is produced by $L\alpha$ (Lyman alpha). radiation from su

→ D-region is due to photo ionization of oxygen molecule (O_2) its first ionization potential.

→ HF (or) Short waves penetrate.

→ LF & VLF (long & very long) reflect back to earth.

→ Critical frequency is 100KHZ.

→ D-layer is also known as absorbing layer for HF. [due to enough electrode

→ Some times, lower D-region which suggests "C-region" for range 50to 70 km.

Normal E-region:-

→ Narrow layer.

→ 90km to 140km range.

→ Kennelly & oliver Heaviside. that densely ionized layer which acts as mirror for turning EM waves back to earth.

→ Kennelly heavyside layer.

→ Maximum electron density at 110km.

→ During night time E-region remains weekly ionized & during day time remains Constant.

→ Electron density low in winter & high in summer.

→ Electron density of E-region ranges from 10^5 to $4.5 * 10^5$ do day & from $5 * 10^3$ to 10^4 at night.

→ It is maximum at noon in summer & increases with incre solar activity (i.e., sunspot cycle).

→ Critical frequency of E-layer lies in range 3MHz to 5MHz at no

→ E-region is formed by ionization of all gases by soft x-ra radiations.

→ E-layer is most useful for long distance radio propagation duri day light hours.

→ Some HF Waves will reflect by E-region.

Sporadic E-region (E_s):-

↳ irregular

→ There exists an anomalous ionisation termed as sporadic E-r denoted by Es.

→ The presence of this layer is irregular.

→ It usually occurs in the form of clouds, varying in size from about one km to several hundred km across.

→ This region presence & ionization has no connection with sun radiation.

→ The sporadic E-layer is quit unpredictable & it may be Observed both in day & night hours. & in any season of the ye

→ Electron density is 10 times to that of normal E-layer.

→ It appears at height 90km to 130km. with normal E-laye

→ Es is fairly stable with height & its height may differ from E-layer by not more than 5 to 10km.

→ Although the cause of sporadic-E-layer formation is stil uncertain, but studies reveal that origin of Es is due to different causes at different times.

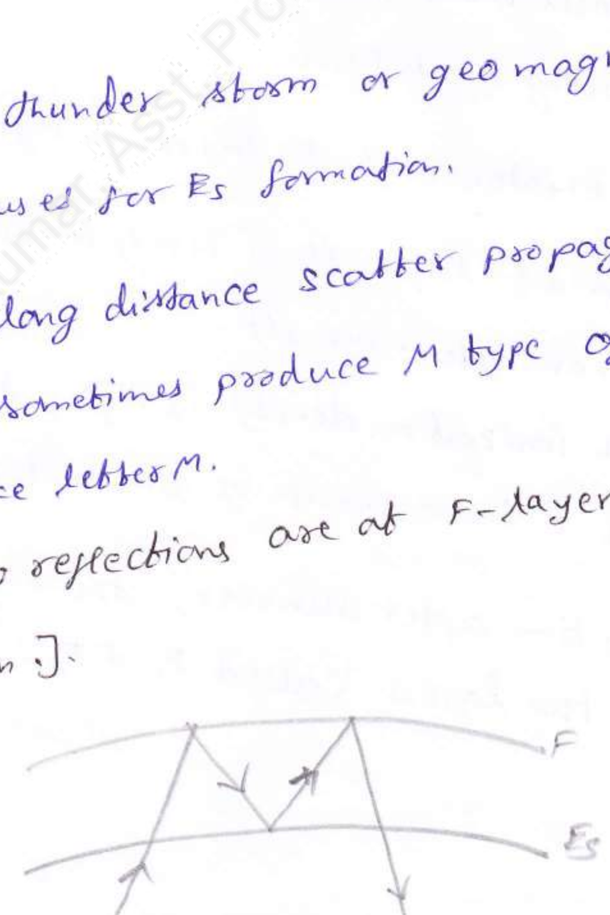
→ Some times they are produced by meteoric ionization then a other times due to vertical transport of ion clouds i.e., form of Es.

→ At times, thunder storm or geomagnetic disturbances a also the causes for Es formation.

→ Es helps long distance scatter propagation of VHF signal

→ Es also sometimes produce M type of reflection i.e., signa path is like letter M.

[Two reflections are at F-layer & one at the top o the Es region].



F_1 & F_2 & F regions (or) Appleton regions:-

→ Average height is 270 Km.

→ Always remains ionized irrespective of hours of day or seasons of the years.

→ Appleton in 1925 showed that there was a further densel ionized ~~ionized~~ layer at a still greater height than the kenelly heaviside layer. He concluded this by hearing ech effects by sending short duration HF signals projected vertically upwards.

These echo signals indicated reflections from higher layer called F-layer (or) Appleton layer

→ F-layer facilitates long distance sky wave propagation of rad signals during night hours.

The existance of F-layer in night hours is due to the fac

i) Being topmost layer, it is highly ionized & hence some ionizati remain even after sun set.

ii) Although ionization density is high, the actual air density is not much & hence most of the molecules of this layer are

During day time after sunrise, the F-region is found to split up in to two layers called F_1 & F_2.

F_1 layer:-

→ Height range is 140Km to 250km.

→ Critical frequency at noon time is order of 5MHz to 7MH

→ Electron density range $2*10^5$ to $4.5*10^5$.

→ ϕ " is lower in winter.

→ This layer is formed by ionization of Oxygen atoms.

→ Some of the HF waves are reflected from F_1 layer but mo penetrate it & is reflected from the F_2 layer. Hence the mai effect of F_1 layer is to provide more absorption for HF waves.

F_2-layer:-

→ Height range 250Km to 400km.

→ It falls to 300km during night time. when it recombines with F

→ Electron density $3*10^5$ to $2*10^6$.

→ Air density is low due to the topmost region.

→ F_2 layer is formed by the ionization of UV, X-rays & Corpus radiation

→ F_2-layer is effected largely by earth's magnetic field, atmospheric (i.e., ionospheric tides & winds), ionospheric storm & geomagnetic disturbances.

→ Its Ionization density shows large changes with solar act & the change from sun spot minimum to sun spot maximum.

→ It is most important reflecting medium for high frequency radi wa

Outer Atmospher or G-region:-

→ Above 400 Km

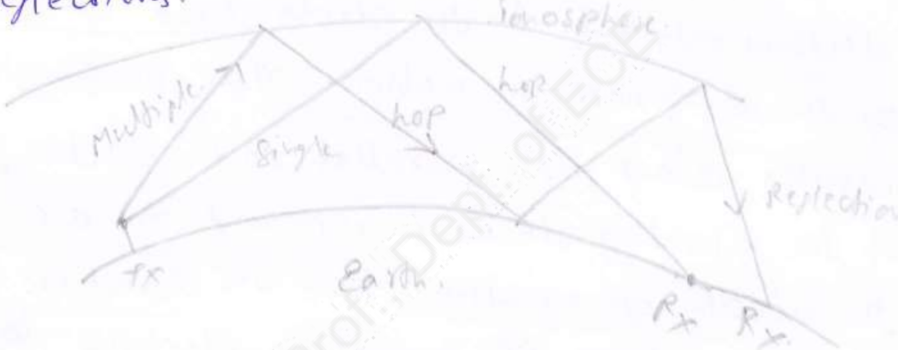
→ Terristrial magnetic fields.

→ Halving shape of magnetic lines of force.

Sky wave propagation:-

The propagation of space & ground waves are limited by the curvature of the earth & hence these ~~methods~~ ~~of~~ modes of propag fail for communication over long distances. Therefore, propagation over long distance of thousand KM or more are almost exclusi performed by the sky waves or ionospheric waves.

The sky waves are reflected from some of the ionized la of ionosphere & return back to earth either in single hop or in mult hops of reflections.



Thus for a sky wave of suitable frequency it is possible to c any distance round the earth. Radio wave of frequency 2 MHz to is ~~to~~ reflected from the ionosphere but in the day time the lower frequencies of 2-30 MHz are highly attenuated & hence ~~different~~ efficient long distance communication or broadcasting is perform in the freq. range of 10 MHz to 30 MHz.

Since in night higher freq. is around 30 MHz is not at a reflected back to earth, so during night some what lower fre is utilized for long distance (or) broad casting.

Further sky waves follows different paths in the ionospher & at receiving point, the received signal is the vector sum of all.

Propagation of radio waves through the Ionosphere. [Neglecting Earth's magnetic Field - Theory] (or) Expression for Refractive index of the Ionosphere:-

In an ionized medium having free electrons, & ions whe the radio waves passes through, it set these charged particles in motion. Since mass of the ions are much heavier than the electro so their motions are negligibly small & neglected for all pract purposes.

The radio waves passing through the ionosphere is influenced by electrons only & the electric field of radio waves set elect of the ionosphere in motion. These electrons then vibrate simulta neously along paths parallel to the electric field of the radio wa & the vibrating electrons represent an a.c. current proportional in the velocity of vibration. Here the effect of earth's magnetic fiel on the vibrations of ionospheric electrons ~~lags~~ lags behind the electric field of the wave, thus resulting electron current is inductive.

The actual current flowing through a volume of the space in ionosphere consists of the components e.g. the usual capacitive cu which leads the voltage by 90° & the electron current which lags the voltage by 90° & hence subtracted from the capacitive current.

Thus free electrons in space leads the dielectric constant reduce The reduction in the dielectric constant due to presence of the elec in the ionosphere causes the path of radio waves to bent towards ea i.e., from high electron density to lower electron density.

Let an electric field of value,

$$E = E_m \sin \omega t \quad \text{volts/metre.}$$

is acting across a cubic metre of space in the ionosphere, wh
ω is the angular velocity & E_m, the maximum amplitude.

Force exerted by the electric field on the each electron is,

$$F = -eE \text{ Newton.} \quad ; \; e = \text{Charge of an electron in coul}$$

Let us assume that there is no collision, then the electron will ha
an instantaneous velocity v meters/sec, in the direction opposite to th
field.

Force = Mass * Acceleration

$$-Ee = m\frac{dv}{dt}. \qquad m \Rightarrow \text{mass of } e^- \text{ in kg.}$$
$$\frac{dv}{dt} \Rightarrow \text{Acceleration.}$$

$$\frac{dv}{dt} = -\frac{Ee}{m}.$$

$$dv = -\frac{Ee}{m}\,dt.$$

Integ. b.s.

$$\int dv = -\int \frac{Ee}{m}\,dt.$$

$$v = -\frac{e}{m}\int E\,dt$$

$$= -\frac{e}{m}\int E_m \sin \omega t\,dt$$

$$= \frac{e}{m}\,\frac{E_m \cos \omega t}{\omega}.$$

$$v = \left(\frac{e}{m\omega}\right) E_m \cos \omega t.$$

If N be the no. of electron per cubic metre, the instantaneous e
current constituted by these N electrons ~~flowing~~ moving with instantane
velocity v is,

$$i_e = -Nev \text{ amp./m}^2.$$

$$i_e = -Ne\left(\frac{e}{m\omega}\right) E_m \cos\omega t.$$

$$i_e = -\left(\frac{Ne^2}{m\omega}\right) E_m \cos\omega t.$$

Which shows current i_e lags behind the electric field $E = (E_m \sin\omega t)$ by

Besides this inductive current (or conduction current component obtained by ionization of air i.e., presence of electron & its motion), there is usual capacitive current (i_c) (or displacement cur exists in an un-ionized air). The capacitance of unit vol

$$K_0 = 8.854 \times 10^{-12} \text{ F/m}.$$

Hence the capacitive (or) displacement current through this capacitance is,

$$i_c = \frac{dD}{dt} = \frac{d}{dt}(K_0 E) = K_0 \frac{d}{dt}(E_m \sin\omega t).$$

$$\because D = \varepsilon_0 E = K_0 E;\ K_0 = \text{constant}.$$

$$i_c = K_0 E_m (\cos\omega t)\,\omega. \quad \text{— (a)}$$

Thus total current i that flows through a cubic metre of ionized medium is,

$$i = i_c + i_e = K_0 E_m \omega \cos\omega t - \frac{Ne^2}{m\omega} \cos\omega t$$

$$= E_m(\cos\omega t).\omega\left[K_0 - \frac{Ne^2}{m\omega^2}\right] \rightarrow \text{(b)}$$

Comparing (a) & (b),

$$K = K_0 - \frac{Ne^2}{m\omega^2} = K_0\left[1 - \frac{Ne^2}{m\omega^2 K_0}\right]$$

Hence the relative dielectric constant w.r.t vaccum (or) air,

$$K_r = \frac{K}{K_0} = 1 - \frac{Ne^2}{m\omega^2 K_0}$$

Thus relative refractive index (μ) of the ionosphere w.r.t. vaccum (or) air (i.e., un-ionized air)

$$\mu = \sqrt{K_r} = \sqrt{\frac{K}{K_0}}$$

$$= \sqrt{1 - \frac{Ne^2}{m\omega^2 K_0}}$$

$$\boxed{\because V = \frac{c}{\sqrt{K_r}} = \frac{c}{\mu} \quad \therefore \mu = \sqrt{K_r}}$$

$m = 9.107 * 10^{-31}$ kg.

$e = 1.1602 * 10^{-18}$ coulombs $\qquad$ & $\omega = 2\pi f$.

$K_0 = 8.854 * 10^{-12} = \frac{1}{36\pi \times 10^9}$ F/m.

Sub. the values in μ,

$$\mu = \sqrt{1 - \frac{81N}{f^2}}.$$

where, N = number of $\bar{e}$'s per cubic metre (or) ionic density &

f = freq. in Hz.

If N is in cubic cm then freq. is in KHz.

The refractive index of ionosphere is less than one where that un-ionized medium is one. Thus presence of electrons in ionosphere reduces the refractive index of the air & reduction is higher if $\bar{e}$'s are more.

Mechanism of Radio waves bending by the ionosphere:-

The bending of radio waves at the ionosphere can readily be understood with the help of refractive index,

$$\mu = \sqrt{K_r} = \sqrt{1 - \frac{81N}{f^2}}.$$

N = ionic density, in m^{-3}

f = freq. in HZ.

If N is expressed in per cubic cc. then f will be in KHz.

The equation shows that real values of refractive index of ionosphere is always less than unity and the deviation of μ from the unity becomes greater, if the ionic density is higher & freq. is lower.

If $f^2 < 81N$, then the refractive index becomes ima[ginary] which means under such condition the radio waves are atten[uated] at this freq. & ionosphere is not able to transmit (or) bend the radio waves.

The bending of radio waves by the ionosphere is gover[ned] by the ordinary optical laws. By snell's law, the angle of incidence (i) & refraction (r) at any point is,

$$\mu = \frac{\sin i}{\sin r}.$$

$\because \mu < 1$ for the ionosphere, so $\sin i < \sin r$, i.e., angle of refraction will go on deviating from the normal as the wave will encounter rarer medium as shown in fig. If successi[ve] layers of the ionosphere are of higher electron density i.e. $N_6 > \cancel{N_5} > N_4 > N_3 > N_2 > N_1$ it means, μ will go on decreasi[ng] and decreasing i.e., $\mu_1 > \mu_2 > \mu_3 > \mu_4 > \mu_5 > \mu_6$. Thus a wave enters at say point P will be deviating more & more & a point will reach where it travels parallel to earth (at Pm). The angle of refraction is 90° & the point Pm is the highest point in the ionosphere reached by the radio wave.

If μ_m, be refractive index & N_m be the maximum electron density at the point Pm then, eqr. μ will become,

$\mu_m = \sin i_m$. $\quad \because \sin r = \sin 90° = 1$.

The point Pm, is usually called as point of reflection although it is actually a point of refraction. At this point total internal reflection takes place & the wave gets bent earthwards & ultimately returns to earth.

"Hence the radio waves once enter at point P, leave ionosphere at point Q after slight penetration in to the ionosphere & thus radio waves are reflected back to earth after successive refraction in the ionosphere".

$\mu_m = \sin i_m$ suggests that smaller the angle of incidence ($\angle i$), the smaller the refractive index μ_m which implies higher should be electron density needed to return the radio wave towards the earth. Further, if the angle of incidence reduces to zero i.e., vertical incidence ($\angle i = 0$), the refractive index also becomes zero for reflection to take place & this corresponds to maximum electron density of the layer & the frequency corresponds to critical frequency – the max. freq. which can be reflected by a layer at vertical incidence.

Critical frequency :-

The critical frequency of an ionized layer of the ionosphere is defined as the highest frequency which can be reflec by a particular layer at vertical incidence. This highest freq. is called critical frequency for that particular layer & it different for different layers. It is usually donoted by f_0 (or)

Critical frequency for a particular regular layer is proportional to the square root of the maximum $\bar{e}$ density in layer.

$$\mu = \frac{\sin i}{\sin r} = \sqrt{1 - \frac{81N}{f^2}}$$

By definition, at vertical incidence,

Angle of incidence, $\angle i = 0$; $N = N_{max}$, $f = f_c$.

As the angle of incidence go on dcreasing & reaches to ze (i.e., vertical incidence) the electron density go on increasing & reaches to max. electron density (N_m). Then the highest fre that can be reflected back by the ionosphere is one for whic refractive index μ becoms zero.

$$\mu = \frac{\sin \theta}{\sin r} = \sqrt{1 - \frac{81 N_m}{f_c^2}} = 0.$$

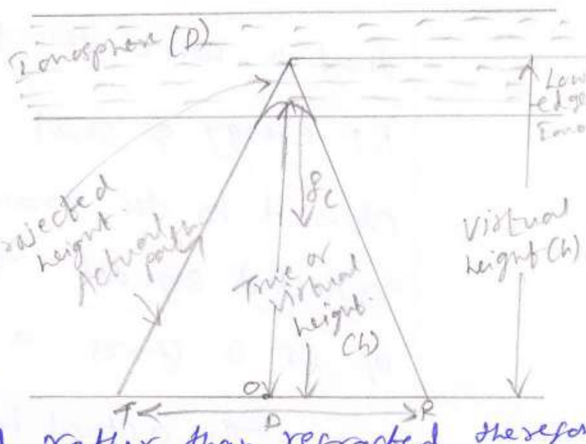
$$1 = \frac{81 N_m}{f_c^2}$$

$$f_c = \sqrt{81 N_m}$$

$$f_c = 9\sqrt{N_m}.$$

Virtual height:-

The actual path of the wave in the ionized layer is a curve and is due to the refraction of the wave, as happens in the case of refraction from the prism.



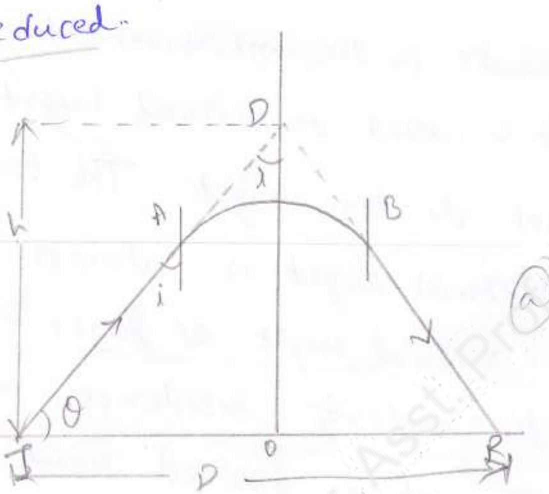
Since it is more convenient to think of the wave being reflected rather than refracted therefore the path can be assumed to be straight lines TD and RD as shown in fig.

This assumption is made in the measurements of the height of a layer. The height OD is called the virtual height of the ionized layer as it is not the true height. The true height is shown in figure. Virtual height is always greater than the actual height. If the virtual height of layer is known, then it is easy to calculate the angle of incidence required for wave to return to earth at a desired point.

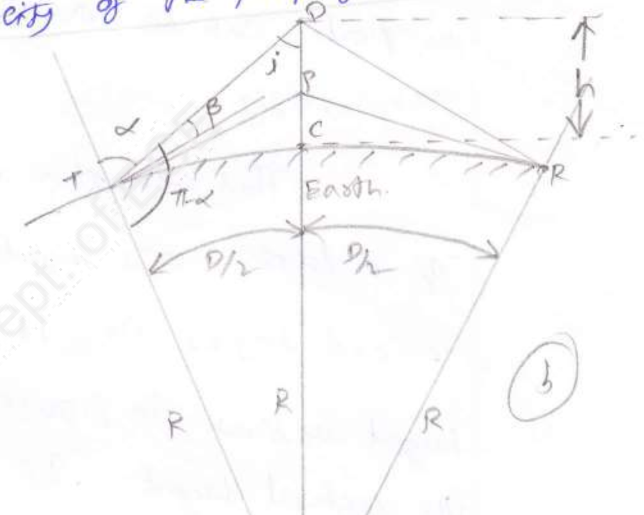
"Virtual height of an ionospheric layer may be defined as the height to which a short pulse of energy sent vertically upward and travelling with the speed of light would reach taking the same two ways travel time as does the actual pulse reflected from the layer. In the measurement of virtual heights the transmitting point (T) and receiving point (R) are usually placed very close together so that the wave sent near vertically upward.

The Commonest method of virtual height measurement is that in which the transmitted signal consists of pulses of RF energy of short duration. The receiver which is located closed to the transmitter, picks up both the direct and the reflected signals. The spacing b/n these signals on the time axis of CR.O gives a measurement of the height of the layer.

"The actual height (h) is less than virtual height because interchange of energy takes place b/n the wave & the electrons of the ionosphere causes the velocity of the propagation to be reduced."



virtual height determination for flat earth.



virtual height determination for curved earth.

The virtual height has the greatest advantage of being easily measured, and it is very useful in transmission-path cal.'s. For flat earth assumption and assuming that the ionospheric conditions are symmetrical for the incident and reflected waves, the transmission-path distance TR is obtained from the fig. (a),

$$\tan\beta = \frac{DO}{TO} = \frac{h}{TR/2}$$

$$\frac{TR}{2} = \frac{h}{\tan\beta} \qquad \because TO = \frac{TR}{2}$$

$$TR = \frac{2h}{\tan\beta} = D.$$

When the curvature of earth is accounted for, then the transmission-path distance may be calculated from the geometry of the figure, (b), from ΔTOD,

$$\frac{\sin i}{R} = \frac{\sin(\pi-\alpha)}{R+h}$$

$\because \sin(\pi-\alpha) = \sin\alpha.$

R = radius of earth = 6370 km

$$\frac{\sin i}{R} = \frac{\sin\alpha}{R+h}$$

$$\angle OTD = \pi - \angle i + \theta \quad (or)$$

$$\pi - \alpha = \pi - (i+\theta)$$

$$i = \alpha - \theta.$$

$$\frac{\sin(\alpha-\theta)}{R} = \frac{\sin\alpha}{R+h}.$$

$$\sin(\alpha-\theta) = \frac{R.\sin\alpha}{R+h}.$$

$$\theta = \alpha - \sin^{-1}\left(\frac{R\sin\alpha}{R+h}\right)$$

from fig (b) $\quad 90^\circ = \alpha + \beta$

$$\alpha = 90 - \beta$$

$$\theta = (90-\beta) - \sin^{-1}\left(\frac{R\sin\alpha}{R+h}\right)$$

$$= 90^\circ - \beta - \sin^{-1}\left(\frac{R\sin(90-\beta)}{R+h}\right)$$

$$\theta = 90^\circ - \beta - \sin^{-1}\left(\frac{R\cos\beta}{R+h}\right) \text{ radians.}$$

$$\text{Angle} = \frac{\text{Arc}}{\text{Radius}}$$

$$\theta = \frac{\text{Arc } TE}{R} = \frac{D/2}{R} = \frac{D}{2R}.$$

$$D = 2R \cdot \theta$$

$$= 2R\left[(90^\circ - \beta) - \sin^{-1}\left(\frac{R\cos\beta}{R+h}\right)\right].$$

Measurement of virtual height is normally carried o[ut] by means of an instrument known as an IONO SONDE.

The basic method is to transmit vertically upward [a] pulse -- modulated radio wave with a pulse duration of about 150 micro-seconds. The reflected signal is received close to the Tx'tion and the time T required for the round trip is measured.

The virtual height is given by,

$$h = \frac{cT}{2} = \text{virtual height}$$

c = velocity of light, in m/s = 3×10^8 m/s.

The ionosonde will have facilities for sweeping over t[he] radio frequency range, typically, it will sweep from 1 MHz [to] 20 MHz in 3 min's.

Skip distance:-

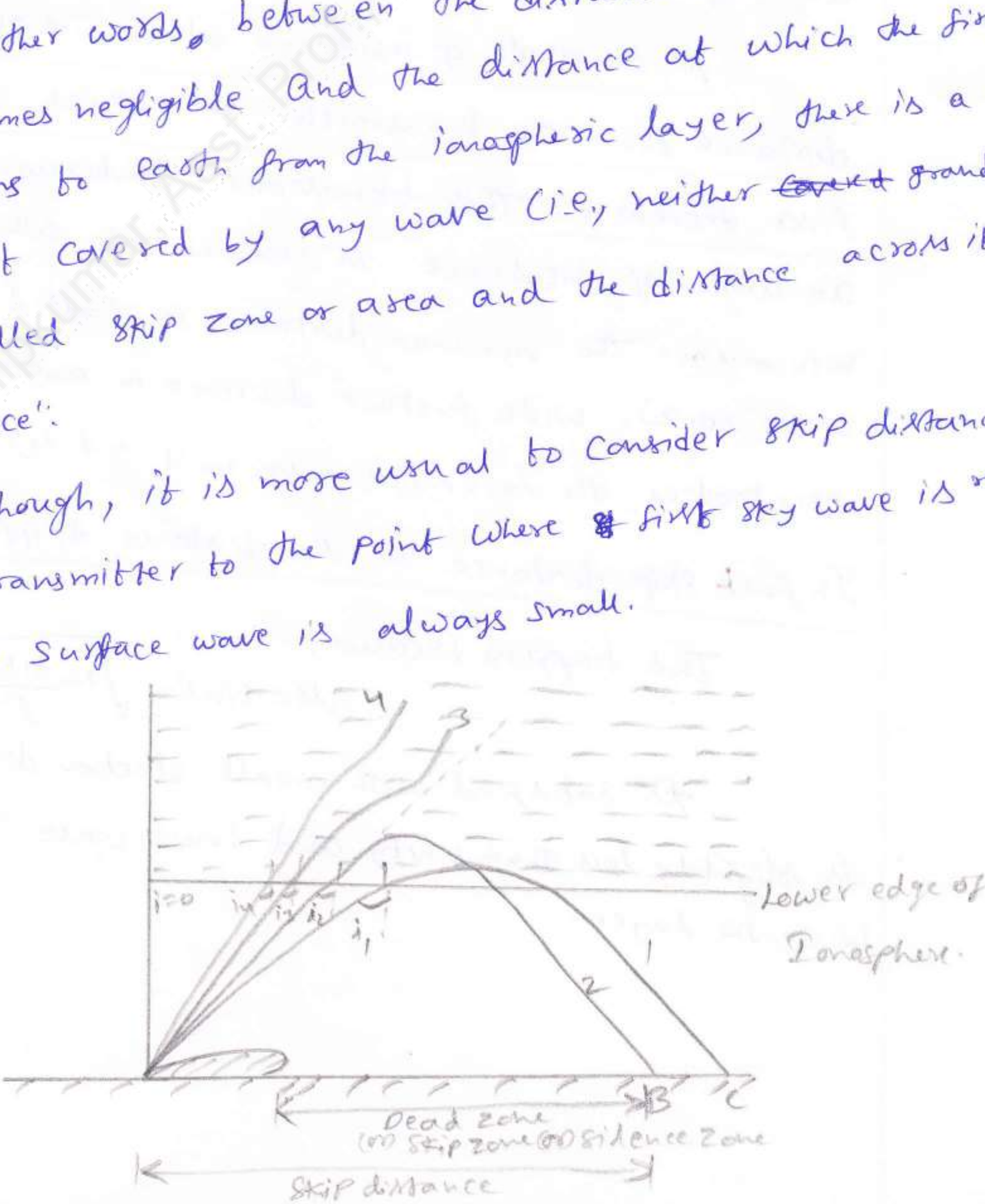
Radio wave radiated horizontally from a transmitter nea the earth's surface is quickly absorbed due to large ground losses and hence only short distance communication is carried out by this horizontal radiations of ground or surface wave.

Radio wave radiated at high angle may not be bent sufficiently at the ionospheric layers to return to earth at all an hence escapes rather penetrates the layer.

Thus radio wave radiated at shallow angle (i.e., angle between horizontal and high angle) just great enough to escape absorption by the earth, will enter the lower layer, ~~be~~ ben at the upper layer and return to earth.

In other words, between the distance at which surface wave becomes negligible and the distance at which the first wave returns to earth from the ionospheric layer, there is a zon which is not covered by any wave (i.e., neither ~~cover~~ ground or This is called skip zone or area and the distance across it is 'skip distance'.

Although, it is more usual to consider skip distance from the transmitter to the point where first sky wave is receiv as range of surface wave is always small.



Skip distance Can be defined as,

The minimum distance from the transmitter at which a sky w[...] of ~~freq.~~ given frequency is returned to earth by the ionosphere.

(or)

The minimum distance from the transmitter to a point where sky wave of a given freq. is first received

The higher the freq., the higher the skip distance and for a frequency less than Critical frequency of a layer skip distance is [...]

As the frequency of a wave exceeds the critical frequency, [...] effect of the ionosphere depends upon the angle of incidence at the ionosphere as shown in fig. Which gives waves of di[...] angle of incidence.

As the angle of incidence at the ionosphere decreases, the distance from the transmitter at which the ray returns to g[...] first decreases. This behaviour Continuous until eventually an angle of incidence is reached at which the distance becom[...] minimum. The minimum distance is called skip distance.D. (as wave no. 2). With further decrease in angle of incidence, the w[...] penetrates the layer (as wave no.'s 3 & 4) & does not return to [...] In fact skip distance is the distance skipped over by the sky w[...]

This happens because,

$$\mu = \sin i = \sqrt{1 - \frac{81N}{f^2}}$$

is satisfied with small electron density, This means μ is slightly less than unity and hence wave returns after slight pen[...] in to the layer.

As the angle of incidence is further decreased (as wave no. 2) (16)
sin i derease still more and so also the μ, as N becomes comparitively more. Hence the wave penetrates still more before reaches to earth.

The frequency which makes a given distance corresponds to the skip distance is the maximum usable frequency for those two poi[nts]

For a given frequency of propagation $f = f_{muf}$ the skip distan[ce] can be calculated,

$$\frac{f_{muf}}{f_c} = \sqrt{1+\left(\frac{D}{2h}\right)^2}$$

$$\left(\frac{f_{muf}}{f_c}\right)^2 - 1 = \left(\frac{D_{skip}}{2h}\right)^2 \Rightarrow D_{skip} = 2h\sqrt{\left(\frac{f_{muf}}{f_c}\right)^2 - 1}$$

Maximum usuable frequency:-

Critical freq. is the max. frequency of the radio wave w[hich] is returned from a ionized layer at vertical incidence. Howe[ver] when the freq. of radio waves exceeds the critical freq., then influence of the ionosphere layer on the path of propagation (i.e., Communication) depends on the angle of incidence at the ionosphere.

The Maximum usable frequency (MUF) is also a limiting frequency which can be reflected back to earth but this time [for a] specific angle of incidence rather than the vertical. The maximu[m] possible value of freq. for which reflection takes place for a gi[ven] distance of propagation, is called as the maximum usable freque[ncy] (MUF) for that distance, and for the given ionospheric layer.

If the frequency is higher than this then the wave penetrate the ionized layer and does not reflect back to the earth.

→ Stating in another way MUF can also be defined as the freq which makes a given receiving point corresponds to a distance from the transmitter equal to the skip distance.

→ The MUF is the highest frequency which can be used for sky wave ~~propagation~~ communication b/n two given points on the earth. This implies that maximum usable frequency is the highest frequency which can be used for sky wave commn b/n given points on the earth and there is a different value of MUF for each pair of points on the globe.

Normal value of MUF vary from 8MHz to 35MHz.

After unusual solar activity it may be as high as 50MHz. At the same time the highest working freq. b/n two particular points on the earth is obviously a bit less than MUF.

For a sky wave to return to earth, angle of refraction, i.e., $\angle r = 90°$, which implies $N = N_{max}$ & $f = f_{max}$, i.e., max. freq

$$\mu = \frac{\sin i}{\sin 90} = \sqrt{1 - \frac{81 N_m}{f^2_{muf}}}$$

$$\mu = \sin i = \sqrt{1 - \frac{81 N_m}{f^2_{muf}}} \qquad \text{but, } f_c^2 = 81 N_m$$

$$\sin i = \sqrt{1 - \frac{f_c^2}{f_{muf}^2}} \quad \text{(or)} \quad \sin^2 i = 1 - \frac{f_c^2}{f_{muf}^2}$$

$$\frac{f_c^2}{f_{muf}^2} = 1 - \sin^2 i = \cos^2 i$$

$$f_{muf}^2 = f_c^2 \sec^2 i \Rightarrow \boxed{f_{muf} = \sec i \, f_c}$$

This shows that muf for a layer is greater than f_c by

factor $\sec i$.

This is known as SECANT LAW and gives the max. freq. w

can be used for sky wave comm. for a given angle of incidence

between two points on the earth.

This eq. can be applied safely to a distance of 1000 km.

However, as the distance b/n two points (i.e., Tx & Rx) is incre

a limit occurs to the curvature of the earth, where the path

the wave is tangent to the surface of the earth at these

points.

The angle (i) corresponding to this limiting distance is ab

74°, for F-layer. Hence maximum usable frequency for this

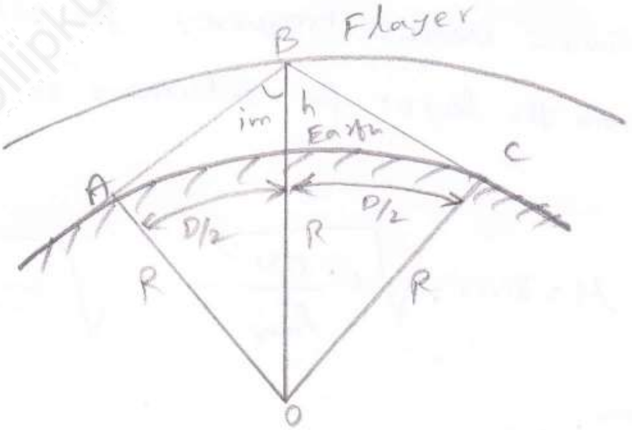
case is given by,

$$f_{muf} = \sec 74\ f_c$$

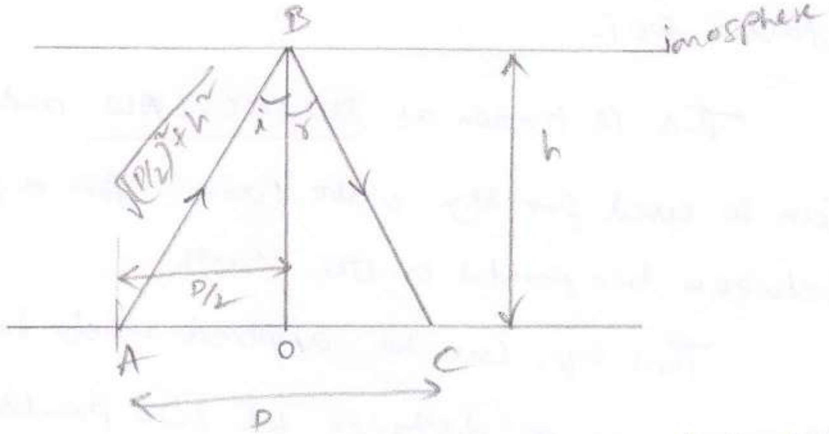
$$f_{muf} = 3.6\ f_c.$$

$i_m = i_{max}$

$\sin i_{max} = \frac{R}{R+h}$.



Calculation of MUF for Thin layer (or Flath Earth):-



The ionized layer may be assumed to be thinlayer with sharp ionization density gradient, which gives mirror li[ke] reflection of radio waves as shown in fig.

For short distance of communication (say up to 500km) the earth can be assumed to be flat.

from fig.

$$\cos i = \frac{BO}{AB} = \frac{h}{\sqrt{h^2+\frac{D^2}{4}}} = \frac{2h}{\sqrt{4h^2+D^2}}$$

h = height of layer, D = propagation distance AC.

The maximum usable frequency for which the wave is to b[e] reflected from the layer for returning to earth, $f = f_m$, $\sin r = [1]$ & $N = N_m$,

$$\mu = \sin i = \sqrt{1-\frac{81N}{f_{muf}^2}} = \sqrt{1-\frac{f_c^2}{f_{muf}^2}}$$

from, ---

$$\cos^2 i = \frac{f_c^2}{f_{muf}^2} = \frac{4h^2}{4h^2+D^2}$$

$$\frac{f_{muf}^2}{f_c^2} = \frac{4h^2+D^2}{4h^2}$$

$$\frac{f_{muf}}{f_c} = \sqrt{\left(1+\frac{D^2}{4h^2}\right)}$$

$$f_{muf} = f_c\sqrt{1+\left(\frac{D}{2h}\right)^2}$$

Lowest Usable frequency (L.U.F (or) L.U.H.F):-

The absorption of an high frequency (HF) radio wave in the D-region of the ionosphere is proportional to the inverse squa of the frequency.

The sensitivity of an HF receiver is normally limited external noise which increases as the frequency is reduced Hence there is a frequency limit below which the signal to noi ratio fails to reach an acceptable value for the service requi

Therefore LUF is dependent upon the engineering charac teristics of the like transmitted power.

In addition to absorption limitations, the signal can loose ene after it has been transmitted by several other mechanism,

i) Free space propagation loss i.e., spatial spreading of the energ
ii) Polarization change caused by the earth's magnetic field.
iii) Scatter processes.
iv) Focusing & defocusing caused by ionospheric curvature.

As the MUF limits the highest permissible freq. for sky wave propagation in a given path, the LUF gives the lowest permissible freq For a lower freq. of transmission the received sky wave signal gets lost in the background noise and no communication is possible.

The LUF is limited by absorption in the D-layer during day light hours. where as at night, it is primarily limited by increased noise at lower frequencies.

The value of day time LUF is normally much higher than the night time

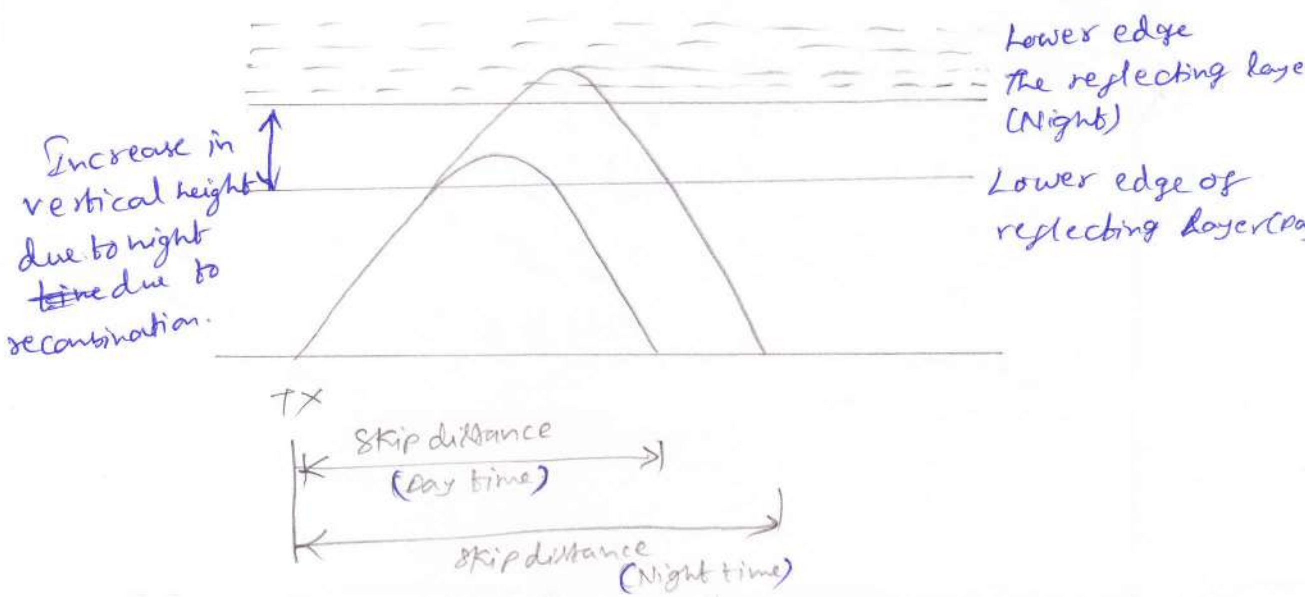
Optimum Working Frequency (OWF) (AND DAY AND Night Freque

In practical radio Communication, for satisfactory reception of signals at the receiving points, it is essentia that the frequency should be less than MUF and absorption waves by the ionosphere be small.

The absorption is dependent on the inverse Square frequency. The highest possible freq. gives the strongest sky wave signal at the receiver and hence it is preferre to work as closely as possible to the MUF.

Optimum frequencies are selected from the predication MUF based on a monthly average and in practice there is daily variations about 15% from this mean value.

Hence it is normal to use a freq. 85% of the pred MUF. Therefore there is a frequency called Optimum work frequency (O.W.F) or Optimum traffic frequency (O.T.F), w is 50% to 85% of MUF is used to accomodate a number Channels i.e., OWF = 85% MUF.



Since MUF for a particular location varies considerab with time of the day, from season to season and from months to months and accordingly the optimum working freq. also follow similar variations.

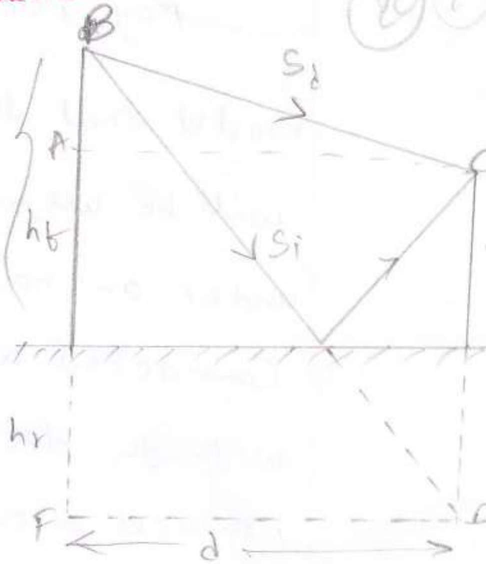
However, in practice it is not possible to change the frequency of communication from hour to hour. ~~to~~

Therefore, for continuous communication, it is necess to use atleast two frequencies, one for day and other for night. Even sometimes a third freq. for transition period i also used. In the night the vertical height of the ionospheric layer increases than in the day time and ~~also~~ so the skip distance too increases. as shown in fig.

Typical freq.'s for day and night are 6.450 MHZ and 5000 MHZ respectively.

The practical freq. for day time is selected 15% to 2 lower than the average of optimum freq. for entire of the day time.

Field strength of space (or) Tropospheric wave:-



If the curvature of earth is neglected the space wave propagation takes place as shown in fig. in which the energy received at the Rx'ing point in two ways i.e., direct & other by the indirect rays. The field strength received at the Rx'ing point is the vector sum of the fields of the two rays.

The direct ray suffers almost negligible attenuation the indirect ray under-going reflection at the ground too will assumed to be of almost same magnitude but of different phase, as the heights of Tx & Rx'ing antennas are small compared to the distance b/n them & so the angle of incidence at ground is also small.

$$E = \frac{88\sqrt{P}\, h_t h_r}{\lambda d^2} \text{ Volt/metre.}$$

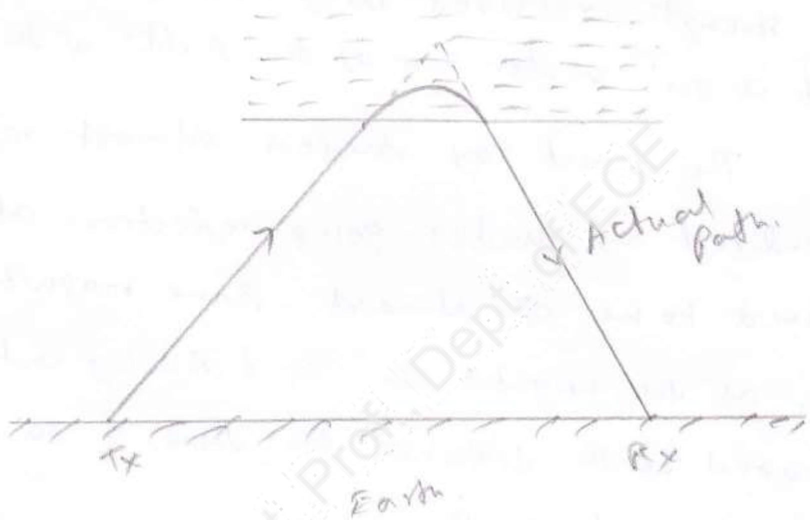
E_R is the resultant field strength at Rx'ing point R.

$\sqrt{P}$: Effective power radiation.

Ray path:-

It is the imaginary st. line under which the path of the EM (or) Radio wave can be approximated to the actual path under the propagation, the path that this follows can be in a spherical or a plane front. At large distances from sources the spherical wavefront can be considered as a plane front.

Ray path is used for the approximation for the Transmissi[on] model that the actual EM wave follows as shown in fig. This will be useful due to the fact that the actual propaga[tion] under an non-ideal or non-uniform atmosphere can be Considered as a series of Short straight paths, due to[…] with the decreasing refractive index. This layer is realit[…] produces a curved path which is approximed as in fig.



Radio Horizon:-

Horizon means visible. It has another meaning, that is, a line at which earth & sky appear to meet.

Radio horizon of an antenna is defined as the locus of t[he] distant points at which direct rays from the antenna beca[me] tangential to a planetary surface. The horizon is a circle on a spherical surface.

Reflection of Radio waves by the surface of the Earth:- (21) (9)

If an EM wave is incident on the earth, it is reflected back, The angle of reflection is equal to the angle of incidence. The reflection co-efficient is the ratio of the reflected wave to the incident wave,

$$\text{i.e., } \rho = \frac{\text{Reflected wave}}{\text{Incident wave}}$$

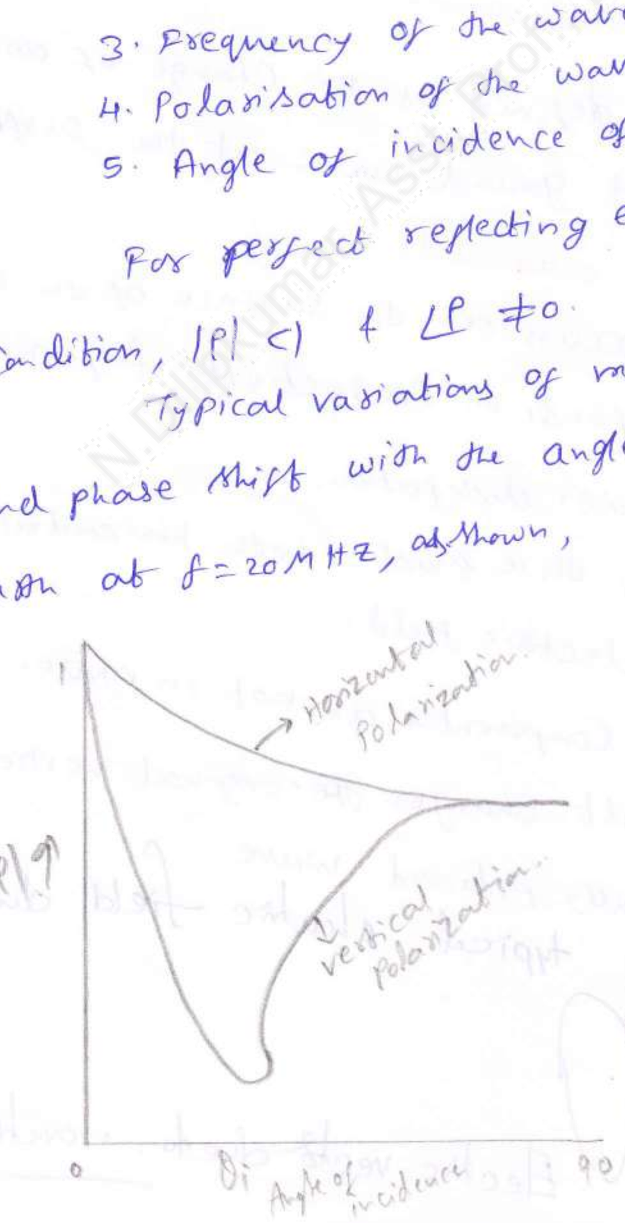
The field strength near the earth is the vectorial sum of the incident and reflected fields.

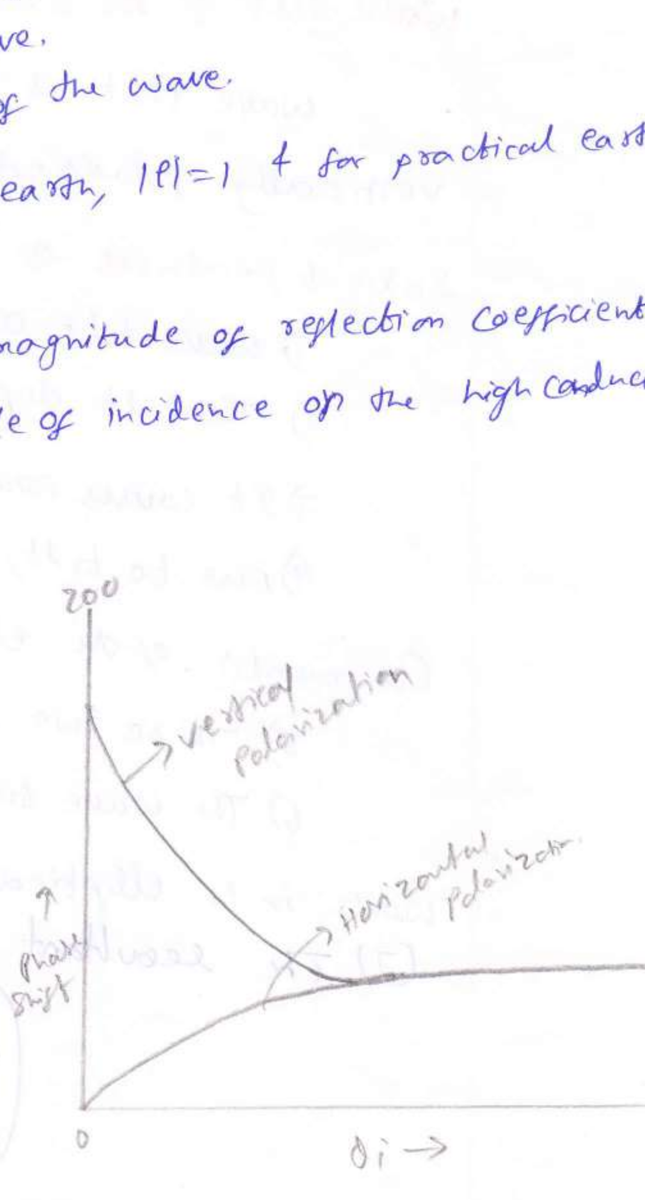
The reflection coefficient, ρ depends on

1. Dielectric Constant, ϵ_r
2. Conductivity of the Earth, σ
3. Frequency of the wave.
4. Polarisation of the wave.
5. Angle of incidence of the wave.

For perfect reflecting earth, $|\rho| = 1$ & for practical earth condition, $|\rho| < 1$ & $\angle \rho \neq 0$.

Typical variations of magnitude of reflection coefficient and phase shift with the angle of incidence on the high conducting earth at $f = 20\,MHz$, as shown,





From fig. that for horizontally polarized waves, the reflect Co-efficient is the same as that of perfect earth for $\theta_i = 0$. That is, $\rho = 1\angle 0$. When θ_i increases, $|\rho|$ reduces from 1 & the phase shift becomes small. The phase is found to be lagging with respect to the perfect earth.

For vertically polarised wave, $\theta_i = 0$, the reflection Coefficient, ρ is $1\angle 180°$. At $\theta = 90°$, the reflection co-efficient for vertical & horizontally polarised waves are identical. The angle of incidence at which there is no reflection is known as Brewster angle.

$$\text{Brewster angle, } \theta_b = \theta_i = \tan^{-1}\sqrt{\frac{\epsilon_{r_2}}{\epsilon_{r_1}}}$$

Wave Tilt of the Ground wave:-

Wave tilt is defined as the change of orientation of the vertically polarised ground wave at the surface of the earth.

Salient features of wave tilt:-

1) wave tilt occurs at the surface of the earth.
2) The tilt depends on Conductivity & permittivity of the earth.
3) It causes power dissipation.
4) Due to tilt, there exists both horizontal & vertical Components of the electric field.
5) These two Components are not in phase.
6) The wave tilt Changes the original vertically polarised wave in to elliptically polarised wave.

(7) The resultant typical electric field due to wavetilt-

Electric vector due to wavetilt

Ground wave Field Strength:-

According to Somerfield analysis, the ground wave field strength for flat earth, is given by,

$$E = \frac{AE_0}{d}.$$

$E \rightarrow$ Field strength at a point, V/m.

E_0 = Field strength of the wave at a unit distance from the Tx'ing antenna, neglecting earth's losses. (V/m).

A = Factor of the ground losses.

d = distance of the point from Transmitting antenna.

E_0 depends on,

1. Power radiated by the transmitting antenna.
2. Directivity of the antenna in vertical & horizontal plane

The factor A depends on,

1) Conductivity, σ mho/m.
2) Permittivity of the earth, ϵ_r.
3) Frequency of the wave, f.
4) Distance from the transmitter, d.

The determination of field strength due to Somerfield analysis mainly consists of determination of A.

This is found from the knowledge of numerical distan & phase constant. b,

For vertical polarisation, these are given by,

$$P = \frac{\pi}{D_f} \cdot \frac{d}{\lambda} \cos b$$

$$b = \tan^{-1}\left(\frac{\epsilon_r + 1}{D_f}\right)$$

where D_f is known as dissipation factor of dielectric.

~~where~~ $D_f = 1.80 * 10^{12} \frac{\sigma}{f}$.

σ = Conductivity, (mho)/cm of earth.

ϵ_r = relative permittivity of earth.

f = frequency in Hz.

d/λ = normalised distance with respect to λ.

λ = wave length.

Sailent Features of Ground wave propagation:-

1) Ground wave propagates by gliding over the surface of earth

2) It exists for vertically polarised antennas.

3) It exists for antennas close to the earth.

4) It is suitable for VLF, LF & MF Communications.

5) It can be used even at 15KHz & up to 2MHz.

6) Ground wave field strength is $E = \frac{AE_0}{d}$.

7) The Ground wave field strength varies with characteristics of the earth

8) Ground waves requires relatively high transmitter power.

9) Ground wave propagation losses vary considerably with type of earth.

10) Ground waves are not affected by the changes in atmospheric conditions.

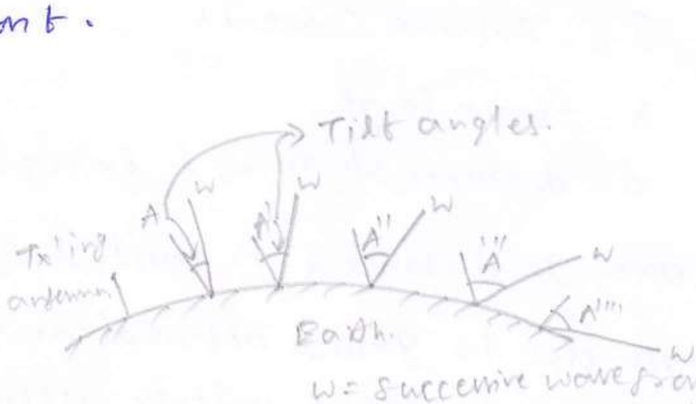
11) Ground waves can be used to communicate between any two points on the globe if there is sufficient transmitter power

12) It can be used for navigation, for ship to ship, ship to shore Communication & maritime mobile Communications.

The factor, A can also be calculated approximately from the following expression:

$$p = \frac{0.582\, d_{km}\, f^2(MHz)}{\sigma\,(mho/m)}$$

Besides ground attenuation, there is still another way in wh surface wave is attenuated i.e., due to diffraction & tilt in wave front.



W = Successive wave fronts.
A, A', A'', A''', A'''' = Tilt angles in increasing order.

Tilting wave fronts in ground wave propaga

As the wave progress over the curvature of the earth, th wave fronts start gradually tilting more & more. This increase in the tilt of wave causes more short circuit of the electric field component & hence the field strength goes on reducing. Ulti at some appreciable distance from the tx'ing antenna in wavele the surface wave dies because of the losses mentioned above.

It may be noted that maximum range of surface wave propagation depends not only on the frequency but power as well. Hence range of transmission can be increased by increasing the power of the transmitter in the VLF band but this method cann be effective at the MF band where the tilting due to diffraction more effective.

The field strength at a distance from the Tx'ing antenna due to ground wave has been calculated from the maxwells eq.

$$E = \frac{120\pi\, h_t \cdot h_r\, I_s}{\lambda d} \text{ Volt/meter}$$

$120\pi = 377\,\Omega$ = Intrinsic impedance of free space.

$h_t = h_r$ = Effective heights of Tx'ing & Rx'ing antennas.

I_s = Antenna Currents.

λ = wave length.

d = distance b/w Tx'ing & Rx'ing antenna.

If, however, the distance d is fairly large, the reduction in the field strength due to ground attenuation & atmospheric absorp increases & thus the actual voltage received at Rx'ing point decreases. This results in less field strength.

A/c to Sommerfield, the field strength for ground wave propagation for a flat earth is given by,

$$E_g = \frac{E_0 A}{d}.$$

E_0 = Ground wave field strength at the surface of earth at unit distance from the Tx'ing antenna. Earth losses not accoun

E_g = Ground wave field strength.

A = factor accounting for earth losses called attenuation facto

d = Distance from Tx'ing antenna expressed in the same unit as E

$\theta = \pi/a$

1) frequency.
100 KHZ to few KHZ.


← Induction.

2) orientation is vertical.

3) Range is limited.
(small) because freq. is less.
∴ Energy is less.

4) Since they are near to the earth there are chances of short circuiting.
∴ Attenuation is high.

5) If freq. is ↑ attenuation becomes higher.

$$A = \frac{2+0.3P}{2+P+0.6P^2} \text{ for } b < 5^\circ.$$

For all values of phase constant, b,

$$A = \frac{2+0.3P}{2+P+0.6P^2} - \sin b \left(\sqrt{\frac{P}{2}}\right)^{-(5/8)P}.$$

P = numerical distance.

Ground wave field strength by Maxwell's Equations:

The field strength at a distance, d is given by,

$$E = \frac{\eta_0 h_t I}{\lambda d} \text{ volts/m.}$$

h_t = effective height of transmitting antenna, m.

η_0 = Characteristics impedance of free space = $120\pi\Omega$.

I = antenna current, A.

d = distance from transmitter, m

λ = wave length, m.

When a receiving antenna of height, h_r is placed at a distance of d the received signal is given by,

$$V = \frac{\eta_0 h_t h_r I}{\lambda d} \text{ volts.}$$

1. What is the critical frequency for reflection at vertical incidence if the maximum value of electron density is $1.24 * 10^6\ cm^{-3}$.

$$f_c = 9\sqrt{N_{max}} = 9 * \sqrt{1.24 * 10^6}$$

f_c should be in MHz.

$$N_m = 1.24 * 10^6\ cm^{-3} = 1.24 \times 10^6 * (10^{-2})^{3}\ m^3$$

$$= 1.24 * 10^6 * 10^{-6} \Rightarrow 1.24$$

$$f_c = 9\sqrt{1.24} = 10.02\ MHz.$$

2) Explain MUF & give any method of calculating. A high frequency radio link has to be established b/n two points at a distance of 2500km on earth surface. Considering the Ionospheric height to be 200km & its f_c is 5MHz. Cal. the MUF for the given path.

$D_{skip} = 2500\,km$; $h = 200\,km$; $f_c = 5MHz$.

$$f_{muf} = f_c \sqrt{1 + \left(\frac{D}{2h}\right)^2} = 5 * 10^6 \sqrt{1 + \left(\frac{2500}{2 * 200}\right)^2} = 31.6$$

3) Two points on earth are 1500km a part, & are to communicate by means of HF. For a single hop transmission, the f_c 7MHz, & conditions are idealized. Cal. the MUF for these two points if the height of the Ionosphere layer is 300KM.

$$f_{muf} = 18.851\ MHz.$$

484,479,
460,75.

4) What do you understand by the team Critical frequency & Calculate the critical frequency for the F_1, F_2 & E layers for which the max. Ionic densities are $2.3*10^6$, $3.5*10^6$ & $1.7*10^6$ electron per cc respectively.

For F_1 ; $N_{max} = 2.3*10^6/cm^3 = 2.3*10^6*10^{-6} = 2.3$

$F_2 = 3.5.$

$E = 1.7.$

$F_1 \Rightarrow f_c = 9\sqrt{N_{max}} = 9\sqrt{2.3} = 13.65$ MHZ.

$F_2 \Rightarrow f_c = 9\sqrt{3.5} = \cancel{9\sqrt{3.5}} = 16.83$ MHZ.

$E \Rightarrow f_c = 9\sqrt{1.7} = 11.73$ MHZ.

5) The observed critical freq. of E & F layers at Gauhati particular times are 2.5 MHZ & 8.4 MHZ. respectively. Calculate max. electron Concentration of the layers.

For E-layer $\Rightarrow f_c = 9\sqrt{N_{max}}$

$$N_{max} = \left(\frac{2.5*10^6}{9}\right)^2 \Rightarrow 77.16*10^9 \Rightarrow 0.07716\ m^{-3}.$$

$$\text{F-layer} \Rightarrow N_{max} = \left(\frac{8.4*10^6}{9}\right)^2 \Rightarrow N_{max} = 0.8711*10^{12}\ m^{-3}$$

6) Assume that reflection takes place at a height of 400 Km & that the max. density in the Ionosphere Corresponds to a 0.9 refractive index at 10 MHZ. What will be range (assume flat earth) for which the MUF is 10 MHZ.

$h = 400$ km

$N_{max} = ?$

$\mu = 0.9$

$f = 10$ MHZ.

$$\mu = \sqrt{1 - \frac{81N}{f^2}} \Rightarrow N_{max} = \frac{[1-(0.9)^2]*(10*10^6)^2}{81}$$

$$= 23.456*10^6\ m^{-3}$$

(c)

$$D_{skip} = 2h\sqrt{\frac{f_{muf}^2}{f_c^2} - 1}$$

$$f_c = 9\sqrt{N_{max}} = 9\sqrt{23.45 * 10^{10}} = 4.3588 * 10^6\ Hz.$$

$$D_{skip} = 2 * 400 * 10^3 \sqrt{\left(\frac{[10*10^6]}{(4.3588*10^6)}\right) - 1}$$

$$D_{skip} = 1651.76\ Km.$$

7) At what frequency a wave must propagate for the D-region to have an index refraction 0.5? $N = 400$ electron/m³ for D-region

$\mu = 0.5$

$N_{max} = 400$ electrons/cm³

$$\mu = \sqrt{1 - \frac{81N}{f^2}} \Rightarrow \mu^2 = 1 - \frac{81N}{f^2}$$

$$f = \sqrt{\frac{81N}{1-\mu^2}} = 207.82\ \cancel{K}Hz$$

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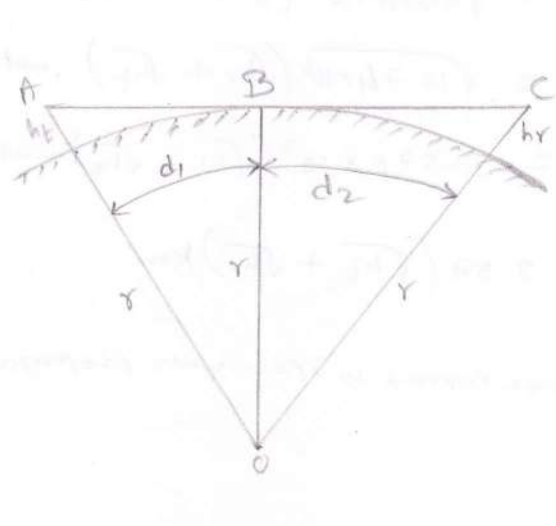
Space wave Propagation:-

As already mentioned, the space wave or line of sight propa is chiefly useful at higher frequencies i.e., VHF, UHF and micro because the sky wave and surface wave propagations both fail such frequencies. The space wave propagation is practically limit to line of sight distance and is also limited by the curvature of earth.

Line of sight distance is that distance between the transmit and receiver, in which if a direct ray passes from the transmitter the receiver without being intercepted by the bulge in the earth's surface, considering variation of refractive index(μ) of earth's atmosphere with height, the transmitting antenna must 'see' atle the top of the receiving antenna.

Line of sight propagation is limited to about few tens of kilo and the propagation occurs in the troposphere - a region 16 km above earth's surface. Now the various aspects of space wave propagation will be described i.e., line of sight communication ra effective earth's radius, field strength etc.,



Optical range of line of Sight (LOS) propagati

Range of space wave propagation or Line of sight Distance (D

In general, space wave Communication is possible only up or slightly beyond the line of sight distance and this distance is determined mainly by the heights of transmitting and receiving antennas as derived below.

Let d be the distance between transmitter and the receiving and heights of the transmitting and receiving antennas are h_t and h_r respectively above ground.

From 11.33, the LOS distance,

$$d = d_1 + d_2 \quad - (a)$$

If r be the radius of earth (equal to 6370 km) then from ΔABO, ΔCB

$$d_1 = \sqrt{(h_t+r)^2 - r^2}$$

$$= \sqrt{h_t^2 + r^2 + 2rh_t - r^2} \approx \sqrt{2rh_t} \text{ metres} \quad / \because h_t^2 << 2rh_t$$

$$\text{III}^{ly} \quad d_2 = \sqrt{(h_r+r)^2 - r^2}$$

$$= \sqrt{h_r^2 + r^2 + 2rh_r - r^2} \approx \sqrt{2rh_r} \text{ metres} \quad / \because h_r^2 \le 2rh$$

Sub d_1 & d_2 in (a),

$$d = \sqrt{2rh_t} + \sqrt{2rh_r}$$

$$= \sqrt{2r}\left(\sqrt{h_r} + \sqrt{h_t}\right)$$

$$= \sqrt{2 \times 6370 \times 10^3}\left(\sqrt{h_r} + \sqrt{h_t}\right) \text{ metres}$$

$$= \sqrt{12.74 \times 10^6}\left(\sqrt{h_r} + \sqrt{h_t}\right) \text{ metres}$$

$$= 3.570 \times 10^3\left(\sqrt{h_t} + \sqrt{h_r}\right) \text{ metres}$$

$$d = 3.57\left(\sqrt{h_t} + \sqrt{h_r}\right) \text{ Km}$$

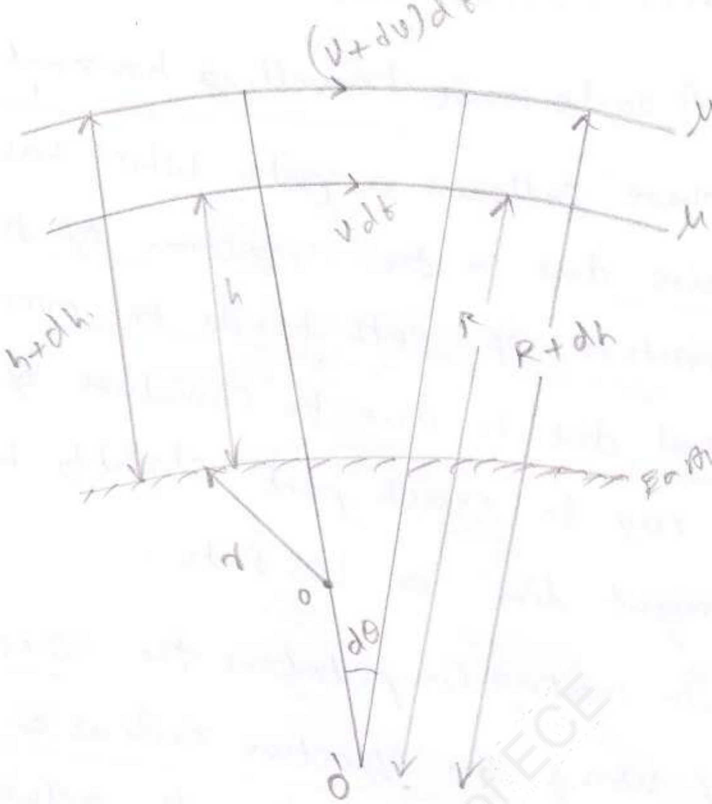
LOS distance covered by Space wave propagation.

Effective Earth's Radius :-

A radio wave travelling horizontally in the earth's atmosphere follows a path which has a slight downward curvature due to the refraction of the wave in the atmosp[here]. This curvature of path tends to overcome partially the los[s] of signal due to due to curvature of earth and permits [the] direct ray to reach point slightly beyond the horizon as f[or] the straight line or LOS path.

In making computations the effect of refraction is accou[nted] for by using an effective radius of curvature of earth which is a bit greater than the actual radius, and then assuming straight line path (i.e., without refraction) in [the] atmosphere.

As the dielectric constant i.e., refractive index of the atmosphere changes with the height above ground and hence [the] refraction of radio wave takes place. The dielectric constant of the atmosphere near the surface of earth is greater th[an] unity but decreases to unity at greater height where air density approaches zero. This decrease in refractive index with height causes refraction of the radio wave w[hich] results in the bending of radio wave towards the region o[f] higher dielectric constant or refractive index i.e., towards t[he] earth.



Let us now derive a relation between the radius o
Curvature of the ray path in the troposphere and change of
refractive index with height by assuming the curvature
the earth.

Considering a radio wave which is travelling nearly hori
in the troposphere and its path is bent into an arc by the
variation of the refractive index with height as shown in

V = velocity of propagation.
h = Height above the earth (of the travelling ray).
R = Radius of Curvature of ray path.
r = Actual radius of earth.

$$\text{Angle} = \frac{\text{Arc}}{\text{radius}}.$$

$$d\theta = \frac{Vdt}{R}.$$

$$R\,d\theta = V\,dt \quad \text{—} (1)$$

$$(R + dh)\,d\theta = (V + dv)\,dt \quad \text{—} (2)$$

② − ① ⇒

$$(R+dh-R)\,d\theta = (v+dv-v)\,dt$$

$$dh\,d\theta = dv\,dt.$$

$$\frac{d\theta}{dt} = \frac{dv}{dh}.$$

$$v = \frac{c}{\sqrt{K_r}} = \frac{c}{\mu}. \quad -(a)$$

K_r = dielectric Constant (relative);

c = velocity of light.

μ = Refractive index at height h.

Hence the Change in refractive index with height is obta[ined] by differentiating eq. (a), w.r.t h.

$$\frac{dv}{dh} = -\frac{c}{\mu^2}\frac{d\mu}{dh}$$

$$\frac{dv}{dh} = -\left(\frac{c}{\mu}\right)\frac{1}{\mu}\frac{d\mu}{dh}$$

$$\frac{dv}{dh} = -\frac{v}{\mu}\frac{d\mu}{dh}.$$

$$\because \mu \simeq 1$$

$$\frac{dv}{dh} \simeq -v\frac{d\mu}{dh}.$$

from above eq's, $R = \frac{v\,dt}{d\theta} = \frac{v}{\left(\frac{d\theta}{dt}\right)}$

$$= \frac{v}{\left(\frac{dv}{dh}\right)} = \frac{\cancel{v}}{-\cancel{v}\left(\frac{d\mu}{dh}\right)}$$

$$\boxed{\therefore R = -\frac{dh}{d\mu}.}$$

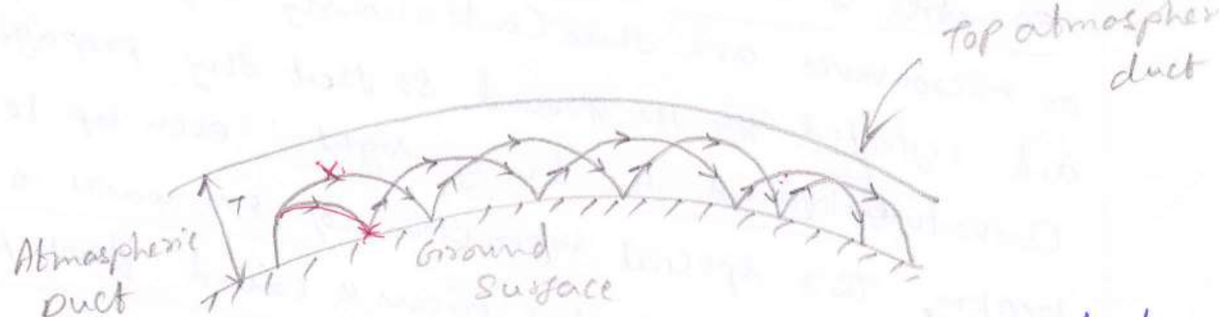
This shows Radius of curvature of the wave path is a function of [the] rate of change of dielectric constant or refractive index with height changes, hour to hour, day to day & season to season.

Duct propagation:-

At VHF, UHF, Microwaves, the waves are neither reflected by Ionosphere nor propagated along earth's surface, but the transmission does occur much beyond the line of sight distance due to the refraction of such high frequency wave (specially microwaves) in the troposphere. As already mentioned troposphere is the region 16 km above the earth's surface, the troposphere temperature falls at the rate of 6.5° per km till reaches at about 50°c at the upper boundary.

Region next higher to troposphere is the stratopause where the temperature almost remains constant to -50°c. Inside the troposphere the atmosphere has a dielectric constant slightly greater than unity at the earth's surface where the density is most dense and this decreases to unity at great heights where the air density approaches zero.

The dielectric constant of dry air is slightly greater than unity and the presence of water vapour increases the dielectric constant still further & hence the dielectric constant depends on air conditions i.e., on the weather.



Super-refraction in the atmospheric duct.

A normal or std. atmosphere is one where the dielectric Constant is assumed to decrease uniformly with height to a value of unity at a height where air density is essentially zero.

However, in actually the condition, of so called std. atm hardly exists. The air is frequently turbulent & at other times there are often layers of air one above the other having different temperatures and water vapour contents. These conditions besides giving phenomena of scattering, refraction and reflection, give a new phenamena called super refraction or duct propagation.

In this two boundary surfaces between layers of air form a duct or a sort of leaky wave guide which guides the EM wave b/n its wall.

When the freq. is sufficiently high, the region where variation of dielectric Constant or refractive index is usually high (or refractive index decreases rapidly with height), actually traps the energy and causes it to travel along the earth surface as it happens in a wave guide.

This happens near the ~~earth~~ ground often with in the 50 metres of the troposphere as shown in fig. The higher freq or microwaves are thus continuously refracted in the duct and reflected by the ground so that they propagate around Curvature for beyond the line of sight, even up to a distance 1000 km. This special refraction of EM waves is called Super refraction and this process is called duct propagation.

turbulence = unusual flow (instab
duct = An enclosed Conduit for a
(Passage)
trap = catch (or by to move force

The main formation of duct is a temperature inversion. In the inversion layer the temperature increases with height rather usual decrease of temperature at the rate of 6.5° in the standard atmosphere.

When the refracting conditions are sufficiently different from the standard to cause trapping of the wave, the concept of an effective earth radius does not hold good. In order to give the necessary curvature the actual refractive index μ at a height h must be replaced by modified refractive index,

N ⇒ variations in the refractive index.

$$N = \mu + \frac{h}{r}.$$

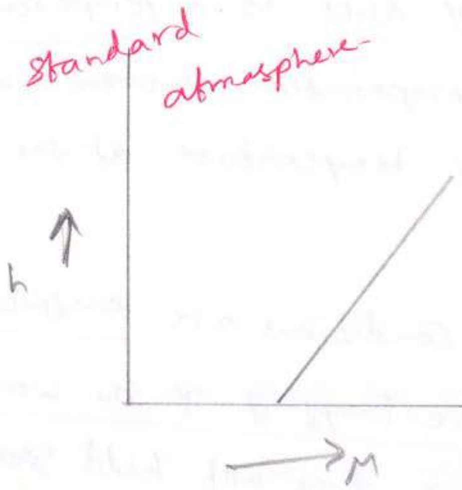
Although N is always close to unity, yet its actual value is important & hence when dealing with numerical values, it is convenient to introduce the excess modified index of refractive modulus M, related to N as,

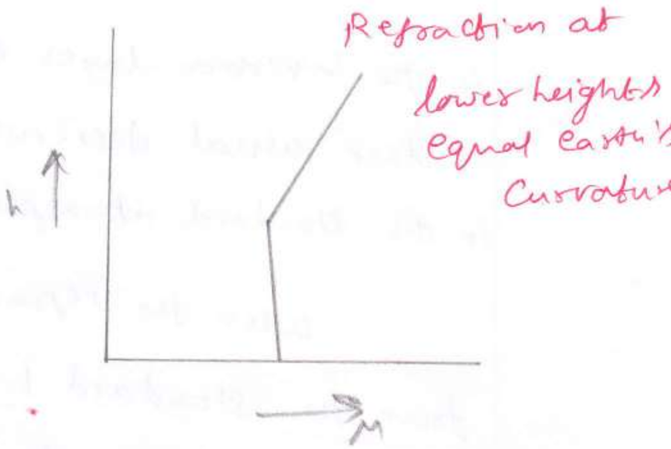
$$N-1 = \left[\mu - 1 + \frac{h}{r}\right]$$

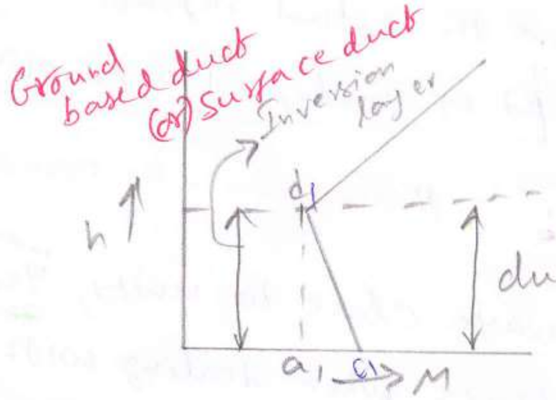
$$\boxed{M = (N-1)\times 10^6 = \left[\mu - 1 + \frac{h}{r}\right]\times 10^6}$$

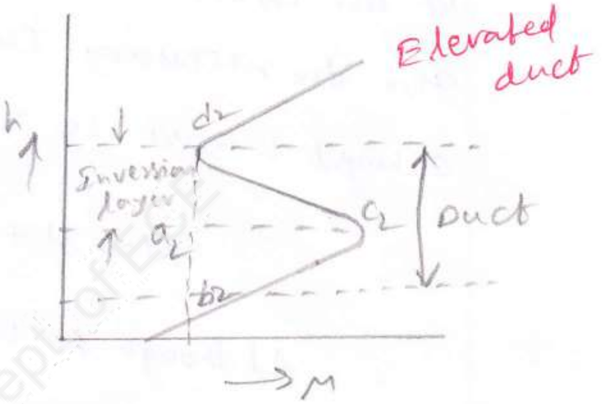
μ = Refractive index
h = height above ground
r = True radius of earth = 6370 km.

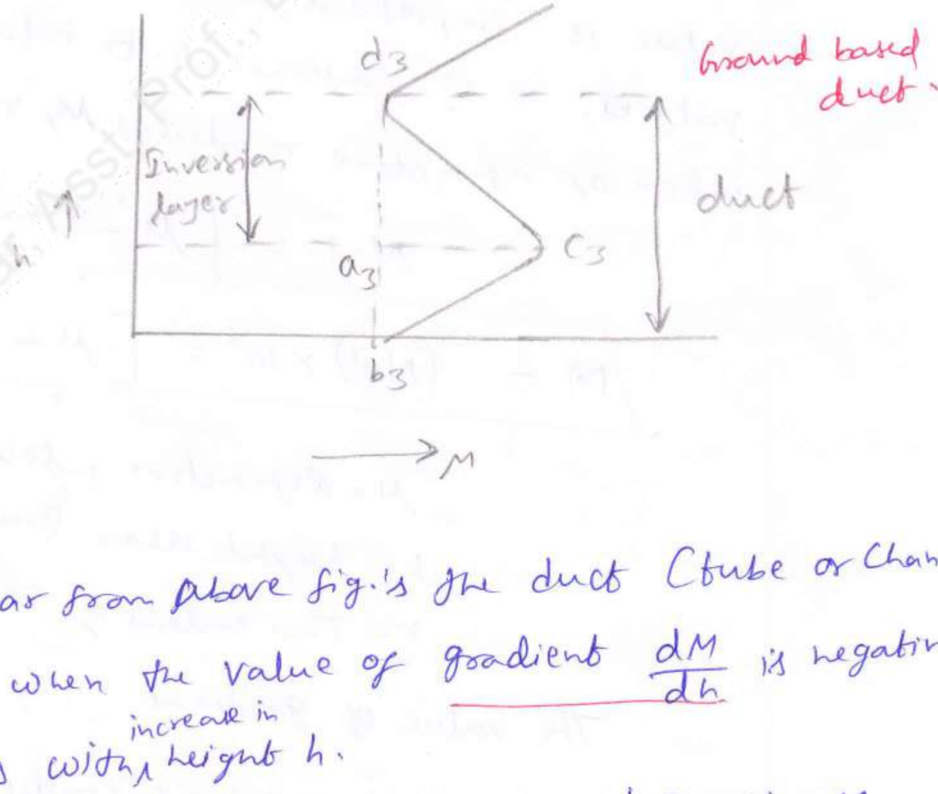
The value of gradient i.e., $\frac{dM}{dh}$ and its sign both depends on the tropospheric conditions. From the measurements, M is plotted against height h, the following different curves are obtained.











As in clear from above fig.'s the duct (tube or Chan

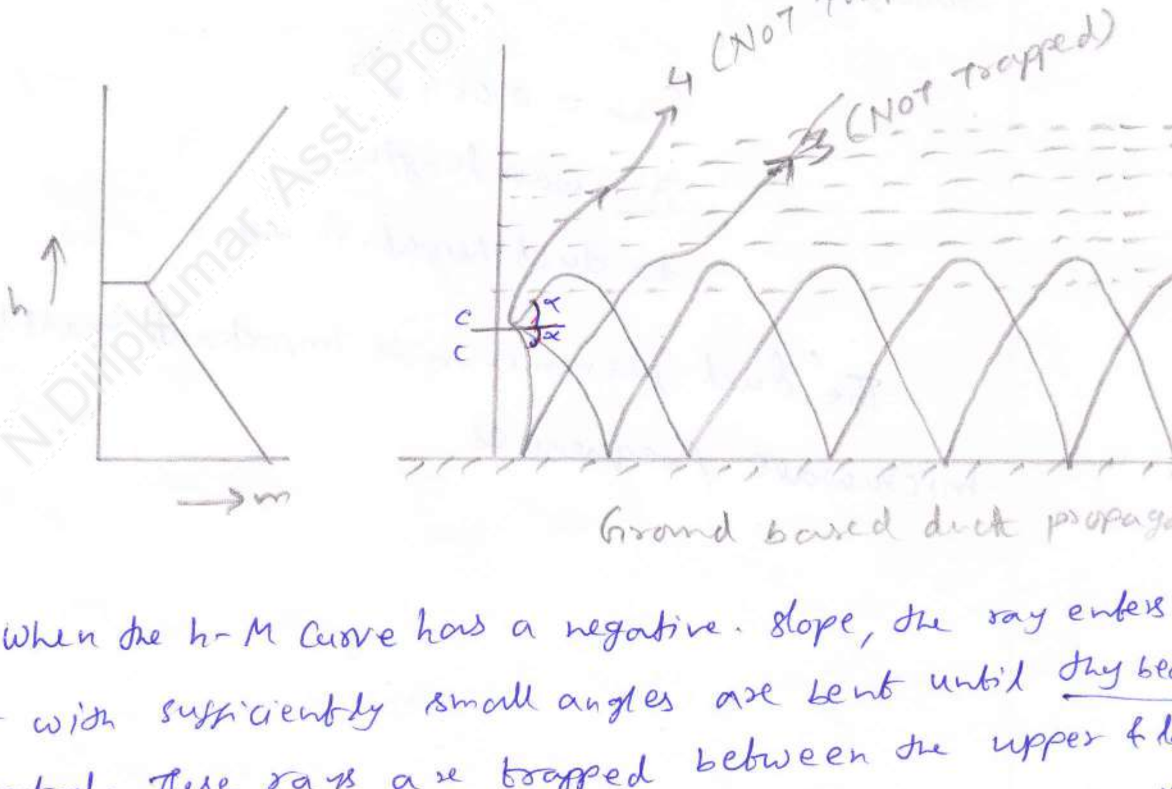
is formed only when the value of gradient $\frac{dM}{dh}$ is negativ

i.e., M decreases with increase in height h.

The height for which this process of decreasing M

Continues forms the inversion layer.

In fig. d_1c_1, d_2c_2, & d_3c_3 are the inversion layers whose heights are respectively d_1a_1, d_2a_2, d_3a_3. The ~~which~~ widths of the duct, however, are determined by t dropping the projections vertically downwards which cut a a_1, b_2 and b_3. The horizontals drawn at these points a b_2 and b_3 give lower end of the duct and horizontal at d_1, d_2, d_3 give the top end of the duct. Thus the duct widths are respectively d_1a_1, d_2b_2, d_3b_3. If inversion layer is just above ground, it gives ground ba duct [Fig. (c, e) and if it is above ground, it gives d to elevated duct as in fig (d)].



Ground based duct propaga

When the h-M curve has a negative slope, the ray enters duct with sufficiently small angles are bent until they be horizontal. These rays are trapped between the upper & l walls of the duct and are oscillating b/n ground & upper wall the duct in the ground based duct & b/n two walls in the atmosphere in case of an elevated duct. This phenomena is called super refraction or duct propagation.

The duct propagation condition depends whether the Tx'ter i inside the duct or above the duct or below the duct. The most favourable condition for duct formation is when the transmitter is inside the duct, through in this condition to not all the waves are trapped.

Ground based duct is formed mostly over water. Infact, it is believed that ground based duct are nea always exist over the ocean specially the trade-wind bel Although ground based duct also occur on land but less frequently & temporarily. The elevated ducts are typically found in coastal areas at an elevation of 1000 5000 feet having width of about ~~ø~~ 1000 feet.

$$\lambda_{max} = 0.084\, d^{3/2}$$

λ_{max} = wavelength.

d = duct height. in mts.

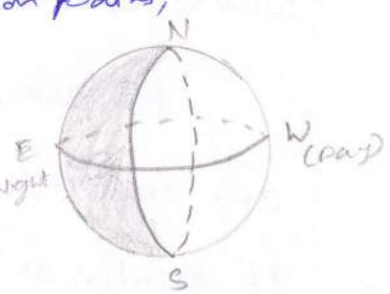
The duct phenomenon is important mostly at UHF & microwave frequencies.

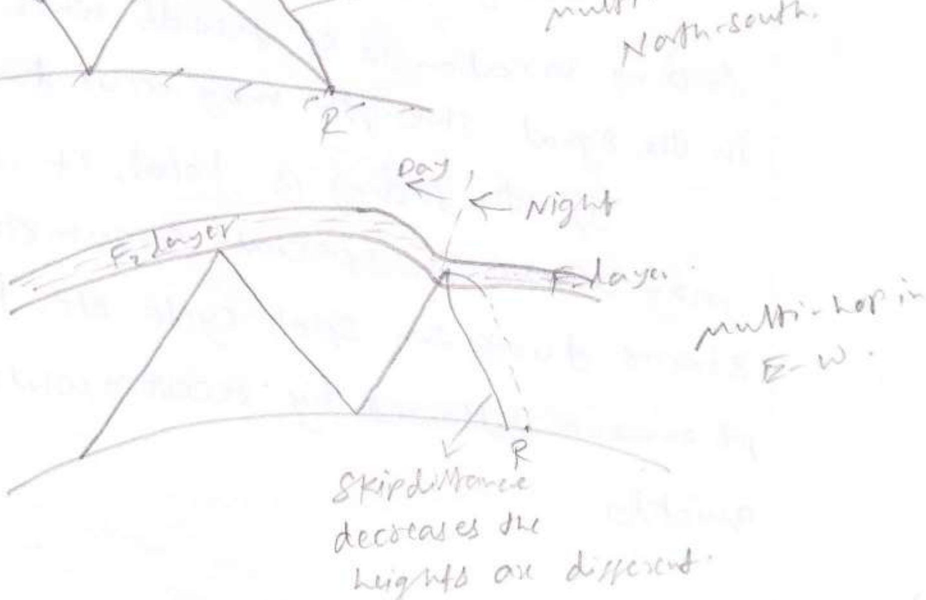
Multi-hop propagation:-

The transmission path is limited by the skip distance and curvature of the earth. The longest single hop propagation is obtained when the transmitted ray is tangential at the earth surface. The maximum practical distance covered by a sky wave in single hop is 2000km for E layer & 4000 for F_2 layer. Since the semi-circumference of the earth is just ~~ab~~ over 20,000km, multi-hop propagation paths are occuring.

Further there is no problem in south-north propagation path as there will be day in the one half portion and night in the other half portion of the globe.

It had been found that long distance short wave comm. generally involves two or four transmission paths, & each contribute appreciable energy to the receiver.





Fadings:-

Fading is the fluctuation in the received signal ~~at~~ stre
at the receiver or a random variation in the received signa
is known as fading.

Fading of radio waves is the name given to undesirab
variations in the intensity or loudness of the waves received at
receiver. It is caused by variation in the heights and
density of ionization in the different layers of the ionosphe
Fading is the common characteristic of the sky wave signals.
Fading may be slow, rapid, frequency selective or general bu
in each case fading caused due to interference between two
waves of different path lengths.

Because, the signal received at any instant and at
any receiving point is the vector sum of all the waves receiv
It results as the signal waves leave the transmitter at same
time but arrive at the receiver following different paths.

Fading is more likely with higher frequencies. In nor
fading variation is of few db while when fading is severe, cha
in the signal strength may occur from 10 db to 20db.

If the fading is total, it is called fade-out which
may occur in special circumstances like S.I.D. Ionosph
storms during sun spot cycle etc. Fading is a gradual
phenomenon followed by recovery while fadeout usually occurs
quickly.

Selective Fading :-

Produces serious distortion of modulated signal. Sinc the fading is frequency selective, it is just possible for adjacent portion of a signal to fade independently, although th frequency separation is only a few dozen Hertz.

very common

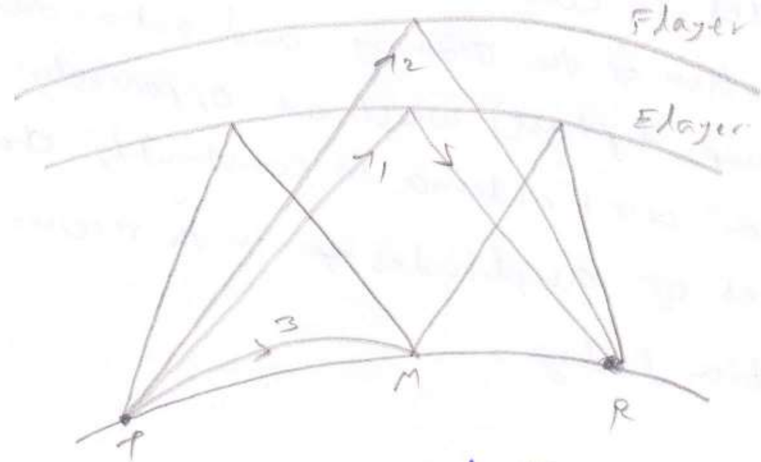
Selective fading is more prevalent at high frequencies for ca sky wave propagation is used. It can play Havoc with ampl modulated signals and at high percentage of modulation. AM signals are more distorted by selective fading rather than SSB s

violent disturbance

Hence selective fading can be reduced by using Exalted Carrie reception and also single side band (SSB) system.

Interference fading :-

Interference fading is the most serious type of fad and it is produced by the interference between upper and lo rays of a sky wave, between sky waves reaching the receiver by different number of hops or different paths and even b/w a ground wave and a sky wave, particularly at the lower end of the HF band.



Interference fading due to lower and upper rays of sky waves (No. 1 & and due to multi-hop propagation

ray 1 = reflected from E-layer.
ray 2 = reflected from F-layer.
ray 3 = ground wave, interfering at point M, producing louder signal depending whether in phase or out of phase

Interference fading also occurs because of the fluctuatio of height (or ionic density) in the ionospheric layers if a singl sky wave frequency is in use.

Since the ionosphere is subject to continual small fluctua and so also the returned sky wave. As the path length of eac wave is subject to continual small variations, the relative phase of the waves reaching at the receiver vary in a randa way. Hence the amplitude of their resultant varies continue and this effect is called as interference fading.

It can be minimized either by space or frequency diversity reception.

Absorption fading:-

This type of fading occurs due to ~~change of polariza~~ variations of signal strength with the different amount of absorption of waves absorbed by the transmission medium.

Polarization fading:-

This type of fading occurs ~~at distances near the ski~~ ~~distance. A~~ due to the change of polarization of the dow coming sky waves. The state of polarization of a down com sky waves is constantly changing. This is caused by a super position of the ordinary and extra-ordinary waves (with random amp. & phase) which are oppositely polarized. The polarization w.r.t antenna is constantly changing, giving rise to changes of amplitudes ~~of~~ in the receiver & producing polarization fading.

Skip Fading :-

This type of fading occurs at distances near the skip distance. Any variation in the height or density of an ionized layer may move the receiving point in-out of the skip zone.

The most common method to minimize fading is to use automatic volume control (AVC ~~or~~ AGC) in the receiver. How AVC is not the complete solution to the problems of fading because the signals usually drop below the noise level and no amount of amplification will make the signal usable. Further AVC can not help selective fading as components of the same signal fed out of different times.

The best way to minimize fading is to employ a diversity reception system.